Demo 8

Demo 8 is another speed test, which can be explicitly analysed using this execution protocol. While running the speed test the LaTeX output is deactivated and the root finding process iterated a large number of times to better measure the average execution time. The measurement results are saved as raw data in the file demo8speed.txt.
In this demo or speed test the following polynomials with two near roots are used (Example 3 from Liu et al.):

$$
\begin{aligned}
h_2 &:= (t - 0.56)(t - 0.57) \\
h_4 &:= (t - 0.4)(t - 0.40000001)(t + 1)(2 - t) \\
h_8 &:= (t - 0.50000002)(t - 0.50000003)(t + 5)^3(t + 7)^3 \\
h_{16} &:= (t - 0.30000008)(t - 0.30000009)(6 - t)^7(t + 5)^6(t + 7)
\end{aligned}
$$

Contents

Part I

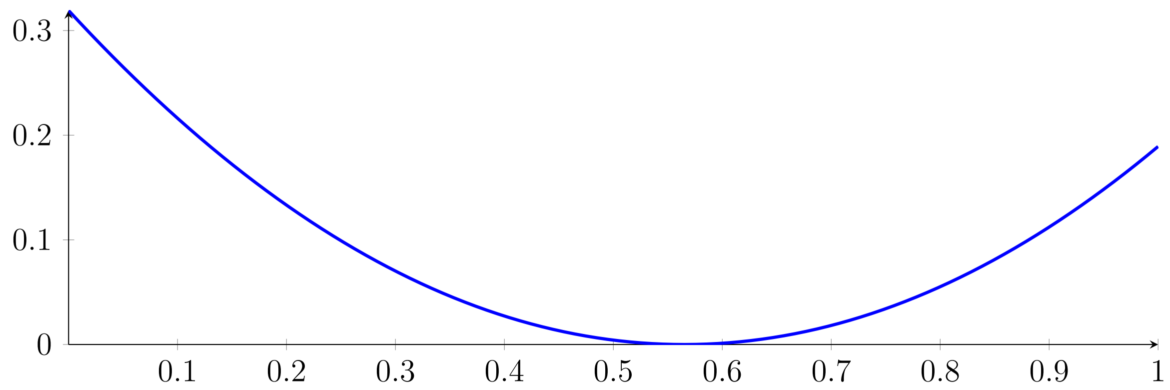
Numeric = MpfrFloat with precision 1024

1 Running `BezClip` on h_2 with epsilon 2

$$1X^2 - 1.13X + 0.3192$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

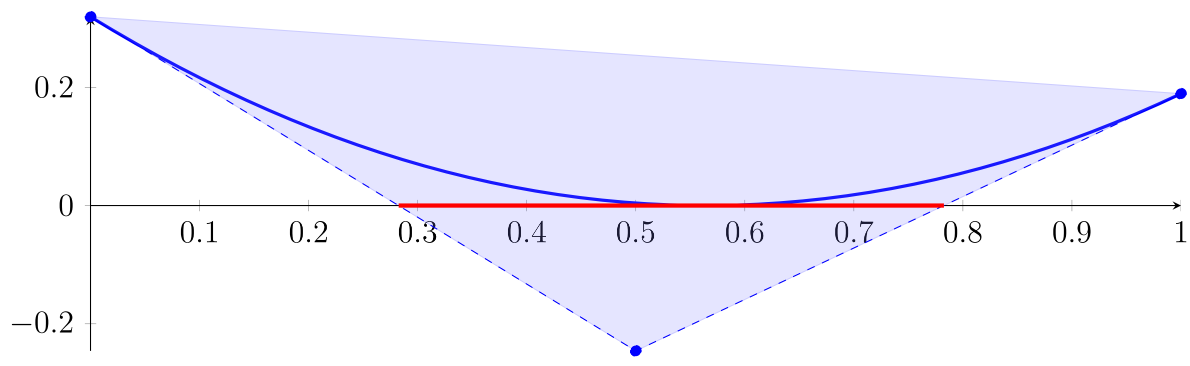
$$p = 1X^2 - 1.13X + 0.3192$$



1.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

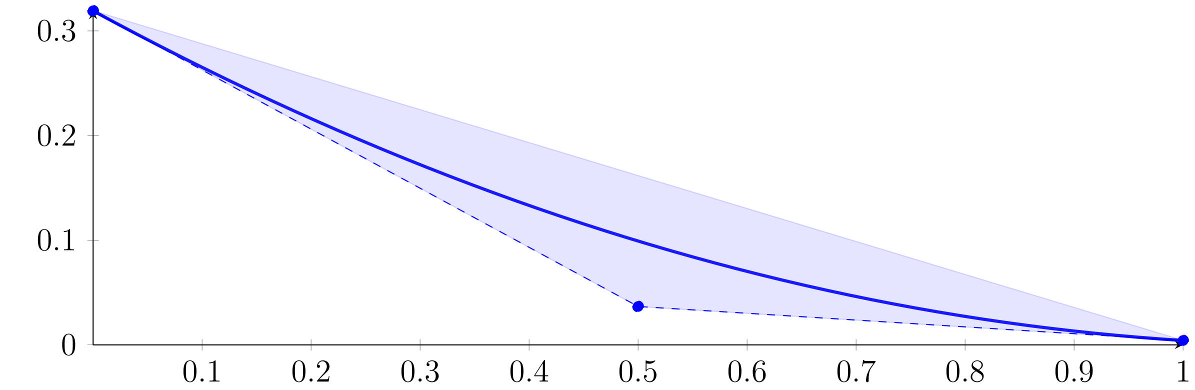
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

1.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

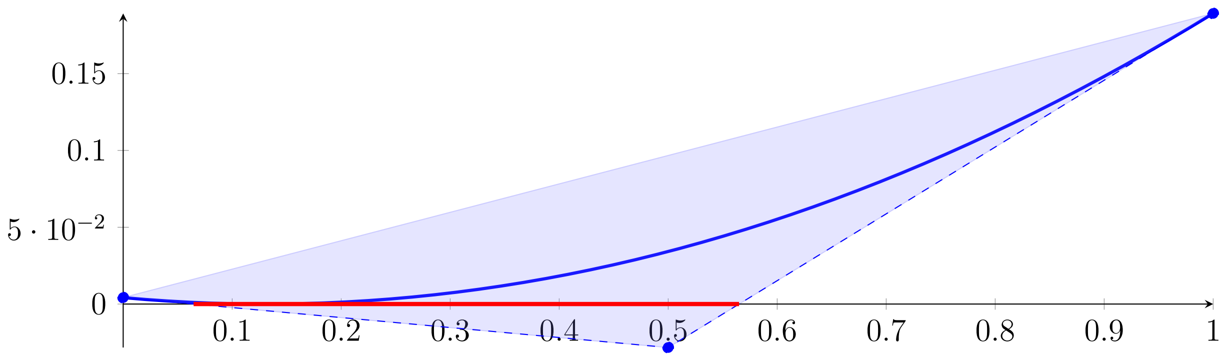
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.065X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

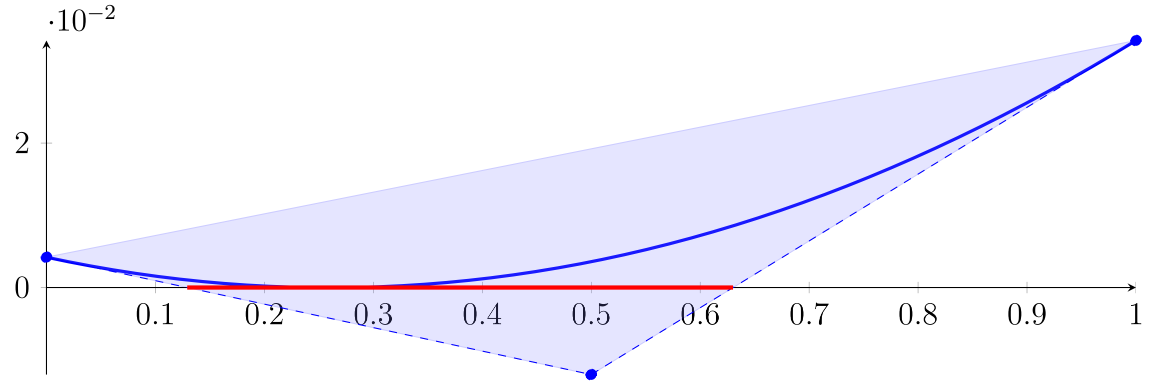
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\Longrightarrow$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

1.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 - 0.0325X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

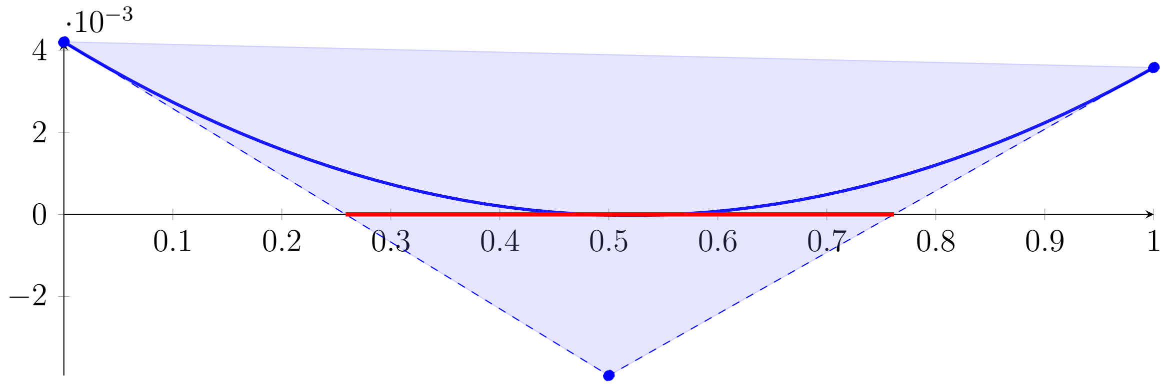
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

1.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

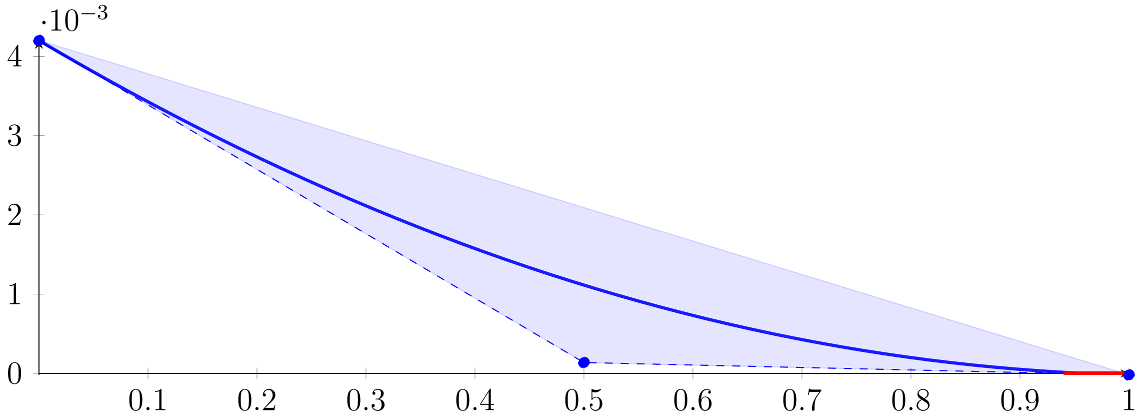
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

1.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) + 0.0001375000000000007B_{1,2}(X) \\ &\quad - 1.874999999999948 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

$$[0.9400000000000017, 0.9955555555555557]$$

Longest intersection interval: 0.05555555555555396
$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

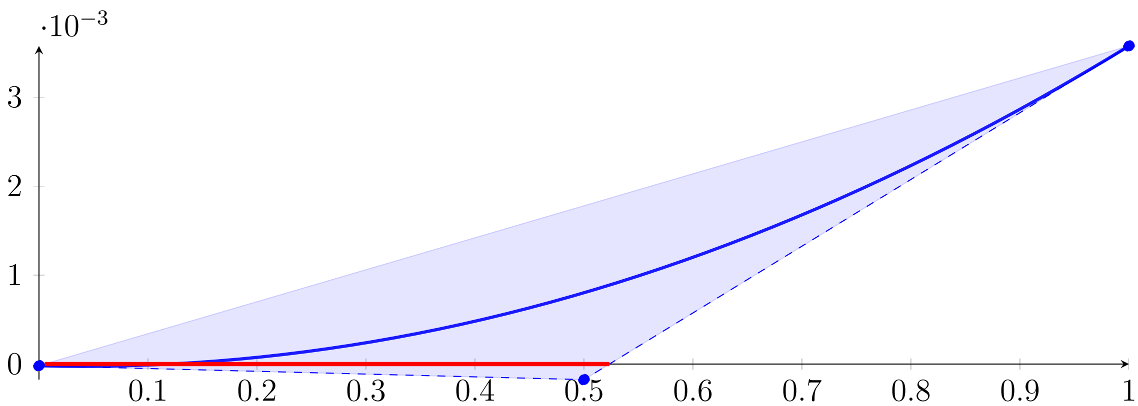
1.7 Recursion Branch 1 2 1 1 1 1 in Interval 1: $[0.5587500000000001, 0.562222222222$

Found root in interval $[0.5587500000000001, 0.5622222222222222]$ at recursion depth 6!

1.8 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625X^2 - 0.0003125000000000003X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05}B_{0,2}(X) - 0.0001749999999999996B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

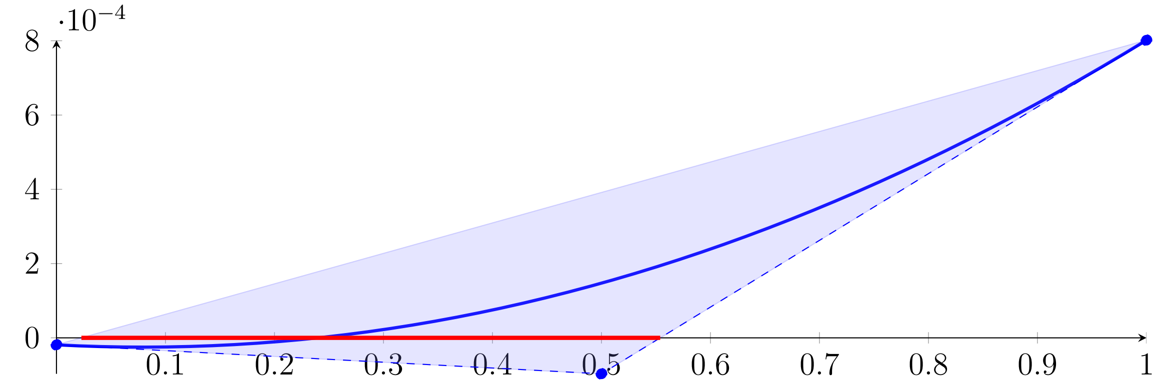
$$[0.005217391304347681, 0.5233333333333333]$$

Longest intersection interval: 0.5181159420289856
$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

1.9 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 - 0.00015625000000000001X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 9.687499999999955 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0008015625000000004 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142222, 0.5539130434782606\}$$

Intersection intervals with the x axis:

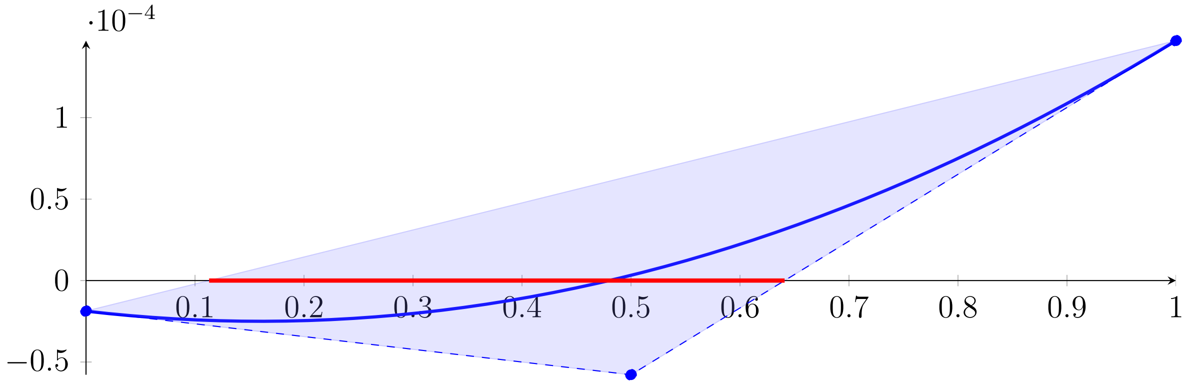
$$[0.022857142857142222, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\Longrightarrow$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

1.10 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 - 7.812500000000007 \cdot 10^{-05} X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 5.781249999999951 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\implies$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

1.11 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Found root in interval $[0.5625, 0.5703125]$ at recursion depth 8!

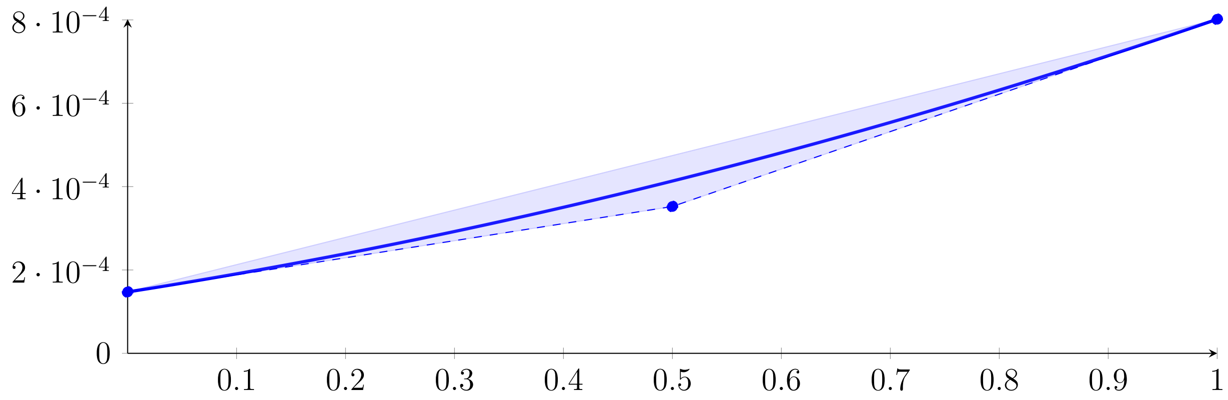
1.12 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Found root in interval $[0.5703125, 0.578125]$ at recursion depth 8!

1.13 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.000244140625X^2 + 0.0004101562499999999X + 0.0001472656250000005\\ &= 0.0001472656250000005B_{0,2}(X) + 0.0003523437500000004B_{1,2}(X)\\ &\quad + 0.0008015625000000004B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

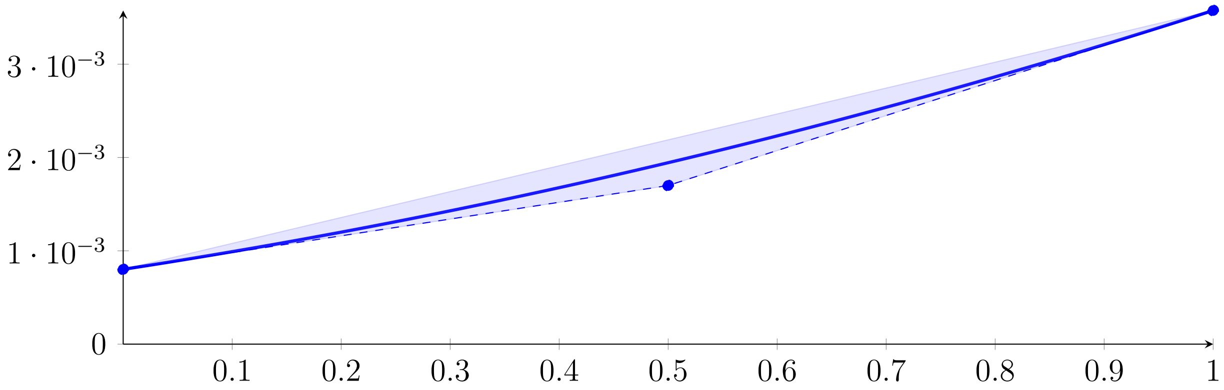
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.14 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.0009765625X^2 + 0.001796875X + 0.0008015625000000004\\ &= 0.0008015625000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

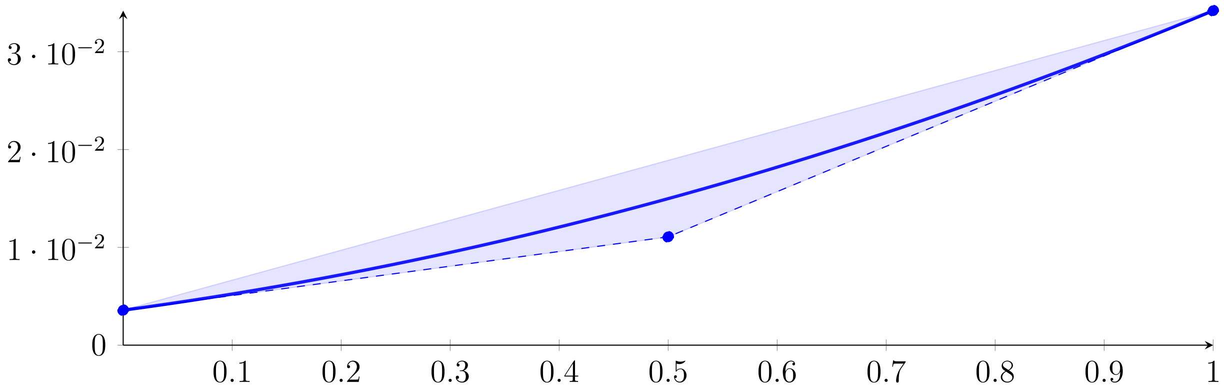
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.15 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

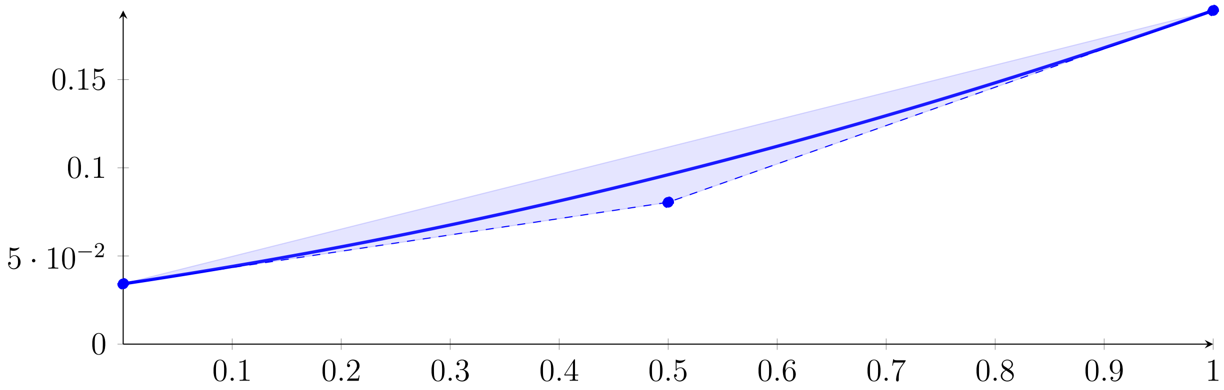
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.16 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

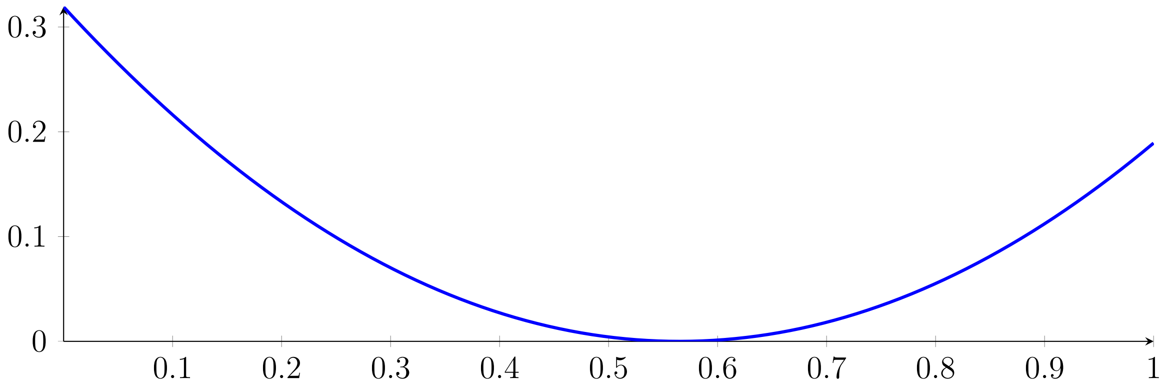
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

1.17 Result: 3 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5587500000000001, 0.5622222222222222],\ [0.5625, 0.5703125],\ [0.5703125, 0.578125]$$

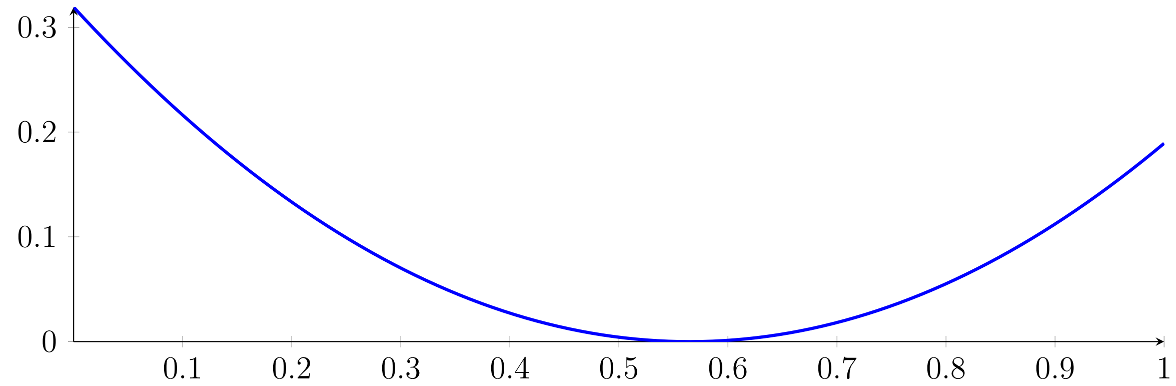
with precision $\varepsilon = 0.01$.

2 Running QuadClip on h_2 with epsilon 2

$$1X^2 - 1.13X + 0.3192$$

Called QuadClip with input polynomial on interval $[0,1]$:

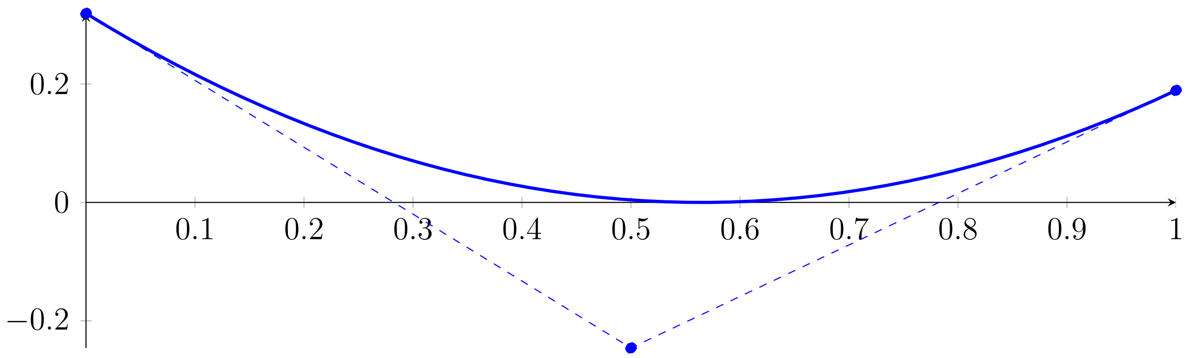
$$p = 1X^2 - 1.13X + 0.3192$$



2.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

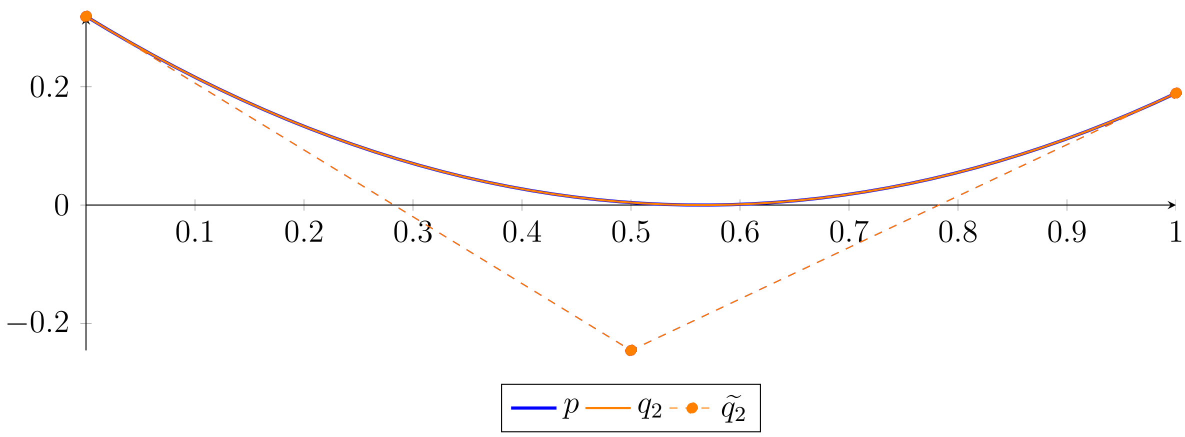
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

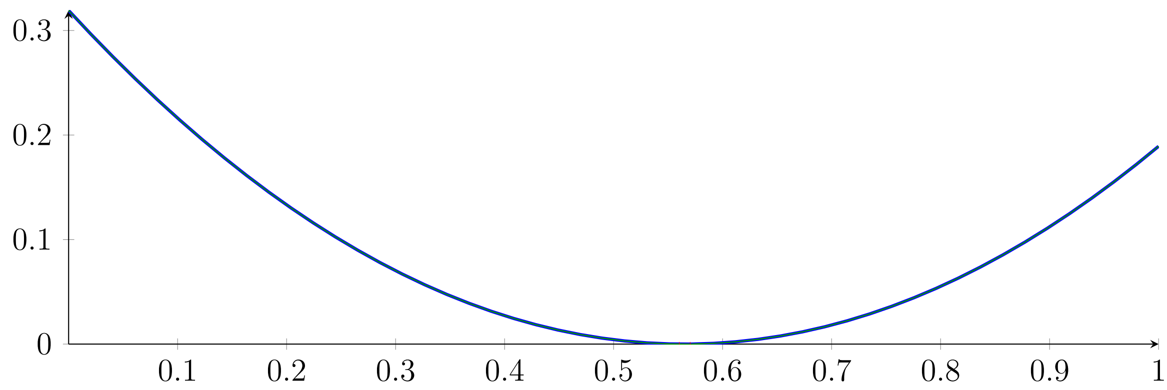
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001], \ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\implies$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

2.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

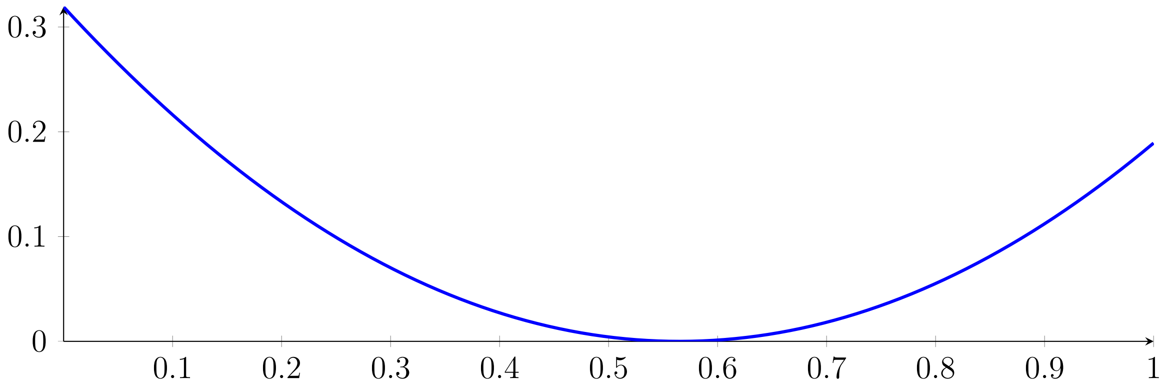
2.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

2.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001], [0.57, 0.57]$$

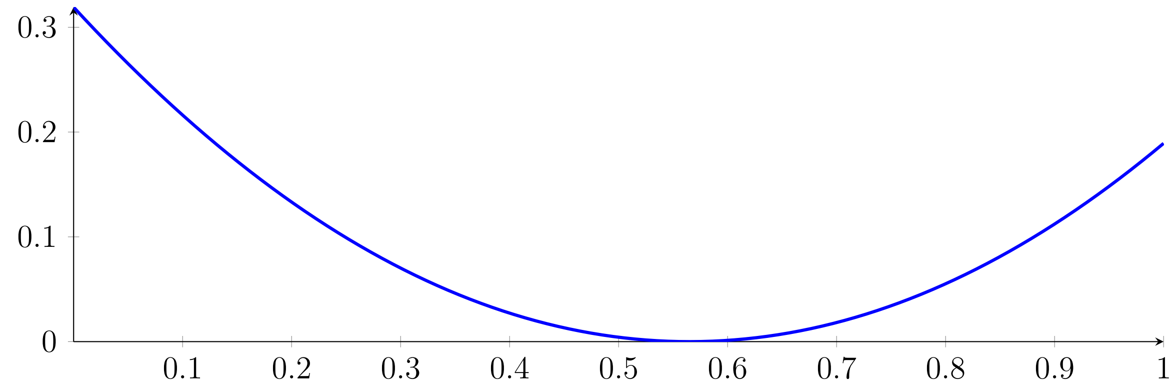
with precision $\varepsilon = 0.01$.

3 Running `CubeClip` on h_2 with epsilon 2

$$1X^2 - 1.13X + 0.3192$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

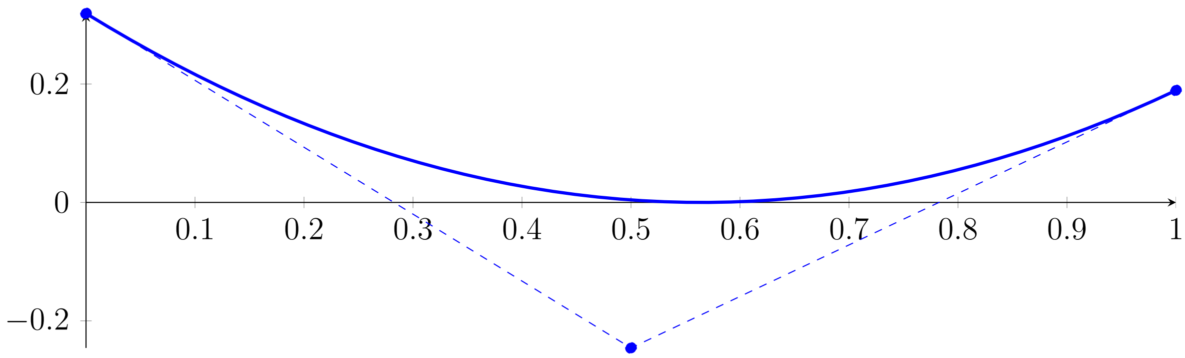
$$p = 1X^2 - 1.13X + 0.3192$$



3.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

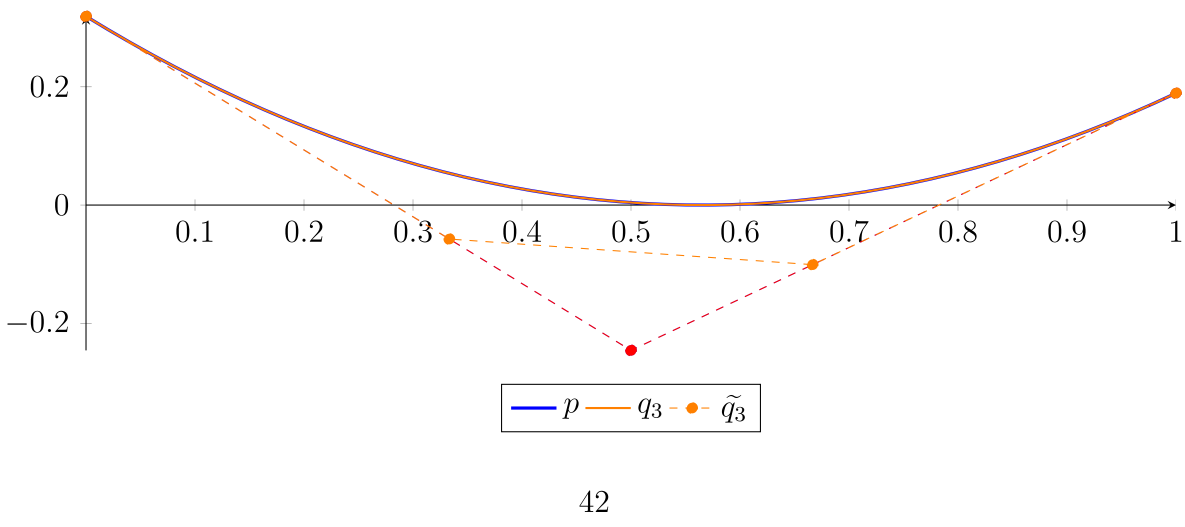
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308}X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501{\cdot}10^{-308}$.
Bounding polynomials M and m:

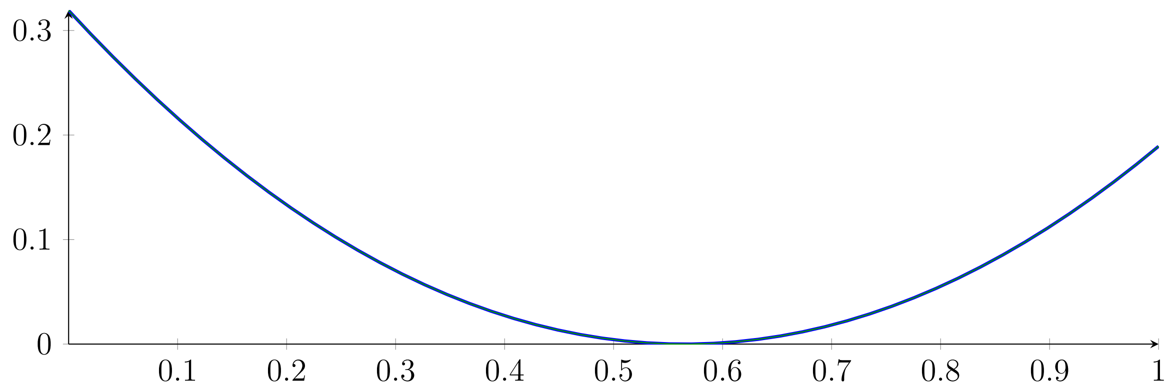
$$M = -1.668805393880401{\cdot}10^{-308}X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801{\cdot}10^{-308}X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783{\cdot}10^{291}, 2.824576038015892{\cdot}10^{291}, 5.992310449541053{\cdot}10^{307}\}$ $\quad N(m) = \{-4.84$

Intersection intervals:

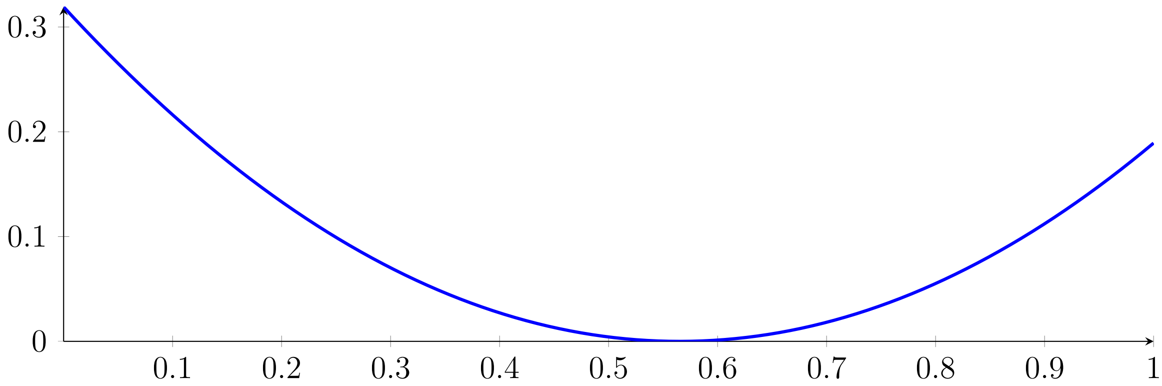


No intersection intervals with the x axis.

3.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

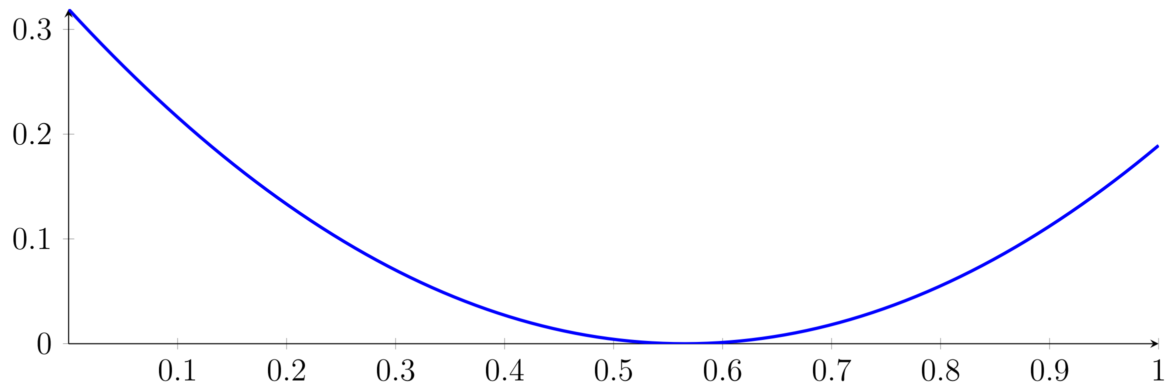
with precision $\varepsilon = 0.01$.

4 Running BezClip on h_2 with epsilon 4

$$1X^2 - 1.13X + 0.3192$$

Called BezClip with input polynomial on interval $[0,1]$:

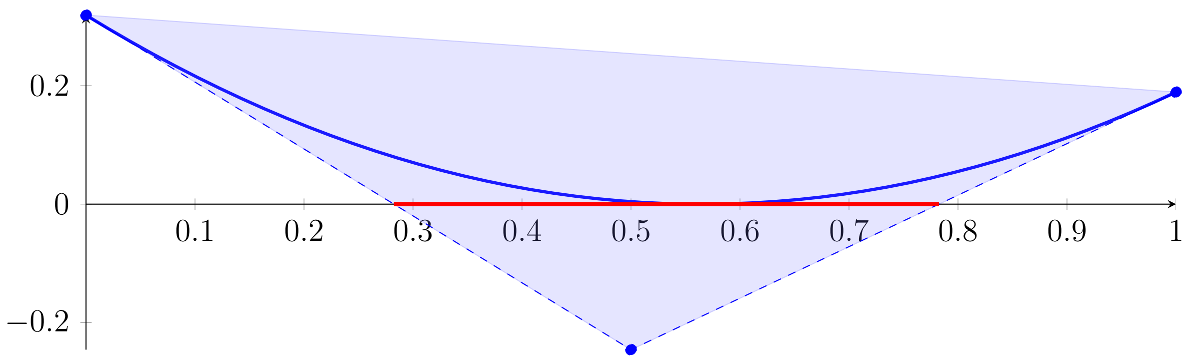
$$p = 1X^2 - 1.13X + 0.3192$$



4.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

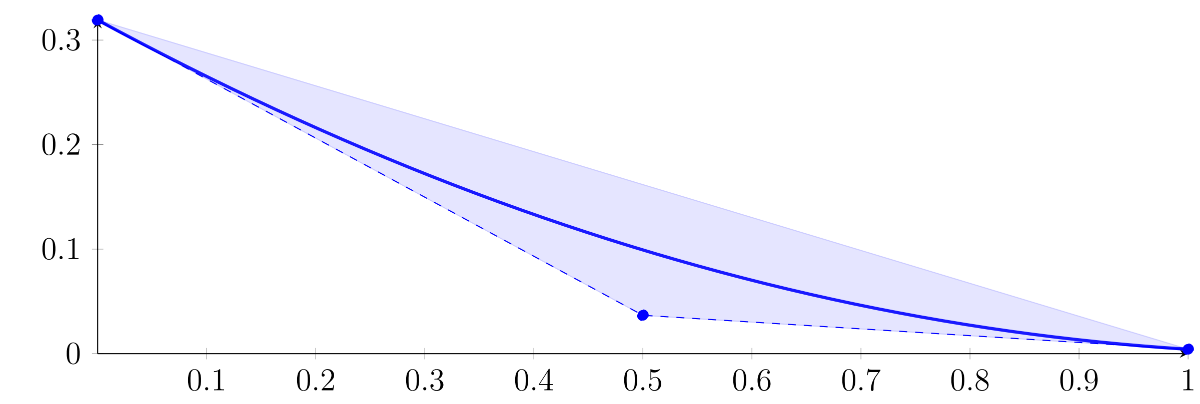
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

4.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

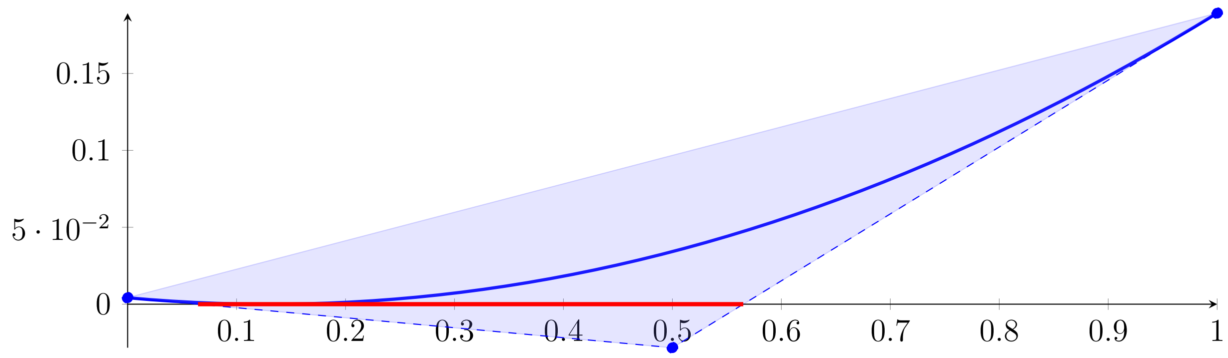
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

4.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.25X^2 - 0.065X + 0.004200000000000001\\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

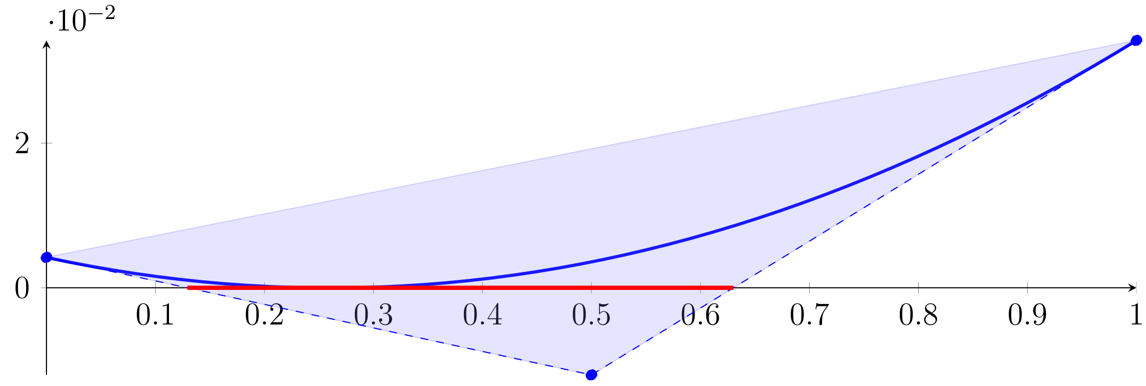
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\Longrightarrow$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

4.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.0625X^2 - 0.0325X + 0.004200000000000001\\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

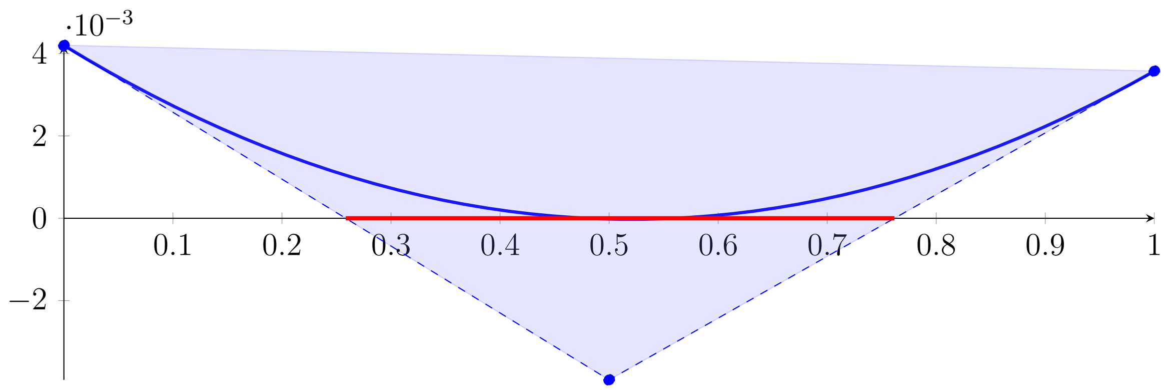
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

4.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

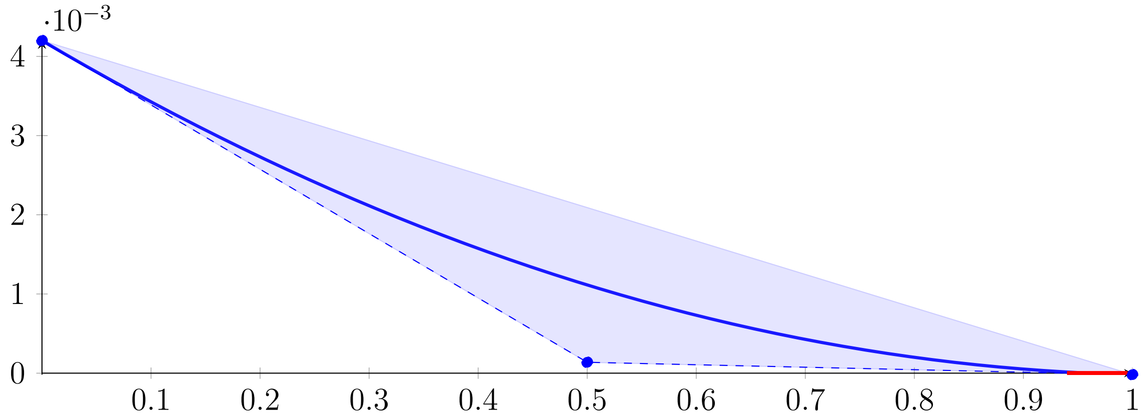
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

4.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001 \\ &= 0.004200000000000001 B_{0,2}(X) + 0.0001375000000000007 B_{1,2}(X) \\ &\quad - 1.874999999999948 \cdot 10^{-05} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

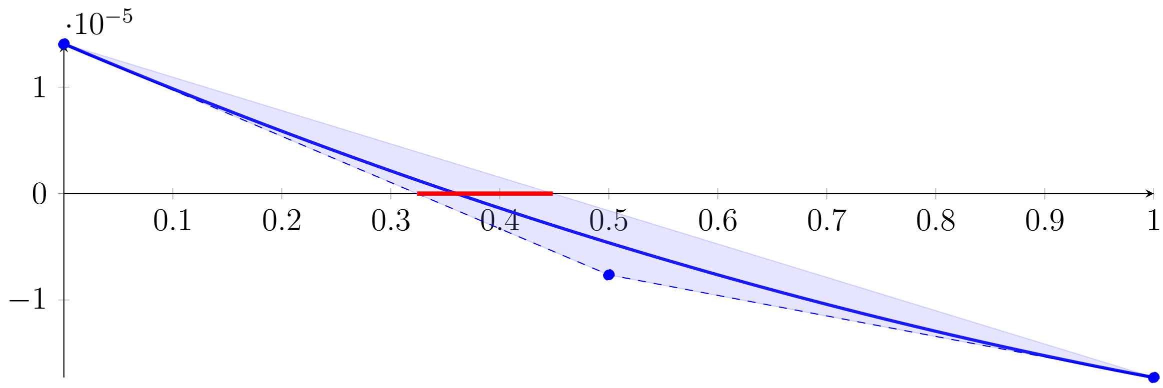
$$[0.9400000000000017, 0.9955555555555557]$$

Longest intersection interval: 0.05555555555555396
$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

4.7 Recursion Branch 1 2 1 1 1 1 in Interval 1: $[0.5587500000000001, 0.56222222222222$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.205632716049313 \cdot 10^{-05} X^2 - 4.340277777777758 \cdot 10^{-05} X + 1.406249999999919 \cdot 10^{-05} \\ &= 1.406249999999919 \cdot 10^{-05} B_{0,2}(X) - 7.638888888888706 \\ &\quad \cdot 10^{-06} B_{1,2}(X) - 1.728395061728347 \cdot 10^{-05} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3239999999999962, 0.4486153846153773\}$$

Intersection intervals with the x axis:

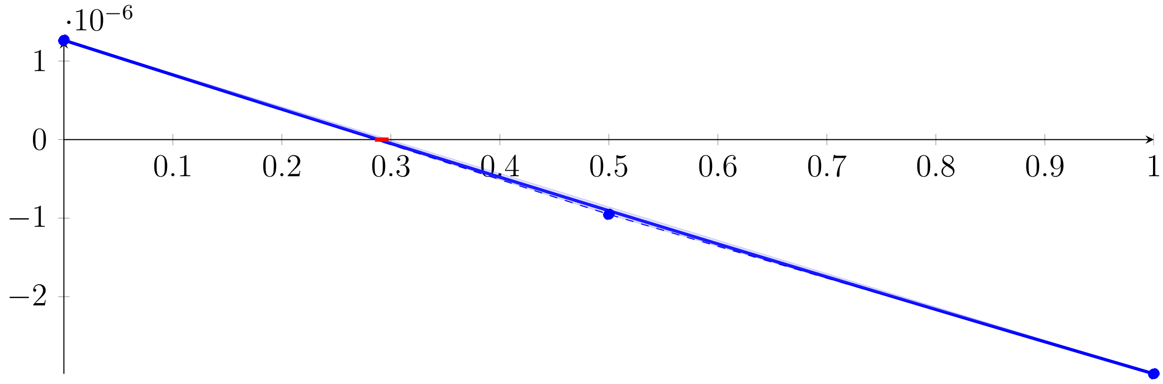
$$[0.3239999999999962, 0.4486153846153773]$$

Longest intersection interval: 0.1246153846153811
$\Longrightarrow$ Selective recursion: interval 1: $[0.5598750000000001, 0.5603076923076923]$,

4.8 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[0.5598750000000001, 0.5603076923$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1.872226331360733 \cdot 10^{-07} X^2 - 4.435096153845849 \cdot 10^{-06} X + 1.265624999999897 \cdot 10^{-06}$$
$$= 1.265624999999897 \cdot 10^{-06} B_{0,2}(X) - 9.519230769230272 \cdot 10^{-07} B_{1,2}(X) - 2.982248520709878 \cdot 10^{-06} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.285365853658533, 0.2979431929480858\}$$

Intersection intervals with the x axis:

$$[0.285365853658533, 0.2979431929480858]$$

Longest intersection interval: 0.01257733928955277
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599984756097562, 0.560003917727718]$,

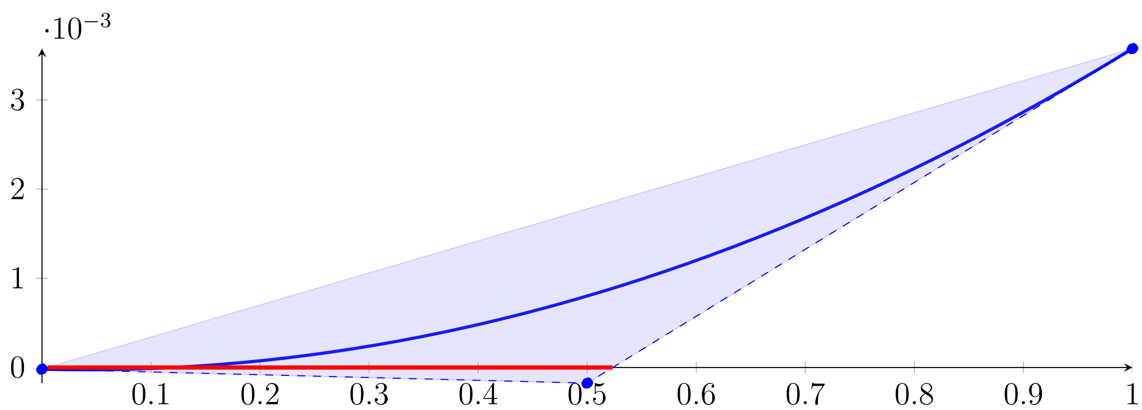
4.9 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[0.5599984756097562, 0.56000391$

Found root in interval $[0.5599984756097562, 0.560003917727718]$ at recursion depth 8!

4.10 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.00390625 X^2 - 0.0003125000000000003 X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 0.0001749999999999996 B_{1,2}(X) + 0.003575 B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

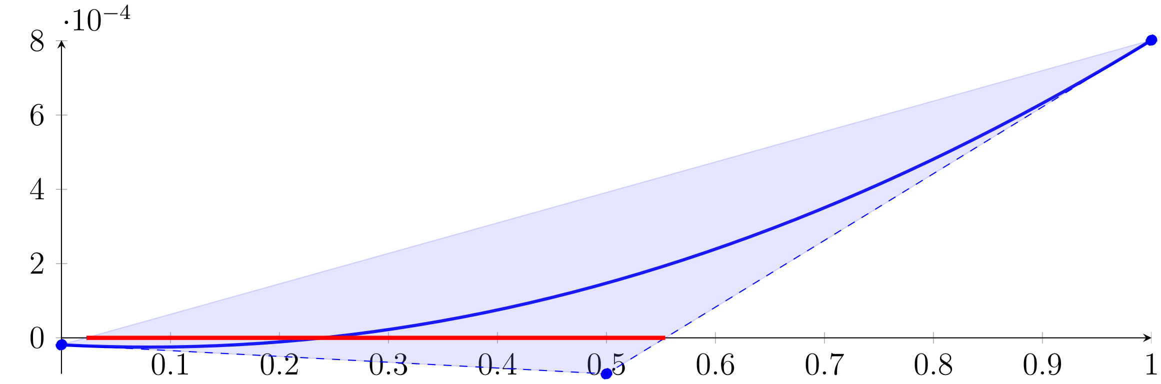
$$[0.005217391304347681, 0.5233333333333333]$$

Longest intersection interval: 0.5181159420289856
$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

4.11 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 - 0.00015625000000000001X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 9.687499999999955 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0008015625000000004 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142222, 0.5539130434782606\}$$

Intersection intervals with the x axis:

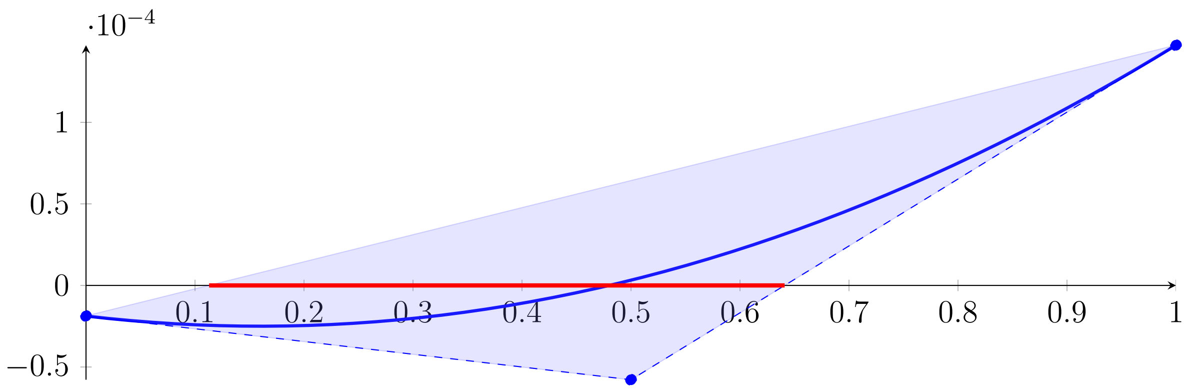
$$[0.022857142857142222, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\Longrightarrow$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

4.12 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 - 7.812500000000007 \cdot 10^{-05} X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 5.781249999999951 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

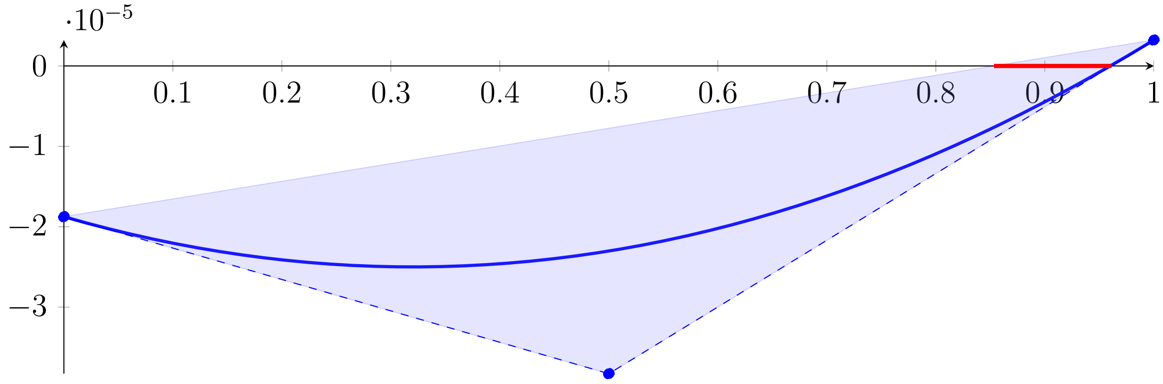
$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\Longrightarrow$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

4.13 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625\cdot 10^{-05}X^2 - 3.906250000000003\cdot 10^{-05}X - 1.874999999999948\cdot 10^{-05} \\ &= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 3.82812499999995 \\ &\quad \cdot 10^{-05}B_{1,2}(X) + 3.222656250000487\cdot 10^{-06}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8533333333333109, 0.9611764705882294\}$$

Intersection intervals with the x axis:

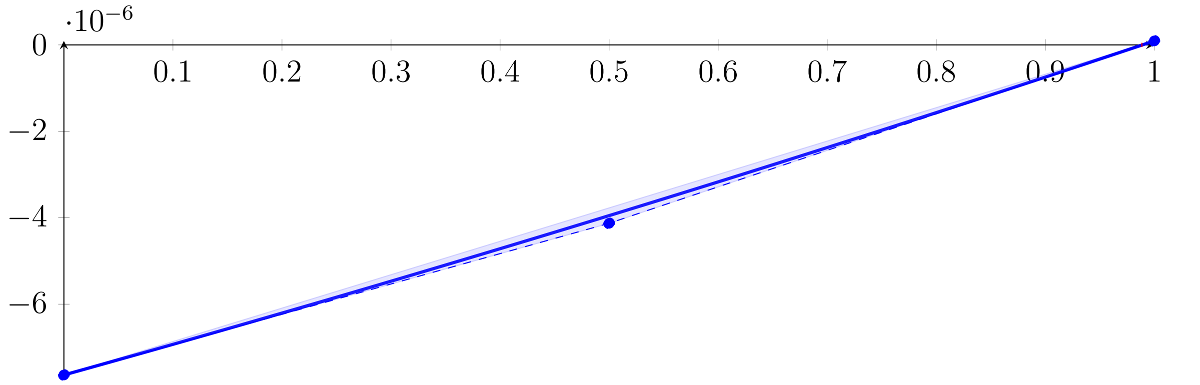
$$[0.8533333333333109, 0.9611764705882294]$$

Longest intersection interval: 0.1078431372549185
$\Longrightarrow$ Selective recursion: interval 1: $[0.5691666666666665, 0.5700091911764705]$,

4.14 Recursion Branch 1 2 1 1 2 1 1 1 1 in Interval 1: $[0.5691666666666665, 0.57000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 7.098475496205558\cdot 10^{-07}X^2 + 7.021037581700123\cdot 10^{-06}X - 7.638888888889855\cdot 10^{-06} \\ &= -7.638888888889855\cdot 10^{-06}B_{0,2}(X) - 4.128370098039794 \\ &\quad \cdot 10^{-06}B_{1,2}(X) + 9.199624243082374\cdot 10^{-08}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9881001669449061, 0.9891009174311909\}$$

Intersection intervals with the x axis:

$$[0.9881001669449061, 0.9891009174311909]$$

Longest intersection interval: 0.001000750486284818
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999165275459, 0.5700000084322719]$,

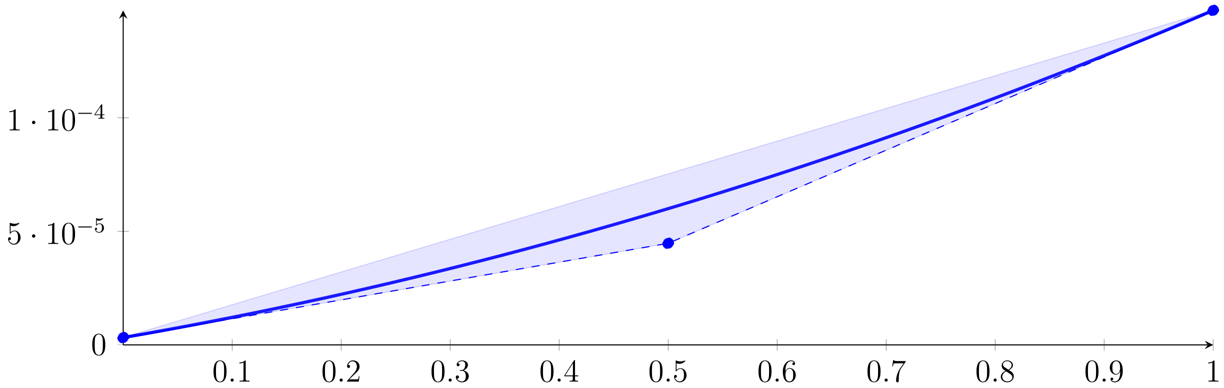
4.15 Recursion Branch 1 2 1 1 2 1 1 1 1 1 in Interval 1: $[0.569999165275459, 0.57000$

Found root in interval $[0.569999165275459, 0.5700000084322719]$ at recursion depth 10!

4.16 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 6.103515625 \cdot 10^{-05} X^2 + 8.300781249999997 \cdot 10^{-05} X + 3.222656250000487 \cdot 10^{-06}\\ &= 3.222656250000487 \cdot 10^{-06} B_{0,2}(X) + 4.472656250000047\\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

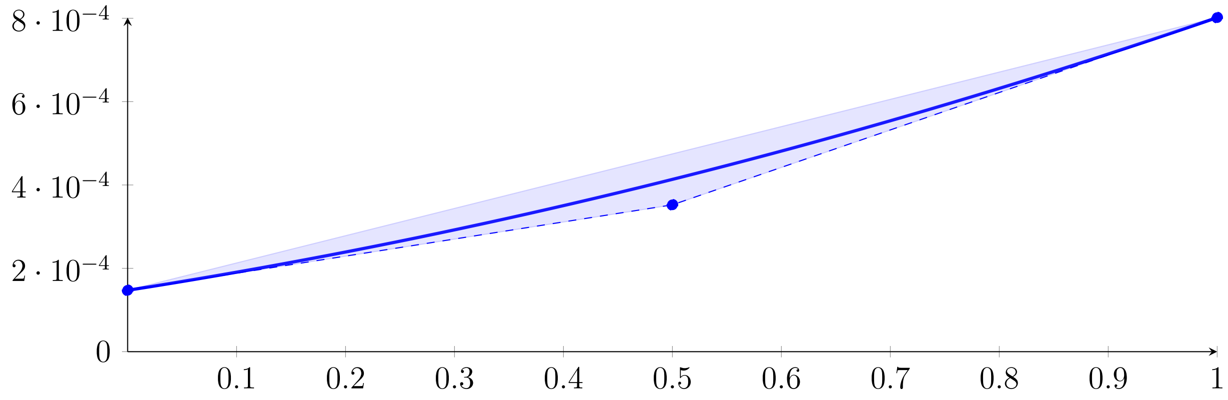
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

4.17 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.000244140625 X^2 + 0.0004101562499999999 X + 0.0001472656250000005\\ &= 0.0001472656250000005 B_{0,2}(X) + 0.0003523437500000004 B_{1,2}(X)\\ &\quad + 0.0008015625000000004 B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

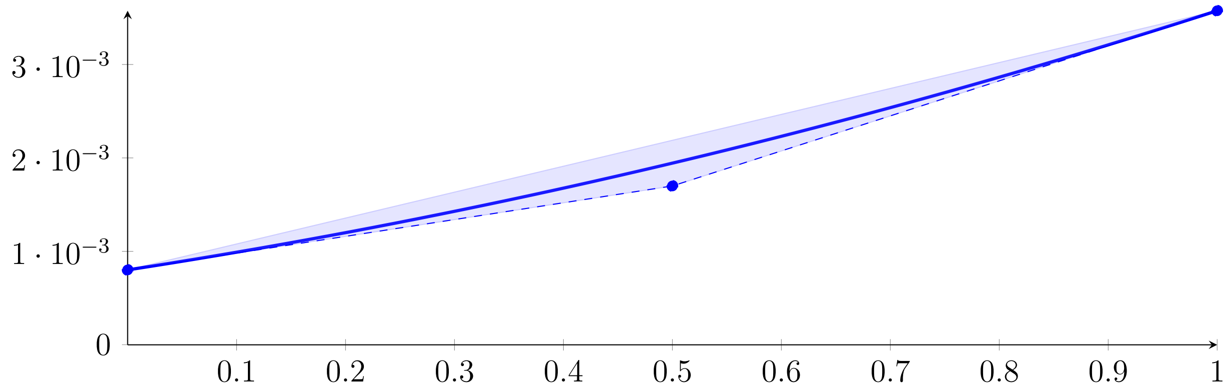
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

4.18 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 + 0.001796875X + 0.0008015625000000004 \\ &= 0.0008015625000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

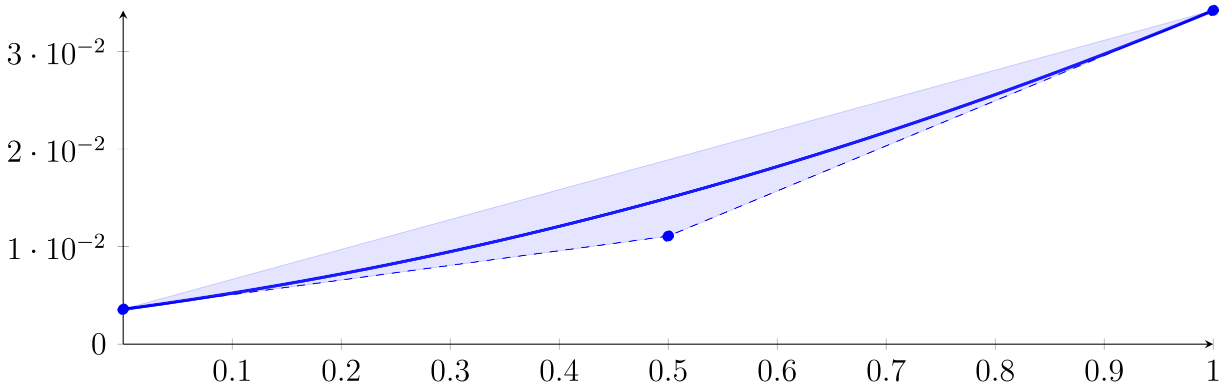
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

4.19 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

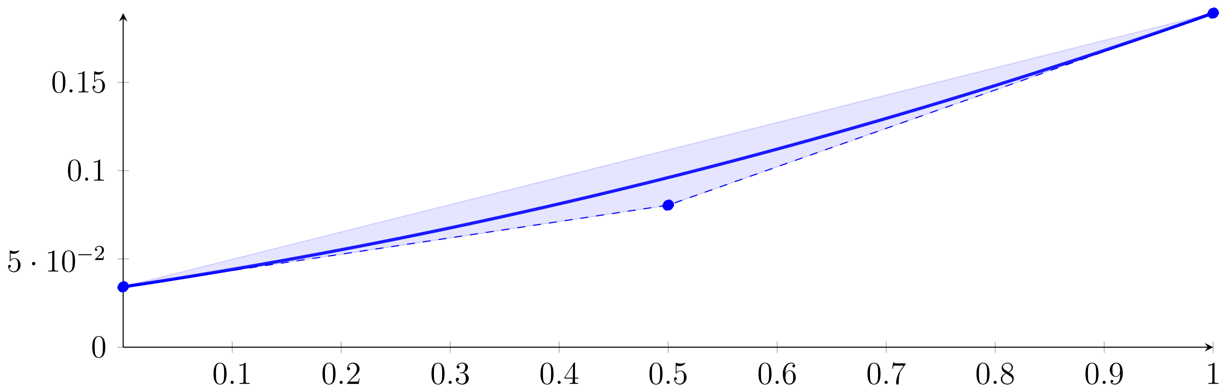
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

4.20 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

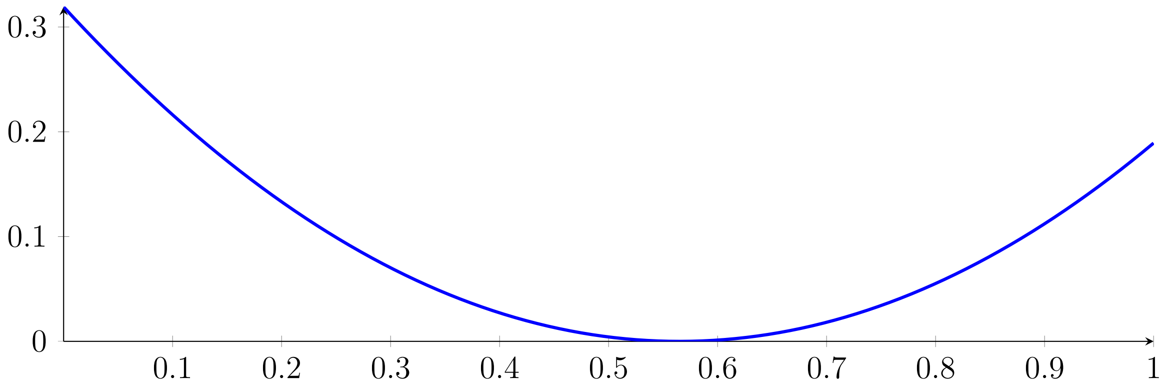
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

4.21 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5599984756097562, 0.560003917727718], \; [0.569999165275459, 0.5700000084322719]$$

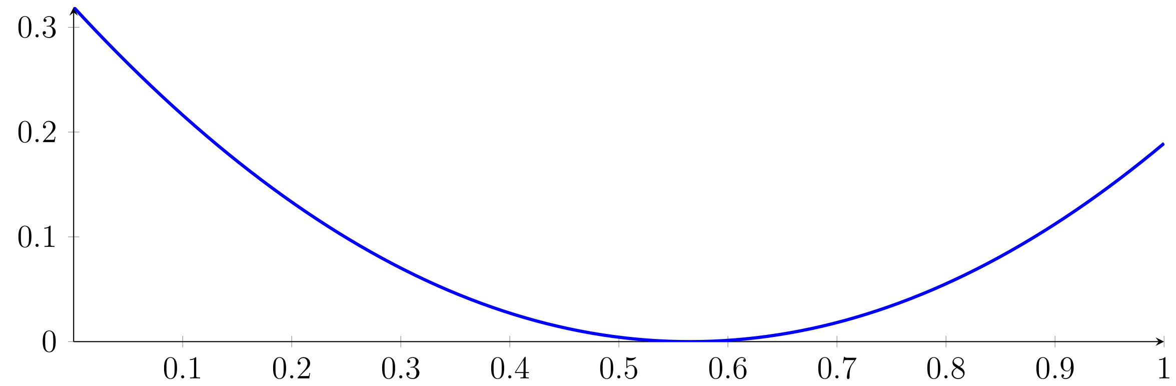
with precision $\varepsilon = 0.0001$.

5 Running `QuadClip` on h_2 with epsilon 4

$$1X^2 - 1.13X + 0.3192$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

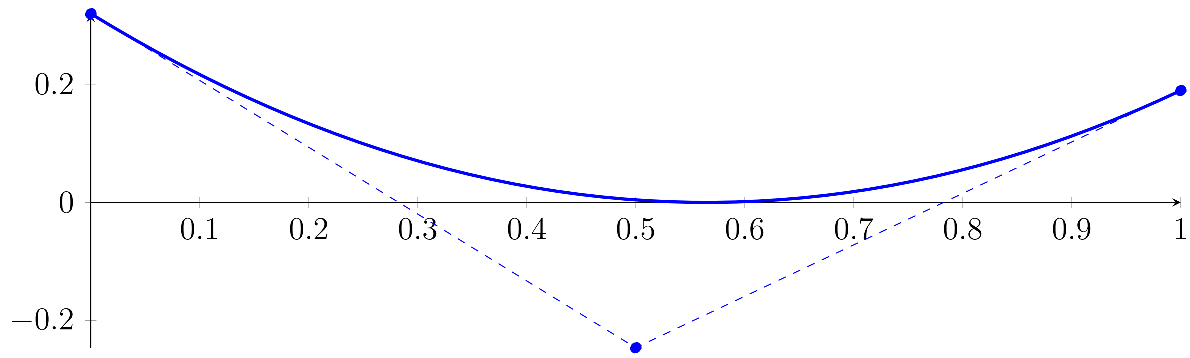
$$p = 1X^2 - 1.13X + 0.3192$$



5.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

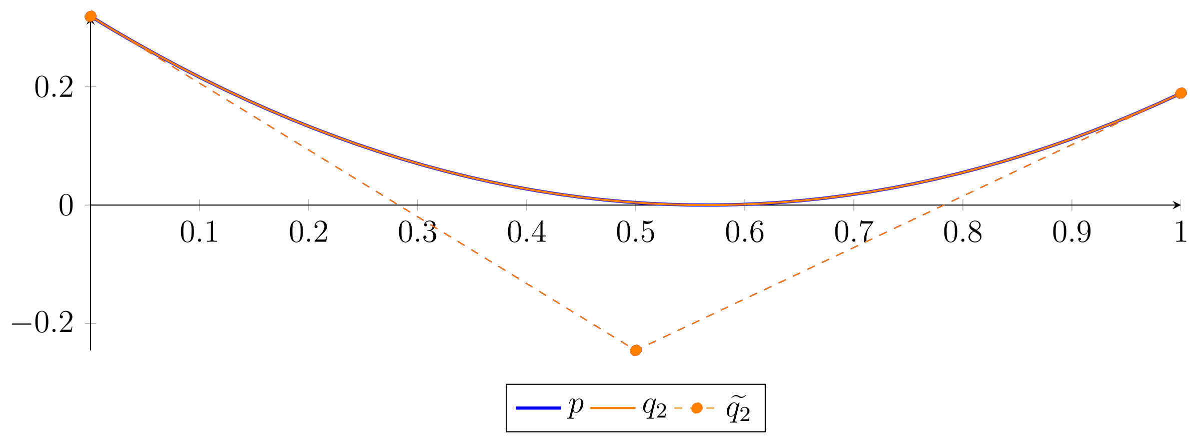
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

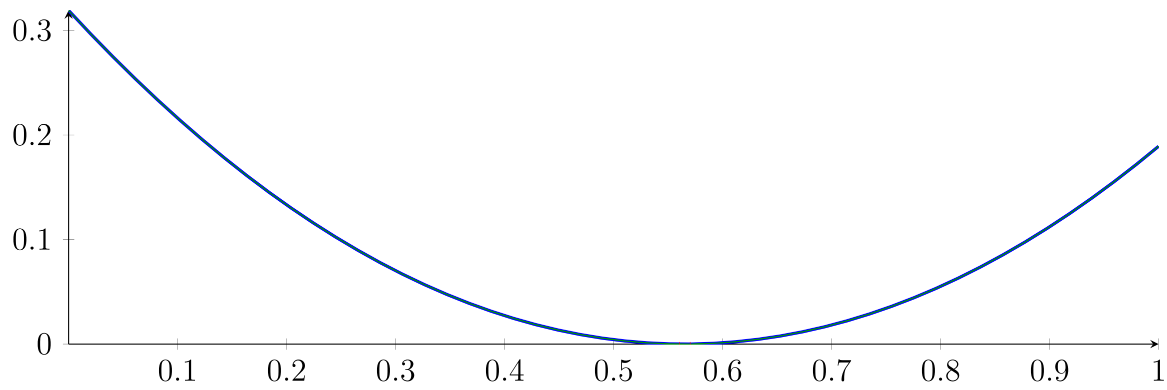
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

5.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

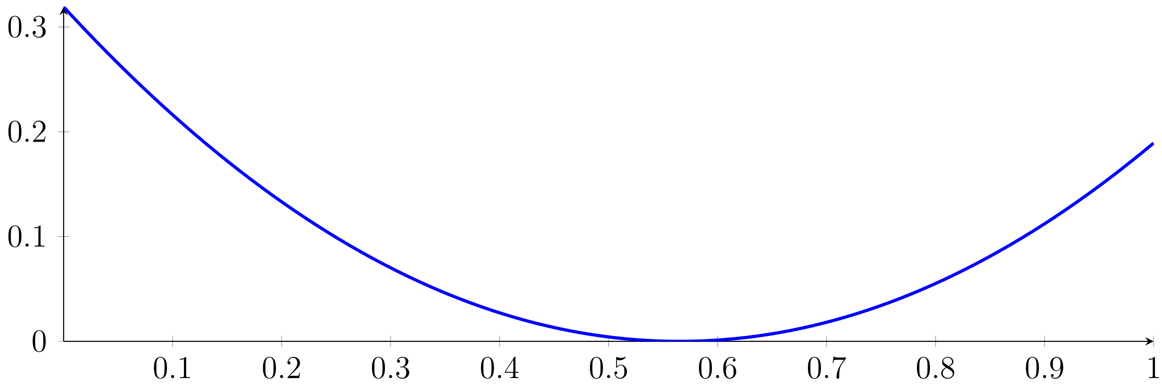
5.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

5.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

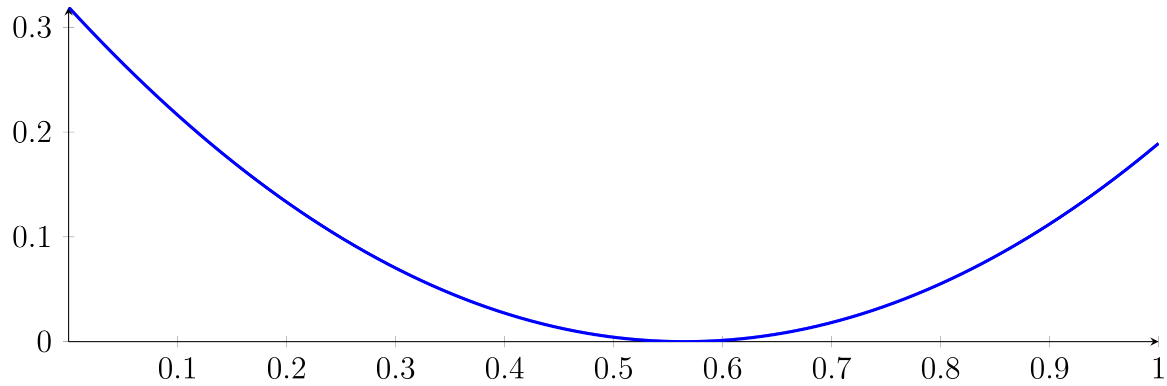
with precision $\varepsilon = 0.0001$.

6 Running `CubeClip` on h_2 with epsilon 4

$$1X^2 - 1.13X + 0.3192$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

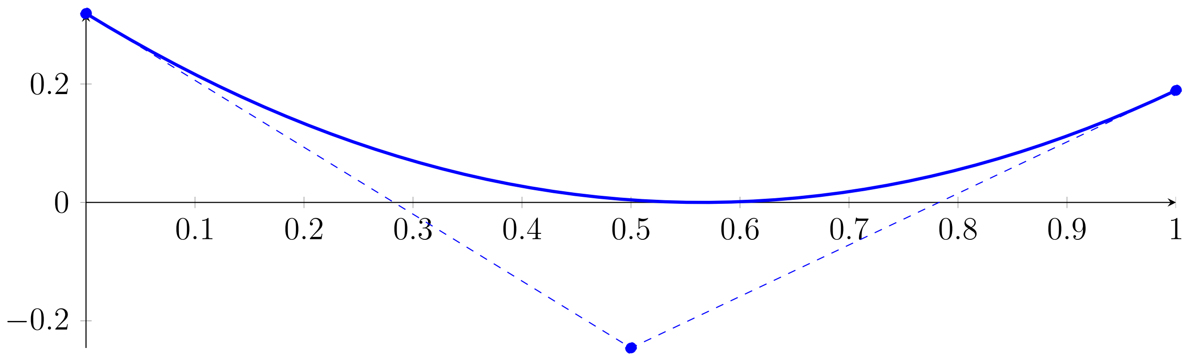
$$p = 1X^2 - 1.13X + 0.3192$$



6.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

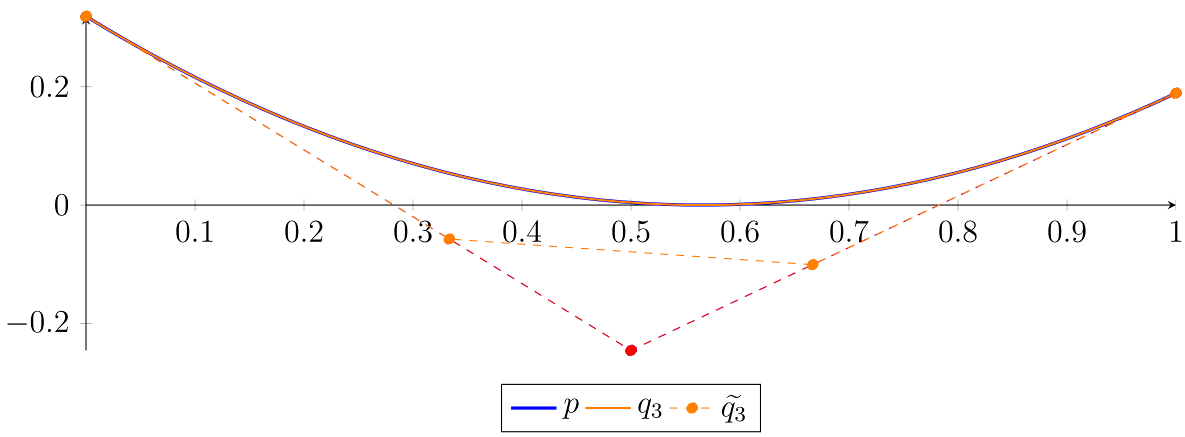
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501 \cdot 10^{-308}$.
Bounding polynomials M and m:

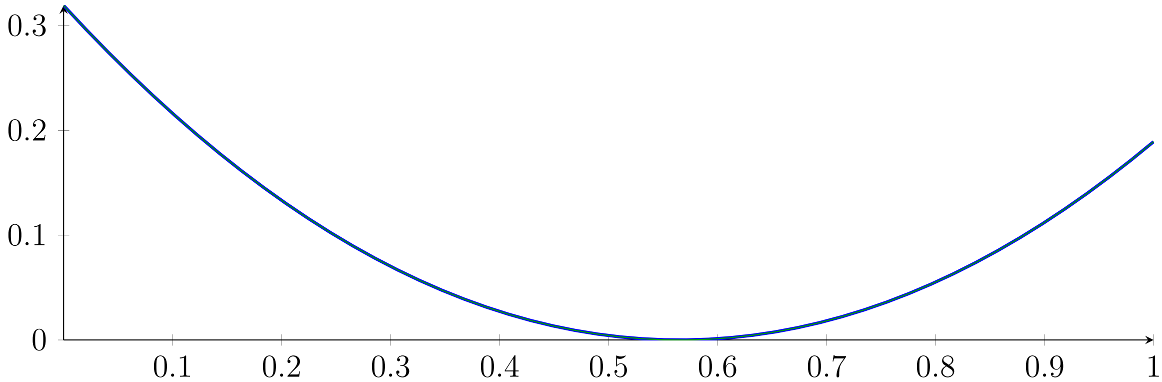
$$M = -1.668805393880401 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783 \cdot 10^{291}, 2.824576038015892 \cdot 10^{291}, 5.992310449541053 \cdot 10^{307}\}$ $N(m) = \{-4.842$

Intersection intervals:

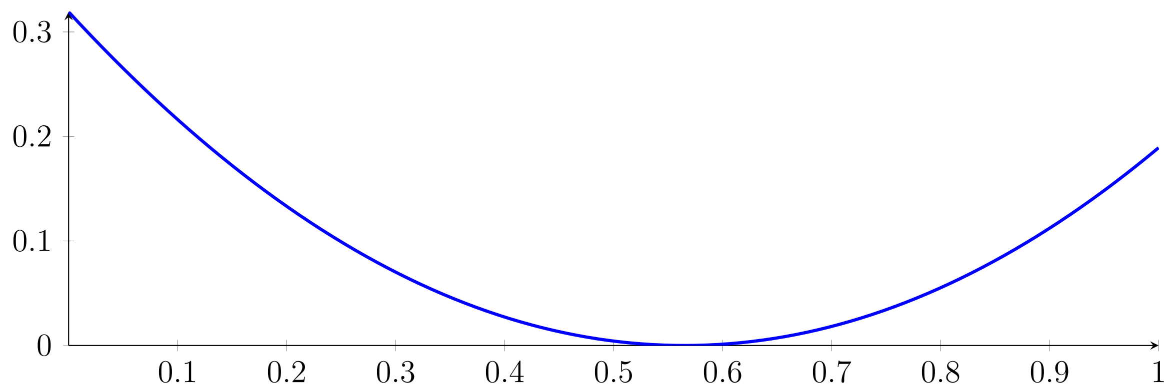


No intersection intervals with the x axis.

6.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

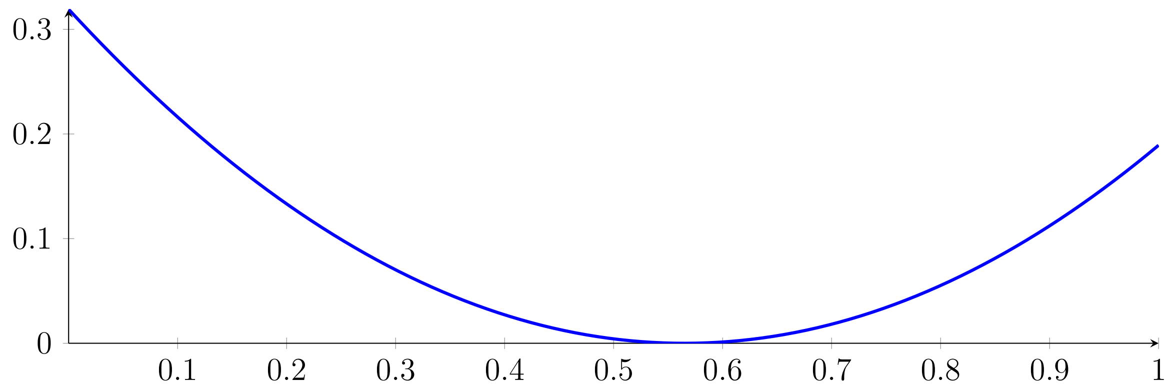
with precision $\varepsilon = 0.0001$.

7 Running `BezClip` on h_2 with epsilon 8

$$1X^2 - 1.13X + 0.3192$$

Called `BezClip` with input polynomial on interval $[0,1]$:

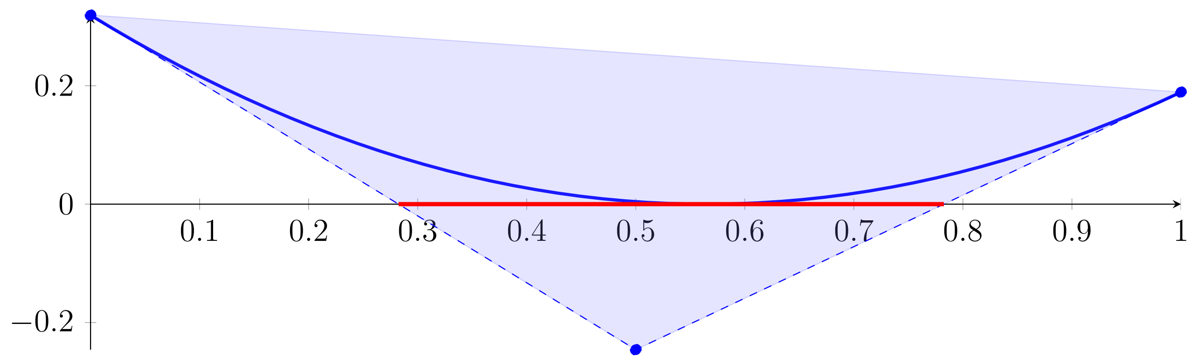
$$p = 1X^2 - 1.13X + 0.3192$$



7.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

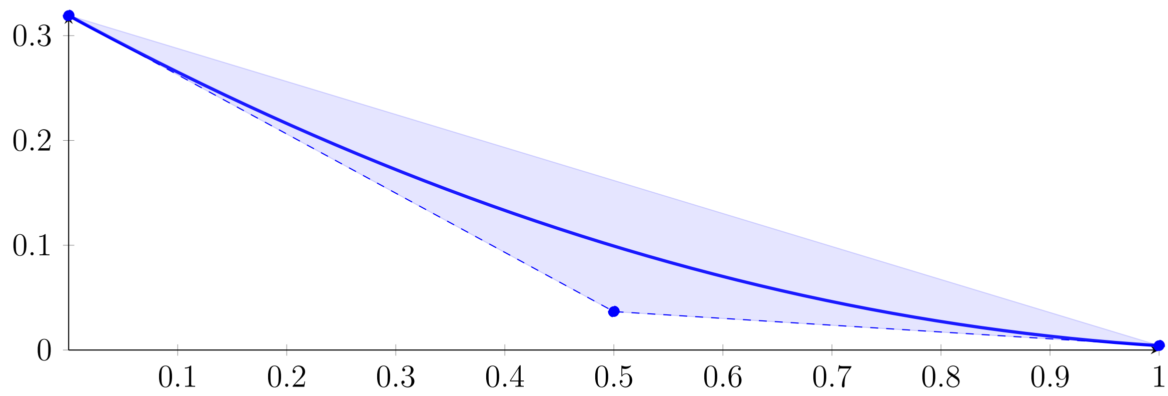
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

7.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

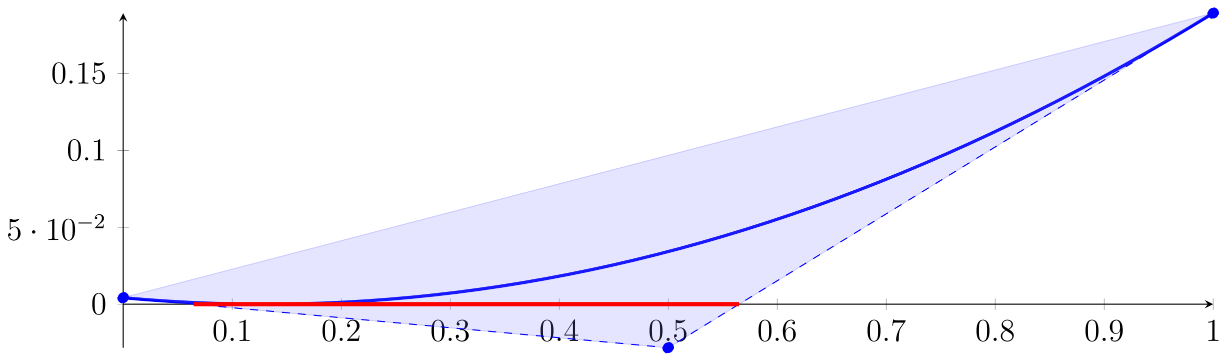
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

7.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.065X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

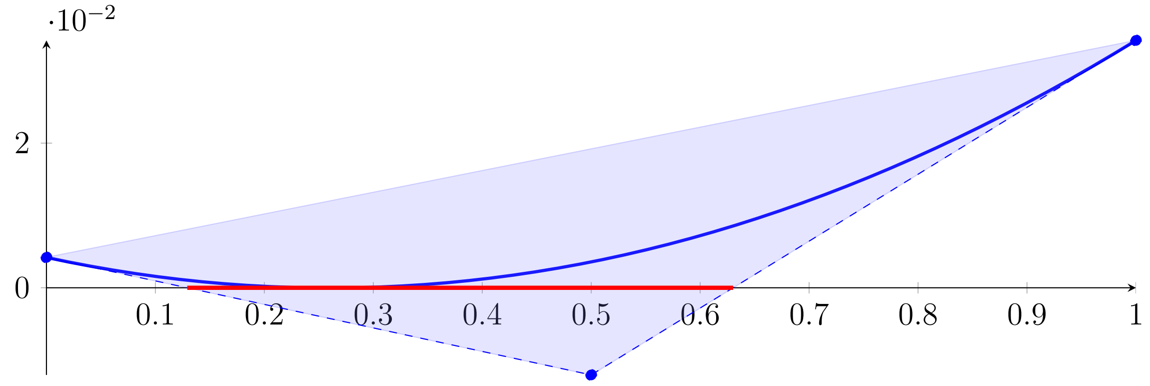
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\Longrightarrow$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

7.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 - 0.0325X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

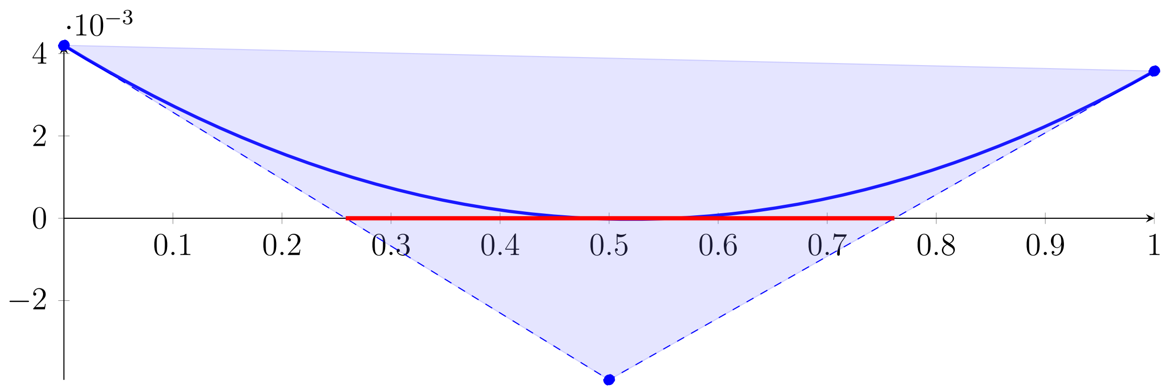
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

7.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

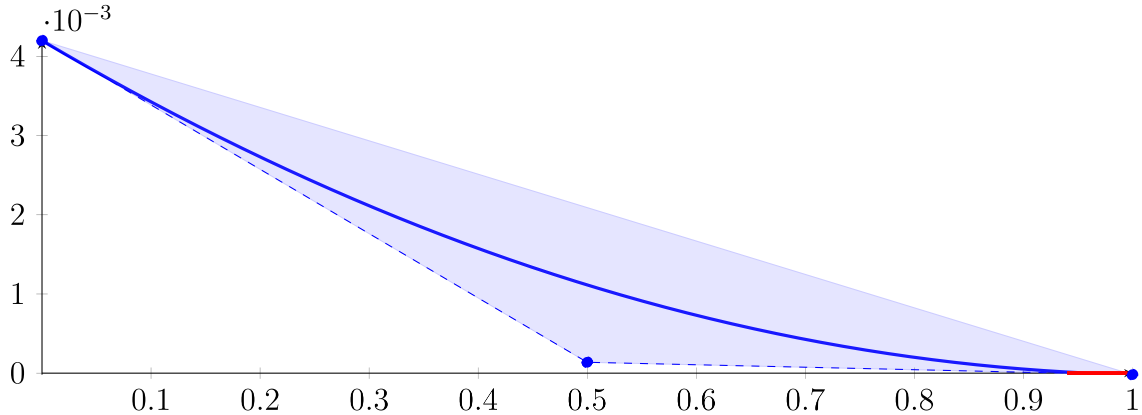
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

7.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001\\ &= 0.004200000000000001B_{0,2}(X) + 0.0001375000000000007B_{1,2}(X)\\ &\quad - 1.874999999999948\cdot 10^{-05}B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

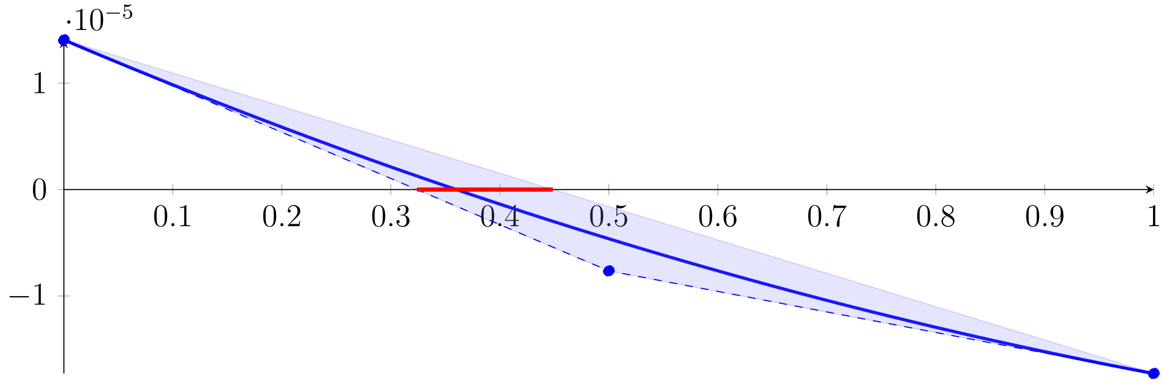
$$[0.9400000000000017, 0.9955555555555557]$$

Longest intersection interval: 0.05555555555555396
$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

7.7 Recursion Branch 1 2 1 1 1 1 in Interval 1: $[0.5587500000000001, 0.562222222222$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 1.205632716049313\cdot 10^{-05}X^2 - 4.340277777777758\cdot 10^{-05}X + 1.406249999999919\cdot 10^{-05}\\ &= 1.406249999999919\cdot 10^{-05}B_{0,2}(X) - 7.638888888888706\\ &\quad \cdot 10^{-06}B_{1,2}(X) - 1.728395061728347\cdot 10^{-05}B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3239999999999962, 0.4486153846153773\}$$

Intersection intervals with the x axis:

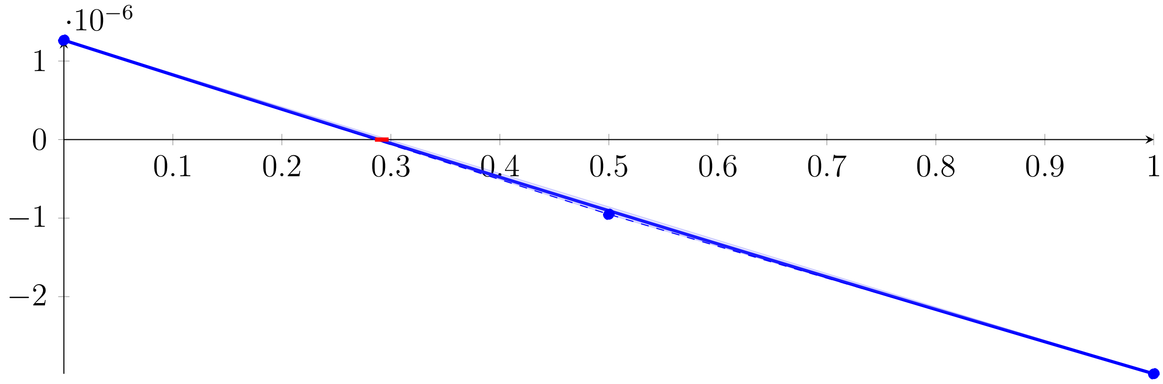
$$[0.3239999999999962, 0.4486153846153773]$$

Longest intersection interval: 0.1246153846153811
$\Longrightarrow$ Selective recursion: interval 1: $[0.5598750000000001, 0.5603076923076923]$,

7.8 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[0.5598750000000001, 0.5603076923$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.872226331360733 \cdot 10^{-07} X^2 - 4.435096153845849 \cdot 10^{-06} X + 1.265624999999897 \cdot 10^{-06} \\ &= 1.265624999999897 \cdot 10^{-06} B_{0,2}(X) - 9.519230769230272 \\ &\quad \cdot 10^{-07} B_{1,2}(X) - 2.982248520709878 \cdot 10^{-06} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2853658536585333, 0.2979431929480858\}$$

Intersection intervals with the x axis:

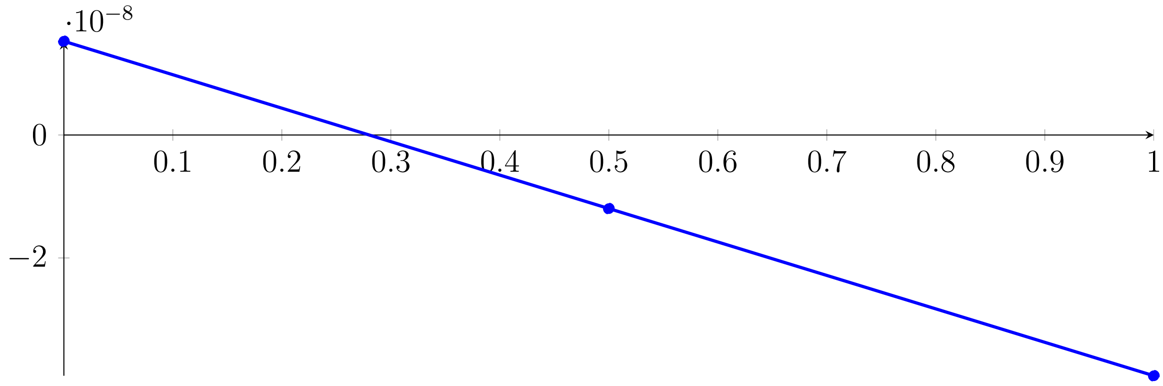
$$[0.2853658536585333, 0.2979431929480858]$$

Longest intersection interval: 0.01257733928955277
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599984756097562, 0.560003917727718]$,

7.9 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[0.5599984756097562, 0.56000391$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 2.961664791042273 \cdot 10^{-11} X^2 - 5.443777144130786 \cdot 10^{-08} X + 1.524622620463798 \cdot 10^{-08} \\ &= 1.524622620463798 \cdot 10^{-08} B_{0,2}(X) - 1.197265951601595 \\ &\quad \cdot 10^{-08} B_{1,2}(X) - 3.916192858875946 \cdot 10^{-08} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2800670527278236, 0.2802195050086158\}$$

Intersection intervals with the x axis:

$$[0.2800670527278236, 0.2802195050086158]$$

Longest intersection interval: 0.0001524522807922228
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599999997676943, 0.5600000005973576]$,

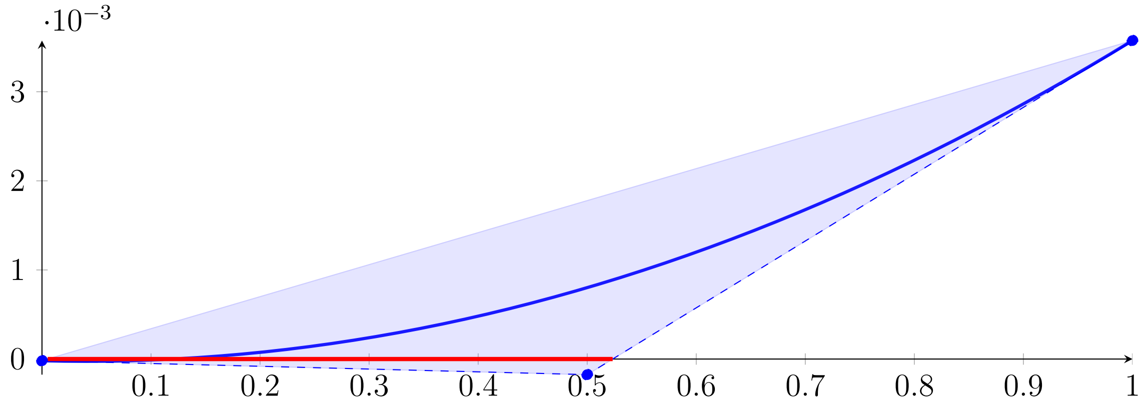
7.10 Recursion Branch 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.5599999997676943, 0.56000$

Found root in interval $[0.5599999997676943, 0.5600000005973576]$ at recursion depth 9!

7.11 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.00390625X^2 - 0.0003125000000000003X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 0.0001749999999999996 B_{1,2}(X) + 0.003575 B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

$$[0.005217391304347681, 0.5233333333333333]$$

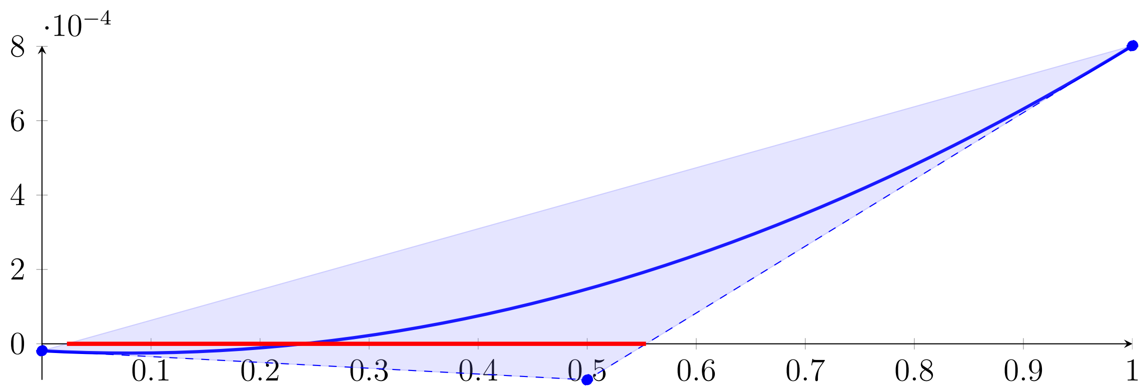
Longest intersection interval: 0.5181159420289856

$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

7.12 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.0009765625X^2 - 0.0001562500000000001X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 9.687499999999955$$
$$\cdot 10^{-05} B_{1,2}(X) + 0.0008015625000000004 B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142\,22, 0.5539130434782606\}$$

Intersection intervals with the x axis:

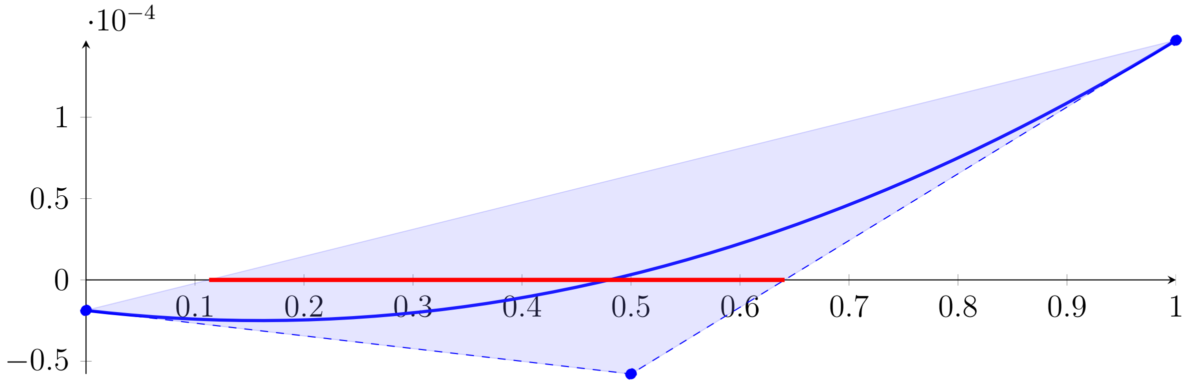
$$[0.02285714285714222, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\Longrightarrow$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

7.13 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 - 7.812500000000007\cdot 10^{-05}X - 1.874999999999948\cdot 10^{-05} \\ &= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 5.781249999999951 \\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0001472656250000005B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

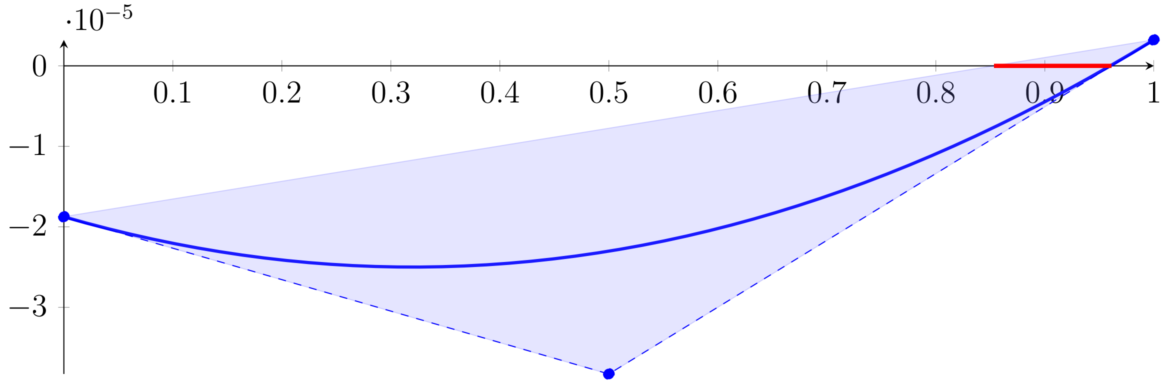
$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\Longrightarrow$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

7.14 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625\cdot 10^{-05}X^2 - 3.906250000000003\cdot 10^{-05}X - 1.874999999999948\cdot 10^{-05} \\ &= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 3.82812499999995 \\ &\quad \cdot 10^{-05}B_{1,2}(X) + 3.222656250000487\cdot 10^{-06}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8533333333333109, 0.9611764705882294\}$$

Intersection intervals with the x axis:

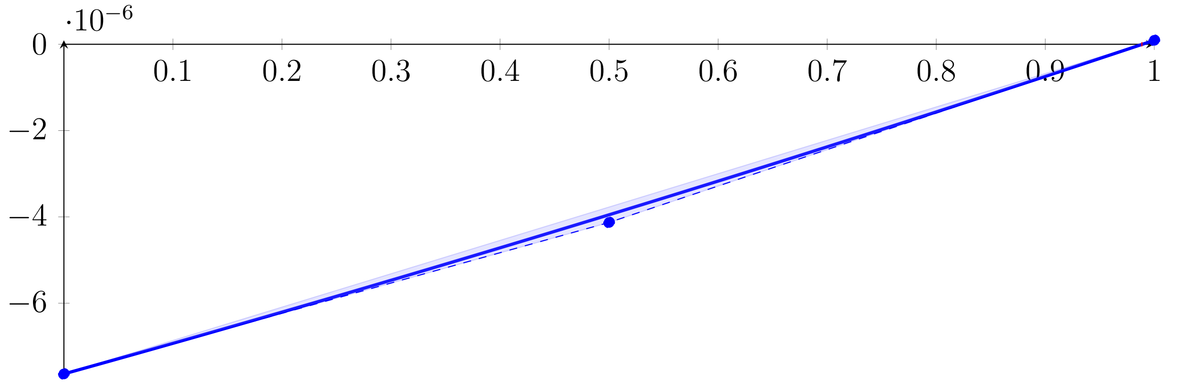
$$[0.8533333333333109, 0.9611764705882294]$$

Longest intersection interval: 0.1078431372549185
$\Longrightarrow$ Selective recursion: interval 1: $[0.5691666666666665, 0.5700091911764705]$,

7.15 Recursion Branch 1 2 1 1 2 1 1 1 1 in Interval 1: $[0.5691666666666665, 0.57000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.098475496205558 \cdot 10^{-07} X^2 + 7.021037581700123 \cdot 10^{-06} X - 7.638888888889855 \cdot 10^{-06}$$
$$= -7.638888888889855 \cdot 10^{-06} B_{0,2}(X) - 4.128370098039794$$
$$\cdot 10^{-06} B_{1,2}(X) + 9.199624243082374 \cdot 10^{-08} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9881001669449061, 0.9891009174311909\}$$

Intersection intervals with the x axis:

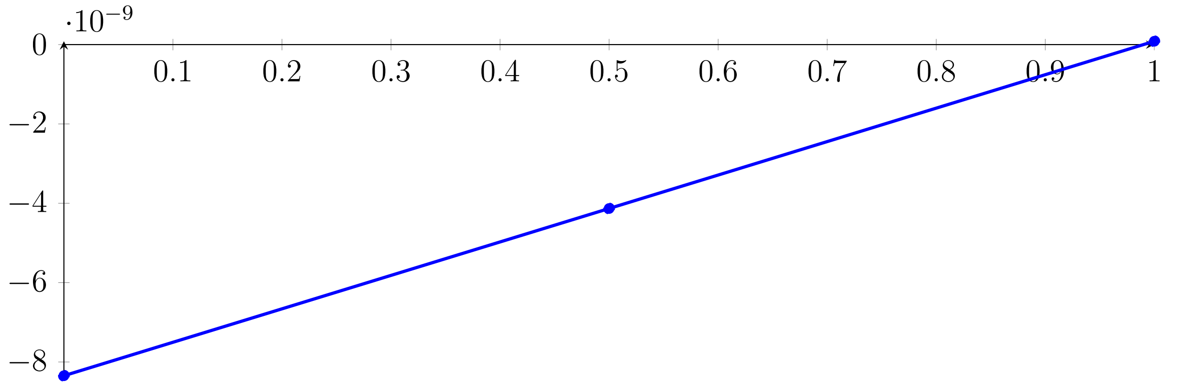
$$[0.9881001669449061, 0.9891009174311909]$$

Longest intersection interval: 0.001000750486284818
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999165275459, 0.5700000084322719]$,

7.16 Recursion Branch 1 2 1 1 2 1 1 1 1 1 in Interval 1: $[0.569999165275459, 0.57000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.109134111283559 \cdot 10^{-13} X^2 + 8.430160521565627 \cdot 10^{-09} X - 8.346548643959838 \cdot 10^{-09}$$
$$= -8.346548643959838 \cdot 10^{-09} B_{0,2}(X) - 4.131468383177024$$
$$\cdot 10^{-09} B_{1,2}(X) + 8.432279101691722 \cdot 10^{-11} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.989998330342568, 0.989999173638737\}$$

Intersection intervals with the x axis:

$$[0.989998330342568, 0.989999173638737]$$

Longest intersection interval: $8.432961690166558 \cdot 10^{-07}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999999999296, 0.5700000000000071]$,

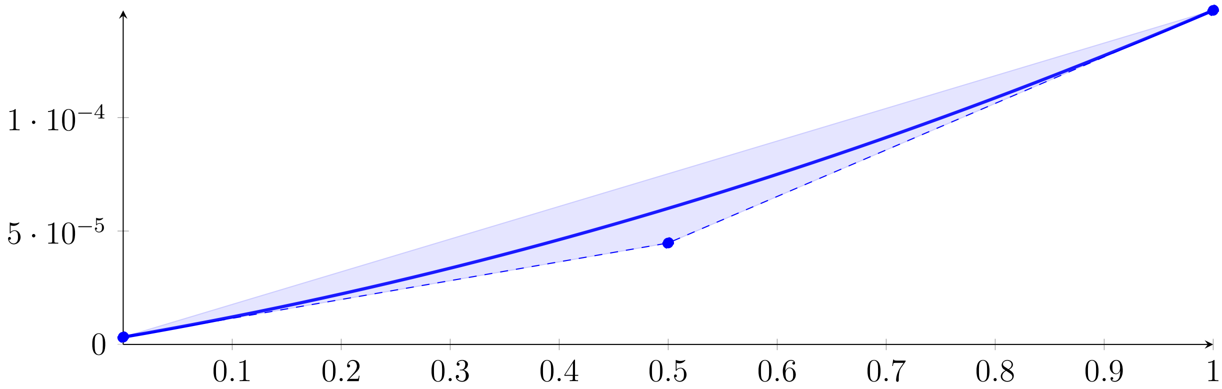
7.17 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.569999999999296, 0.570$

Found root in interval $[0.569999999999296, 0.5700000000000071]$ at recursion depth 11!

7.18 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625 \cdot 10^{-05} X^2 + 8.300781249999997 \cdot 10^{-05} X + 3.222656250000487 \cdot 10^{-06} \\ &= 3.222656250000487 \cdot 10^{-06} B_{0,2}(X) + 4.472656250000047 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

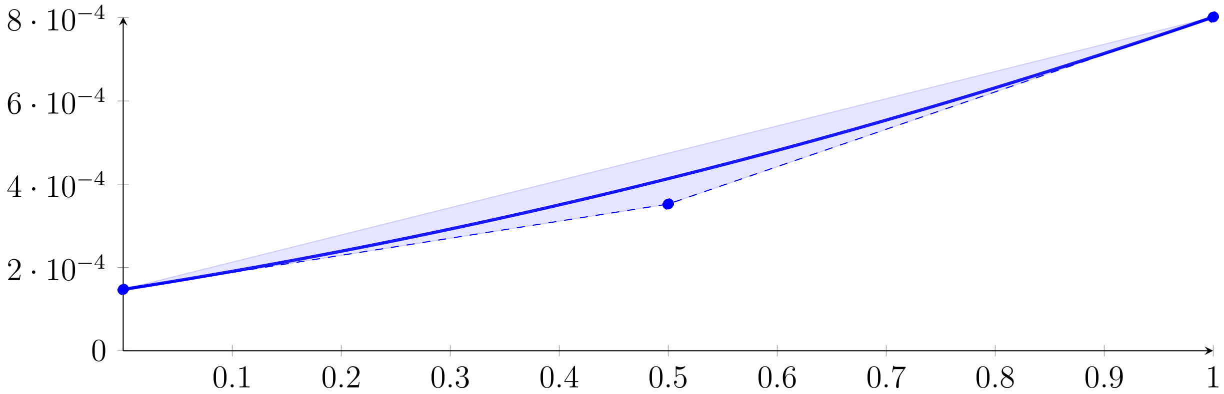
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

7.19 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625 X^2 + 0.0004101562499999999 X + 0.0001472656250000005 \\ &= 0.0001472656250000005 B_{0,2}(X) + 0.0003523437500000004 B_{1,2}(X) \\ &\quad + 0.0008015625000000004 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

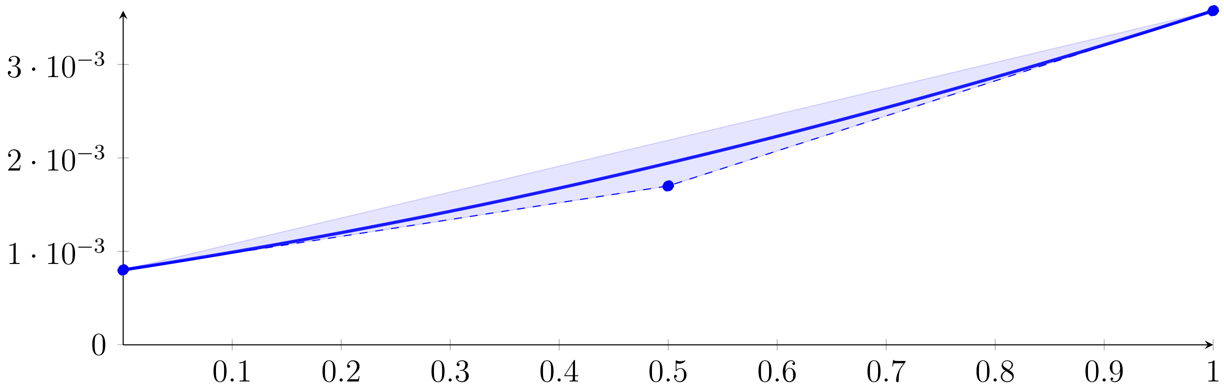
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

7.20 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 + 0.001796875X + 0.0008015625000000004 \\ &= 0.0008015625000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

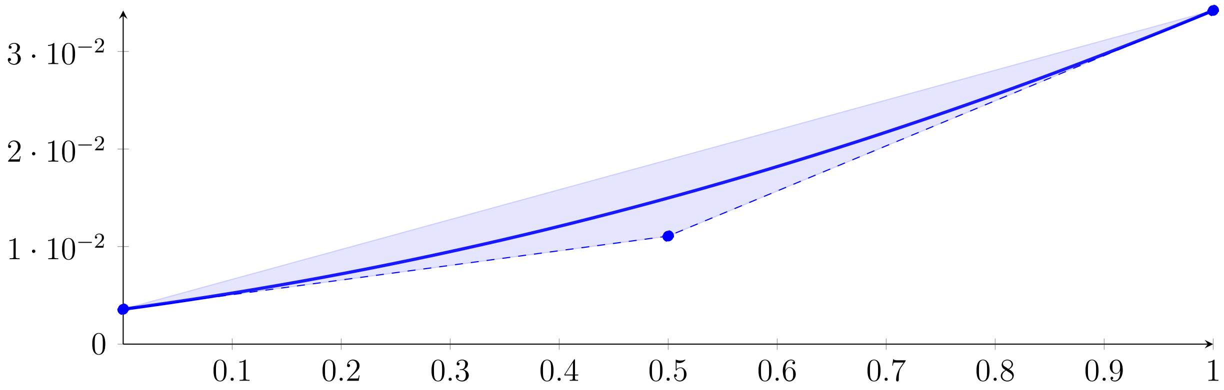
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

7.21 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

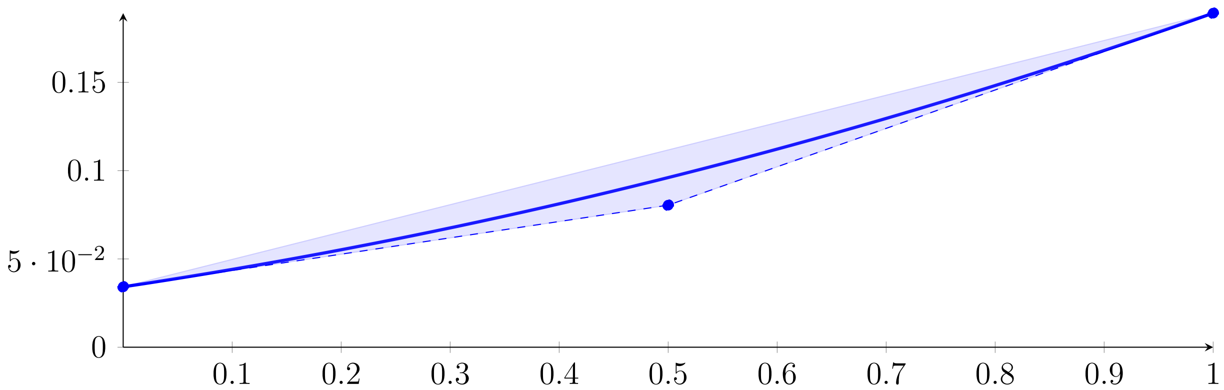
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

7.22 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

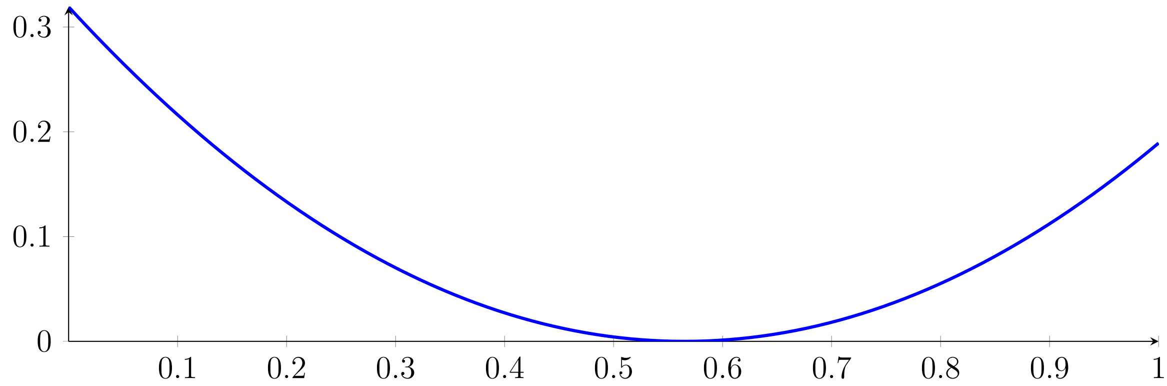
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

7.23 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5599999997676943, 0.5600000005973576],\ [0.569999999999296, 0.5700000000000071]$$

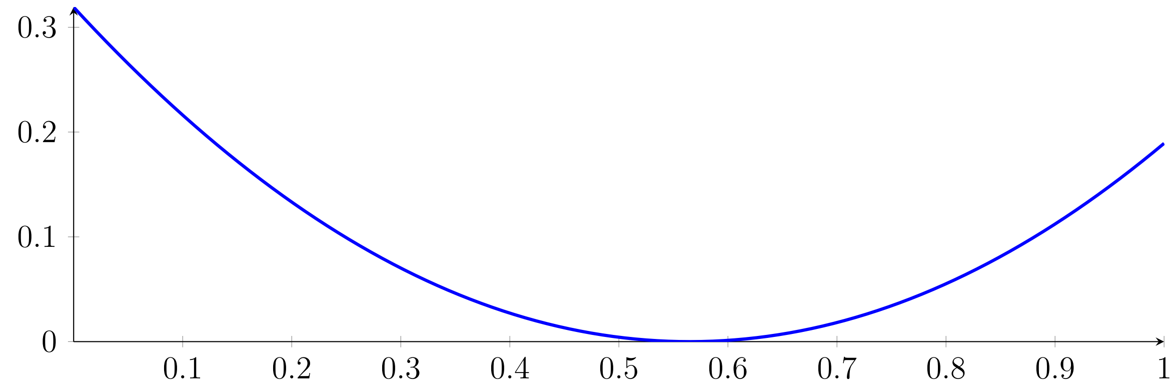
with precision $\varepsilon = 1 \cdot 10^{-08}$.

8 Running QuadClip on h_2 with epsilon 8

$$1X^2 - 1.13X + 0.3192$$

Called QuadClip with input polynomial on interval $[0, 1]$:

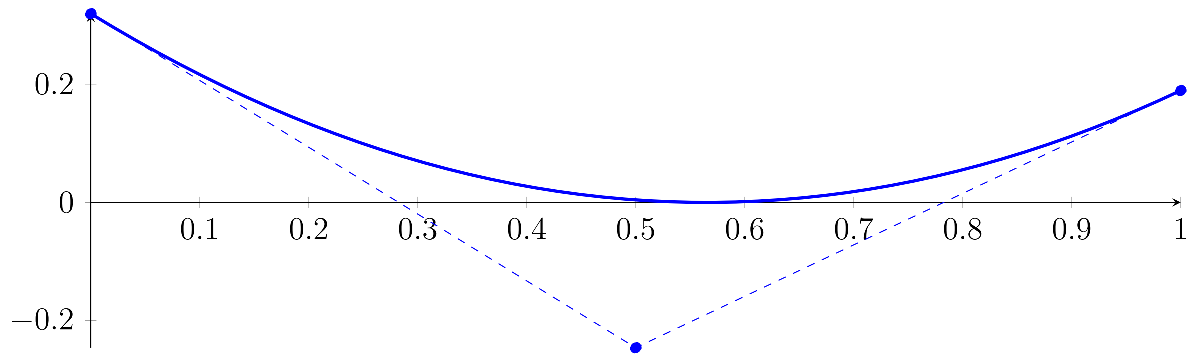
$$p = 1X^2 - 1.13X + 0.3192$$



8.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

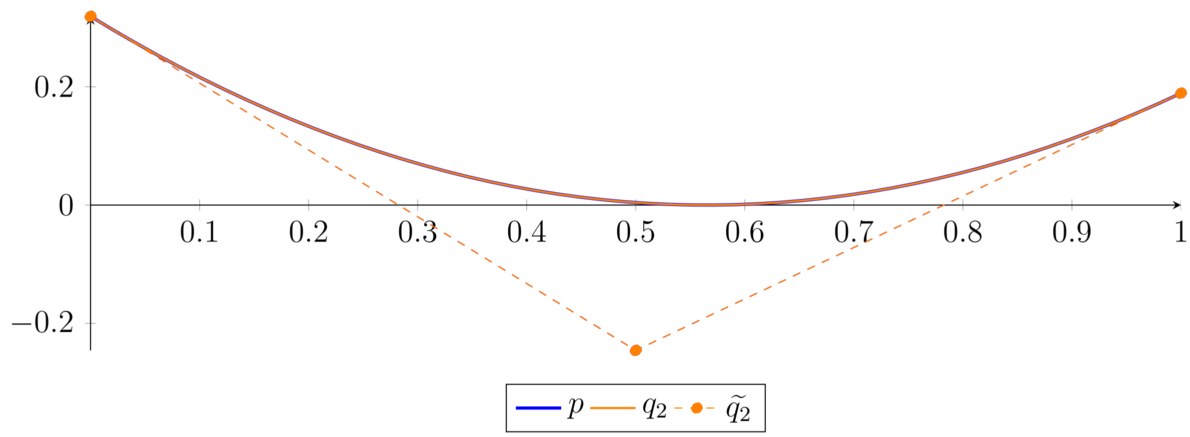
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

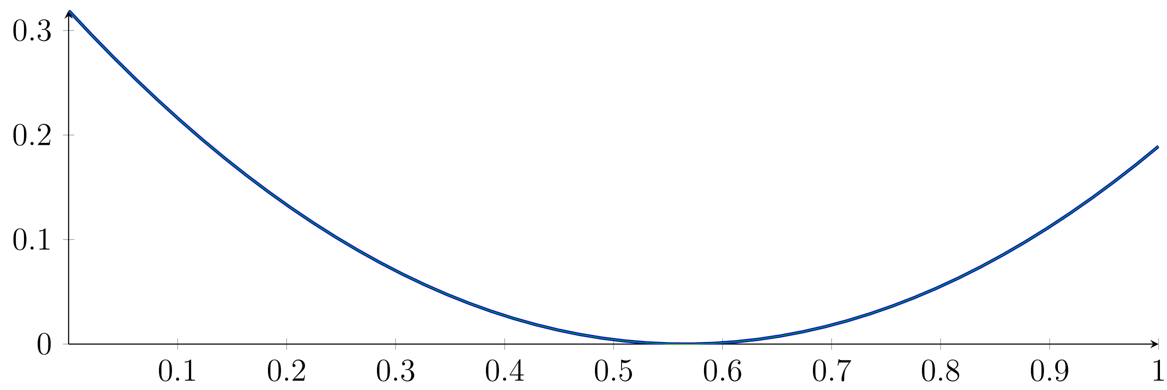
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

8.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

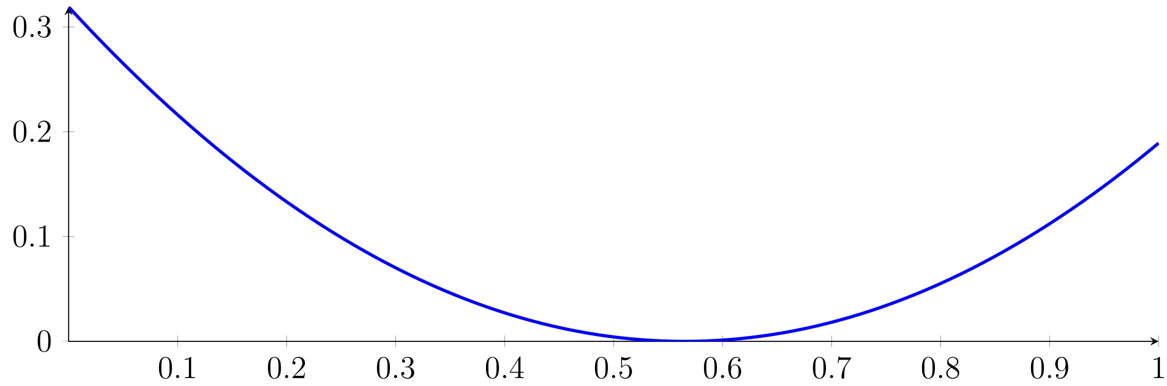
8.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

8.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

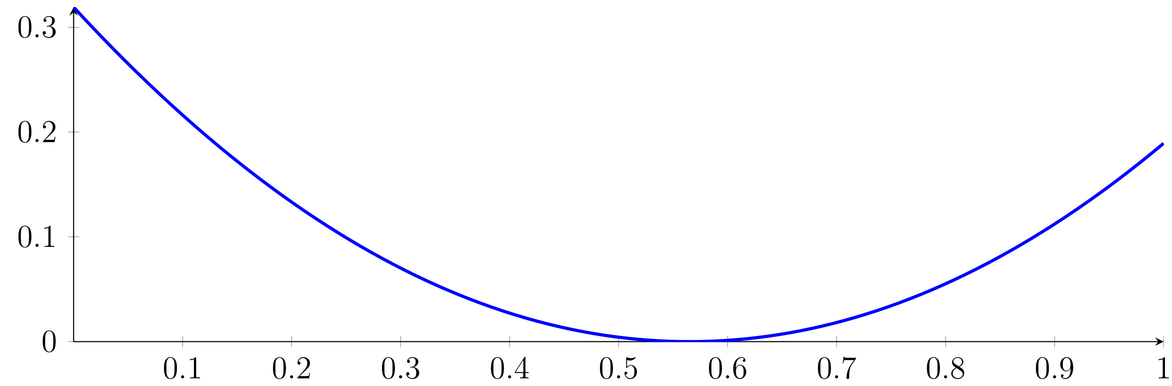
with precision $\varepsilon = 1 \cdot 10^{-08}$.

9 Running CubeClip on h_2 with epsilon 8

$$1X^2 - 1.13X + 0.3192$$

Called CubeClip with input polynomial on interval $[0,1]$:

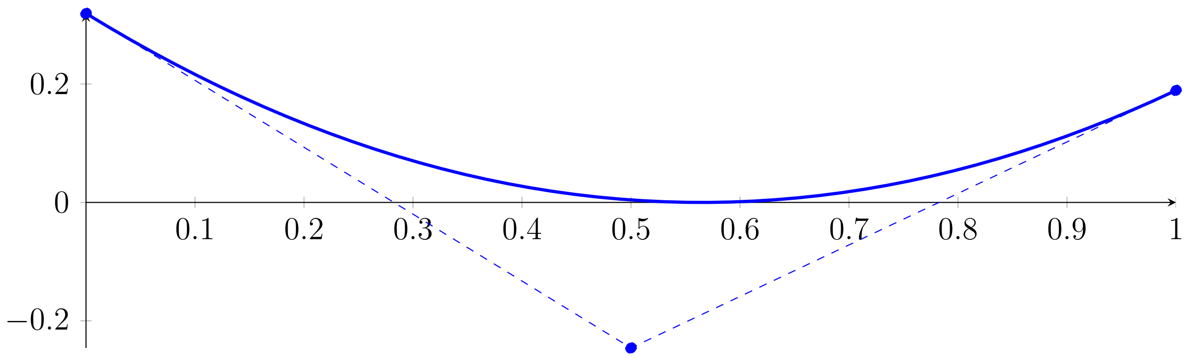
$$p = 1X^2 - 1.13X + 0.3192$$



9.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

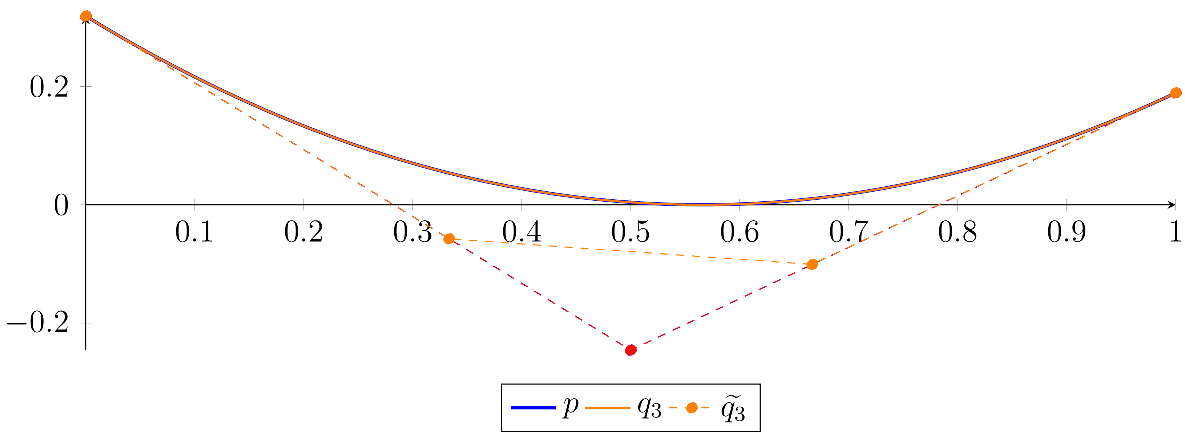
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308}X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501 \cdot 10^{-308}$.

Bounding polynomials M and m:

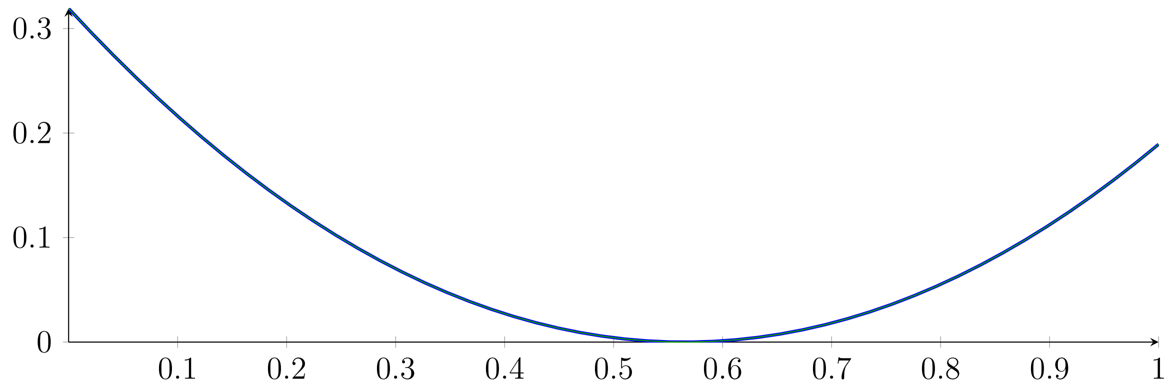
$$M = -1.668805393880401 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783 \cdot 10^{291}, 2.824576038015892 \cdot 10^{291}, 5.992310449541053 \cdot 10^{307}\} \quad N(m) = \{-4.84$

Intersection intervals:

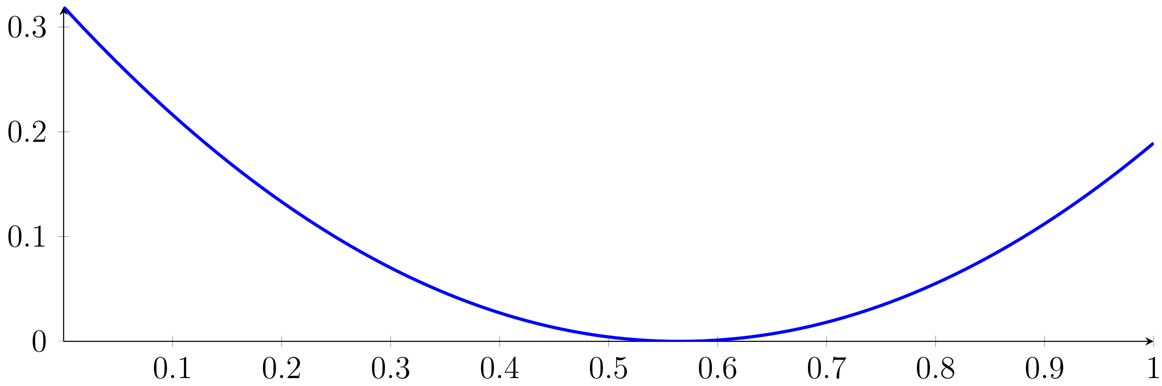


No intersection intervals with the x axis.

9.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

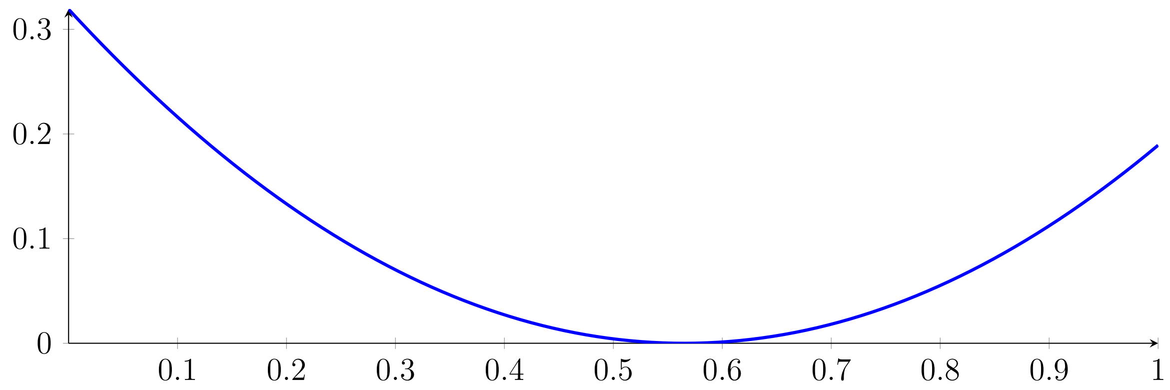
with precision $\varepsilon = 1 \cdot 10^{-08}$.

10 Running `BezClip` on h_2 with epsilon 16

$$1X^2 - 1.13X + 0.3192$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

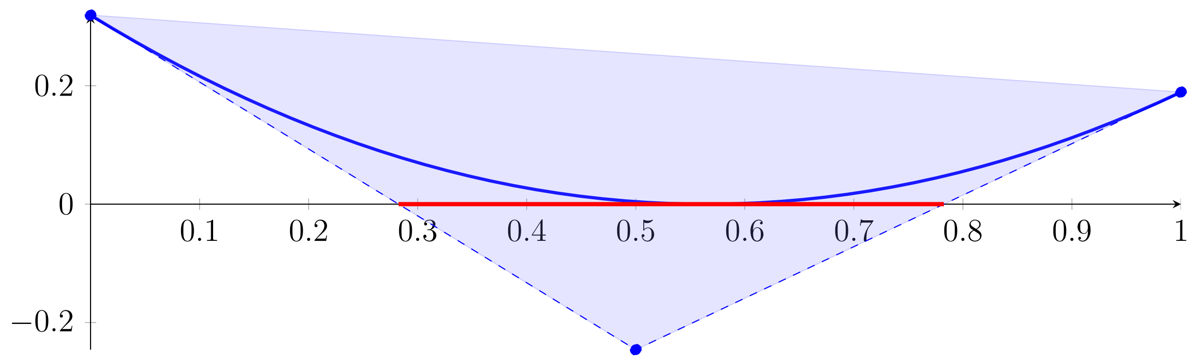
$$p = 1X^2 - 1.13X + 0.3192$$



10.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

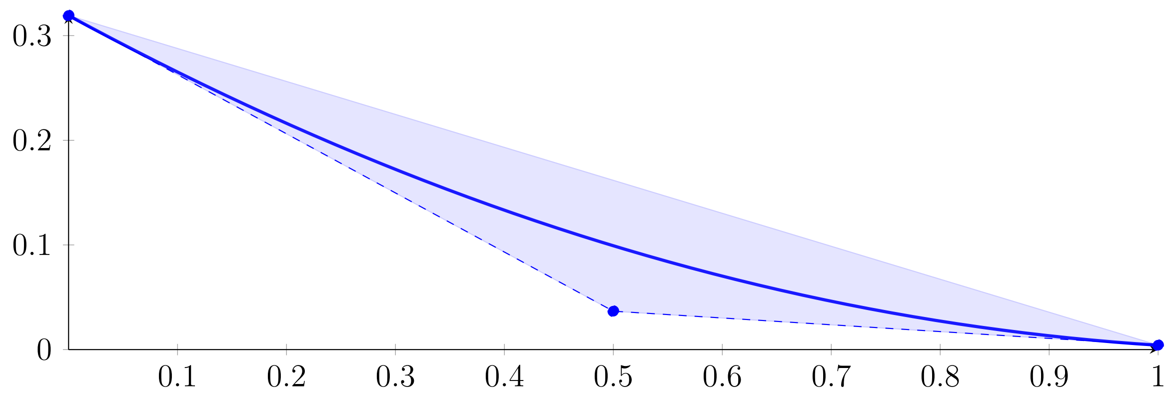
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

10.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

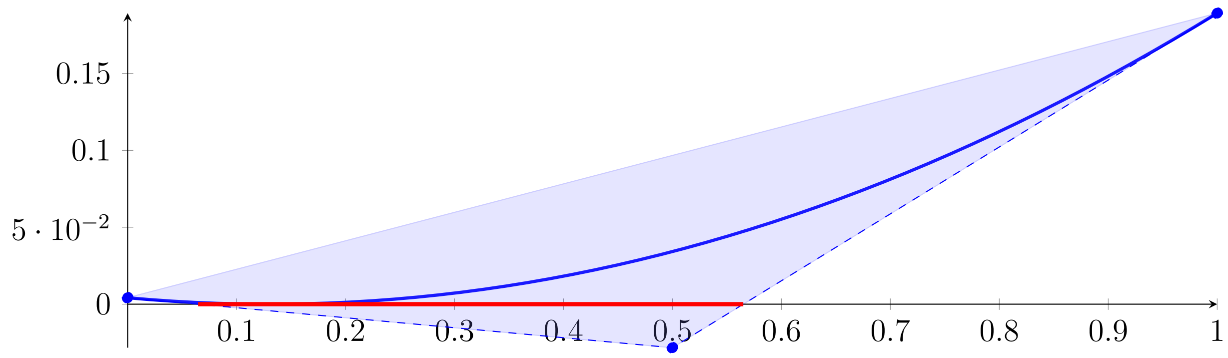
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

10.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.25X^2 - 0.065X + 0.004200000000000001\\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

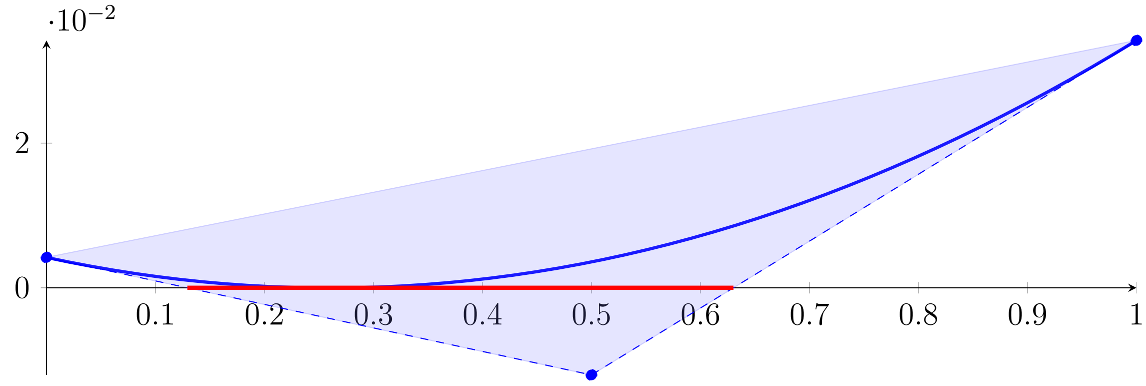
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\implies$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

10.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.0625X^2 - 0.0325X + 0.004200000000000001\\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

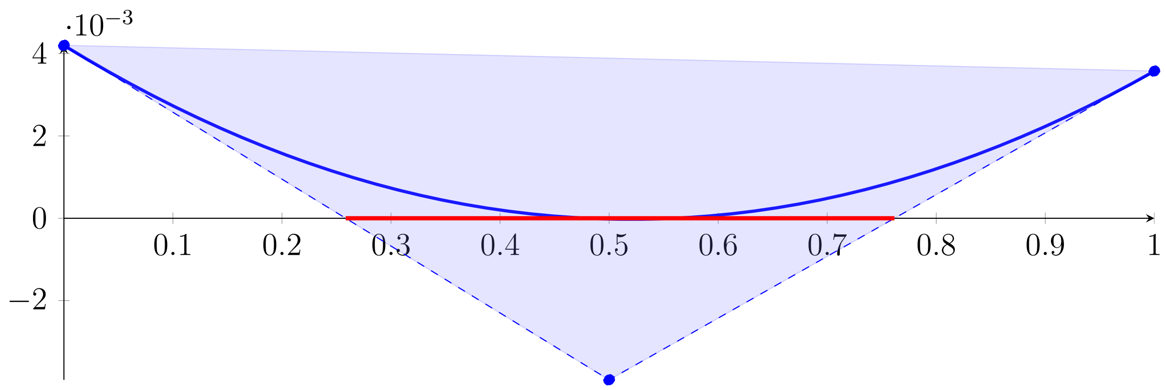
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

10.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

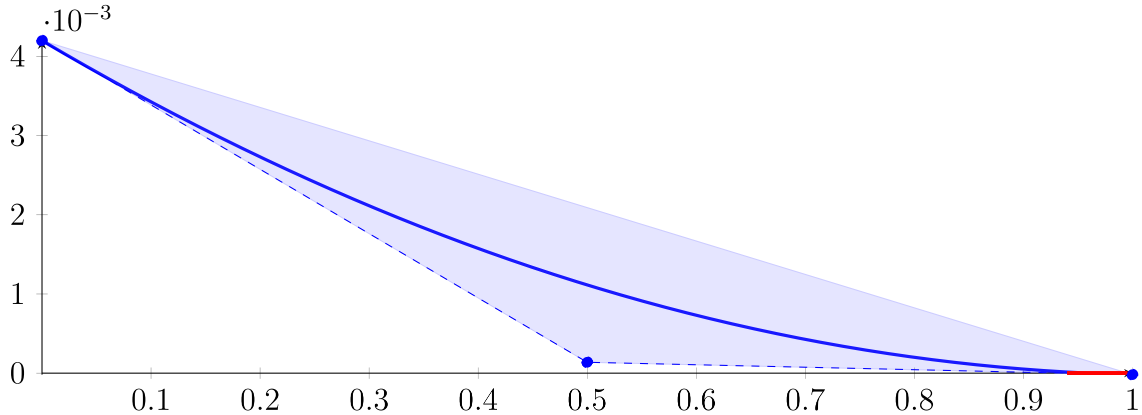
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

10.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001\\ &= 0.004200000000000001B_{0,2}(X) + 0.0001375000000000007B_{1,2}(X)\\ &\quad - 1.874999999999948\cdot 10^{-05}B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

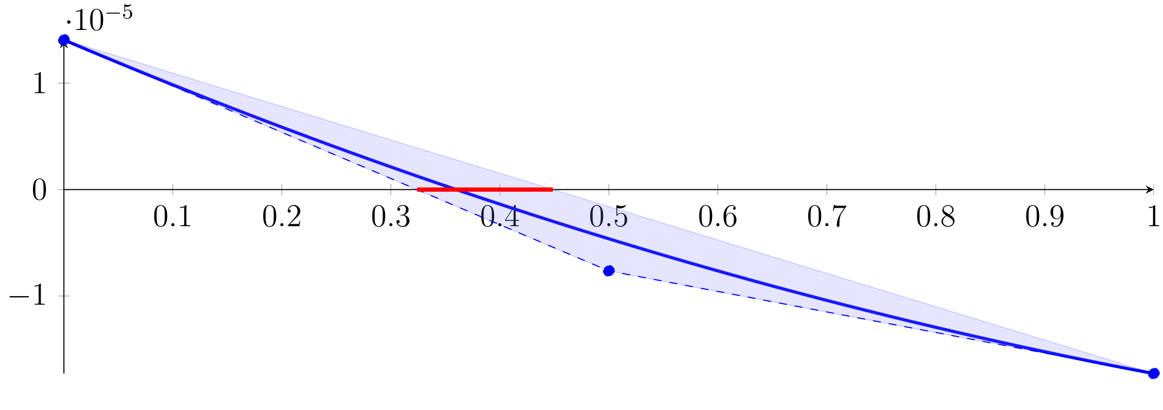
$$[0.9400000000000017, 0.9955555555555557]$$

Longest intersection interval: 0.05555555555555396
$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

10.7 Recursion Branch 1 2 1 1 1 in Interval 1: $[0.5587500000000001, 0.56222222222$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 1.205632716049313\cdot 10^{-05}X^2 - 4.340277777777758\cdot 10^{-05}X + 1.406249999999919\cdot 10^{-05}\\ &= 1.406249999999919\cdot 10^{-05}B_{0,2}(X) - 7.638888888888706\\ &\quad \cdot 10^{-06}B_{1,2}(X) - 1.728395061728347\cdot 10^{-05}B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3239999999999962, 0.4486153846153773\}$$

Intersection intervals with the x axis:

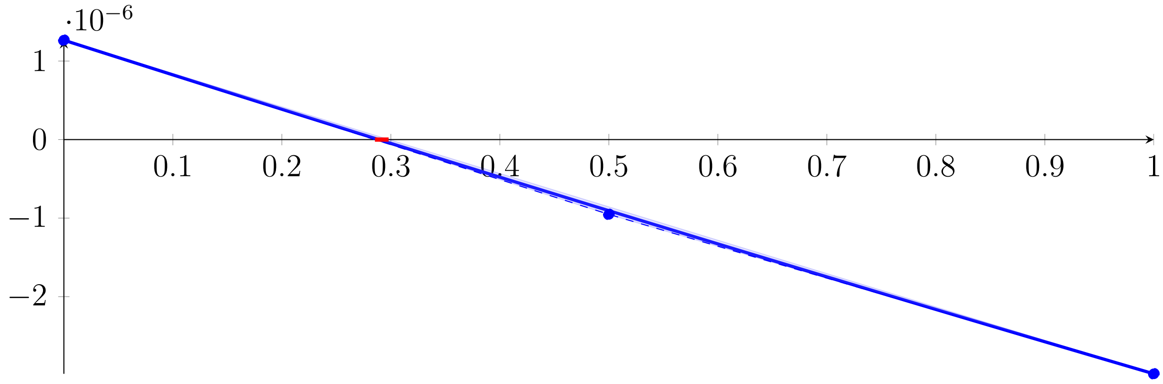
$$[0.3239999999999962, 0.4486153846153773]$$

Longest intersection interval: 0.1246153846153811
$\Longrightarrow$ Selective recursion: interval 1: $[0.5598750000000001, 0.5603076923076923]$,

10.8 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[0.5598750000000001, 0.560307692$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1.872226331360733 \cdot 10^{-07} X^2 - 4.435096153845849 \cdot 10^{-06} X + 1.265624999999897 \cdot 10^{-06}$$
$$= 1.265624999999897 \cdot 10^{-06} B_{0,2}(X) - 9.519230769230272 \cdot 10^{-07} B_{1,2}(X) - 2.982248520709878 \cdot 10^{-06} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2853658536585333, 0.2979431929480858\}$$

Intersection intervals with the x axis:

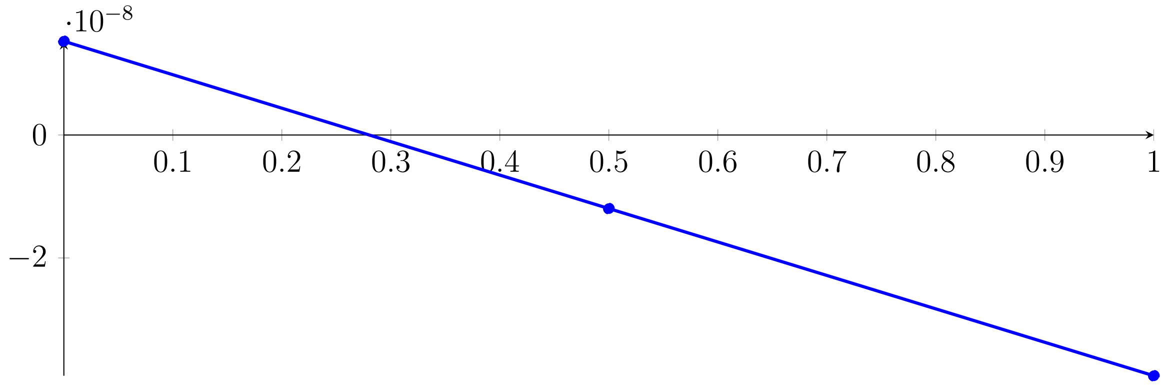
$$[0.2853658536585333, 0.2979431929480858]$$

Longest intersection interval: 0.01257733928955277
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599984756097562, 0.560003917727718]$,

10.9 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[0.5599984756097562, 0.5600039$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 2.961664791042273 \cdot 10^{-11} X^2 - 5.443777144130786 \cdot 10^{-08} X + 1.524622620463798 \cdot 10^{-08}$$
$$= 1.524622620463798 \cdot 10^{-08} B_{0,2}(X) - 1.197265951601595 \cdot 10^{-08} B_{1,2}(X) - 3.916192858875946 \cdot 10^{-08} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2800670527278236, 0.2802195050086158\}$$

Intersection intervals with the x axis:

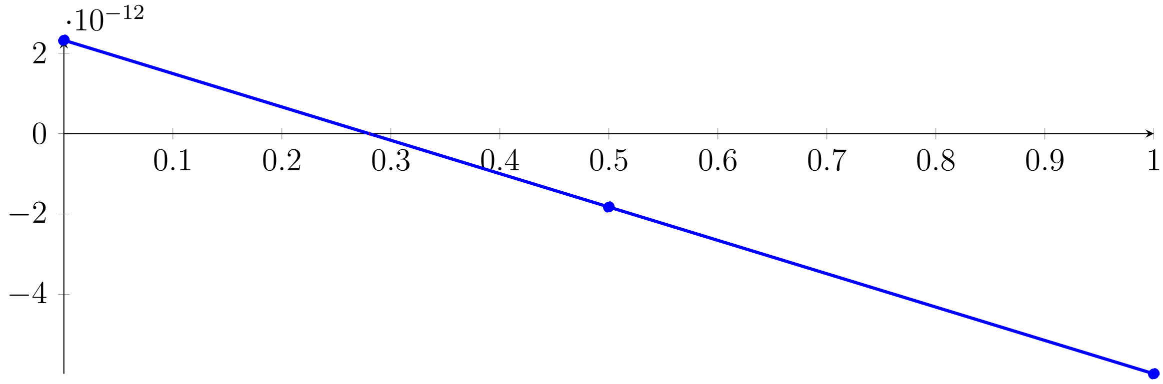
$$[0.2800670527278236, 0.2802195050086158]$$

Longest intersection interval: 0.0001524522807922228
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599999997676943, 0.5600000005973576]$,

10.10 Recursion Branch 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.5599999997676943, 0.5600$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.883411841000453 \cdot 10^{-19} X^2 - 8.296633341677063 \cdot 10^{-12} X + 2.323057420473189 \cdot 10^{-12}$$
$$= 2.323057420473189 \cdot 10^{-12} B_{0,2}(X) - 1.825259250365342$$
$$\cdot 10^{-12} B_{1,2}(X) - 5.97357523286269 \cdot 10^{-12} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2800000102214486, 0.2800000334520226\}$$

Intersection intervals with the x axis:

$$[0.2800000102214486, 0.2800000334520226]$$

Longest intersection interval: $2.323057397344962 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.56, 0.5600000000000001]$,

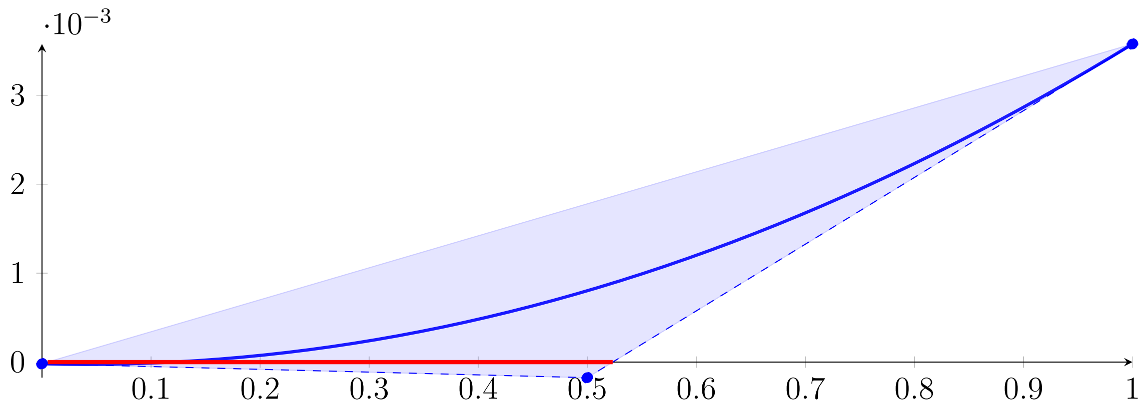
10.11 Recursion Branch 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.56, 0.5600000000000001]$

Found root in interval $[0.56, 0.5600000000000001]$ at recursion depth 10!

10.12 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.00390625 X^2 - 0.0003125000000000003 X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 0.0001749999999999996 B_{1,2}(X) + 0.003575 B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

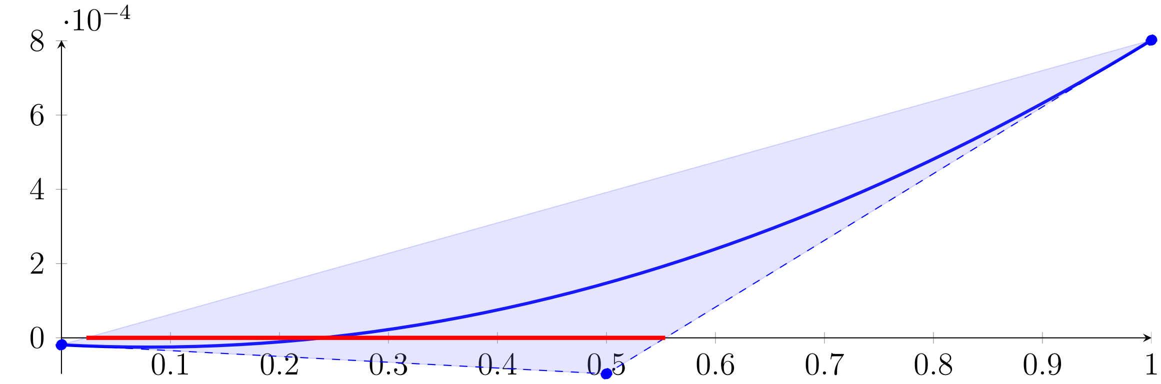
$$[0.005217391304347681, 0.5233333333333333]$$

Longest intersection interval: 0.5181159420289856
$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

10.13 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 - 0.00015625000000000001X - 1.874999999999948\cdot 10^{-05} \\ &= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 9.687499999999955 \\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0008015625000000004B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142222, 0.5539130434782606\}$$

Intersection intervals with the x axis:

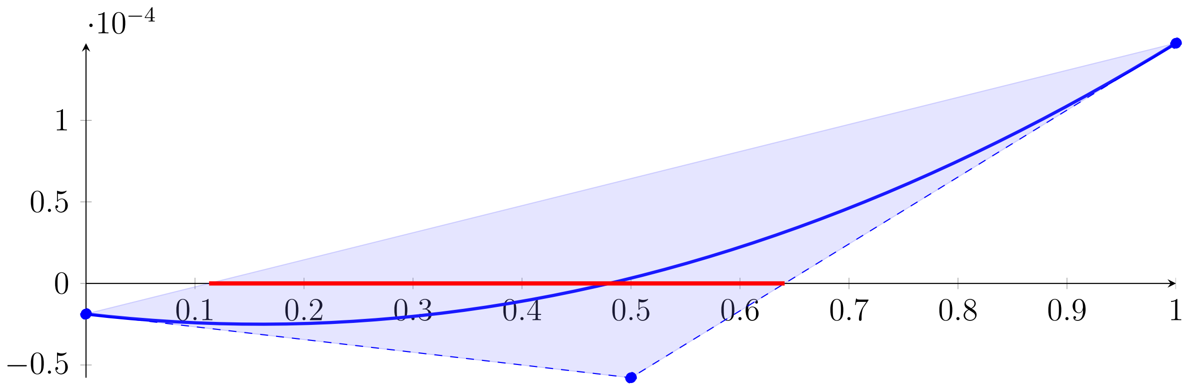
$$[0.022857142857142222, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\Longrightarrow$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

10.14 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 - 7.812500000000007\cdot 10^{-05}X - 1.874999999999948\cdot 10^{-05} \\ &= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 5.781249999999951 \\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0001472656250000005B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

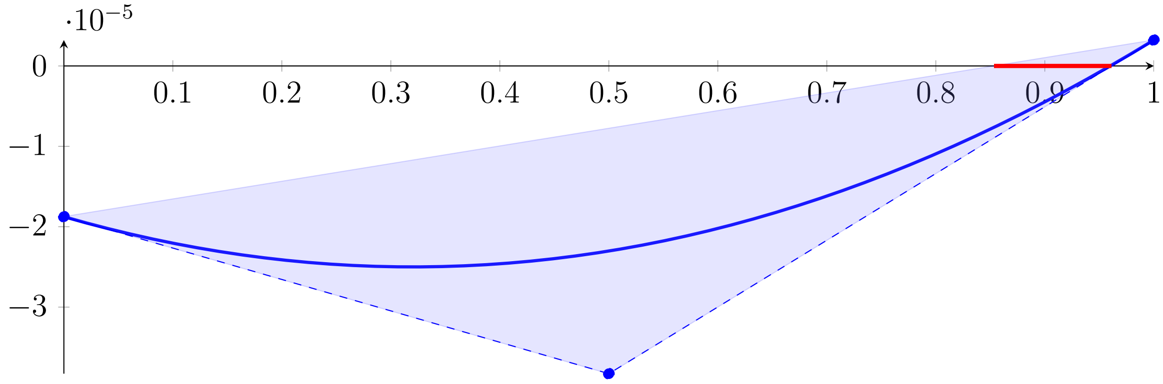
$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\Longrightarrow$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

10.15 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.103515625 \cdot 10^{-05} X^2 - 3.906250000000003 \cdot 10^{-05} X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 3.82812499999995$$
$$\cdot 10^{-05} B_{1,2}(X) + 3.222656250000487 \cdot 10^{-06} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.8533333333333109, 0.9611764705882294\}$$

Intersection intervals with the x axis:

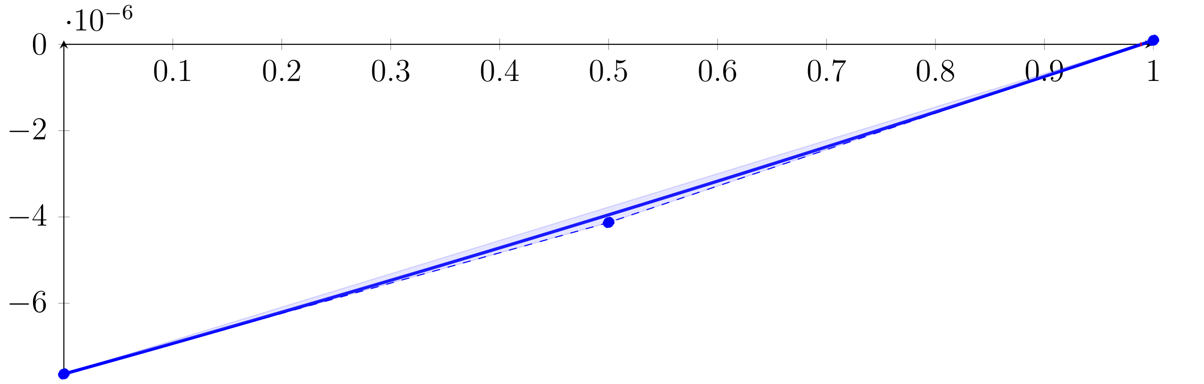
$$[0.8533333333333109, 0.9611764705882294]$$

Longest intersection interval: 0.1078431372549185
$\Longrightarrow$ Selective recursion: interval 1: $[0.5691666666666665, 0.5700091911764705]$,

10.16 Recursion Branch 1 2 1 1 2 1 1 1 1 in Interval 1: $[0.5691666666666665, 0.5700$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.098475496205558 \cdot 10^{-07} X^2 + 7.021037581700123 \cdot 10^{-06} X - 7.638888888889855 \cdot 10^{-06}$$
$$= -7.638888888889855 \cdot 10^{-06} B_{0,2}(X) - 4.128370098039794$$
$$\cdot 10^{-06} B_{1,2}(X) + 9.199624243082374 \cdot 10^{-08} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9881001669449061, 0.9891009174311909\}$$

Intersection intervals with the x axis:

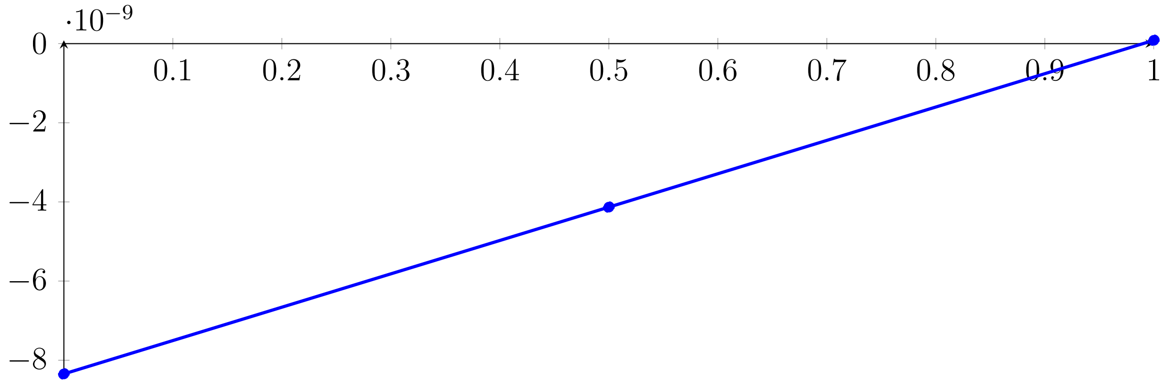
$$[0.9881001669449061, 0.9891009174311909]$$

Longest intersection interval: 0.001000750486284818
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999165275459, 0.5700000084322719]$,

10.17 Recursion Branch 1 2 1 1 2 1 1 1 1 1 in Interval 1: $[0.569999165275459, 0.5700$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.109134111283559 \cdot 10^{-13} X^2 + 8.430160521565627 \cdot 10^{-09} X - 8.346548643959838 \cdot 10^{-09}$$
$$= -8.346548643959838 \cdot 10^{-09} B_{0,2}(X) - 4.131468383177024$$
$$\cdot 10^{-09} B_{1,2}(X) + 8.432279101691722 \cdot 10^{-11} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.989998330342568, 0.989999173638737\}$$

Intersection intervals with the x axis:

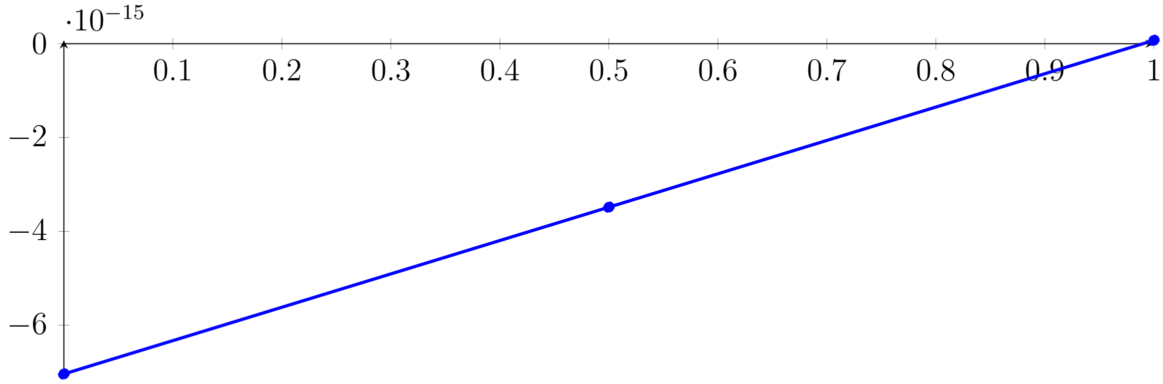
$$[0.989998330342568, 0.989999173638737]$$

Longest intersection interval: $8.432961690166558 \cdot 10^{-07}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999999999296, 0.5700000000000071]$,

10.18 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.569999999999296, 0.57$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 5.055649552501669 \cdot 10^{-25} X^2 + 7.110309100930884 \cdot 10^{-15} X - 7.039206010412064 \cdot 10^{-15}$$
$$= -7.039206010412064 \cdot 10^{-15} B_{0,2}(X) - 3.484051459946622$$
$$\cdot 10^{-15} B_{1,2}(X) + 7.1103091024385 \cdot 10^{-17} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999985907, 0.9899999999993017\}$$

Intersection intervals with the x axis:

$$[0.9899999999985907, 0.9899999999993017]$$

Longest intersection interval: $7.110309102923987 \cdot 10^{-13}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

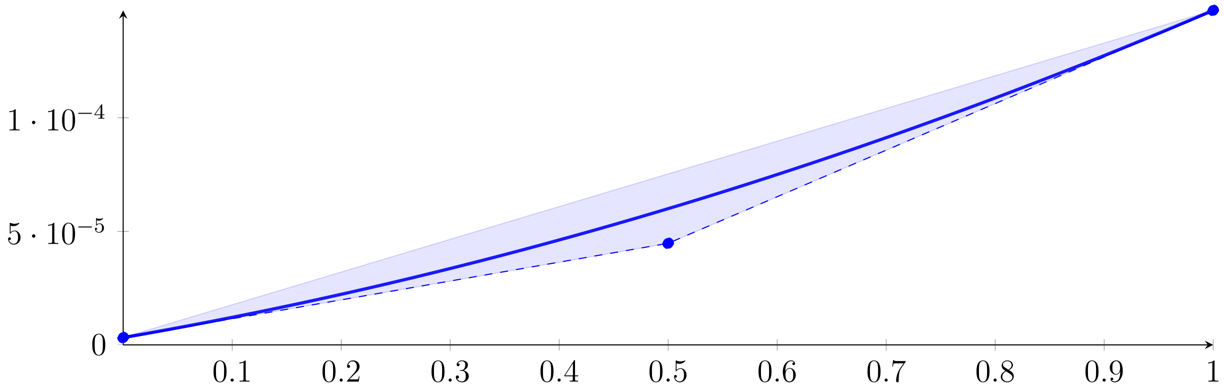
10.19 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 12!

10.20 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625 \cdot 10^{-05} X^2 + 8.300781249999997 \cdot 10^{-05} X + 3.222656250000487 \cdot 10^{-06} \\ &= 3.222656250000487 \cdot 10^{-06} B_{0,2}(X) + 4.472656250000047 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

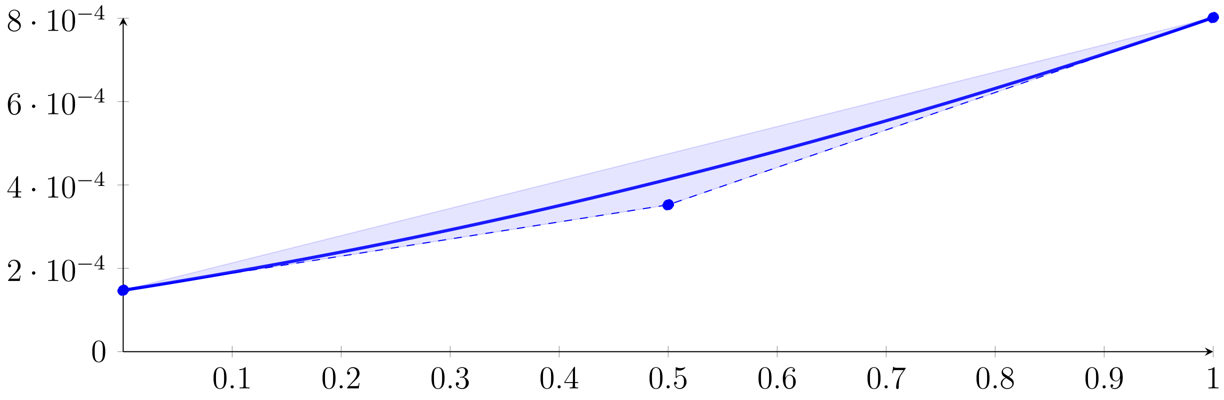
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

10.21 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625 X^2 + 0.0004101562499999999 X + 0.0001472656250000005 \\ &= 0.0001472656250000005 B_{0,2}(X) + 0.0003523437500000004 B_{1,2}(X) \\ &\quad + 0.0008015625000000004 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

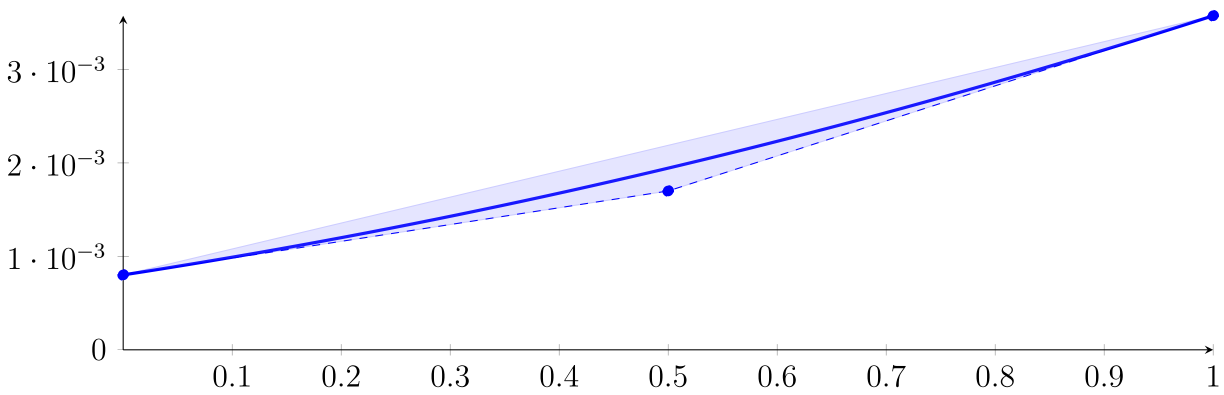
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

10.22 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 + 0.001796875X + 0.0008015625000000004 \\ &= 0.0008015625000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

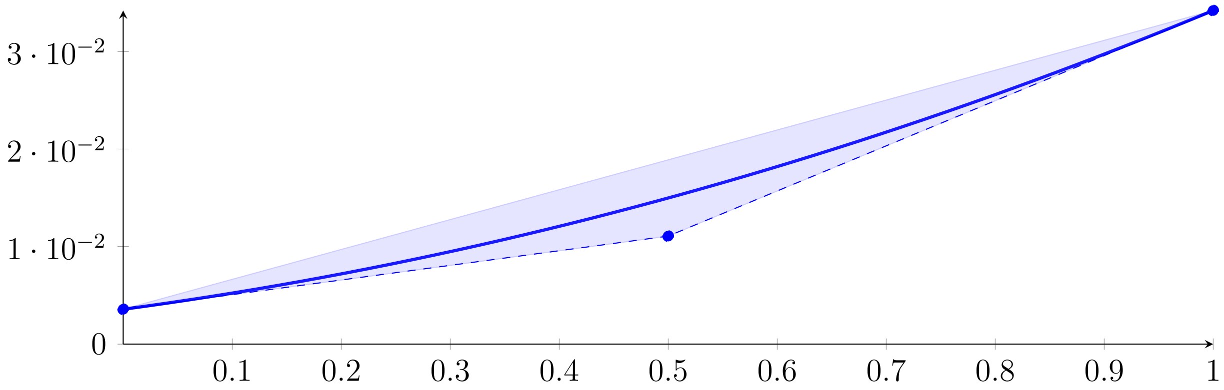
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

10.23 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

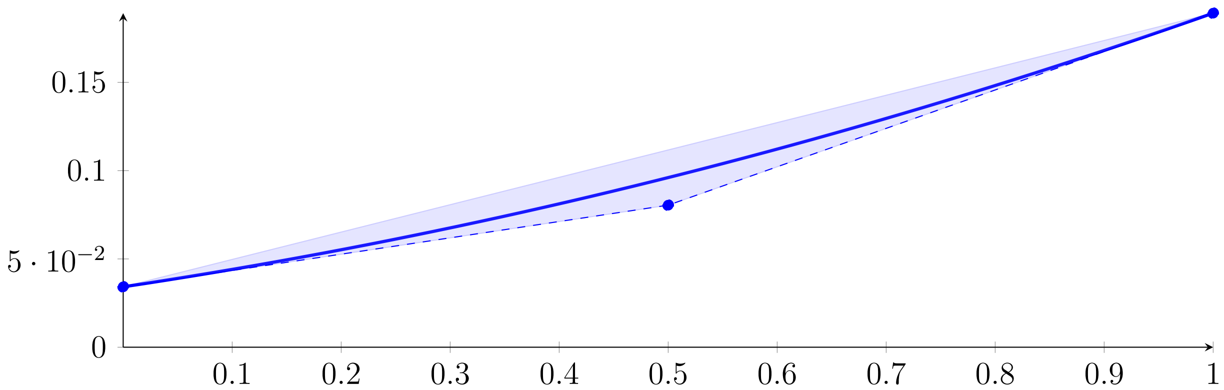
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

10.24 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

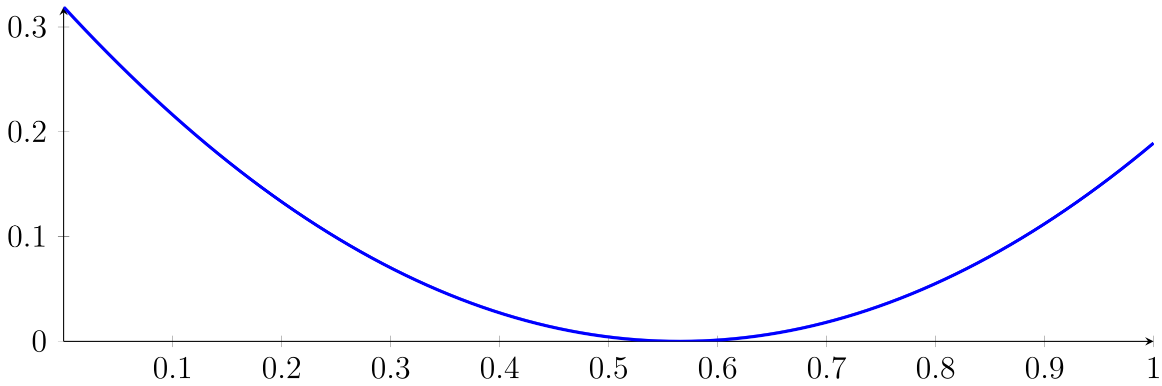
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

10.25 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.56, 0.5600000000000001],\ [0.57, 0.57]$$

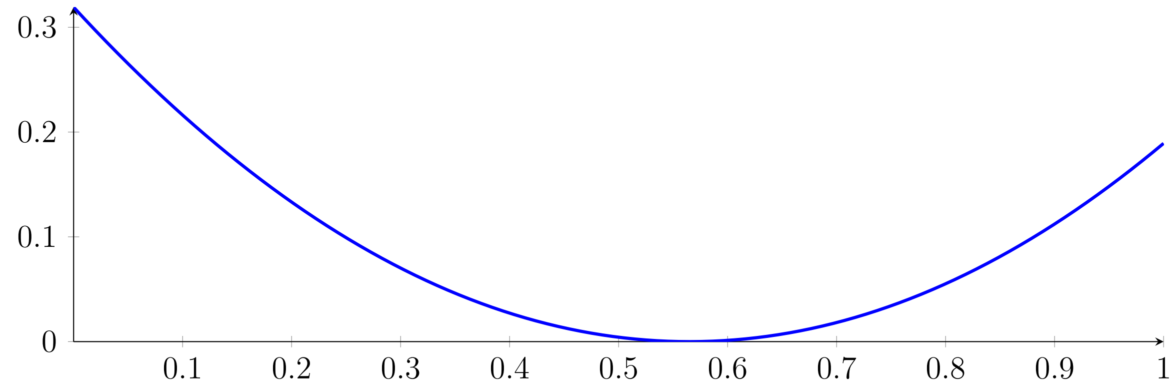
with precision $\varepsilon = 1 \cdot 10^{-16}$.

11 Running `QuadClip` on h_2 with epsilon 16

$$1X^2 - 1.13X + 0.3192$$

Called `QuadClip` with input polynomial on interval $[0,1]$:

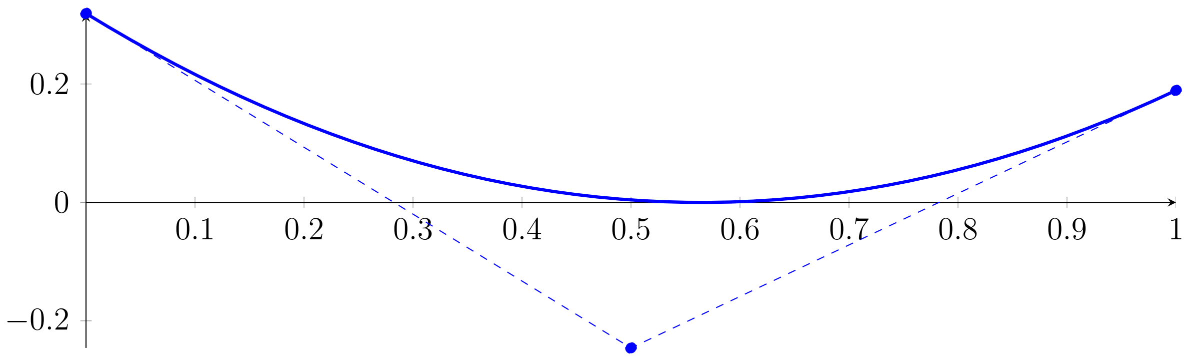
$$p = 1X^2 - 1.13X + 0.3192$$



11.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

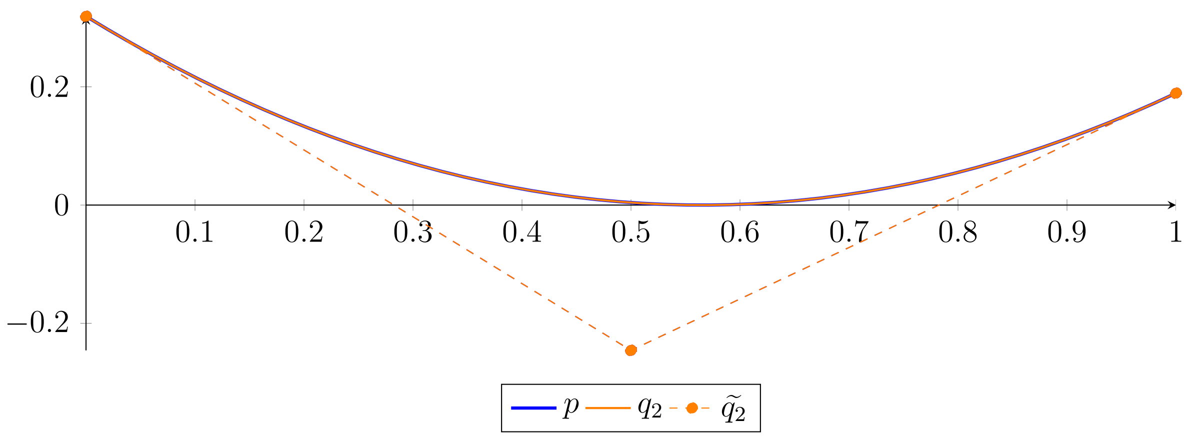
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

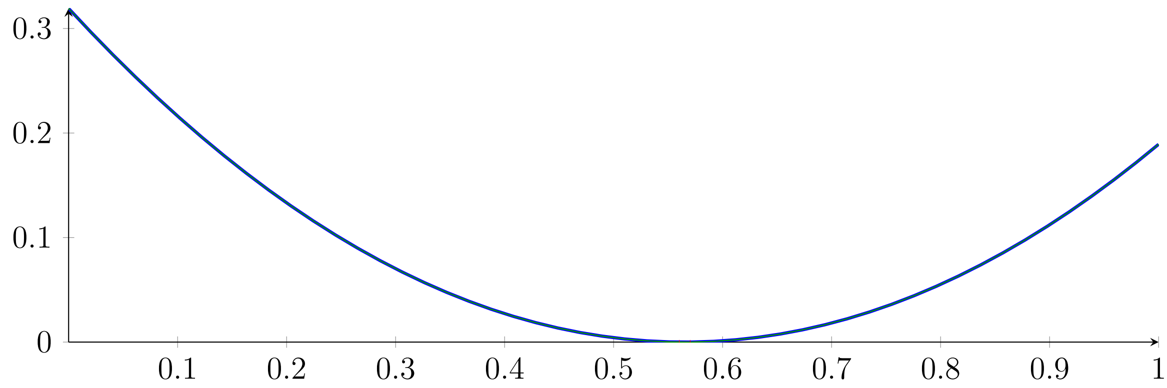
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

11.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

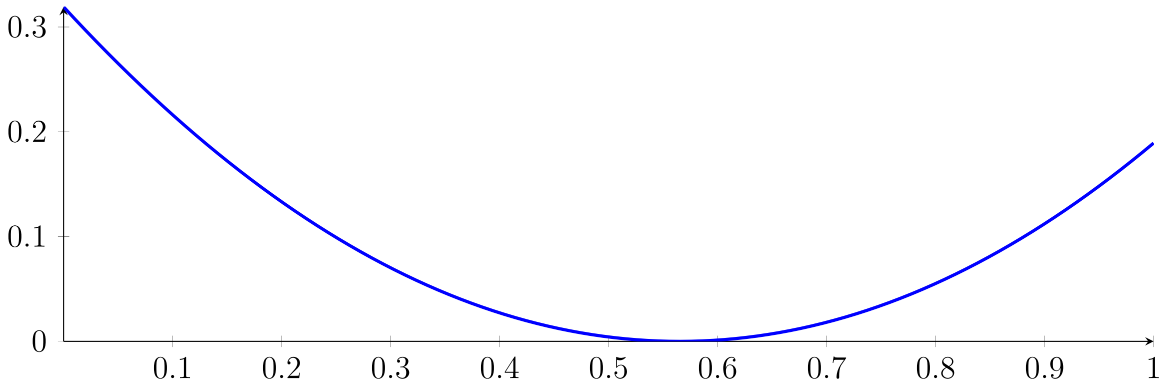
11.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

11.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001], [0.57, 0.57]$$

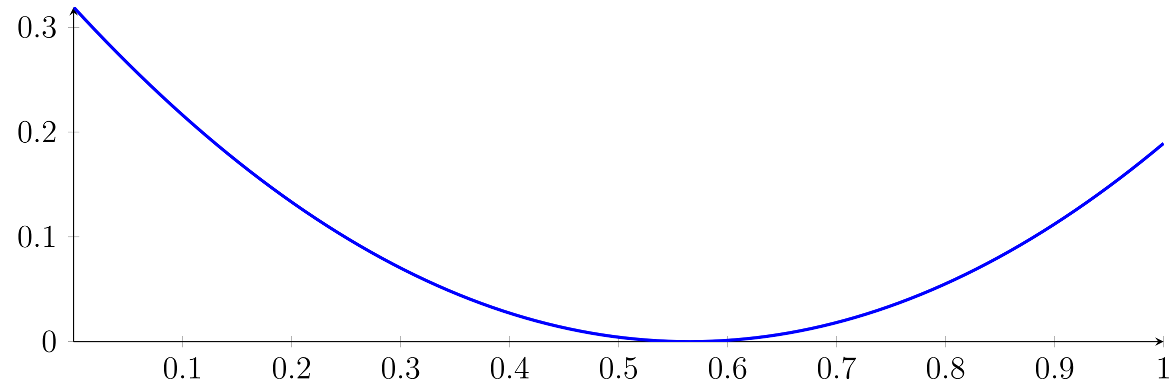
with precision $\varepsilon = 1 \cdot 10^{-16}$.

12 Running CubeClip on h_2 with epsilon 16

$$1X^2 - 1.13X + 0.3192$$

Called CubeClip with input polynomial on interval $[0, 1]$:

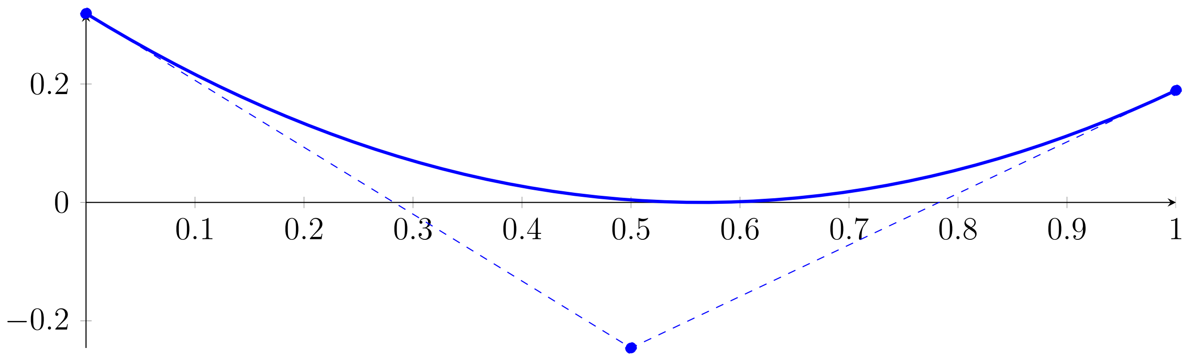
$$p = 1X^2 - 1.13X + 0.3192$$



12.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

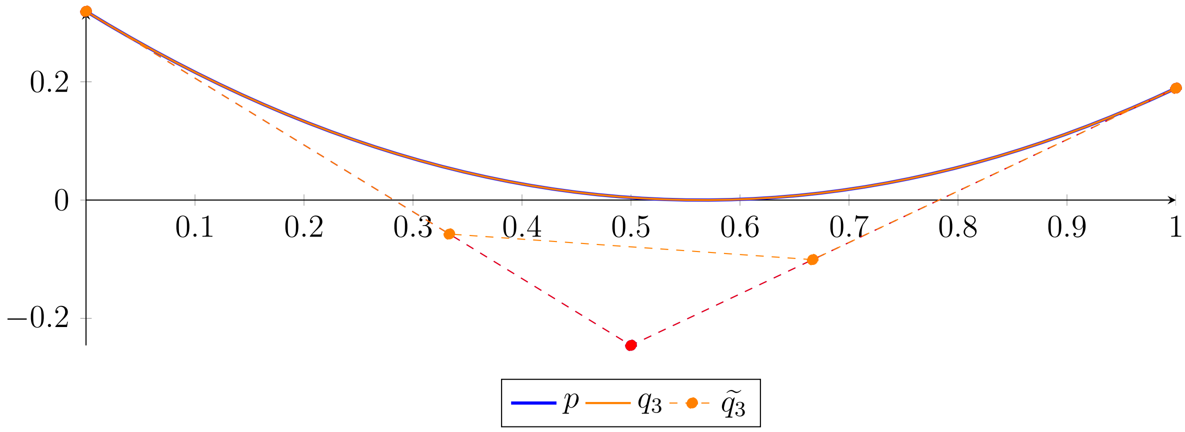
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308}X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501 \cdot 10^{-308}$.
Bounding polynomials M and m:

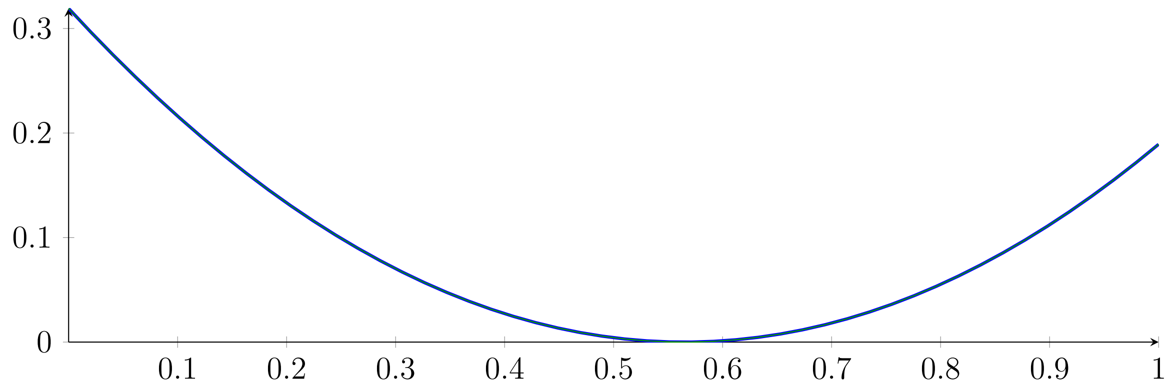
$$M = -1.668805393880401 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783 \cdot 10^{291}, 2.824576038015892 \cdot 10^{291}, 5.992310449541053 \cdot 10^{307}\}$ $N(m) = \{-4.84$

Intersection intervals:

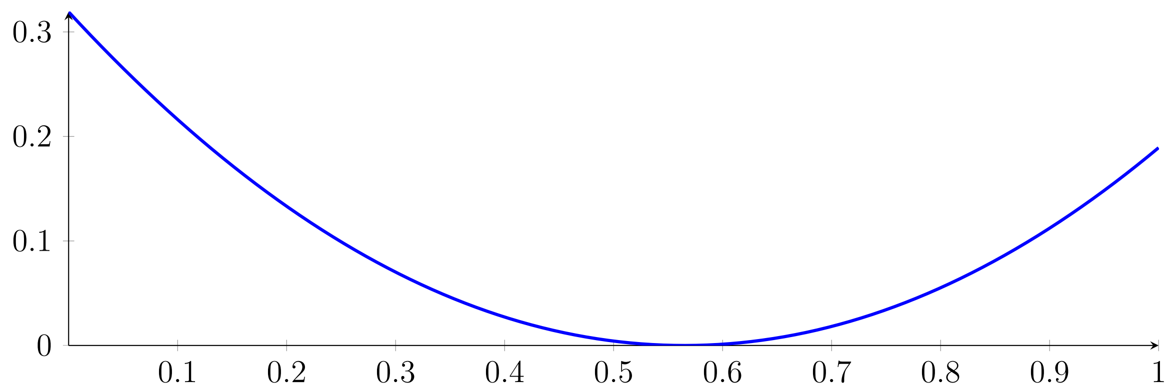


No intersection intervals with the x axis.

12.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

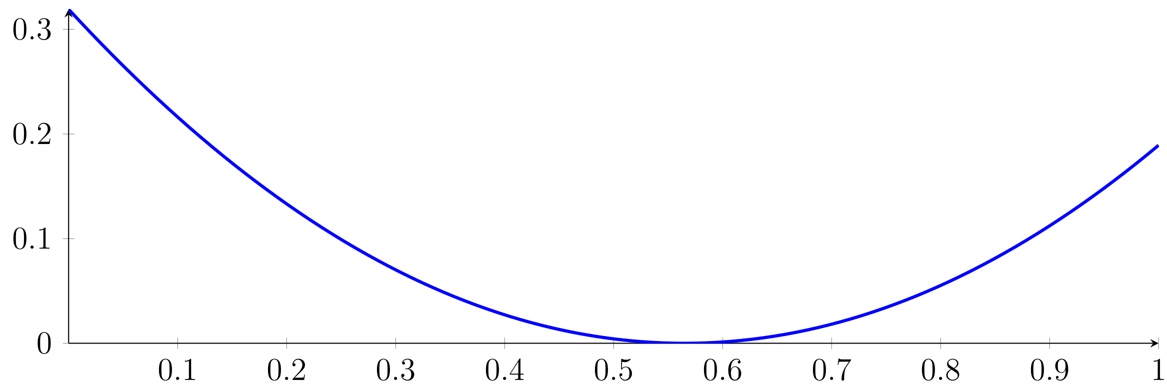
with precision $\varepsilon = 1 \cdot 10^{-16}$.

13 Running `BezClip` on h_2 with epsilon 32

$$1X^2 - 1.13X + 0.3192$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

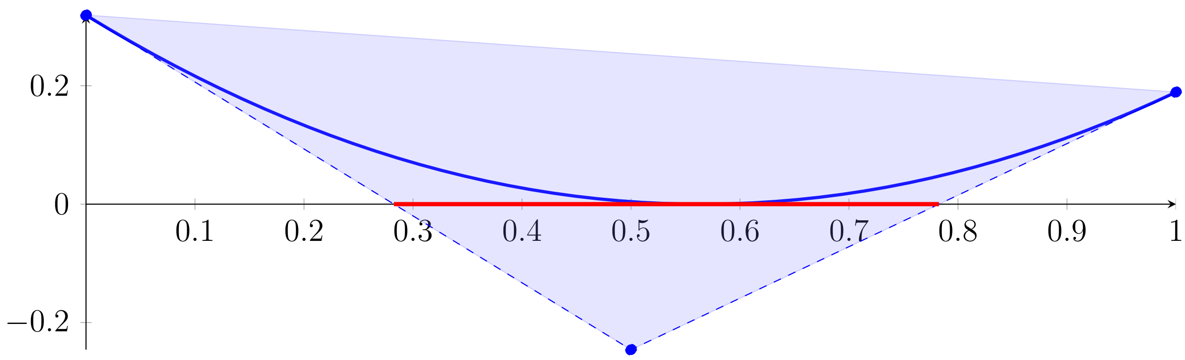
$$p = 1X^2 - 1.13X + 0.3192$$



13.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

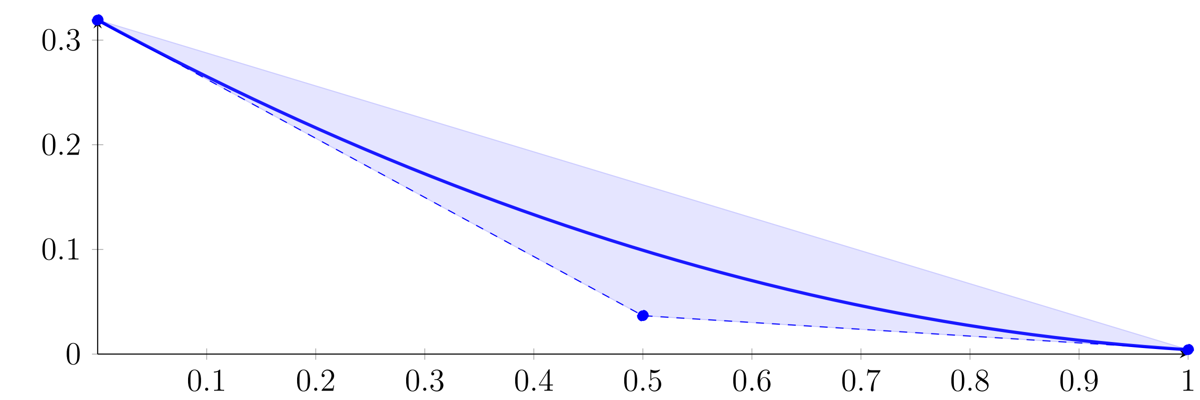
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

13.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

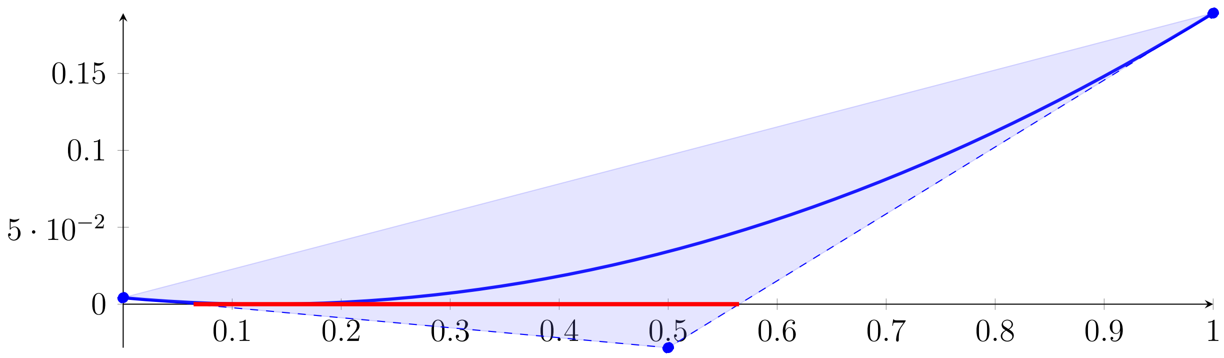
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

13.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.065X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

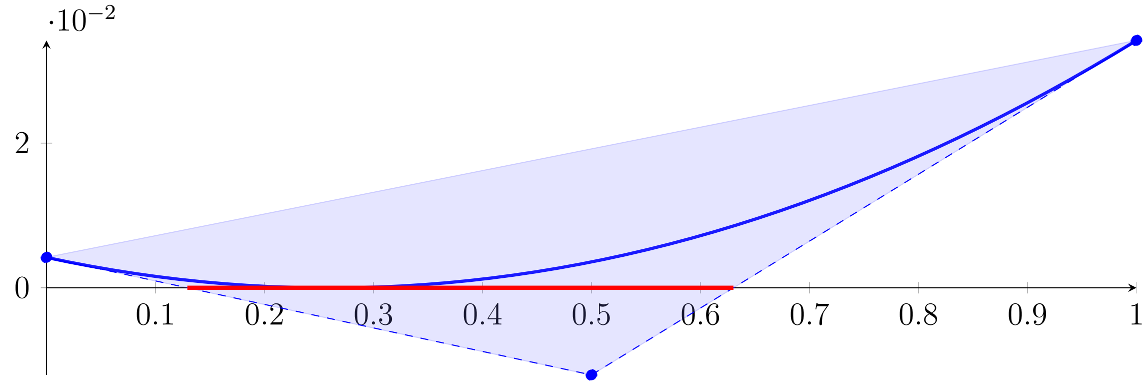
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\Longrightarrow$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

13.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 - 0.0325X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

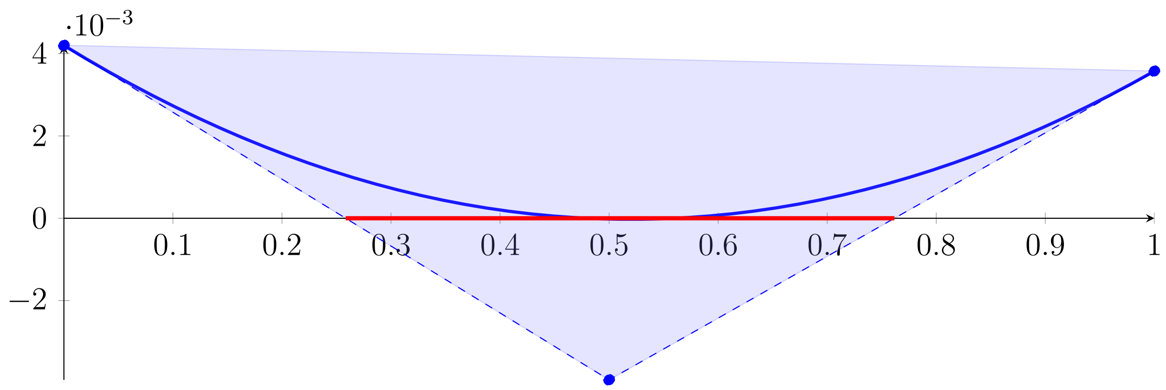
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

13.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

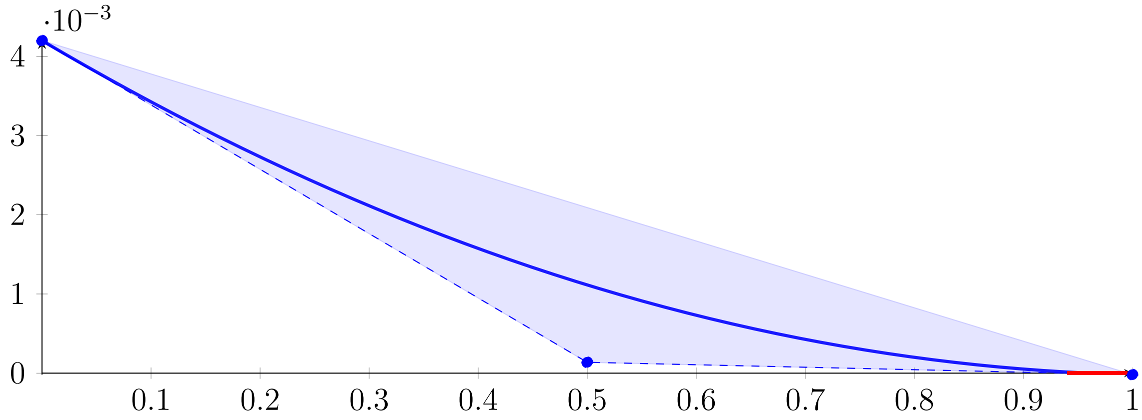
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

13.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) + 0.0001375000000000007B_{1,2}(X) \\ &\quad - 1.874999999999948 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

$$[0.9400000000000017, 0.9955555555555557]$$

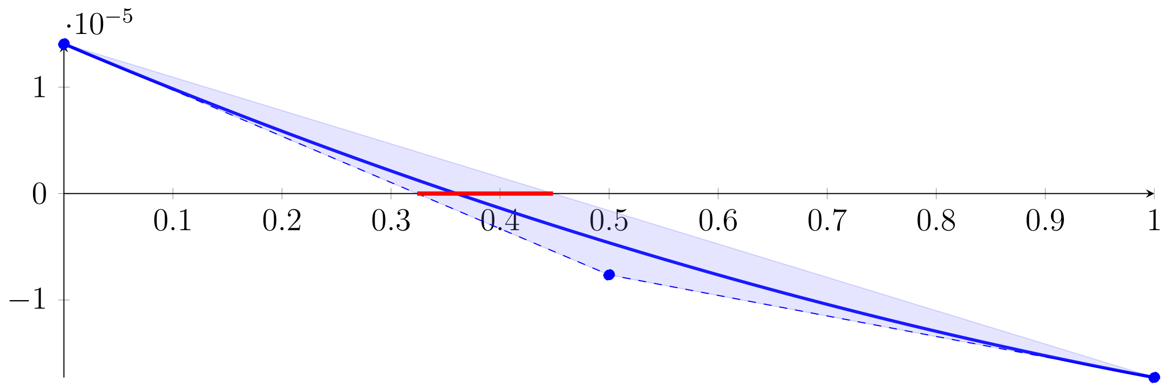
Longest intersection interval: 0.05555555555555396

$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

13.7 Recursion Branch 1 2 1 1 1 in Interval 1: $[0.5587500000000001, 0.562222222222$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.205632716049313 \cdot 10^{-05}X^2 - 4.340277777777758 \cdot 10^{-05}X + 1.406249999999919 \cdot 10^{-05} \\ &= 1.406249999999919 \cdot 10^{-05}B_{0,2}(X) - 7.638888888888706 \\ &\quad \cdot 10^{-06}B_{1,2}(X) - 1.728395061728347 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3239999999999962, 0.4486153846153773\}$$

Intersection intervals with the x axis:

$$[0.3239999999999962, 0.4486153846153773]$$

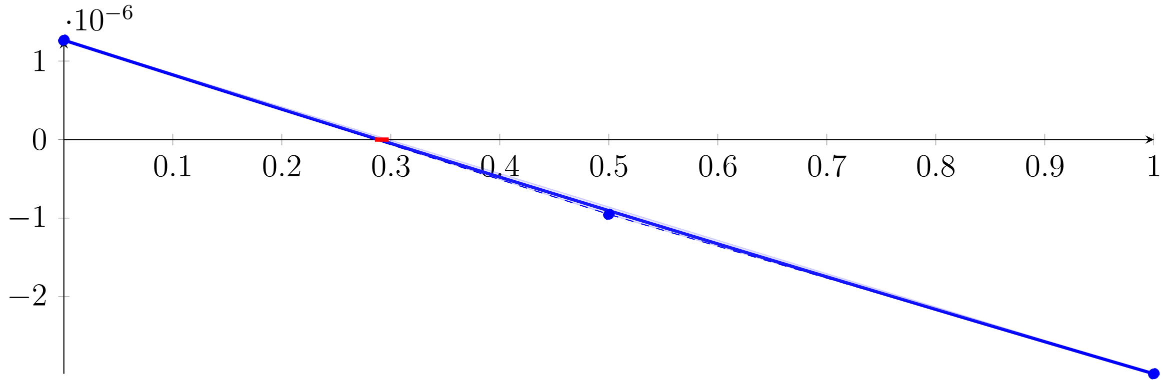
Longest intersection interval: 0.1246153846153811

$\Longrightarrow$ Selective recursion: interval 1: $[0.5598750000000001, 0.5603076923076923]$,

13.8 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[0.5598750000000001, 0.560307692$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.872226331360733 \cdot 10^{-07} X^2 - 4.435096153845849 \cdot 10^{-06} X + 1.265624999999897 \cdot 10^{-06} \\ &= 1.265624999999897 \cdot 10^{-06} B_{0,2}(X) - 9.519230769230272 \\ &\quad \cdot 10^{-07} B_{1,2}(X) - 2.982248520709878 \cdot 10^{-06} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2853658536585333, 0.2979431929480858\}$$

Intersection intervals with the x axis:

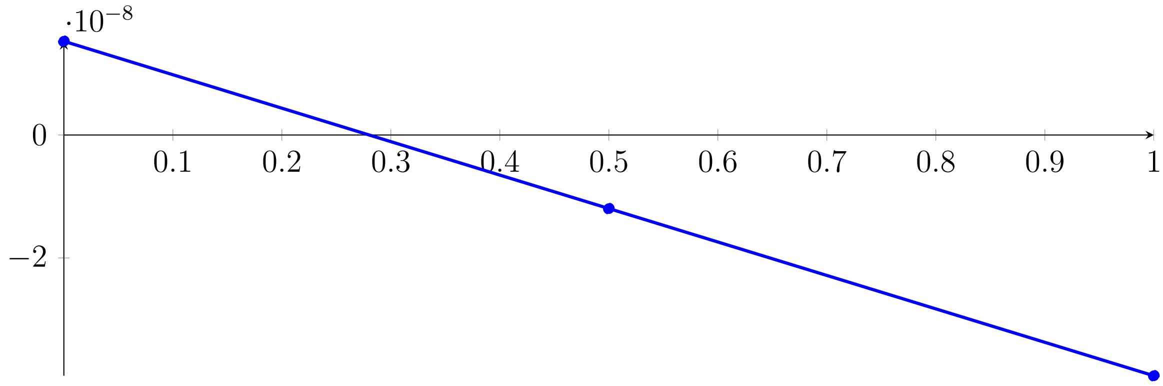
$$[0.2853658536585333, 0.2979431929480858]$$

Longest intersection interval: 0.01257733928955277
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599984756097562, 0.560003917727718]$,

13.9 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[0.5599984756097562, 0.5600039$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 2.961664791042273 \cdot 10^{-11} X^2 - 5.443777144130786 \cdot 10^{-08} X + 1.524622620463798 \cdot 10^{-08} \\ &= 1.524622620463798 \cdot 10^{-08} B_{0,2}(X) - 1.197265951601595 \\ &\quad \cdot 10^{-08} B_{1,2}(X) - 3.916192858875946 \cdot 10^{-08} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2800670527278236, 0.2802195050086158\}$$

Intersection intervals with the x axis:

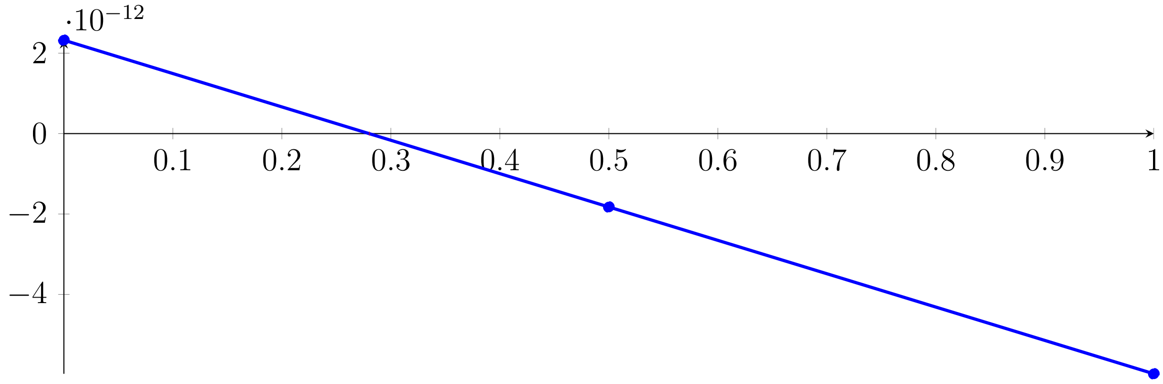
$$[0.2800670527278236, 0.2802195050086158]$$

Longest intersection interval: 0.0001524522807922228
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599999997676943, 0.5600000005973576]$,

13.10 Recursion Branch 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.5599999997676943, 0.5600$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.883411841000453 \cdot 10^{-19} X^2 - 8.296633341677063 \cdot 10^{-12} X + 2.323057420473189 \cdot 10^{-12}$$
$$= 2.323057420473189 \cdot 10^{-12} B_{0,2}(X) - 1.825259250365342$$
$$\cdot 10^{-12} B_{1,2}(X) - 5.97357523286269 \cdot 10^{-12} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2800000102214486, 0.2800000334520226\}$$

Intersection intervals with the x axis:

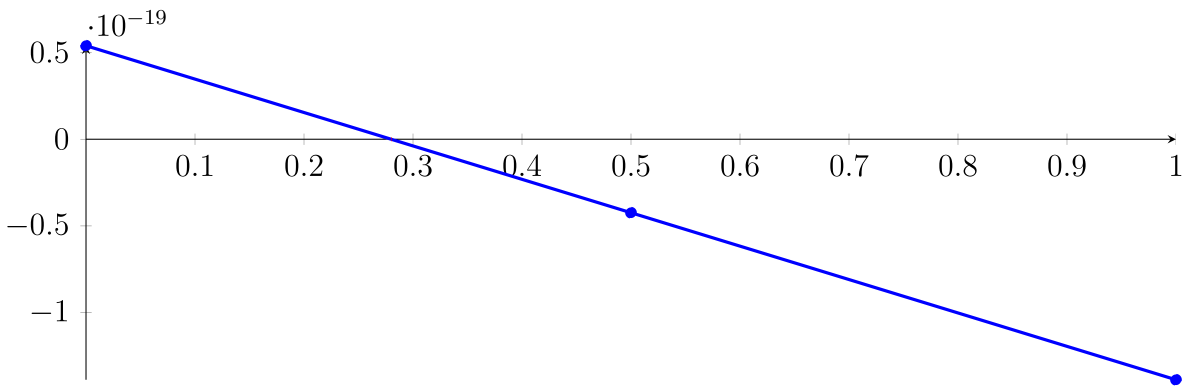
$$[0.2800000102214486, 0.2800000334520226]$$

Longest intersection interval: $2.323057397344962 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.56, 0.5600000000000001]$,

13.11 Recursion Branch 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.56, 0.5600000000000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 3.714699054532536 \cdot 10^{-34} X^2 - 1.927355456197032 \cdot 10^{-19} X + 5.396595277351629 \cdot 10^{-20}$$
$$= 5.396595277351629 \cdot 10^{-20} B_{0,2}(X) - 4.240182003633529$$
$$\cdot 10^{-20} B_{1,2}(X) - 1.387695928461865 \cdot 10^{-19} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999969, 0.2799999999999975\}$$

Intersection intervals with the x axis:

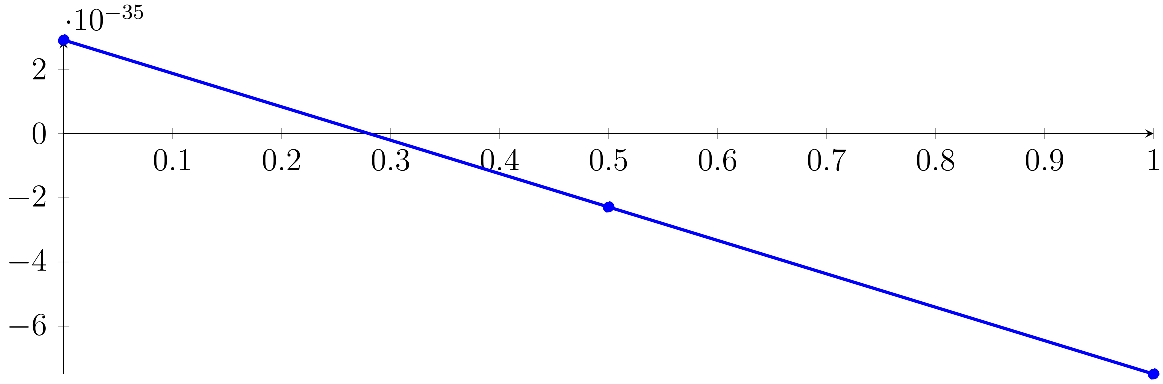
$$[0.2799999999999969, 0.2799999999999975]$$

Longest intersection interval: $5.396595277351738 \cdot 10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

13.12 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0.5$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.081840742754402 \cdot 10^{-64} X^2 - 1.040115735269099 \cdot 10^{-34} X + 2.912324058753444 \cdot 10^{-35} \\ &= 2.912324058753444 \cdot 10^{-35} B_{0,2}(X) - 2.288254617592053 \\ &\quad \cdot 10^{-35} B_{1,2}(X) - 7.488833293937551 \cdot 10^{-35} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999967, 0.2799999999999967\}$$

Intersection intervals with the x axis:

$$[0.2799999999999967, 0.2799999999999967]$$

Longest intersection interval: $2.912324058753504 \cdot 10^{-31}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

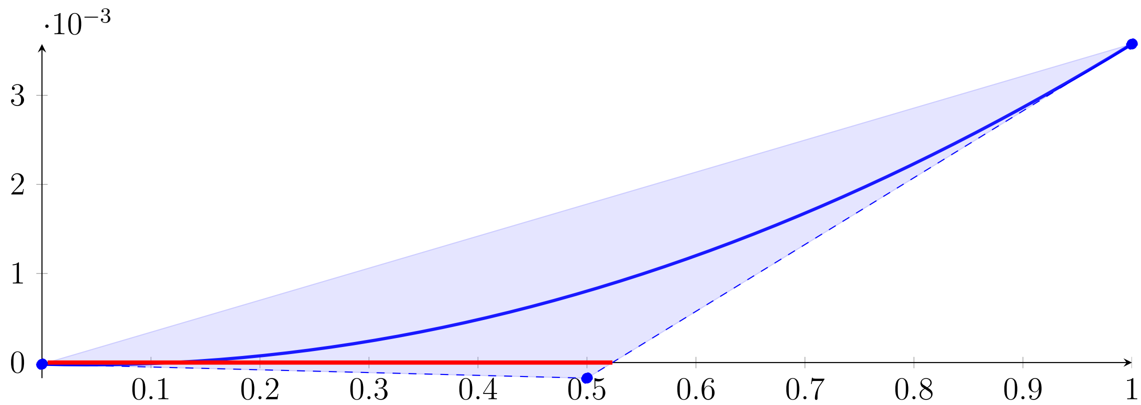
13.13 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 12!

13.14 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625 X^2 - 0.0003125000000000003 X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 0.0001749999999999996 B_{1,2}(X) + 0.003575 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

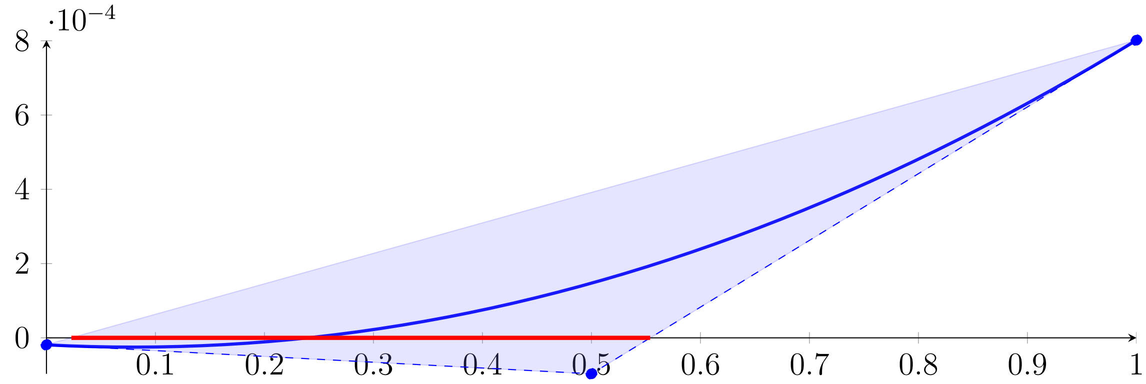
$$[0.005217391304347681, 0.5233333333333333]$$

Longest intersection interval: 0.5181159420289856
$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

13.15 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.0009765625X^2 - 0.00015625000000000001X - 1.874999999999948 \cdot 10^{-05}\\ &= -1.874999999999948 \cdot 10^{-05}B_{0,2}(X) - 9.687499999999955\\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0008015625000000004B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142222, 0.5539130434782606\}$$

Intersection intervals with the x axis:

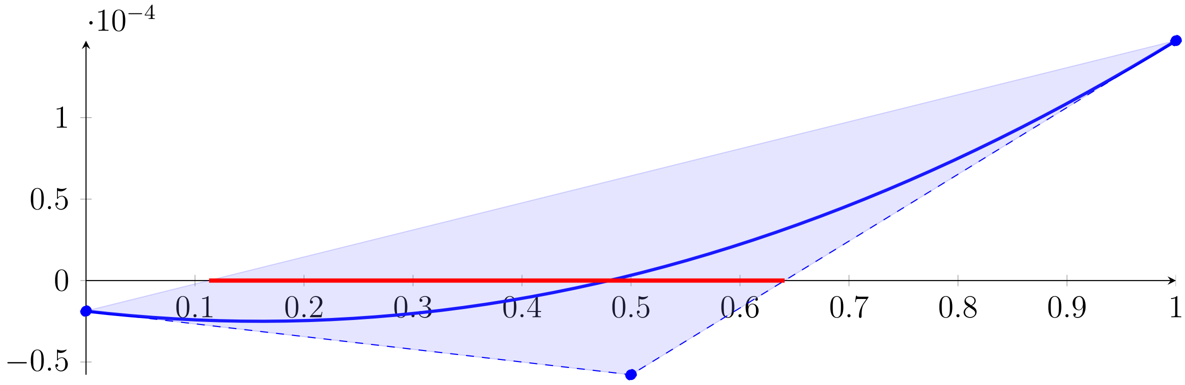
$$[0.022857142857142222, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\Longrightarrow$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

13.16 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.000244140625X^2 - 7.812500000000007 \cdot 10^{-05}X - 1.874999999999948 \cdot 10^{-05}\\ &= -1.874999999999948 \cdot 10^{-05}B_{0,2}(X) - 5.781249999999951\\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0001472656250000005B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

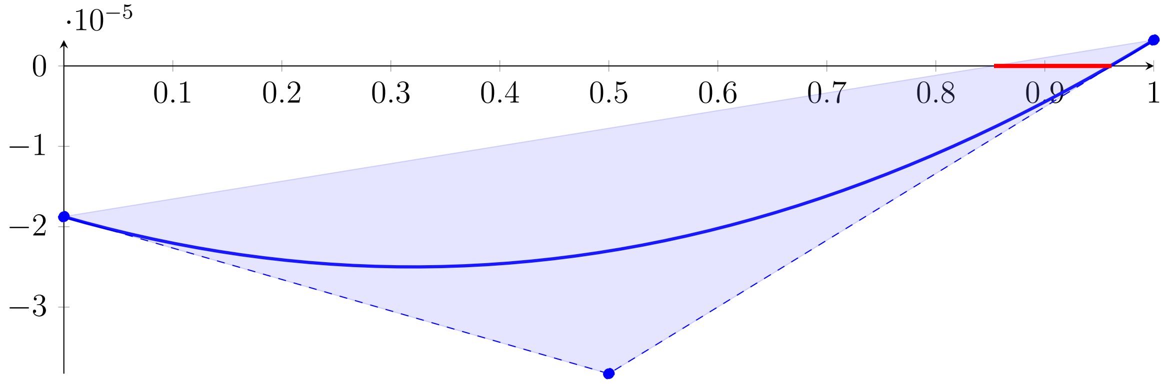
$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\Longrightarrow$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

13.17 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625 \cdot 10^{-05} X^2 - 3.906250000000003 \cdot 10^{-05} X - 1.874999999999948 \cdot 10^{-05} \\ &= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 3.82812499999995 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 3.222656250000487 \cdot 10^{-06} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8533333333333109, 0.9611764705882294\}$$

Intersection intervals with the x axis:

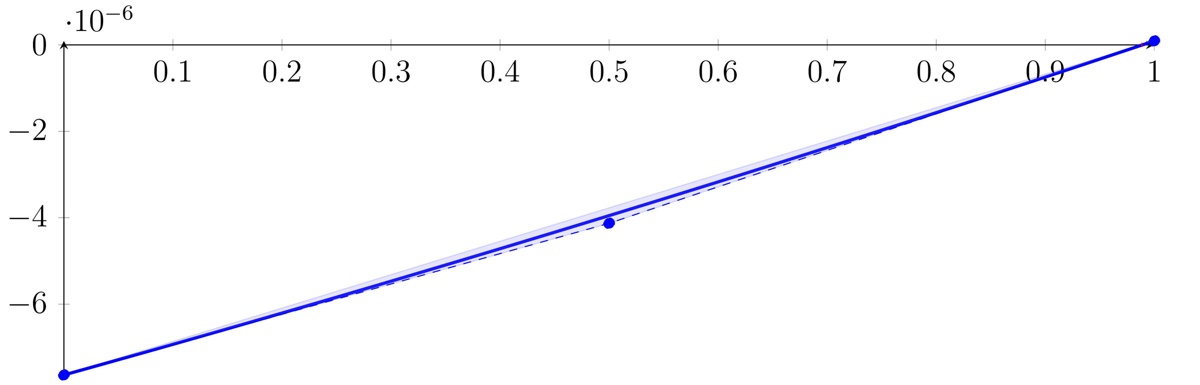
$$[0.8533333333333109, 0.9611764705882294]$$

Longest intersection interval: 0.1078431372549185
$\Longrightarrow$ Selective recursion: interval 1: $[0.5691666666666665, 0.5700091911764705]$,

13.18 Recursion Branch 1 2 1 1 2 1 1 1 1 in Interval 1: $[0.5691666666666665, 0.5700$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 7.098475496205558 \cdot 10^{-07} X^2 + 7.021037581700123 \cdot 10^{-06} X - 7.638888888889855 \cdot 10^{-06} \\ &= -7.638888888889855 \cdot 10^{-06} B_{0,2}(X) - 4.128370098039794 \\ &\quad \cdot 10^{-06} B_{1,2}(X) + 9.199624243082374 \cdot 10^{-08} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9881001669449061, 0.9891009174311909\}$$

Intersection intervals with the x axis:

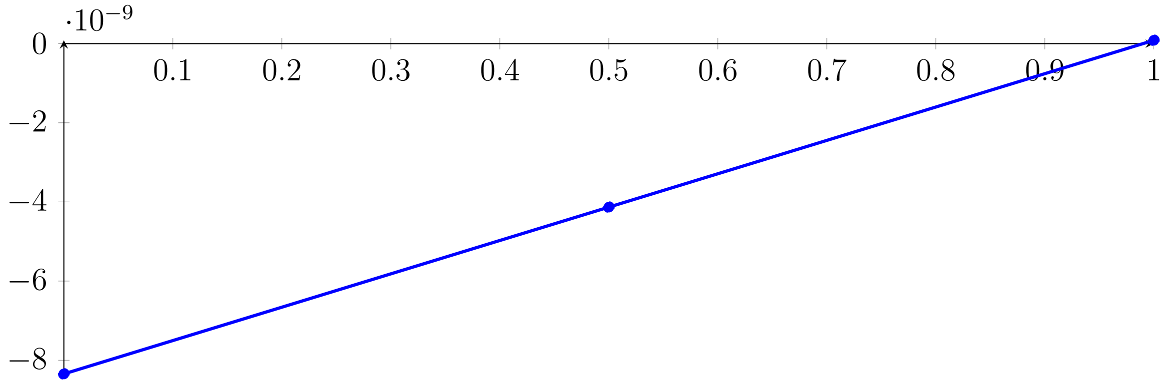
$$[0.9881001669449061, 0.9891009174311909]$$

Longest intersection interval: 0.001000750486284818
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999165275459, 0.5700000084322719]$,

13.19 Recursion Branch 1 2 1 1 2 1 1 1 1 1 in Interval 1: $[0.569999165275459, 0.570$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 7.109134111283559\cdot10^{-13}X^2 + 8.430160521565627\cdot10^{-09}X - 8.346548643959838\cdot10^{-09} \\ &= -8.346548643959838\cdot10^{-09}B_{0,2}(X) - 4.131468383177024 \\ &\quad\cdot10^{-09}B_{1,2}(X) + 8.432279101691722\cdot10^{-11}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.989998330342568, 0.989999173638737\}$$

Intersection intervals with the x axis:

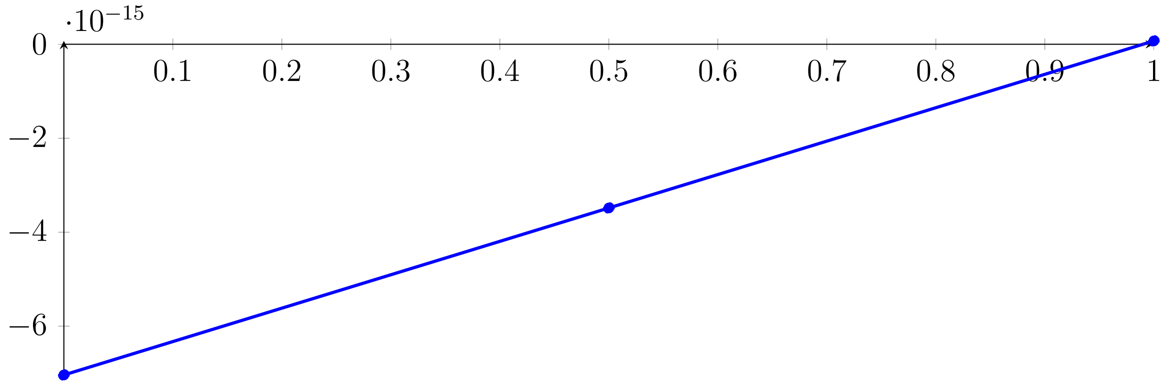
$$[0.989998330342568, 0.989999173638737]$$

Longest intersection interval: $8.432961690166558\cdot10^{-07}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999999999296, 0.5700000000000071]$,

13.20 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.569999999999296, 0.57$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 5.055649552501669\cdot10^{-25}X^2 + 7.110309100930884\cdot10^{-15}X - 7.039206010412064\cdot10^{-15} \\ &= -7.039206010412064\cdot10^{-15}B_{0,2}(X) - 3.484051459946622 \\ &\quad\cdot10^{-15}B_{1,2}(X) + 7.1103091024385\cdot10^{-17}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999985907, 0.9899999999993017\}$$

Intersection intervals with the x axis:

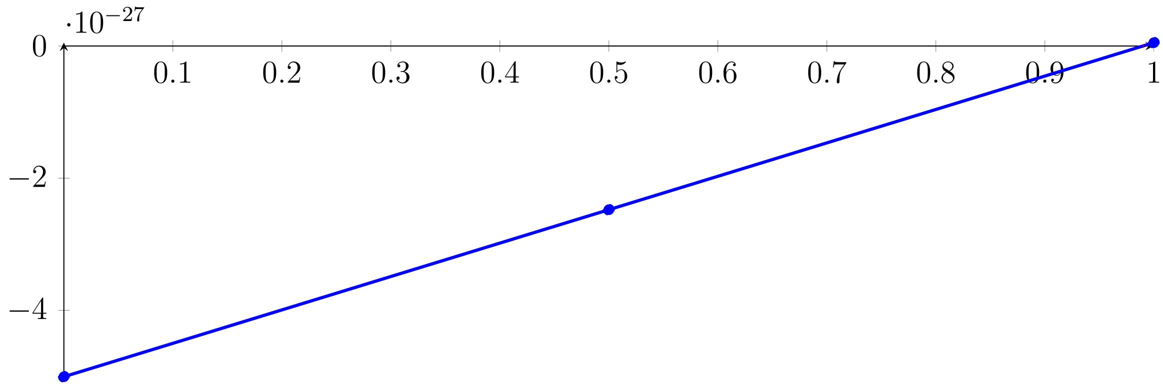
$$[0.9899999999985907, 0.9899999999993017]$$

Longest intersection interval: $7.110309102923987 \cdot 10^{-13}$
$\implies$ Selective recursion: interval 1: $[0.57, 0.57]$,

13.21 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 2.555959240484234 \cdot 10^{-49} X^2 + 5.055649553206969 \cdot 10^{-27} X - 5.005093057674892 \cdot 10^{-27} \\ &= -5.005093057674892 \cdot 10^{-27} B_{0,2}(X) - 2.477268281071407 \\ &\quad \cdot 10^{-27} B_{1,2}(X) + 5.055649553207703 \cdot 10^{-29} B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999999985, 0.9899999999999985\}$$

Intersection intervals with the x axis:

$$[0.9899999999999985, 0.9899999999999985]$$

Longest intersection interval: $5.055649553207806 \cdot 10^{-25}$
$\implies$ Selective recursion: interval 1: $[0.57, 0.57]$,

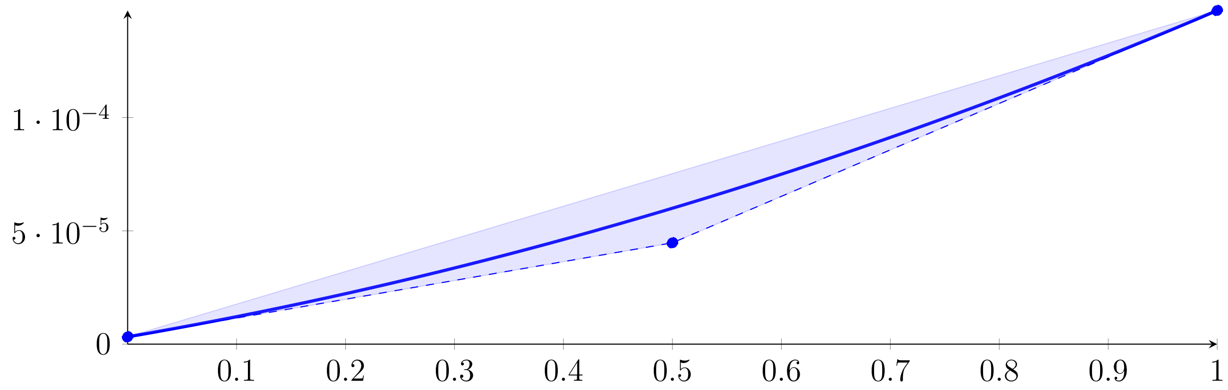
13.22 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 13!

13.23 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 6.103515625 \cdot 10^{-05} X^2 + 8.300781249999997 \cdot 10^{-05} X + 3.222656250000487 \cdot 10^{-06} \\ &= 3.222656250000487 \cdot 10^{-06} B_{0,2}(X) + 4.472656250000047 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

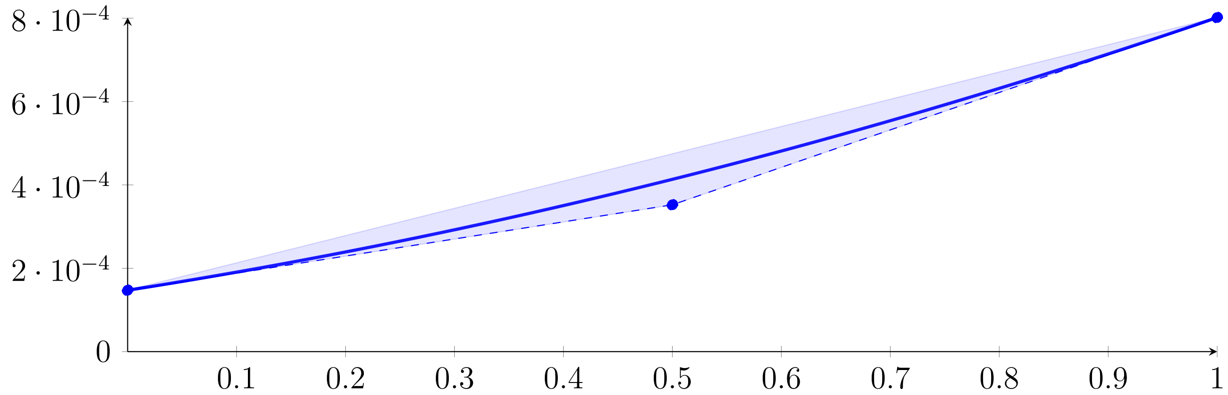
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

13.24 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.000244140625X^2 + 0.0004101562499999999X + 0.0001472656250000005\\ &= 0.0001472656250000005B_{0,2}(X) + 0.0003523437500000004B_{1,2}(X)\\ &\quad + 0.0008015625000000004B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

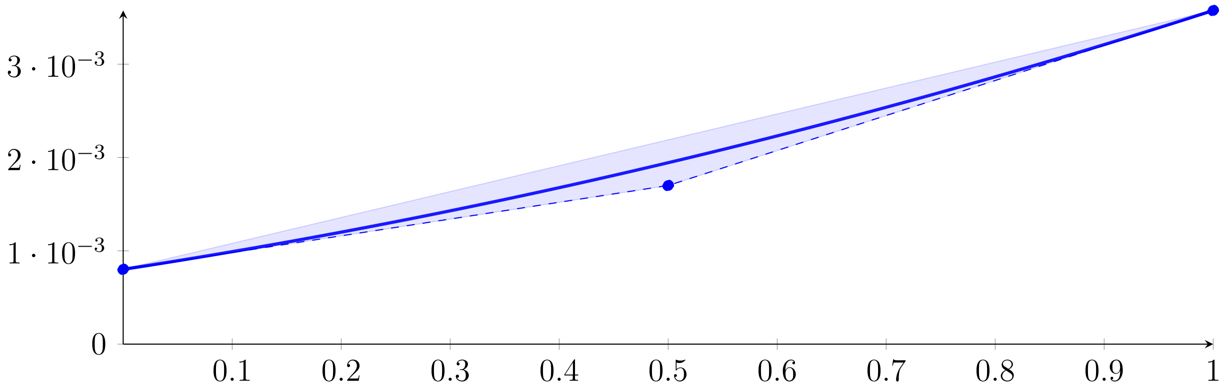
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

13.25 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.0009765625X^2 + 0.001796875X + 0.0008015625000000004\\ &= 0.0008015625000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

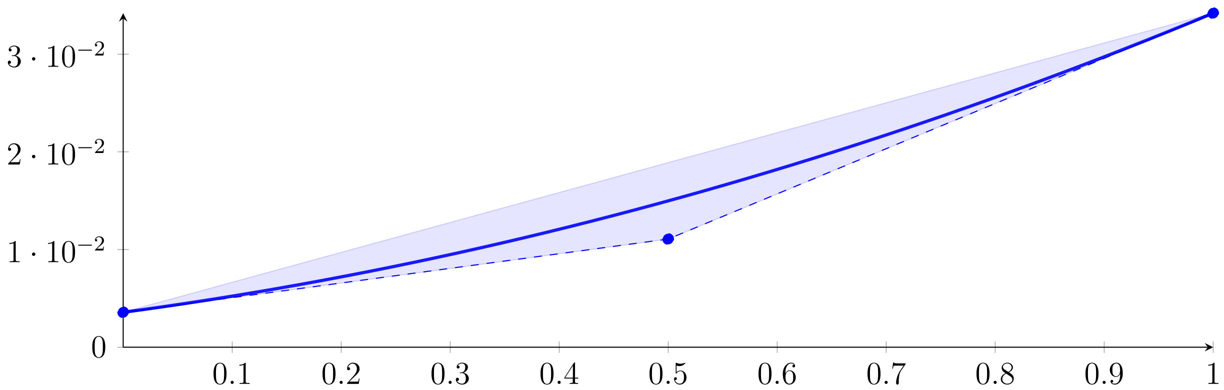
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

13.26 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

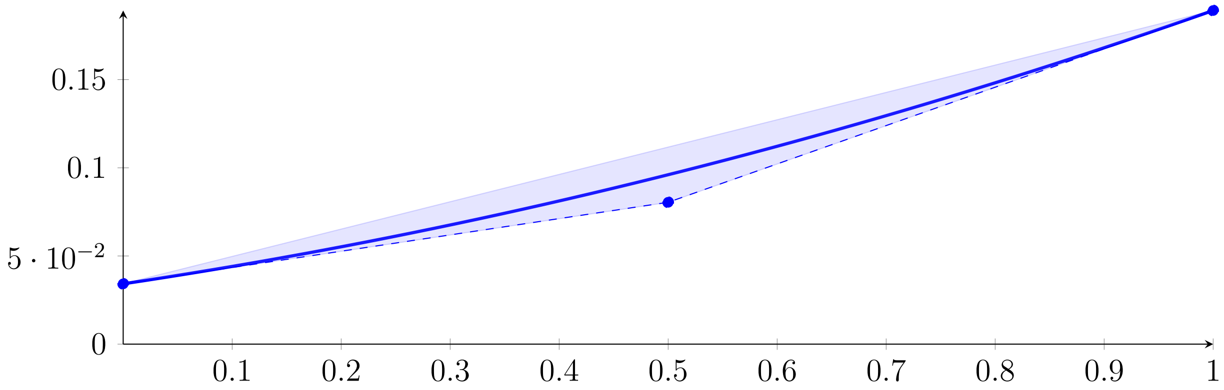
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

13.27 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

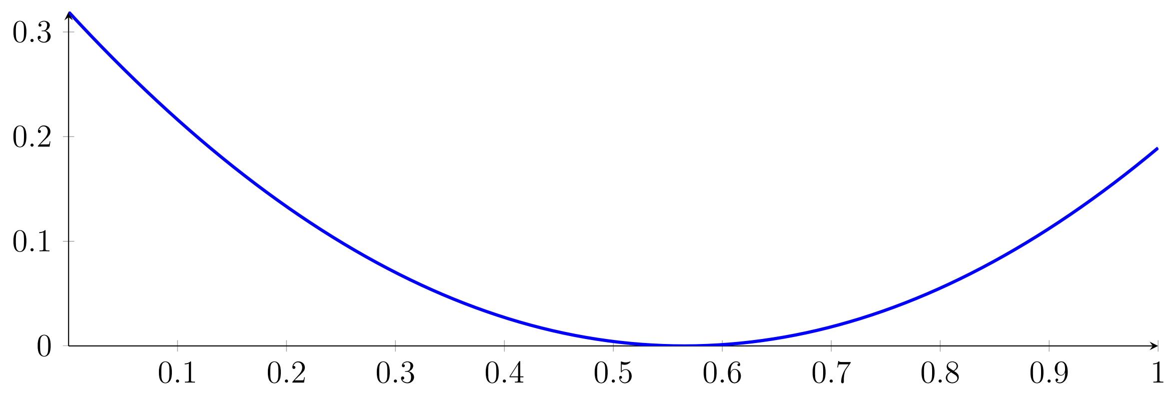
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

13.28 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

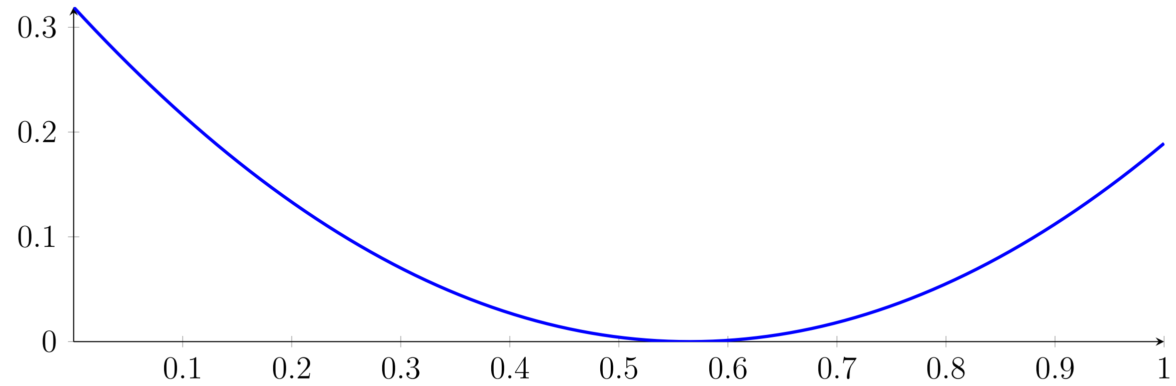
with precision $\varepsilon = 1 \cdot 10^{-32}$.

14 Running `QuadClip` on h_2 with epsilon 32

$$1X^2 - 1.13X + 0.3192$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

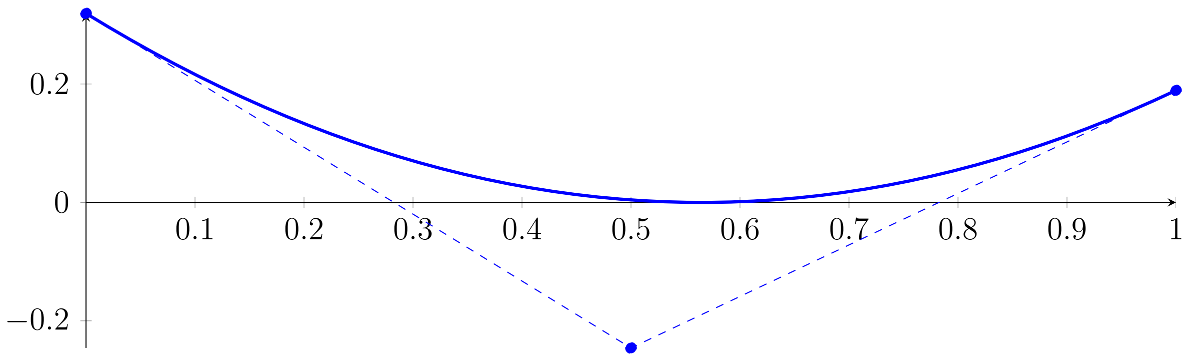
$$p = 1X^2 - 1.13X + 0.3192$$



14.1 Recursion Branch 1 for Input Interval $[0, 1]$

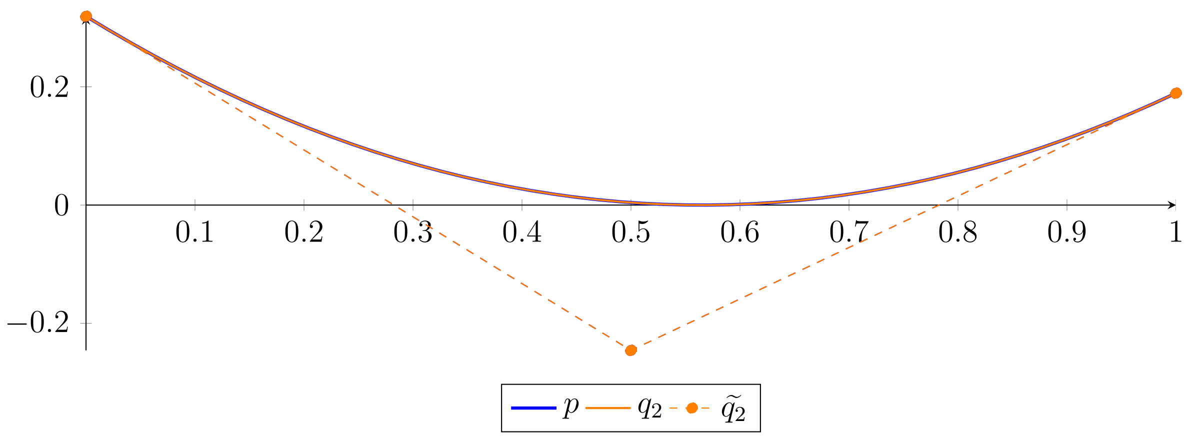
Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \\ \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

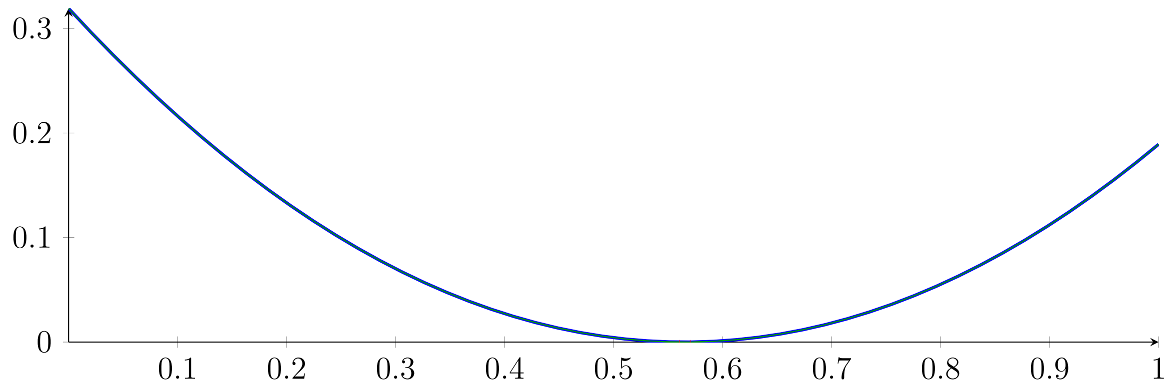
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

14.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

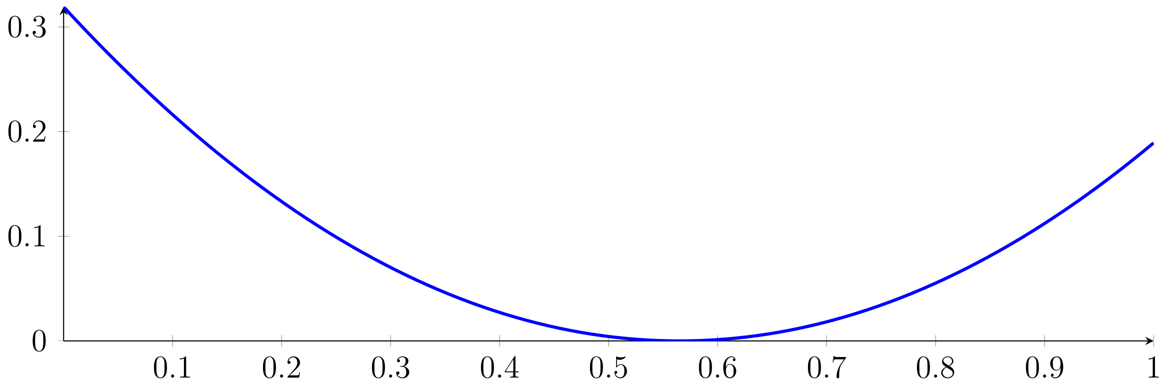
14.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

14.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

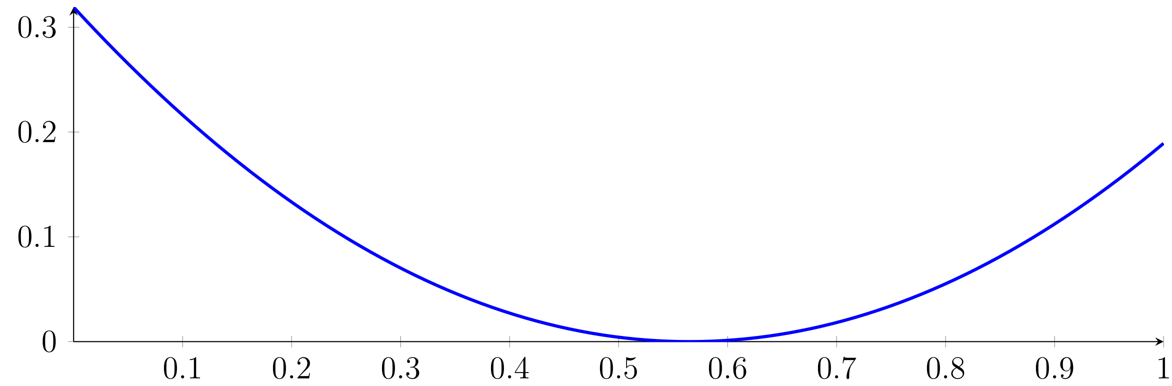
with precision $\varepsilon = 1 \cdot 10^{-32}$.

15 Running `CubeClip` on h_2 with epsilon 32

$$1X^2 - 1.13X + 0.3192$$

Called `CubeClip` with input polynomial on interval $[0,1]$:

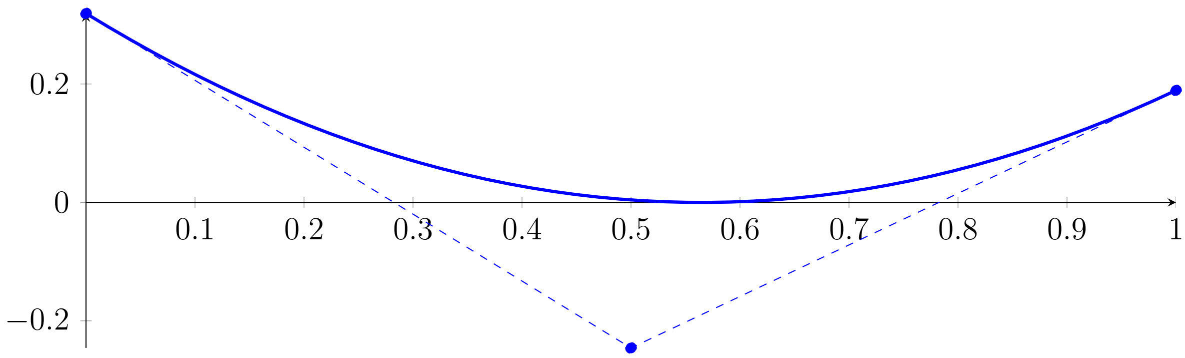
$$p = 1X^2 - 1.13X + 0.3192$$



15.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

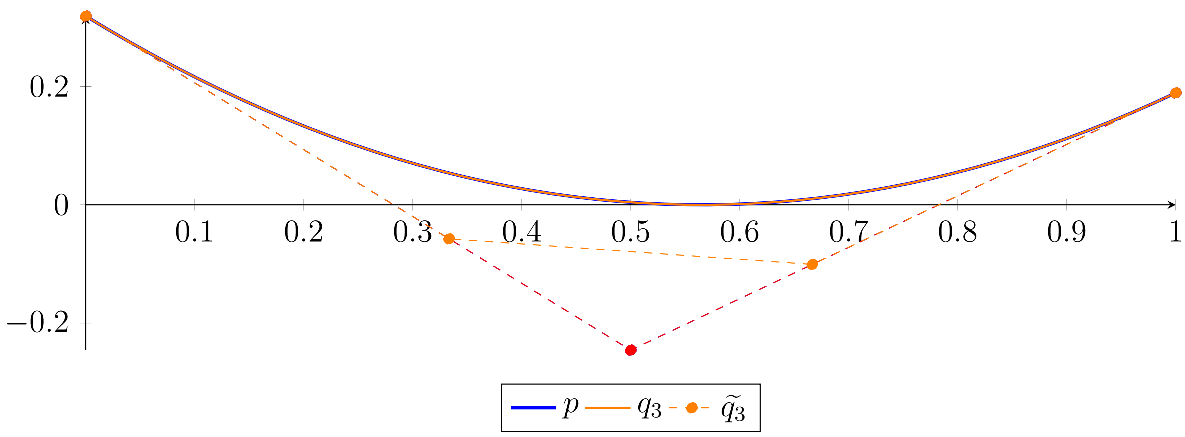
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308}X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501 \cdot 10^{-308}$.

Bounding polynomials M and m:

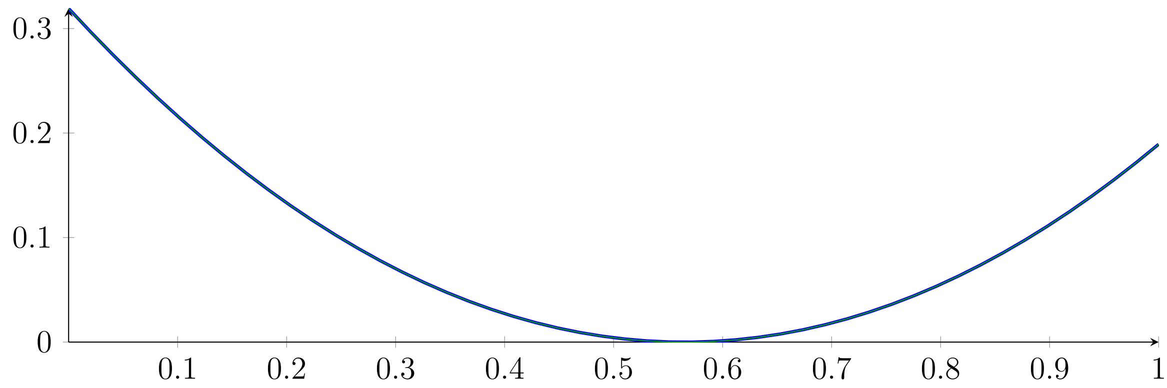
$$M = -1.668805393880401 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783 \cdot 10^{291}, 2.824576038015892 \cdot 10^{291}, 5.992310449541053 \cdot 10^{307}\}$ $N(m) = \{-4.84$

Intersection intervals:

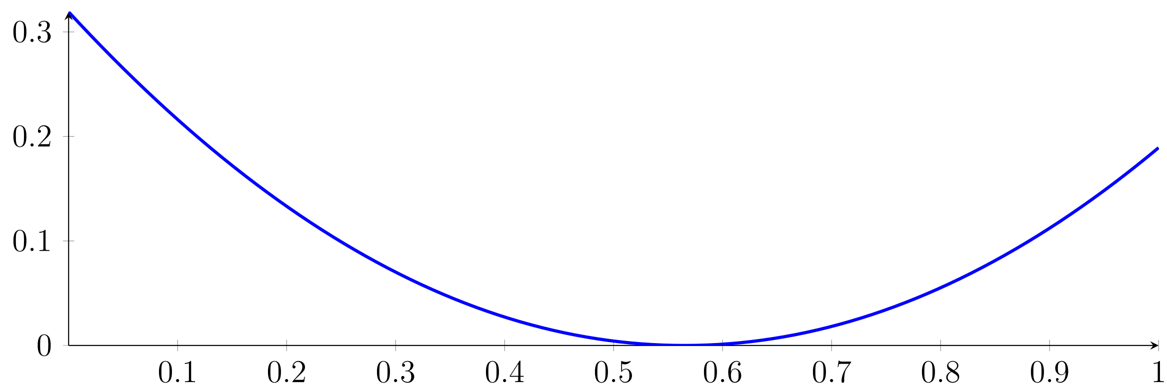


No intersection intervals with the x axis.

15.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

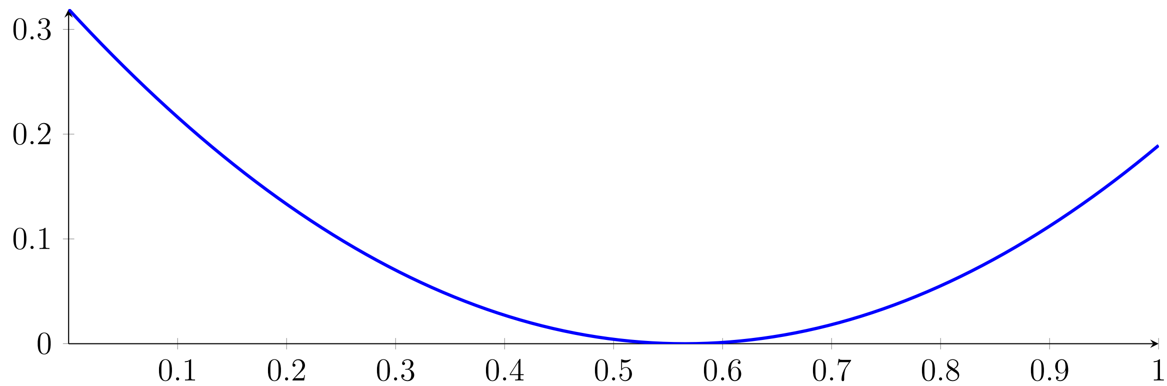
with precision $\varepsilon = 1 \cdot 10^{-32}$.

16 Running BezClip on h_2 with epsilon 64

$$1X^2 - 1.13X + 0.3192$$

Called BezClip with input polynomial on interval $[0, 1]$:

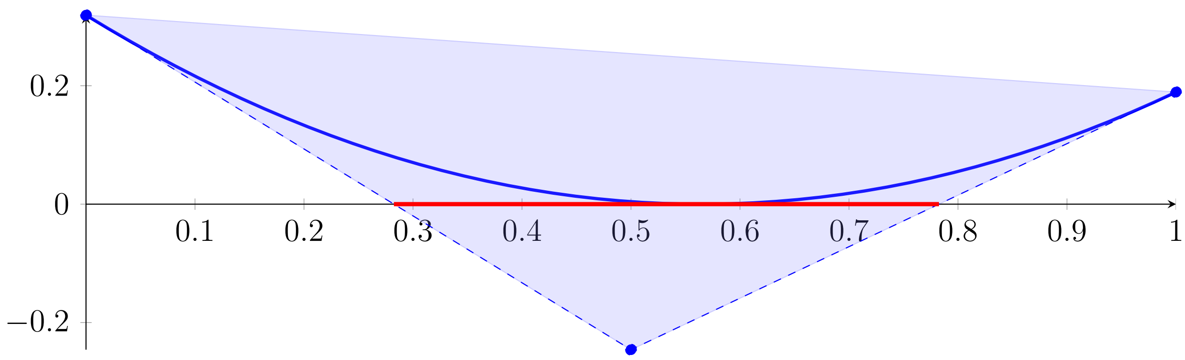
$$p = 1X^2 - 1.13X + 0.3192$$



16.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

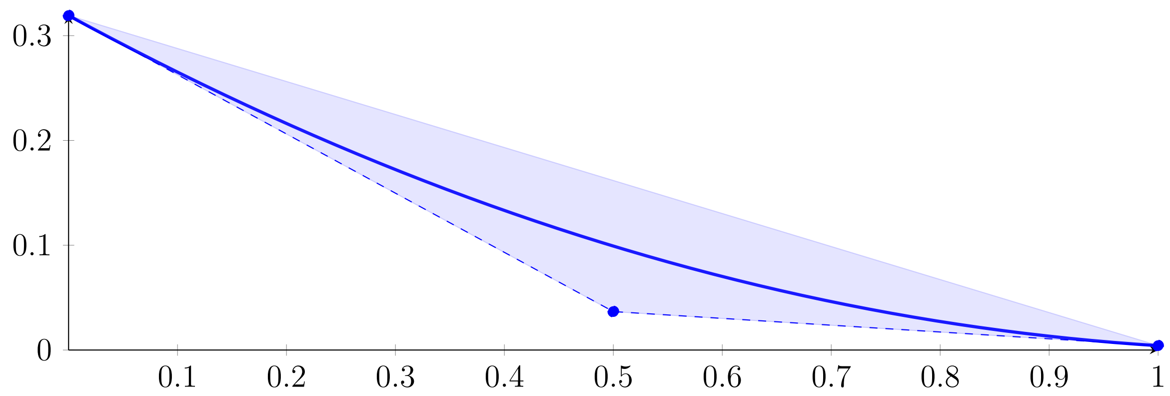
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

16.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

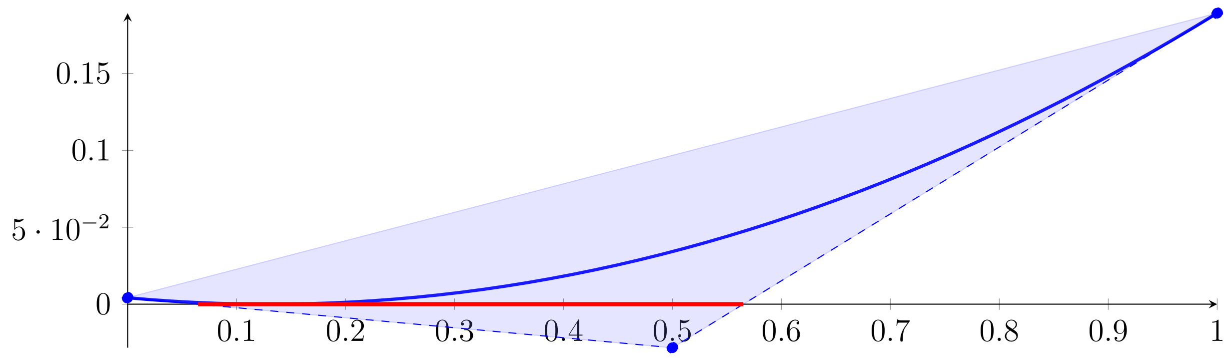
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

16.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.065X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

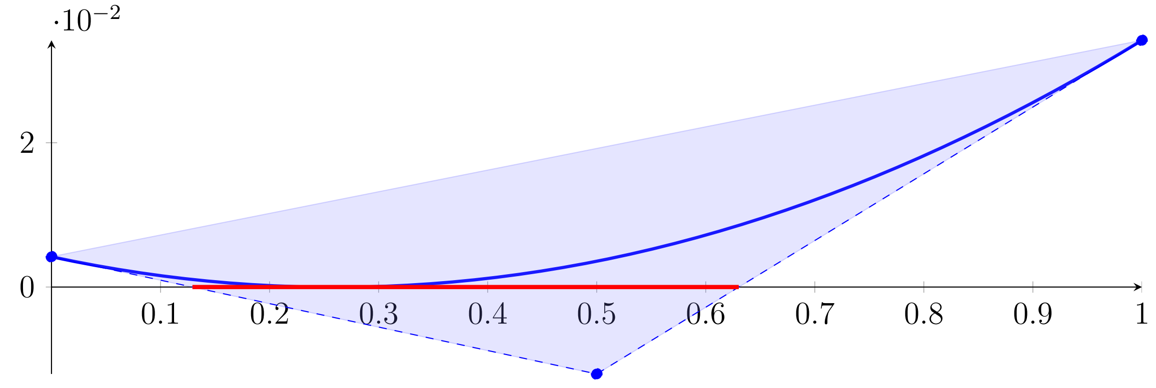
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\Longrightarrow$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

16.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 - 0.0325X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

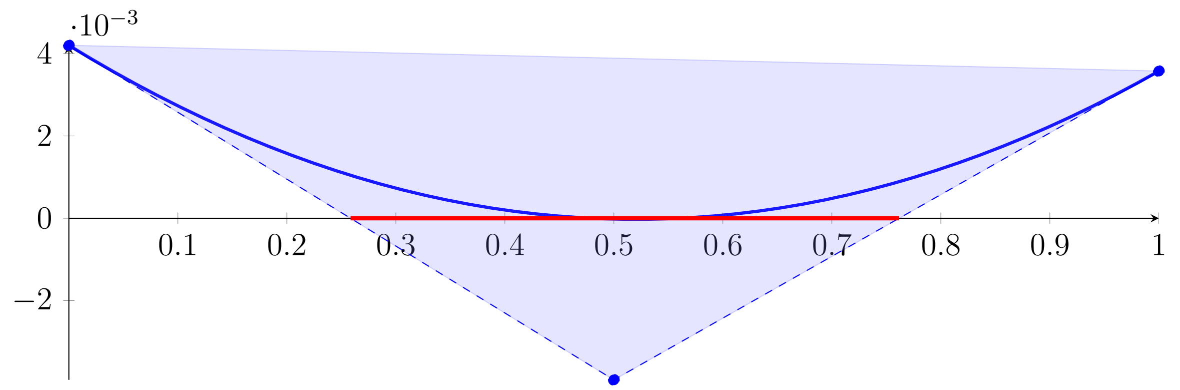
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

16.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

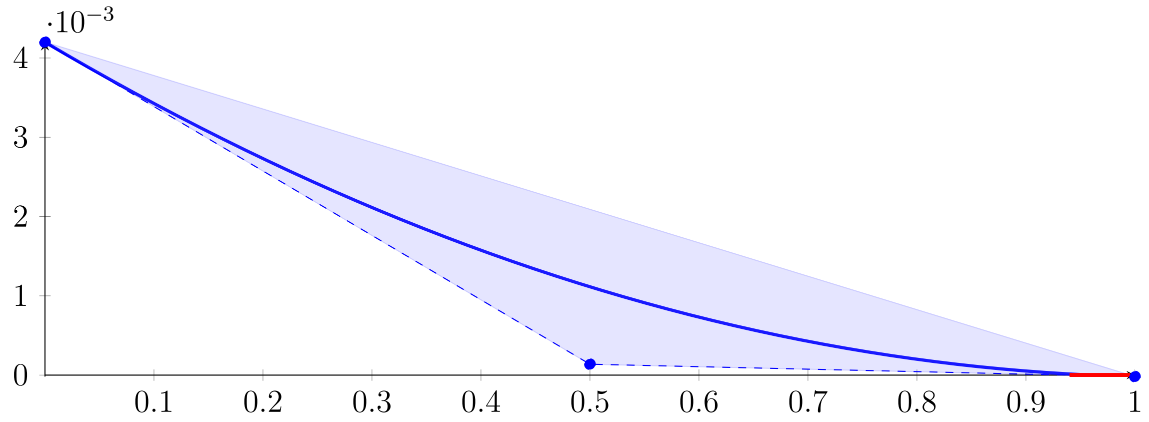
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

16.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) + 0.0001375000000000007B_{1,2}(X) \\ &\quad - 1.874999999999948 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

$$[0.9400000000000017, 0.9955555555555557]$$

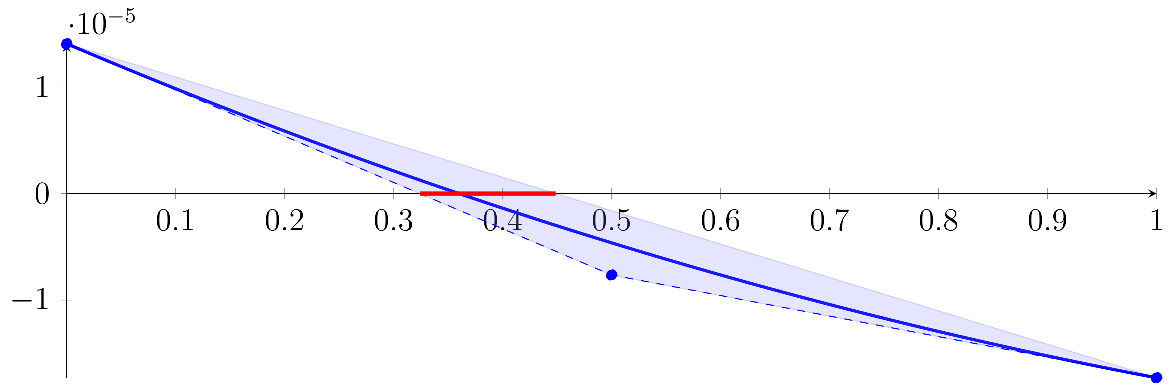
Longest intersection interval: 0.05555555555555396

$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

16.7 Recursion Branch 1 2 1 1 1 in Interval 1: $[0.5587500000000001, 0.56222222222$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.205632716049313 \cdot 10^{-05}X^2 - 4.340277777777758 \cdot 10^{-05}X + 1.406249999999919 \cdot 10^{-05} \\ &= 1.406249999999919 \cdot 10^{-05}B_{0,2}(X) - 7.638888888888706 \\ &\quad \cdot 10^{-06}B_{1,2}(X) - 1.728395061728347 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3239999999999962, 0.4486153846153773\}$$

Intersection intervals with the x axis:

$$[0.3239999999999962, 0.4486153846153773]$$

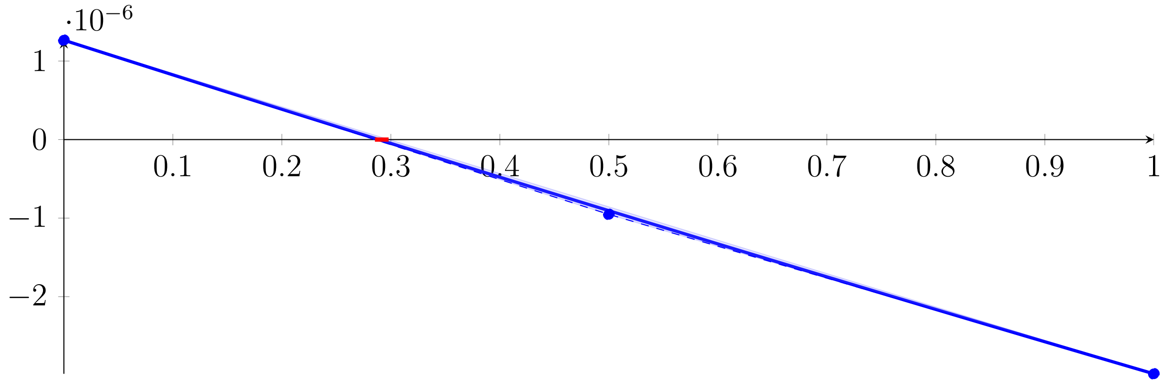
Longest intersection interval: 0.1246153846153811

$\Longrightarrow$ Selective recursion: interval 1: $[0.5598750000000001, 0.5603076923076923]$,

16.8 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[0.5598750000000001, 0.560307692$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1.872226331360733 \cdot 10^{-07} X^2 - 4.435096153845849 \cdot 10^{-06} X + 1.265624999999897 \cdot 10^{-06}$$
$$= 1.265624999999897 \cdot 10^{-06} B_{0,2}(X) - 9.519230769230272$$
$$\cdot 10^{-07} B_{1,2}(X) - 2.982248520709878 \cdot 10^{-06} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2853658536585533, 0.2979431929480858\}$$

Intersection intervals with the x axis:

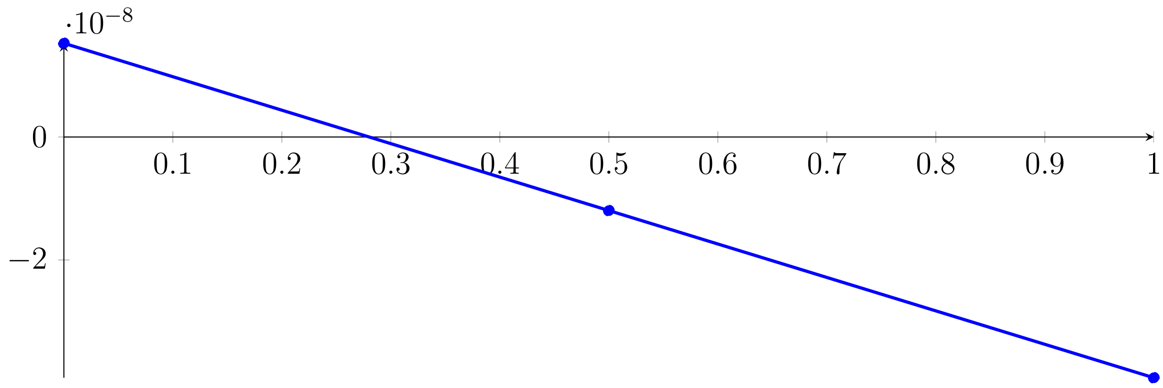
$$[0.2853658536585533, 0.2979431929480858]$$

Longest intersection interval: 0.01257733928955277
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599984756097562, 0.560003917727718]$,

16.9 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[0.5599984756097562, 0.5600039$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 2.961664791042273 \cdot 10^{-11} X^2 - 5.443777144130786 \cdot 10^{-08} X + 1.524622620463798 \cdot 10^{-08}$$
$$= 1.524622620463798 \cdot 10^{-08} B_{0,2}(X) - 1.197265951601595$$
$$\cdot 10^{-08} B_{1,2}(X) - 3.916192858875946 \cdot 10^{-08} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2800670527278236, 0.2802195050086158\}$$

Intersection intervals with the x axis:

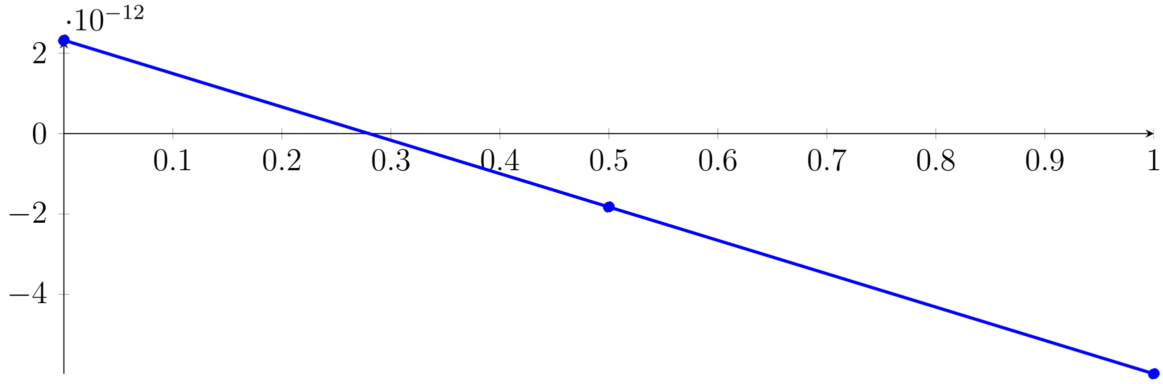
$$[0.2800670527278236, 0.2802195050086158]$$

Longest intersection interval: 0.0001524522807922228
$\Longrightarrow$ Selective recursion: interval 1: $[0.5599999997676943, 0.5600000005973576]$,

16.10 Recursion Branch 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.5599999997676943, 0.5600$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.883411841000453 \cdot 10^{-19} X^2 - 8.296633341677063 \cdot 10^{-12} X + 2.323057420473189 \cdot 10^{-12}$$
$$= 2.323057420473189 \cdot 10^{-12} B_{0,2}(X) - 1.825259250365342 \cdot 10^{-12} B_{1,2}(X) - 5.97357523286269 \cdot 10^{-12} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2800000102214486, 0.2800000334520226\}$$

Intersection intervals with the x axis:

$$[0.2800000102214486, 0.2800000334520226]$$

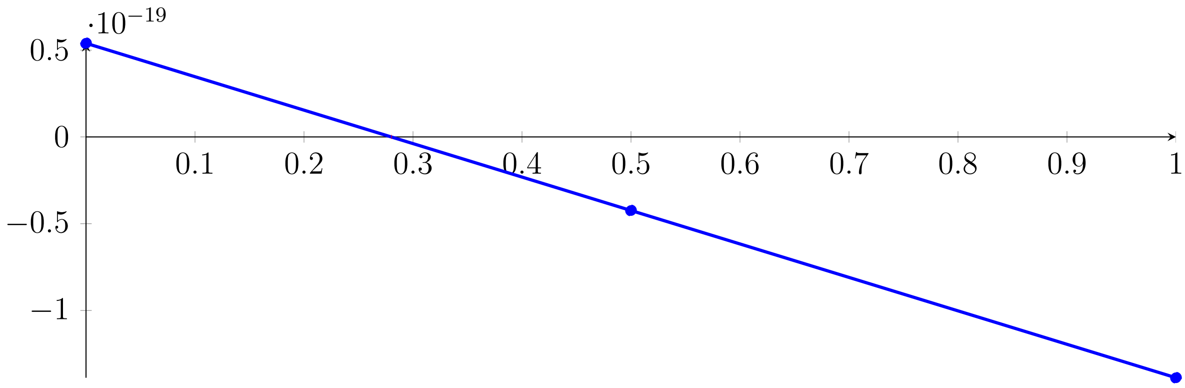
Longest intersection interval: $2.323057397344962 \cdot 10^{-08}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.56, 0.5600000000000001]$,

16.11 Recursion Branch 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.56, 0.5600000000000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 3.714699054532536 \cdot 10^{-34} X^2 - 1.927355456197032 \cdot 10^{-19} X + 5.396595277351629 \cdot 10^{-20}$$
$$= 5.396595277351629 \cdot 10^{-20} B_{0,2}(X) - 4.240182003633529 \cdot 10^{-20} B_{1,2}(X) - 1.387695928461865 \cdot 10^{-19} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999969, 0.2799999999999975\}$$

Intersection intervals with the x axis:

$$[0.2799999999999969, 0.2799999999999975]$$

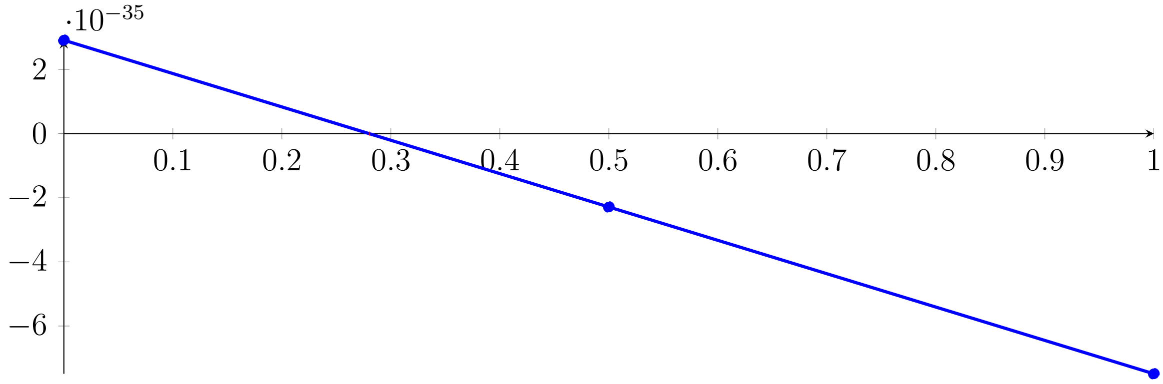
Longest intersection interval: $5.396595277351738 \cdot 10^{-16}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

16.12 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0.5$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1.081840742754402 \cdot 10^{-64} X^2 - 1.040115735269099 \cdot 10^{-34} X + 2.912324058753444 \cdot 10^{-35}$$
$$= 2.912324058753444 \cdot 10^{-35} B_{0,2}(X) - 2.288254617592053$$
$$\cdot 10^{-35} B_{1,2}(X) - 7.488833293937551 \cdot 10^{-35} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999967, 0.2799999999999967\}$$

Intersection intervals with the x axis:

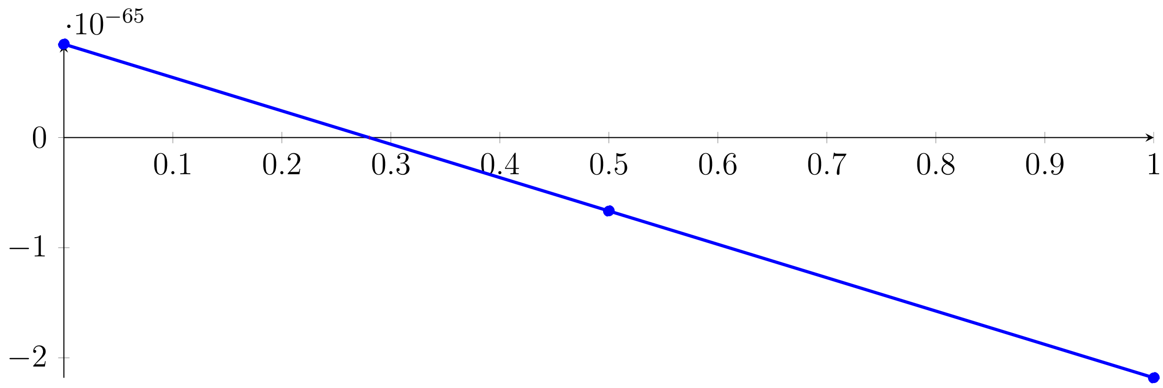
$$[0.2799999999999967, 0.2799999999999967]$$

Longest intersection interval: $2.912324058753504 \cdot 10^{-31}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

16.13 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 9.175774438637789 \cdot 10^{-126} X^2 - 3.029154079712289 \cdot 10^{-65} X + 8.481631423194308 \cdot 10^{-66}$$
$$= 8.481631423194308 \cdot 10^{-66} B_{0,2}(X) - 6.664138975367135$$
$$\cdot 10^{-66} B_{1,2}(X) - 2.180990937392858 \cdot 10^{-65} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999967, 0.2799999999999967\}$$

Intersection intervals with the x axis:

$$[0.2799999999999967, 0.2799999999999967]$$

Longest intersection interval: $8.481631423194481 \cdot 10^{-62}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

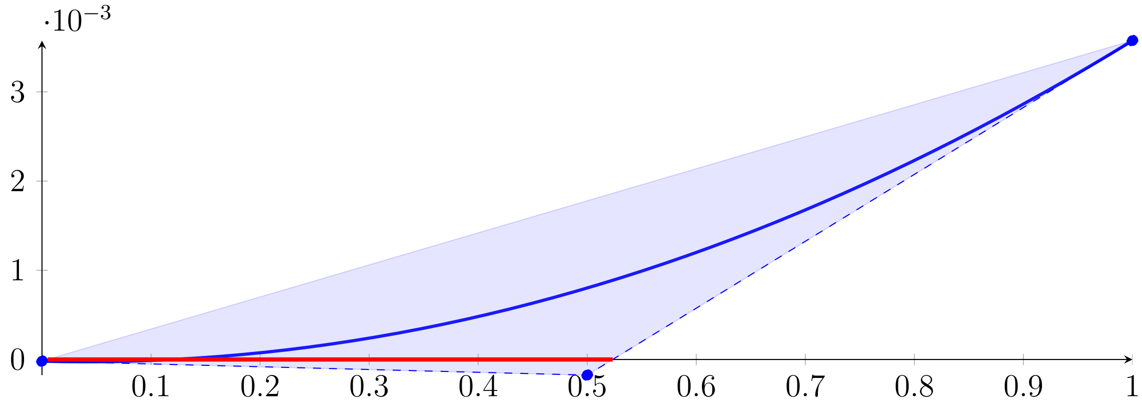
16.14 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 13!

16.15 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.00390625X^2 - 0.0003125000000000003X - 1.874999999999948\cdot 10^{-05}$$
$$= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 0.0001749999999999996B_{1,2}(X) + 0.003575B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

$$[0.005217391304347681, 0.5233333333333333]$$

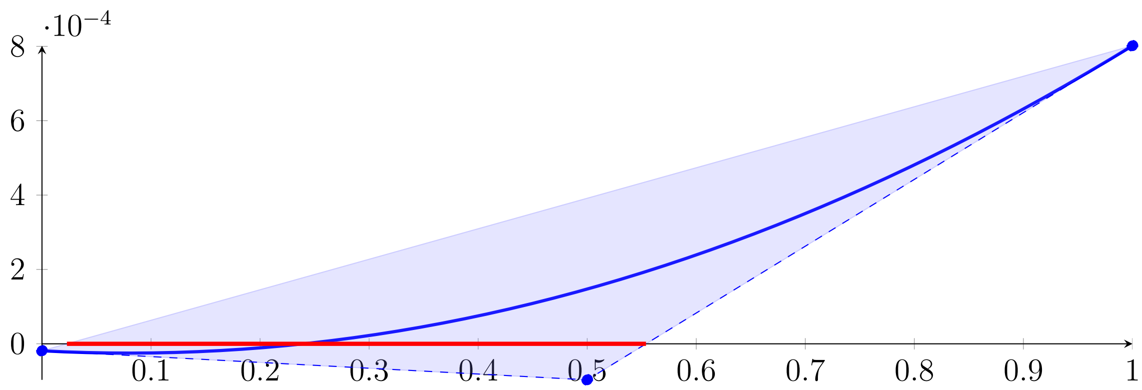
Longest intersection interval: 0.5181159420289856

$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

16.16 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.0009765625X^2 - 0.0001562500000000001X - 1.874999999999948\cdot 10^{-05}$$
$$= -1.874999999999948\cdot 10^{-05}B_{0,2}(X) - 9.687499999999955$$
$$\cdot 10^{-05}B_{1,2}(X) + 0.0008015625000000004B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142224, 0.5539130434782606\}$$

Intersection intervals with the x axis:

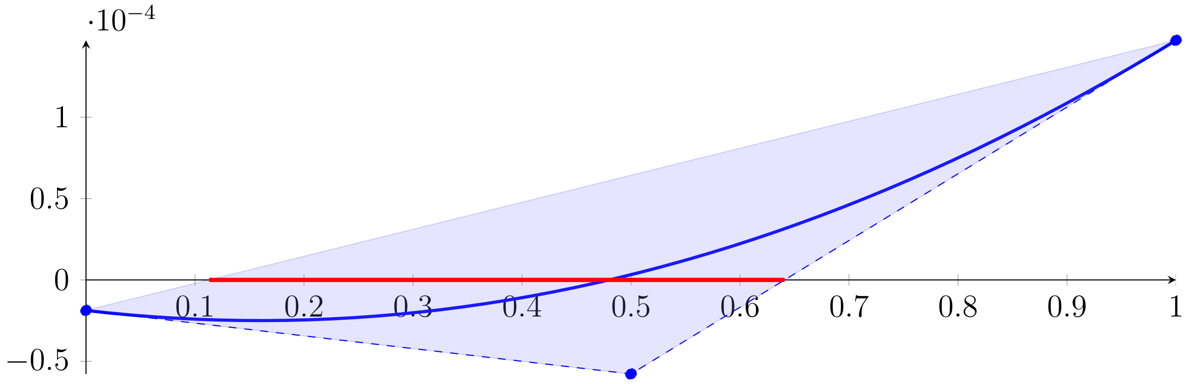
$$[0.022857142857142224, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\implies$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

16.17 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 0.000244140625X^2 - 7.812500000000007 \cdot 10^{-05}X - 1.874999999999948 \cdot 10^{-05}\\ &= -1.874999999999948 \cdot 10^{-05}B_{0,2}(X) - 5.781249999999951\\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0001472656250000005B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

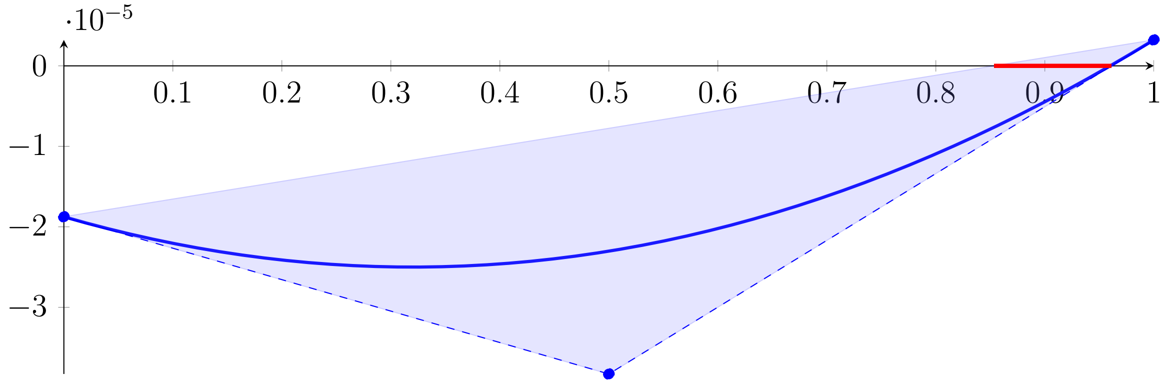
$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\implies$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

16.18 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 6.103515625 \cdot 10^{-05}X^2 - 3.906250000000003 \cdot 10^{-05}X - 1.874999999999948 \cdot 10^{-05}\\ &= -1.874999999999948 \cdot 10^{-05}B_{0,2}(X) - 3.82812499999995\\ &\quad \cdot 10^{-05}B_{1,2}(X) + 3.222656250000487 \cdot 10^{-06}B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8533333333333109, 0.9611764705882294\}$$

Intersection intervals with the x axis:

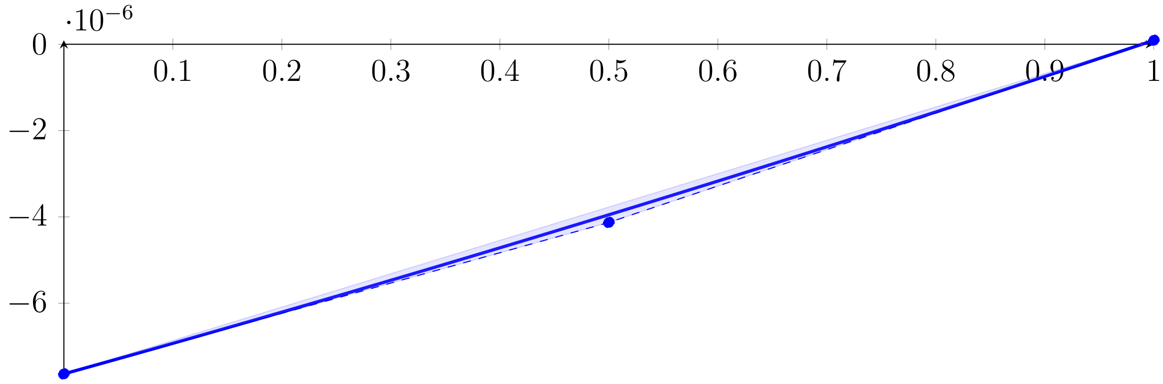
$$[0.8533333333333109, 0.9611764705882294]$$

Longest intersection interval: 0.1078431372549185
$\Longrightarrow$ Selective recursion: interval 1: $[0.5691666666666665, 0.5700091911764705]$,

16.19 Recursion Branch 1 2 1 1 2 1 1 1 1 in Interval 1: $[0.5691666666666665, 0.5700$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.098475496205558 \cdot 10^{-07} X^2 + 7.021037581700123 \cdot 10^{-06} X - 7.638888888889855 \cdot 10^{-06}$$
$$= -7.638888888889855 \cdot 10^{-06} B_{0,2}(X) - 4.128370098039794$$
$$\cdot 10^{-06} B_{1,2}(X) + 9.199624243082374 \cdot 10^{-08} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9881001669449061, 0.9891009174311909\}$$

Intersection intervals with the x axis:

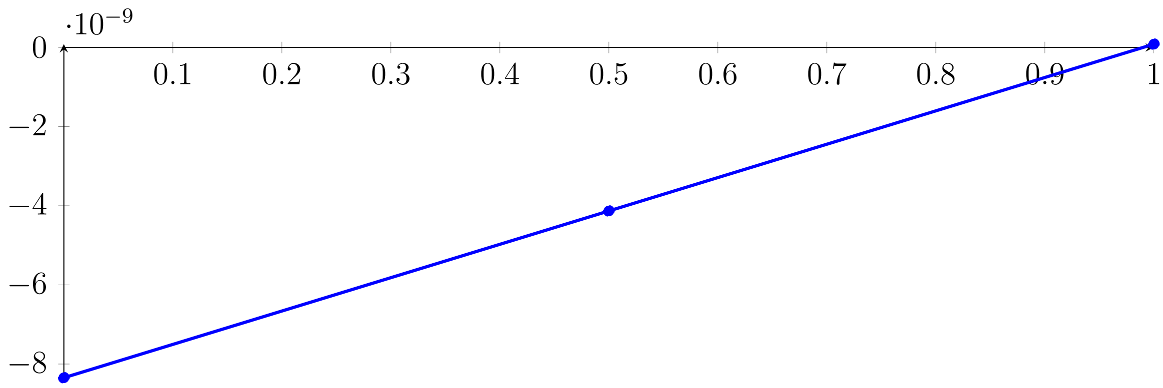
$$[0.9881001669449061, 0.9891009174311909]$$

Longest intersection interval: 0.001000750486284818
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999165275459, 0.5700000084322719]$,

16.20 Recursion Branch 1 2 1 1 2 1 1 1 1 1 in Interval 1: $[0.569999165275459, 0.5700$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.109134111283559 \cdot 10^{-13} X^2 + 8.430160521565627 \cdot 10^{-09} X - 8.346548643959838 \cdot 10^{-09}$$
$$= -8.346548643959838 \cdot 10^{-09} B_{0,2}(X) - 4.131468383177024$$
$$\cdot 10^{-09} B_{1,2}(X) + 8.432279101691722 \cdot 10^{-11} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.989998330342568, 0.989999173638737\}$$

Intersection intervals with the x axis:

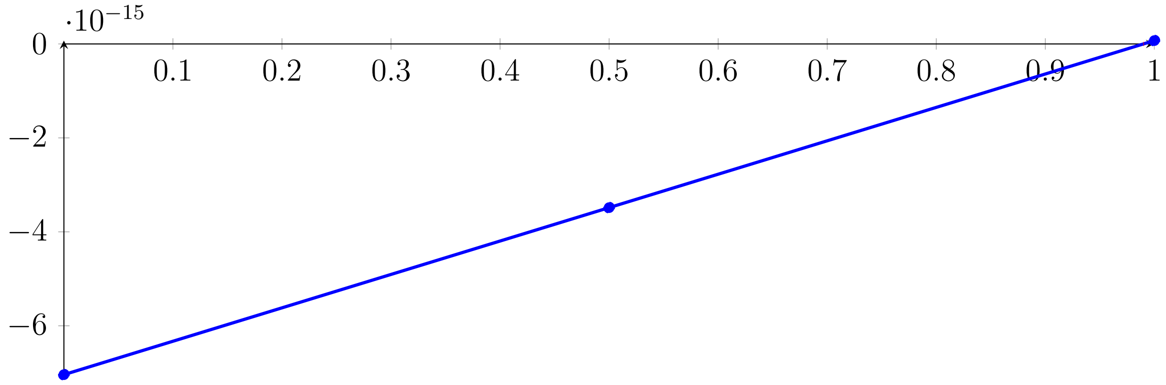
$$[0.989998330342568, 0.989999173638737]$$

Longest intersection interval: $8.432961690166558 \cdot 10^{-07}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999999999296, 0.5700000000000071]$,

16.21 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.569999999999296, 0.57$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 5.055649552501669 \cdot 10^{-25} X^2 + 7.110309100930884 \cdot 10^{-15} X - 7.039206010412064 \cdot 10^{-15}$$
$$= -7.039206010412064 \cdot 10^{-15} B_{0,2}(X) - 3.484051459946622$$
$$\cdot 10^{-15} B_{1,2}(X) + 7.1103091024385 \cdot 10^{-17} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999985907, 0.9899999999993017\}$$

Intersection intervals with the x axis:

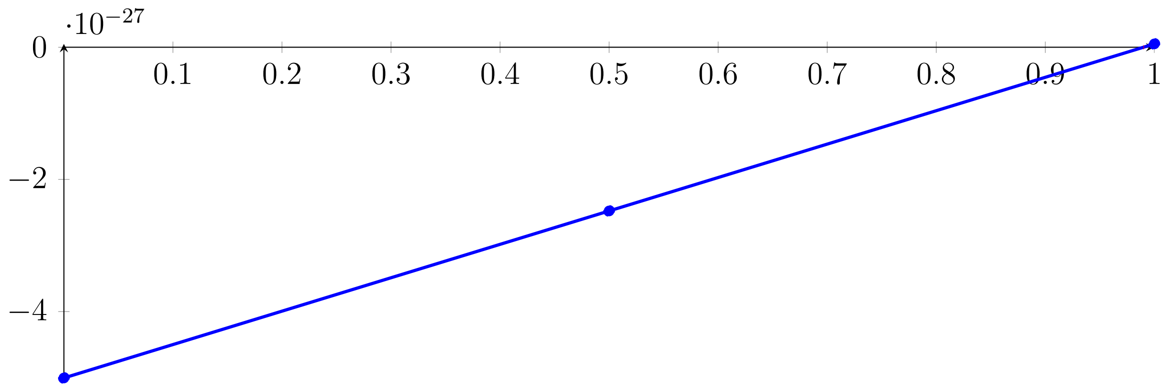
$$[0.9899999999985907, 0.9899999999993017]$$

Longest intersection interval: $7.110309102923987 \cdot 10^{-13}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

16.22 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 2.555959240484234 \cdot 10^{-49} X^2 + 5.055649553206969 \cdot 10^{-27} X - 5.005093057674892 \cdot 10^{-27}$$
$$= -5.005093057674892 \cdot 10^{-27} B_{0,2}(X) - 2.477268281071407$$
$$\cdot 10^{-27} B_{1,2}(X) + 5.055649553207703 \cdot 10^{-29} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999999985, 0.9899999999999985\}$$

Intersection intervals with the x axis:

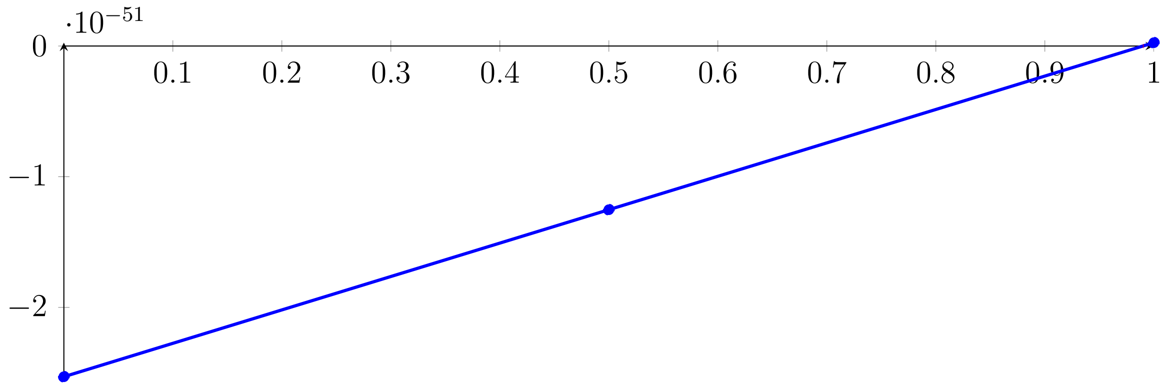
$$[0.9899999999999985, 0.9899999999999985]$$

Longest intersection interval: $5.055649553207806\cdot 10^{-25}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

16.23 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.532927639018775\cdot 10^{-98}X^2 + 2.555959240484605\cdot 10^{-51}X - 2.530399648079756\cdot 10^{-51} \\ &= -2.530399648079756\cdot 10^{-51}B_{0,2}(X) - 1.252420027837453 \\ &\quad \cdot 10^{-51}B_{1,2}(X) + 2.555959240484977\cdot 10^{-53}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999999985, 0.9899999999999985\}$$

Intersection intervals with the x axis:

$$[0.9899999999999985, 0.9899999999999985]$$

Longest intersection interval: $2.555959240485029\cdot 10^{-49}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

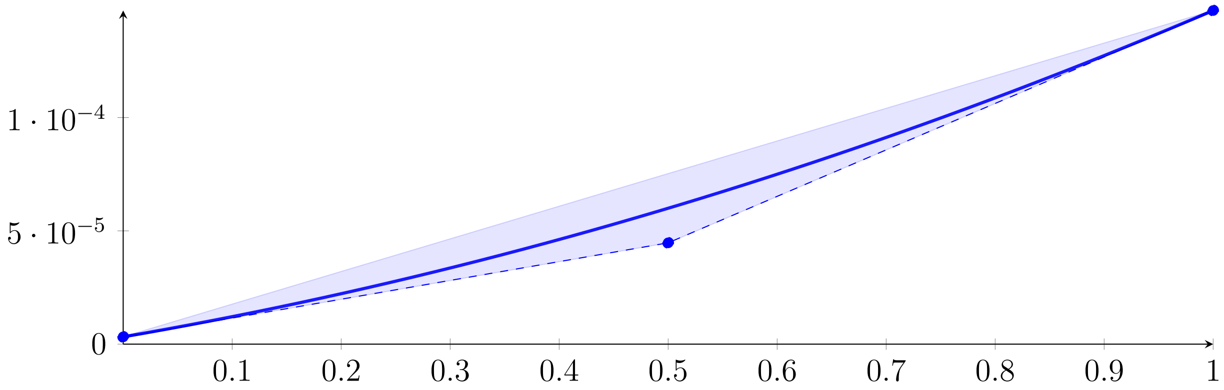
16.24 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 14!

16.25 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625\cdot 10^{-05}X^2 + 8.300781249999997\cdot 10^{-05}X + 3.222656250000487\cdot 10^{-06} \\ &= 3.222656250000487\cdot 10^{-06}B_{0,2}(X) + 4.472656250000047 \\ &\quad \cdot 10^{-05}B_{1,2}(X) + 0.0001472656250000005B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

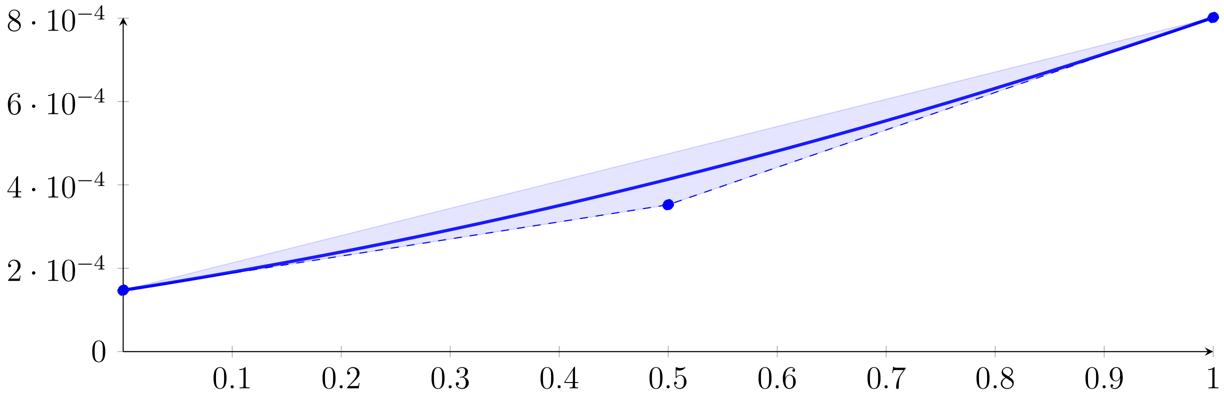
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

16.26 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 + 0.0004101562499999999X + 0.0001472656250000005 \\ &= 0.0001472656250000005B_{0,2}(X) + 0.0003523437500000004B_{1,2}(X) \\ &\quad + 0.0008015625000000004B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

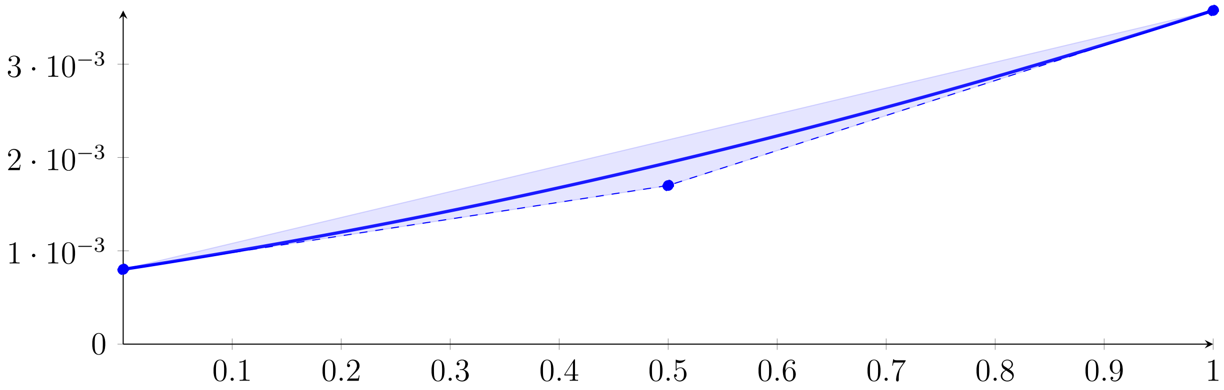
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

16.27 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 + 0.001796875X + 0.0008015625000000004 \\ &= 0.0008015625000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

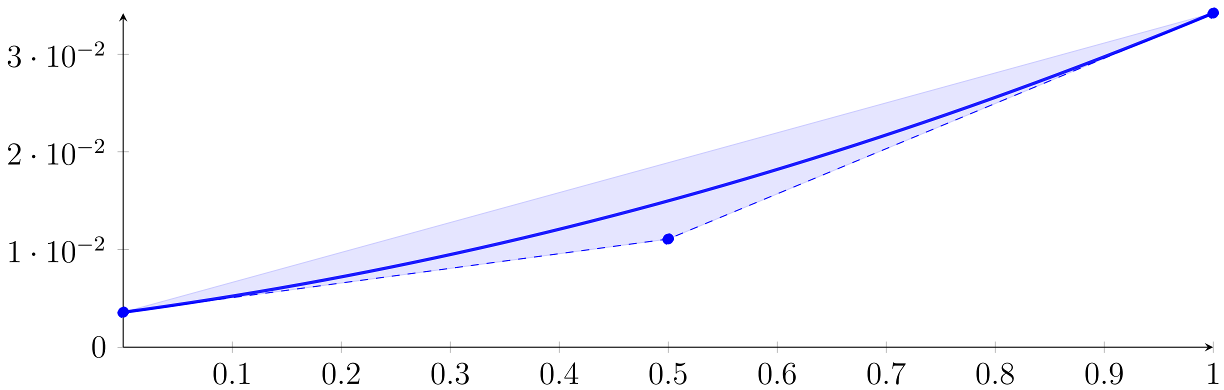
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

16.28 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

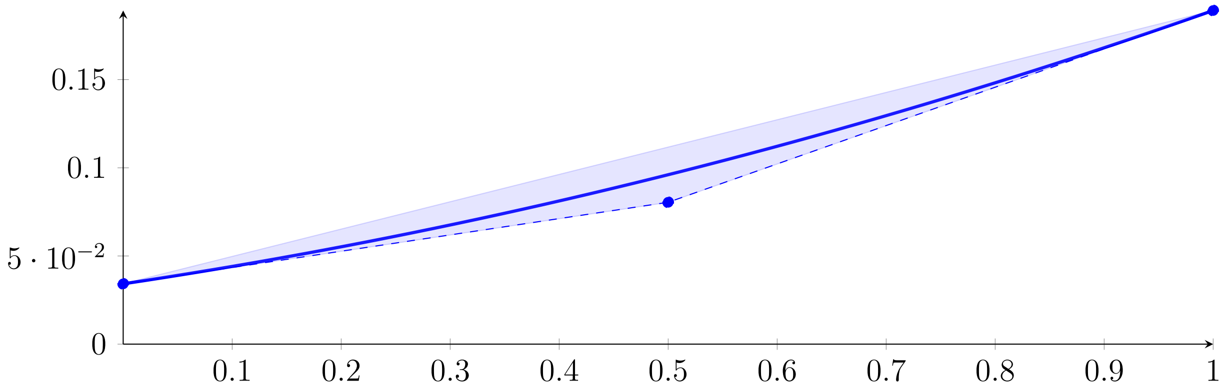
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

16.29 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

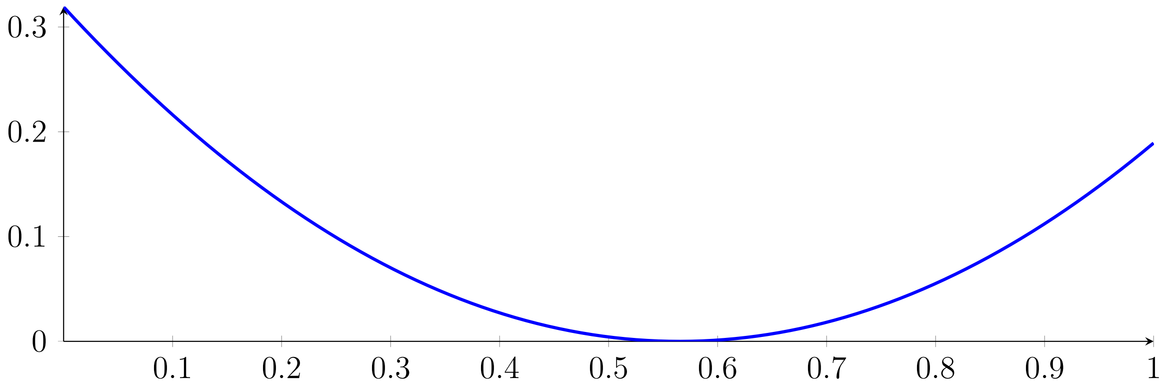
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

16.30 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

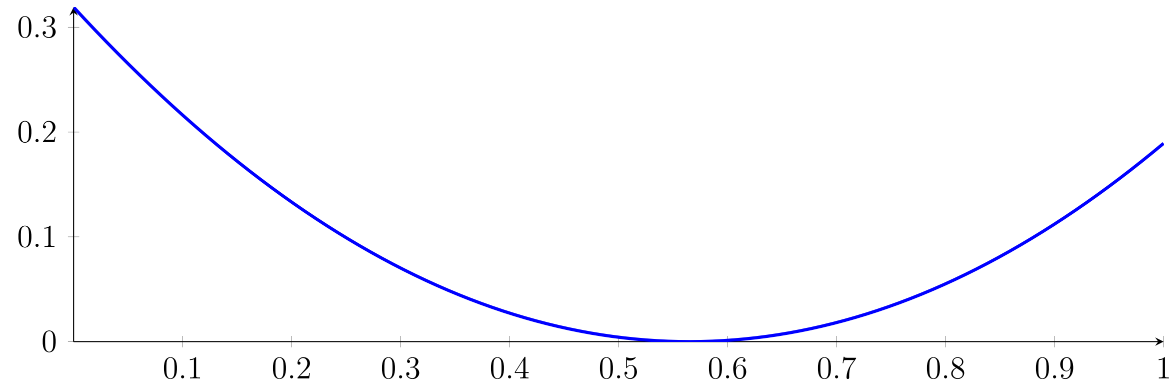
with precision $\varepsilon = 1 \cdot 10^{-64}$.

17 Running QuadClip on h_2 with epsilon 64

$$1X^2 - 1.13X + 0.3192$$

Called QuadClip with input polynomial on interval $[0, 1]$:

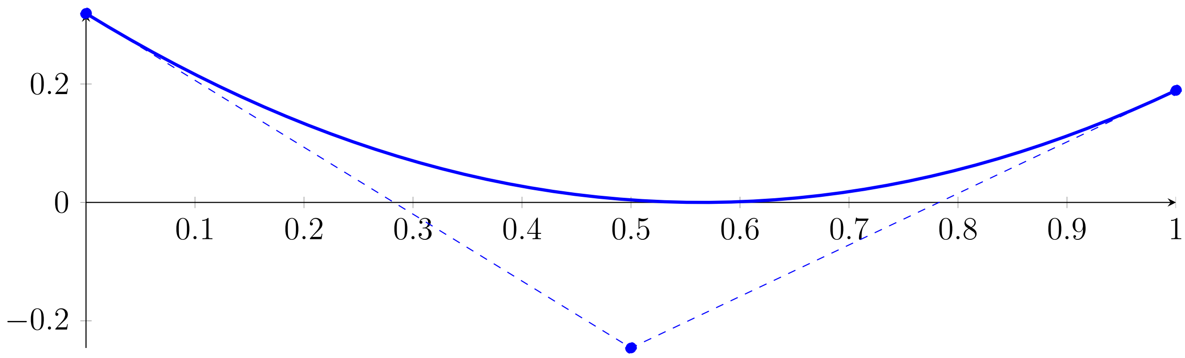
$$p = 1X^2 - 1.13X + 0.3192$$



17.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

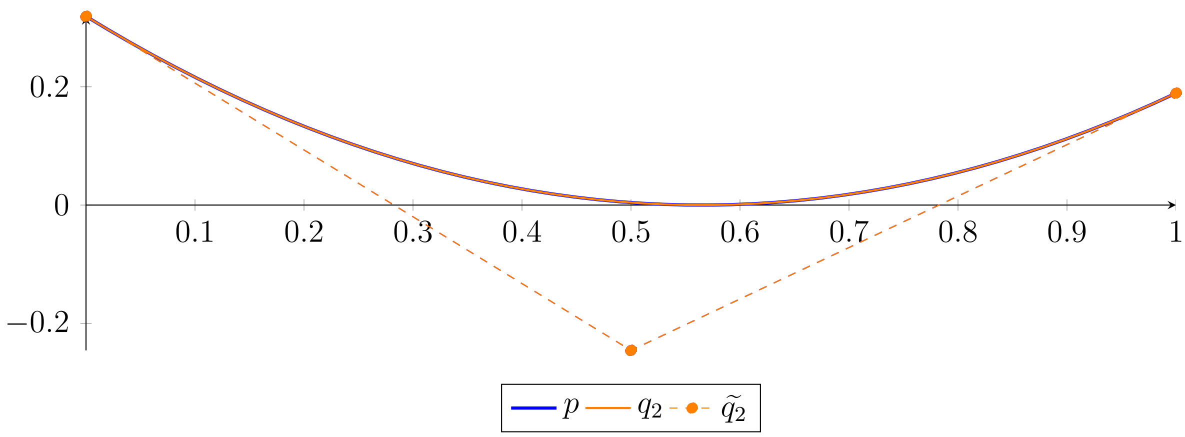
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

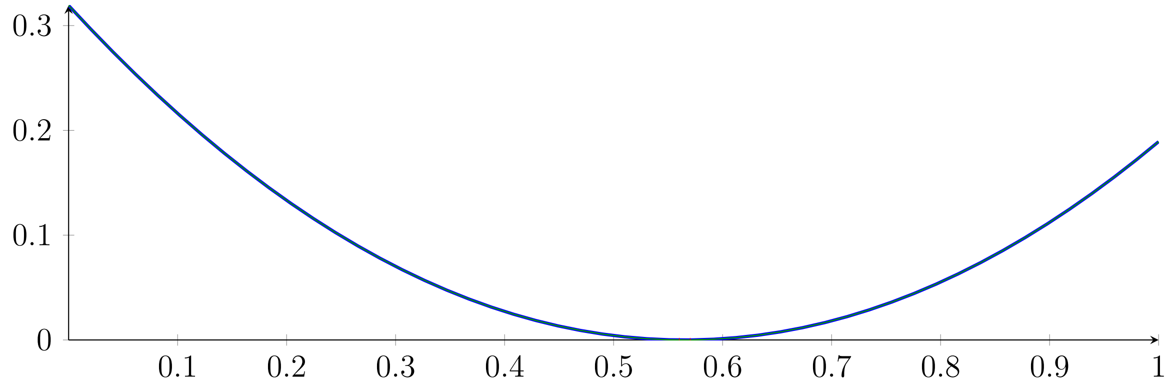
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

17.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

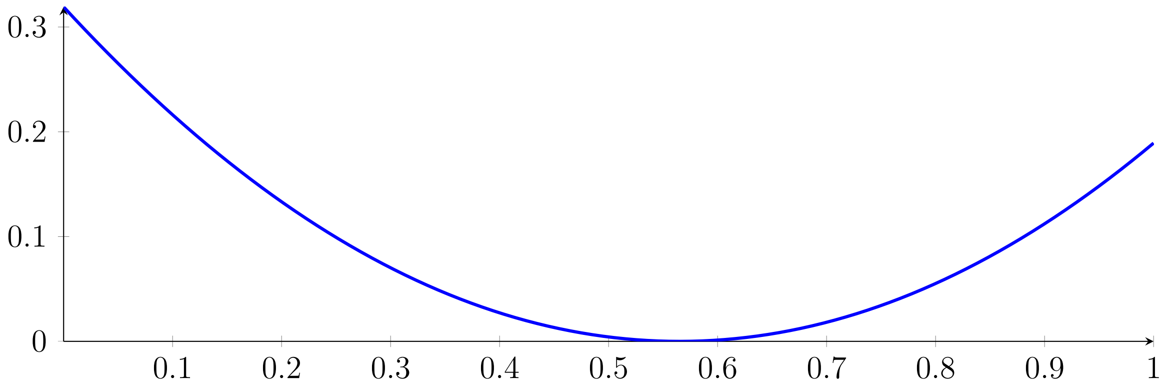
17.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

17.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001], [0.57, 0.57]$$

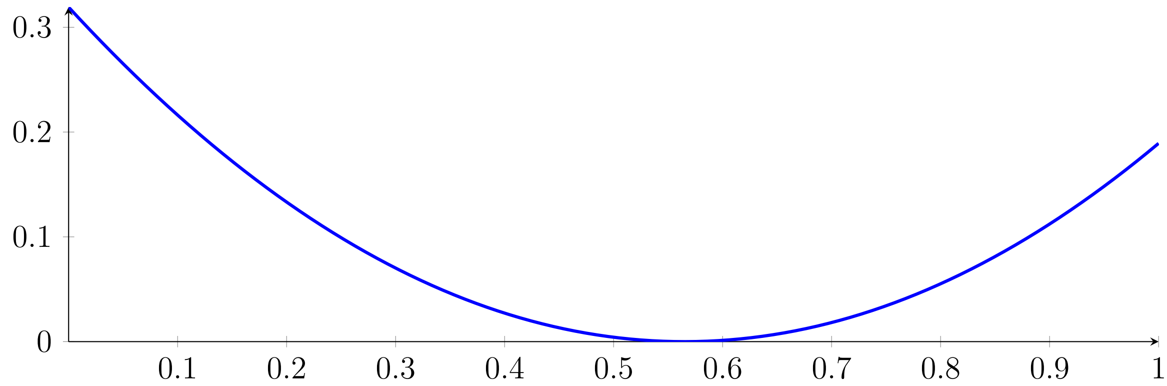
with precision $\varepsilon = 1 \cdot 10^{-64}$.

18 Running CubeClip on h_2 with epsilon 64

$$1X^2 - 1.13X + 0.3192$$

Called CubeClip with input polynomial on interval $[0, 1]$:

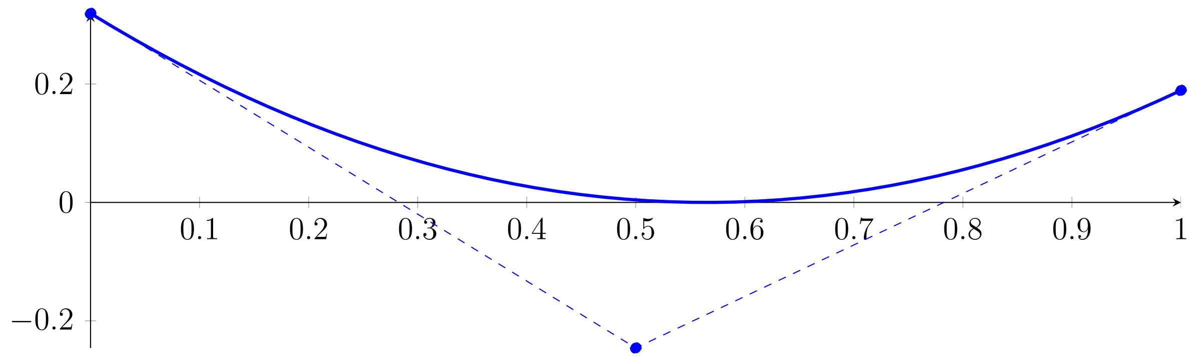
$$p = 1X^2 - 1.13X + 0.3192$$



18.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

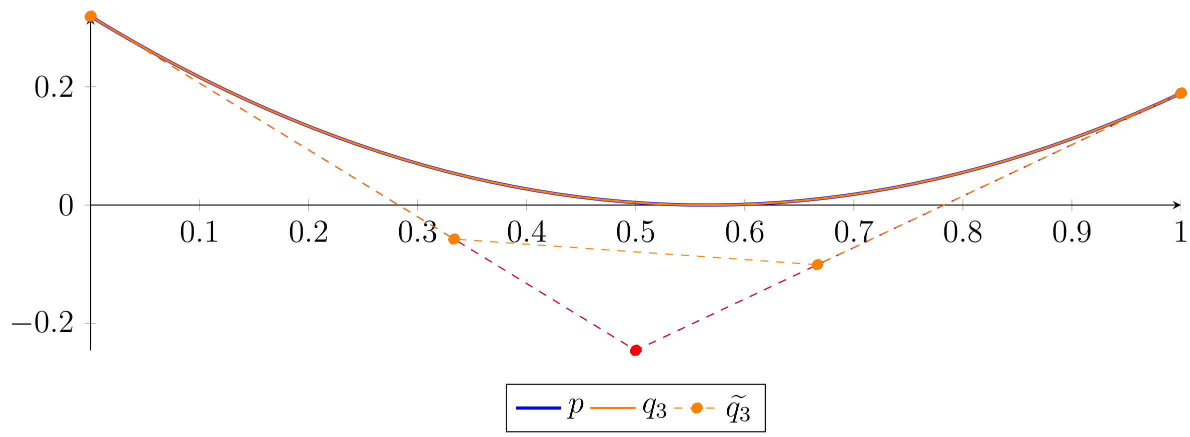
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \tilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501 \cdot 10^{-308}$.
Bounding polynomials M and m:

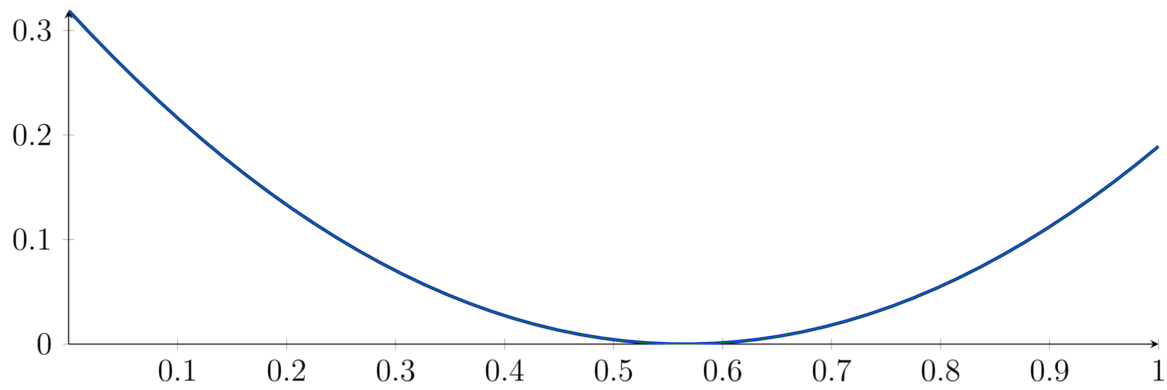
$$M = -1.668805393880401 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783 \cdot 10^{291}, 2.824576038015892 \cdot 10^{291}, 5.992310449541053 \cdot 10^{307}\}$ $N(m) = \{-4.842$

Intersection intervals:

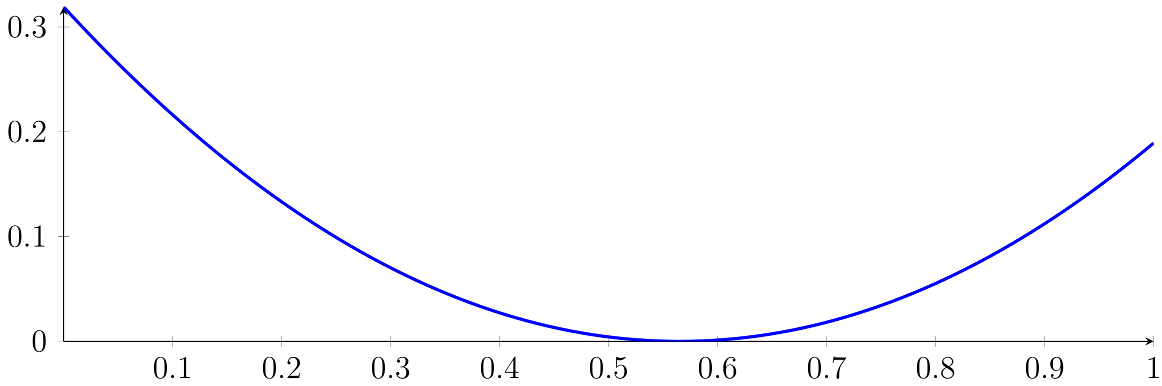


No intersection intervals with the x axis.

18.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

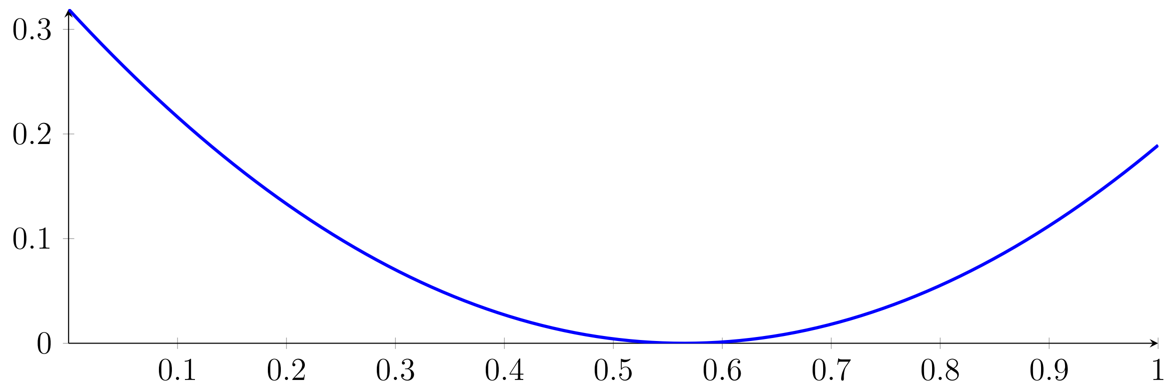
with precision $\varepsilon = 1 \cdot 10^{-64}$.

19 Running `BezClip` on h_2 with epsilon 128

$$1X^2 - 1.13X + 0.3192$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

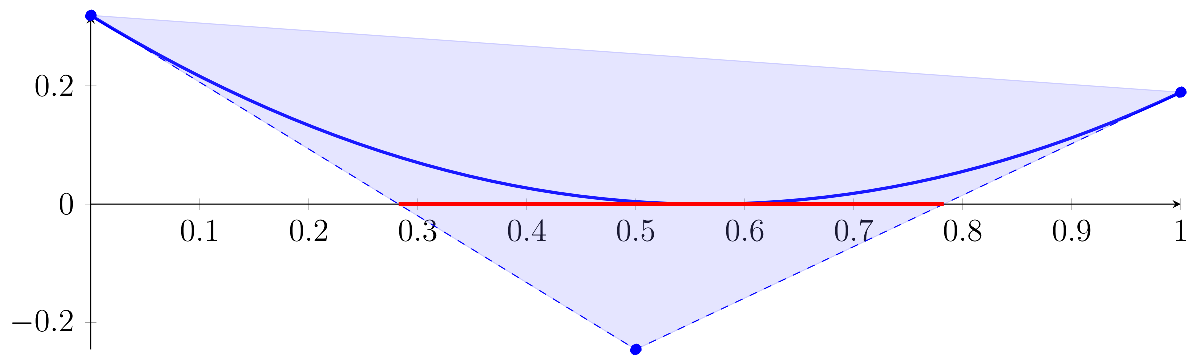
$$p = 1X^2 - 1.13X + 0.3192$$



19.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2824778761061947, 0.7825287356321839\}$$

Intersection intervals with the x axis:

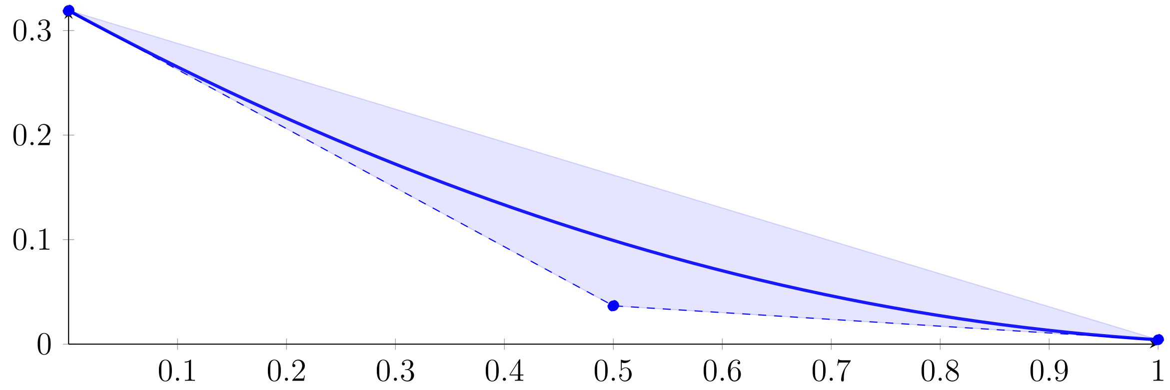
$$[0.2824778761061947, 0.7825287356321839]$$

Longest intersection interval: 0.5000508595259892
$\Longrightarrow$ Bisection: first half $[0, 0.5]$ und second half $[0.5, 1]$

19.2 Recursion Branch 1 1 on the First Half $[0, 0.5]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.565X + 0.3192 \\ &= 0.3192B_{0,2}(X) + 0.0367B_{1,2}(X) + 0.004200000000000001B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

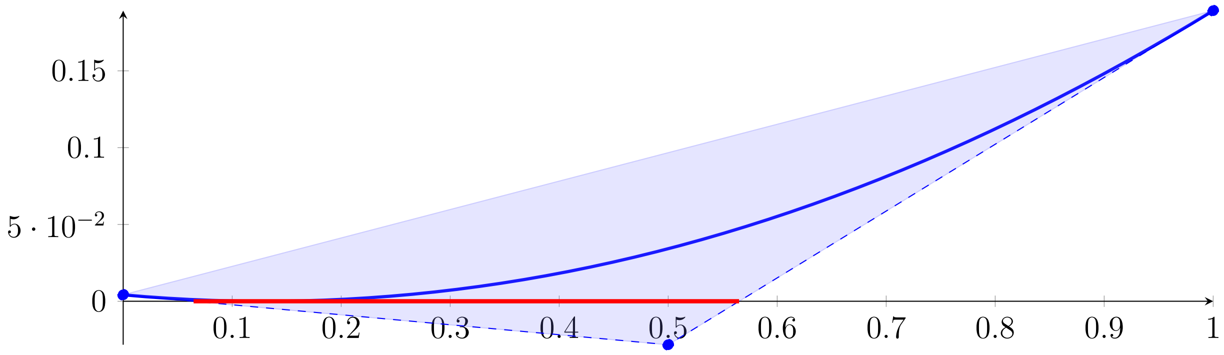
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

19.3 Recursion Branch 1 2 on the Second Half $[0.5, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.25X^2 - 0.065X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.0283B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06461538461538463, 0.5650574712643678\}$$

Intersection intervals with the x axis:

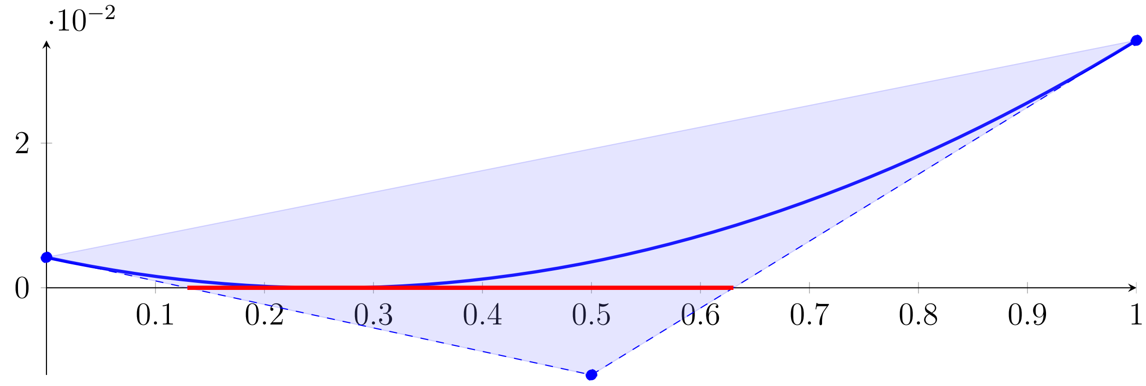
$$[0.06461538461538463, 0.5650574712643678]$$

Longest intersection interval: 0.5004420866489832
$\Longrightarrow$ Bisection: first half $[0.5, 0.75]$ und second half $[0.75, 1]$

19.4 Recursion Branch 1 2 1 on the First Half $[0.5, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 - 0.0325X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.01205B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1292307692307693, 0.6302702702702703\}$$

Intersection intervals with the x axis:

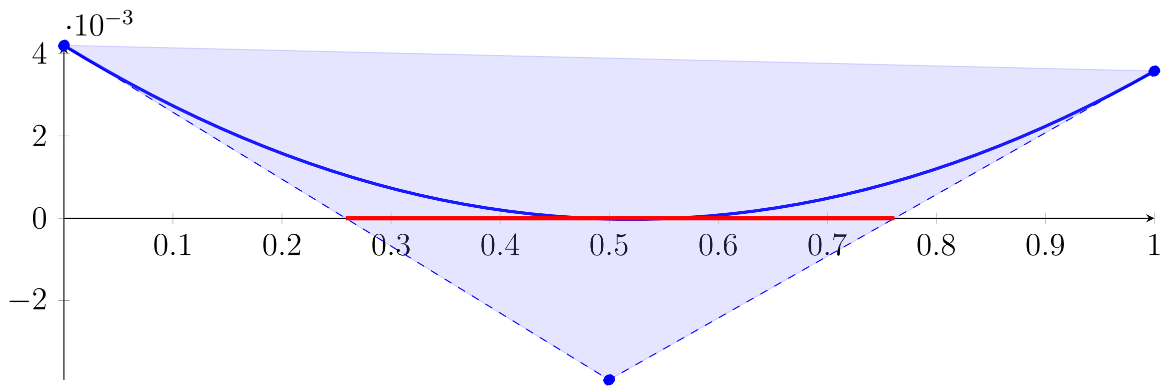
$$[0.1292307692307693, 0.6302702702702703]$$

Longest intersection interval: 0.501039501039501
$\Longrightarrow$ Bisection: first half $[0.5, 0.625]$ und second half $[0.625, 0.75]$

19.5 Recursion Branch 1 2 1 1 on the First Half $[0.5, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 - 0.01625X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) - 0.003924999999999999B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2584615384615385, 0.7616666666666666\}$$

Intersection intervals with the x axis:

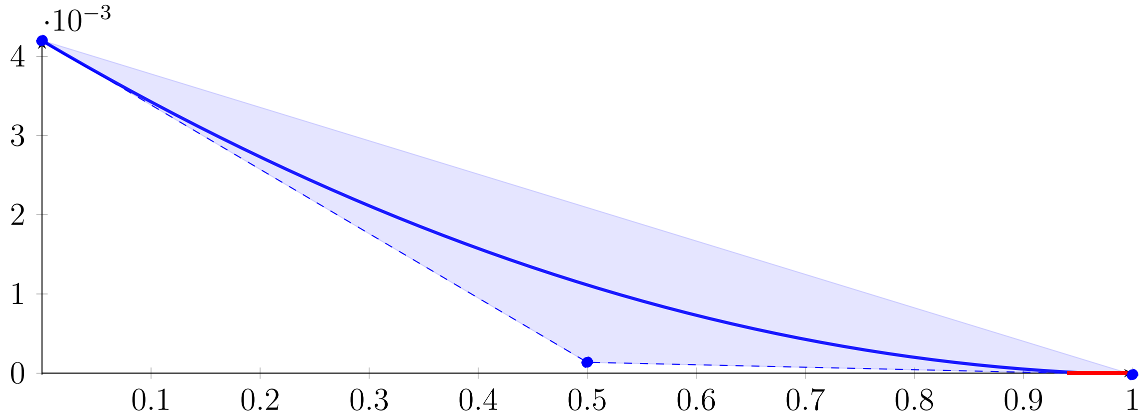
$$[0.2584615384615385, 0.7616666666666666]$$

Longest intersection interval: 0.5032051282051281
$\Longrightarrow$ Bisection: first half $[0.5, 0.5625]$ und second half $[0.5625, 0.625]$

19.6 Recursion Branch 1 2 1 1 1 on the First Half $[0.5, 0.5625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.00390625X^2 - 0.008125X + 0.004200000000000001 \\ &= 0.004200000000000001B_{0,2}(X) + 0.0001375000000000007B_{1,2}(X) \\ &\quad - 1.874999999999948 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9400000000000017, 0.9955555555555557\}$$

Intersection intervals with the x axis:

$$[0.9400000000000017, 0.9955555555555557]$$

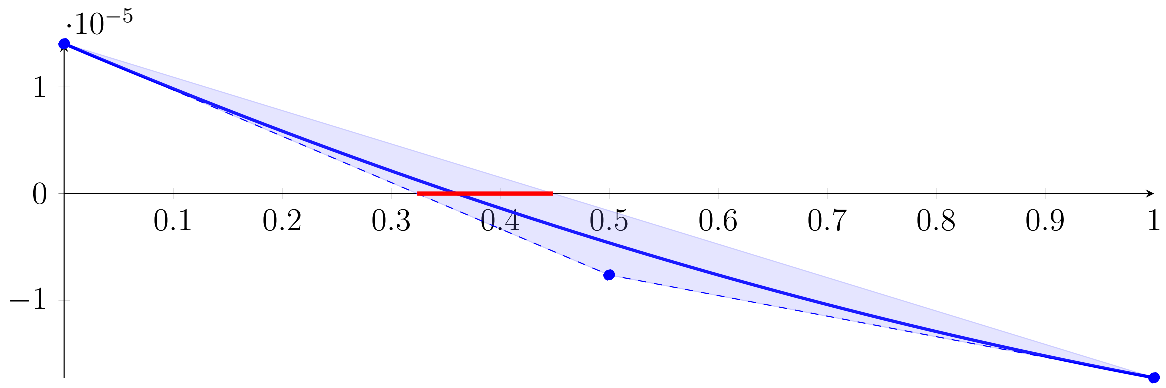
Longest intersection interval: 0.05555555555555396

$\Longrightarrow$ Selective recursion: interval 1: $[0.5587500000000001, 0.5622222222222222]$,

19.7 Recursion Branch 1 2 1 1 1 in Interval 1: $[0.5587500000000001, 0.56222222222$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.205632716049313 \cdot 10^{-05}X^2 - 4.340277777777758 \cdot 10^{-05}X + 1.406249999999919 \cdot 10^{-05} \\ &= 1.406249999999919 \cdot 10^{-05}B_{0,2}(X) - 7.638888888888706 \\ &\quad \cdot 10^{-06}B_{1,2}(X) - 1.728395061728347 \cdot 10^{-05}B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3239999999999962, 0.4486153846153773\}$$

Intersection intervals with the x axis:

$$[0.3239999999999962, 0.4486153846153773]$$

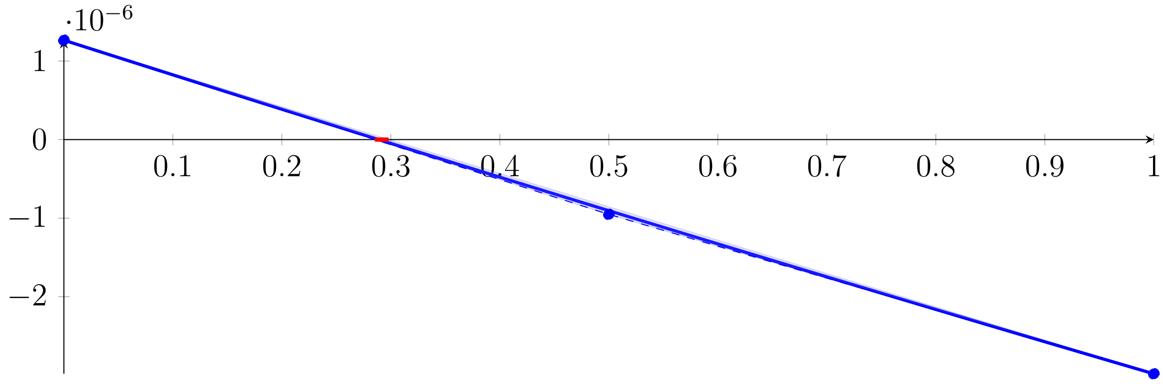
Longest intersection interval: 0.1246153846153811

$\Longrightarrow$ Selective recursion: interval 1: $[0.5598750000000001, 0.5603076923076923]$,

19.8 Recursion Branch 1 2 1 1 1 1 1 in Interval 1: $[0.5598750000000001, 0.56030769$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 1.872226331360733 \cdot 10^{-07} X^2 - 4.435096153845849 \cdot 10^{-06} X + 1.265624999999897 \cdot 10^{-06} \\ &= 1.265624999999897 \cdot 10^{-06} B_{0,2}(X) - 9.519230769230272 \\ &\quad \cdot 10^{-07} B_{1,2}(X) - 2.982248520709878 \cdot 10^{-06} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.285365853658533, 0.2979431929480858\}$$

Intersection intervals with the x axis:

$$[0.285365853658533, 0.2979431929480858]$$

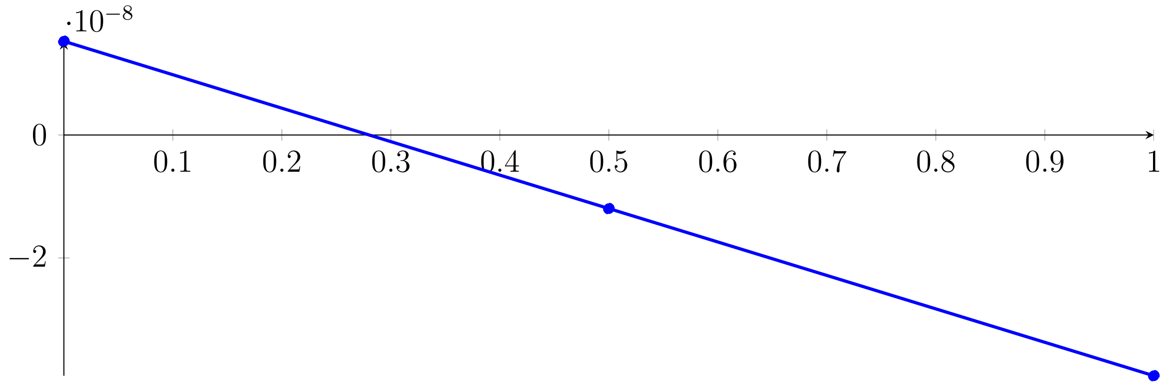
Longest intersection interval: 0.01257733928955277

$\Longrightarrow$ Selective recursion: interval 1: $[0.5599984756097562, 0.560003917727718]$,

19.9 Recursion Branch 1 2 1 1 1 1 1 1 in Interval 1: $[0.5599984756097562, 0.5600039$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 2.961664791042273 \cdot 10^{-11} X^2 - 5.443777144130786 \cdot 10^{-08} X + 1.524622620463798 \cdot 10^{-08} \\ &= 1.524622620463798 \cdot 10^{-08} B_{0,2}(X) - 1.197265951601595 \\ &\quad \cdot 10^{-08} B_{1,2}(X) - 3.916192858875946 \cdot 10^{-08} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2800670527278236, 0.2802195050086158\}$$

Intersection intervals with the x axis:

$$[0.2800670527278236, 0.2802195050086158]$$

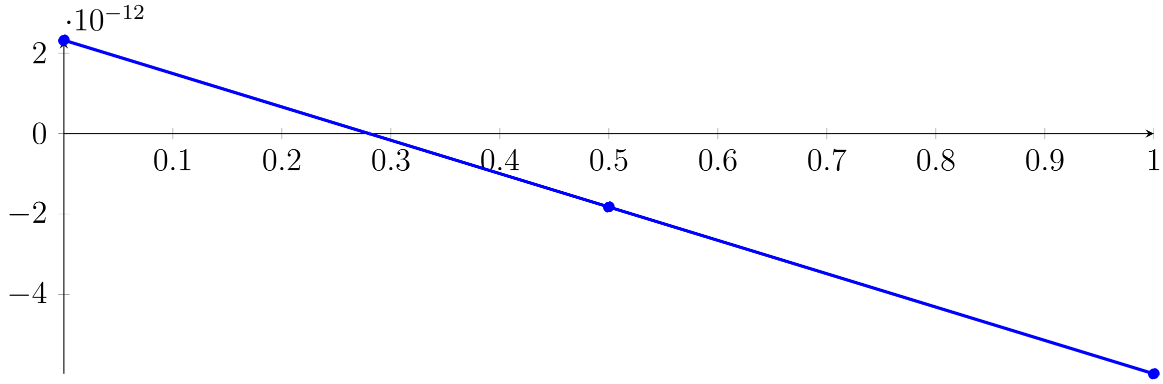
Longest intersection interval: 0.0001524522807922228

$\Longrightarrow$ Selective recursion: interval 1: $[0.5599999997676943, 0.5600000005973576]$,

19.10 Recursion Branch 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.5599999997676943, 0.5600$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.883411841000453 \cdot 10^{-19} X^2 - 8.296633341677063 \cdot 10^{-12} X + 2.323057420473189 \cdot 10^{-12}$$
$$= 2.323057420473189 \cdot 10^{-12} B_{0,2}(X) - 1.825259250365342$$
$$\cdot 10^{-12} B_{1,2}(X) - 5.97357523286269 \cdot 10^{-12} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2800000102214486, 0.2800000334520226\}$$

Intersection intervals with the x axis:

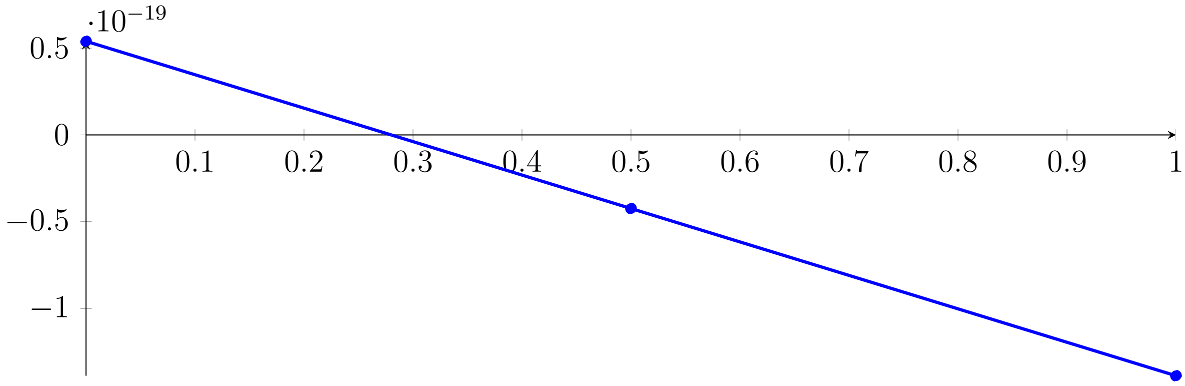
$$[0.2800000102214486, 0.2800000334520226]$$

Longest intersection interval: $2.323057397344962 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.56, 0.5600000000000001]$,

19.11 Recursion Branch 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.56, 0.5600000000000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 3.714699054532536 \cdot 10^{-34} X^2 - 1.927355456197032 \cdot 10^{-19} X + 5.396595277351629 \cdot 10^{-20}$$
$$= 5.396595277351629 \cdot 10^{-20} B_{0,2}(X) - 4.240182003633529$$
$$\cdot 10^{-20} B_{1,2}(X) - 1.387695928461865 \cdot 10^{-19} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999969, 0.2799999999999975\}$$

Intersection intervals with the x axis:

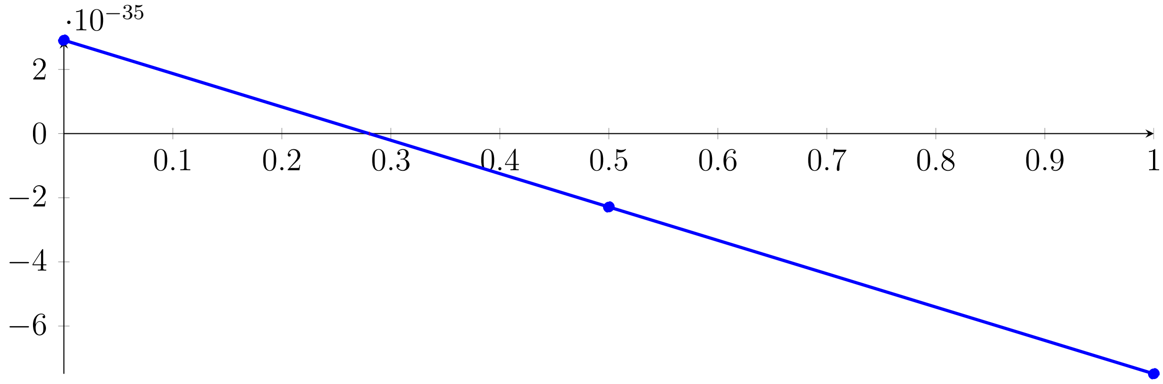
$$[0.2799999999999969, 0.2799999999999975]$$

Longest intersection interval: $5.396595277351738 \cdot 10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

19.12 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0.5$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 1.081840742754402 \cdot 10^{-64} X^2 - 1.040115735269099 \cdot 10^{-34} X + 2.912324058753444 \cdot 10^{-35}$$
$$= 2.912324058753444 \cdot 10^{-35} B_{0,2}(X) - 2.288254617592053$$
$$\cdot 10^{-35} B_{1,2}(X) - 7.488833293937551 \cdot 10^{-35} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999967, 0.2799999999999967\}$$

Intersection intervals with the x axis:

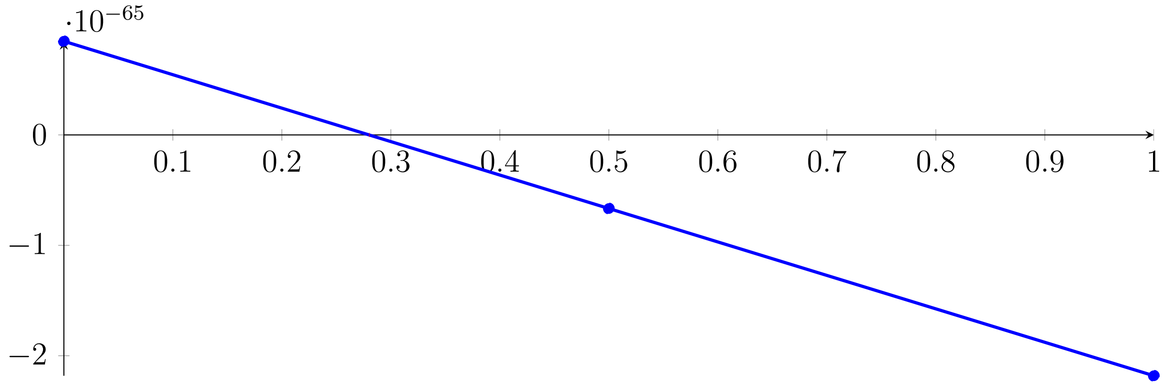
$$[0.2799999999999967, 0.2799999999999967]$$

Longest intersection interval: $2.912324058753504 \cdot 10^{-31}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

19.13 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 9.175774438637789 \cdot 10^{-126} X^2 - 3.029154079712289 \cdot 10^{-65} X + 8.481631423194308 \cdot 10^{-66}$$
$$= 8.481631423194308 \cdot 10^{-66} B_{0,2}(X) - 6.664138975367135$$
$$\cdot 10^{-66} B_{1,2}(X) - 2.180990937392858 \cdot 10^{-65} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999967, 0.2799999999999967\}$$

Intersection intervals with the x axis:

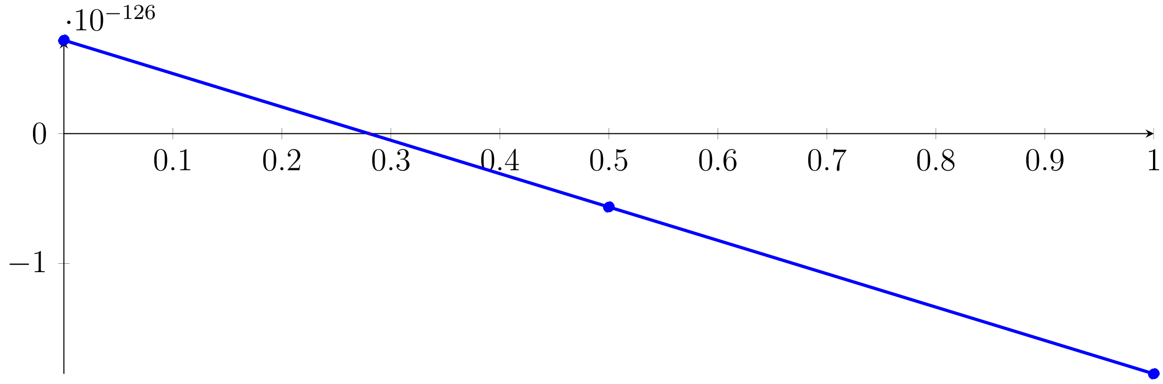
$$[0.2799999999999967, 0.2799999999999967]$$

Longest intersection interval: $8.481631423194481 \cdot 10^{-62}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

19.14 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.600875185422656 \cdot 10^{-248} X^2 - 2.569216842818551 \cdot 10^{-126} X + 7.193807159891857 \cdot 10^{-127}$$
$$= 7.193807159891857 \cdot 10^{-127} B_{0,2}(X) - 5.652277054200896$$
$$\cdot 10^{-127} B_{1,2}(X) - 1.849836126829365 \cdot 10^{-126} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2799999999999967, 0.2799999999999967\}$$

Intersection intervals with the x axis:

$$[0.2799999999999967, 0.2799999999999967]$$

Longest intersection interval: $7.193807159892004 \cdot 10^{-123}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$,

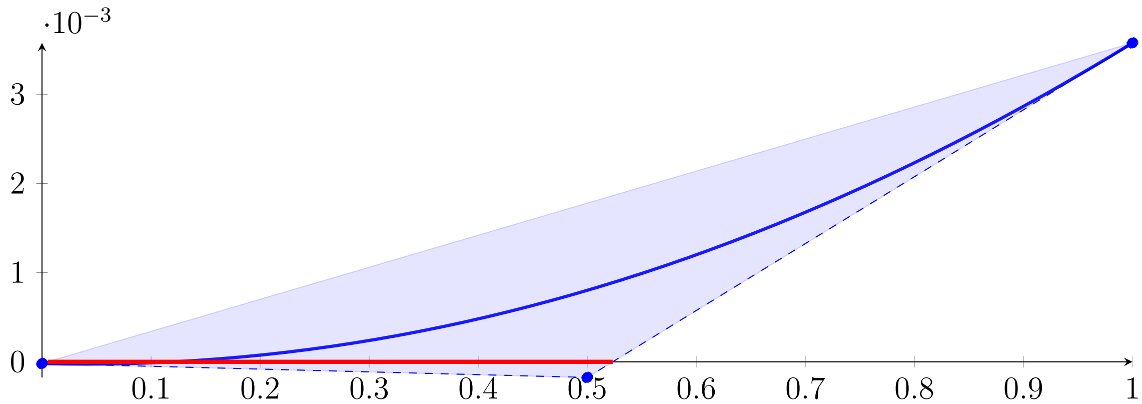
19.15 Recursion Branch 1 2 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 14!

19.16 Recursion Branch 1 2 1 1 2 on the Second Half $[0.5625, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 0.00390625 X^2 - 0.0003125000000000003 X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 0.0001749999999999996 B_{1,2}(X) + 0.003575 B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.005217391304347681, 0.5233333333333333\}$$

Intersection intervals with the x axis:

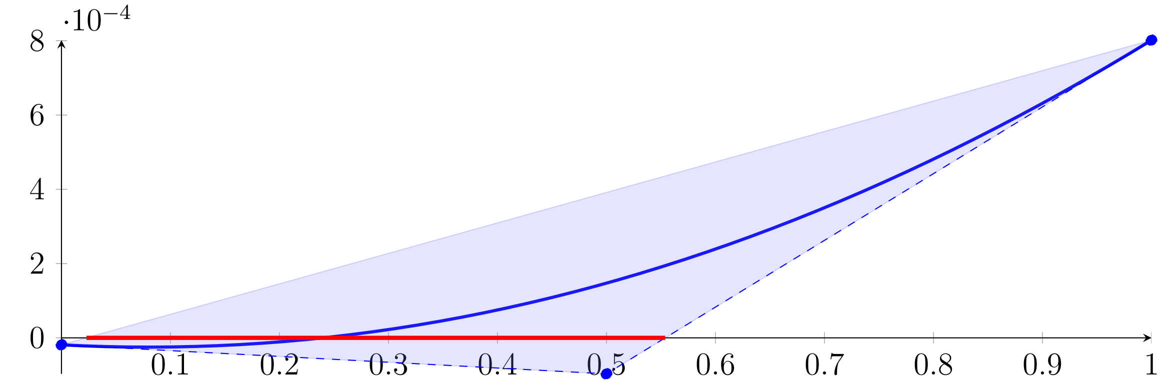
$$[0.005217391304347681, 0.5233333333333333]$$

Longest intersection interval: 0.5181159420289856
$\Longrightarrow$ Bisection: first half $[0.5625, 0.59375]$ und second half $[0.59375, 0.625]$

19.17 Recursion Branch 1 2 1 1 2 1 on the First Half $[0.5625, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 - 0.00015625000000000001X - 1.874999999999948\cdot10^{-05} \\ &= -1.874999999999948\cdot10^{-05}B_{0,2}(X) - 9.687499999999955 \\ &\quad \cdot10^{-05}B_{1,2}(X) + 0.0008015625000000004B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.022857142857142222, 0.5539130434782606\}$$

Intersection intervals with the x axis:

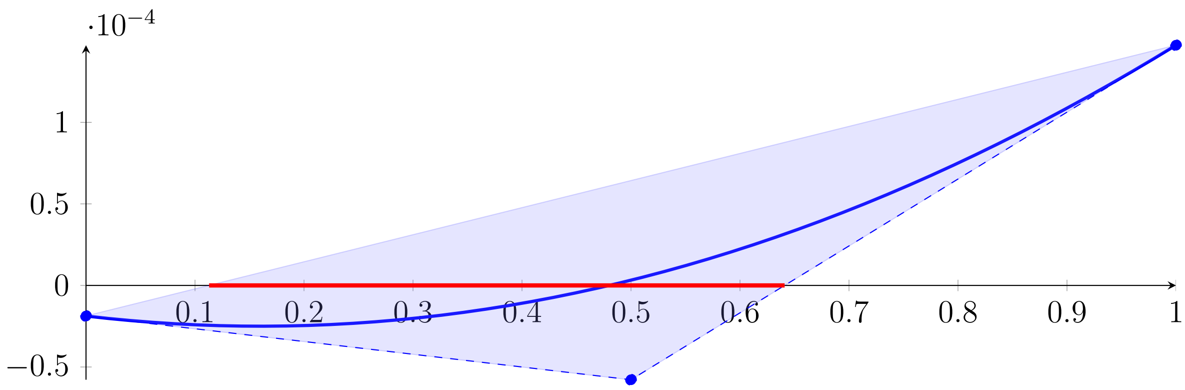
$$[0.022857142857142222, 0.5539130434782606]$$

Longest intersection interval: 0.5310559006211184
$\Longrightarrow$ Bisection: first half $[0.5625, 0.578125]$ und second half $[0.578125, 0.59375]$

19.18 Recursion Branch 1 2 1 1 2 1 1 on the First Half $[0.5625, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 - 7.812500000000007\cdot10^{-05}X - 1.874999999999948\cdot10^{-05} \\ &= -1.874999999999948\cdot10^{-05}B_{0,2}(X) - 5.781249999999951 \\ &\quad \cdot10^{-05}B_{1,2}(X) + 0.0001472656250000005B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1129411764705851, 0.6409523809523798\}$$

Intersection intervals with the x axis:

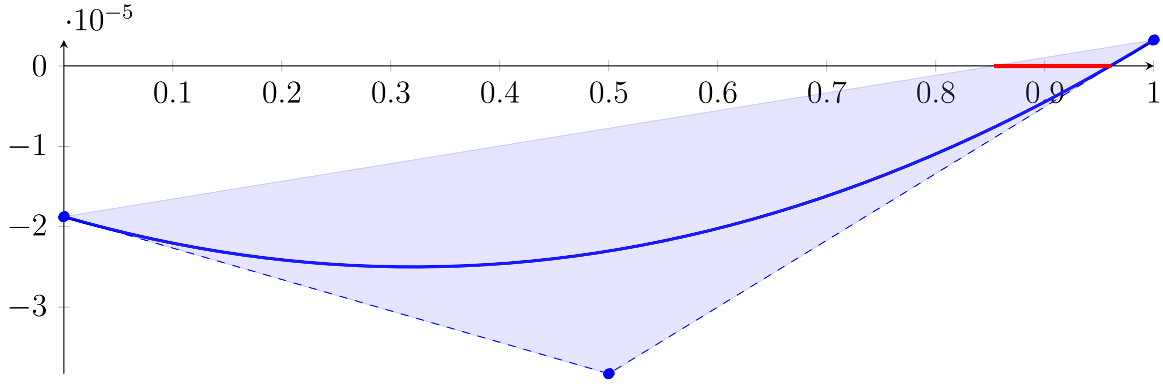
$$[0.1129411764705851, 0.6409523809523798]$$

Longest intersection interval: 0.5280112044817946
$\Longrightarrow$ Bisection: first half $[0.5625, 0.5703125]$ und second half $[0.5703125, 0.578125]$

19.19 Recursion Branch 1 2 1 1 2 1 1 1 on the First Half $[0.5625, 0.5703125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.103515625 \cdot 10^{-05} X^2 - 3.906250000000003 \cdot 10^{-05} X - 1.874999999999948 \cdot 10^{-05}$$
$$= -1.874999999999948 \cdot 10^{-05} B_{0,2}(X) - 3.82812499999995$$
$$\cdot 10^{-05} B_{1,2}(X) + 3.222656250000487 \cdot 10^{-06} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.8533333333333109, 0.9611764705882294\}$$

Intersection intervals with the x axis:

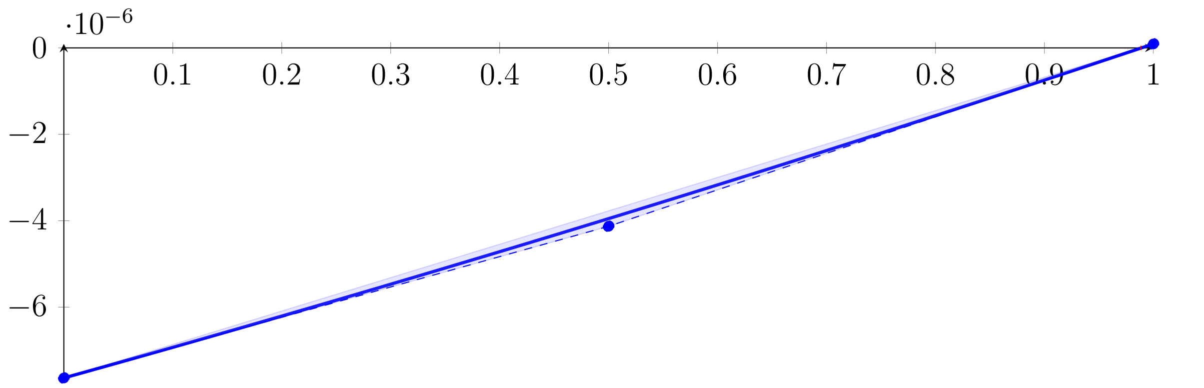
$$[0.8533333333333109, 0.9611764705882294]$$

Longest intersection interval: 0.1078431372549185
$\Longrightarrow$ Selective recursion: interval 1: $[0.5691666666666665, 0.5700091911764705]$,

19.20 Recursion Branch 1 2 1 1 2 1 1 1 1 in Interval 1: $[0.5691666666666665, 0.5700$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 7.098475496205558 \cdot 10^{-07} X^2 + 7.021037581700123 \cdot 10^{-06} X - 7.638888888889855 \cdot 10^{-06}$$
$$= -7.638888888889855 \cdot 10^{-06} B_{0,2}(X) - 4.128370098039794$$
$$\cdot 10^{-06} B_{1,2}(X) + 9.199624243082374 \cdot 10^{-08} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9881001669449061, 0.9891009174311909\}$$

Intersection intervals with the x axis:

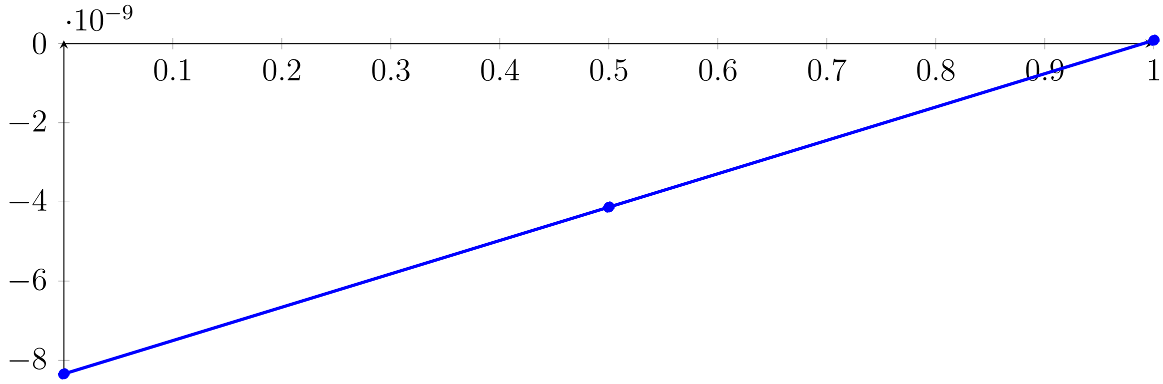
$$[0.9881001669449061, 0.9891009174311909]$$

Longest intersection interval: 0.001000750486284818
$\Longrightarrow$ Selective recursion: interval 1: $[0.569999165275459, 0.5700000084322719]$,

19.21 Recursion Branch 1 2 1 1 2 1 1 1 1 1 in Interval 1: $[0.569999165275459, 0.570$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 7.109134111283559 \cdot 10^{-13} X^2 + 8.430160521565627 \cdot 10^{-09} X - 8.346548643959838 \cdot 10^{-09} \\ &= -8.346548643959838 \cdot 10^{-09} B_{0,2}(X) - 4.131468383177024 \\ &\quad \cdot 10^{-09} B_{1,2}(X) + 8.432279101691722 \cdot 10^{-11} B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.989998330342568, 0.989999173638737\}$$

Intersection intervals with the x axis:

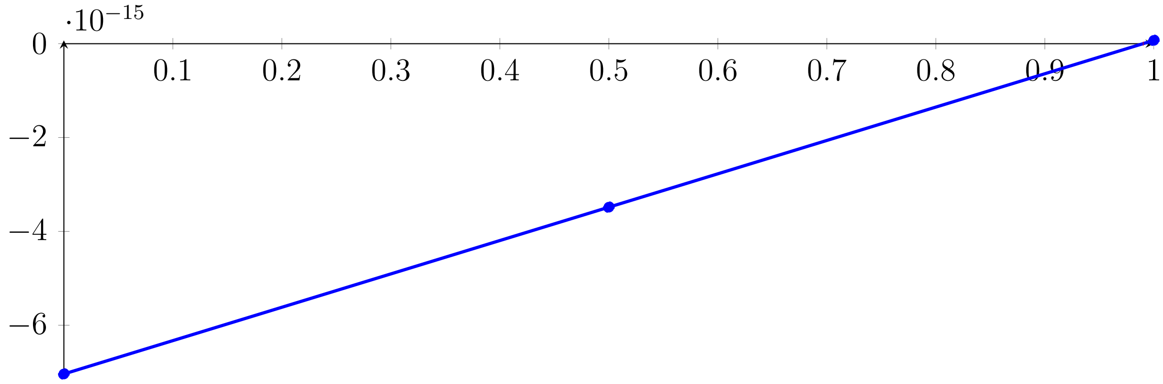
$$[0.989998330342568, 0.989999173638737]$$

Longest intersection interval: $8.432961690166558 \cdot 10^{-07}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5699999999999296, 0.5700000000000071]$,

19.22 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.5699999999999296, 0.57$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 5.055649552501669 \cdot 10^{-25} X^2 + 7.110309100930884 \cdot 10^{-15} X - 7.039206010412064 \cdot 10^{-15} \\ &= -7.039206010412064 \cdot 10^{-15} B_{0,2}(X) - 3.484051459946622 \\ &\quad \cdot 10^{-15} B_{1,2}(X) + 7.1103091024385 \cdot 10^{-17} B_{2,2}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999985907, 0.9899999999993017\}$$

Intersection intervals with the x axis:

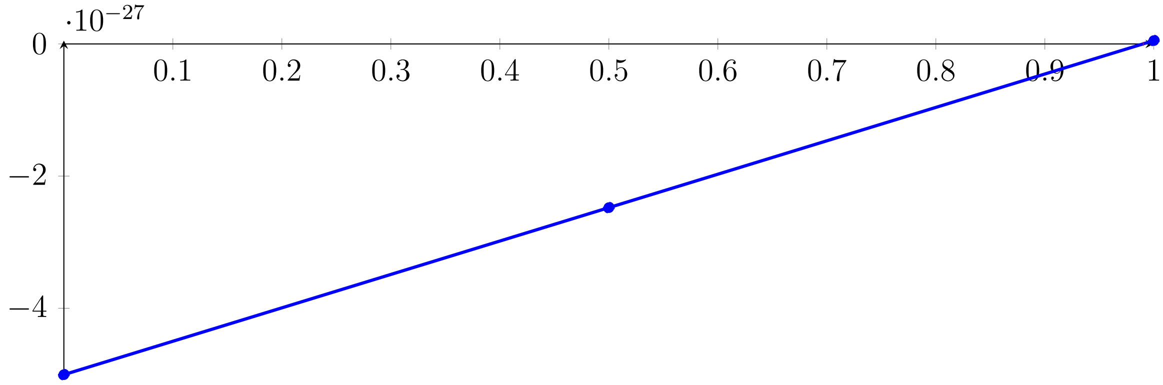
$$[0.9899999999985907, 0.9899999999993017]$$

Longest intersection interval: $7.110309102923987 \cdot 10^{-13}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

19.23 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 2.555959240484234 \cdot 10^{-49} X^2 + 5.055649553206969 \cdot 10^{-27} X - 5.005093057674892 \cdot 10^{-27}$$
$$= -5.005093057674892 \cdot 10^{-27} B_{0,2}(X) - 2.477268281071407$$
$$\cdot 10^{-27} B_{1,2}(X) + 5.055649553207703 \cdot 10^{-29} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999999985, 0.9899999999999985\}$$

Intersection intervals with the x axis:

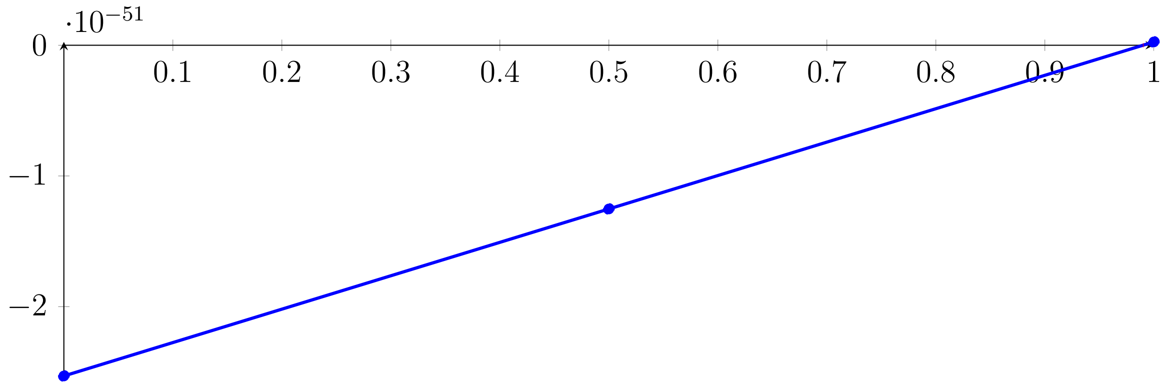
$$[0.9899999999999985, 0.9899999999999985]$$

Longest intersection interval: $5.055649553207806 \cdot 10^{-25}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

19.24 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 6.532927639018775 \cdot 10^{-98} X^2 + 2.555959240484605 \cdot 10^{-51} X - 2.530399648079756 \cdot 10^{-51}$$
$$= -2.530399648079756 \cdot 10^{-51} B_{0,2}(X) - 1.252420027837453$$
$$\cdot 10^{-51} B_{1,2}(X) + 2.555959240484977 \cdot 10^{-53} B_{2,2}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999999985, 0.9899999999999985\}$$

Intersection intervals with the x axis:

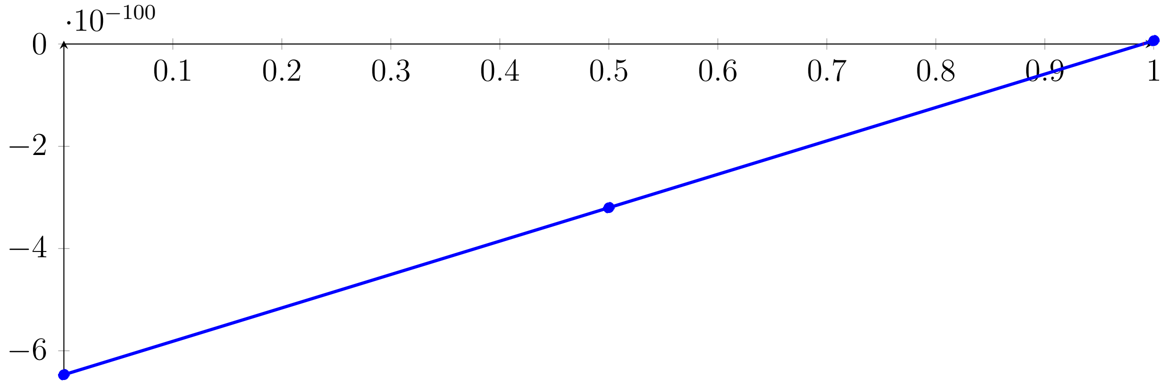
$$[0.9899999999999985, 0.9899999999999985]$$

Longest intersection interval: $2.555959240485029 \cdot 10^{-49}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

19.25 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 4.267914353666869 \cdot 10^{-195} X^2 + 6.532927639019723 \cdot 10^{-100} X - 6.467598362629517 \cdot 10^{-100} \\ &= -6.467598362629517 \cdot 10^{-100} B_{0,2}(X) - 3.201134543119655 \\ &\quad \cdot 10^{-100} B_{1,2}(X) + 6.532927639020672 \cdot 10^{-102} B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9899999999999985, 0.9899999999999985\}$$

Intersection intervals with the x axis:

$$[0.9899999999999985, 0.9899999999999985]$$

Longest intersection interval: $6.532927639020805 \cdot 10^{-98}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.57, 0.57]$,

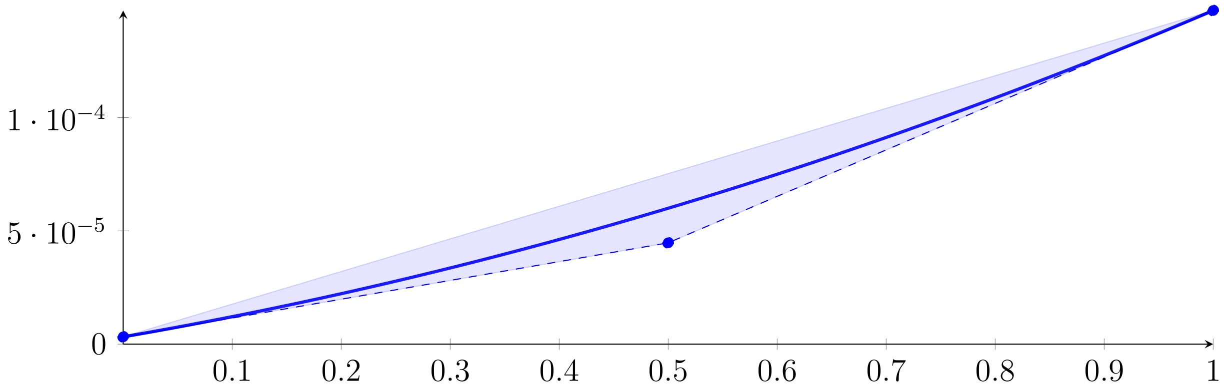
19.26 Recursion Branch 1 2 1 1 2 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 15!

19.27 Recursion Branch 1 2 1 1 2 1 1 2 on the Second Half $[0.5703125, 0.578125]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 6.103515625 \cdot 10^{-05} X^2 + 8.300781249999997 \cdot 10^{-05} X + 3.222656250000487 \cdot 10^{-06} \\ &= 3.222656250000487 \cdot 10^{-06} B_{0,2}(X) + 4.472656250000047 \\ &\quad \cdot 10^{-05} B_{1,2}(X) + 0.0001472656250000005 B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

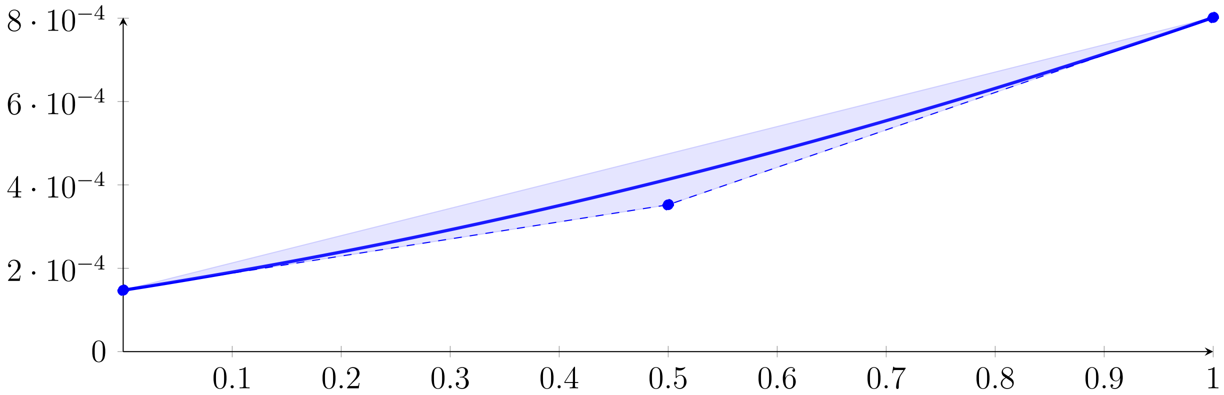
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

19.28 Recursion Branch 1 2 1 1 2 1 2 on the Second Half $[0.578125, 0.59375]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.000244140625X^2 + 0.00041015624999999999X + 0.00014726562500000005 \\ &= 0.00014726562500000005B_{0,2}(X) + 0.00035234375000000004B_{1,2}(X) \\ &\quad + 0.00080156250000000004B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

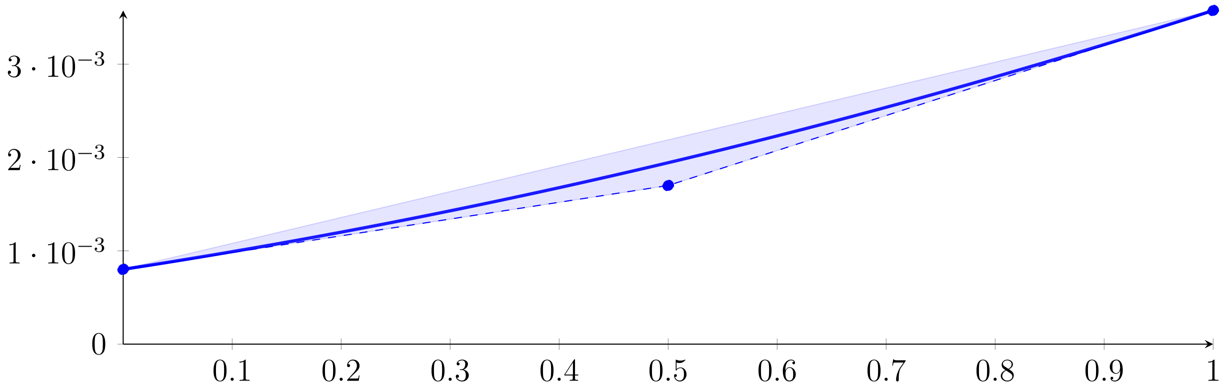
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

19.29 Recursion Branch 1 2 1 1 2 2 on the Second Half $[0.59375, 0.625]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0009765625X^2 + 0.001796875X + 0.00080156250000000004 \\ &= 0.00080156250000000004B_{0,2}(X) + 0.0017B_{1,2}(X) + 0.003575B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

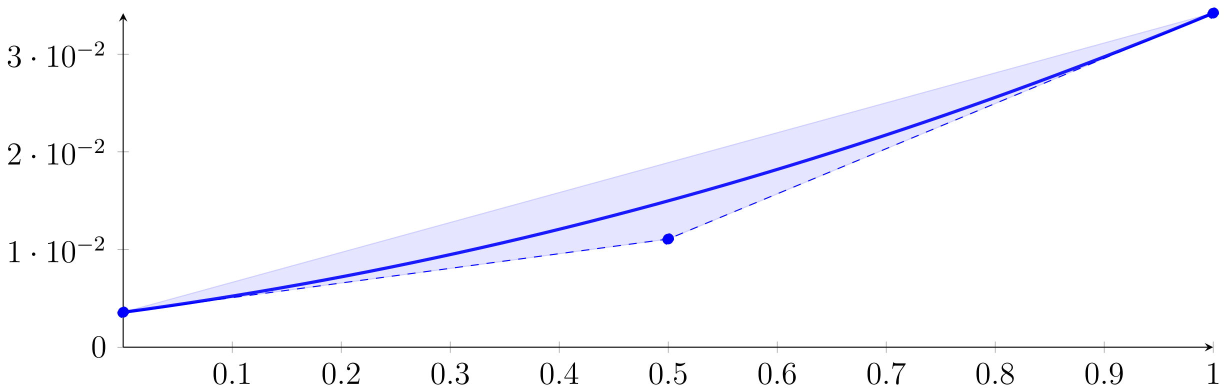
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

19.30 Recursion Branch 1 2 1 2 on the Second Half $[0.625, 0.75]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.015625X^2 + 0.015X + 0.003575 \\ &= 0.003575B_{0,2}(X) + 0.011075B_{1,2}(X) + 0.0342B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

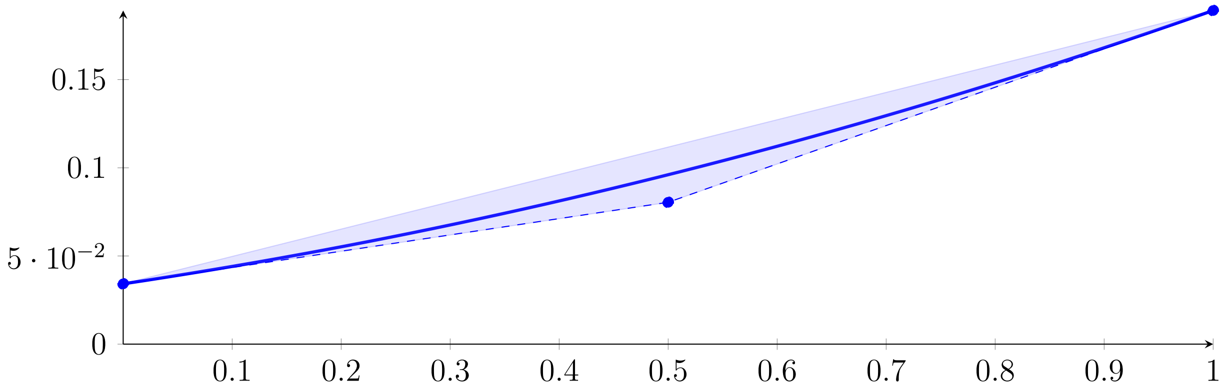
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

19.31 Recursion Branch 1 2 2 on the Second Half $[0.75, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= 0.0625X^2 + 0.0925X + 0.0342 \\ &= 0.0342B_{0,2}(X) + 0.08045B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

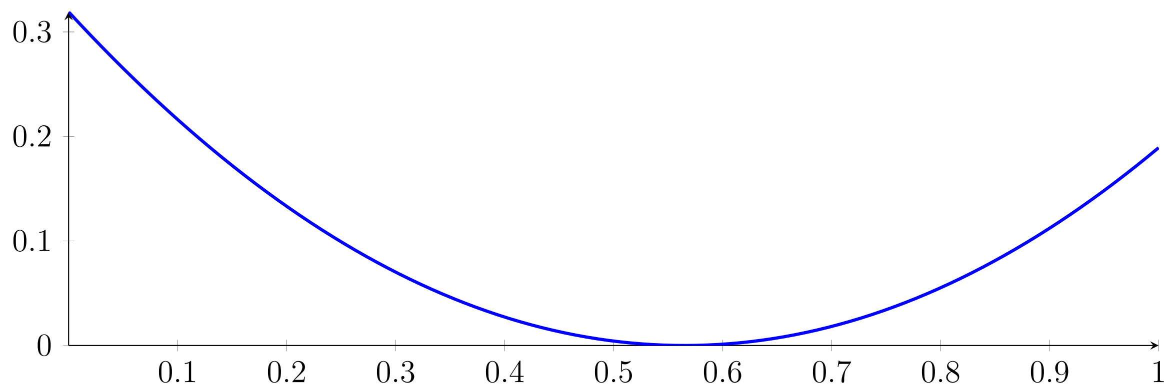
$$\{\}$$

Intersection intervals with the x axis:
No intersection with the x axis. Done.

19.32 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

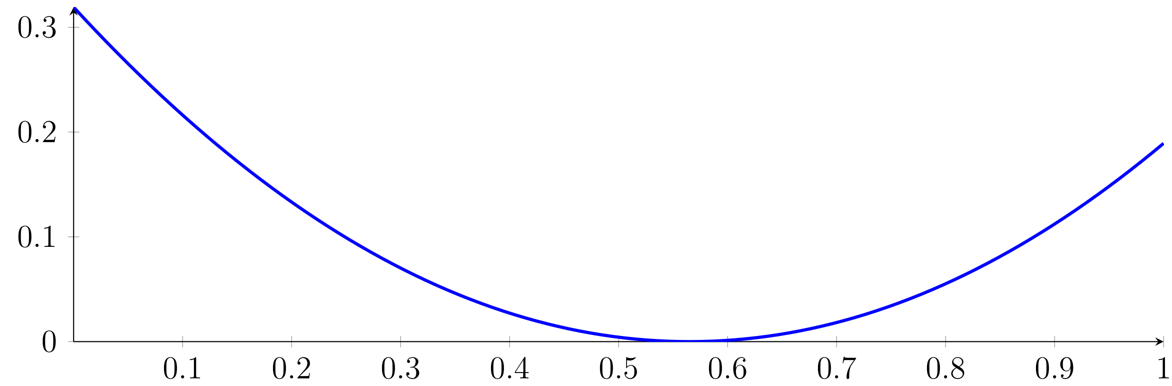
with precision $\varepsilon = 1 \cdot 10^{-128}$.

20 Running QuadClip on h_2 with epsilon 128

$$1X^2 - 1.13X + 0.3192$$

Called QuadClip with input polynomial on interval $[0,1]$:

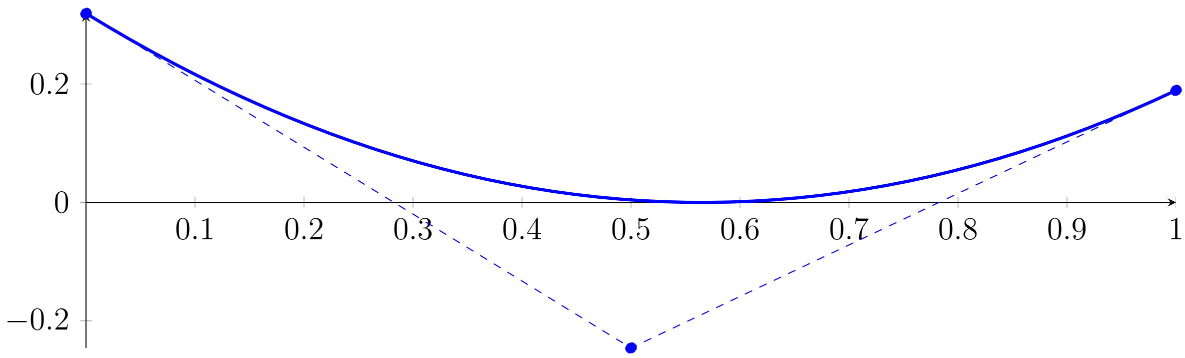
$$p = 1X^2 - 1.13X + 0.3192$$



20.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

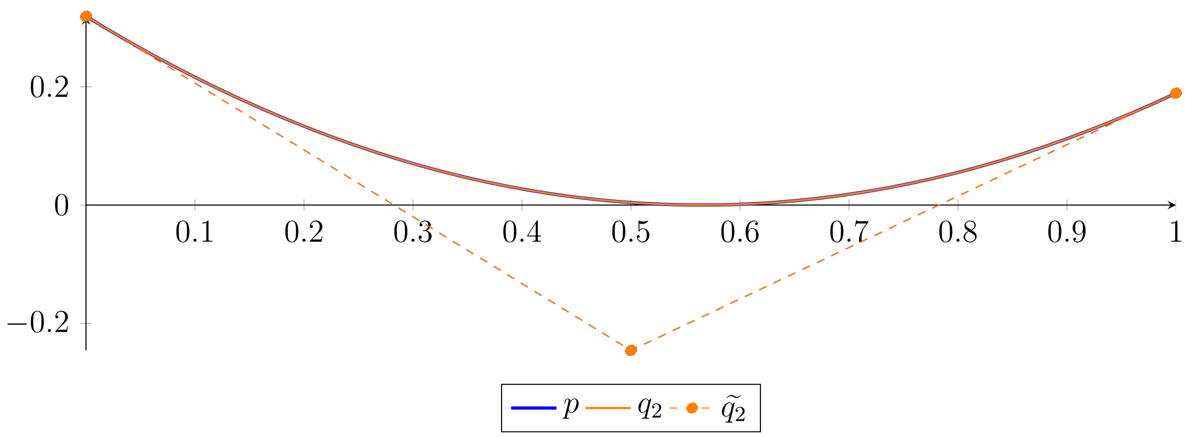
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.344026969402005 \cdot 10^{-309}$.
Bounding polynomials M and m:

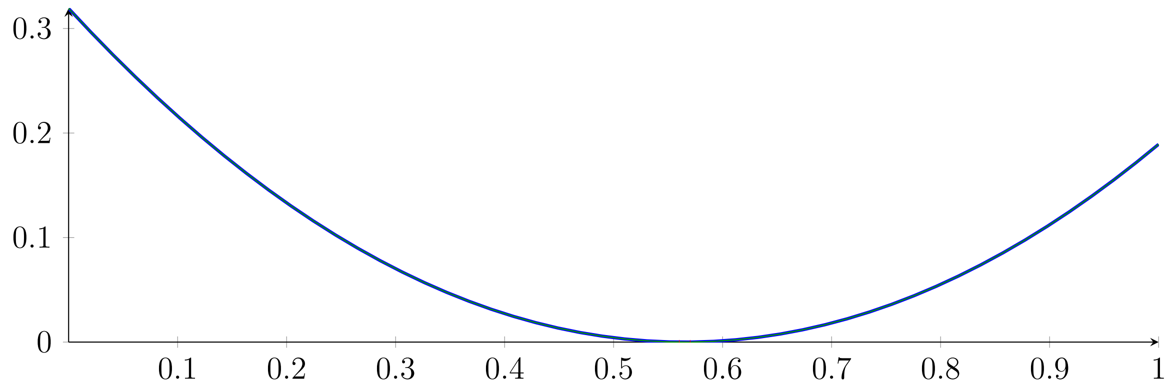
$$M = 1X^2 - 1.13X + 0.3192$$

$$m = 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$$N(M) = \{0.5600000000000001, 0.57\} \qquad N(m) = \{0.5600000000000001, 0.57\}$$

Intersection intervals:



$$[0.5600000000000001, 0.5600000000000001],\ [0.57, 0.57]$$

Longest intersection interval: $1.668805393880401 \cdot 10^{-306}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5600000000000001, 0.5600000000000001]$, interval 2: $[0.57, 0.57]$,

20.2 Recursion Branch 1 1 in Interval 1: $[0.5600000000000001, 0.5600000000000001]$

Found root in interval $[0.5600000000000001, 0.5600000000000001]$ at recursion depth 2!

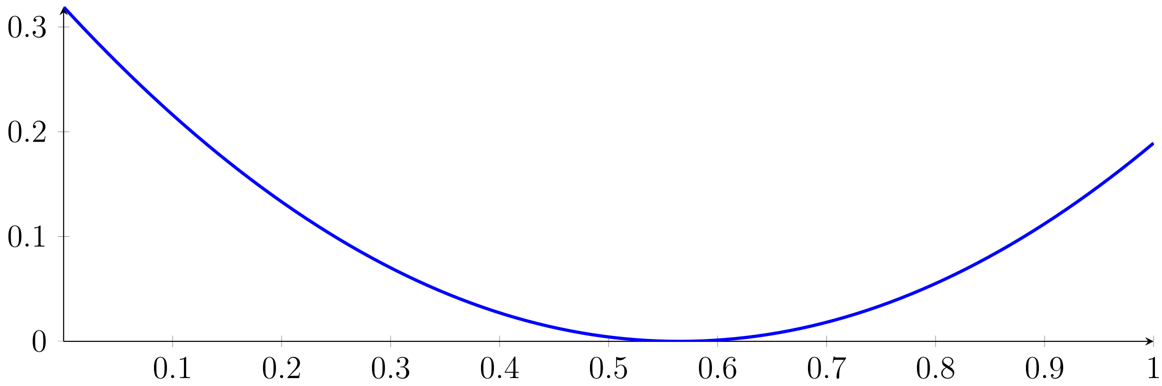
20.3 Recursion Branch 1 2 in Interval 2: $[0.57, 0.57]$

Found root in interval $[0.57, 0.57]$ at recursion depth 2!

20.4 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

$$[0.5600000000000001, 0.5600000000000001], [0.57, 0.57]$$

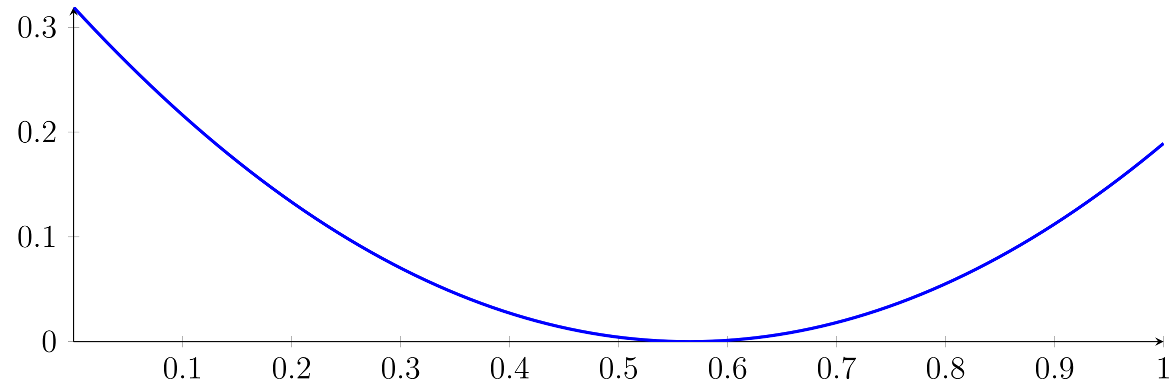
with precision $\varepsilon = 1 \cdot 10^{-128}$.

21 Running `CubeClip` on h_2 with epsilon 128

$$1X^2 - 1.13X + 0.3192$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

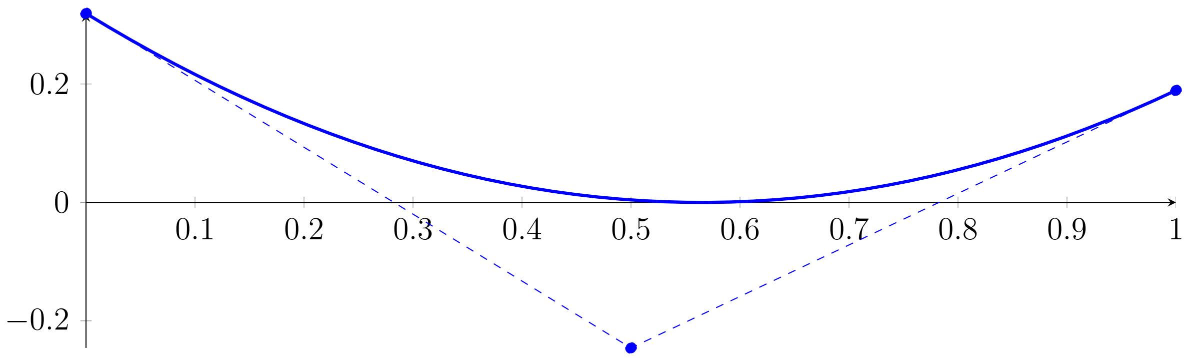
$$p = 1X^2 - 1.13X + 0.3192$$



21.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

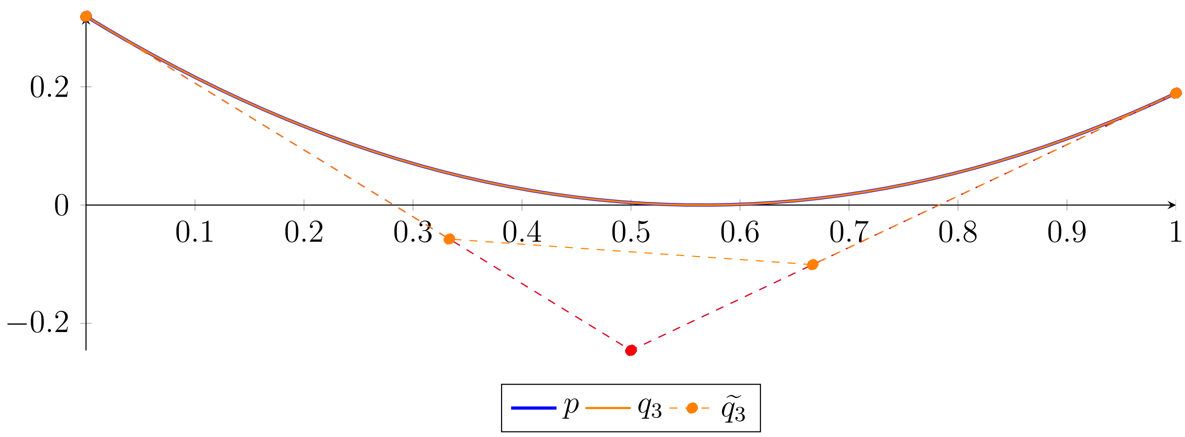
$$\begin{aligned} p &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2}(X) - 0.2458B_{1,2}(X) + 0.1892B_{2,2}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -2.225073858507201 \cdot 10^{-308}X^3 + 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,3} - 0.05746666666666667B_{1,3} - 0.1008B_{2,3} + 0.1892B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1X^2 - 1.13X + 0.3192 \\ &= 0.3192B_{0,2} - 0.2458B_{1,2} + 0.1892B_{2,2} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.086006742350501 \cdot 10^{-308}$.
Bounding polynomials M and m:

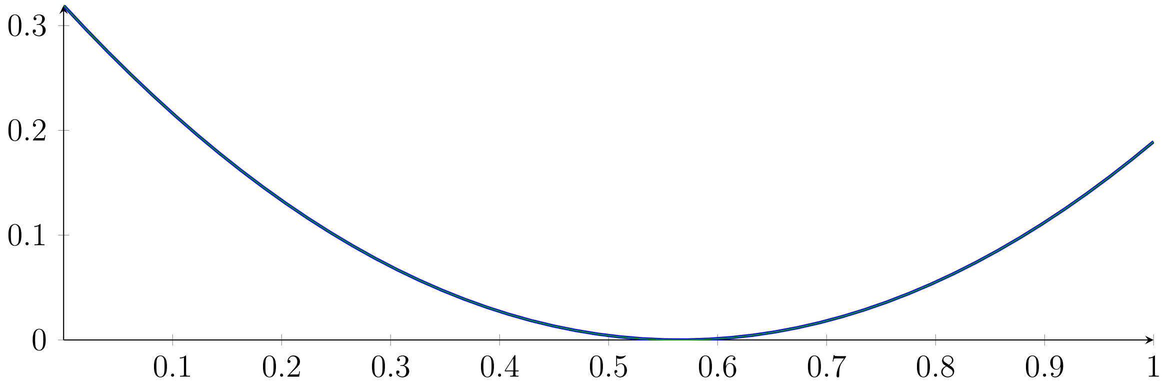
$$M = -1.668805393880401 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

$$m = -1.946939626193801 \cdot 10^{-308} X^3 + 1X^2 - 1.13X + 0.3192$$

Root of M and m:

$N(M) = \{-5.649152076031783 \cdot 10^{291}, 2.824576038015892 \cdot 10^{291}, 5.992310449541053 \cdot 10^{307}\}$ $\quad N(m) = \{-4.84$

Intersection intervals:

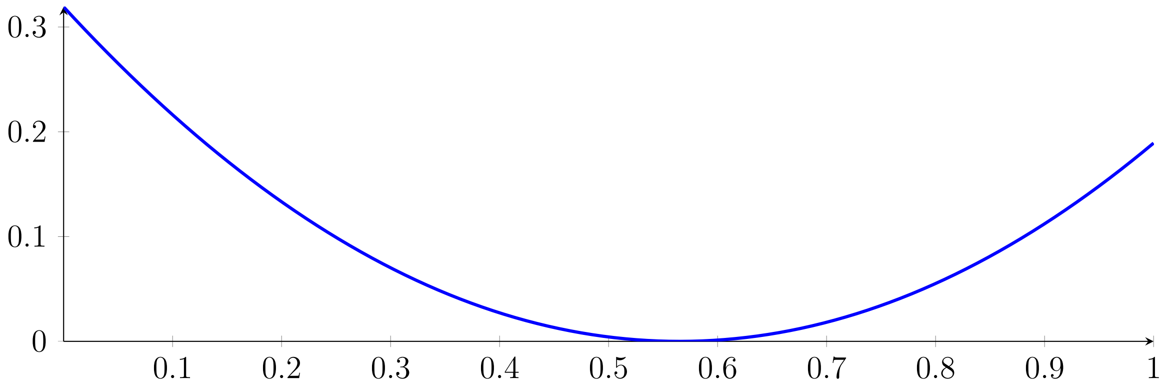


No intersection intervals with the x axis.

21.2 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = 1X^2 - 1.13X + 0.3192$$



Result: Root Intervals

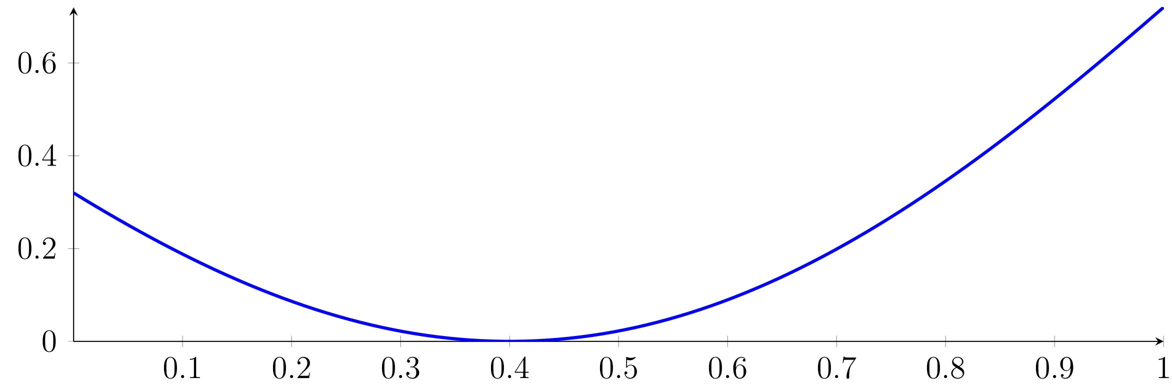
with precision $\varepsilon = 1 \cdot 10^{-128}$.

22 Running `BezClip` on h_4 with epsilon 2

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

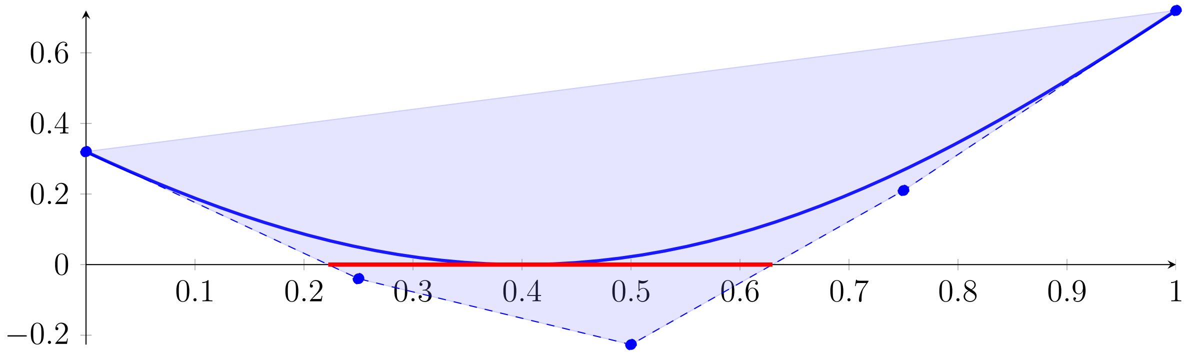
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



22.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

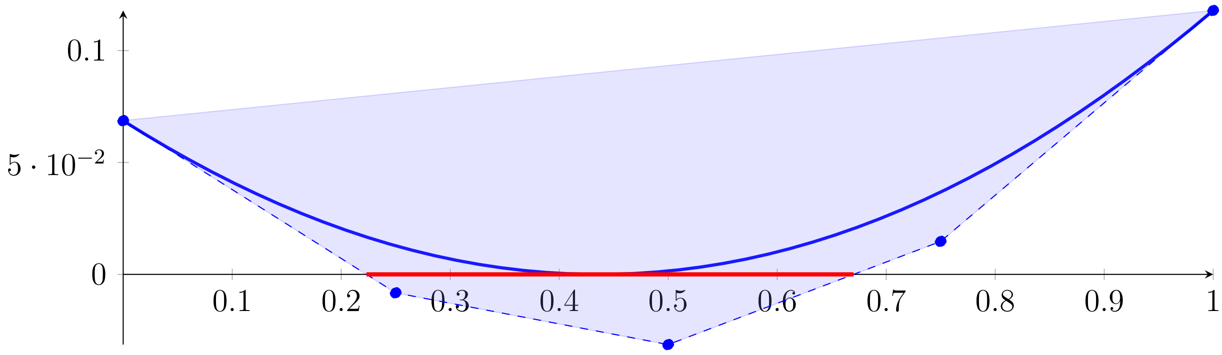
$$[0.222222225308642, 0.6297709955349339]$$

Longest intersection interval: 0.407548770226292
$\Longrightarrow$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

22.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &\quad + 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ &= 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &\quad + 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

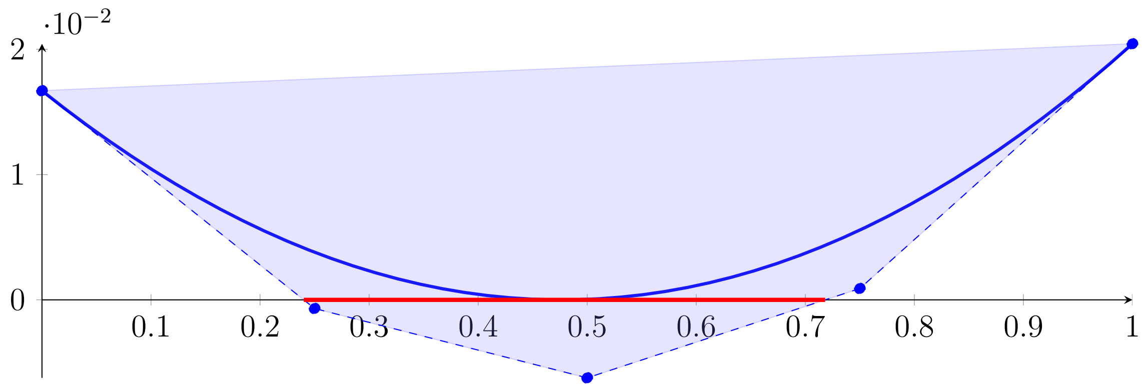
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\implies$ Selective recursion: interval 1: $[0.3131782986367004, 0.4953216655933138]$,

22.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.495321665593313$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -0.0011006606529341 82X^4 + 0.003307158935864367X^3 \\ &\quad + 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377 \\ &= 0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X) \\ &\quad + 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

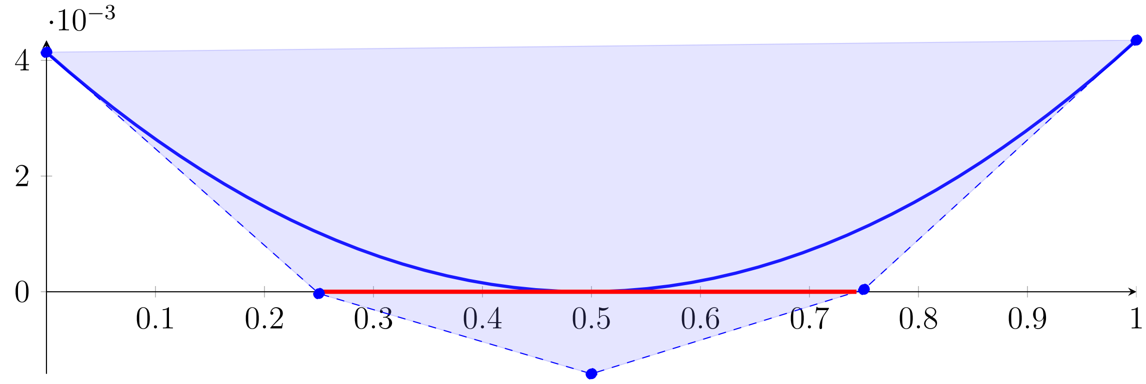
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\implies$ Selective recursion: interval 1: $[0.3569093751201798, 0.4439937820951003]$,

22.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.44399378209510$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -5.751241374789061\cdot 10^{-05}X^4 + 0.0002459162030906773X^3 \\ &\quad + 0.01670691422385101X^2 - 0.01668640845758443X + 0.004139787469617577 \\ &= 0.004139787469617577B_{0,4}(X) - 3.181464477852956\cdot 10^{-05}B_{1,4}(X) \\ &\quad - 0.001418931055199467B_{2,4}(X) + 3.991728912743387 \\ &\quad \cdot 10^{-05}B_{3,4}(X) + 0.004348697025226952B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

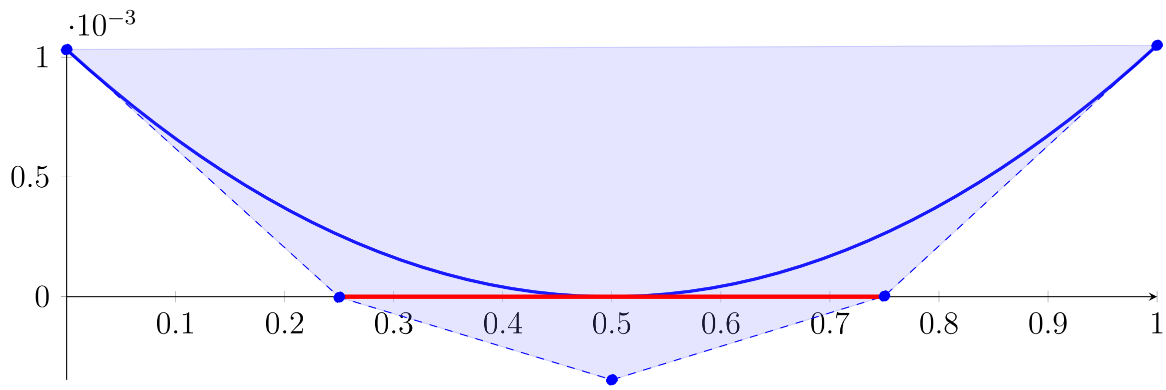
$$[0.2480933797192248, 0.7431594518918532]$$

Longest intersection interval: 0.4950660721726284
$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

22.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.454731120985874\cdot 10^{-06}X^4 + 2.291337271769576\cdot 10^{-05}X^3 \\ &\quad + 0.004134358001933594X^2 - 0.004136159738306538X + 0.001031853315293836 \\ &= 0.001031853315293836B_{0,4}(X) - 2.186619282798525\cdot 10^{-06}B_{1,4}(X) \\ &\quad - 0.0003471668868705005B_{2,4}(X) + 2.640855710153786 \\ &\quad \cdot 10^{-06}B_{3,4}(X) + 0.001049510220517602B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

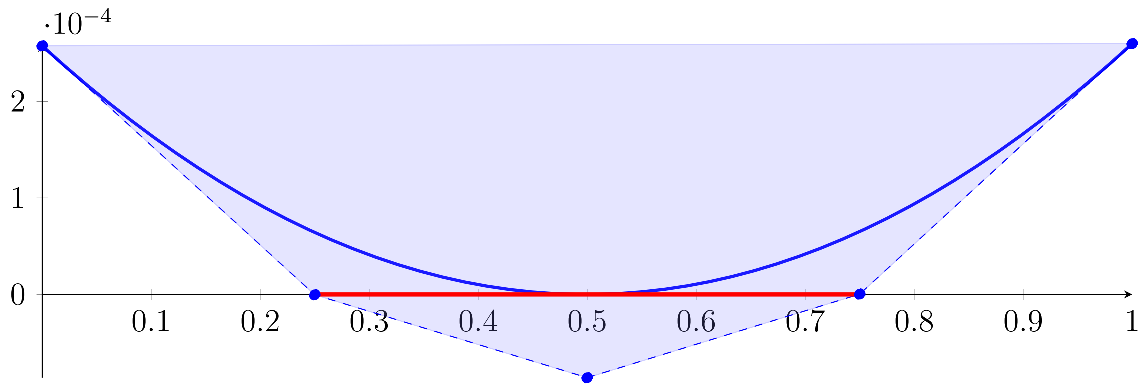
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\Longrightarrow$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

22.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.4107674724$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.135832675829806\cdot10^{-07}X^4 + 2.413460912088167\cdot10^{-06}X^3\\ &+ 0.001031922910124016X^2 - 0.001031832710654165X + 0.0002576480709401398\\ = &\ 0.0002576480709401398B_{0,4}(X) - 3.101067234014613\cdot10^{-07}B_{1,4}(X) - 8.628113269960673\\ &\cdot10^{-05}B_{2,4}(X) + 3.383582395460159\cdot10^{-07}B_{3,4}(X) + 0.0002599381480544958B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

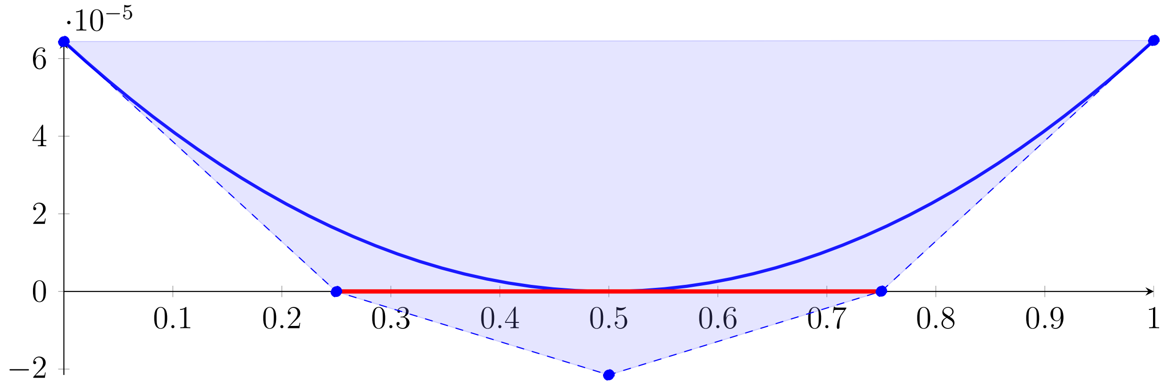
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\Longrightarrow$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

22.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.405372055$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.327690666746824\cdot10^{-08}X^4 + 2.739027984571277\cdot10^{-07}X^3\\ &+ 0.000257714430407974X^2 - 0.0002576778286693267X + 6.437695235811399\cdot10^{-05}\\ = &\ 6.437695235811399\cdot10^{-05}B_{0,4}(X) - 4.250480921768141\cdot10^{-08}B_{1,4}(X) - 2.150955690855369\\ &\cdot10^{-05}B_{2,4}(X) + 4.427175972025921\cdot10^{-08}B_{3,4}(X) + 6.467417998855097\cdot10^{-05}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\Longrightarrow$ Selective recursion: interval 1: $[0.3973195510833879, 0.4026829657906133]$,

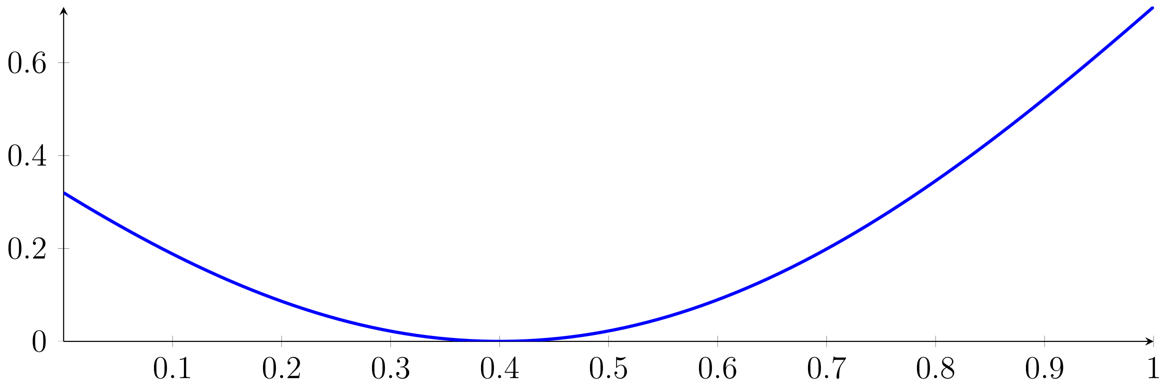
22.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Found root in interval $[0.3973195510833879, 0.4026829657906133]$ at recursion depth 8!

22.9 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3973195510833879, 0.4026829657906133]$$

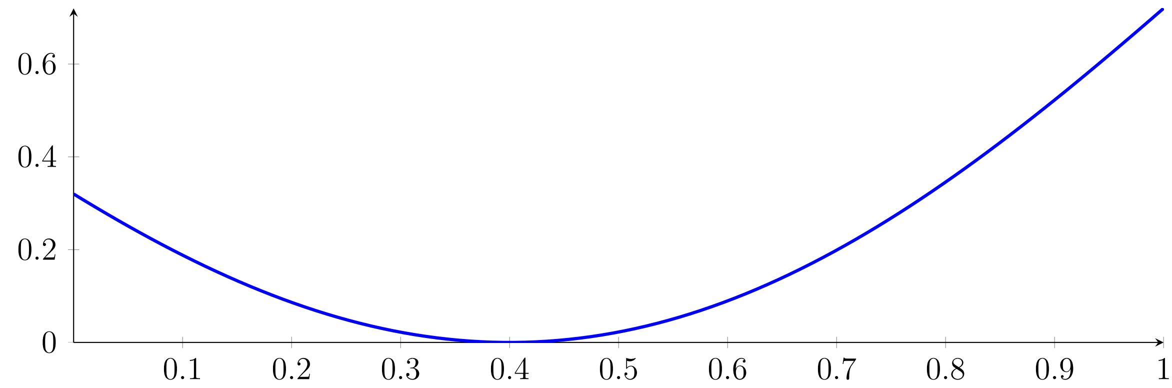
with precision $\varepsilon = 0.01$.

23 Running QuadClip on h_4 with epsilon 2

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called QuadClip with input polynomial on interval $[0, 1]$:

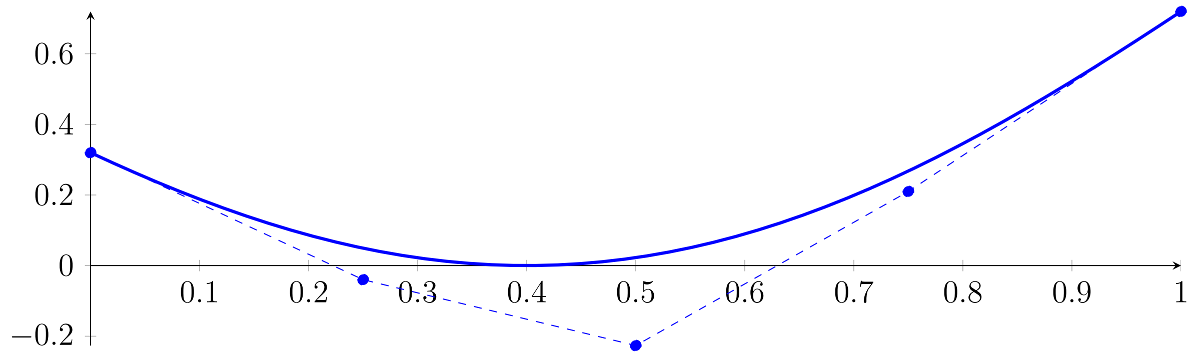
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



23.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

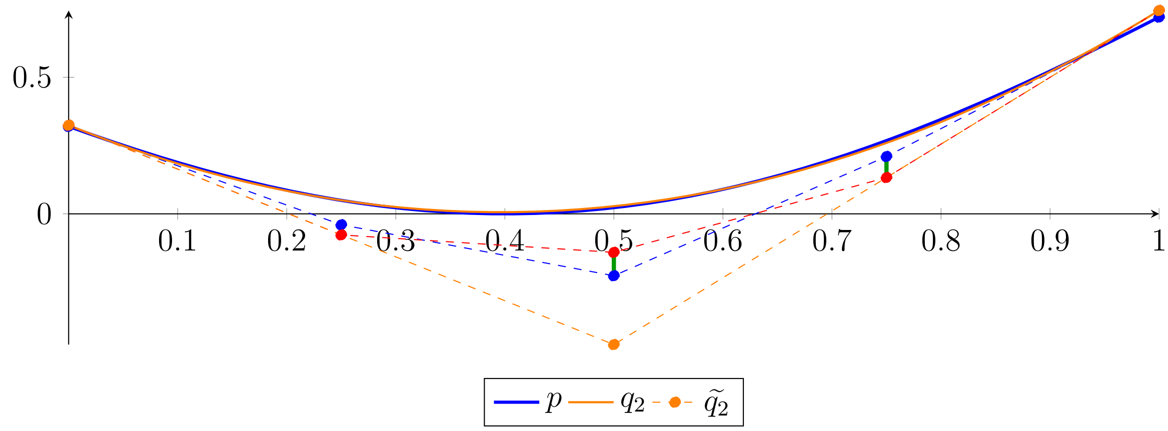
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.810653852360235 \cdot 10^{-306}X^4 - 3.682497235829418 \cdot 10^{-306}X^3 \\ &\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\ &\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

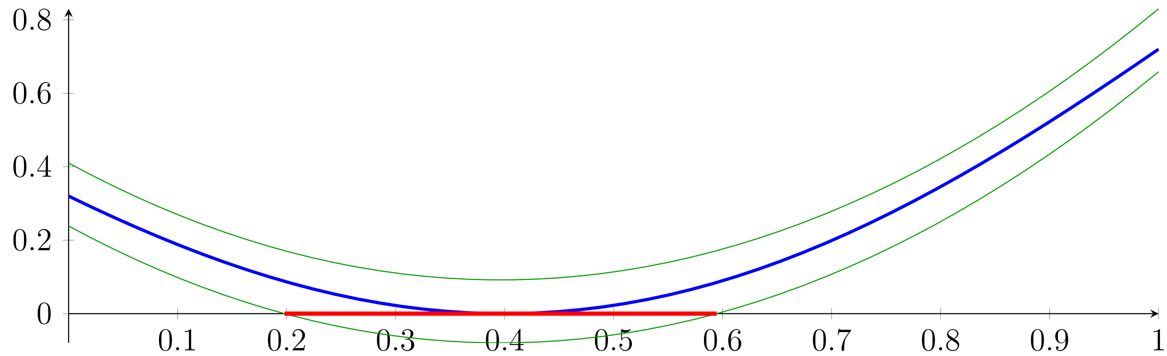
$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$

$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



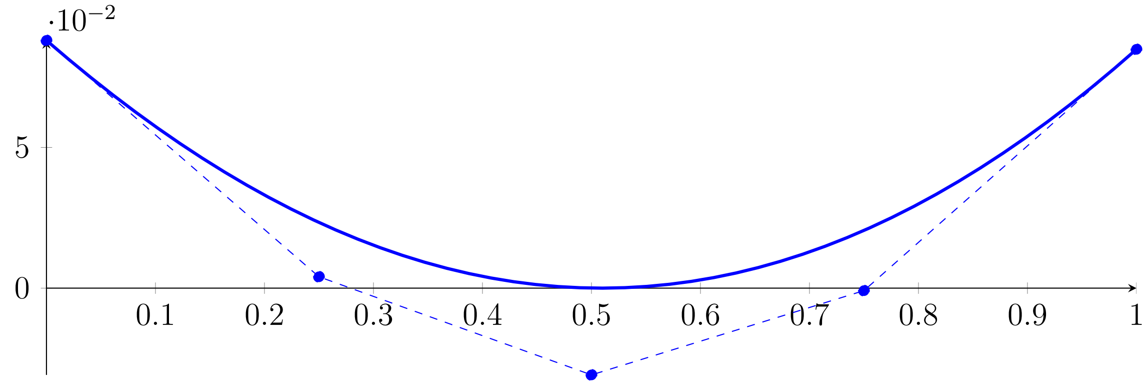
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

23.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

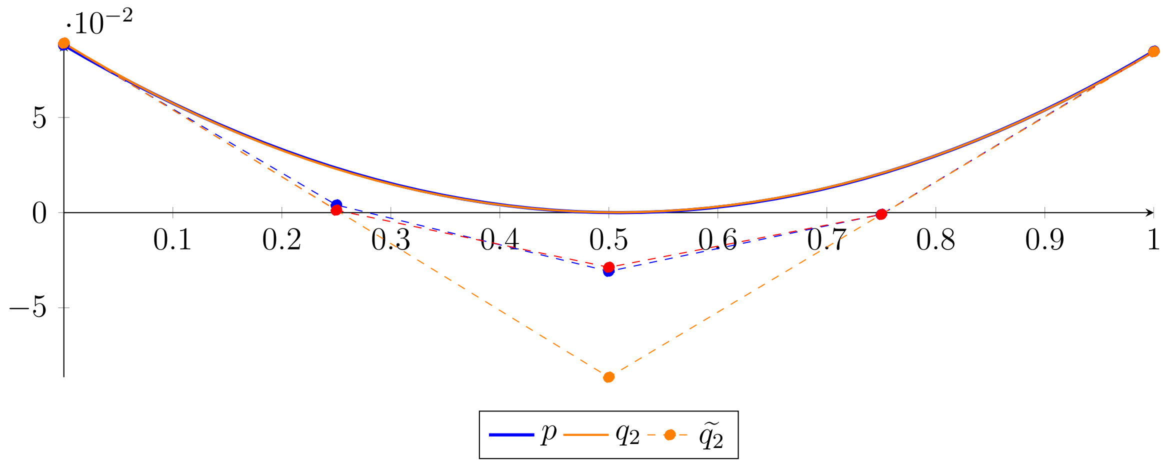
$$\begin{aligned} p = &-0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &+ 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ = &\ 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &- 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402 \cdot 10^{-307}X^4 - 6.258020227051504 \cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

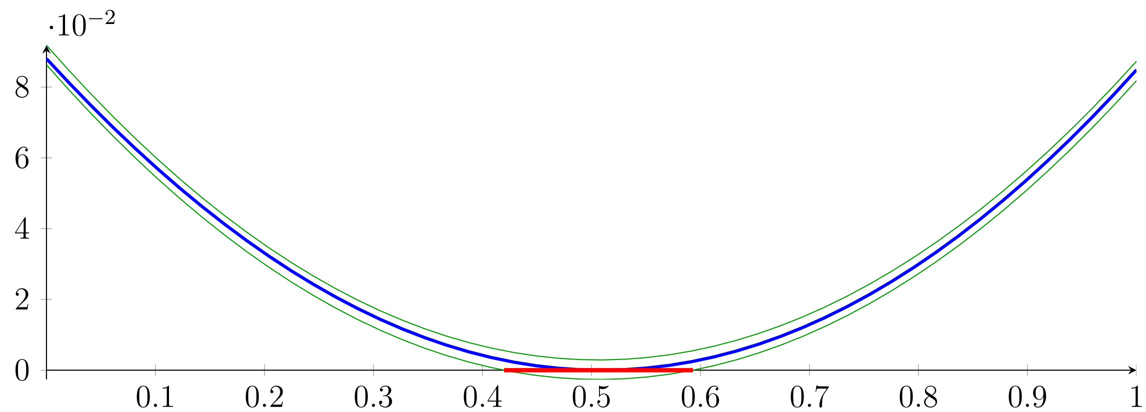
Bounding polynomials M and m:

$$\begin{aligned} M &= 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423 \\ m &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473 \end{aligned}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



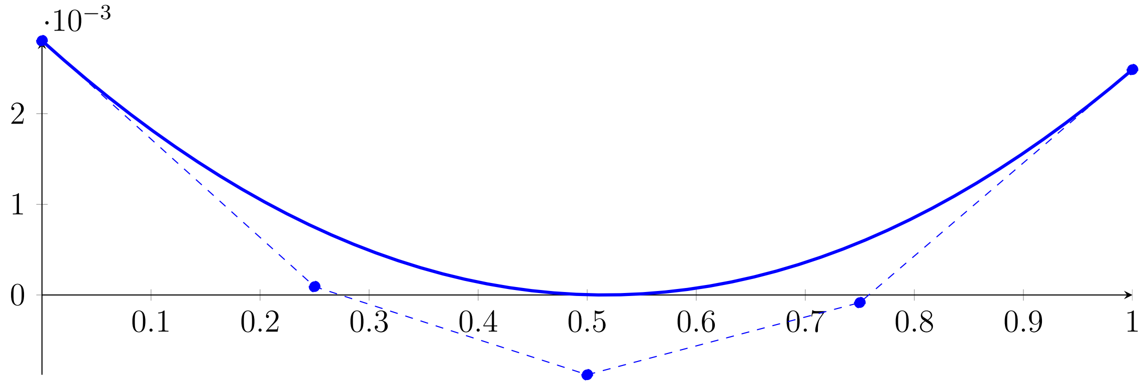
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

23.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

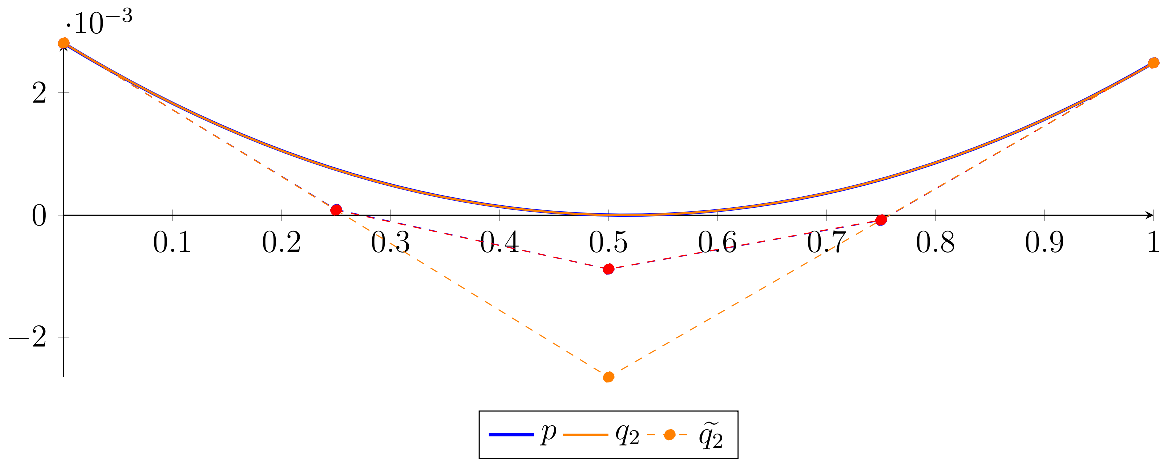
$$\begin{aligned}p = &-2.231982021693433\cdot 10^{-05}X^4 + 0.0001110015522974976X^3\\&+ 0.01044647623937015X^2 - 0.01085434180562325X + 0.002805735592529265\\= &\,0.002805735592529265B_{0,4}(X) + 9.215014112345361\cdot 10^{-05}B_{1,4}(X)\\&- 0.0008803559370539994B_{2,4}(X) - 8.403225392871914\\&\cdot 10^{-05}B_{3,4}(X) + 0.002486551758356734B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 0.01057471601887308X^2 - 0.01090053604423198X + 0.002809372542696975\\&= 0.002809372542696975B_{0,2} - 0.002640895479419014B_{1,2} + 0.002483552517338081B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &\,9.278384156079834\cdot 10^{-309}X^4 - 1.903481152394832\cdot 10^{-308}X^3\\&+ 0.01057471601887308X^2 - 0.01090053604423198X + 0.002809372542696975\\= &\,0.002809372542696975B_{0,4} + 8.423853163898018\cdot 10^{-05}B_{1,4} - 0.0008784428096068336B_{2,4}\\&- 7.867148104046678\cdot 10^{-05}B_{3,4} + 0.002483552517338081B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429\cdot 10^{-06}$.
Bounding polynomials M and m:

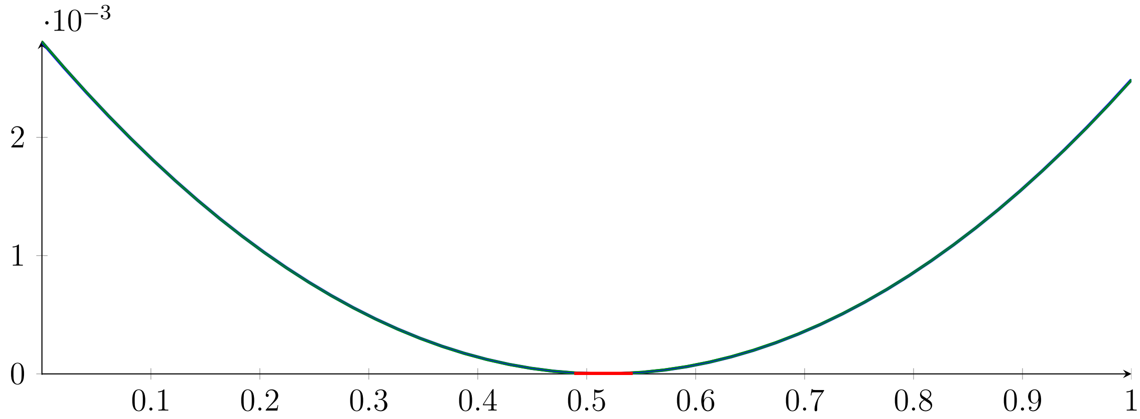
$$M = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



$$[0.4885305113706042, 0.542280720323693]$$

Longest intersection interval: 0.05375020895308878
$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

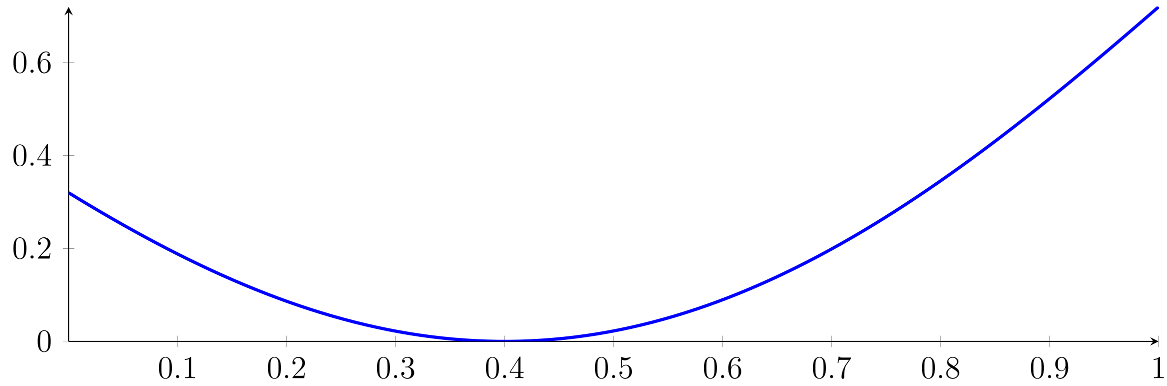
23.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.40181554717343$

Found root in interval $[0.3981210713411766, 0.4018155471734359]$ at recursion depth 4!

23.5 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3981210713411766, 0.4018155471734359]$$

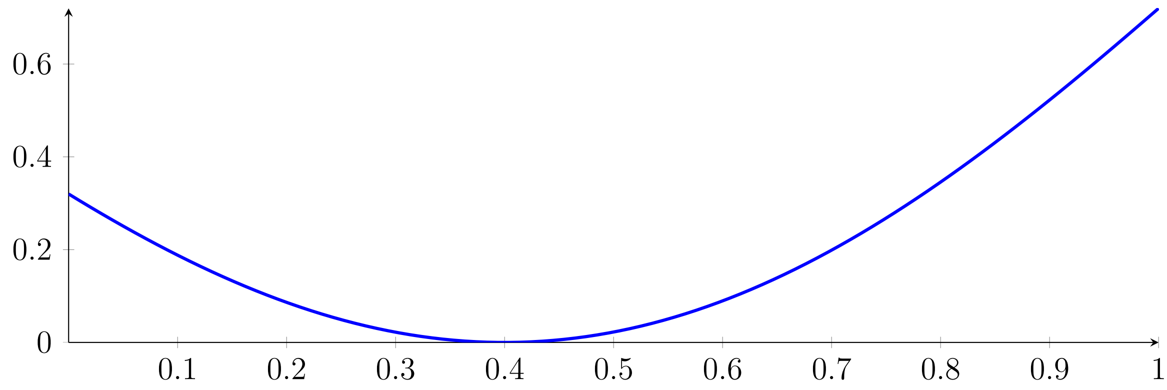
with precision $\varepsilon = 0.01$.

24 Running `CubeClip` on h_4 with epsilon 2

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

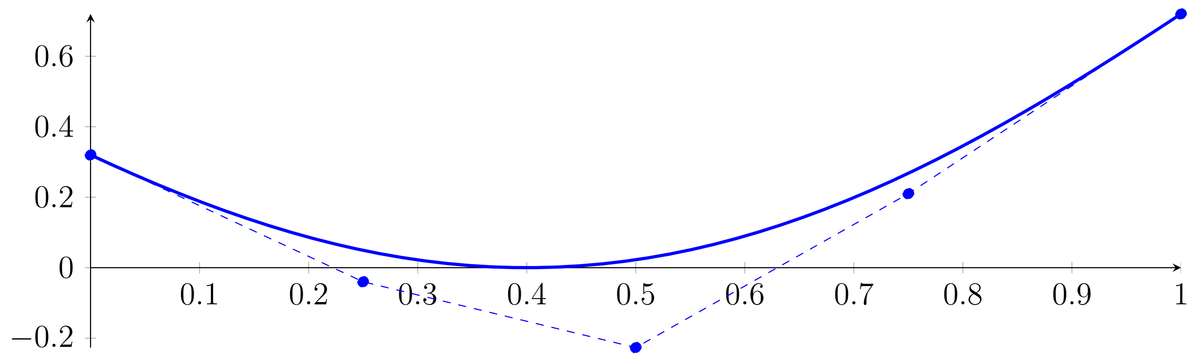
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



24.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

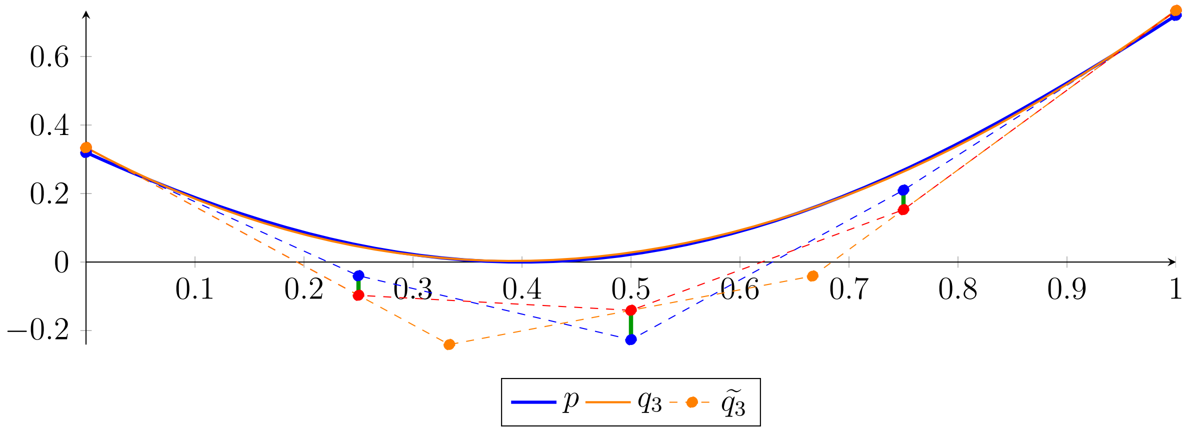
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3} \\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -2.781342323134002 \cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2 \\ &\quad - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4} \\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

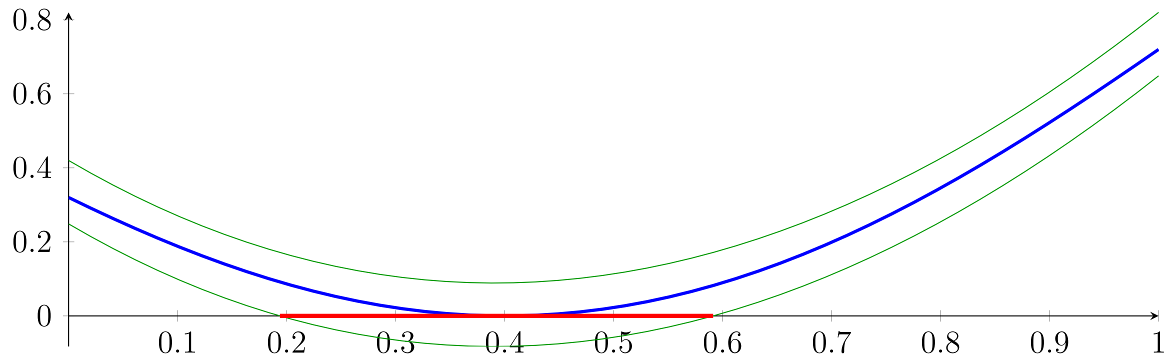
$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$

$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



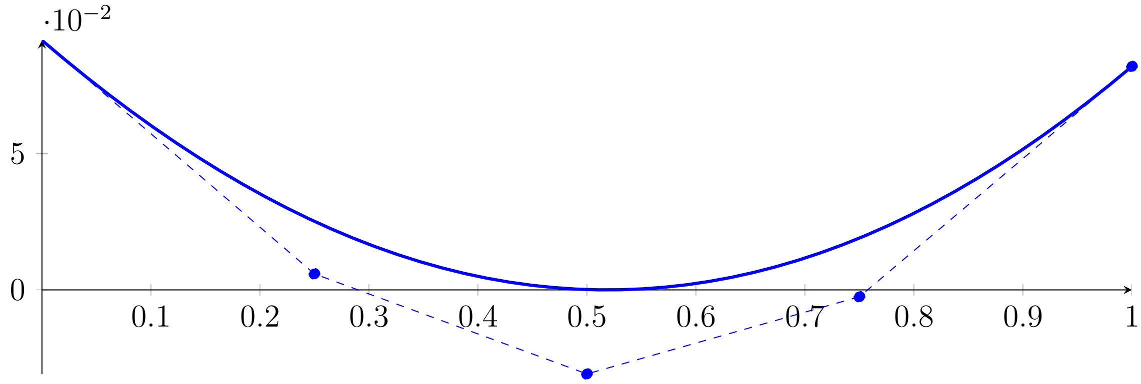
$[0.1938265012447289, 0.5913474001559605]$

Longest intersection interval: 0.3975208989112316
$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

24.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

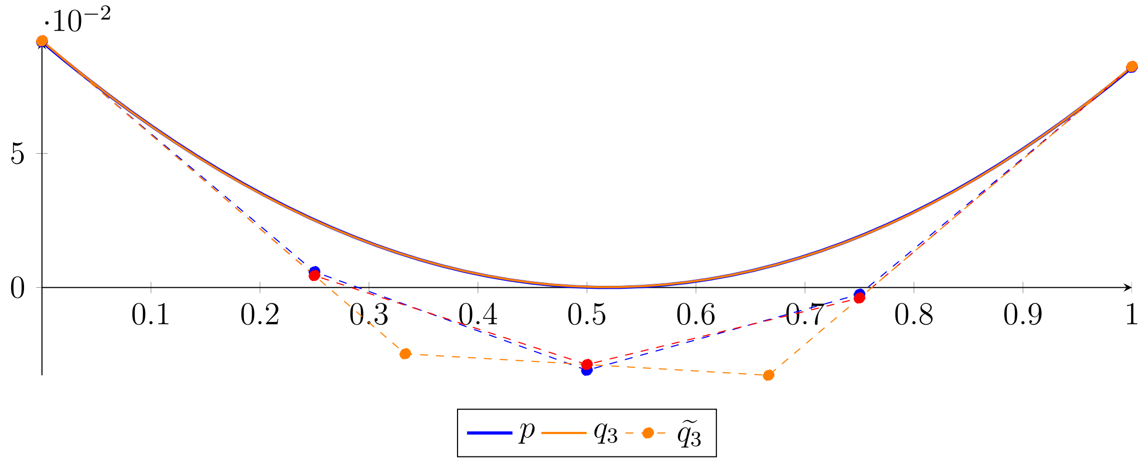
$$\begin{aligned} p = &-0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &+ 0.2941201876709534X^2 - 0.34309913433192X + 0.09165715738594469 \\ = &\; 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &- 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502\cdot 10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455446609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.

Bounding polynomials M and m:

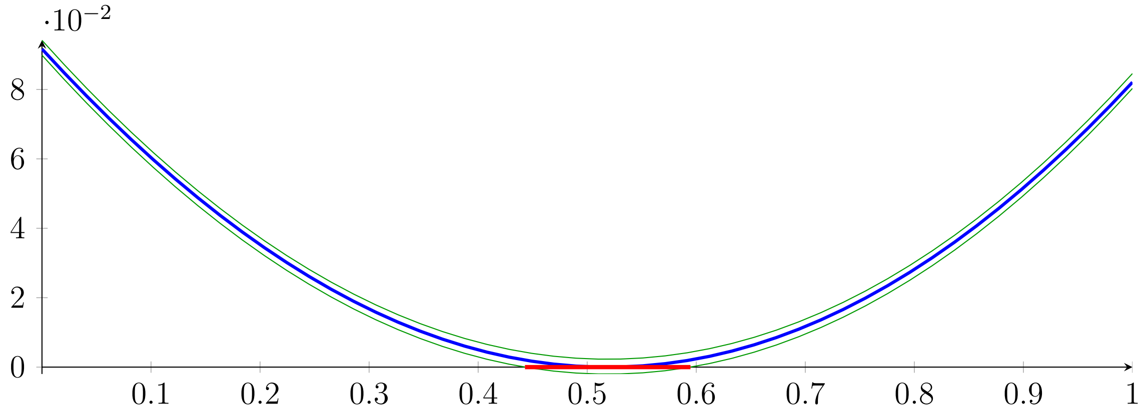
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



$[0.4429176870393937, 0.5947109255489168]$

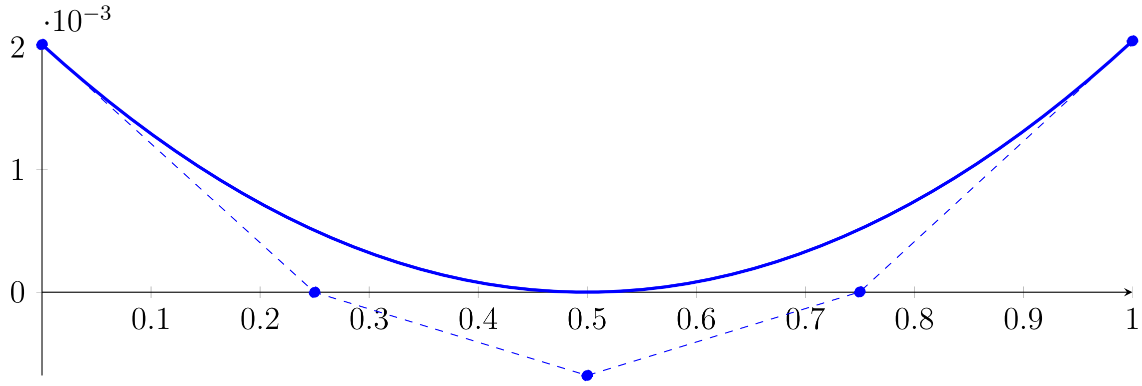
Longest intersection interval: 0.1517932385095231

$\Longrightarrow$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

24.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

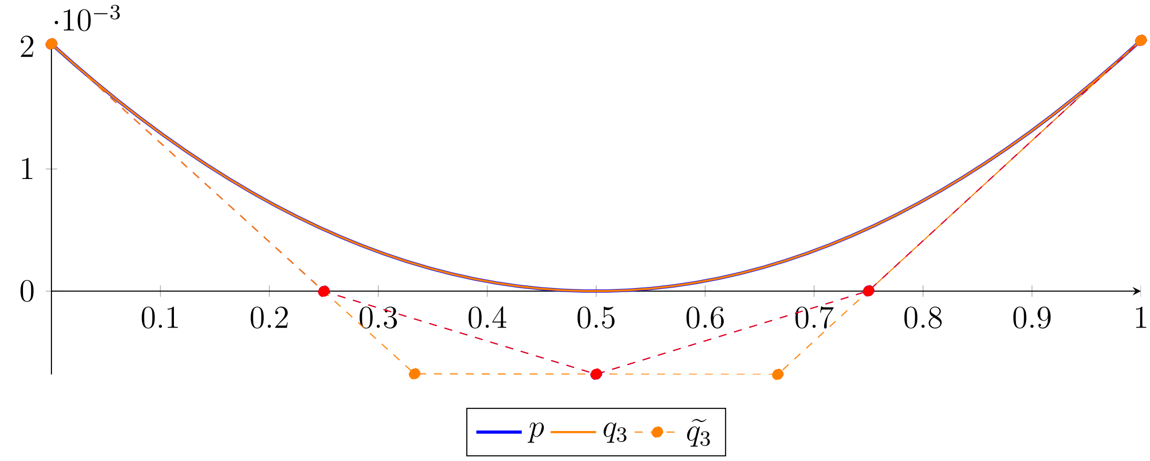
$$\begin{aligned}
p = &-1.325713168422468\cdot 10^{-05}X^4 + 7.039695732710913\cdot 10^{-05}X^3 \\
&+ 0.008070351522992233X^2 - 0.008098671960928044X + 0.002023786815863734 \\
= &\ 0.002023786815863734B_{0,4}(X) - 8.811743682771854\cdot 10^{-07}B_{1,4}(X) \\
&- 0.0006804905774349116B_{2,4}(X) + 2.557845995594498 \\
&\cdot 10^{-06}B_{3,4}(X) + 0.002052606203570807B_{4,4}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &\ 4.388269395865976\cdot 10^{-05}X^3 + 0.008087396406586236X^2 \\
&- 0.008102459712837822X + 0.002023976203459223 \\
= &\ 0.002023976203459223B_{0,3} - 0.0006768437008200514B_{1,3} \\
&- 0.0006818648029039136B_{2,3} + 0.002052795591166296B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q_3} = &-6.518771069845317\cdot 10^{-311}X^4 + 4.388269395865976\cdot 10^{-05}X^3 \\
&+ 0.008087396406586236X^2 - 0.008102459712837822X + 0.002023976203459223 \\
= &\ 0.002023976203459223B_{0,4} - 1.638724750232882\cdot 10^{-06}B_{1,4} - 0.0006793542518619825B_{2,4} \\
&+ 1.800295613638801\cdot 10^{-06}B_{3,4} + 0.002052795591166296B_{4,4}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544 \cdot 10^{-06}$.
Bounding polynomials M and m:

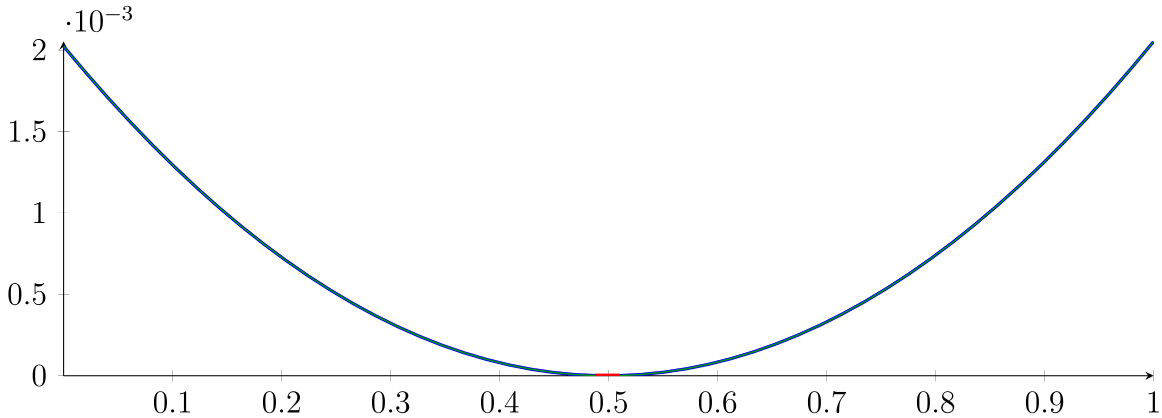
$$M = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002025112529032156$$

$$m = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002022839877886289$$

Root of M and m:

$N(M) = \{-185.2936165687423\}$ $N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649$

Intersection intervals:



$$[0.4874742490566883, 0.5103358670775649]$$

Longest intersection interval: 0.02286161802087665
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

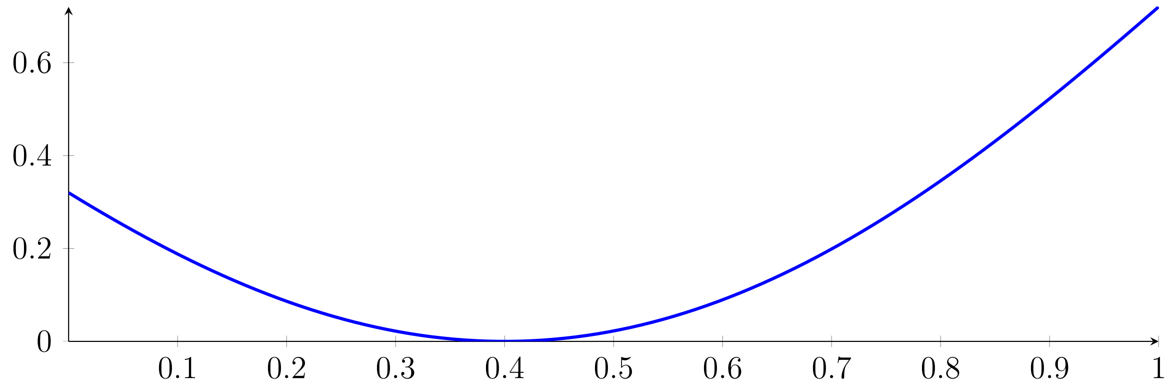
24.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.40068970704716$

Found root in interval $[0.3993102145057523, 0.4006897070471601]$ at recursion depth 4!

24.5 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3993102145057523, 0.4006897070471601]$$

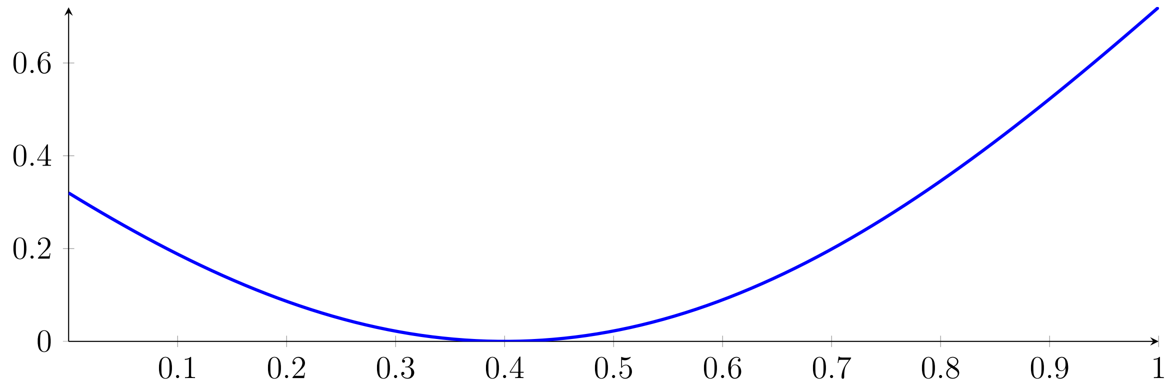
with precision $\varepsilon = 0.01$.

25 Running `BezClip` on h_4 with epsilon 4

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

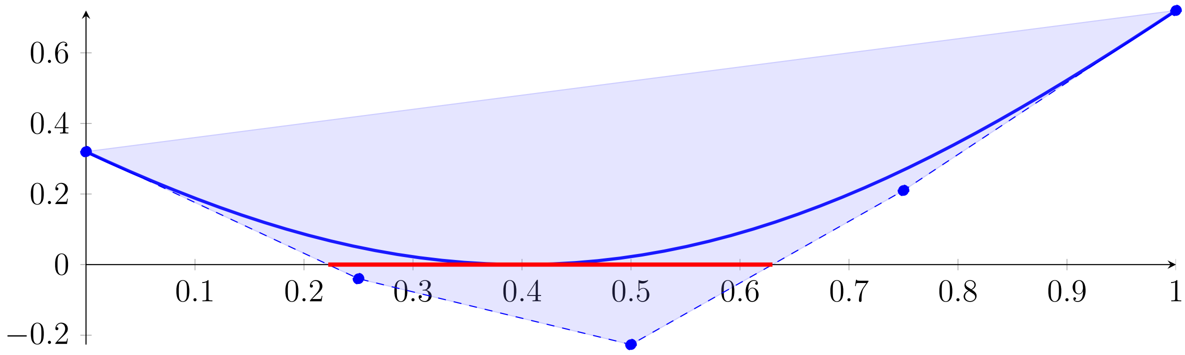
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



25.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

$$[0.222222225308642, 0.6297709955349339]$$

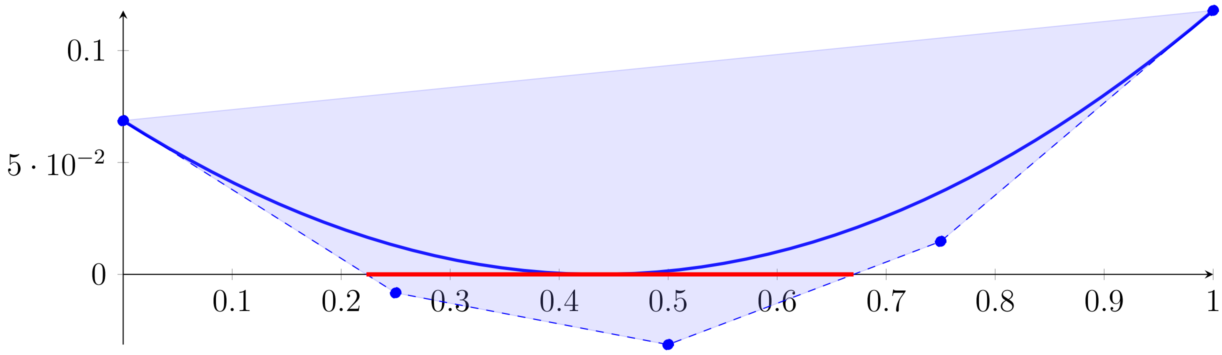
Longest intersection interval: 0.407548770226292

$\Longrightarrow$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

25.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &\quad + 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ &= 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &\quad + 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

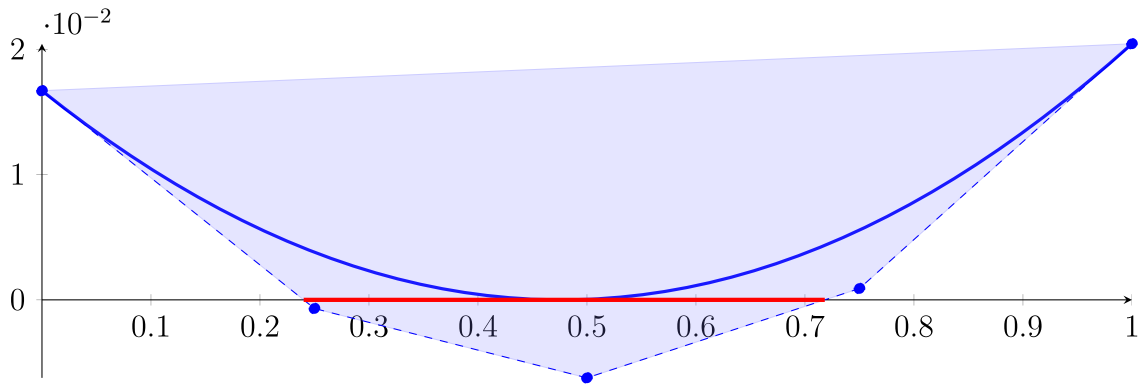
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\Longrightarrow$ Selective recursion: interval 1: $[0.3131782986367004, 0.4953216655933138]$,

25.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.495321665593313$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -0.001100660652934182X^4 + 0.003307158935864367X^3\\ &\quad + 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377\\ &= 0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X)\\ &\quad + 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

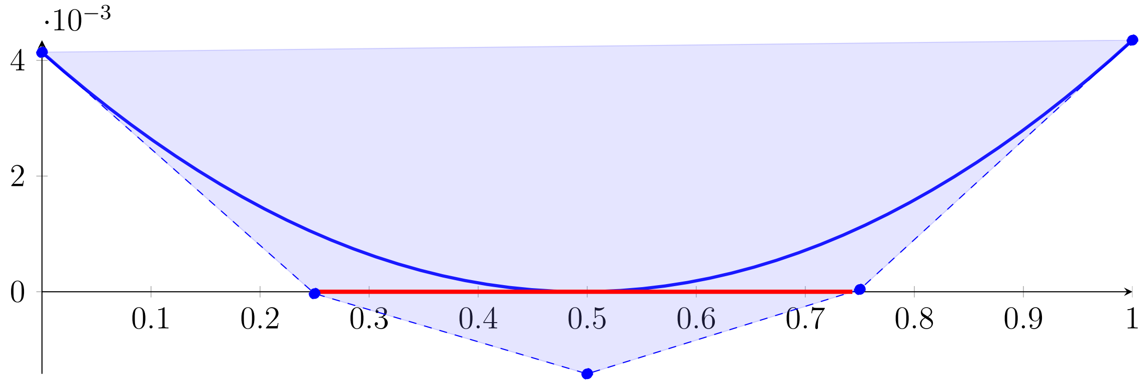
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\Longrightarrow$ Selective recursion: interval 1: $[0.3569093751201798, 0.4439937820951003]$,

25.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.44399378209510$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-5.751241374789061 \cdot 10^{-05} X^4 + 0.0002459162030906773 X^3 \\ &+ 0.01670691422385101 X^2 - 0.01668640845758443 X + 0.004139787469617577 \\ = &\, 0.004139787469617577 B_{0,4}(X) - 3.181464477852956 \cdot 10^{-05} B_{1,4}(X) \\ &- 0.001418931055199467 B_{2,4}(X) + 3.991728912743387 \\ &\cdot 10^{-05} B_{3,4}(X) + 0.004348697025226952 B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

$$[0.2480933797192248, 0.7431594518918532]$$

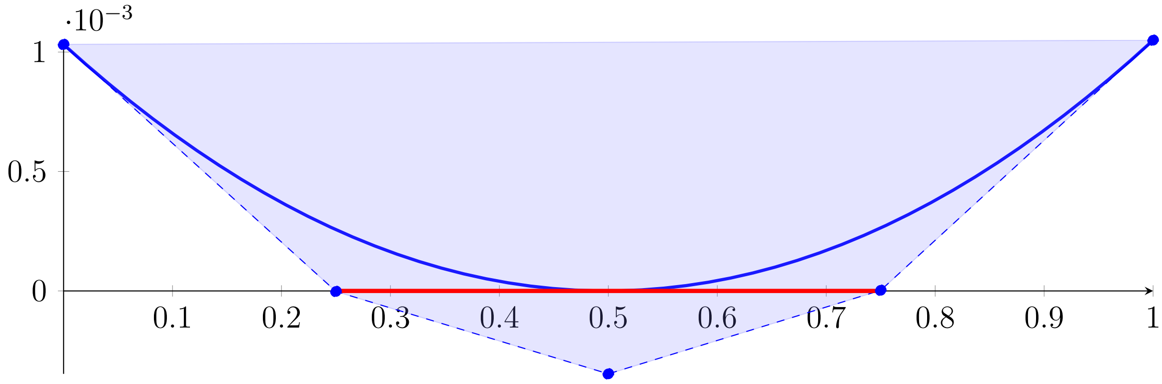
Longest intersection interval: 0.4950660721726284

$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

25.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-3.454731120985874 \cdot 10^{-06} X^4 + 2.291337271769576 \cdot 10^{-05} X^3 \\ &+ 0.004134358001933594 X^2 - 0.004136159738306538 X + 0.001031853315293836 \\ = &\, 0.001031853315293836 B_{0,4}(X) - 2.186619282798525 \cdot 10^{-06} B_{1,4}(X) \\ &- 0.0003471668868705005 B_{2,4}(X) + 2.640855710153786 \\ &\cdot 10^{-06} B_{3,4}(X) + 0.001049510220517602 B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

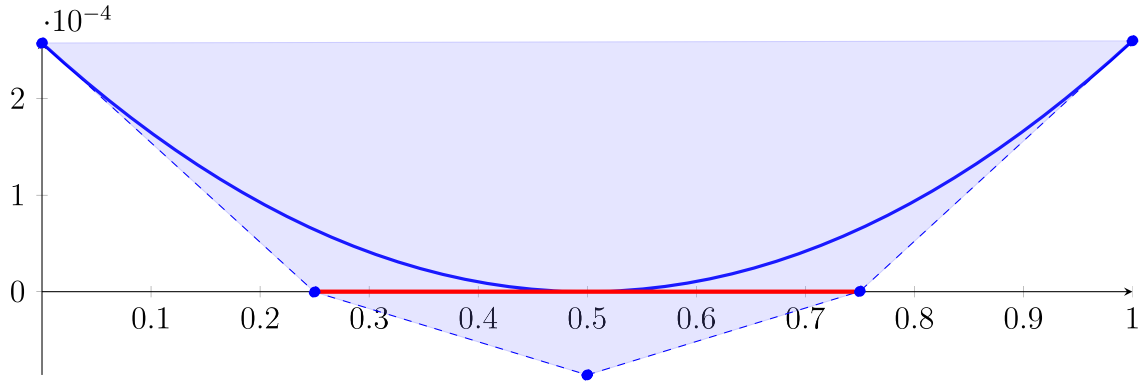
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\implies$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

25.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.410767472477$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.135832675829806\cdot 10^{-07}X^4 + 2.413460912088167\cdot 10^{-06}X^3\\ &+ 0.001031922910124016X^2 - 0.001031832710654165X + 0.0002576480709401398\\ = &\ 0.0002576480709401398B_{0,4}(X) - 3.101067234014613\cdot 10^{-07}B_{1,4}(X) - 8.628113269960673\\ &\cdot 10^{-05}B_{2,4}(X) + 3.383582395460159\cdot 10^{-07}B_{3,4}(X) + 0.0002599381480544958B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

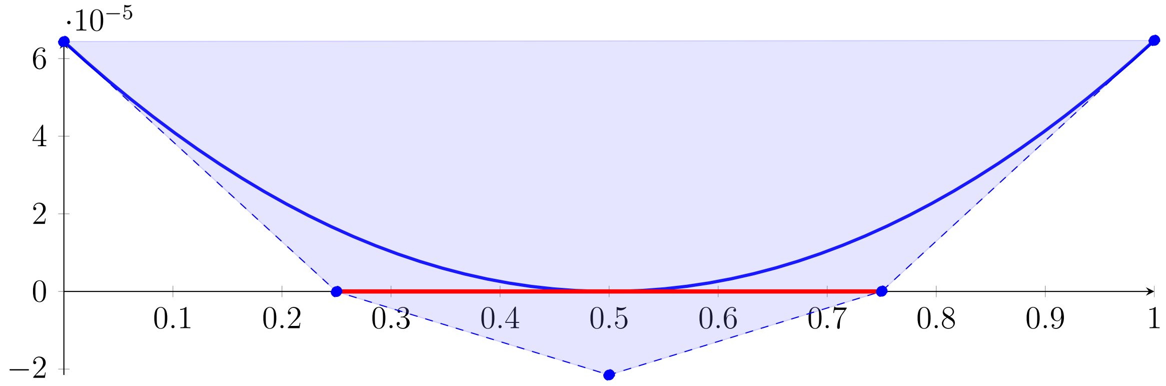
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\implies$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

25.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.405372055$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.327690666746824\cdot 10^{-08}X^4 + 2.739027984571277\cdot 10^{-07}X^3\\ &+ 0.0002577144304079\,74X^2 - 0.0002576778286693267X + 6.437695235811399\cdot 10^{-05}\\ = &\ 6.437695235811399\cdot 10^{-05}B_{0,4}(X) - 4.250480921768141\cdot 10^{-08}B_{1,4}(X) - 2.150955690855369\\ &\cdot 10^{-05}B_{2,4}(X) + 4.427175972025921\cdot 10^{-08}B_{3,4}(X) + 6.467417998855097\cdot 10^{-05}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

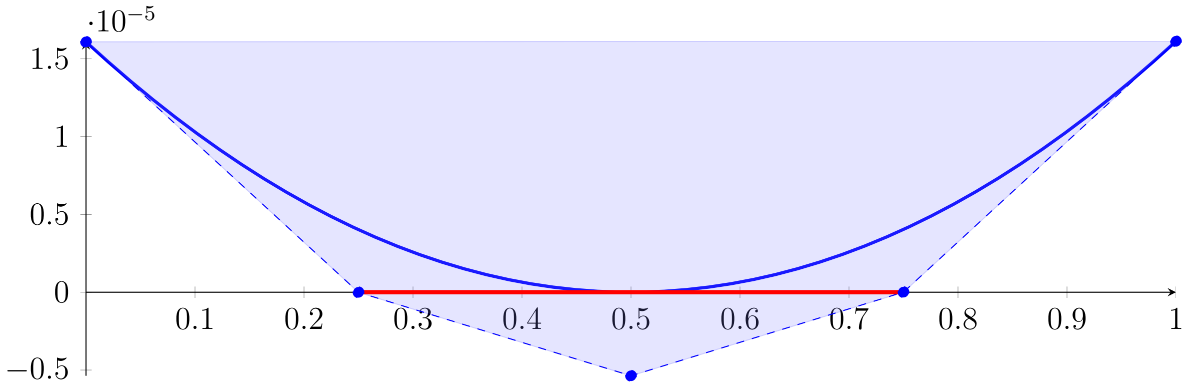
$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\implies$ Selective recursion: interval 1: $[0.3973195510833879, 0.4026829657906133]$,

25.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -8.274952589981882 \cdot 10^{-10} X^4 + 3.251124603913595 \cdot 10^{-08} X^3 + 6.438882283687421 \\ & \cdot 10^{-05} X^2 - 6.438267481175771 \cdot 10^{-05} X + 1.609012302857716 \cdot 10^{-05} \\ = & \, 1.609012302857716 \cdot 10^{-05} B_{0,4}(X) - 5.545674362270907 \cdot 10^{-09} B_{1,4}(X) - 5.369743904489329 \\ & \cdot 10^{-06} B_{2,4}(X) + 5.656149705765249 \cdot 10^{-09} B_{3,4}(X) + 1.61279548044738 \cdot 10^{-05} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499138638713212, 0.7497369428484978\}$$

Intersection intervals with the x axis:

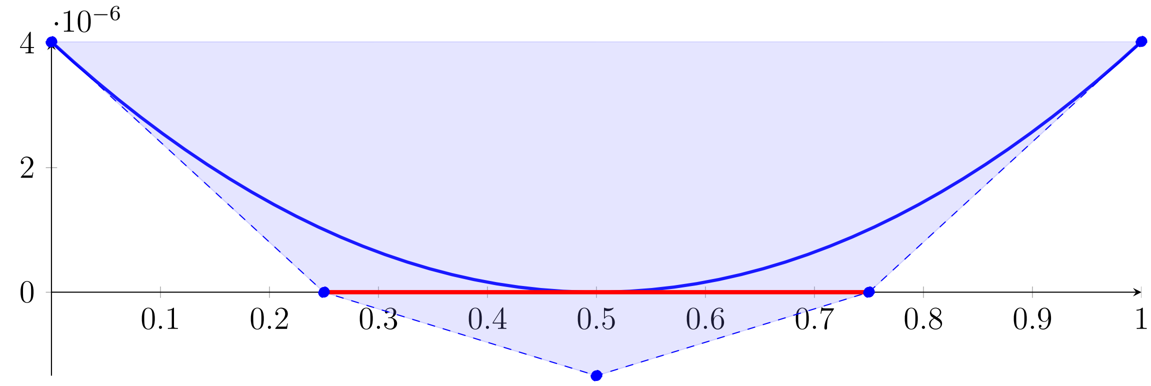
$$[0.2499138638713212, 0.7497369428484978]$$

Longest intersection interval: 0.4998230789771766
$\implies$ Selective recursion: interval 1: $[0.3986599427764149, 0.4013407012292117]$,

25.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3986599427764149, 0.40134$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -5.164529187662443 \cdot 10^{-11} X^4 + 3.956301794548619 \cdot 10^{-09} X^3 + 1.609182796552748 \cdot 10^{-05} X^2 - 1.609096222935268 \cdot 10^{-05} X + 4.022033038444256 \cdot 10^{-06}$$
$$= 4.022033038444256 \cdot 10^{-06} B_{0,4}(X) - 7.075188939141538 \cdot 10^{-10} B_{1,4}(X) - 1.341476748644171 \cdot 10^{-06} B_{2,4}(X) + 7.14424642122371 \cdot 10^{-10} B_{3,4}(X) + 4.026803431121726 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499560300444542, 0.7498669294180401\}$$

Intersection intervals with the x axis:

$$[0.2499560300444542, 0.7498669294180401]$$

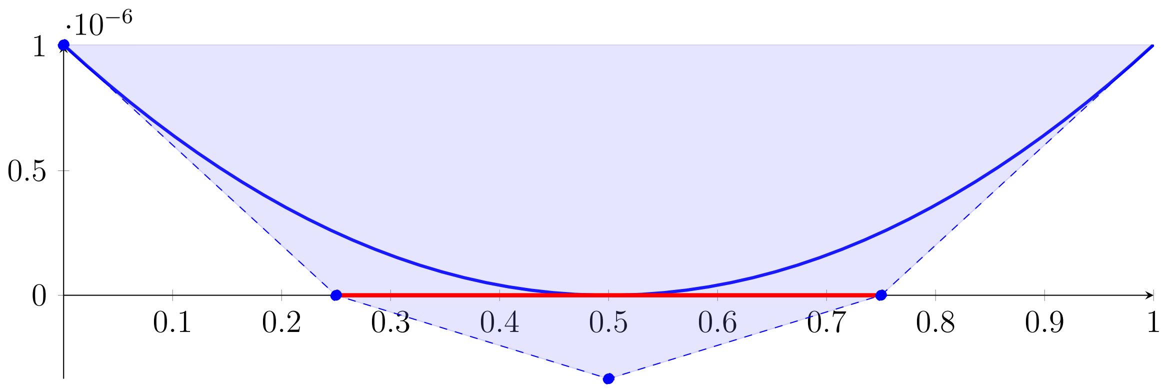
Longest intersection interval: 0.499910899373586

$\Longrightarrow$ Selective recursion: interval 1: $[0.3993300145167841, 0.4006701548859251]$,

25.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3993300145167841, 0.400$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.225530543299082 \cdot 10^{-12} X^4 + 4.878223136378425 \cdot 10^{-10} X^3 + 4.022259900669466 \cdot 10^{-06} X^2 - 4.022145641341714 \cdot 10^{-06} X + 1.00544708348311 \cdot 10^{-06}$$
$$= 1.00544708348311 \cdot 10^{-06} B_{0,4}(X) - 8.932685231834447 \cdot 10^{-11} B_{1,4}(X) - 3.352490870761692 \cdot 10^{-07} B_{2,4}(X) + 8.975838996701131 \cdot 10^{-11} B_{3,4}(X) + 1.006045939593956 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499777912437083, 0.7499330838112102\}$$

Intersection intervals with the x axis:

$$[0.2499777912437083, 0.7499330838112102]$$

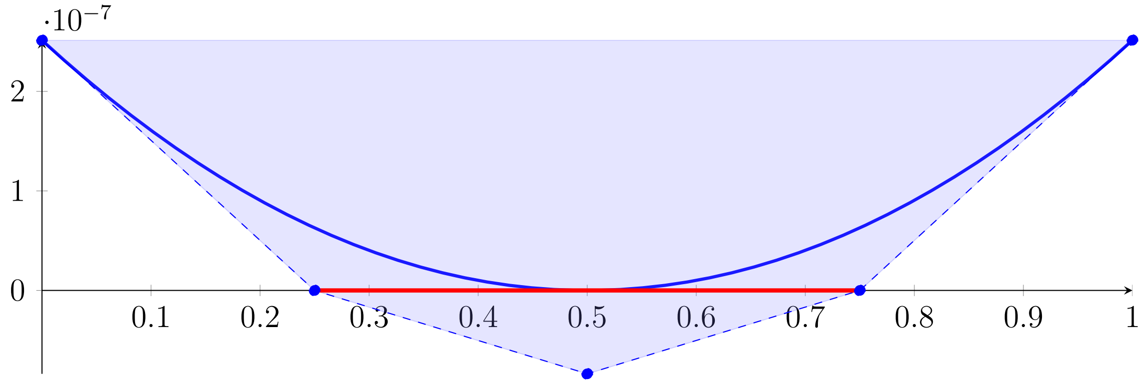
Longest intersection interval: 0.499955292567502

$\Longrightarrow$ Selective recursion: interval 1: $[0.3996650198462185, 0.4003350301165539]$,

25.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3996650198462185, 0.4$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.015235660316643\cdot10^{-13}X^4 + 6.055838633908679\cdot10^{-11}X^3 + 1.005476298211762\cdot10^{-06}X^2 - 1.005461639191386\cdot10^{-06}X + 2.51354188678016\cdot10^{-07}$$
$$= 2.51354188678016\cdot10^{-07}B_{0,4}(X) - 1.122111983053638\cdot10^{-11}B_{1,4}(X) - 8.379724788238336\cdot10^{-08}B_{2,4}(X) + 1.124798694225224\cdot10^{-11}B_{3,4}(X) + 2.51429204561165\cdot10^{-07}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499888398329751, 0.7499664473546936\}$$

Intersection intervals with the x axis:

$$[0.2499888398329751, 0.7499664473546936]$$

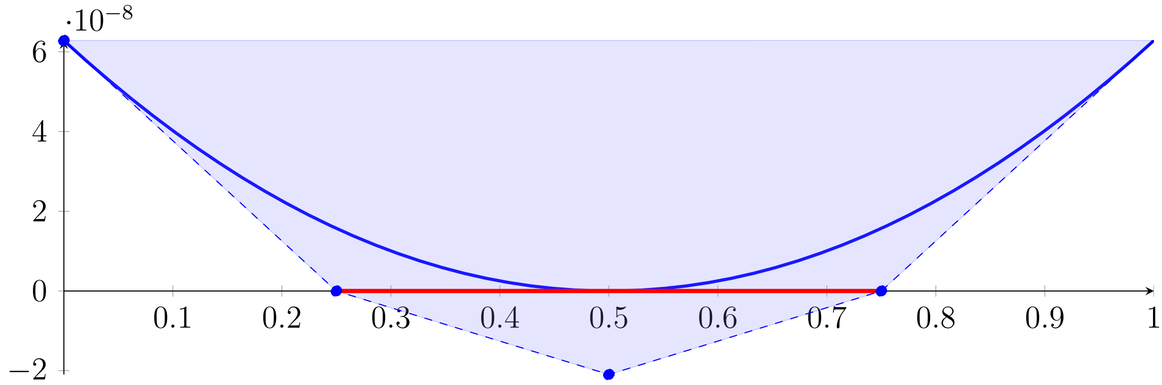
Longest intersection interval: 0.4999776075217186

$\implies$ Selective recursion: interval 1: $[0.3998325149363758, 0.4001675050683531]$,

25.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3998325149363758, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.259296672250957\cdot10^{-14}X^4 + 7.543595361590864\cdot10^{-12}X^3 + 2.5135789423507\cdot10^{-07}X^2 - 2.51356038307676\cdot10^{-07}X + 6.283760343276371\cdot10^{-08}$$
$$= 6.283760343276371\cdot10^{-08}B_{0,4}(X) - 1.406144155297956\cdot10^{-12}B_{1,4}(X) - 2.094743334856264\cdot10^{-08}B_{2,4}(X) + 1.407718382090997\cdot10^{-12}B_{3,4}(X) + 6.284699036255256\cdot10^{-08}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499944057673539, 0.7499832005219574\}$$

Intersection intervals with the x axis:

$$[0.2499944057673539, 0.7499832005219574]$$

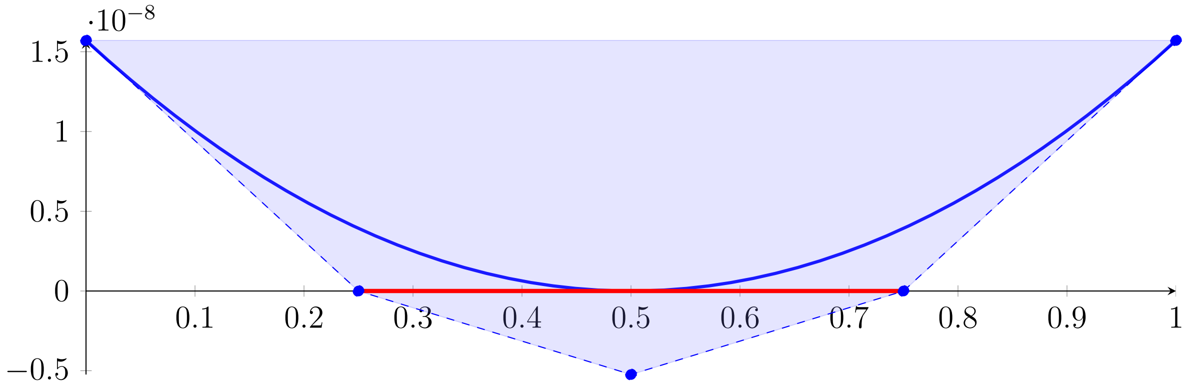
Longest intersection interval: 0.4999887947546035

$\implies$ Selective recursion: interval 1: $[0.3999162605953574, 0.4000837519076994]$,

25.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999162605953574$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.869898688873272\cdot 10^{-16}X^4 + 9.413120459510821\cdot 10^{-13}X^3 + 6.283807021204131\\&\quad\cdot 10^{-08}X^2 - 6.283783670437638\cdot 10^{-08}X + 1.570928313186755\cdot 10^{-08}\\&= 1.570928313186755\cdot 10^{-08}B_{0,4}(X) - 1.760442265467946\cdot 10^{-13}B_{1,4}(X) - 5.236623518313756\\&\quad\cdot 10^{-09}B_{2,4}(X) + 1.76037617409066\cdot 10^{-13}B_{3,4}(X) + 1.571045716458857\cdot 10^{-08}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499971984359141, 0.7499915961258623\}$$

Intersection intervals with the x axis:

$$[0.2499971984359141, 0.7499915961258623]$$

Longest intersection interval: 0.4999943976899482
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999581329542053, 0.400041877672038]$,

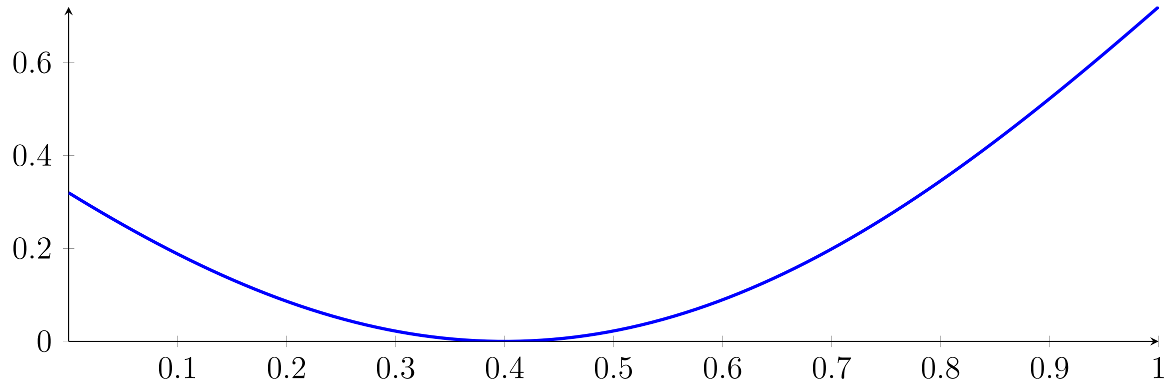
25.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999581329542053, 0.400041877672038]$

Found root in interval $[0.3999581329542053, 0.400041877672038]$ at recursion depth 14!

25.15 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3999581329542053, 0.400041877672038]$$

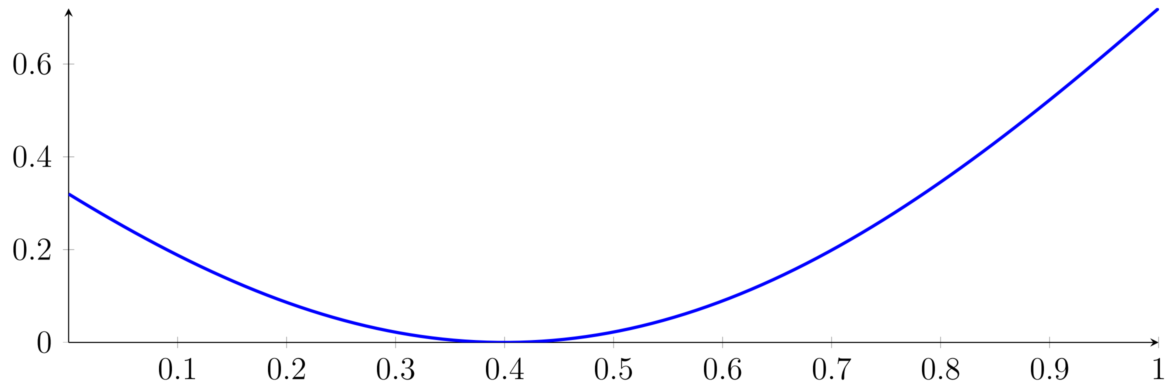
with precision $\varepsilon = 0.0001$.

26 Running `QuadClip` on h_4 with epsilon 4

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

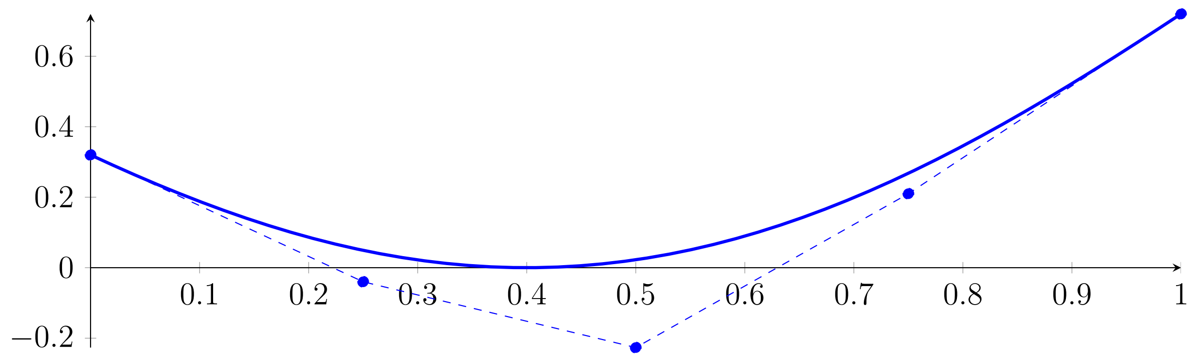
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



26.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

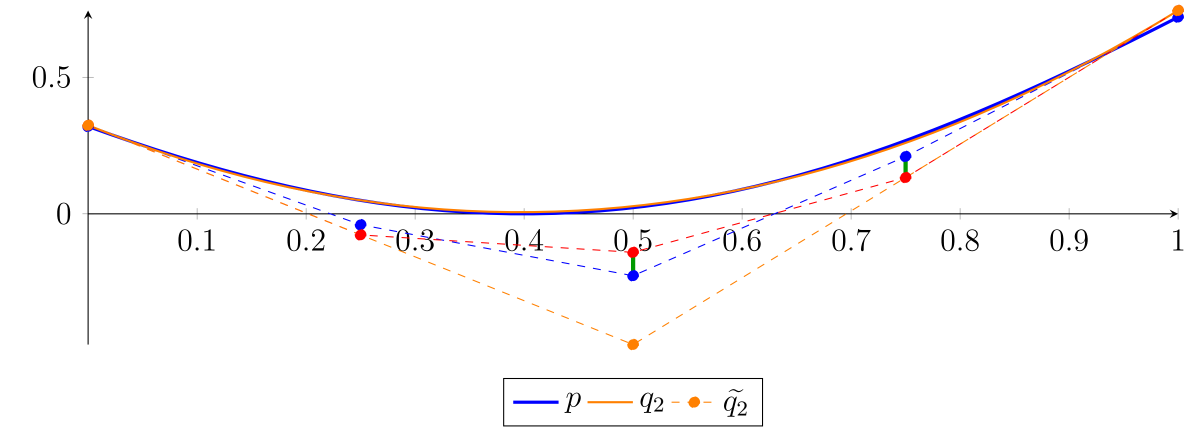
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.810653852360235 \cdot 10^{-306}X^4 - 3.682497235829418 \cdot 10^{-306}X^3 \\ &\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\ &\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

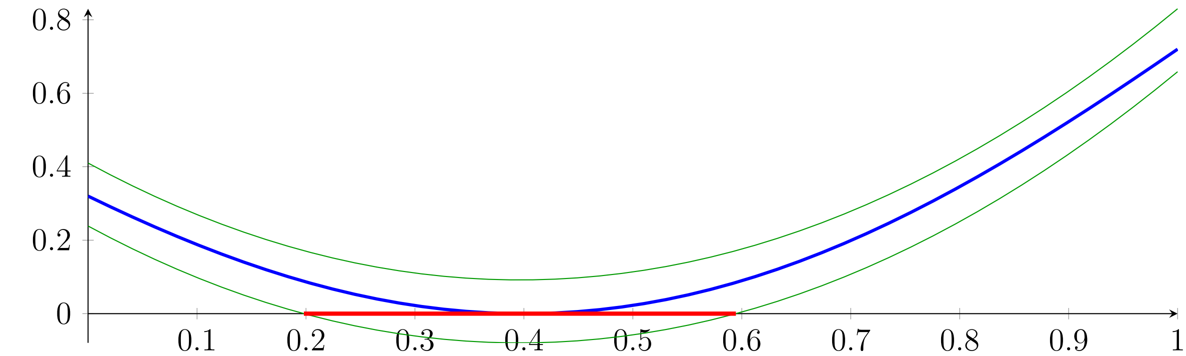
$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$

$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



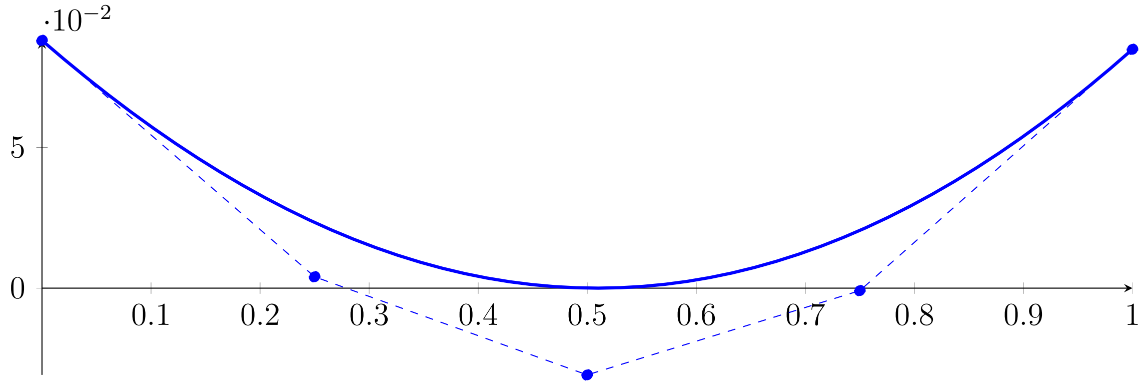
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

26.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

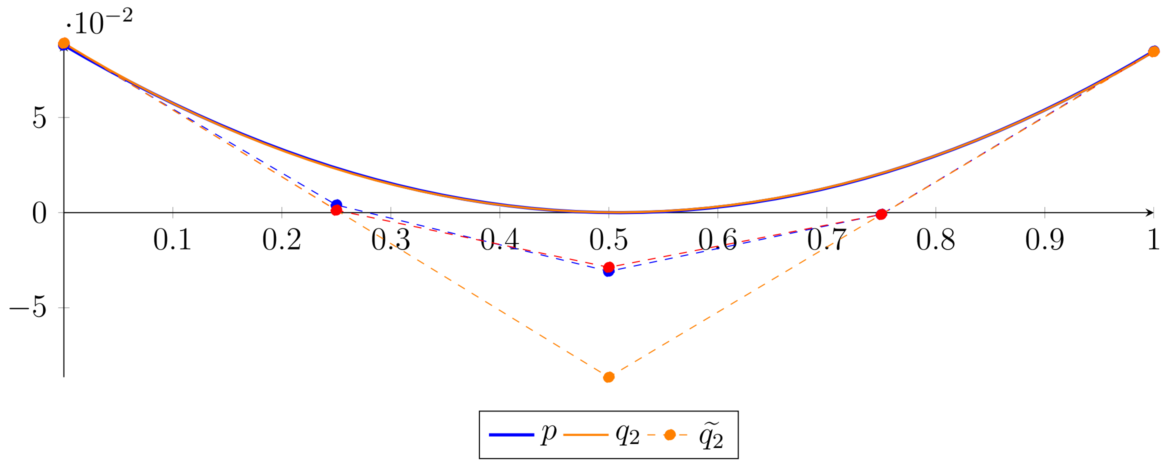
$$\begin{aligned} p = &-0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &+ 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ = &\ 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &- 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402 \cdot 10^{-307}X^4 - 6.258020227051504 \cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

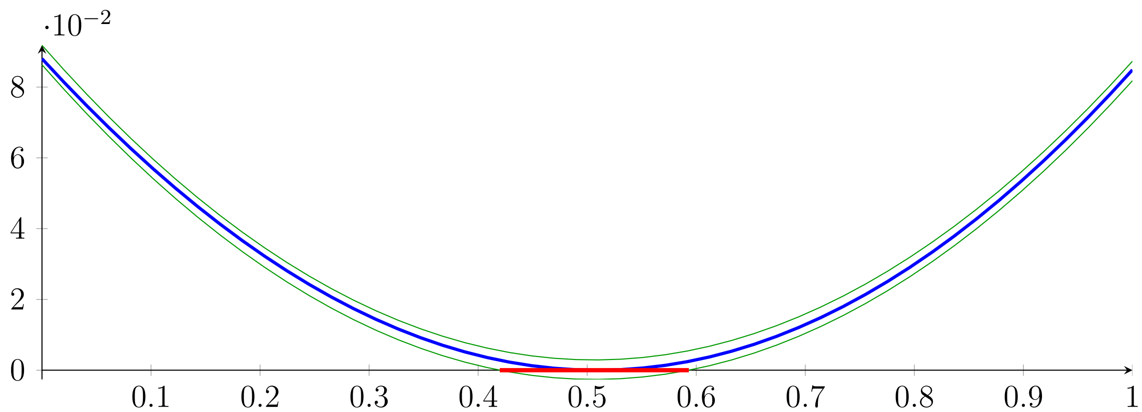
Bounding polynomials M and m:

$$M = 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423$$
$$m = 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



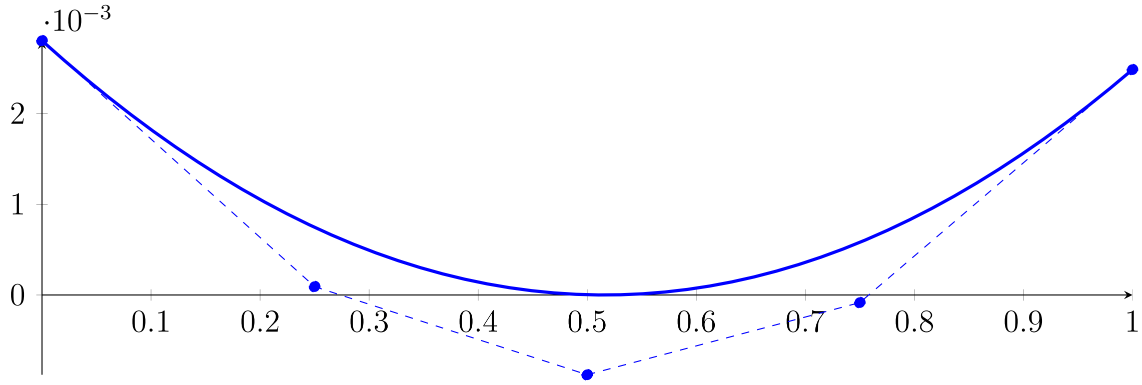
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

26.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

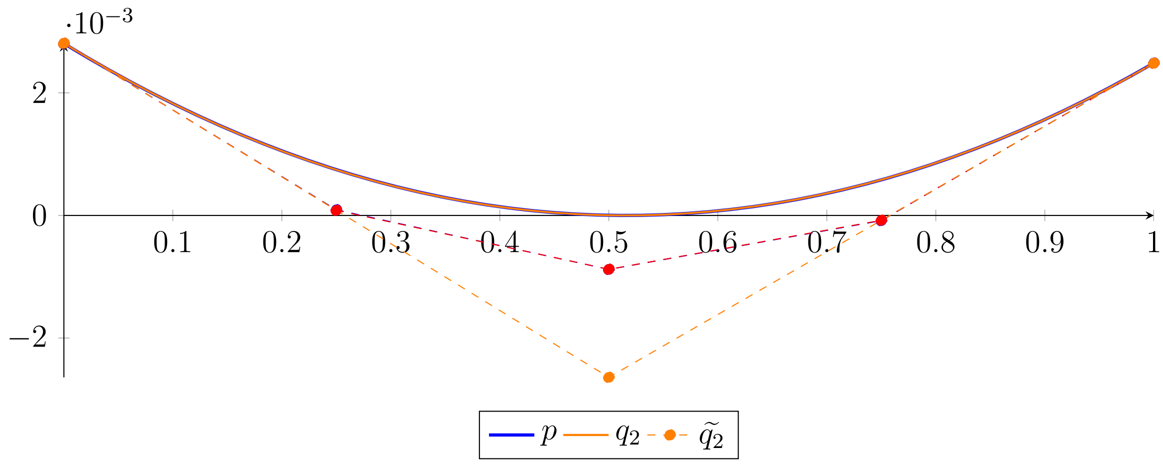
$$\begin{aligned}p &= -2.231982021693433 \cdot 10^{-05} X^4 + 0.0001110015522974976 X^3 \\&\quad + 0.01044647623937015 X^2 - 0.01085434180562325 X + 0.002805735592529265 \\&= 0.002805735592529265 B_{0,4}(X) + 9.215014112345361 \cdot 10^{-05} B_{1,4}(X) \\&\quad - 0.0008803559370539994 B_{2,4}(X) - 8.403225392871914 \\&\quad \cdot 10^{-05} B_{3,4}(X) + 0.002486551758356734 B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\&= 0.002809372542696975 B_{0,2} - 0.002640895479419014 B_{1,2} + 0.002483552517338081 B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 9.278384156079834 \cdot 10^{-309} X^4 - 1.903481152394832 \cdot 10^{-308} X^3 \\&\quad + 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\&= 0.002809372542696975 B_{0,4} + 8.423853163898018 \cdot 10^{-05} B_{1,4} - 0.0008784428096068336 B_{2,4} \\&\quad - 7.867148104046678 \cdot 10^{-05} B_{3,4} + 0.002483552517338081 B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429 \cdot 10^{-06}$.
Bounding polynomials M and m:

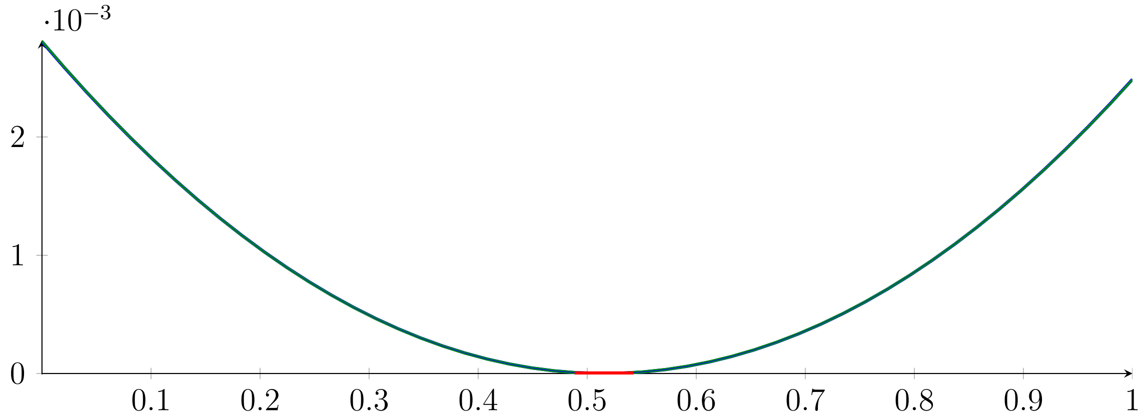
$$M = 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



$[0.4885305113706042, 0.542280720323693]$

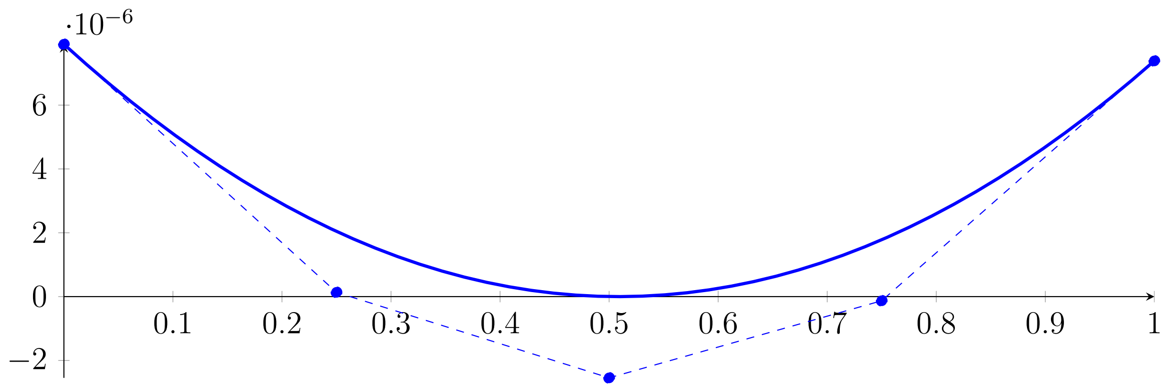
Longest intersection interval: 0.05375020895308878

$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

26.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.40181554717343$

Normalized monomial und Bézier representations and the Bézier polygon:

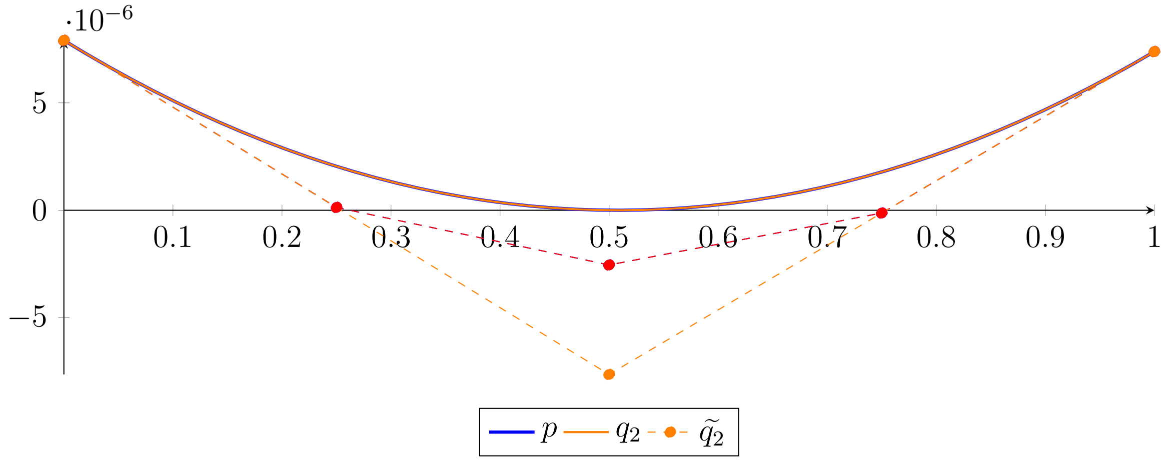
$$p = -1.862993414511891 \cdot 10^{-10} X^4 + 1.046428359410323 \cdot 10^{-08} X^3 + 3.055842313534119 \cdot 10^{-05} X^2 - 3.109078018724982 \cdot 10^{-05} X + 7.906738260659747 \cdot 10^{-06}$$
$$= 7.906738260659747 \cdot 10^{-06} B_{0,4}(X) + 1.340432138472922 \cdot 10^{-07} B_{1,4}(X) - 2.545581310408299 \cdot 10^{-06} B_{2,4}(X) - 1.295192412084993 \cdot 10^{-07} B_{3,4}(X) + 7.384659193003765 \cdot 10^{-06} B_{4,4}(X)$$



Degree reduction and raising:

$$q_2 = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06}$$
$$= 7.907245506324471 \cdot 10^{-06} B_{0,2} - 7.641198706965435 \cdot 10^{-06} B_{1,2} + 7.384157270177368 \cdot 10^{-06} B_{2,2}$$

$$\widetilde{q}_2 = 2.673714696616243 \cdot 10^{-311} X^4 - 5.466261157526541 \cdot 10^{-311} X^3 + 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06}$$
$$= 7.907245506324471 \cdot 10^{-06} B_{0,4} + 1.330233996795178 \cdot 10^{-07} B_{1,4} - 2.545565341893317 \cdot 10^{-06} B_{2,4} - 1.285207183940336 \cdot 10^{-07} B_{3,4} + 7.384157270177368 \cdot 10^{-06} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 1.019814167774439 \cdot 10^{-09}$.
Bounding polynomials M and m:

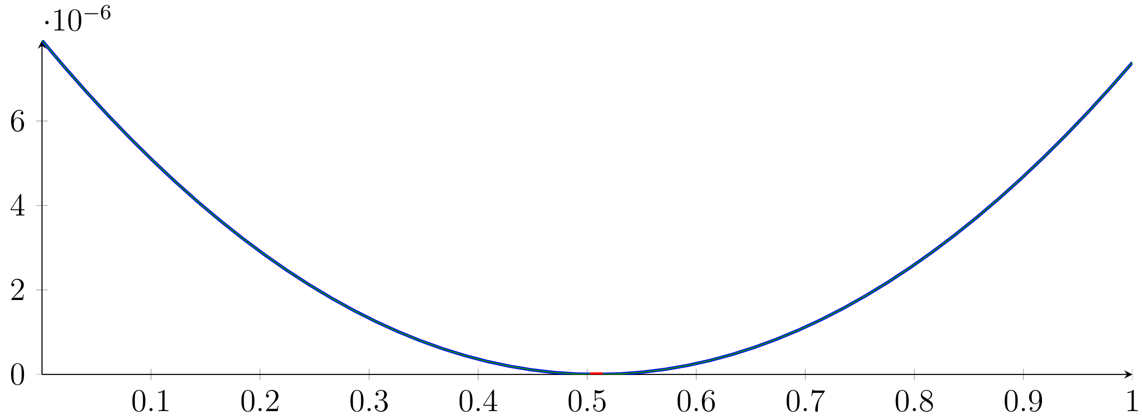
$$M = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.908265320492245 \cdot 10^{-06}$$

$$m = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.906225692156696 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.50281872462556, 0.5142903109829997\}$$

Intersection intervals:



$$[0.50281872462556, 0.5142903109829997]$$

Longest intersection interval: 0.01147158635743977
$\implies$ Selective recursion: interval 1: $[0.3999787229673132, 0.4000211044658684]$,

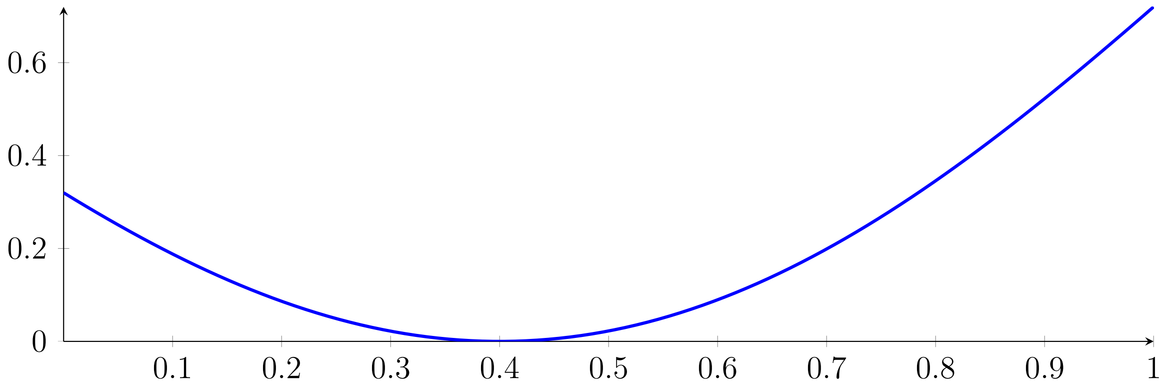
26.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999787229673132, 0.400021104465$

Found root in interval $[0.3999787229673132, 0.4000211044658684]$ at recursion depth 5!

26.6 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3999787229673132, 0.4000211044658684]$$

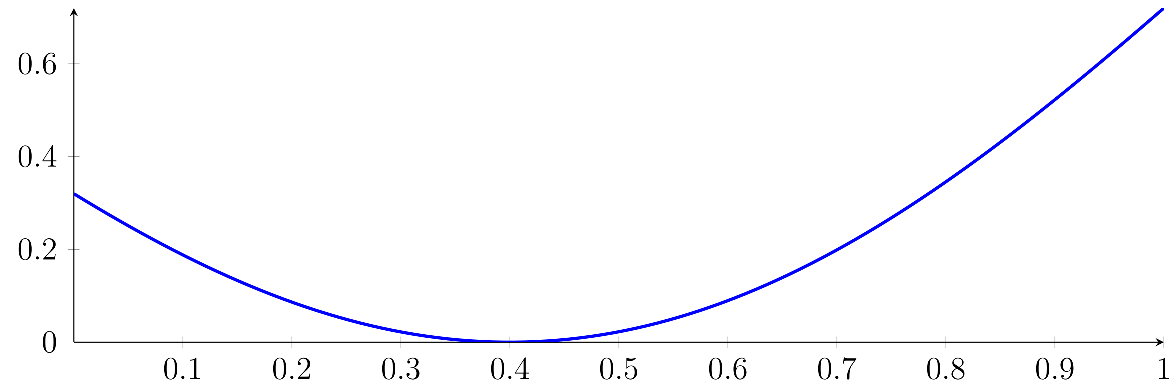
with precision $\varepsilon = 0.0001$.

27 Running `CubeClip` on h_4 with epsilon 4

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `CubeClip` with input polynomial on interval $[0,1]$:

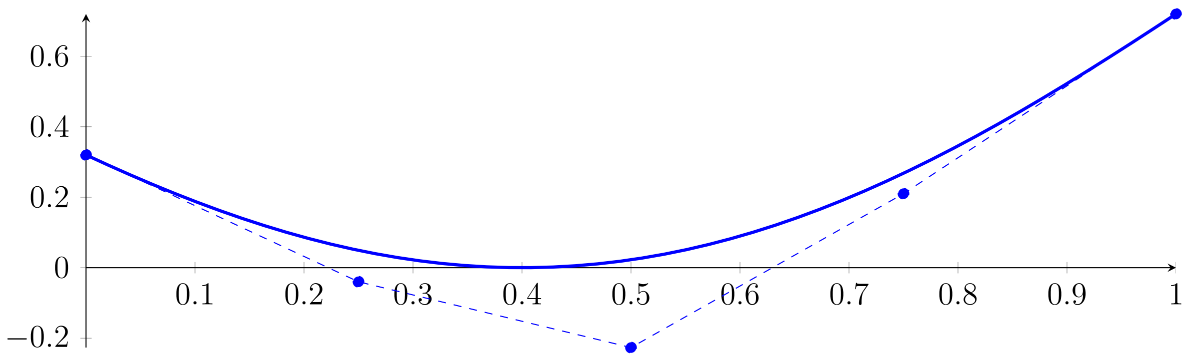
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



27.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

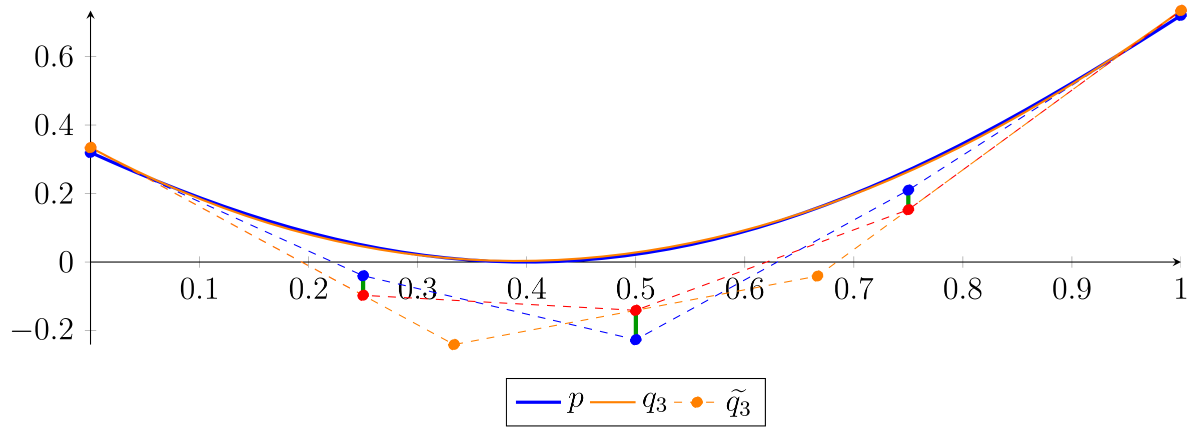
$$\begin{aligned}p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008\\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X)\\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143\\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3}\\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= -2.781342323134002\cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2\\ &\quad - 1.725714301714286X + 0.3342857222857143\\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4}\\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.

Bounding polynomials M and m:

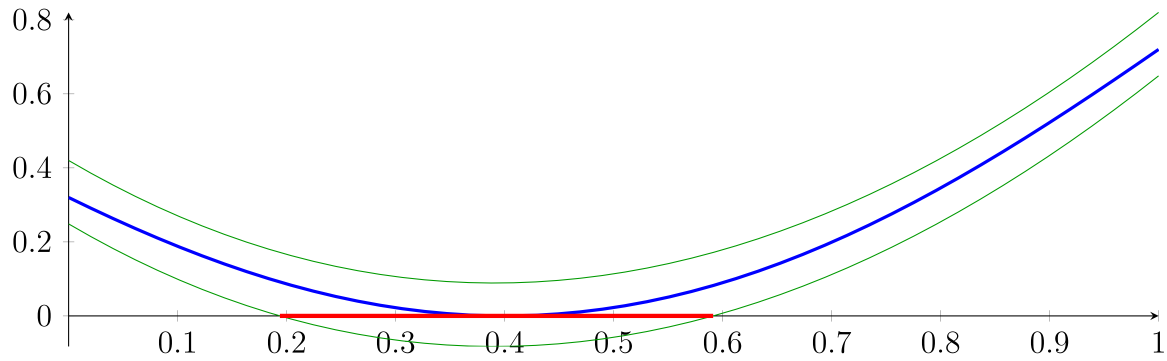
$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$

$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $\quad N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



$[0.1938265012447289, 0.5913474001559605]$

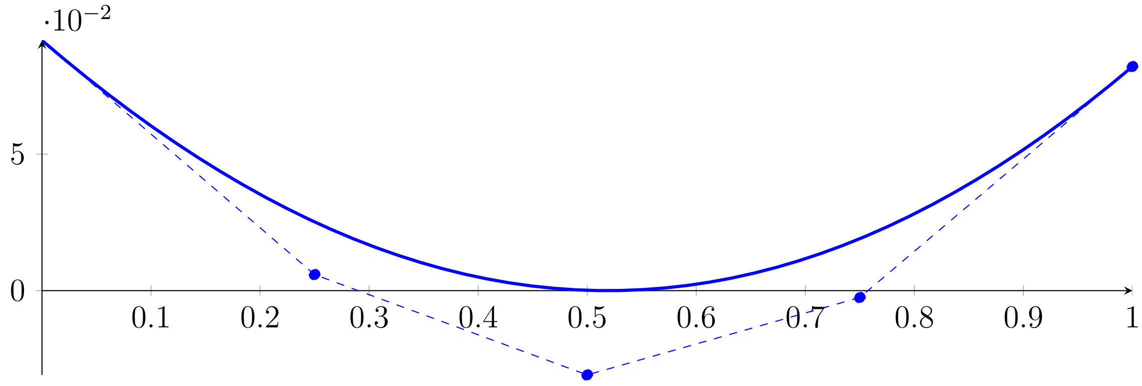
Longest intersection interval: 0.3975208989112316

$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

27.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

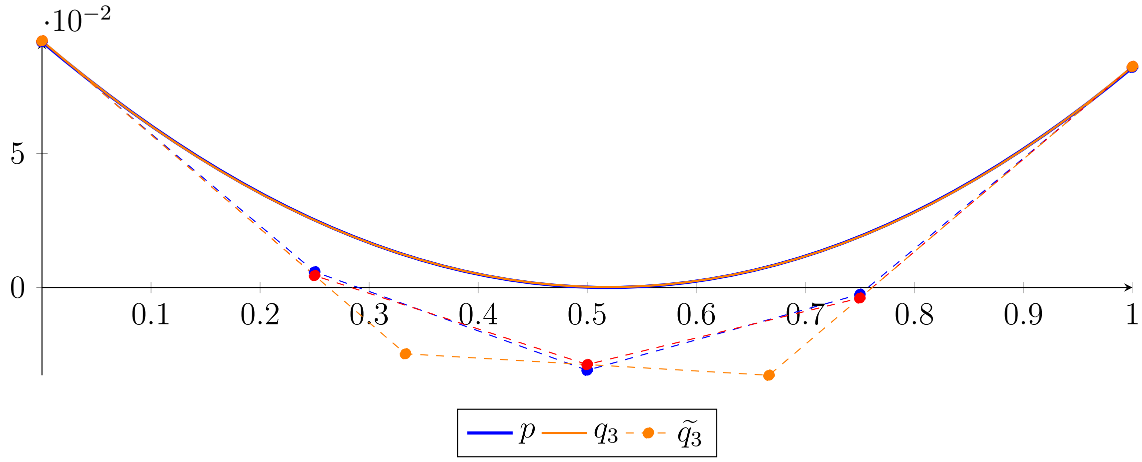
$$\begin{aligned} p &= -0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &\quad + 0.2941201876709534X^2 - 0.343099134331 92X + 0.09165715738594469 \\ &= 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &\quad - 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502{\cdot}10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455466609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.
Bounding polynomials M and m:

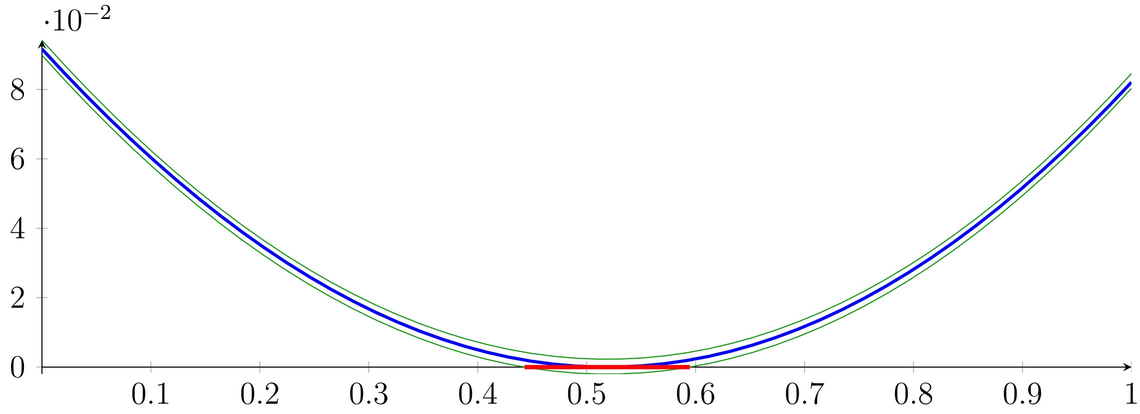
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



$$[0.4429176870393937, 0.5947109255489168]$$

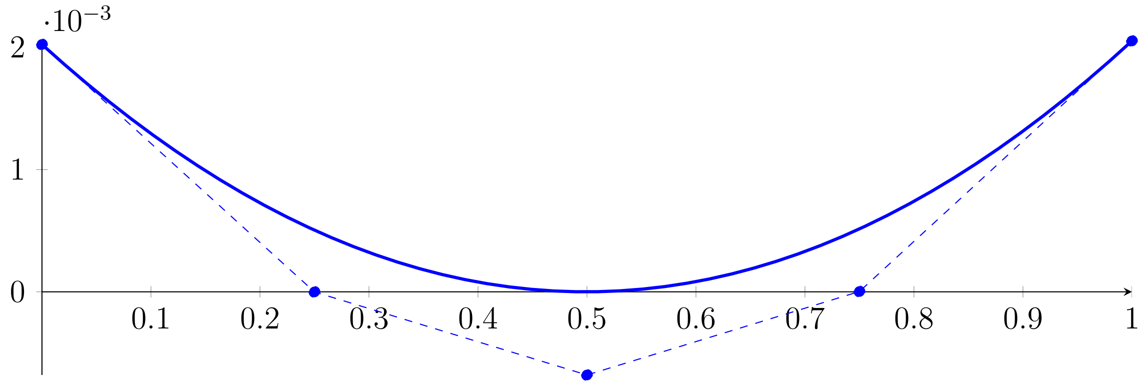
Longest intersection interval: 0.1517932385095231

$\Longrightarrow$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

27.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

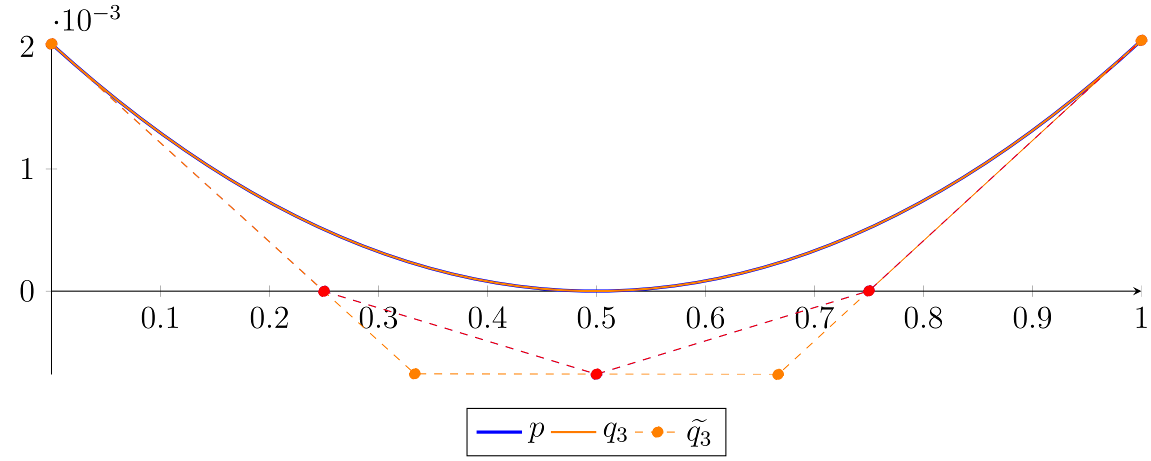
$$\begin{aligned} p = &-1.325713168422468 \cdot 10^{-05} X^4 + 7.039695732710913 \cdot 10^{-05} X^3 \\ &+ 0.008070351522992233 X^2 - 0.008098671960928044 X + 0.002023786815863734 \\ = &\, 0.002023786815863734 B_{0,4}(X) - 8.811743682771854 \cdot 10^{-07} B_{1,4}(X) \\ &- 0.0006804905774349 16 B_{2,4}(X) + 2.557845995594498 \\ &\cdot 10^{-06} B_{3,4}(X) + 0.002052606203570807 B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &\, 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ &- 0.008102459712837822 X + 0.002023976203459223 \\ = &\, 0.002023976203459223 B_{0,3} - 0.0006768437008200514 B_{1,3} \\ &- 0.0006818648029039136 B_{2,3} + 0.002052795591166296 B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = &-6.518771069845317 \cdot 10^{-311} X^4 + 4.388269395865976 \cdot 10^{-05} X^3 \\ &+ 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002023976203459223 \\ = &\, 0.002023976203459223 B_{0,4} - 1.638724750232882 \cdot 10^{-06} B_{1,4} - 0.0006793542518619825 B_{2,4} \\ &+ 1.800295613638801 \cdot 10^{-06} B_{3,4} + 0.002052795591166296 B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544 \cdot 10^{-06}$.
Bounding polynomials M and m:

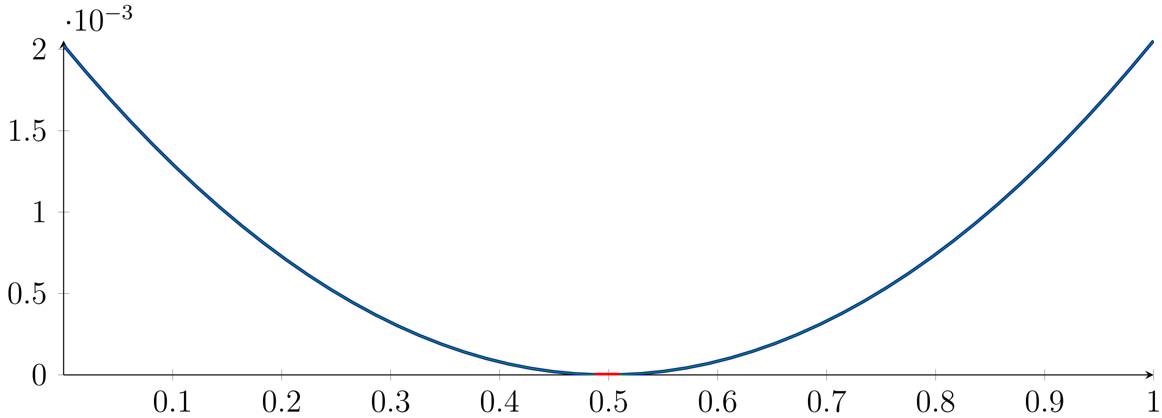
$$M = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002025112529032156$$

$$m = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002022839877886289$$

Root of M and m:

$N(M) = \{-185.2936165687423\}$ $N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649\}$

Intersection intervals:



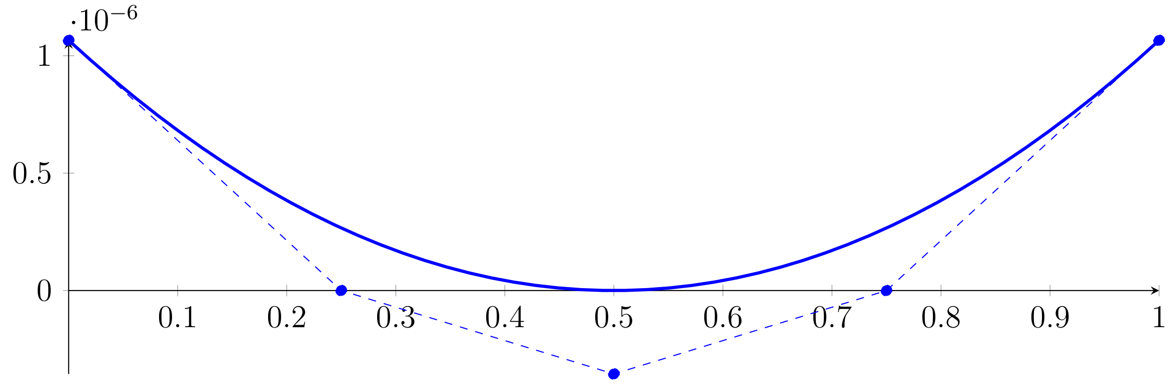
$$[0.4874742490566883, 0.5103358670775649]$$

Longest intersection interval: 0.02286161802087665
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

27.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.40068970704716$

Normalized monomial und Bézier representations and the Bézier polygon:

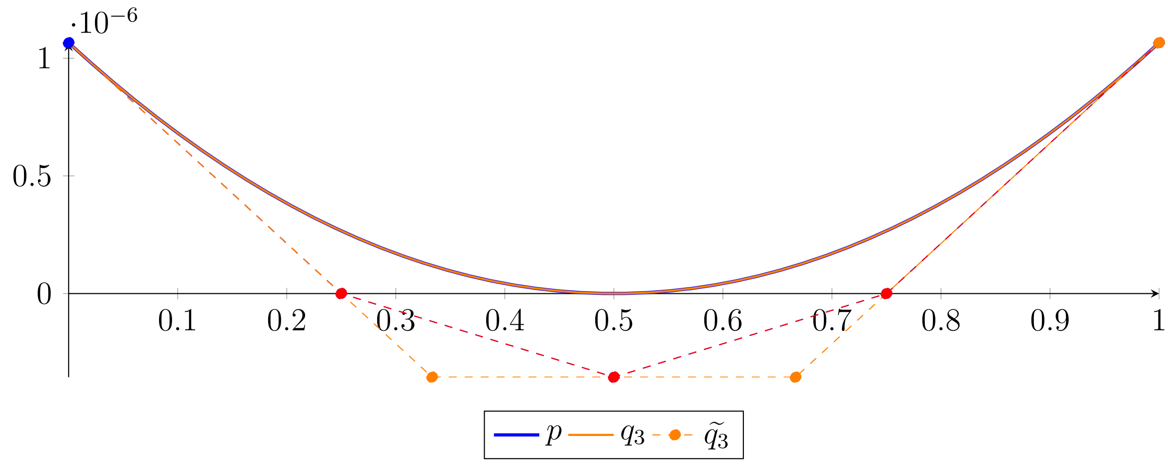
$$\begin{aligned} p &= -3.621407750870066 \cdot 10^{-12} X^4 + 5.322780243378263 \cdot 10^{-10} X^3 + 4.261926231315171 \\ &\quad \cdot 10^{-06} X^2 - 4.262596936226815 \cdot 10^{-06} X + 1.065750606194374 \cdot 10^{-06} \\ &= 1.065750606194374 \cdot 10^{-06} B_{0,4}(X) + 1.013721376702048 \cdot 10^{-10} B_{1,4}(X) - 3.552268233665051 \\ &\quad \cdot 10^{-07} B_{2,4}(X) - 1.009108120675939 \cdot 10^{-10} B_{3,4}(X) + 1.065608557899316 \cdot 10^{-06} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\
&\quad - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06} \\
&= 1.06575065792877\cdot 10^{-06}B_{0,3} - 3.551153323761442\cdot 10^{-07}B_{1,3} \\
&\quad - 3.553376935441088\cdot 10^{-07}B_{2,3} + 1.065608609633713\cdot 10^{-06}B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -5.304989477413181\cdot 10^{-314}X^4 + 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085 \\
&\quad \cdot 10^{-06}X^2 - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06} \\
&= 1.06575065792877\cdot 10^{-06}B_{0,4} + 1.011652000844408\cdot 10^{-10}B_{1,4} - 3.552265129601265 \\
&\quad \cdot 10^{-07}B_{2,4} - 1.011177496533579\cdot 10^{-10}B_{3,4} + 1.065608609633713\cdot 10^{-06}B_{4,4}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.104063786460057\cdot 10^{-13}$.

Bounding polynomials M and m:

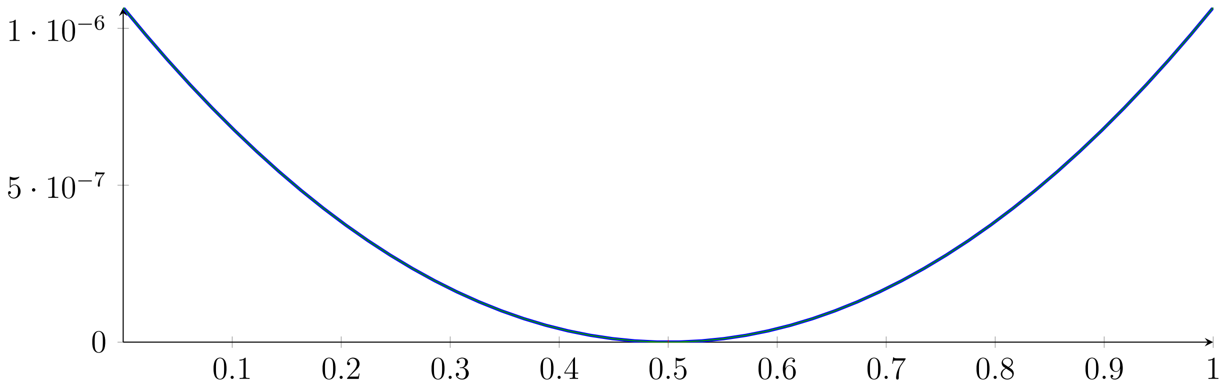
$$\begin{aligned}
M &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\
&\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750968335149\cdot 10^{-06}
\end{aligned}$$

$$\begin{aligned}
m &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\
&\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750347522392\cdot 10^{-06}
\end{aligned}$$

Root of M and m:

$N(M) = \{-8118.41926893212\}$ $\quad N(m) = \{-8118.419268932102, 0.4997707518163204, 0.5002933638554905\}$

Intersection intervals:



$$[0.4997707518163204, 0.5002933638554905]$$

Longest intersection interval: 0.0005226120391701048
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996445302967, 0.4000003654697068]$,

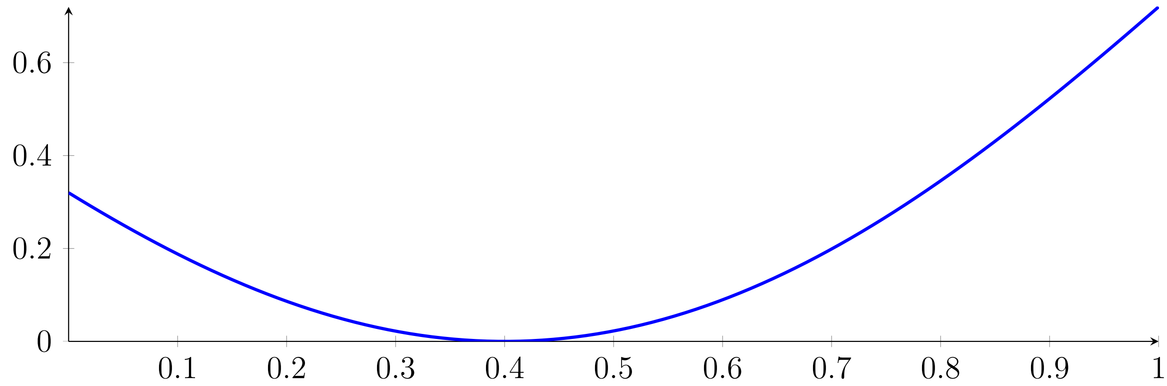
27.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999996445302967, 0.400000365469$

Found root in interval $[0.3999996445302967, 0.4000003654697068]$ at recursion depth 5!

27.6 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3999996445302967, 0.4000003654697068]$$

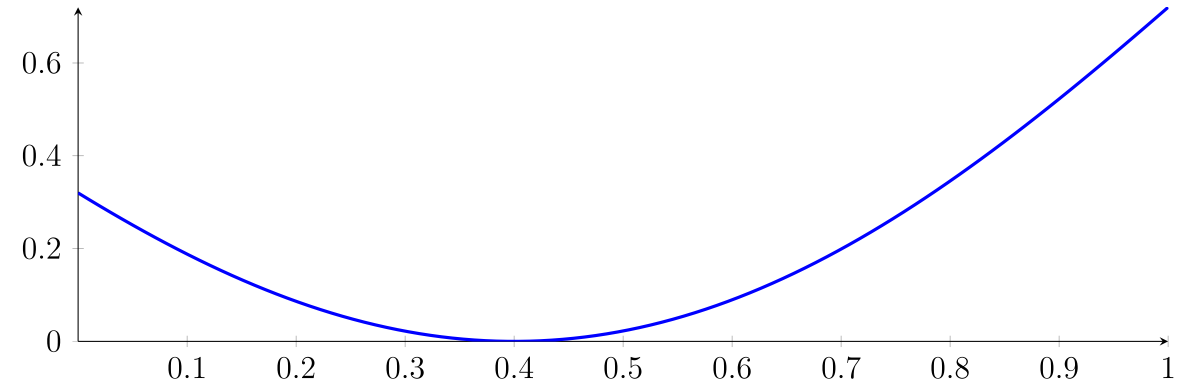
with precision $\varepsilon = 0.0001$.

28 Running BezClip on h_4 with epsilon 8

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called BezClip with input polynomial on interval $[0, 1]$:

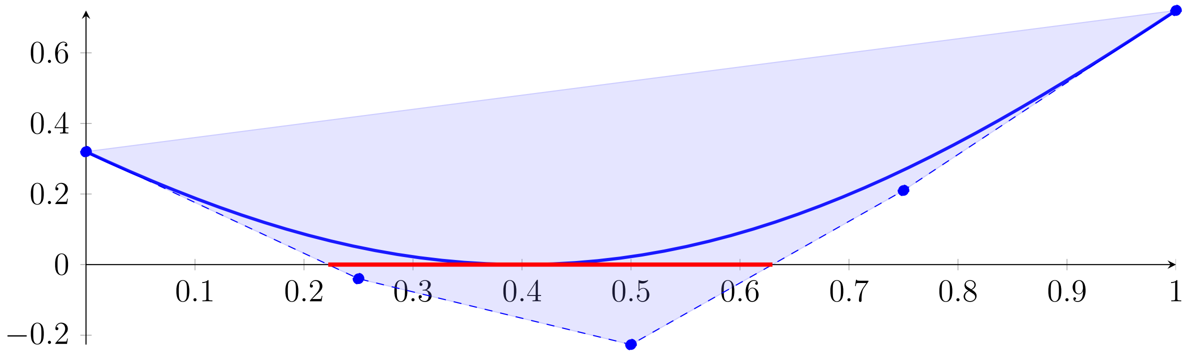
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



28.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

$$[0.222222225308642, 0.6297709955349339]$$

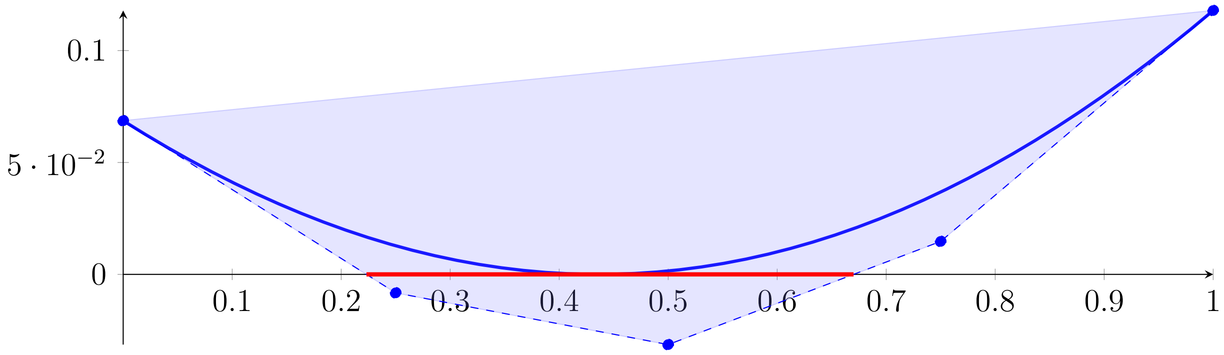
Longest intersection interval: 0.4075487702262920

$\implies$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

28.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &\quad + 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ &= 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &\quad + 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

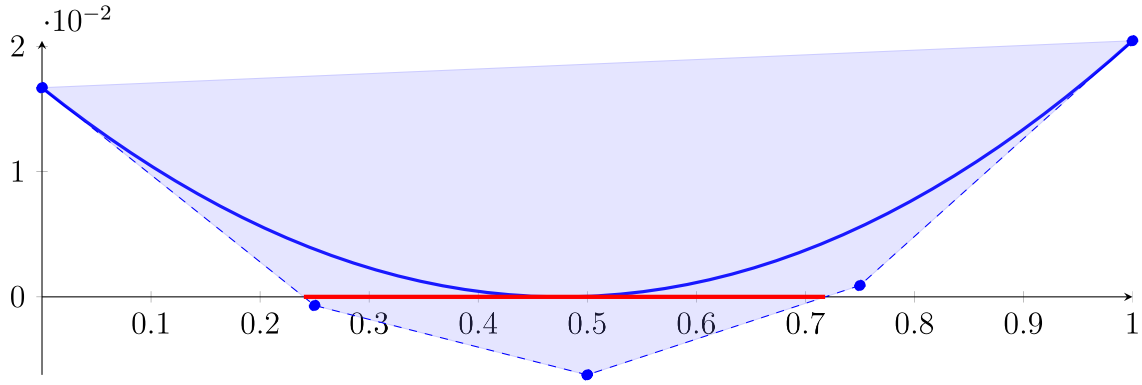
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\Longrightarrow$ Selective recursion: interval 1: [0.3131782986367004, 0.4953216655933138],

28.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.49532166559331$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -0.001100660652934182X^4 + 0.003307158935864367X^3 \\ & + 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377 \\ = & \, 0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X) \\ & + 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

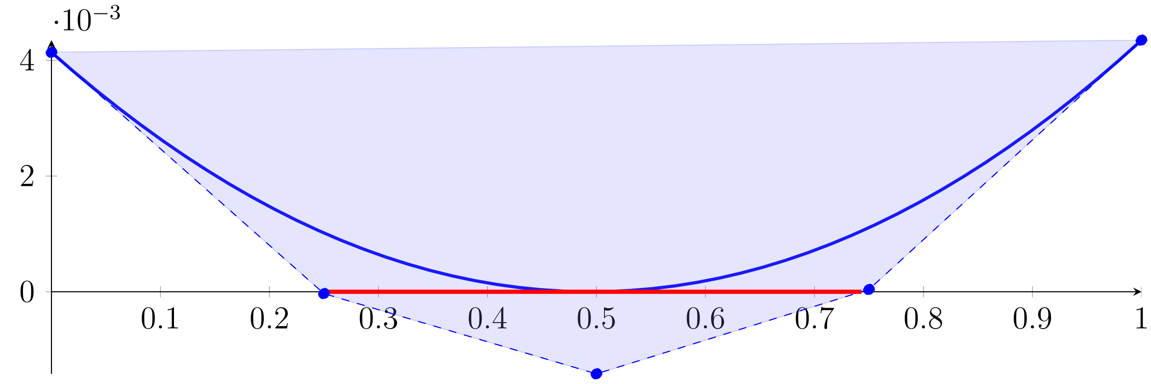
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\Longrightarrow$ Selective recursion: interval 1: [0.3569093751201798, 0.4439937820951003],

28.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.44399378209510$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p &= -5.751241374789061 \cdot 10^{-05} X^4 + 0.0002459162030906773 X^3 \\
&\quad + 0.01670691422385101 X^2 - 0.01668640845758443 X + 0.004139787469617577 \\
&= 0.004139787469617577 B_{0,4}(X) - 3.181464477852956 \cdot 10^{-05} B_{1,4}(X) \\
&\quad - 0.001418931055199467 B_{2,4}(X) + 3.991728912743387 \\
&\quad \cdot 10^{-05} B_{3,4}(X) + 0.004348697025226952 B_{4,4}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

$$[0.2480933797192248, 0.7431594518918532]$$

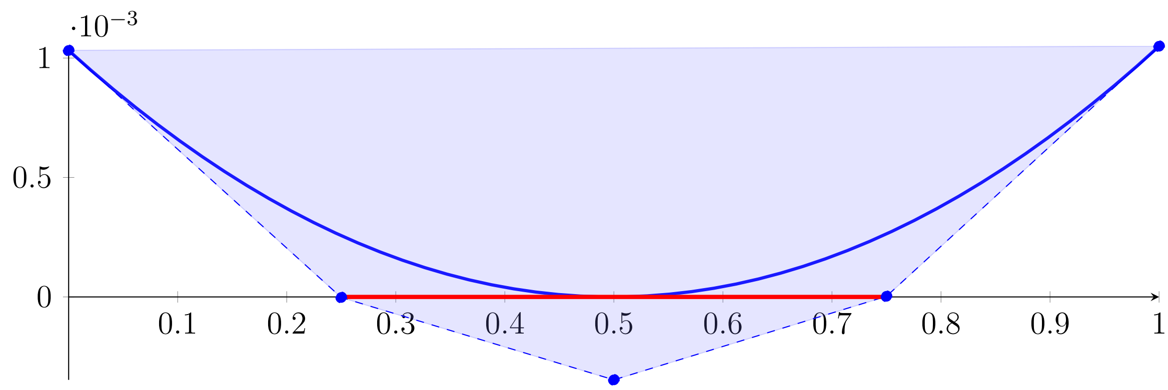
Longest intersection interval: 0.4950660721726284

$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

28.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p &= -3.454731120985874 \cdot 10^{-06} X^4 + 2.291337271769576 \cdot 10^{-05} X^3 \\
&\quad + 0.004134358001933594 X^2 - 0.004136159738306538 X + 0.001031853315293836 \\
&= 0.001031853315293836 B_{0,4}(X) - 2.186619282798525 \cdot 10^{-06} B_{1,4}(X) \\
&\quad - 0.0003471668868705005 B_{2,4}(X) + 2.640855710153786 \\
&\quad \cdot 10^{-06} B_{3,4}(X) + 0.001049510220517602 B_{4,4}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

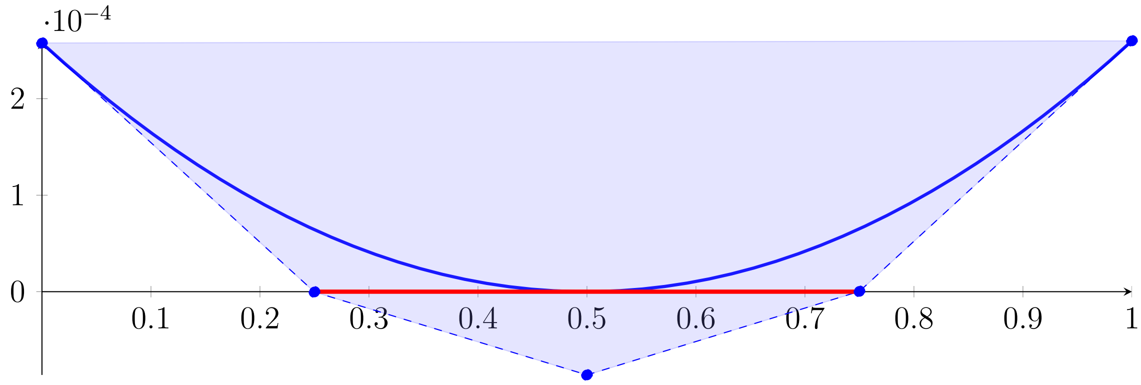
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\Longrightarrow$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

28.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.4107674724$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -2.135832675829806 \cdot 10^{-07} X^4 + 2.413460912088167 \cdot 10^{-06} X^3 \\ &\quad + 0.001031922910124016 X^2 - 0.001031832710654165 X + 0.0002576480709401398 \\ &= 0.0002576480709401398 B_{0,4}(X) - 3.101067234014613 \cdot 10^{-07} B_{1,4}(X) - 8.628113269960673 \\ &\quad \cdot 10^{-05} B_{2,4}(X) + 3.383582395460159 \cdot 10^{-07} B_{3,4}(X) + 0.0002599381480544958 B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

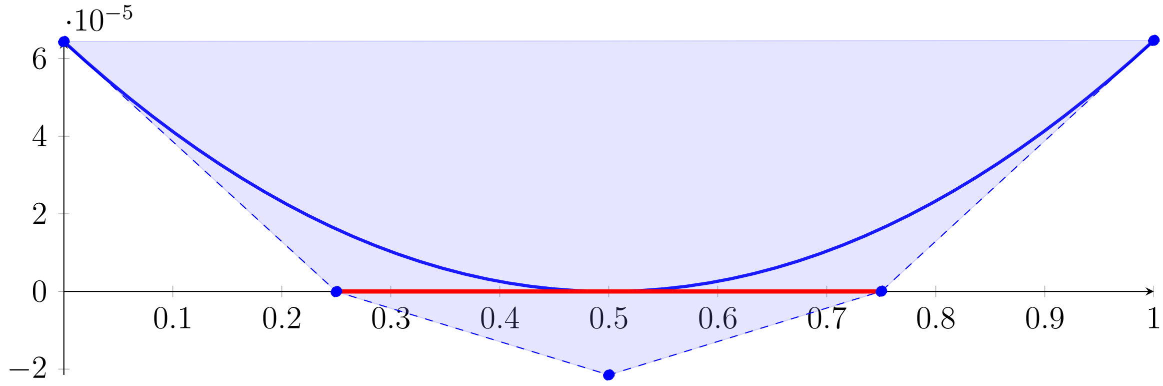
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\Longrightarrow$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

28.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.405372055$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.327690666746824 \cdot 10^{-08} X^4 + 2.739027984571277 \cdot 10^{-07} X^3 \\ &\quad + 0.0002577144304079 74 X^2 - 0.0002576778286693267 X + 6.437695235811399 \cdot 10^{-05} \\ &= 6.437695235811399 \cdot 10^{-05} B_{0,4}(X) - 4.250480921768141 \cdot 10^{-08} B_{1,4}(X) - 2.150955690855369 \\ &\quad \cdot 10^{-05} B_{2,4}(X) + 4.427175972025921 \cdot 10^{-08} B_{3,4}(X) + 6.467417998855097 \cdot 10^{-05} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

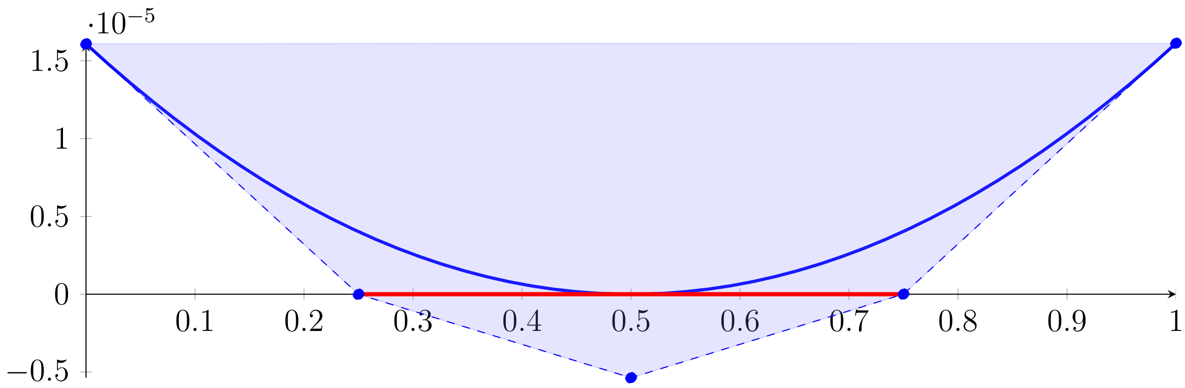
$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\Longrightarrow$ Selective recursion: interval 1: $[0.3973195510833879, 0.4026829657906133]$,

28.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.274952589981882\cdot 10^{-10}X^4 + 3.251124603913595\cdot 10^{-08}X^3 + 6.438882283687421\\&\quad\cdot 10^{-05}X^2 - 6.438267481175771\cdot 10^{-05}X + 1.609012302857716\cdot 10^{-05}\\&= 1.609012302857716\cdot 10^{-05}B_{0,4}(X) - 5.545674362270907\cdot 10^{-09}B_{1,4}(X) - 5.369743904489329\\&\quad\cdot 10^{-06}B_{2,4}(X) + 5.656149705765249\cdot 10^{-09}B_{3,4}(X) + 1.61279548044738\cdot 10^{-05}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499138638713212, 0.7497369428484978\}$$

Intersection intervals with the x axis:

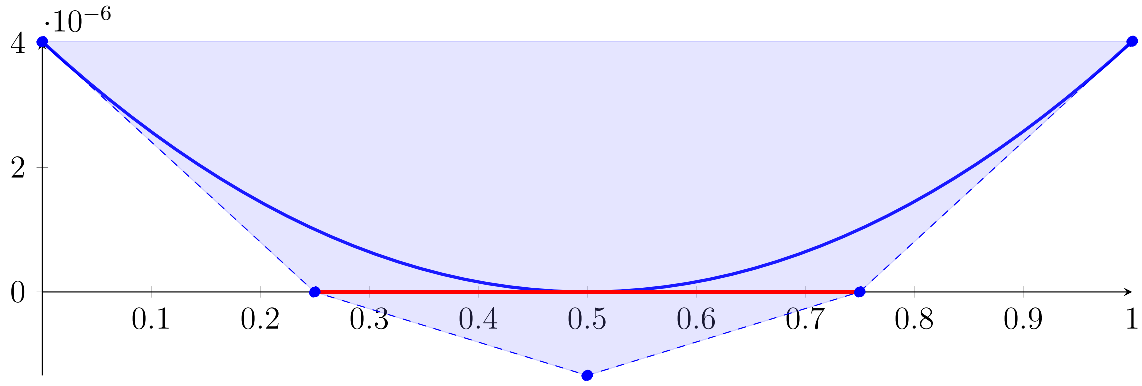
$$[0.2499138638713212, 0.7497369428484978]$$

Longest intersection interval: 0.4998230789771766
$\Longrightarrow$ Selective recursion: interval 1: $[0.3986599427764149, 0.4013407012292117]$,

28.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3986599427764149, 0.40134

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -5.164529187662443 \cdot 10^{-11} X^4 + 3.956301794548619 \cdot 10^{-09} X^3 + 1.609182796552748 \cdot 10^{-05} X^2 - 1.609096222935268 \cdot 10^{-05} X + 4.022033038444256 \cdot 10^{-06}$$
$$= 4.022033038444256 \cdot 10^{-06} B_{0,4}(X) - 7.075188939141538 \cdot 10^{-10} B_{1,4}(X) - 1.341476748644171 \cdot 10^{-06} B_{2,4}(X) + 7.14424642122371 \cdot 10^{-10} B_{3,4}(X) + 4.026803431121726 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499560300444542, 0.7498669294180401\}$$

Intersection intervals with the x axis:

$$[0.2499560300444542, 0.7498669294180401]$$

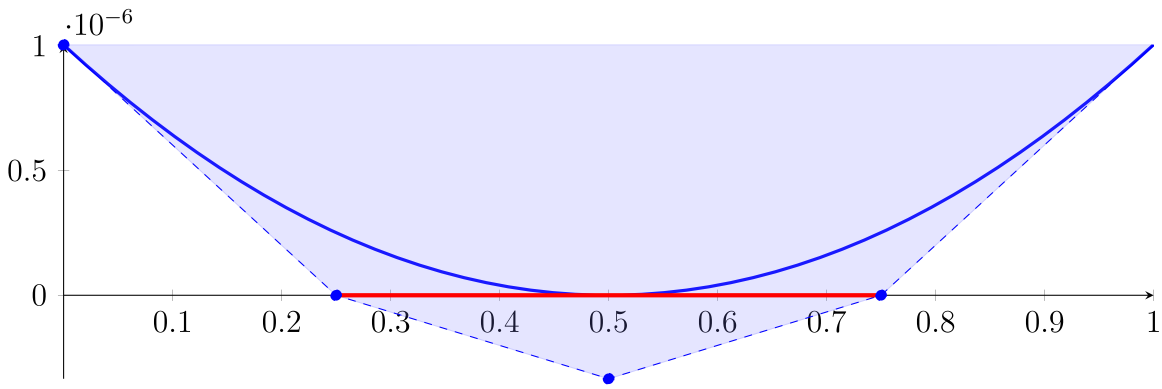
Longest intersection interval: 0.499910899373586

$\Longrightarrow$ Selective recursion: interval 1: [0.3993300145167841, 0.4006701548859251],

28.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3993300145167841, 0.400

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.225530543299082 \cdot 10^{-12} X^4 + 4.878223136378425 \cdot 10^{-10} X^3 + 4.022259900669466 \cdot 10^{-06} X^2 - 4.022145641341714 \cdot 10^{-06} X + 1.00544708348311 \cdot 10^{-06}$$
$$= 1.00544708348311 \cdot 10^{-06} B_{0,4}(X) - 8.932685231834447 \cdot 10^{-11} B_{1,4}(X) - 3.352490870761692 \cdot 10^{-07} B_{2,4}(X) + 8.975838996701131 \cdot 10^{-11} B_{3,4}(X) + 1.006045939593956 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499777912437083, 0.7499330838112102\}$$

Intersection intervals with the x axis:

$$[0.2499777912437083, 0.7499330838112102]$$

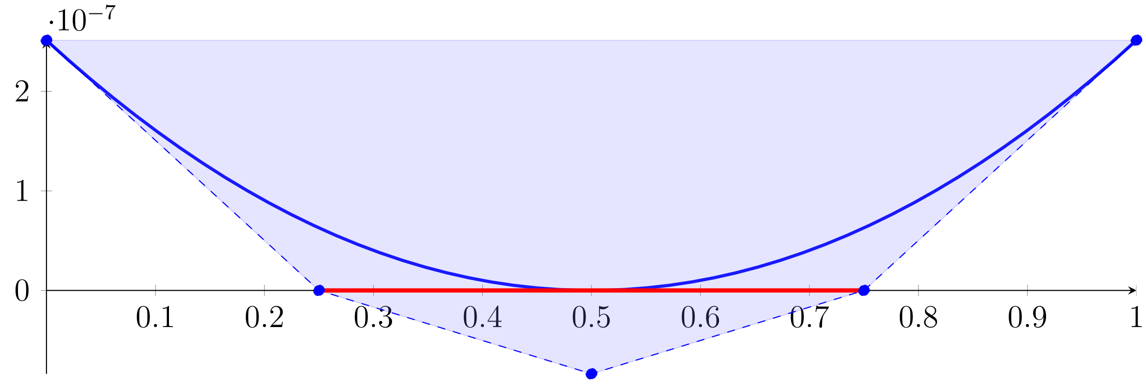
Longest intersection interval: 0.499955292567502

$\Longrightarrow$ Selective recursion: interval 1: [0.3996650198462185, 0.4003350301165539],

28.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3996650198462185, 0.4$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.015235660316643\cdot 10^{-13}X^4 + 6.055838633908679\cdot 10^{-11}X^3 + 1.005476298211762 \cdot 10^{-06}X^2 - 1.005461639191386\cdot 10^{-06}X + 2.51354188678016\cdot 10^{-07}$$
$$= 2.51354188678016\cdot 10^{-07}B_{0,4}(X) - 1.122111983053638\cdot 10^{-11}B_{1,4}(X) - 8.379724788238336 \cdot 10^{-08}B_{2,4}(X) + 1.124798694225224\cdot 10^{-11}B_{3,4}(X) + 2.51429204561165\cdot 10^{-07}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499888398329751, 0.7499664473546936\}$$

Intersection intervals with the x axis:

$$[0.2499888398329751, 0.7499664473546936]$$

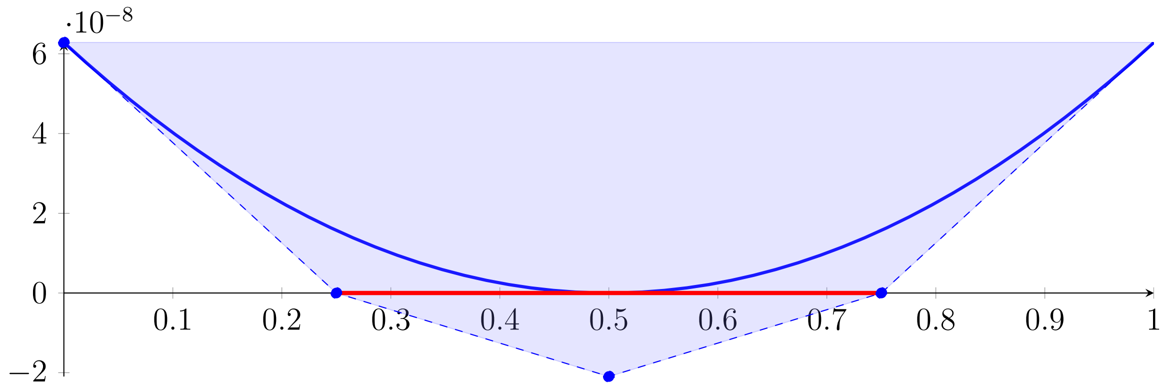
Longest intersection interval: 0.4999776075217186

$\Longrightarrow$ Selective recursion: interval 1: $[0.3998325149363758, 0.4001675050683531]$,

28.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3998325149363758, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.259296672250957\cdot 10^{-14}X^4 + 7.543595361590864\cdot 10^{-12}X^3 + 2.5135789423507 \cdot 10^{-07}X^2 - 2.51356038307676\cdot 10^{-07}X + 6.283760343276371\cdot 10^{-08}$$
$$= 6.283760343276371\cdot 10^{-08}B_{0,4}(X) - 1.406144155297956\cdot 10^{-12}B_{1,4}(X) - 2.094743334856264 \cdot 10^{-08}B_{2,4}(X) + 1.407718382090997\cdot 10^{-12}B_{3,4}(X) + 6.284699036255256\cdot 10^{-08}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499944057673539, 0.7499832005219574\}$$

Intersection intervals with the x axis:

$$[0.2499944057673539, 0.7499832005219574]$$

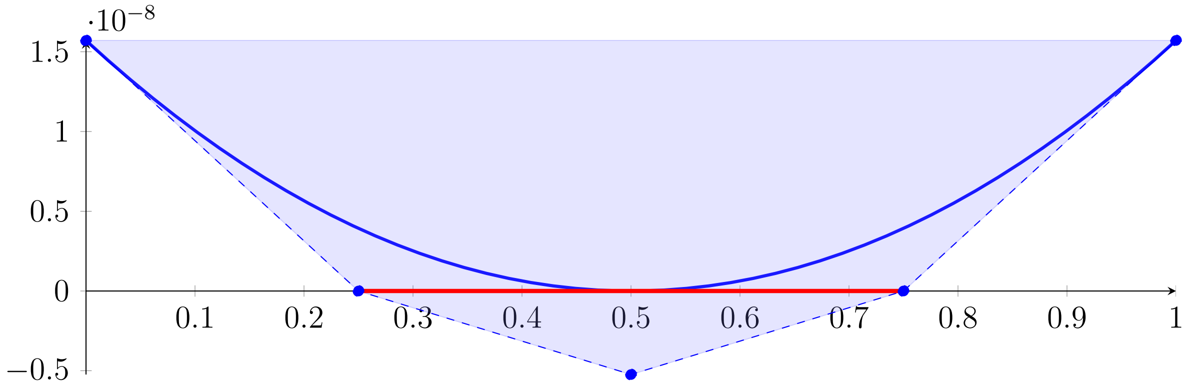
Longest intersection interval: 0.4999887947546035

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999162605953574, 0.4000837519076994]$,

28.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999162605953574

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.869898688873272\cdot 10^{-16}X^4 + 9.413120459510821\cdot 10^{-13}X^3 + 6.283807021204131\\&\quad\cdot 10^{-08}X^2 - 6.283783670437638\cdot 10^{-08}X + 1.570928313186755\cdot 10^{-08}\\&= 1.570928313186755\cdot 10^{-08}B_{0,4}(X) - 1.760442265467946\cdot 10^{-13}B_{1,4}(X) - 5.236623518313756\\&\quad\cdot 10^{-09}B_{2,4}(X) + 1.76037617409066\cdot 10^{-13}B_{3,4}(X) + 1.571045716458857\cdot 10^{-08}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499971984359141, 0.7499915961258623\}$$

Intersection intervals with the x axis:

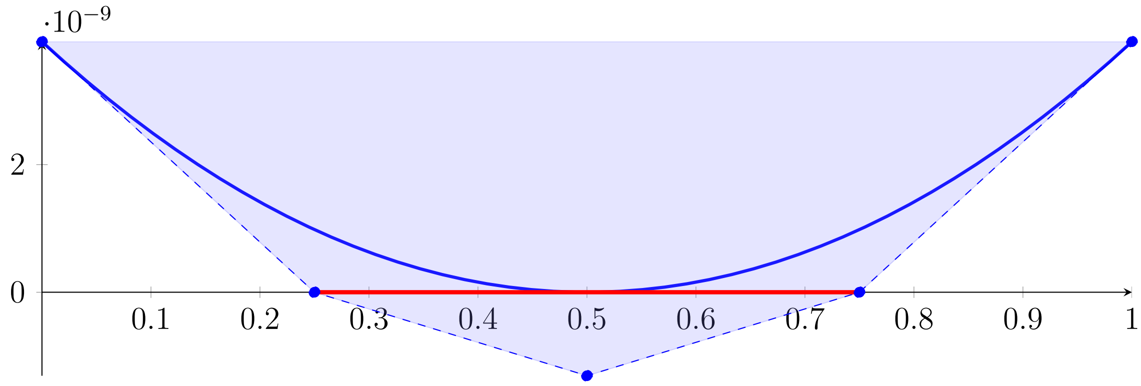
$$[0.2499971984359141, 0.7499915961258623]$$

Longest intersection interval: 0.4999943976899482
$\Longrightarrow$ Selective recursion: interval 1: [0.3999581329542053, 0.400041877672038],

28.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999581329542053, 0.400041877672038]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.918466236188167\cdot 10^{-17}X^4 + 1.175616813221855\cdot 10^{-13}X^3 + 1.570934193292886\\&\quad\cdot 10^{-08}X^2 - 1.57093126037727\cdot 10^{-08}X + 3.927306070737634\cdot 10^{-09}\\&= 3.927306070737634\cdot 10^{-09}B_{0,4}(X) - 2.208020554223638\cdot 10^{-14}B_{1,4}(X) - 1.309126575660575\\&\quad\cdot 10^{-09}B_{2,4}(X) + 2.19747928673869\cdot 10^{-14}B_{3,4}(X) + 3.927452912390452\cdot 10^{-09}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.249998594451196, 0.749995803609747\}$$

Intersection intervals with the x axis:

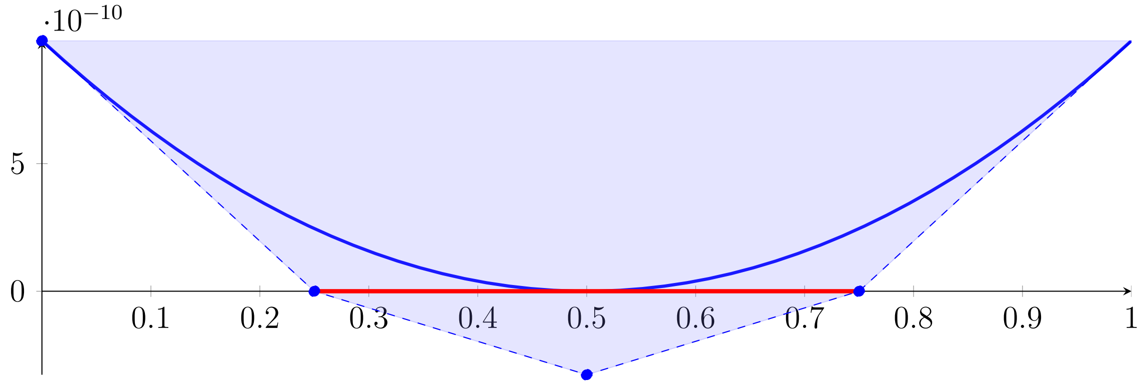
$$[0.249998594451196, 0.749995803609747]$$

Longest intersection interval: 0.499997209158551
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999790690159562, 0.4000209411411543]$,

28.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999790690159562, 0.4000209411411543]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -3.073972764895049 \cdot 10^{-18} X^4 + 1.468881614935617 \cdot 10^{-14} X^3 + 3.92731367890626 \\ &\quad \cdot 10^{-09} X^2 - 3.927309958033017 \cdot 10^{-09} X + 9.818246673938711 \cdot 10^{-10} \\ &= 9.818246673938711 \cdot 10^{-10} B_{0,4}(X) - 2.822114383159451 \cdot 10^{-15} B_{1,4}(X) - 3.272780318049275 \\ &\quad \cdot 10^{-10} B_{2,4}(X) + 2.710526275559223 \cdot 10^{-15} B_{3,4}(X) + 9.818430740092905 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499992814128721, 0.7499979295098023\}$$

Intersection intervals with the x axis:

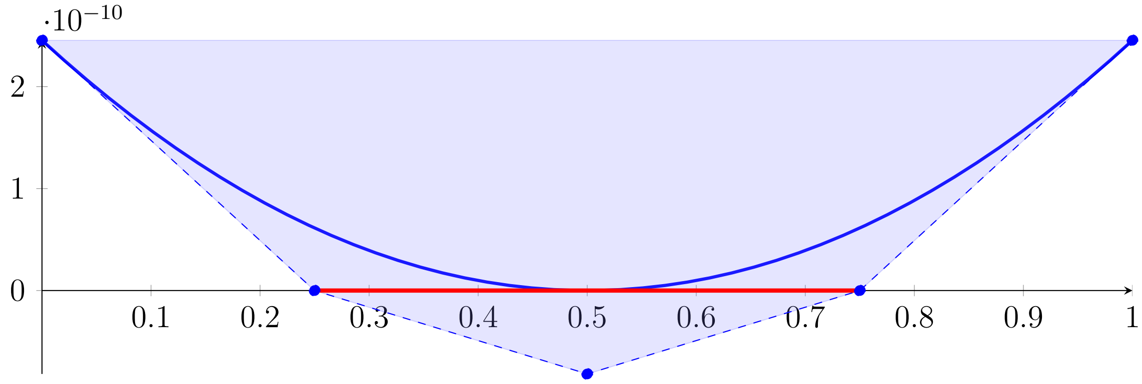
$$[0.2499992814128721, 0.7499979295098023]$$

Longest intersection interval: 0.4999986480969302
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999895370171669, 0.400010473023159]$,

28.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999895370171669, 0.400010473023159]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.921212199577591 \cdot 10^{-19} X^4 + 1.835702882943672 \cdot 10^{-15} X^3 + 9.818258642283608 \\ &\quad \cdot 10^{-10} X^2 - 9.818253497618774 \cdot 10^{-10} X + 2.454559233739064 \cdot 10^{-10} \\ &= 2.454559233739064 \cdot 10^{-10} B_{0,4}(X) - 4.140665629492134 \cdot 10^{-16} B_{1,4}(X) - 8.181910746897216 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.02092399485025 \cdot 10^{-16} B_{3,4}(X) + 2.454582733511515 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995782686167, 0.7499990769537415\}$$

Intersection intervals with the x axis:

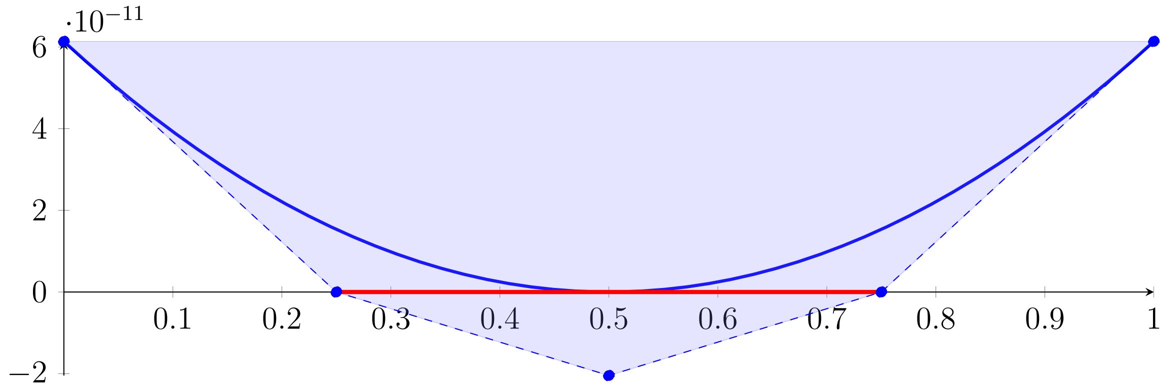
$$[0.2499995782686167, 0.7499990769537415]$$

Longest intersection interval: 0.4999994986851248
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999947710098356, 0.400005239002336]$,

28.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999947710098356, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.200752809081967\cdot 10^{-20} X^4 + 2.294381551300316\cdot 10^{-16} X^3 + 2.454563180284348 \\ &\cdot 10^{-10} X^2 - 2.454562046981275\cdot 10^{-10} X + 6.136393816301913\cdot 10^{-11} \\ = &\ 6.136393816301913\cdot 10^{-11} B_{0,4}(X) - 1.130115127449974\cdot 10^{-16} B_{1,4}(X) - 2.045477784797216 \\ &\cdot 10^{-11} B_{2,4}(X) + 1.013179678980307\cdot 10^{-18} B_{3,4}(X) + 6.136428091947402\cdot 10^{-11} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995395858382, 0.7499999876168341\}$$

Intersection intervals with the x axis:

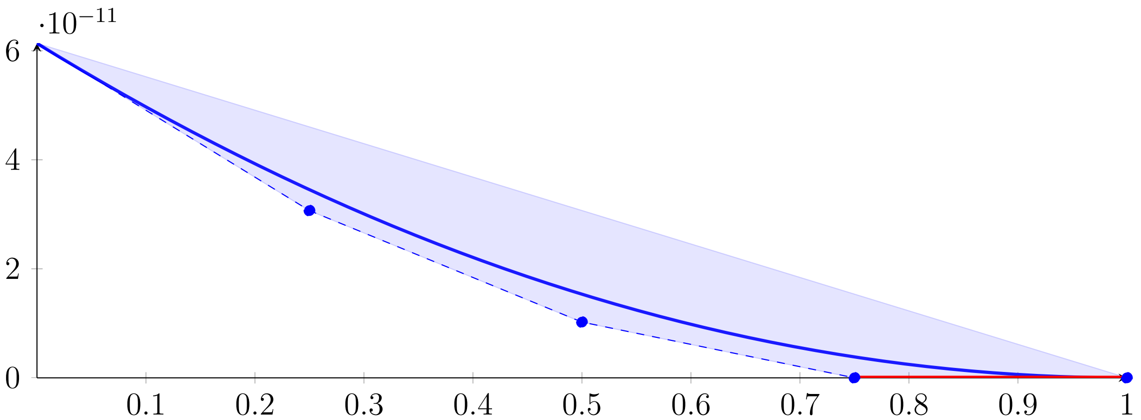
$$[0.2499995395858382, 0.7499999876168341]$$

Longest intersection interval: 0.5000004480309959
$\Longrightarrow$ Bisection: first half $[0.3999947710098356, 0.4000000050060858]$ und second half $[0.4000000050060858, 0.40$

28.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3999947710098356, 0.4000000050060858]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -7.504705056762291 \cdot 10^{-22} X^4 + 2.867976939125394 \cdot 10^{-17} X^3 + 6.13640795071087 \cdot 10^{-11} X^2 - 1.227281023490638 \cdot 10^{-10} X + 6.136393816301913 \cdot 10^{-11}$$
$$= 6.136393816301913 \cdot 10^{-11} B_{0,4}(X) + 3.068191257575319 \cdot 10^{-11} B_{1,4}(X) + 1.022723357300537 \cdot 10^{-11} B_{2,4}(X) - 9.167528198681642 \cdot 10^{-17} B_{3,4}(X) - 5.59999170035142 \cdot 10^{-17} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.7499977590601706, 0.9999990874140811\}$$

Intersection intervals with the x axis:

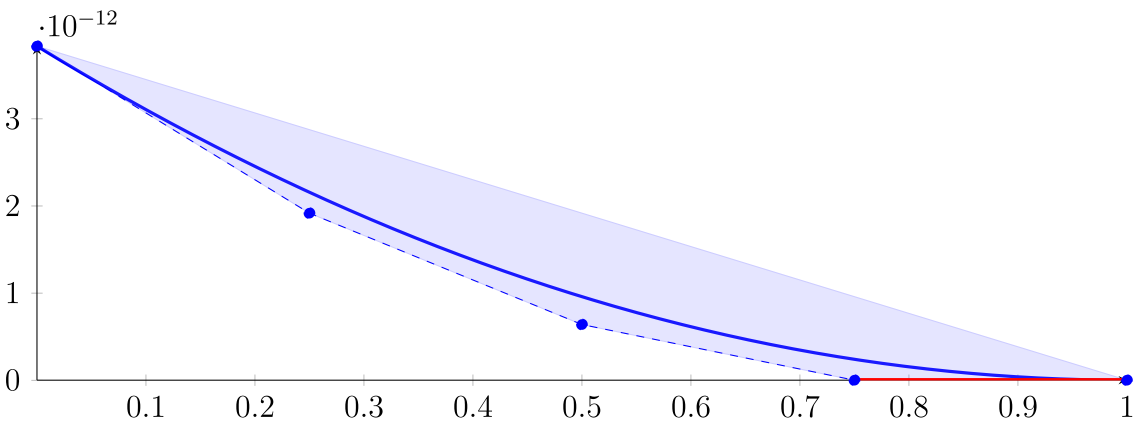
$$[0.7499977590601706, 0.9999990874140811]$$

Longest intersection interval: 0.2500013283539104
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999986964952942, 0.4000000050013093]$,

28.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999986964952942, 0.4000000050013093]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.931587718946285 \cdot 10^{-24} X^4 + 4.480933611814181 \cdot 10^{-19} X^3 + 3.835299758875134 \cdot 10^{-12} X^2 - 7.670593186649561 \cdot 10^{-12} X + 3.835236979687871 \cdot 10^{-12}$$
$$= 3.835236979687871 \cdot 10^{-12} B_{0,4}(X) + 1.917588683025481 \cdot 10^{-12} B_{1,4}(X) + 6.391570128422797 \cdot 10^{-13} B_{2,4}(X) - 5.791883839216654 \cdot 10^{-17} B_{3,4}(X) - 5.59999961259787 \cdot 10^{-17} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.7499773476668325, 0.9999853987696839\}$$

Intersection intervals with the x axis:

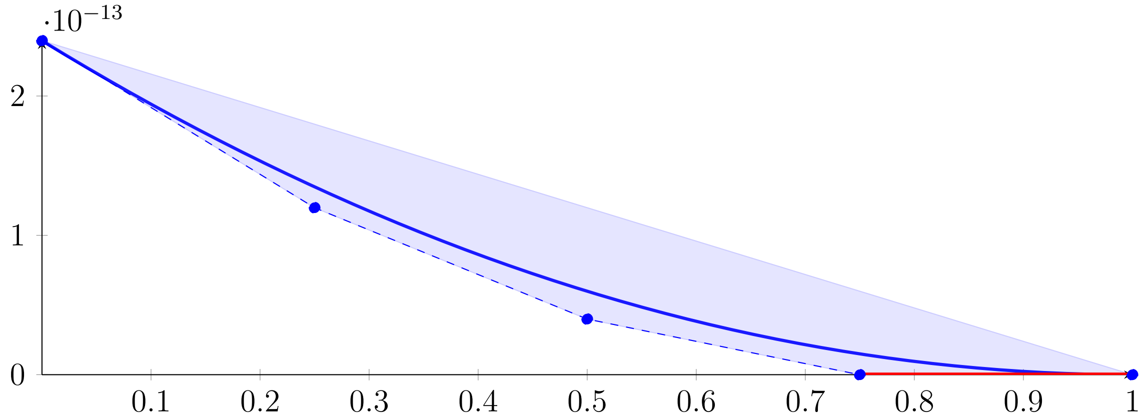
$$[0.7499773476668325, 0.9999853987696839]$$

Longest intersection interval: 0.2500080511028514
$\implies$ Selective recursion: interval 1: $[0.3999996778451648, 0.4000000049822035]$,

28.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999996778451648, 0.4000000049822035]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.145298975553555\cdot 10^{-26}X^4 + 7.001997796519582\cdot 10^{-21}X^3 + 2.397217373893792 \\ &\quad \cdot 10^{-13}X^2 - 4.794695778397391\cdot 10^{-13}X + 2.396918341578483\cdot 10^{-13} \\ &= 2.396918341578483\cdot 10^{-13}B_{0,4}(X) + 1.198244396979135\cdot 10^{-13}B_{1,4}(X) + 3.991066813620863 \\ &\quad \cdot 10^{-14}B_{2,4}(X) - 4.947877676695723\cdot 10^{-17}B_{3,4}(X) - 5.599929052523716\cdot 10^{-17}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7496904492313635, 0.9997664242061345\}$$

Intersection intervals with the x axis:

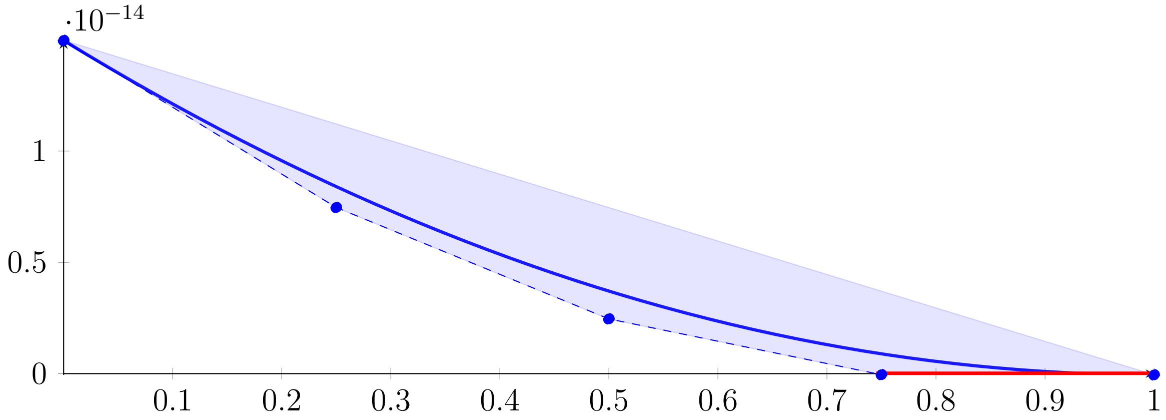
$$[0.7496904492313635, 0.9997664242061345]$$

Longest intersection interval: 0.250075974974771
$\implies$ Selective recursion: interval 1: $[0.3999999230966783, 0.4000000049057922]$,

28.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999230966783, 0.4000000049057922]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -4.479264981640815\cdot 10^{-29}X^4 + 1.095054543732688\cdot 10^{-22}X^3 + 1.499171738187779 \\ &\quad \cdot 10^{-14}X^2 - 3.001796269578916\cdot 10^{-14}X + 1.497026508467358\cdot 10^{-14} \\ &= 1.497026508467358\cdot 10^{-14}B_{0,4}(X) + 7.465774410726289\cdot 10^{-15}B_{1,4}(X) + 2.459903300425297 \\ &\quad \cdot 10^{-15}B_{2,4}(X) - 4.734821885303232\cdot 10^{-17}B_{3,4}(X) - 5.598011973238056\cdot 10^{-17}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7452788722542423, 0.9962745104335203\}$$

Intersection intervals with the x axis:

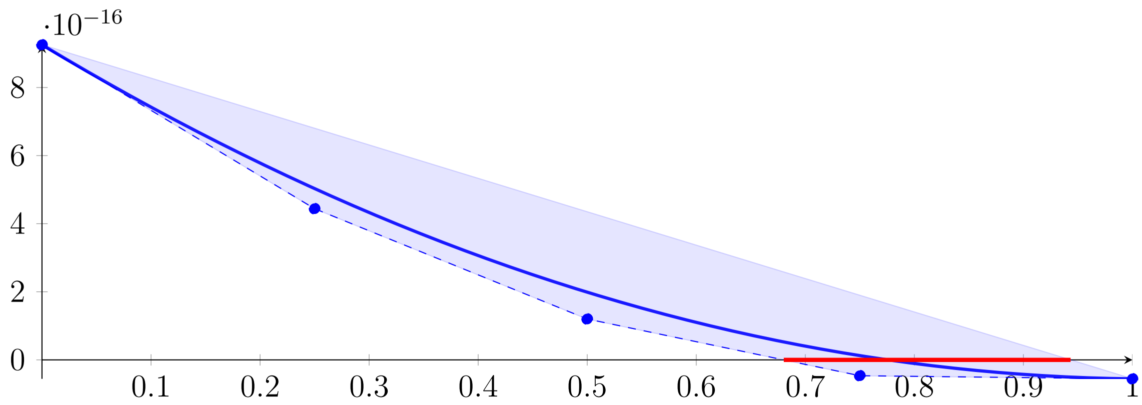
$$[0.7452788722542423, 0.9962745104335203]$$

Longest intersection interval: 0.250995638179278
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999840672825, 0.4000000046010132]$,

28.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999840672825, 0.4000000046010132]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.777753131478217 \cdot 10^{-31} X^4 + 1.731544849986219 \cdot 10^{-24} X^3 + 9.444603761111594 \\ &\quad \cdot 10^{-16} X^2 - 1.925623994999771 \cdot 10^{-15} X + 9.25520203786835 \cdot 10^{-16} \\ &= 9.25520203786835 \cdot 10^{-16} B_{0,4}(X) + 4.441142050368923 \cdot 10^{-16} B_{1,4}(X) + 1.201182689721428 \\ &\quad \cdot 10^{-16} B_{2,4}(X) - 4.646760397452725 \cdot 10^{-17} B_{3,4}(X) - 5.564341337023184 \cdot 10^{-17} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.6802647890355577, 0.9432883441688765\}$$

Intersection intervals with the x axis:

$$[0.6802647890355577, 0.9432883441688765]$$

Longest intersection interval: 0.2630235551333187
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999980356565, 0.4000000034365114]$,

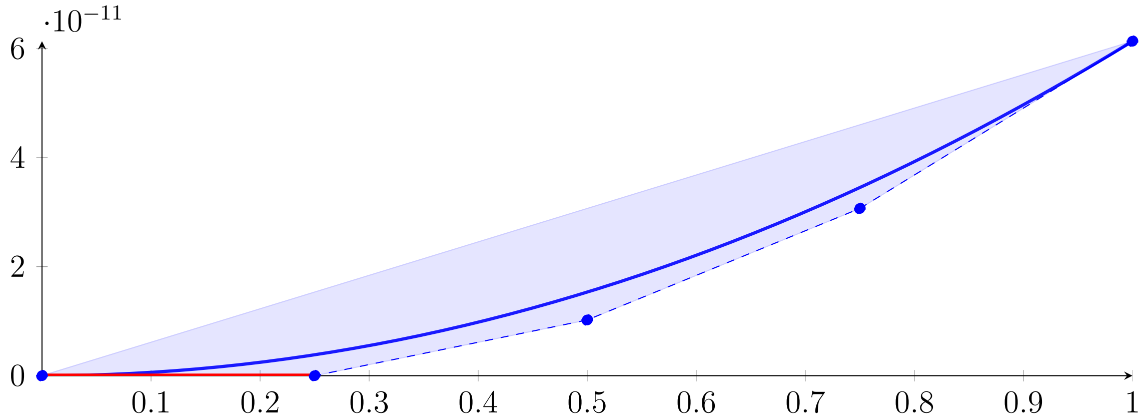
28.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999980356565, 0.4000000034365114]$

Found root in interval $[0.3999999980356565, 0.4000000034365114]$ at recursion depth 23!

28.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.4000000050060858, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-7.504705056762291\cdot 10^{-22}X^4 + 2.867676750923124\cdot 10^{-17}X^3 + 6.136416554191405\\ &\cdot 10^{-11}X^2 + 1.427014599332089\cdot 10^{-16}X - 5.59999170035142\cdot 10^{-17}\\ = &-5.59999170035142\cdot 10^{-17}B_{0,4}(X) - 2.032455202021197\cdot 10^{-17}B_{1,4}(X) + 1.02273762744653\\ &\cdot 10^{-11}B_{2,4}(X) + 3.068214096632685\cdot 10^{-11}B_{3,4}(X) + 6.136428091947402\cdot 10^{-11}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{9.125808216110345e-07, 0.2500004968163653\}$$

Intersection intervals with the x axis:

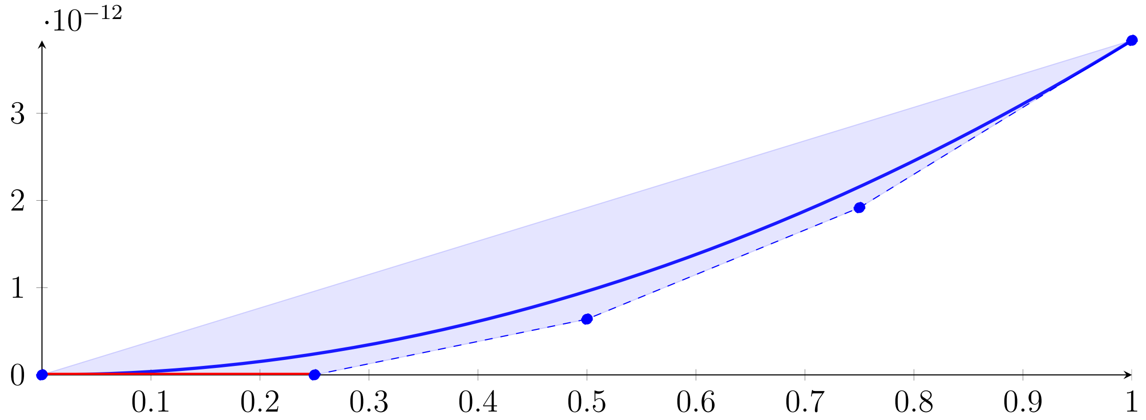
$$[9.125808216110345e-07, 0.2500004968163653]$$

Longest intersection interval: 0.2499995842355437
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050108623, 0.4000013135077487]$,

28.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.4000000050108623, 0.4000013135077487]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.931505911661308\cdot 10^{-24}X^4 + 4.480722567712799\cdot 10^{-19}X^3 + 3.835247589865681\\ &\cdot 10^{-12}X^2 + 6.367513939156292\cdot 10^{-17}X - 5.59997356725911\cdot 10^{-17}\\ = &-5.59997356725911\cdot 10^{-17}B_{0,4}(X) - 4.008095082470037\cdot 10^{-17}B_{1,4}(X) + 6.391837694783034\\ &\cdot 10^{-13}B_{2,4}(X) + 1.917615663569776\cdot 10^{-12}B_{3,4}(X) + 3.835255713338726\cdot 10^{-12}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{1.460109108777016e-05, 0.2500156756317829\}$$

Intersection intervals with the x axis:

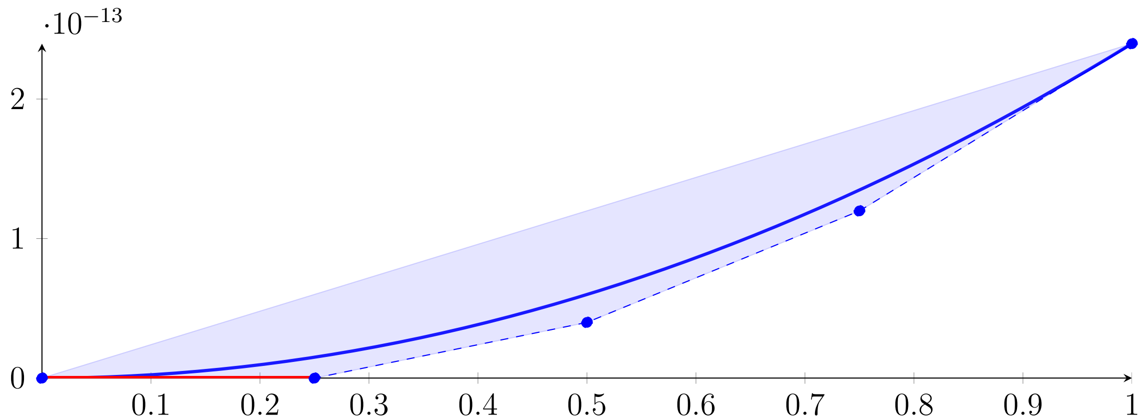
$$[1.460109108777016e-05, 0.2500156756317829]$$

Longest intersection interval: 0.2500010745406951
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050299677, 0.4000003321555954]$,

28.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.4000000050299677, 0.4000003321555954]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.145139184509 63\cdot 10^{-26}X^4 + 7.00121928574046\cdot 10^{-21}X^3 + 2.397050349370657\\ &\cdot 10^{-13}X^2 + 4.391837331735849\cdot 10^{-17}X - 5.599798830250992\cdot 10^{-17}\\ = &-5.599798830250992\cdot 10^{-17}B_{0,4}(X) - 4.50183949731703\cdot 10^{-17}B_{1,4}(X) + 3.991680035453379\\ &\cdot 10^{-14}B_{2,4}(X) + 1.198294600105232\cdot 10^{-13}B_{3,4}(X) + 2.396929623232884\cdot 10^{-13}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0002335692644077866, 0.2502816337968459\}$$

Intersection intervals with the x axis:

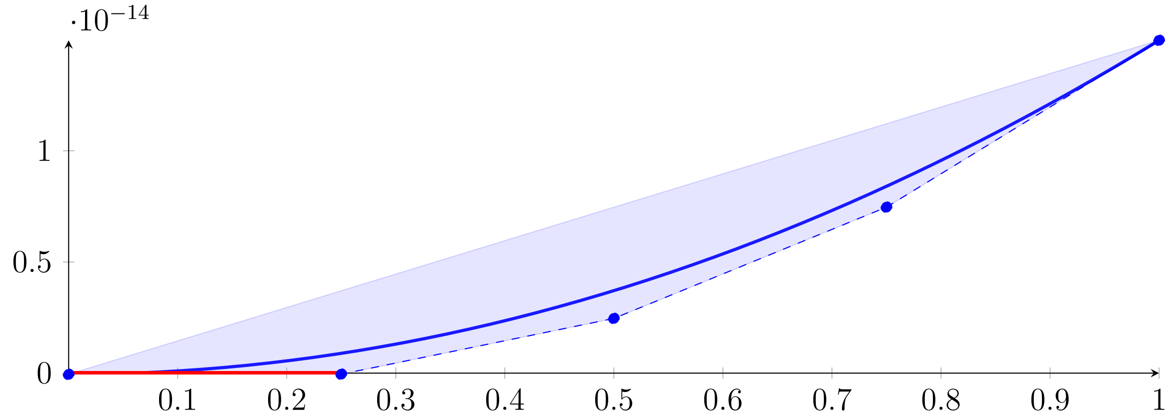
$$[0.0002335692644077866, 0.2502816337968459]$$

Longest intersection interval: 0.2500480645324381
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000051063742, 0.4000000869035043]$,

28.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.4000000051063742, 0.4000000869035043]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-4.476640967897802\cdot 10^{-29}X^4 + 1.09457158991016\cdot 10^{-22}X^3 + 1.49873258928534\\ &\cdot 10^{-14}X^2 + 3.898095063625474\cdot 10^{-17}X - 5.597465330775527\cdot 10^{-17}\\ = &-5.597465330775527\cdot 10^{-17}B_{0,4}(X) - 4.622941564869158\cdot 10^{-17}B_{1,4}(X) + 2.461403470819272\\ &\cdot 10^{-15}B_{2,4}(X) + 7.466924033460424\cdot 10^{-15}B_{3,4}(X) + 1.497033229963901\cdot 10^{-14}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.003725110466798912, 0.2546088699723713\}$$

Intersection intervals with the x axis:

$$[0.003725110466798912, 0.2546088699723713]$$

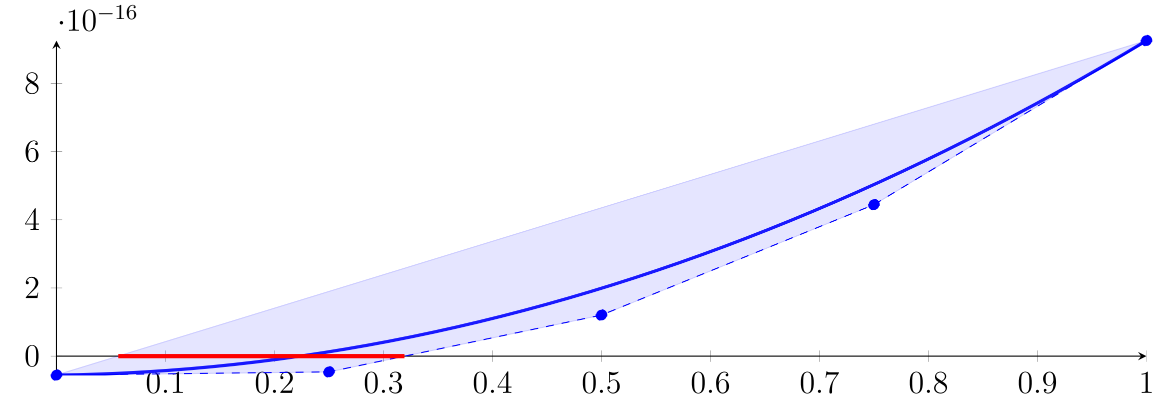
Longest intersection interval: 0.2508837595055724

$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000054110776, 0.4000000259326491]$,

28.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.4000000054110776, 0.4000000259326491]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.773546014711518\cdot 10^{-31}X^4 + 1.728469879701229\cdot 10^{-24}X^3 + 9.433421698048765\\ &\cdot 10^{-16}X^2 + 3.779308932689389\cdot 10^{-17}X - 5.562147411226954\cdot 10^{-17}\\ = &-5.562147411226954\cdot 10^{-17}B_{0,4}(X) - 4.617320178054607\cdot 10^{-17}B_{1,4}(X) + 1.204987655186568\\ &\cdot 10^{-16}B_{2,4}(X) + 4.443944282174566\cdot 10^{-16}B_{3,4}(X) + 9.255137867479705\cdot 10^{-16}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.05669093378980359, 0.3192576000162909\}$$

Intersection intervals with the x axis:

$$[0.05669093378980359, 0.3192576000162909]$$

Longest intersection interval: 0.2625666662264874
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000065744646, 0.4000000119627452]$,

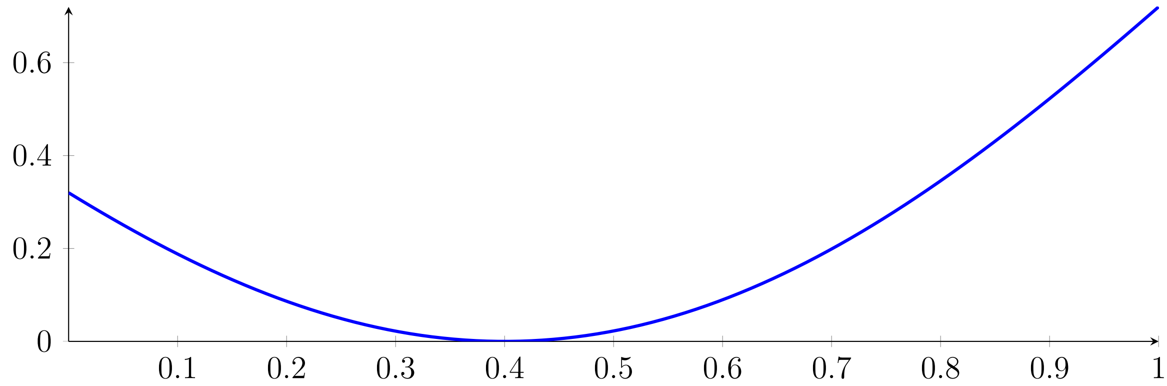
28.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.4000000065744646, 0.4000000119627452]$

Found root in interval $[0.4000000065744646, 0.4000000119627452]$ at recursion depth 23!

28.30 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3999999980356565, 0.4000000034365114],\ [0.4000000065744646, 0.4000000119627452]$$

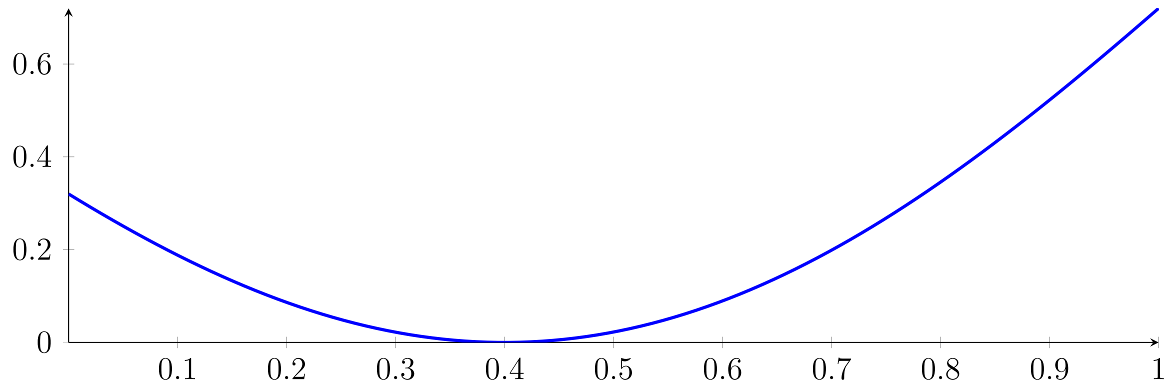
with precision $\varepsilon = 1 \cdot 10^{-08}$.

29 Running QuadClip on h_4 with epsilon 8

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called QuadClip with input polynomial on interval $[0, 1]$:

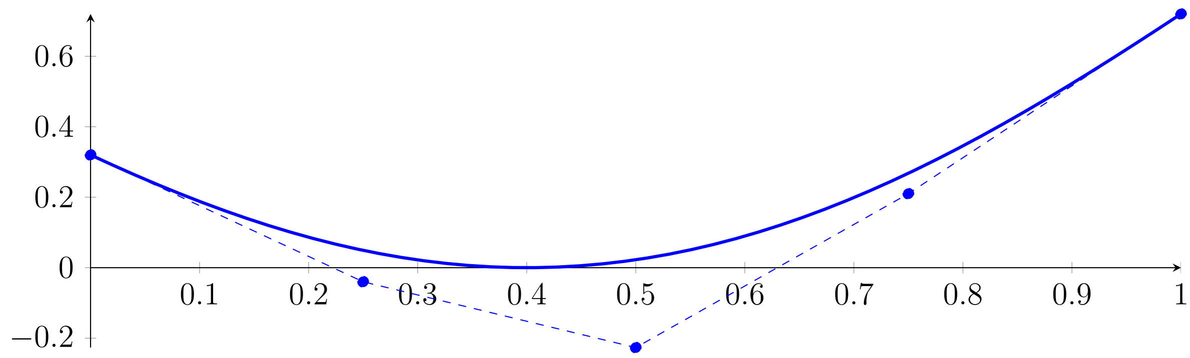
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



29.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

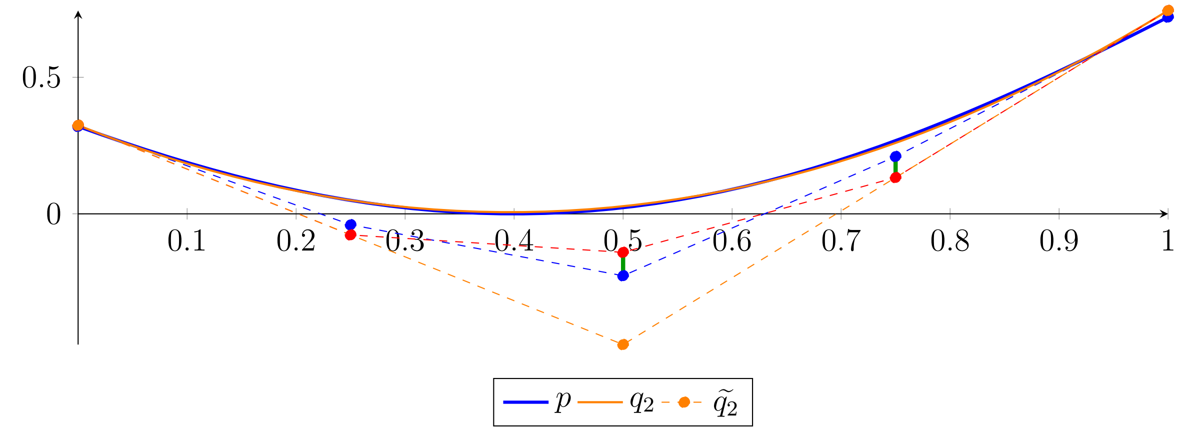
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.810653852360235 \cdot 10^{-306}X^4 - 3.682497235829418 \cdot 10^{-306}X^3 \\ &\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\ &\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

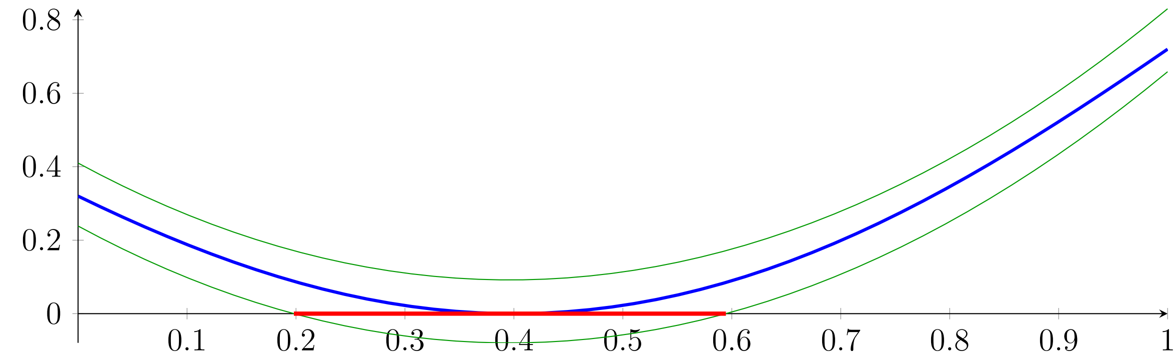
$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$

$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



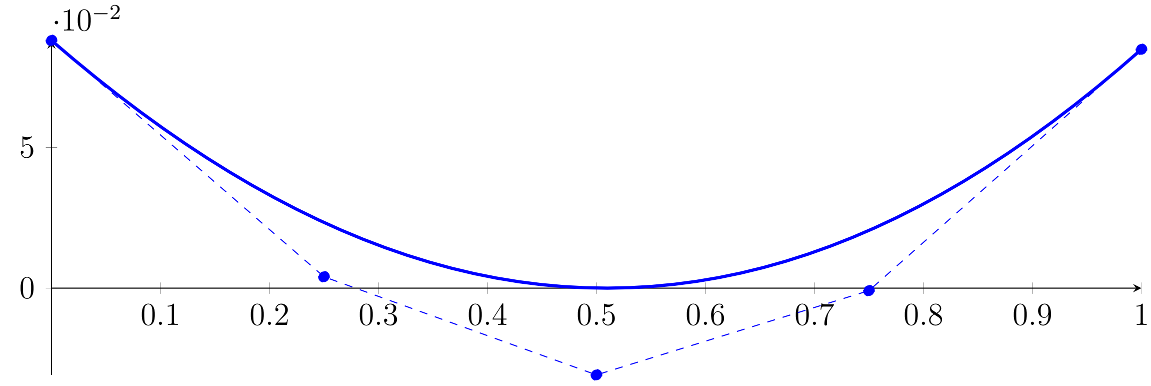
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

29.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

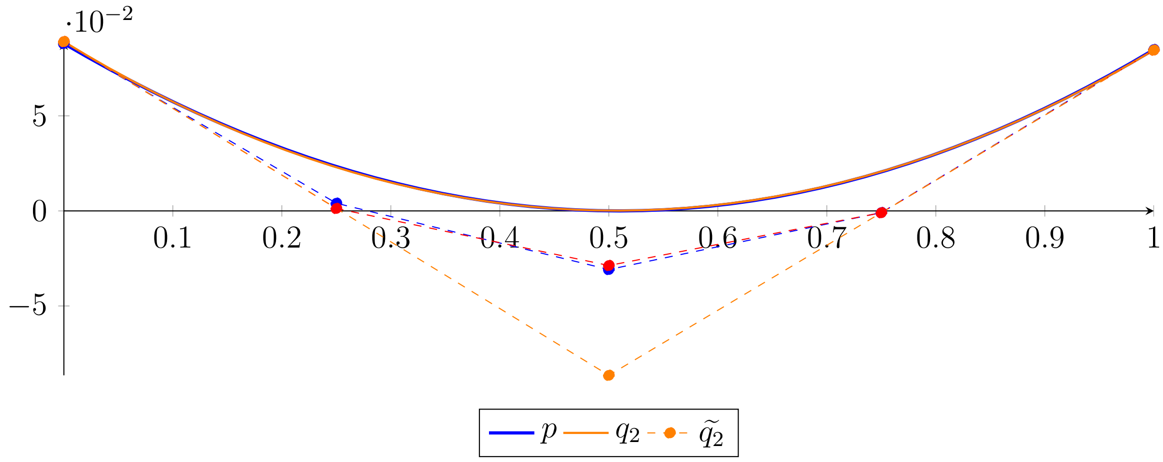
$$\begin{aligned}p &= -0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &\quad + 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ &= 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &\quad - 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402\cdot 10^{-307}X^4 - 6.258020227051504\cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

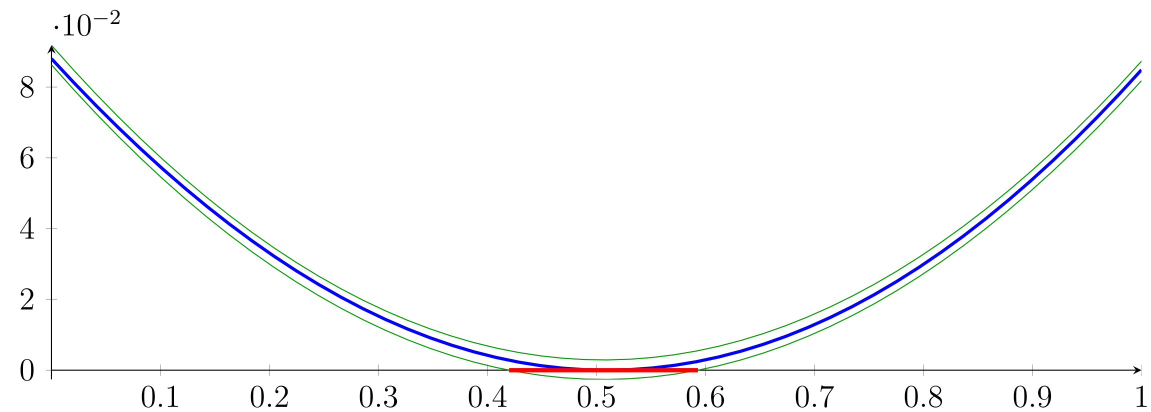
Bounding polynomials M and m:

$$\begin{aligned} M &= 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423 \\ m &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473 \end{aligned}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



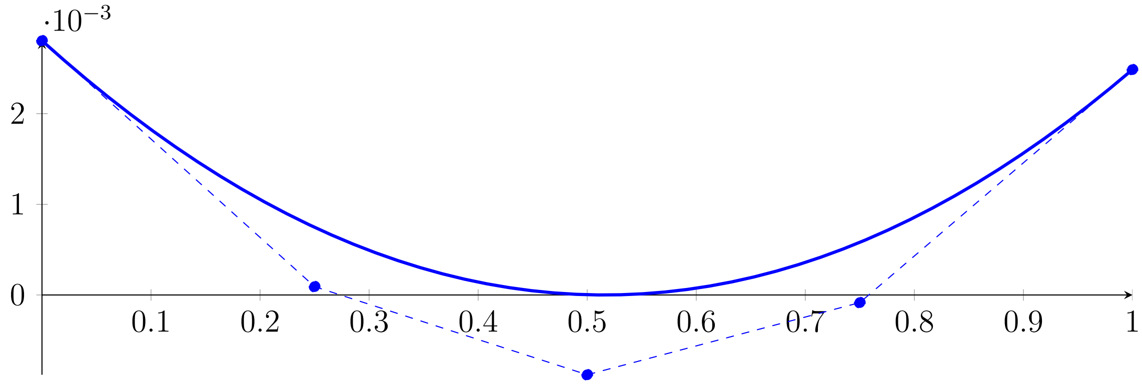
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

29.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

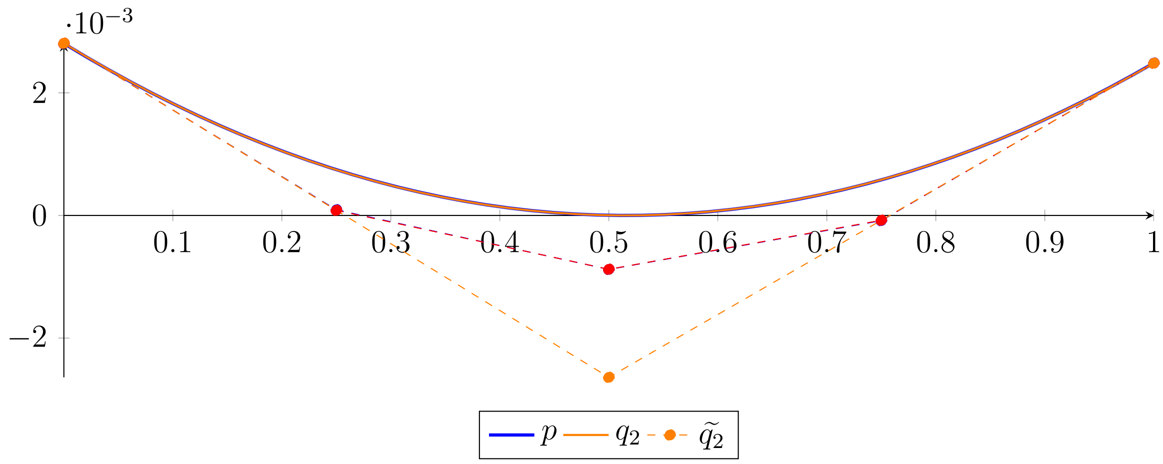
$$\begin{aligned} p &= -2.231982021693433 \cdot 10^{-05} X^4 + 0.0001110015522974976 X^3 \\ &\quad + 0.01044647623937015 X^2 - 0.01085434180562325 X + 0.002805735592529265 \\ &= 0.002805735592529265 B_{0,4}(X) + 9.215014112345361 \cdot 10^{-05} B_{1,4}(X) \\ &\quad - 0.0008803559370539994 B_{2,4}(X) - 8.403225392871914 \\ &\quad \cdot 10^{-05} B_{3,4}(X) + 0.002486551758356734 B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,2} - 0.002640895479419014 B_{1,2} + 0.002483552517338081 B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 9.278384156079834 \cdot 10^{-309} X^4 - 1.903481152394832 \cdot 10^{-308} X^3 \\ &\quad + 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,4} + 8.423853163898018 \cdot 10^{-05} B_{1,4} - 0.0008784428096068336 B_{2,4} \\ &\quad - 7.867148104046678 \cdot 10^{-05} B_{3,4} + 0.002483552517338081 B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429 \cdot 10^{-06}$.
Bounding polynomials M and m:

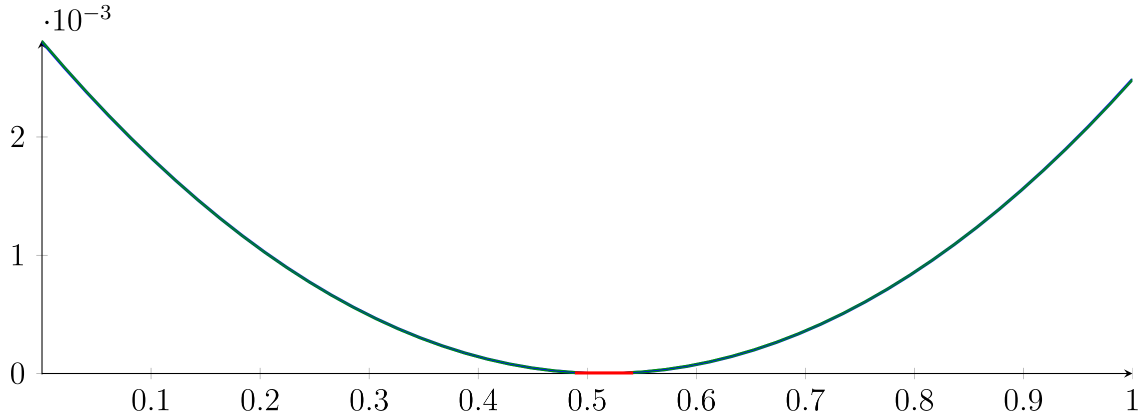
$$M = 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



$$[0.4885305113706042, 0.542280720323693]$$

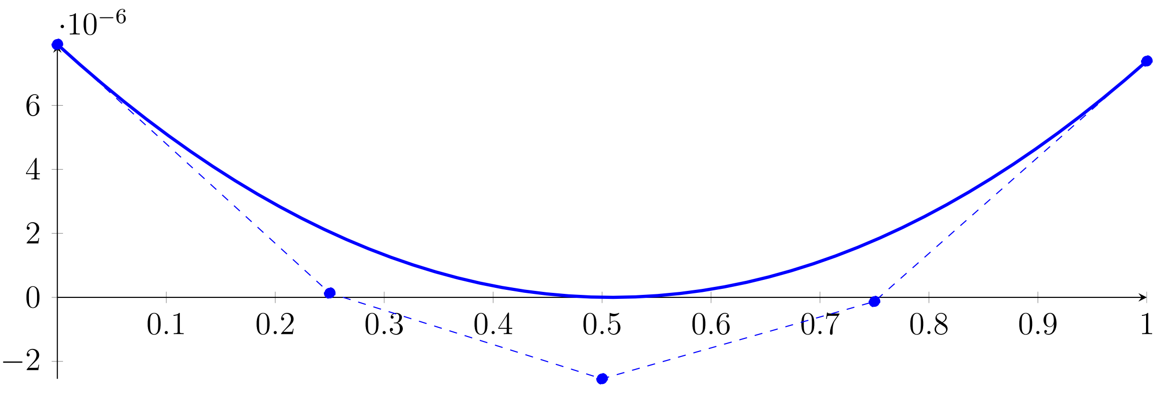
Longest intersection interval: 0.05375020895308878

$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

29.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.40181554717343$

Normalized monomial und Bézier representations and the Bézier polygon:

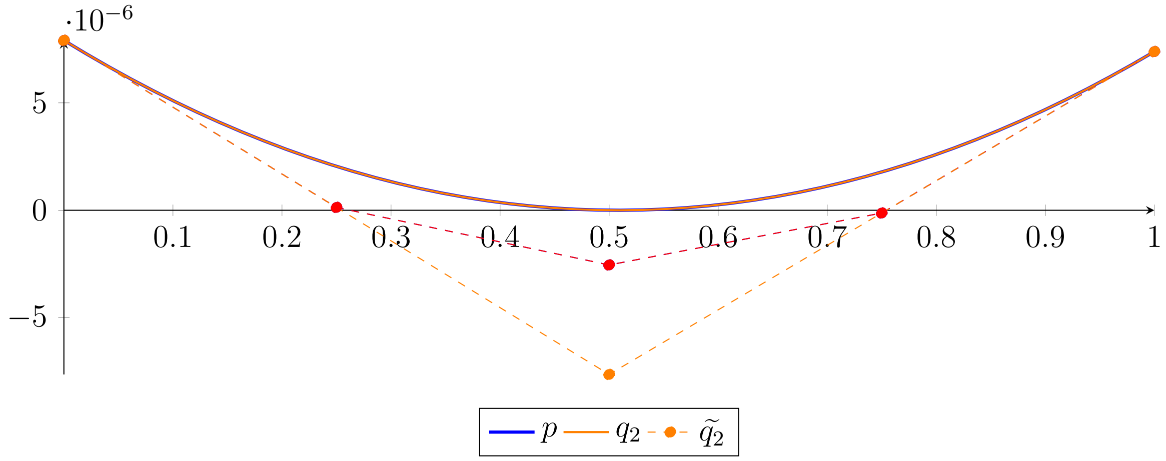
$$p = -1.862993414511891 \cdot 10^{-10} X^4 + 1.046428359410323 \cdot 10^{-08} X^3 + 3.055842313534119 \cdot 10^{-05} X^2 - 3.109078018724982 \cdot 10^{-05} X + 7.906738260659747 \cdot 10^{-06}$$
$$= 7.906738260659747 \cdot 10^{-06} B_{0,4}(X) + 1.340432138472922 \cdot 10^{-07} B_{1,4}(X) - 2.545581310408299 \cdot 10^{-06} B_{2,4}(X) - 1.295192412084993 \cdot 10^{-07} B_{3,4}(X) + 7.384659193003765 \cdot 10^{-06} B_{4,4}(X)$$



Degree reduction and raising:

$$q_2 = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06}$$
$$= 7.907245506324471 \cdot 10^{-06} B_{0,2} - 7.641198706965435 \cdot 10^{-06} B_{1,2} + 7.384157270177368 \cdot 10^{-06} B_{2,2}$$

$$\widetilde{q_2} = 2.673714696616243 \cdot 10^{-311} X^4 - 5.466261157526541 \cdot 10^{-311} X^3 + 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06}$$
$$= 7.907245506324471 \cdot 10^{-06} B_{0,4} + 1.330233996795178 \cdot 10^{-07} B_{1,4} - 2.545565341893317 \cdot 10^{-06} B_{2,4} - 1.285207183940336 \cdot 10^{-07} B_{3,4} + 7.384157270177368 \cdot 10^{-06} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 1.019814167774439 \cdot 10^{-09}$.

Bounding polynomials M and m:

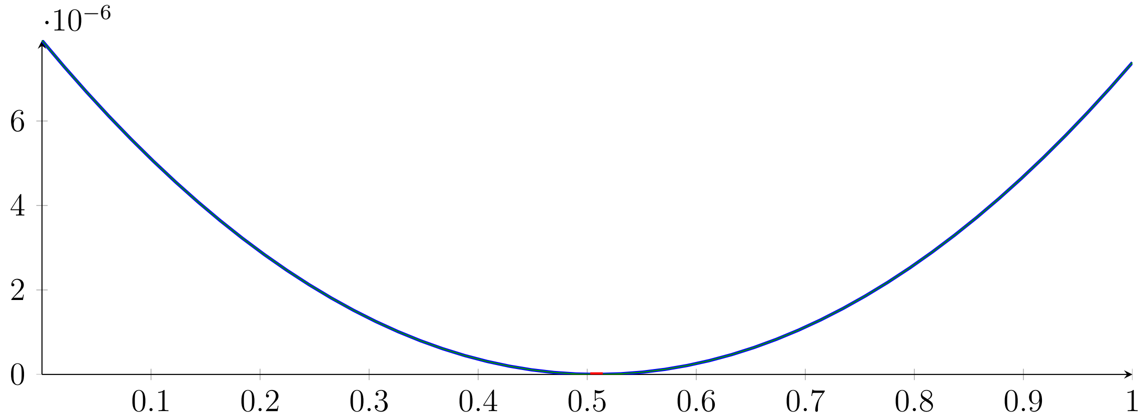
$$M = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.908265320492245 \cdot 10^{-06}$$

$$m = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.906225692156696 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.50281872462556, 0.5142903109829997\}$$

Intersection intervals:



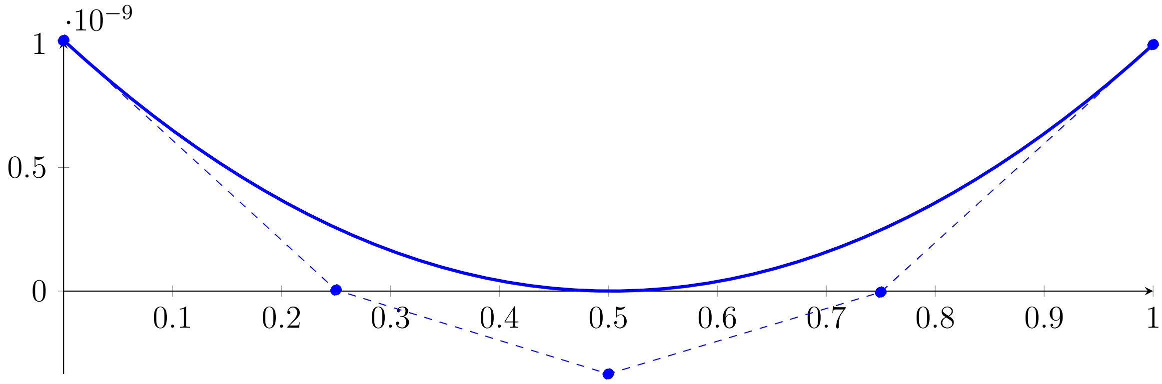
$$[0.50281872462556, 0.5142903109829997]$$

Longest intersection interval: 0.01147158635743977
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999787229673132, 0.4000211044658684]$,

29.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999787229673132, 0.400021104465$

Normalized monomial und Bézier representations and the Bézier polygon:

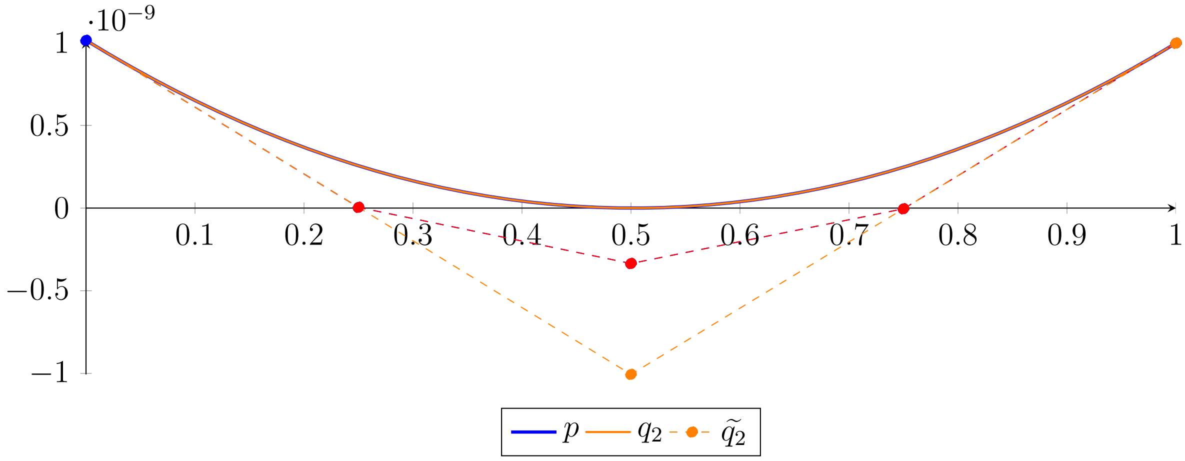
$$\begin{aligned} p = & -3.22630361651822 \cdot 10^{-18} X^4 + 1.523153645441234 \cdot 10^{-14} X^3 + 4.023445841277385 \\ & \cdot 10^{-09} X^2 - 4.040789163099954 \cdot 10^{-09} X + 1.014549826650262 \cdot 10^{-09} \\ = & \ 1.014549826650262 \cdot 10^{-09} B_{0,4}(X) + 4.352535875273264 \cdot 10^{-12} B_{1,4}(X) - 3.352704480201511 \\ & \cdot 10^{-10} B_{2,4}(X) - 4.315317151897678 \cdot 10^{-12} B_{3,4}(X) + 9.972217331378436 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 4.023468683051261\cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014550587950544\cdot 10^{-09} \\ &= 1.014550587950544\cdot 10^{-09}B_{0,2} - 1.005848561585488\cdot 10^{-09}B_{1,2} + 9.972209719297413\cdot 10^{-10}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.491760652125473\cdot 10^{-315}X^4 - 7.128579610273962\cdot 10^{-315}X^3 + 4.023468683051261 \\ &\quad \cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014550587950544\cdot 10^{-09} \\ &= 1.014550587950544\cdot 10^{-09}B_{0,4} + 4.35101318252834\cdot 10^{-12}B_{1,4} - 3.352704477436108 \\ &\quad \cdot 10^{-10}B_{2,4} - 4.313794827873167\cdot 10^{-12}B_{3,4} + 9.972209719297413\cdot 10^{-10}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.522692744924588\cdot 10^{-15}$.

Bounding polynomials M and m:

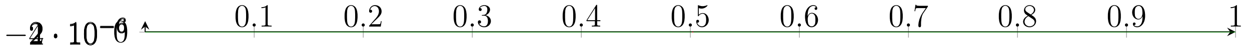
$$M = 4.023468683051261\cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014552110643289\cdot 10^{-09}$$

$$m = 4.023468683051261\cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014549065257799\cdot 10^{-09}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.5015281513143551, 0.5027789820041677\}$$

Intersection intervals:



$$[0.5015281513143551, 0.5027789820041677]$$

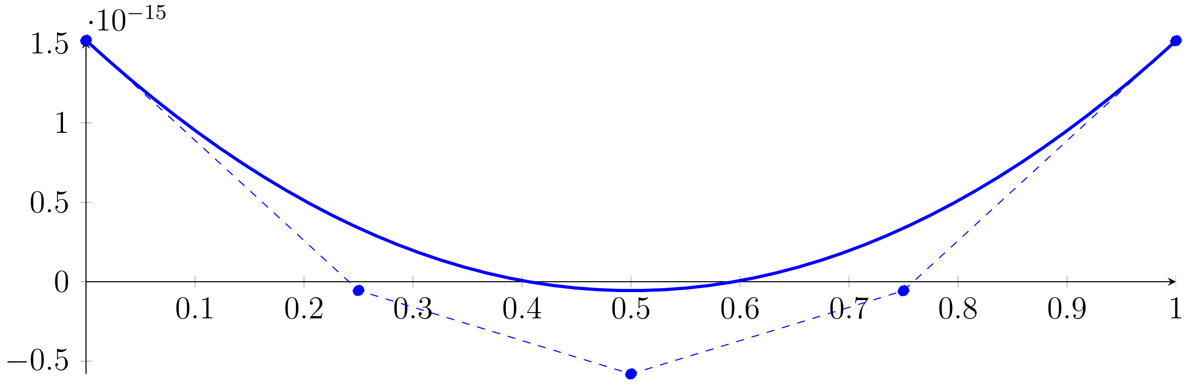
Longest intersection interval: 0.00125083068981256

$\implies$ Selective recursion: interval 1: $[0.3999999784819335, 0.4000000314940126]$,

29.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999999784819335, 0.4000000314$

Normalized monomial und Bézier representations and the Bézier polygon:

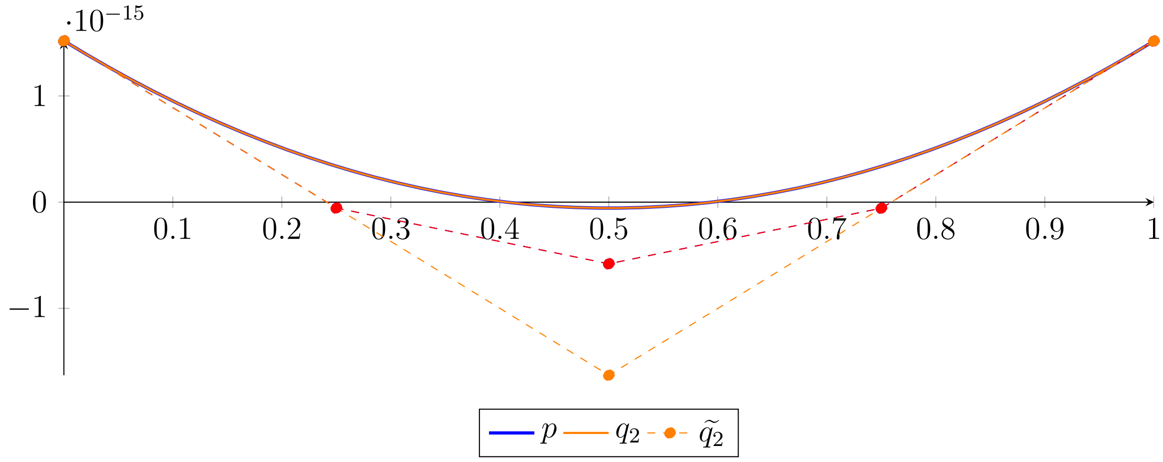
$$\begin{aligned}p &= -7.897676644125011\cdot 10^{-30}X^4 + 2.979577702269594\cdot 10^{-23}X^3 + 6.295028340046794\\ &\quad \cdot 10^{-15}X^2 - 6.297884694019149\cdot 10^{-15}X + 1.519185582318221\cdot 10^{-15}\\ &= 1.519185582318221\cdot 10^{-15}B_{0,4}(X) - 5.52855911865664\cdot 10^{-17}B_{1,4}(X) - 5.805853746835548\\ &\quad \cdot 10^{-16}B_{2,4}(X) - 5.671376072379998\cdot 10^{-17}B_{3,4}(X) + 1.516329258141634\cdot 10^{-15}B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185583808009\cdot 10^{-15}\\ &= 1.519185583808009\cdot 10^{-15}B_{0,2} - 1.629756772140295\cdot 10^{-15}B_{1,2} + 1.516329256651846\cdot 10^{-15}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 5.424840791336887\cdot 10^{-321}X^4 - 1.106707046684392\cdot 10^{-320}X^3 + 6.295028384740446\\ &\quad \cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185583808009\cdot 10^{-15}\\ &= 1.519185583808009\cdot 10^{-15}B_{0,4} - 5.528559416614297\cdot 10^{-17}B_{1,4} - 5.805853746835541\\ &\quad \cdot 10^{-16}B_{2,4} - 5.671375774422431\cdot 10^{-17}B_{3,4} + 1.516329256651846\cdot 10^{-15}B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.979576574030073\cdot 10^{-24}$.

Bounding polynomials M and m:

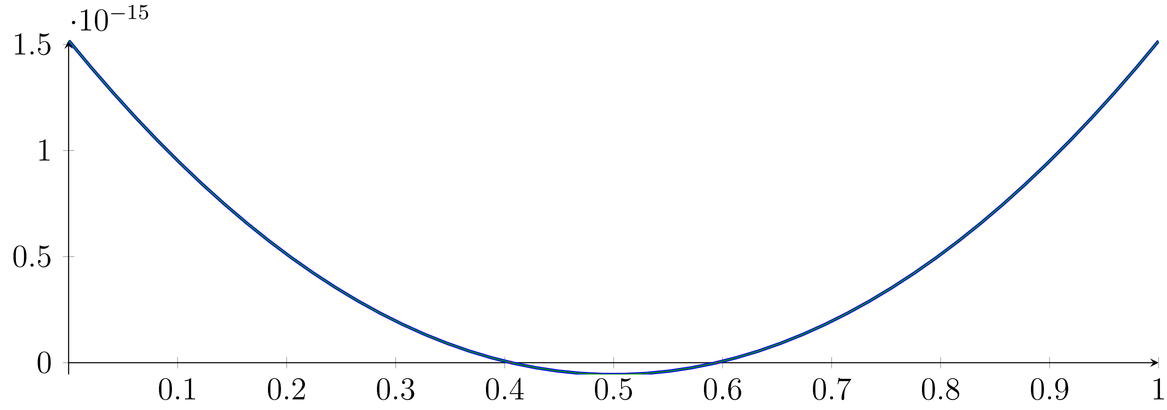
$$M = 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185586787586\cdot 10^{-15}$$

$$m = 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185580828433\cdot 10^{-15}$$

Root of M and m:

$N(M) = \{0.4059087473493063, 0.5945449959829854\}$ $N(m) = \{0.4059087423309477, 0.594545001001344\}$

Intersection intervals:



$$[0.4059087423309477, 0.4059087473493063],\ [0.5945449959829854, 0.594545001001344]$$

Longest intersection interval: $5.01835859594062 \cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.4000000000000001]$, interval 2: $[0.400000009999999$

29.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.40000000$

Found root in interval $[0.3999999999999999, 0.4000000000000001]$ at recursion depth 7!

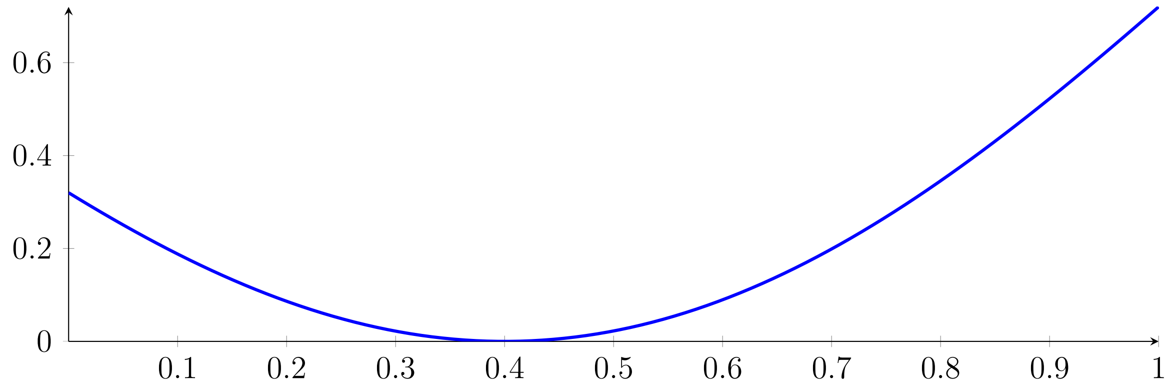
29.8 Recursion Branch 1 1 1 1 1 1 2 in Interval 2: $[0.4000000099999999, 0.40000001$

Found root in interval $[0.4000000099999999, 0.4000000100000001]$ at recursion depth 7!

29.9 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.3999999999999999, 0.4000000000000001],\ [0.4000000099999999, 0.4000000100000001]$$

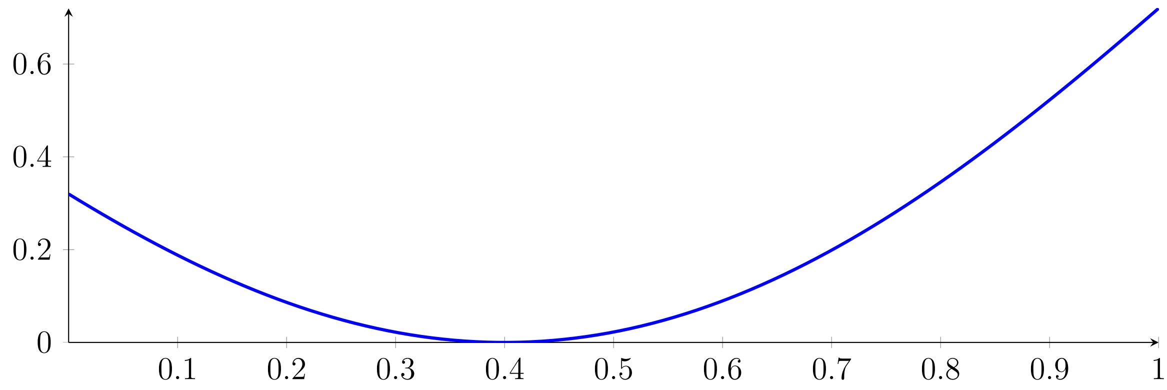
with precision $\varepsilon = 1 \cdot 10^{-08}$.

30 Running CubeClip on h_4 with epsilon 8

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called CubeClip with input polynomial on interval $[0, 1]$**:**

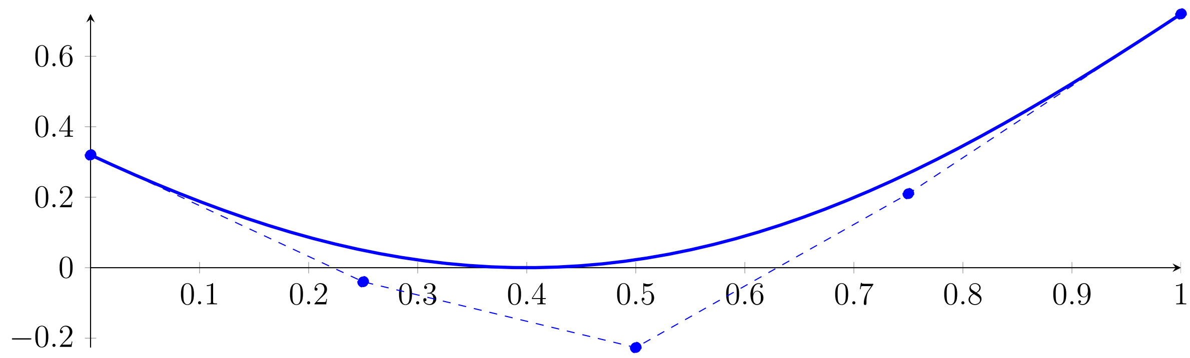
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



30.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

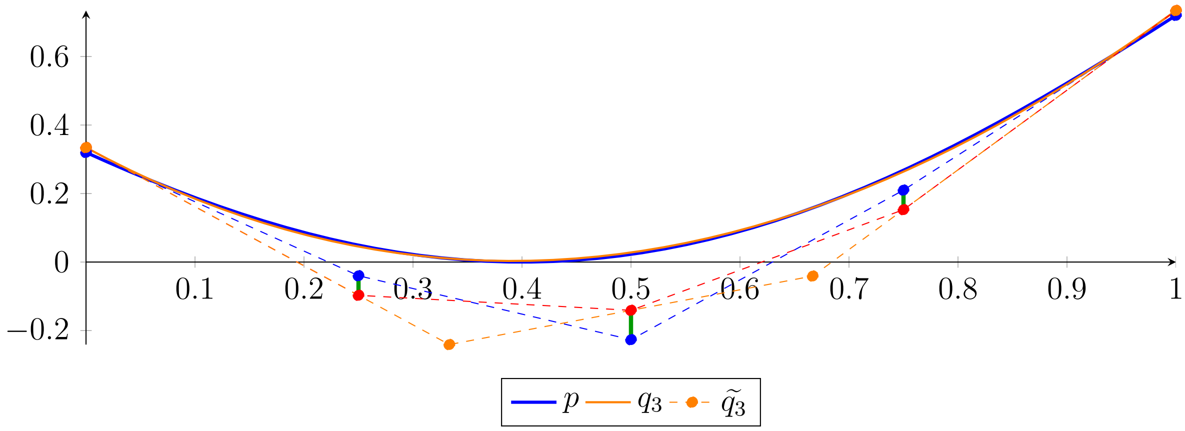
$$\begin{aligned}p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008\\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X)\\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143\\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3}\\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= -2.781342323134002 \cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2\\ &\quad - 1.725714301714286X + 0.3342857222857143\\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4}\\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4}\end{aligned}$$



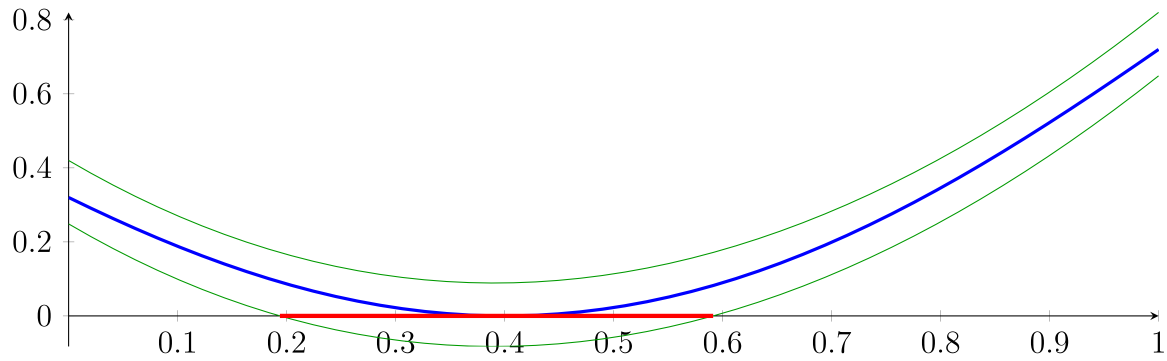
The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$
$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



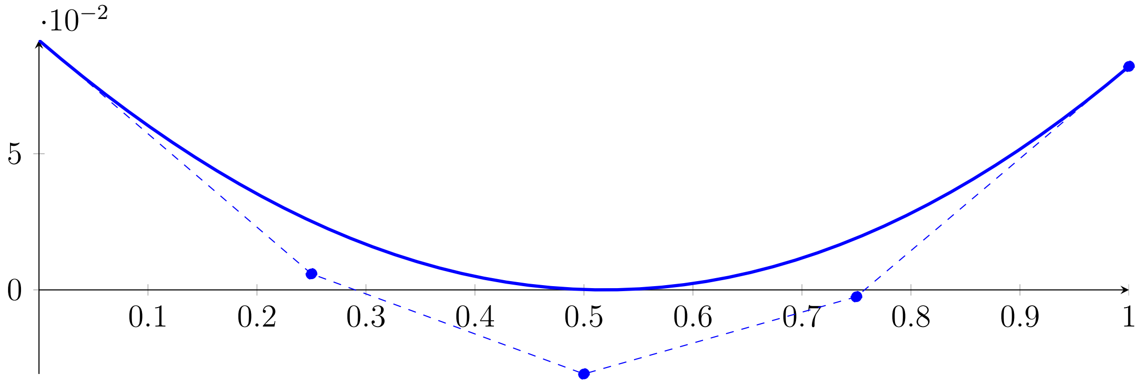
$[0.1938265012447289, 0.5913474001559605]$

Longest intersection interval: 0.3975208989112316
$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

30.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

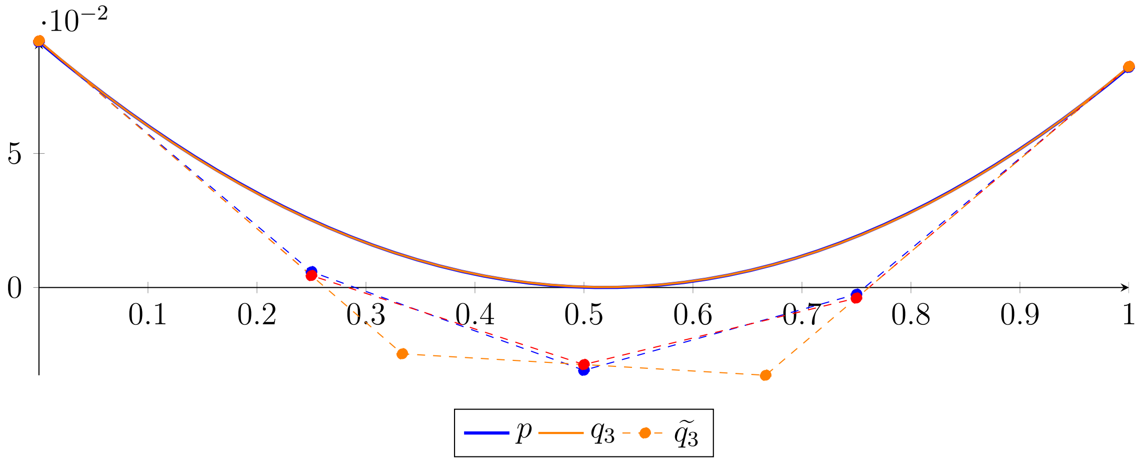
$$\begin{aligned} p = &-0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &+ 0.2941201876709534X^2 - 0.34309913433192X + 0.09165715738594469 \\ = &\ 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &- 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502\cdot 10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455446609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.
Bounding polynomials M and m:

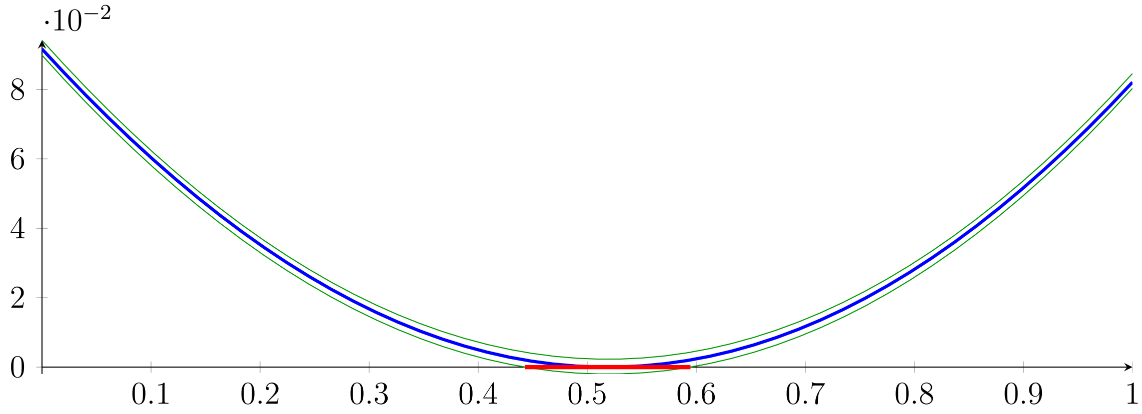
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



$$[0.4429176870393937, 0.5947109255489168]$$

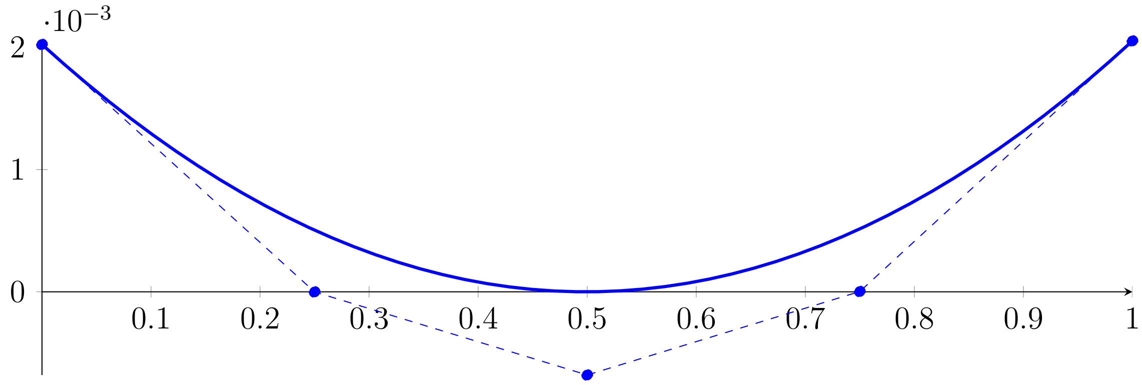
Longest intersection interval: 0.1517932385095231

$\Longrightarrow$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

30.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

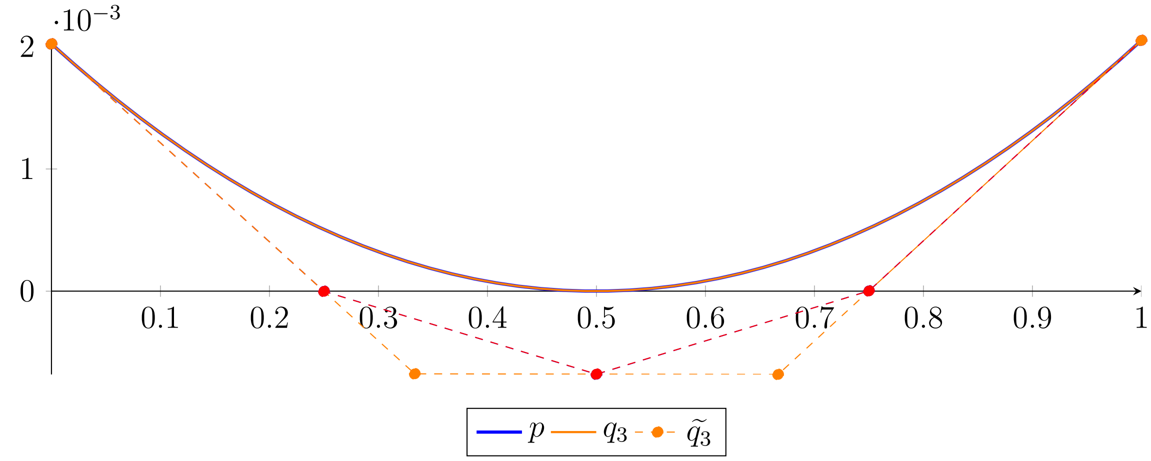
$$\begin{aligned} p = &-1.325713168422468 \cdot 10^{-05} X^4 + 7.039695732710913 \cdot 10^{-05} X^3 \\ &+ 0.008070351522992233 X^2 - 0.008098671960928044 X + 0.002023786815863734 \\ = &\ 0.002023786815863734 B_{0,4}(X) - 8.811743682771854 \cdot 10^{-07} B_{1,4}(X) \\ &- 0.0006804905774349 16 B_{2,4}(X) + 2.557845995594498 \\ &\cdot 10^{-06} B_{3,4}(X) + 0.002052606203570807 B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &\ 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ &- 0.008102459712837822 X + 0.002023976203459223 \\ = &\ 0.002023976203459223 B_{0,3} - 0.0006768437008200514 B_{1,3} \\ &- 0.0006818648029039136 B_{2,3} + 0.002052795591166296 B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} = &-6.518771069845317 \cdot 10^{-311} X^4 + 4.388269395865976 \cdot 10^{-05} X^3 \\ &+ 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002023976203459223 \\ = &\ 0.002023976203459223 B_{0,4} - 1.638724750232882 \cdot 10^{-06} B_{1,4} - 0.0006793542518619825 B_{2,4} \\ &+ 1.800295613638801 \cdot 10^{-06} B_{3,4} + 0.002052795591166296 B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544 \cdot 10^{-06}$.
Bounding polynomials M and m:

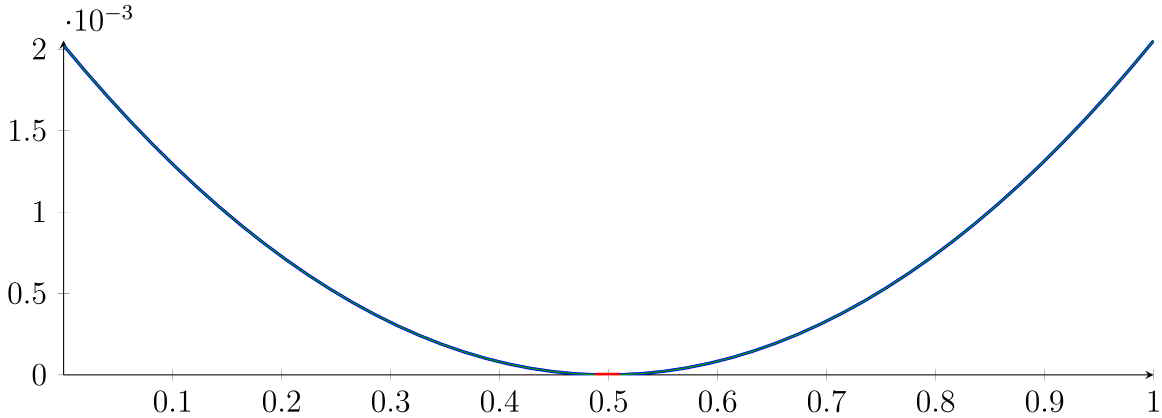
$$\begin{aligned} M &= 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ &\quad - 0.008102459712837822 X + 0.002025112529032156 \end{aligned}$$

$$\begin{aligned} m &= 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ &\quad - 0.008102459712837822 X + 0.002022839877886289 \end{aligned}$$

Root of M and m:

$N(M) = \{-185.2936165687423\}$ $N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649\}$

Intersection intervals:



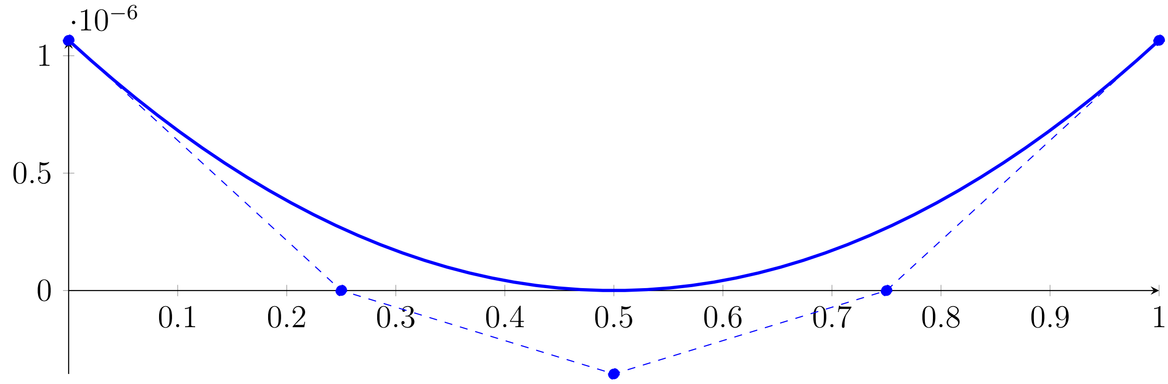
$$[0.4874742490566883, 0.5103358670775649]$$

Longest intersection interval: 0.02286161802087665
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

30.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.40068970704716$

Normalized monomial und Bézier representations and the Bézier polygon:

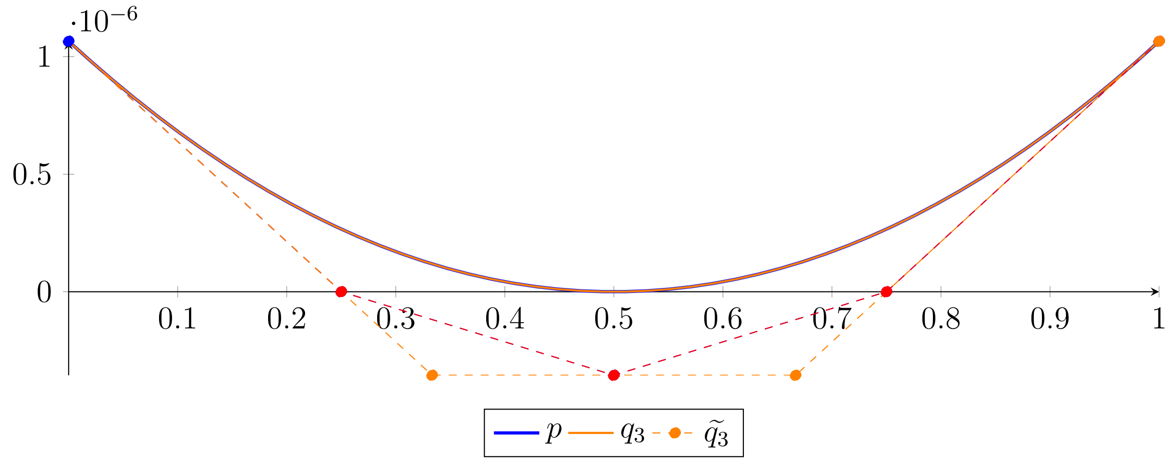
$$\begin{aligned} p &= -3.621407750870066 \cdot 10^{-12} X^4 + 5.322780243378263 \cdot 10^{-10} X^3 + 4.261926231315171 \\ &\quad \cdot 10^{-06} X^2 - 4.262596936226815 \cdot 10^{-06} X + 1.065750606194374 \cdot 10^{-06} \\ &= 1.065750606194374 \cdot 10^{-06} B_{0,4}(X) + 1.013721376702048 \cdot 10^{-10} B_{1,4}(X) - 3.552268233665051 \\ &\quad \cdot 10^{-07} B_{2,4}(X) - 1.009108120675939 \cdot 10^{-10} B_{3,4}(X) + 1.065608557899316 \cdot 10^{-06} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \cdot 10^{-06} X^2 \\ &\quad - 4.262597970914744 \cdot 10^{-06} X + 1.06575065792877 \cdot 10^{-06} \\ &= 1.06575065792877 \cdot 10^{-06} B_{0,3} - 3.551153323761442 \cdot 10^{-07} B_{1,3} \\ &\quad - 3.553376935441088 \cdot 10^{-07} B_{2,3} + 1.065608609633713 \cdot 10^{-06} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -5.304989477413181 \cdot 10^{-314} X^4 + 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \\ &\quad \cdot 10^{-06} X^2 - 4.262597970914744 \cdot 10^{-06} X + 1.06575065792877 \cdot 10^{-06} \\ &= 1.06575065792877 \cdot 10^{-06} B_{0,4} + 1.011652000844408 \cdot 10^{-10} B_{1,4} - 3.552265129601265 \\ &\quad \cdot 10^{-07} B_{2,4} - 1.011177496533579 \cdot 10^{-10} B_{3,4} + 1.065608609633713 \cdot 10^{-06} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.104063786460057 \cdot 10^{-13}$.

Bounding polynomials M and m:

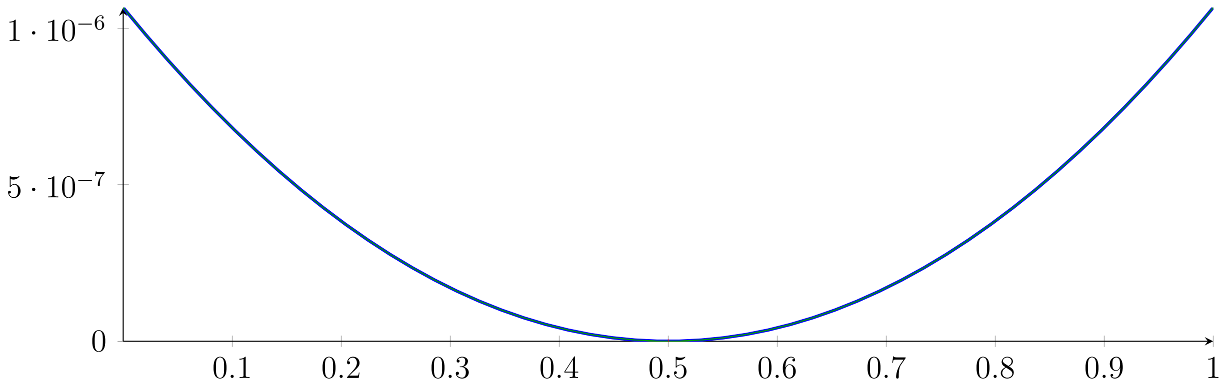
$$\begin{aligned} M &= 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \cdot 10^{-06} X^2 \\ &\quad - 4.262597970914744 \cdot 10^{-06} X + 1.065750968335149 \cdot 10^{-06} \end{aligned}$$

$$\begin{aligned} m &= 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \cdot 10^{-06} X^2 \\ &\quad - 4.262597970914744 \cdot 10^{-06} X + 1.065750347522392 \cdot 10^{-06} \end{aligned}$$

Root of M and m:

$N(M) = \{-8118.41926893212\}$ $\quad N(m) = \{-8118.419268932102, 0.4997707518163204, 0.5002933638554905\}$

Intersection intervals:



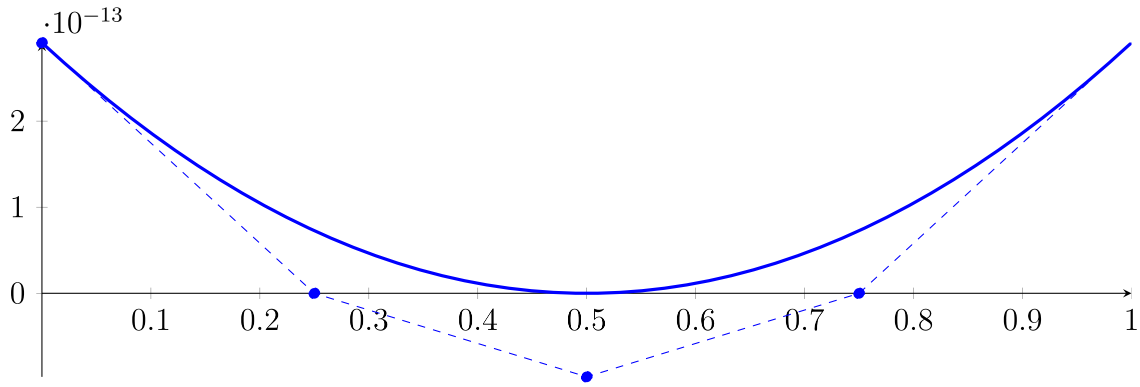
$[0.4997707518163204, 0.5002933638554905]$

Longest intersection interval: 0.0005226120391701048
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996445302967, 0.4000003654697068]$,

30.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999996445302967, 0.400000365469$

Normalized monomial und Bézier representations and the Bézier polygon:

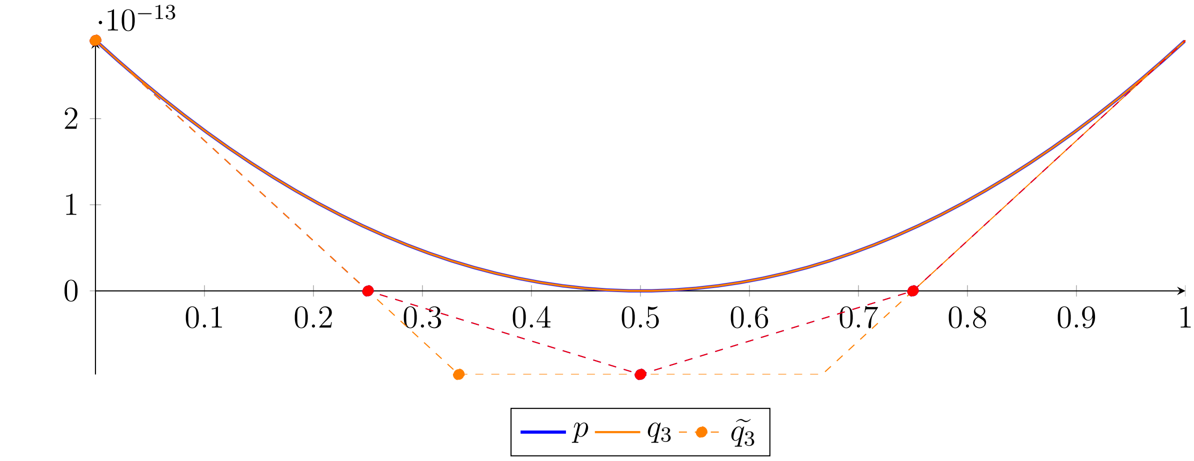
$$\begin{aligned} p = &-2.701438390310769{\cdot}10^{-25}X^4 + 7.494271205548112{\cdot}10^{-20}X^3 + 1.164248026057075 \\ &\cdot 10^{-12}X^2 - 1.164248076702198{\cdot}10^{-12}X + 2.91006022469042{\cdot}10^{-13} \\ = &\ 2.91006022469042{\cdot}10^{-13}B_{0,4}(X) - 5.599670650739874{\cdot}10^{-17}B_{1,4}(X) - 9.707667820587771 \\ &\cdot 10^{-14}B_{2,4}(X) - 5.600329339091629{\cdot}10^{-17}B_{3,4}(X) + 2.910060467663608{\cdot}10^{-13}B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &\ 7.494217176780306{\cdot}10^{-20}X^3 + 1.164248026057422{\cdot}10^{-12}X^2 \\ &- 1.164248076702275{\cdot}10^{-12}X + 2.910060224690459{\cdot}10^{-13} \\ = &\ 2.910060224690459{\cdot}10^{-13}B_{0,3} - 9.707666976504573{\cdot}10^{-14}B_{1,3} \\ &- 9.707668664666337{\cdot}10^{-14}B_{2,3} + 2.910060467663647{\cdot}10^{-13}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} = &-1.011846442682873{\cdot}10^{-320}X^4 + 7.494217176780306{\cdot}10^{-20}X^3 + 1.164248026057422 \\ &\cdot 10^{-12}X^2 - 1.164248076702275{\cdot}10^{-12}X + 2.910060224690459{\cdot}10^{-13} \\ = &\ 2.910060224690459{\cdot}10^{-13}B_{0,4} - 5.599670652283553{\cdot}10^{-17}B_{1,4} - 9.707667820585455 \\ &\cdot 10^{-14}B_{2,4} - 5.600329340635308{\cdot}10^{-17}B_{3,4} + 2.910060467663647{\cdot}10^{-13}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.315518620266374 \cdot 10^{-26}$.
Bounding polynomials M and m:

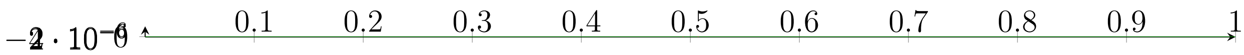
$$\begin{aligned} M &= 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ &\quad - 1.164248076702275 \cdot 10^{-12} X + 2.91006022469069 \cdot 10^{-13} \end{aligned}$$

$$\begin{aligned} m &= 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ &\quad - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690227 \cdot 10^{-13} \end{aligned}$$

Root of M and m:

$N(M) = \{-15535286.38864162, 0.4930646005679288, 0.5069353931872718\}$ $N(m) = \{-15535286.38864162,$

Intersection intervals:



$$[0.4930646005650611, 0.4930646005679288],\ [0.5069353931872718, 0.5069353931901395]$$

Longest intersection interval: $2.86768528910705 \cdot 10^{-12}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.3999999999999999]$, interval 2: $[0.40000001, 0.400000$

30.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.39999999999999$

Found root in interval $[0.3999999999999999, 0.3999999999999999]$ at recursion depth 6!

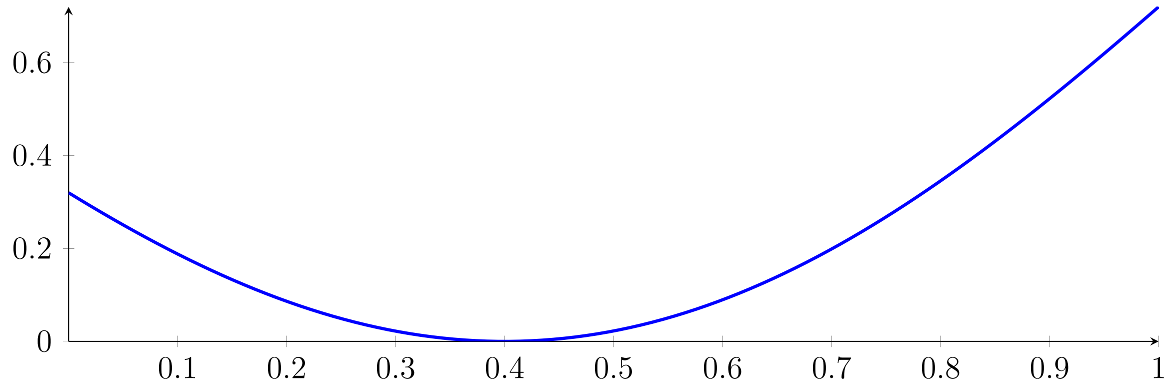
30.7 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 6!

30.8 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.399999999999999, 0.399999999999999],\ [0.40000001, 0.40000001]$$

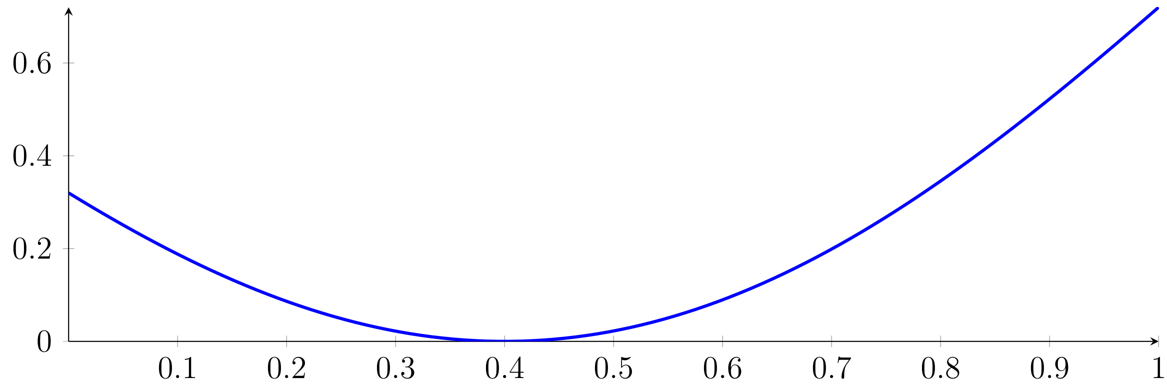
with precision $\varepsilon = 1 \cdot 10^{-08}$.

31 Running BezClip on h_4 with epsilon 16

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called BezClip with input polynomial on interval $[0, 1]$:

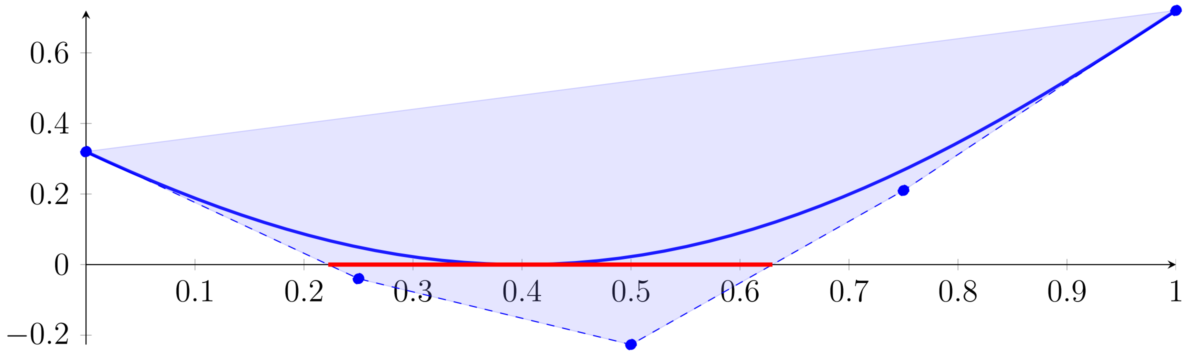
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



31.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

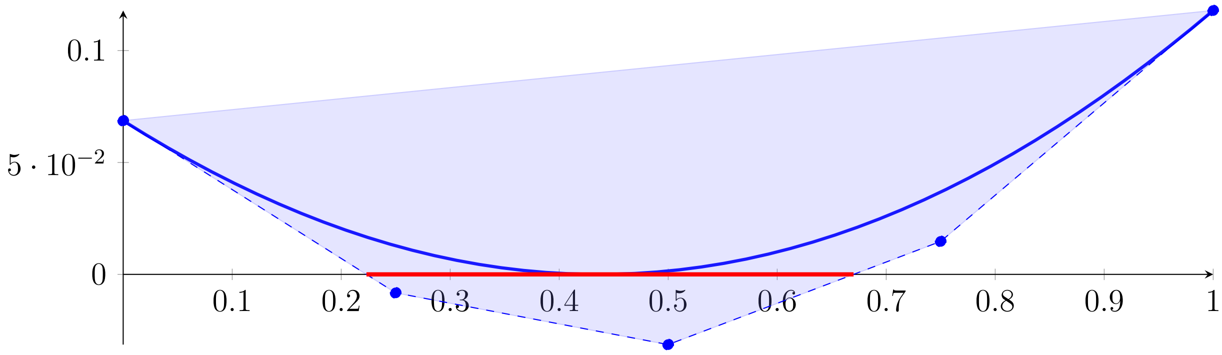
$$[0.222222225308642, 0.6297709955349339]$$

Longest intersection interval: 0.407548770226292
$\Longrightarrow$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

31.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &\quad + 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ &= 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &\quad + 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

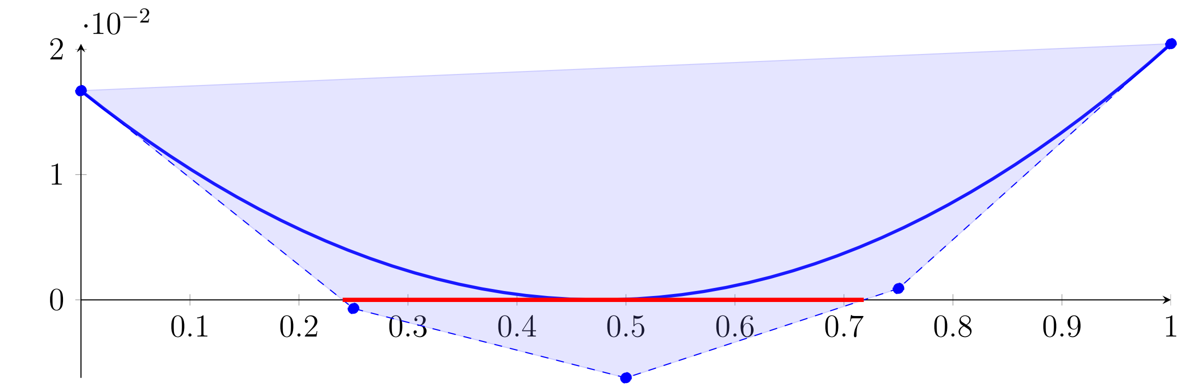
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\Longrightarrow$ Selective recursion: interval 1: [0.3131782986367004, 0.4953216655933138],

31.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.495321665593313$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-0.0011006606529341182X^4 + 0.003307158935864367X^3 \\ &+ 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377 \\ = &\, 0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X) \\ &+ 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

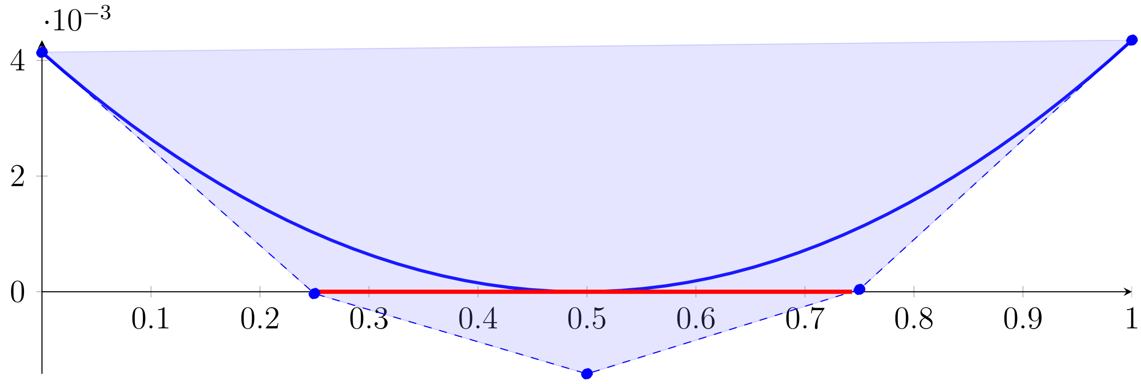
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\Longrightarrow$ Selective recursion: interval 1: [0.3569093751201798, 0.4439937820951003],

31.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.4439937820951$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -5.751241374789061\cdot 10^{-05}X^4 + 0.0002459162030906773X^3\\ &\quad + 0.01670691422385101X^2 - 0.01668640845758443X + 0.004139787469617577\\ &= 0.004139787469617577B_{0,4}(X) - 3.181464477852956\cdot 10^{-05}B_{1,4}(X)\\ &\quad - 0.001418931055199467B_{2,4}(X) + 3.991728912743387\\ &\quad \cdot 10^{-05}B_{3,4}(X) + 0.004348697025226952B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

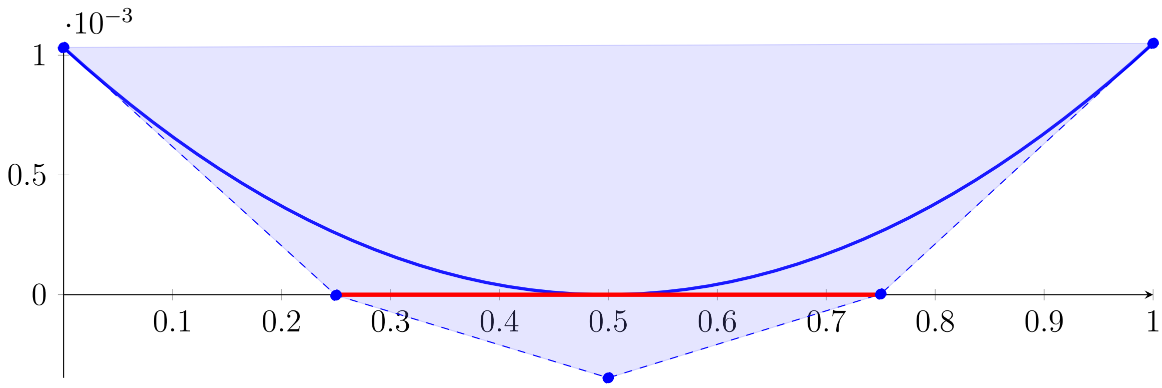
$$[0.2480933797192248, 0.7431594518918532]$$

Longest intersection interval: 0.4950660721726284
$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

31.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.454731120985874\cdot 10^{-06}X^4 + 2.291337271769576\cdot 10^{-05}X^3\\ &\quad + 0.004134358001933594X^2 - 0.004136159738306538X + 0.001031853315293836\\ &= 0.001031853315293836B_{0,4}(X) - 2.186619282798525\cdot 10^{-06}B_{1,4}(X)\\ &\quad - 0.0003471668868705005B_{2,4}(X) + 2.640855710153786\\ &\quad \cdot 10^{-06}B_{3,4}(X) + 0.001049510220517602B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

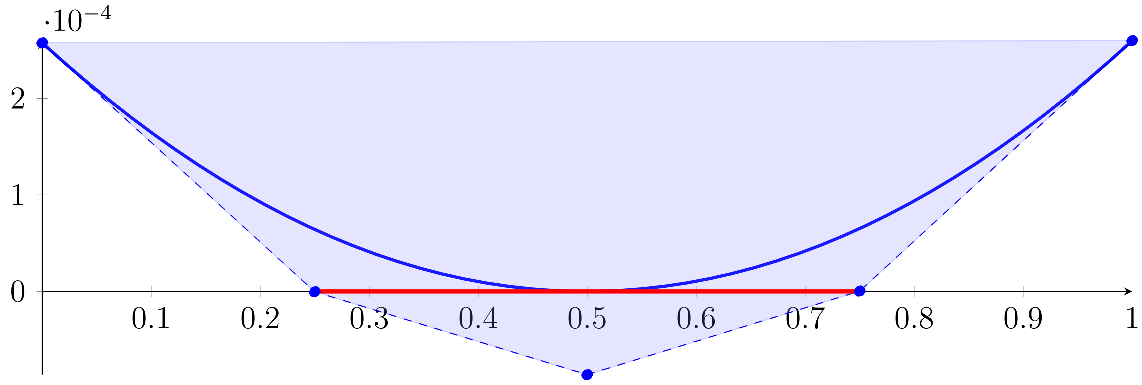
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\implies$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

31.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.41076747247$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.135832675829806\cdot 10^{-07}X^4 + 2.413460912088167\cdot 10^{-06}X^3\\ &+ 0.001031922910124016X^2 - 0.001031832710654165X + 0.0002576480709401398\\ = &\ 0.0002576480709401398B_{0,4}(X) - 3.101067234014613\cdot 10^{-07}B_{1,4}(X) - 8.628113269960673\\ &\cdot 10^{-05}B_{2,4}(X) + 3.383582395460159\cdot 10^{-07}B_{3,4}(X) + 0.0002599381480544958B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

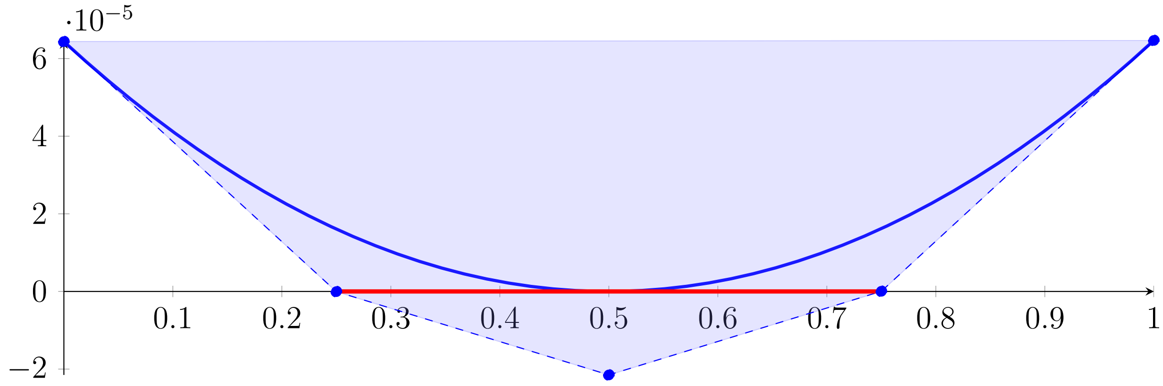
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\implies$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

31.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.40537205$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.327690666746824\cdot 10^{-08}X^4 + 2.739027984571277\cdot 10^{-07}X^3\\ &+ 0.0002577144304079774X^2 - 0.0002576778286693267X + 6.437695235811399\cdot 10^{-05}\\ = &\ 6.437695235811399\cdot 10^{-05}B_{0,4}(X) - 4.250480921768141\cdot 10^{-08}B_{1,4}(X) - 2.150955690855369\\ &\cdot 10^{-05}B_{2,4}(X) + 4.427175972025921\cdot 10^{-08}B_{3,4}(X) + 6.467417998855097\cdot 10^{-05}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

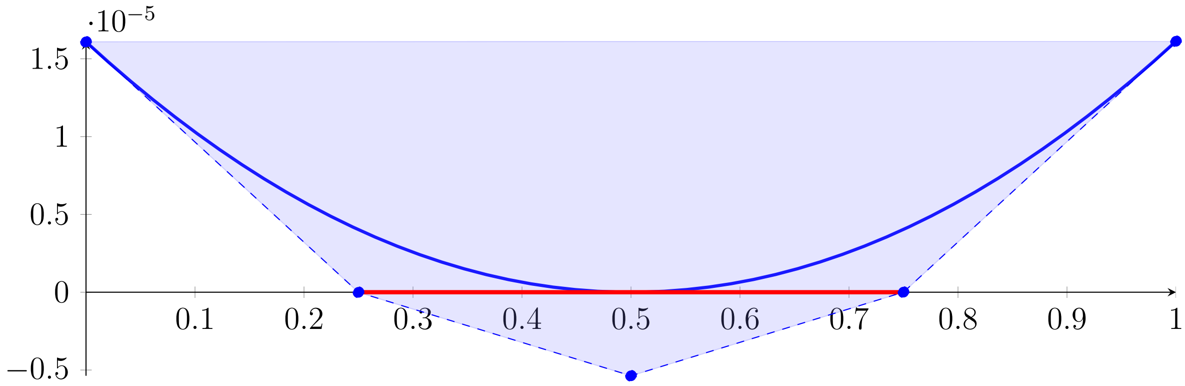
$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\Longrightarrow$ Selective recursion: interval 1: [0.3973195510833879, 0.4026829657906133],

31.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -8.274952589981882 \cdot 10^{-10} X^4 + 3.251124603913595 \cdot 10^{-08} X^3 + 6.438882283687421 \\ & \cdot 10^{-05} X^2 - 6.438267481175771 \cdot 10^{-05} X + 1.609012302857716 \cdot 10^{-05} \\ = & \; 1.609012302857716 \cdot 10^{-05} B_{0,4}(X) - 5.545674362270907 \cdot 10^{-09} B_{1,4}(X) - 5.369743904489329 \\ & \cdot 10^{-06} B_{2,4}(X) + 5.656149705765249 \cdot 10^{-09} B_{3,4}(X) + 1.61279548044738 \cdot 10^{-05} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499138638713212, 0.7497369428484978\}$$

Intersection intervals with the x axis:

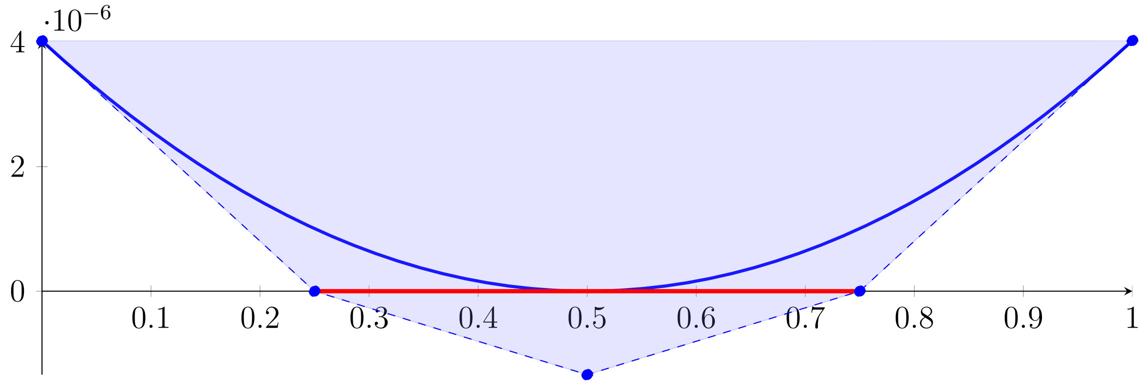
$$[0.2499138638713212, 0.7497369428484978]$$

Longest intersection interval: 0.4998230789771766
$\Longrightarrow$ Selective recursion: interval 1: [0.3986599427764149, 0.4013407012292117],

31.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3986599427764149, 0.40134$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -5.164529187662443\cdot 10^{-11}X^4 + 3.956301794548619\cdot 10^{-09}X^3 + 1.609182796552748 \cdot 10^{-05}X^2 - 1.609096222935268\cdot 10^{-05}X + 4.022033038444256\cdot 10^{-06}$$
$$= 4.022033038444256\cdot 10^{-06}B_{0,4}(X) - 7.075188939141538\cdot 10^{-10}B_{1,4}(X) - 1.341476748644171 \cdot 10^{-06}B_{2,4}(X) + 7.14424642122371\cdot 10^{-10}B_{3,4}(X) + 4.026803431121726\cdot 10^{-06}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499560300444542, 0.7498669294180401\}$$

Intersection intervals with the x axis:

$$[0.2499560300444542, 0.7498669294180401]$$

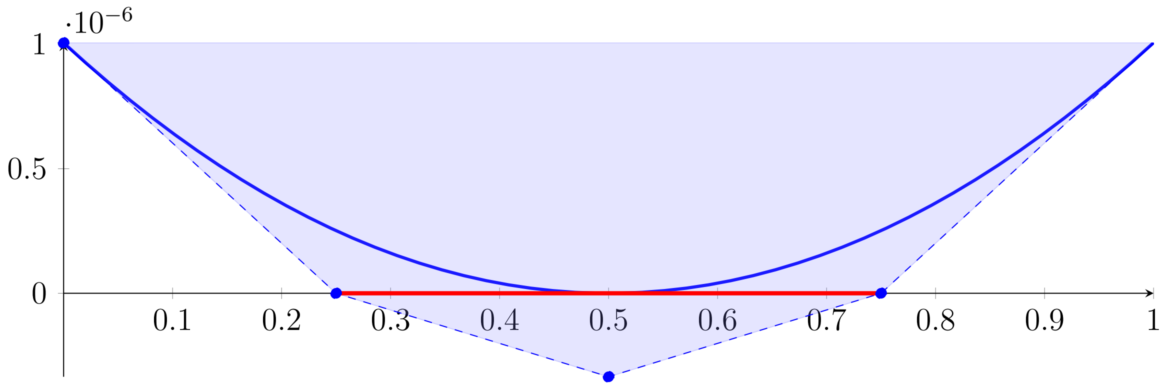
Longest intersection interval: 0.499910899373586

$\Longrightarrow$ Selective recursion: interval 1: $[0.3993300145167841, 0.4006701548859251]$,

31.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3993300145167841, 0.400$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.225530543299082\cdot 10^{-12}X^4 + 4.878223136378425\cdot 10^{-10}X^3 + 4.022259900669466 \cdot 10^{-06}X^2 - 4.022145641341714\cdot 10^{-06}X + 1.00544708348311\cdot 10^{-06}$$
$$= 1.00544708348311\cdot 10^{-06}B_{0,4}(X) - 8.932685231834447\cdot 10^{-11}B_{1,4}(X) - 3.352490870761692 \cdot 10^{-07}B_{2,4}(X) + 8.975838996701131\cdot 10^{-11}B_{3,4}(X) + 1.006045939593956\cdot 10^{-06}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499777912437083, 0.7499330838112102\}$$

Intersection intervals with the x axis:

$$[0.2499777912437083, 0.7499330838112102]$$

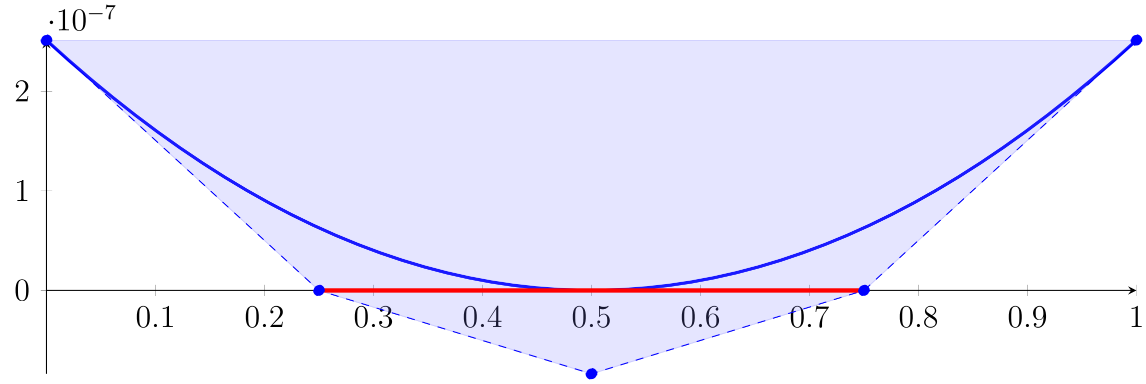
Longest intersection interval: 0.499955292567502

$\Longrightarrow$ Selective recursion: interval 1: $[0.3996650198462185, 0.4003350301165539]$,

31.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3996650198462185, 0.4$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.015235660316643 \cdot 10^{-13} X^4 + 6.055838633908679 \cdot 10^{-11} X^3 + 1.005476298211762 \cdot 10^{-06} X^2 - 1.005461639191386 \cdot 10^{-06} X + 2.51354188678016 \cdot 10^{-07}$$
$$= 2.51354188678016 \cdot 10^{-07} B_{0,4}(X) - 1.122111983053638 \cdot 10^{-11} B_{1,4}(X) - 8.379724788238336 \cdot 10^{-08} B_{2,4}(X) + 1.124798694225224 \cdot 10^{-11} B_{3,4}(X) + 2.51429204561165 \cdot 10^{-07} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499888398329751, 0.7499664473546936\}$$

Intersection intervals with the x axis:

$$[0.2499888398329751, 0.7499664473546936]$$

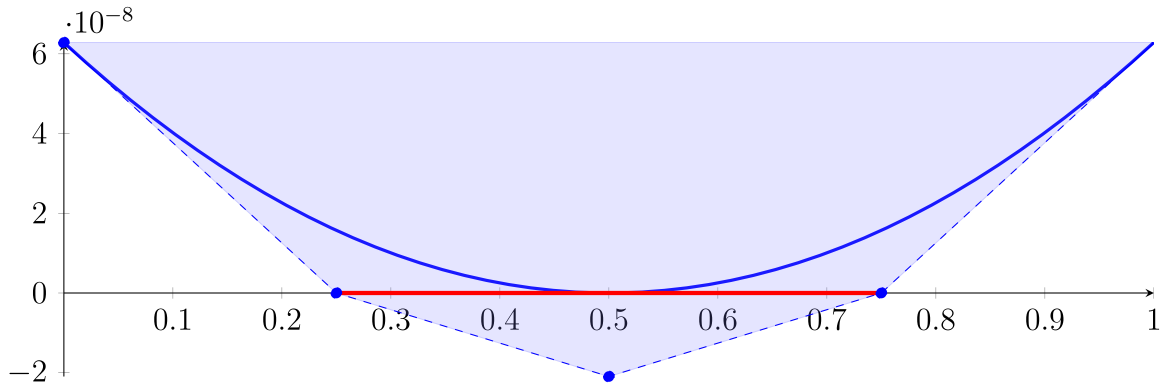
Longest intersection interval: 0.4999776075217186

$\Longrightarrow$ Selective recursion: interval 1: $[0.3998325149363758, 0.4001675050683531]$,

31.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3998325149363758, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.259296672250957 \cdot 10^{-14} X^4 + 7.543595361590864 \cdot 10^{-12} X^3 + 2.5135789423507 \cdot 10^{-07} X^2 - 2.51356038307676 \cdot 10^{-07} X + 6.283760343276371 \cdot 10^{-08}$$
$$= 6.283760343276371 \cdot 10^{-08} B_{0,4}(X) - 1.406144155297956 \cdot 10^{-12} B_{1,4}(X) - 2.094743334856264 \cdot 10^{-08} B_{2,4}(X) + 1.407718382090997 \cdot 10^{-12} B_{3,4}(X) + 6.284699036255256 \cdot 10^{-08} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499944057673539, 0.7499832005219574\}$$

Intersection intervals with the x axis:

$$[0.2499944057673539, 0.7499832005219574]$$

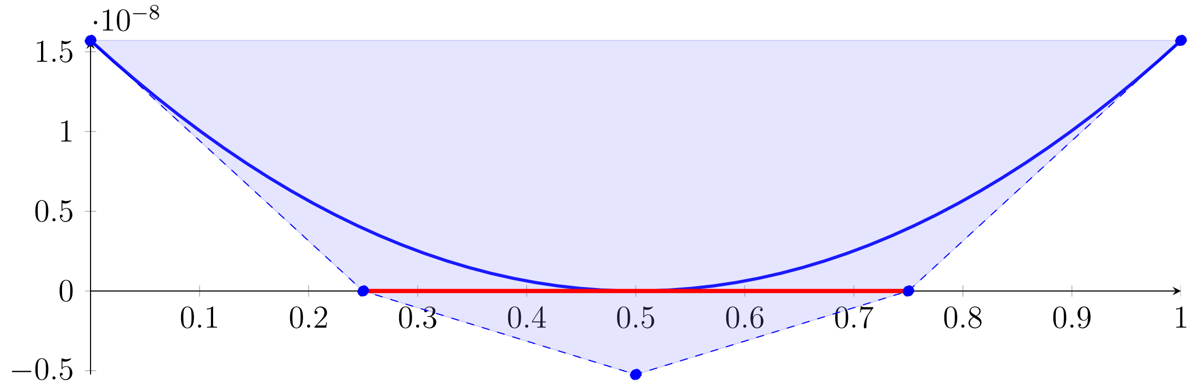
Longest intersection interval: 0.4999887947546035

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999162605953574, 0.4000837519076994]$,

31.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999162605953574

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -7.869898688873272\cdot 10^{-16} X^4 + 9.413120459510821\cdot 10^{-13} X^3 + 6.283807021204131 \cdot 10^{-08} X^2 - 6.283783670437638\cdot 10^{-08} X + 1.570928313186755\cdot 10^{-08}$$
$$= 1.570928313186755\cdot 10^{-08} B_{0,4}(X) - 1.760442265467946\cdot 10^{-13} B_{1,4}(X) - 5.236623518313756 \cdot 10^{-09} B_{2,4}(X) + 1.76037617409066\cdot 10^{-13} B_{3,4}(X) + 1.571045716458857\cdot 10^{-08} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499971984359141, 0.7499915961258623\}$$

Intersection intervals with the x axis:

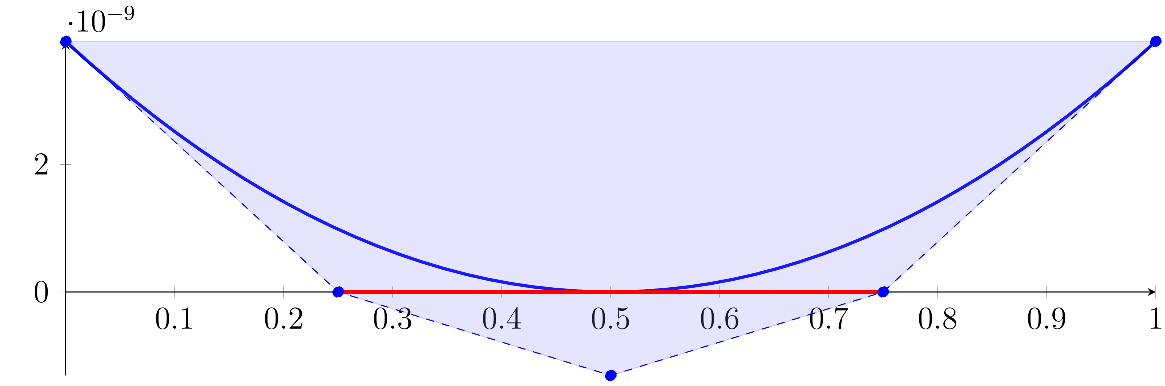
$$[0.2499971984359141, 0.7499915961258623]$$

Longest intersection interval: 0.4999943976899482
$\Longrightarrow$ Selective recursion: interval 1: [0.3999581329542053, 0.400041877672038],

31.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999581329542053, 0.400041877672038]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -4.918466236188167\cdot 10^{-17} X^4 + 1.175616813221855\cdot 10^{-13} X^3 + 1.570934193292886 \cdot 10^{-08} X^2 - 1.57093126037727\cdot 10^{-08} X + 3.927306070737634\cdot 10^{-09}$$
$$= 3.927306070737634\cdot 10^{-09} B_{0,4}(X) - 2.208020554223638\cdot 10^{-14} B_{1,4}(X) - 1.309126575660575 \cdot 10^{-09} B_{2,4}(X) + 2.19747928673869\cdot 10^{-14} B_{3,4}(X) + 3.927452912390452\cdot 10^{-09} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.249998594451196, 0.749995803609747\}$$

Intersection intervals with the x axis:

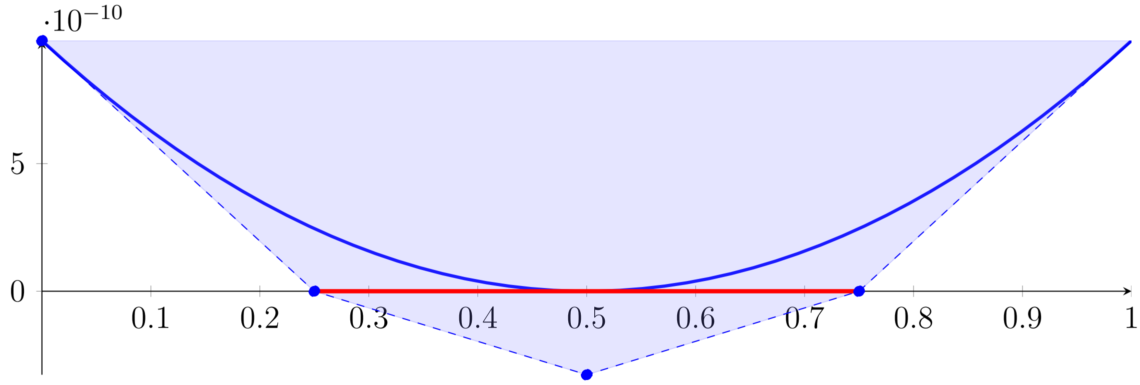
$$[0.249998594451196, 0.749995803609747]$$

Longest intersection interval: 0.499997209158551
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999790690159562, 0.4000209411411543]$,

31.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999790690159562, 0.4000209411411543]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -3.073972764895049 \cdot 10^{-18} X^4 + 1.468881614935617 \cdot 10^{-14} X^3 + 3.92731367890626 \\ &\quad \cdot 10^{-09} X^2 - 3.927309958033017 \cdot 10^{-09} X + 9.818246673938711 \cdot 10^{-10} \\ &= 9.818246673938711 \cdot 10^{-10} B_{0,4}(X) - 2.822114383159451 \cdot 10^{-15} B_{1,4}(X) - 3.272780318049275 \\ &\quad \cdot 10^{-10} B_{2,4}(X) + 2.710526275559223 \cdot 10^{-15} B_{3,4}(X) + 9.818430740092905 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499992814128721, 0.7499979295098023\}$$

Intersection intervals with the x axis:

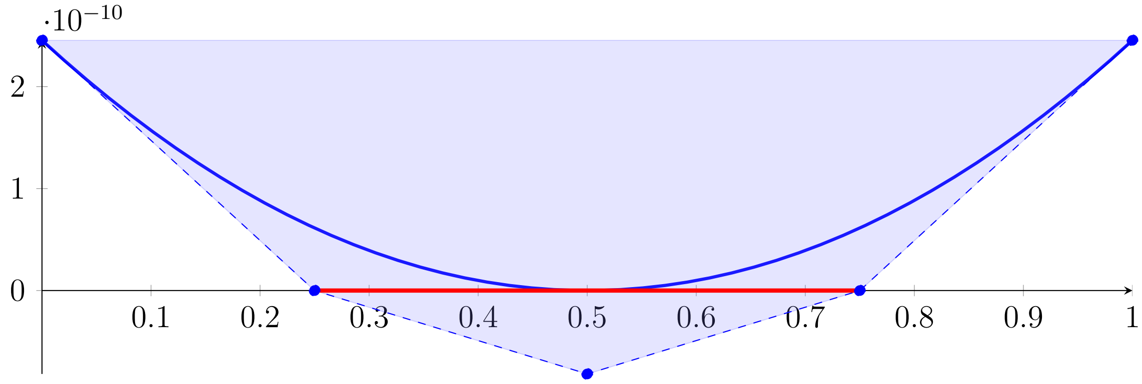
$$[0.2499992814128721, 0.7499979295098023]$$

Longest intersection interval: 0.4999986480969302
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999895370171669, 0.400010473023159]$,

31.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999895370171669, 0.400010473023159]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.921212199577591 \cdot 10^{-19} X^4 + 1.835702882943672 \cdot 10^{-15} X^3 + 9.818258642283608 \\ &\quad \cdot 10^{-10} X^2 - 9.818253497618774 \cdot 10^{-10} X + 2.454559233739064 \cdot 10^{-10} \\ &= 2.454559233739064 \cdot 10^{-10} B_{0,4}(X) - 4.140665629492134 \cdot 10^{-16} B_{1,4}(X) - 8.181910746897216 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.02092399485025 \cdot 10^{-16} B_{3,4}(X) + 2.454582733511515 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995782686167, 0.7499990769537415\}$$

Intersection intervals with the x axis:

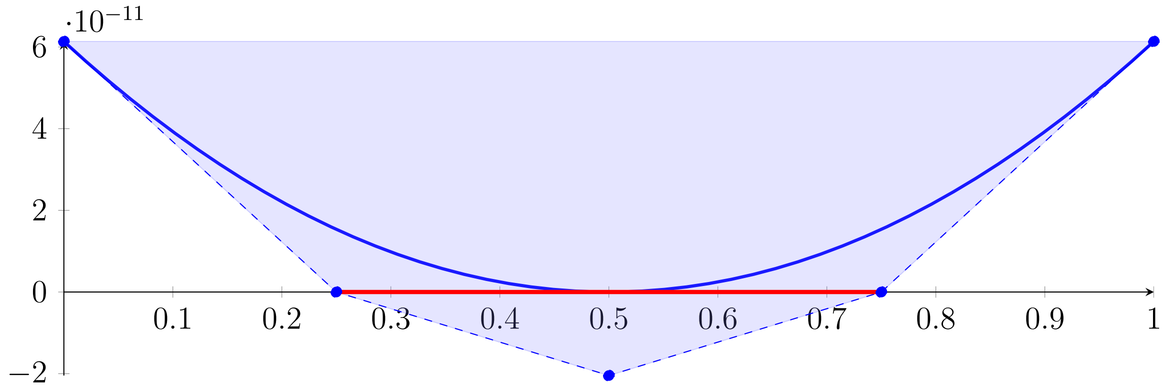
$$[0.2499995782686167, 0.7499990769537415]$$

Longest intersection interval: 0.4999994986851248
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999947710098356, 0.400005239002336]$,

31.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999947710098356, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.200752809081967 \cdot 10^{-20} X^4 + 2.294381551300316 \cdot 10^{-16} X^3 + 2.454563180284348 \\ &\quad \cdot 10^{-10} X^2 - 2.454562046981275 \cdot 10^{-10} X + 6.136393816301913 \cdot 10^{-11} \\ &= 6.136393816301913 \cdot 10^{-11} B_{0,4}(X) - 1.130115127449974 \cdot 10^{-16} B_{1,4}(X) - 2.045477784797216 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 1.013179678980307 \cdot 10^{-18} B_{3,4}(X) + 6.136428091947402 \cdot 10^{-11} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995395858382, 0.7499999876168341\}$$

Intersection intervals with the x axis:

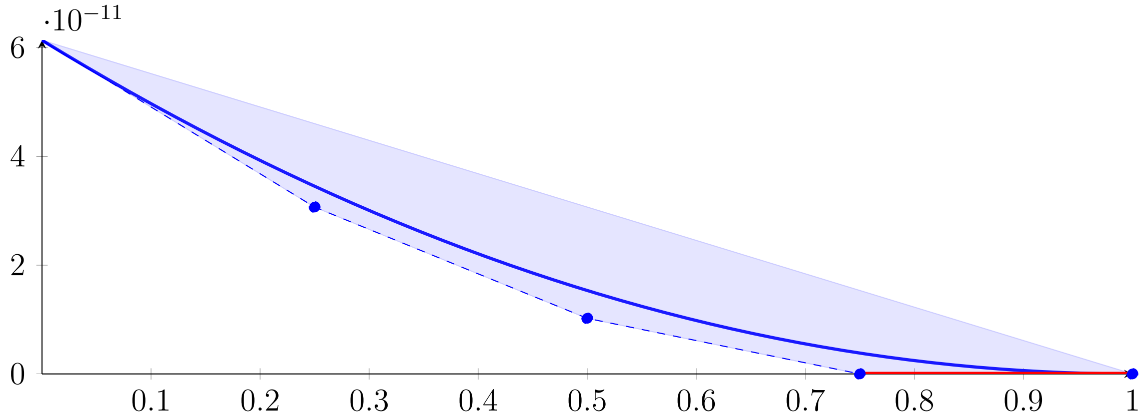
$$[0.2499995395858382, 0.7499999876168341]$$

Longest intersection interval: 0.5000004480309959
$\Longrightarrow$ Bisection: first half $[0.3999947710098356, 0.4000000050060858]$ und second half $[0.4000000050060858, 0.40$

31.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3999947710098356, 0.4000000050060858]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.504705056762291\cdot 10^{-22}X^4 + 2.867976939125394\cdot 10^{-17}X^3 + 6.13640795071087\\&\quad\cdot 10^{-11}X^2 - 1.227281023490638\cdot 10^{-10}X + 6.136393816301913\cdot 10^{-11}\\&= 6.136393816301913\cdot 10^{-11}B_{0,4}(X) + 3.068191257575319\cdot 10^{-11}B_{1,4}(X) + 1.022723357300537\\&\quad\cdot 10^{-11}B_{2,4}(X) - 9.167528198681642\cdot 10^{-17}B_{3,4}(X) - 5.59999170035142\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7499977590601706, 0.9999990874140811\}$$

Intersection intervals with the x axis:

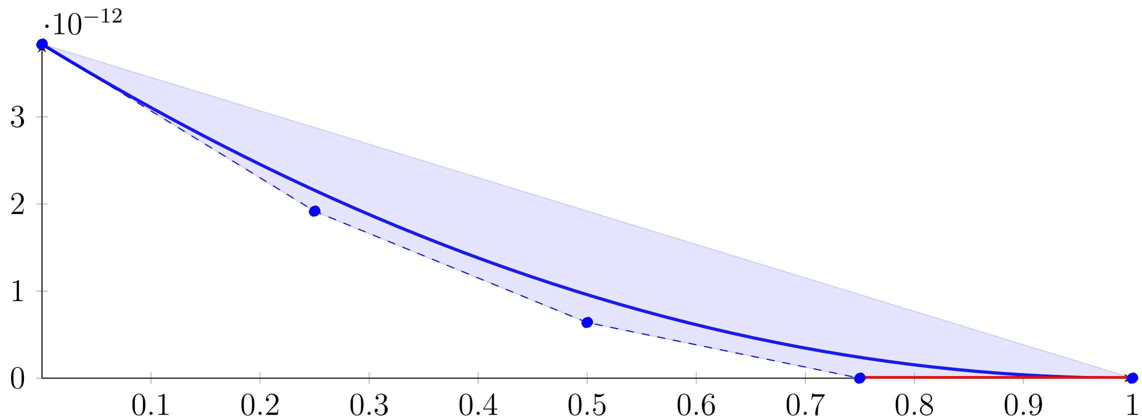
$$[0.7499977590601706, 0.9999990874140811]$$

Longest intersection interval: 0.2500013283539104
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999986964952942, 0.4000000050013093]$,

31.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999986964952942, 0.4000000050013093]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.931587718946285\cdot 10^{-24}X^4 + 4.480933611814181\cdot 10^{-19}X^3 + 3.835299758875134\\&\quad\cdot 10^{-12}X^2 - 7.670593186649561\cdot 10^{-12}X + 3.835236979687871\cdot 10^{-12}\\&= 3.835236979687871\cdot 10^{-12}B_{0,4}(X) + 1.917588683025481\cdot 10^{-12}B_{1,4}(X) + 6.391570128422797\\&\quad\cdot 10^{-13}B_{2,4}(X) - 5.791883839216654\cdot 10^{-17}B_{3,4}(X) - 5.59999961259787\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7499773476668325, 0.9999853987696839\}$$

Intersection intervals with the x axis:

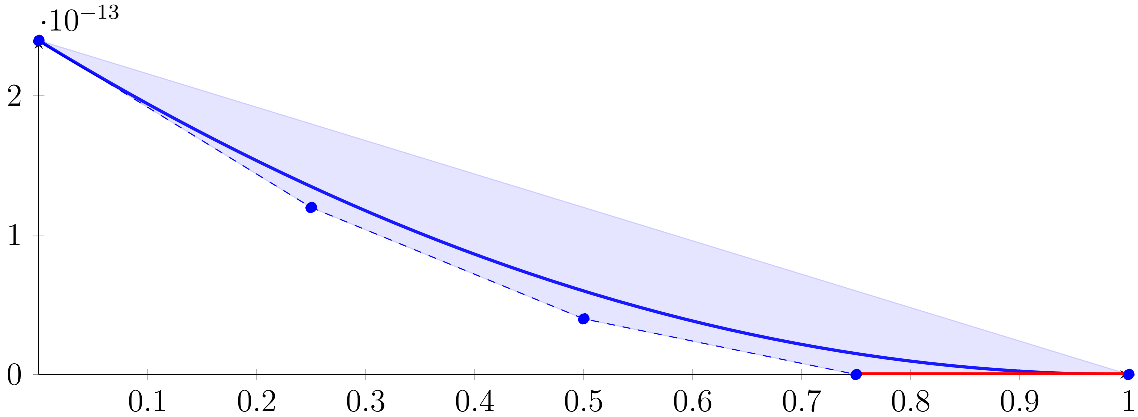
$$[0.7499773476668325, 0.9999853987696839]$$

Longest intersection interval: 0.2500080511028514
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996778451648, 0.4000000049822035]$,

31.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999996778451648, 0.4000000049822035]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.14529897555355 \cdot 10^{-26} X^4 + 7.001997796519582 \cdot 10^{-21} X^3 + 2.397217373893792 \\ &\quad \cdot 10^{-13} X^2 - 4.794695778397391 \cdot 10^{-13} X + 2.396918341578483 \cdot 10^{-13} \\ &= 2.396918341578483 \cdot 10^{-13} B_{0,4}(X) + 1.198244396979135 \cdot 10^{-13} B_{1,4}(X) + 3.991066813620863 \\ &\quad \cdot 10^{-14} B_{2,4}(X) - 4.947877676695723 \cdot 10^{-17} B_{3,4}(X) - 5.599929052523716 \cdot 10^{-17} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7496904492313635, 0.9997664242061345\}$$

Intersection intervals with the x axis:

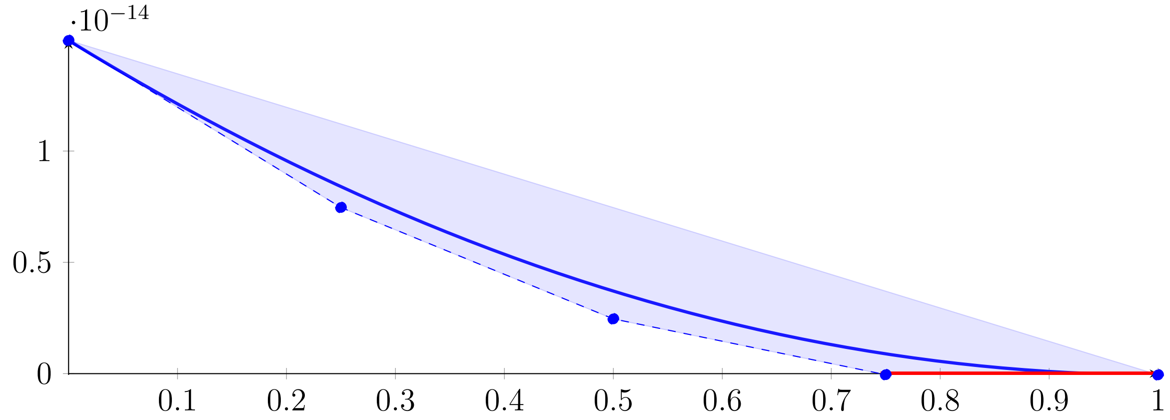
$$[0.7496904492313635, 0.9997664242061345]$$

Longest intersection interval: 0.250075974974771
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999230966783, 0.4000000049057922]$,

31.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999230966783, 0.4000000049057922]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.479264981640815 \cdot 10^{-29} X^4 + 1.095054543732688 \cdot 10^{-22} X^3 + 1.499171738187779 \\ &\quad \cdot 10^{-14} X^2 - 3.001796269578916 \cdot 10^{-14} X + 1.497026508467358 \cdot 10^{-14} \\ &= 1.497026508467358 \cdot 10^{-14} B_{0,4}(X) + 7.465774410726289 \cdot 10^{-15} B_{1,4}(X) + 2.459903300425297 \\ &\quad \cdot 10^{-15} B_{2,4}(X) - 4.734821885303232 \cdot 10^{-17} B_{3,4}(X) - 5.598011973238056 \cdot 10^{-17} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7452788722542423, 0.9962745104335203\}$$

Intersection intervals with the x axis:

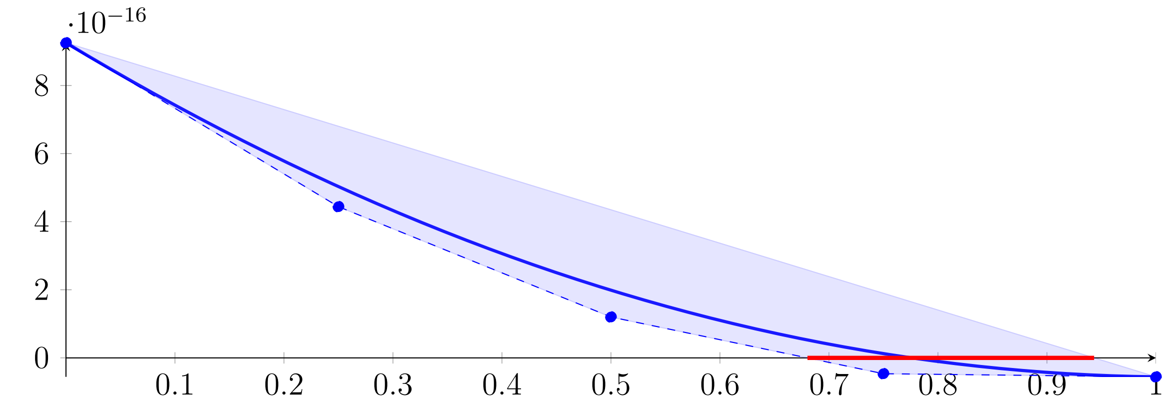
$$[0.7452788722542423, 0.9962745104335203]$$

Longest intersection interval: 0.250995638179278
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999840672825, 0.4000000046010132]$,

31.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999840672825, 0.4000000046010132]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.777753131478217\cdot 10^{-31}X^4 + 1.731544849986219\cdot 10^{-24}X^3 + 9.444603761111594\\&\quad\cdot 10^{-16}X^2 - 1.925623994999771\cdot 10^{-15}X + 9.25520203786835\cdot 10^{-16}\\&= 9.25520203786835\cdot 10^{-16}B_{0,4}(X) + 4.441142050368923\cdot 10^{-16}B_{1,4}(X) + 1.201182689721428\\&\quad\cdot 10^{-16}B_{2,4}(X) - 4.646760397452725\cdot 10^{-17}B_{3,4}(X) - 5.564341337023184\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.6802647890355577, 0.9432883441688765\}$$

Intersection intervals with the x axis:

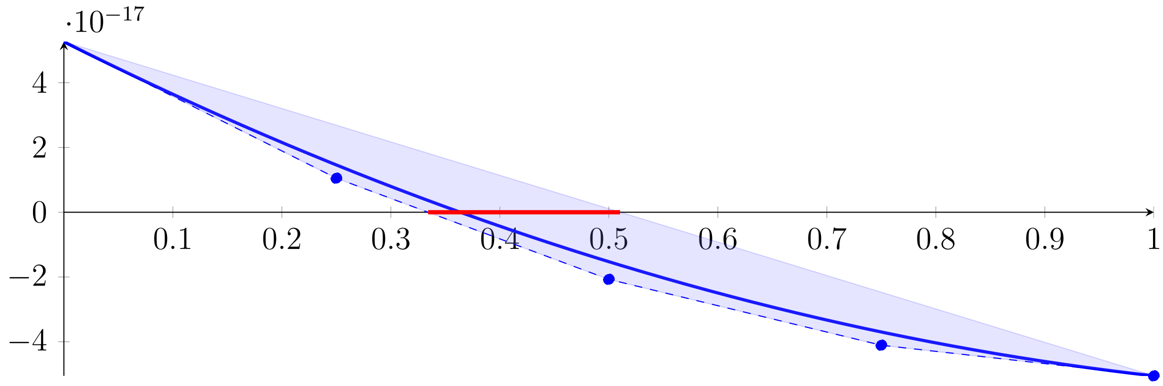
$$[0.6802647890355577, 0.9432883441688765]$$

Longest intersection interval: 0.2630235551333187
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999980356565, 0.4000000034365114]$,

31.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999980356565, 0.4000000034365114]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.508441684091975\cdot 10^{-34}X^4 + 3.150776186268732\cdot 10^{-26}X^3 + 6.533908238790805\\&\quad\cdot 10^{-17}X^2 - 1.685080699755733\cdot 10^{-16}X + 5.264466028744181\cdot 10^{-17}\\&= 5.264466028744181\cdot 10^{-17}B_{0,4}(X) + 1.05176427935485\cdot 10^{-17}B_{1,4}(X) - 2.071952763569348\\&\quad\cdot 10^{-17}B_{2,4}(X) - 4.106685099240717\cdot 10^{-17}B_{3,4}(X) - 5.052432726871564\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3341757003677151, 0.5102760193201083\}$$

Intersection intervals with the x axis:

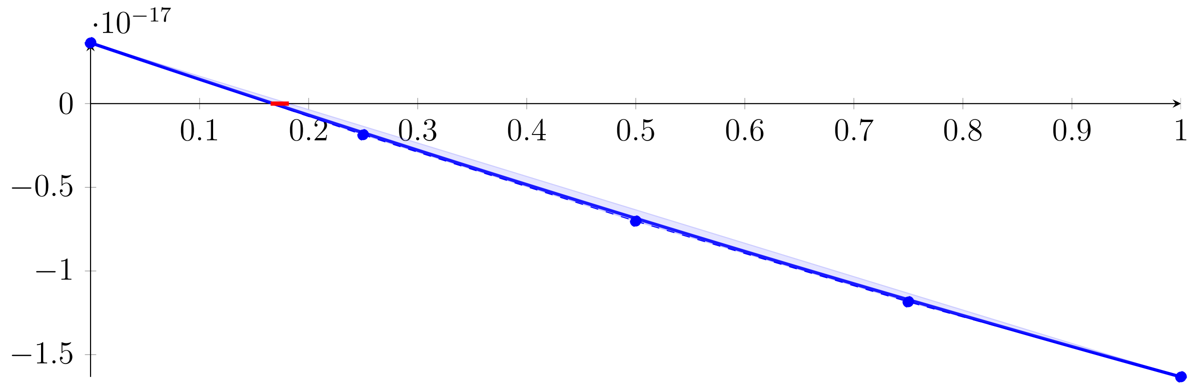
$$[0.3341757003677151, 0.5102760193201083]$$

Longest intersection interval: 0.1761003189523932
$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999840491, 0.4000000007915832]$,

31.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.399999999840491, 0.4000000007915832]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.182586345704061\cdot 10^{-37}X^4 + 1.720671503885177\cdot 10^{-28}X^3 + 2.026251345992899\\&\quad\cdot 10^{-18}X^2 - 2.198411775802323\cdot 10^{-17}X + 3.629995386177444\cdot 10^{-18}\\&= 3.629995386177444\cdot 10^{-18}B_{0,4}(X) - 1.866034053328363\cdot 10^{-18}B_{1,4}(X) - 7.024354935168688\\&\quad\cdot 10^{-18}B_{2,4}(X) - 1.184496725930051\cdot 10^{-17}B_{3,4}(X) - 1.632787102568082\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1651189929990553, 0.1818829383495937\}$$

Intersection intervals with the x axis:

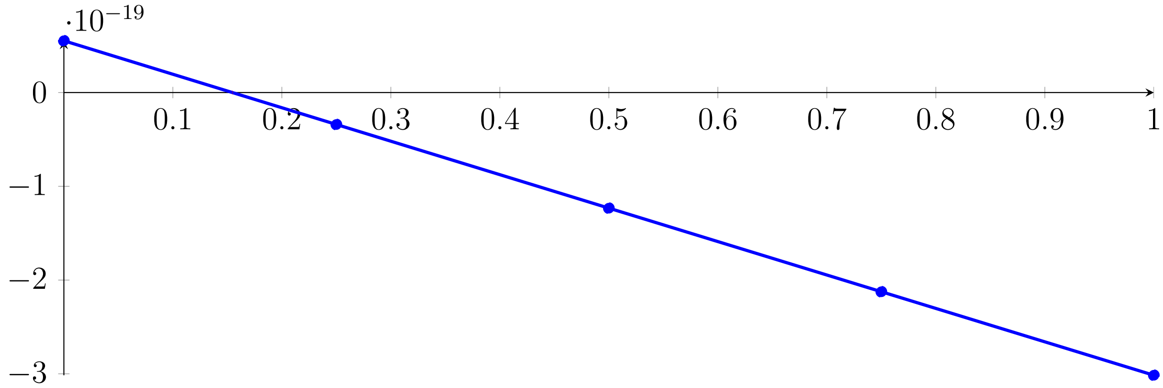
$$[0.1651189929990553, 0.1818829383495937]$$

Longest intersection interval: 0.01676394535053848
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999975344, 0.4000000000134784]$,

31.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999975344, 0.4000000000134784]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -6.462425394283182\cdot 10^{-44}X^4 + 8.106374699799114\cdot 10^{-34}X^3 + 5.694371396423764\\ &\quad \cdot 10^{-22}X^2 - 3.573230357204536\cdot 10^{-19}X + 5.524428779488534\cdot 10^{-20}\\ &= 5.524428779488534\cdot 10^{-20}B_{0,4}(X) - 3.408647113522805\cdot 10^{-20}B_{1,4}(X) - 1.23322323875401\\ &\quad \cdot 10^{-19}B_{2,4}(X) - 2.124632704256334\cdot 10^{-19}B_{3,4}(X) - 3.01509310785925\cdot 10^{-19}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1546060070924308, 0.1548527835869093\}$$

Intersection intervals with the x axis:

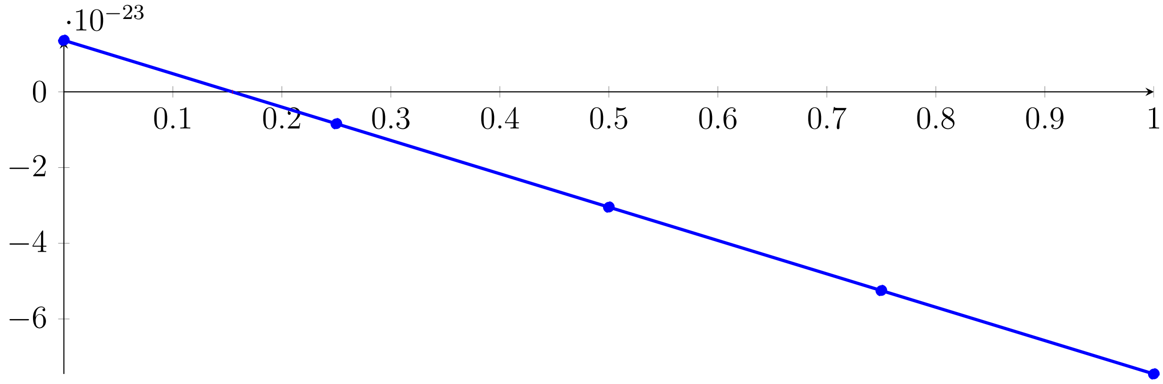
$$[0.1546060070924308, 0.1548527835869093]$$

Longest intersection interval: 0.0002467764944478516
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999994, 0.4000000000000033]$,

31.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999994, 0.4000000000000033]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.396683605522666\cdot 10^{-58}X^4 + 1.218254561570257\cdot 10^{-44}X^3 + 3.467794636018148\\ &\quad \cdot 10^{-29}X^2 - 8.813547453484276\cdot 10^{-23}X + 1.36112658736315\cdot 10^{-23}\\ &= 1.36112658736315\cdot 10^{-23}B_{0,4}(X) - 8.422602760079189\cdot 10^{-24}B_{1,4}(X) - 3.045646561413215\\ &\quad \cdot 10^{-23}B_{2,4}(X) - 5.249032268852739\cdot 10^{-23}B_{3,4}(X) - 7.45241739832649\cdot 10^{-23}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544357246099643, 0.1544357853745533\}$$

Intersection intervals with the x axis:

$$[0.1544357246099643, 0.1544357853745533]$$

Longest intersection interval: $6.076458894192547 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

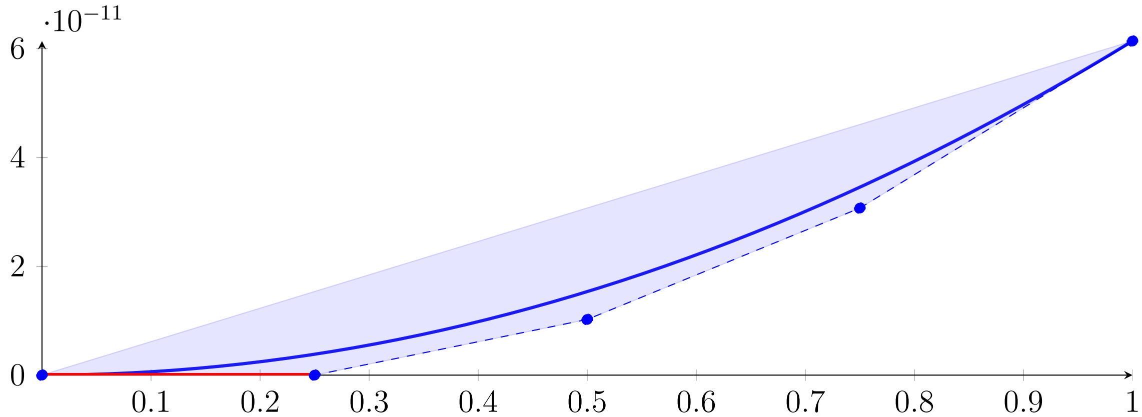
31.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 27!

31.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.4000000050060858, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -7.504705056762291 \cdot 10^{-22} X^4 + 2.867676750923124 \cdot 10^{-17} X^3 + 6.136416554191405 \\ &\quad \cdot 10^{-11} X^2 + 1.427014599332089 \cdot 10^{-16} X - 5.59999170035142 \cdot 10^{-17} \\ &= -5.59999170035142 \cdot 10^{-17} B_{0,4}(X) - 2.032455202021197 \cdot 10^{-17} B_{1,4}(X) + 1.02273762744653 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.068214096632685 \cdot 10^{-11} B_{3,4}(X) + 6.136428091947402 \cdot 10^{-11} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{9.125808216110345e-07, 0.2500004968163653\}$$

Intersection intervals with the x axis:

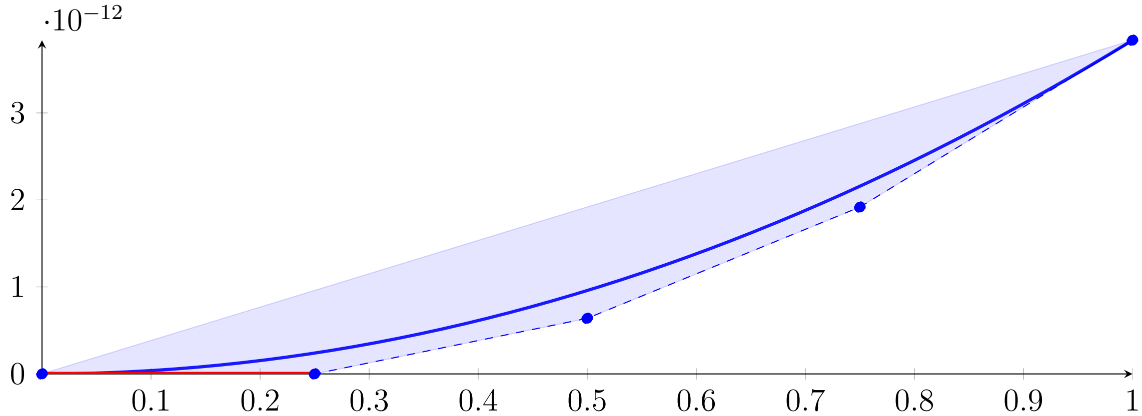
$$[9.125808216110345e-07, 0.2500004968163653]$$

Longest intersection interval: 0.2499995842355437
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050108623, 0.4000013135077487]$,

31.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.4000000050108623, 0.4000013135077487]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.931505911661308 \cdot 10^{-24} X^4 + 4.480722567712799 \cdot 10^{-19} X^3 + 3.835247589865681 \cdot 10^{-12} X^2 + 6.367513939156292 \cdot 10^{-17} X - 5.59997356725911 \cdot 10^{-17}$$
$$= -5.59997356725911 \cdot 10^{-17} B_{0,4}(X) - 4.008095082470037 \cdot 10^{-17} B_{1,4}(X) + 6.391837694783034 \cdot 10^{-13} B_{2,4}(X) + 1.917615663569776 \cdot 10^{-12} B_{3,4}(X) + 3.835255713338726 \cdot 10^{-12} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{1.460109108777016e - 05, 0.2500156756317829\}$$

Intersection intervals with the x axis:

$$[1.460109108777016e - 05, 0.2500156756317829]$$

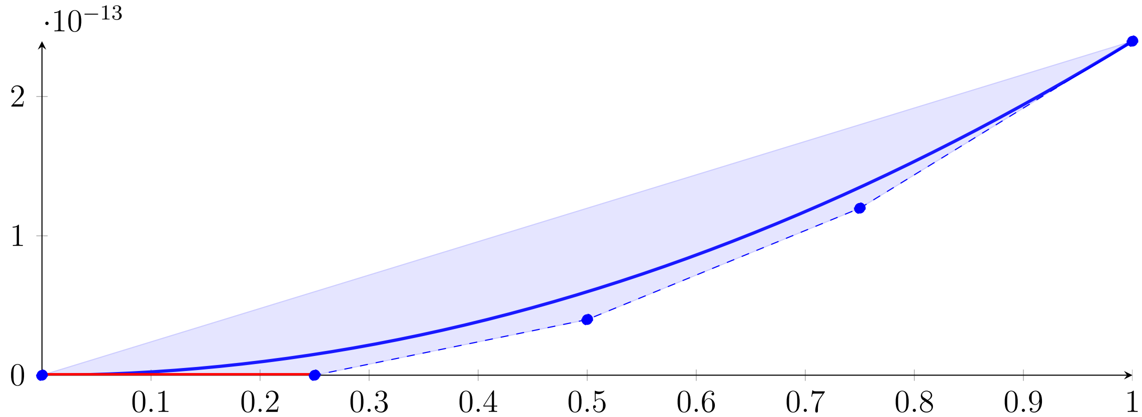
Longest intersection interval: 0.2500010745406951

$\implies$ Selective recursion: interval 1: $[0.4000000050299677, 0.4000003321555954]$,

31.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.4000000050299677, 0.4000003321555954]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.14513918450963 \cdot 10^{-26} X^4 + 7.00121928574046 \cdot 10^{-21} X^3 + 2.397050349370657 \cdot 10^{-13} X^2 + 4.391837331735849 \cdot 10^{-17} X - 5.599798830250992 \cdot 10^{-17}$$
$$= -5.599798830250992 \cdot 10^{-17} B_{0,4}(X) - 4.50183949731703 \cdot 10^{-17} B_{1,4}(X) + 3.991680035453379 \cdot 10^{-14} B_{2,4}(X) + 1.198294600105232 \cdot 10^{-13} B_{3,4}(X) + 2.396929623232884 \cdot 10^{-13} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0002335692644077866, 0.2502816337968459\}$$

Intersection intervals with the x axis:

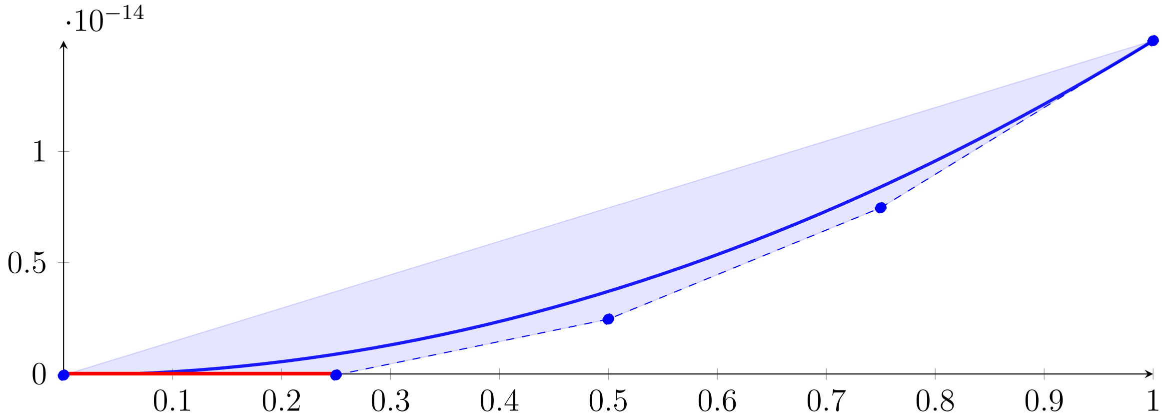
$$[0.0002335692644077866, 0.2502816337968459]$$

Longest intersection interval: 0.2500480645324381
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000051063742, 0.4000000869035043]$,

31.31 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.4000000051063742, 0.4000000869035043]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -4.476640967897802 \cdot 10^{-29} X^4 + 1.09457158991016 \cdot 10^{-22} X^3 + 1.49873258928534 \\ & \cdot 10^{-14} X^2 + 3.898095063625474 \cdot 10^{-17} X - 5.597465330775527 \cdot 10^{-17} \\ = & -5.597465330775527 \cdot 10^{-17} B_{0,4}(X) - 4.622941564869158 \cdot 10^{-17} B_{1,4}(X) + 2.461403470819272 \\ & \cdot 10^{-15} B_{2,4}(X) + 7.466924033460424 \cdot 10^{-15} B_{3,4}(X) + 1.497033229963901 \cdot 10^{-14} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.003725110466798912, 0.2546088699723713\}$$

Intersection intervals with the x axis:

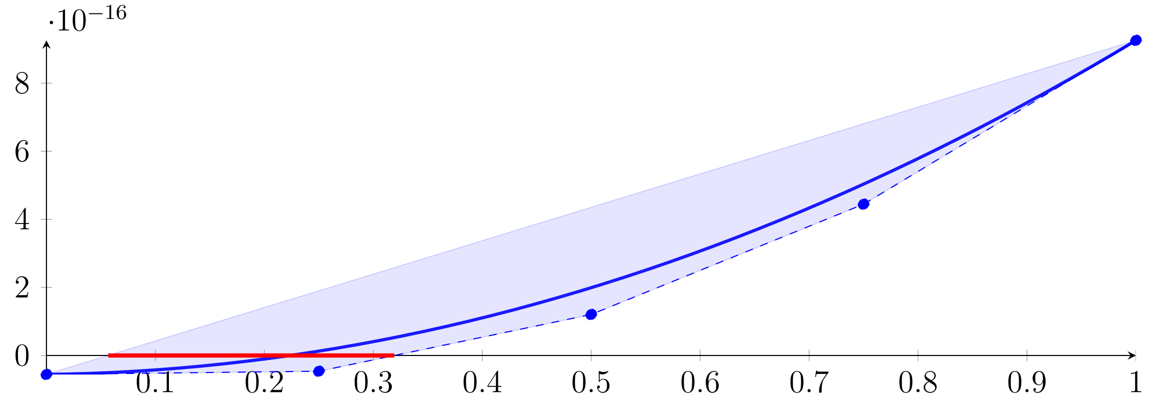
$$[0.003725110466798912, 0.2546088699723713]$$

Longest intersection interval: 0.2508837595055724
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000054110776, 0.4000000259326491]$,

31.32 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.4000000054110776, 0.4000000259326491]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1.773546014711518 \cdot 10^{-31} X^4 + 1.728469879701229 \cdot 10^{-24} X^3 + 9.433421698048765 \\ & \cdot 10^{-16} X^2 + 3.779308932689389 \cdot 10^{-17} X - 5.562147411226954 \cdot 10^{-17} \\ = & -5.562147411226954 \cdot 10^{-17} B_{0,4}(X) - 4.617320178054607 \cdot 10^{-17} B_{1,4}(X) + 1.204987655186568 \\ & \cdot 10^{-16} B_{2,4}(X) + 4.443944282174566 \cdot 10^{-16} B_{3,4}(X) + 9.255137867479705 \cdot 10^{-16} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.05669093378980359, 0.3192576000162909\}$$

Intersection intervals with the x axis:

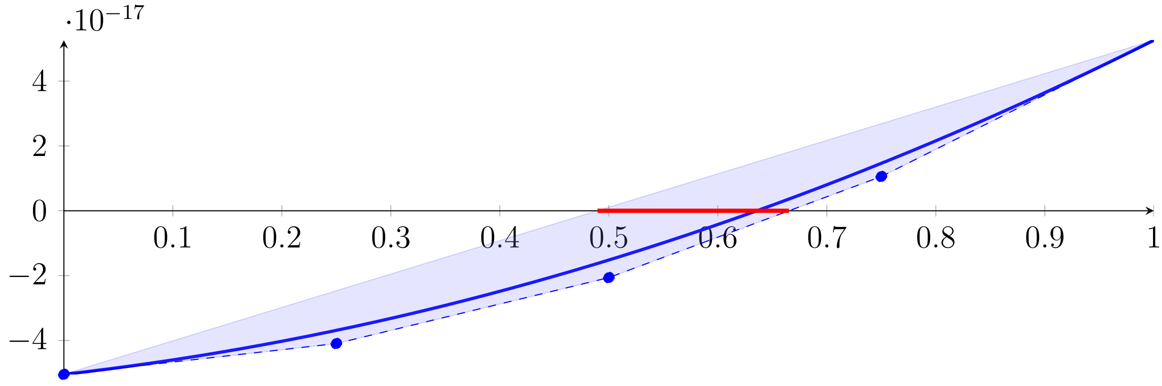
$$[0.05669093378980359, 0.3192576000162909]$$

Longest intersection interval: 0.2625666662264874
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000065744646, 0.4000000119627452]$,

31.33 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.4000000065744646, 0.4000000119627452]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -8.42948070355458 \cdot 10^{-34} X^4 + 3.128819977296419 \cdot 10^{-26} X^3 + 6.503519235890246 \\ &\quad \cdot 10^{-17} X^2 + 3.800678391173011 \cdot 10^{-17} X - 5.044717705924284 \cdot 10^{-17} \\ &= -5.044717705924284 \cdot 10^{-17} B_{0,4}(X) - 4.094548108131031 \cdot 10^{-17} B_{1,4}(X) - 2.060458637689404 \\ &\quad \cdot 10^{-17} B_{2,4}(X) + 1.057550706182803 \cdot 10^{-17} B_{3,4}(X) + 5.259479924267794 \cdot 10^{-17} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4895788965792814, 0.6652062590622766\}$$

Intersection intervals with the x axis:

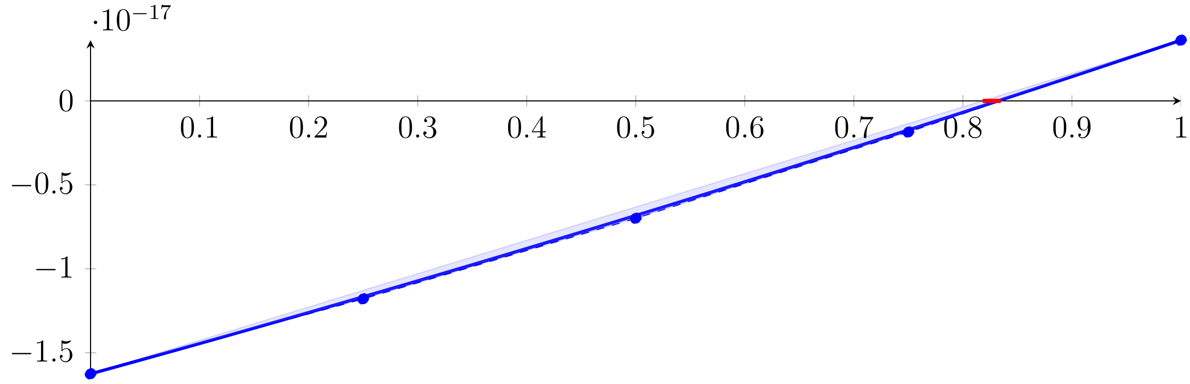
$$[0.4895788965792814, 0.6652062590622766]$$

Longest intersection interval: 0.1756273624829952
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000092124531, 0.4000000101587826]$,

31.34 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.4000000092124531, 0.4000000101587826]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -8.01991079982412 \cdot 10^{-37} X^4 + 1.694950778648659 \cdot 10^{-28} X^3 + 2.006008588115633 \\ &\quad \cdot 10^{-18} X^2 + 1.785893168307927 \cdot 10^{-17} X - 1.625173531873033 \cdot 10^{-17} \\ &= -1.625173531873033 \cdot 10^{-17} B_{0,4}(X) - 1.178700239796051 \cdot 10^{-17} B_{1,4}(X) - 6.987934712504758 \\ &\quad \cdot 10^{-18} B_{2,4}(X) - 1.85453226232069 \cdot 10^{-18} B_{3,4}(X) + 3.613204952634064 \cdot 10^{-18} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8181114615359527, 0.8347943211886801\}$$

Intersection intervals with the x axis:

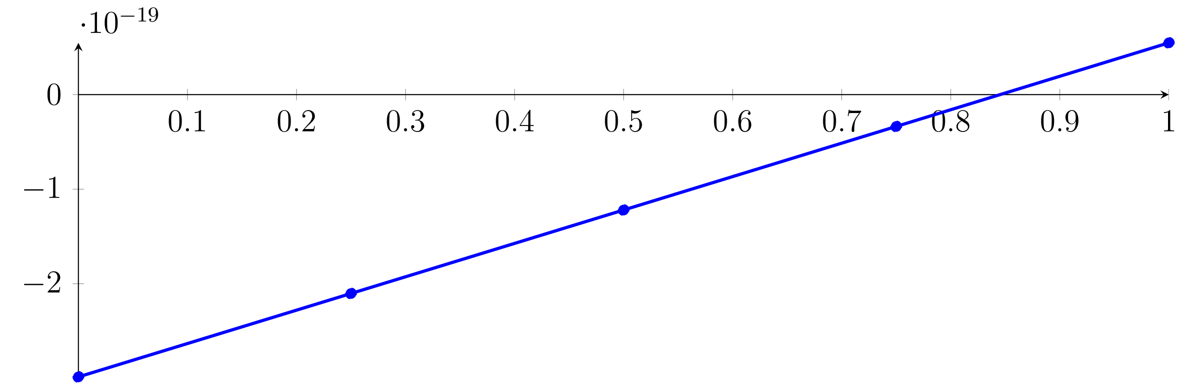
$$[0.8181114615359527, 0.8347943211886801]$$

Longest intersection interval: 0.01668285965272735
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099866561, 0.4000000100024436]$,

31.35 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.4000000099866561, 0.4000000100024436]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -6.212287164586666 \cdot 10^{-44} X^4 + 7.869888381434003 \cdot 10^{-34} X^3 + 5.583079095636332 \\ &\quad \cdot 10^{-22} X^2 + 3.526958212875592 \cdot 10^{-19} X - 2.985043046684015 \cdot 10^{-19} \\ &= -2.985043046684015 \cdot 10^{-19} B_{0,4}(X) - 2.103303493465117 \cdot 10^{-19} B_{1,4}(X) - 1.220633427063613 \\ &\quad \cdot 10^{-19} B_{2,4}(X) - 3.370328474795011 \cdot 10^{-20} B_{3,4}(X) + 5.474982452872209 \cdot 10^{-20} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845012924114553, 0.8452574901650142\}$$

Intersection intervals with the x axis:

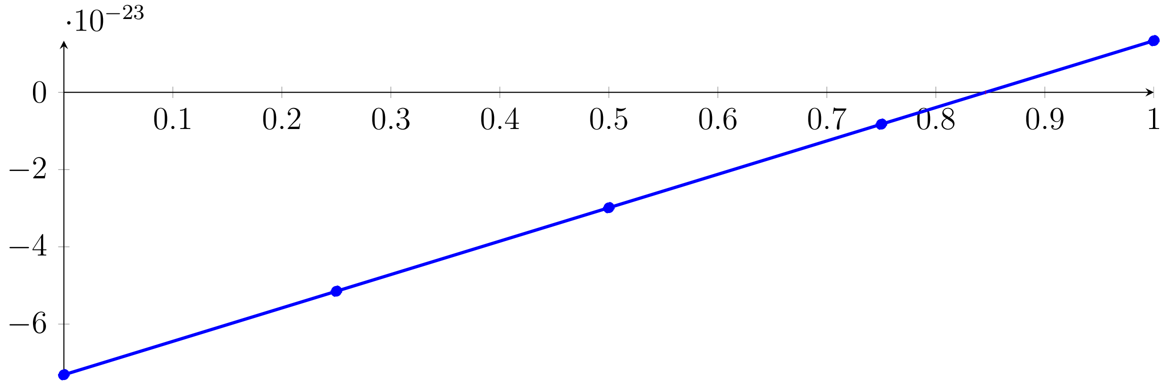
$$[0.845012924114553, 0.8452574901650142]$$

Longest intersection interval: 0.0002445660504611815
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099999968, 0.4000000100000006]$,

31.36 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.4000000099999968, 0.4000000100000006]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.22247151470982 \cdot 10^{-58} X^4 + 1.151216705335136 \cdot 10^{-44} X^3 + 3.33938214525302 \\ &\quad \cdot 10^{-29} X^2 + 8.648818549690798 \cdot 10^{-23} X - 7.311939957361979 \cdot 10^{-23} \\ &= -7.311939957361979 \cdot 10^{-23} B_{0,4}(X) - 5.149735319939279 \cdot 10^{-23} B_{1,4}(X) - 2.987530125952889 \\ &\quad \cdot 10^{-23} B_{2,4}(X) - 8.253243754028074 \cdot 10^{-24} B_{3,4}(X) + 1.336881931710965 \cdot 10^{-23} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454261229035133, 0.8454261825857513\}$$

Intersection intervals with the x axis:

$$[0.8454261229035133, 0.8454261825857513]$$

Longest intersection interval: $5.968223795299727 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

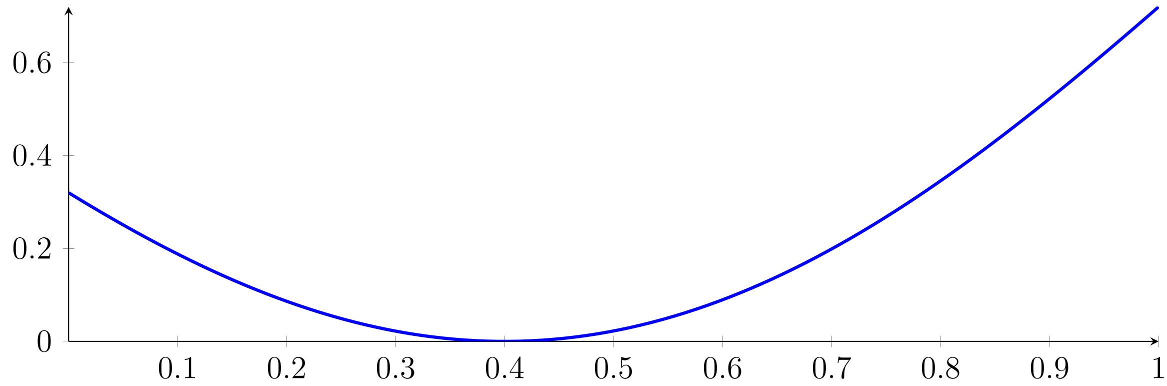
31.37 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 27!

31.38 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

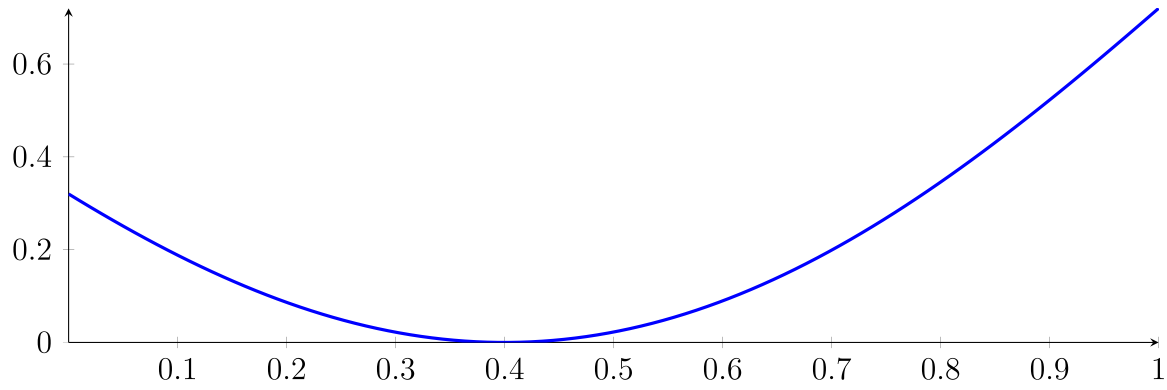
with precision $\varepsilon = 1 \cdot 10^{-16}$.

32 Running `QuadClip` on h_4 with epsilon 16

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `QuadClip` with input polynomial on interval $[0,1]$:

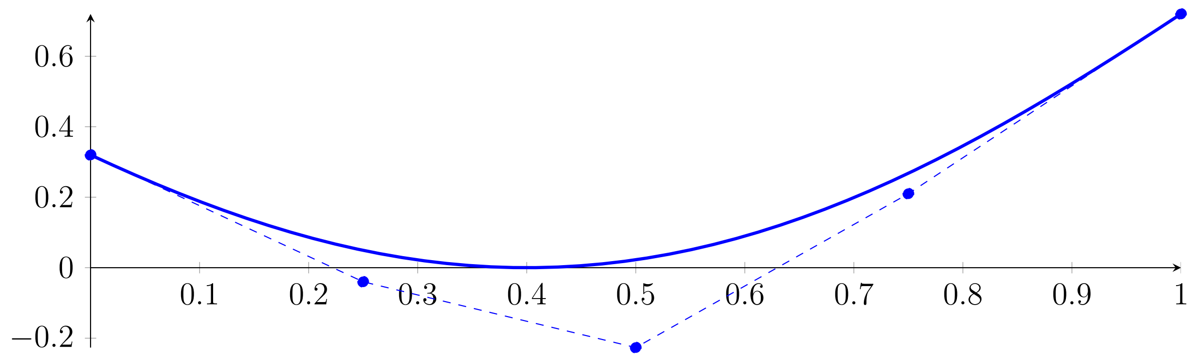
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



32.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

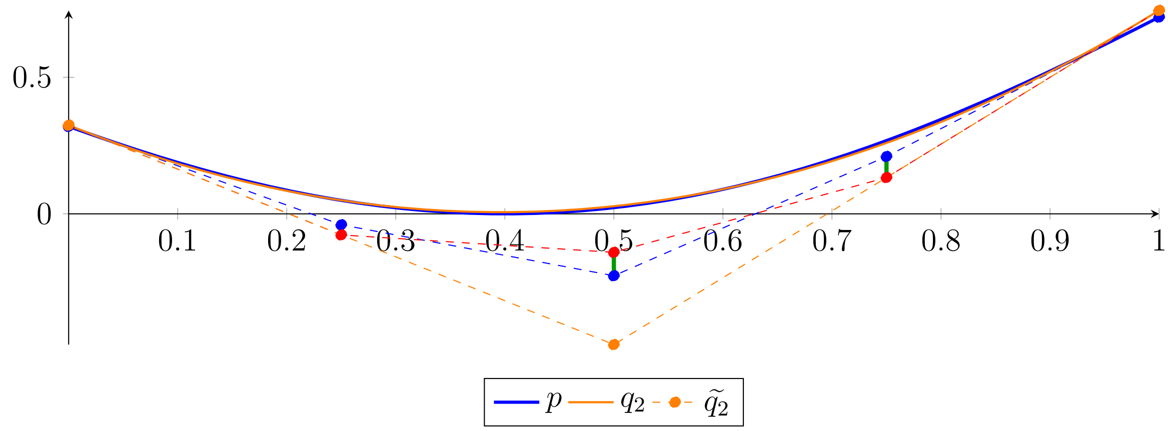
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.810653852360235 \cdot 10^{-306}X^4 - 3.682497235829418 \cdot 10^{-306}X^3 \\ &\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\ &\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

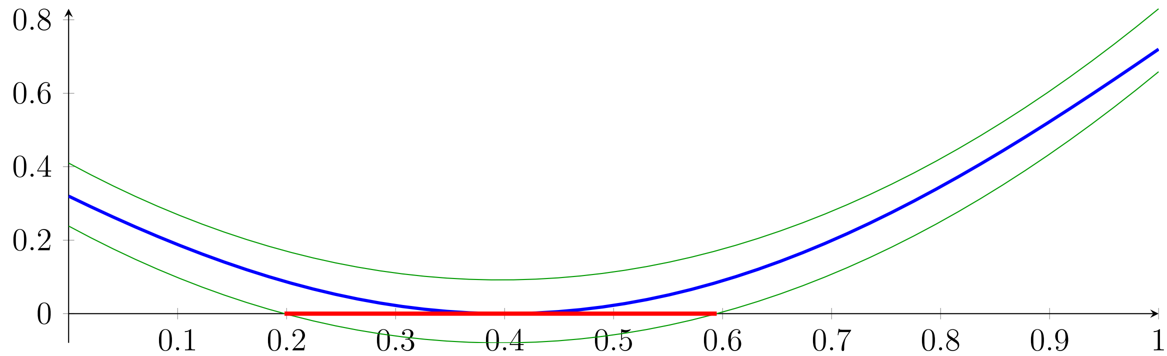
$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$

$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



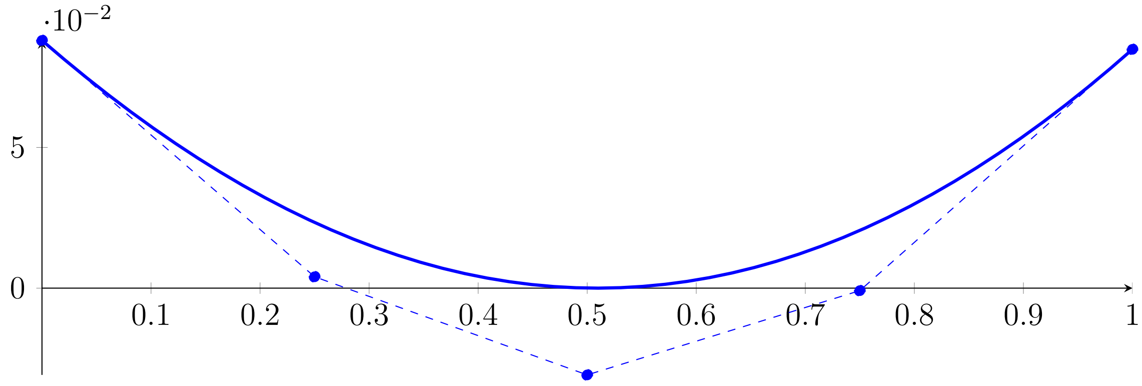
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

32.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

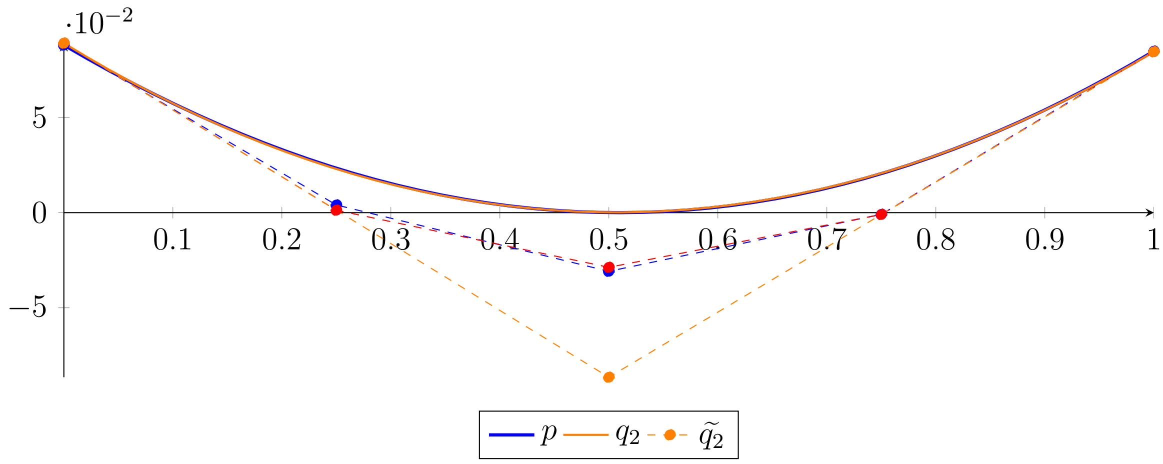
$$\begin{aligned} p = &-0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &+ 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ = &\ 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &- 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402 \cdot 10^{-307}X^4 - 6.258020227051504 \cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

Bounding polynomials M and m:

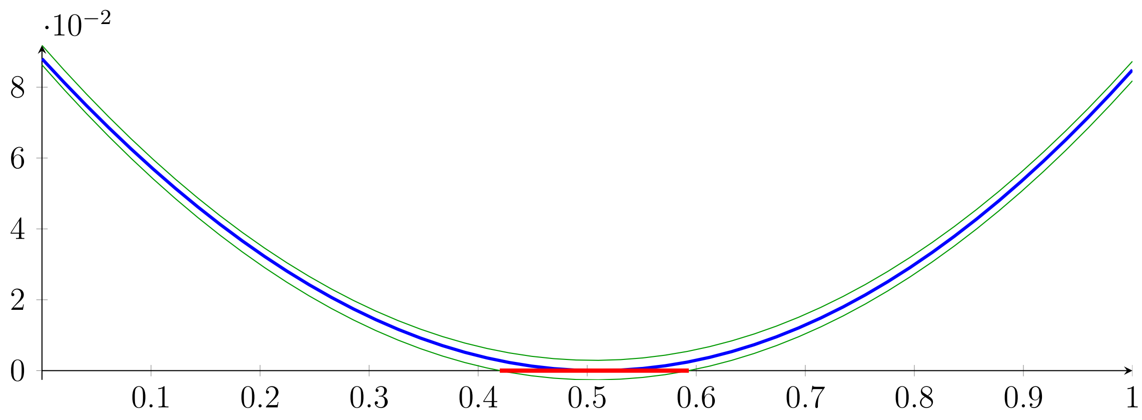
$$M = 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423$$

$$m = 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



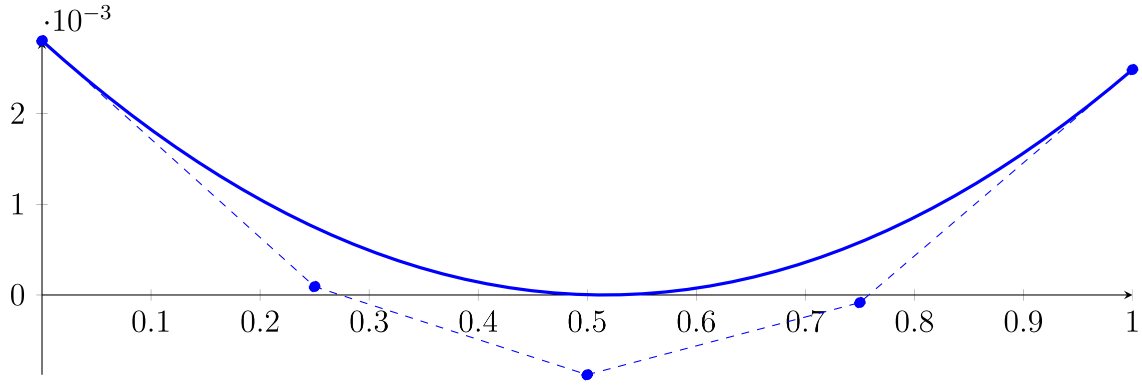
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

32.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

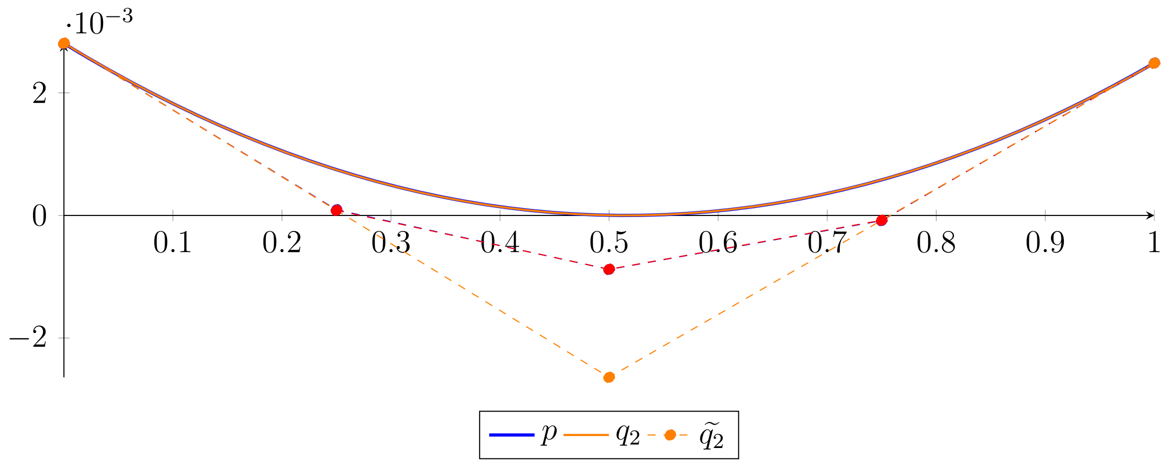
$$\begin{aligned} p = &-2.231982021693433 \cdot 10^{-05} X^4 + 0.0001110015522974976 X^3 \\ &+ 0.01044647623937015 X^2 - 0.01085434180562325 X + 0.002805735592529265 \\ = &\ 0.002805735592529265 B_{0,4}(X) + 9.215014112345361 \cdot 10^{-05} B_{1,4}(X) \\ &- 0.0008803559370539994 B_{2,4}(X) - 8.403225392871914 \\ &\cdot 10^{-05} B_{3,4}(X) + 0.002486551758356734 B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,2} - 0.002640895479419014 B_{1,2} + 0.002483552517338081 B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &\ 9.278384156079834 \cdot 10^{-309} X^4 - 1.903481152394832 \cdot 10^{-308} X^3 \\ &+ 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ = &\ 0.002809372542696975 B_{0,4} + 8.423853163898018 \cdot 10^{-05} B_{1,4} - 0.0008784428096068336 B_{2,4} \\ &- 7.867148104046678 \cdot 10^{-05} B_{3,4} + 0.002483552517338081 B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429 \cdot 10^{-06}$.
Bounding polynomials M and m:

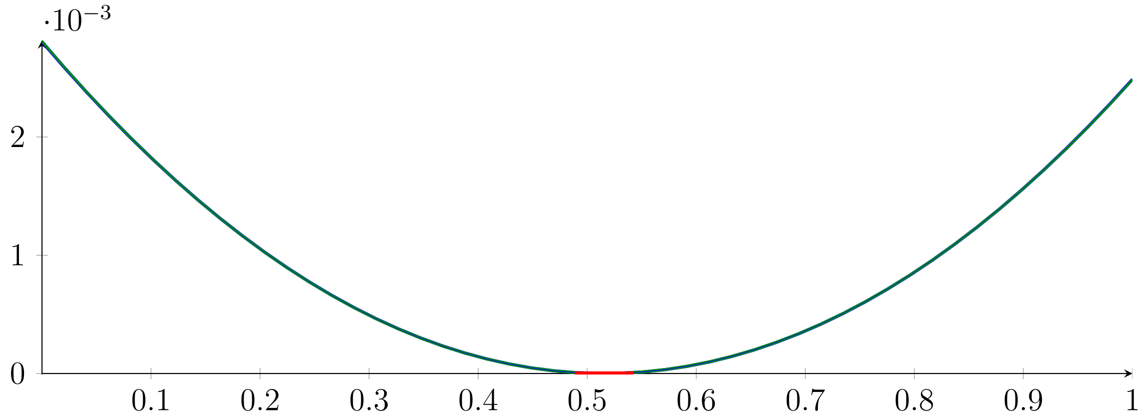
$$M = 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



$$[0.4885305113706042, 0.542280720323693]$$

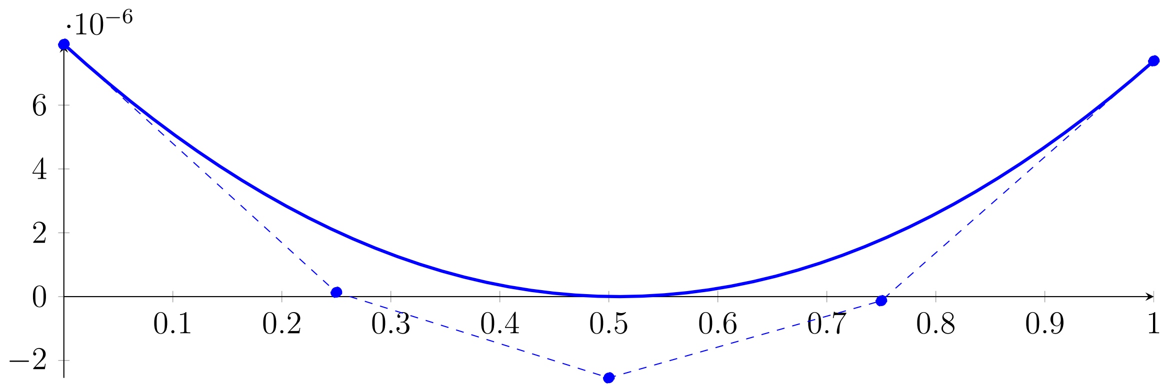
Longest intersection interval: 0.05375020895308878

$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

32.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.4018155471734359]$

Normalized monomial und Bézier representations and the Bézier polygon:

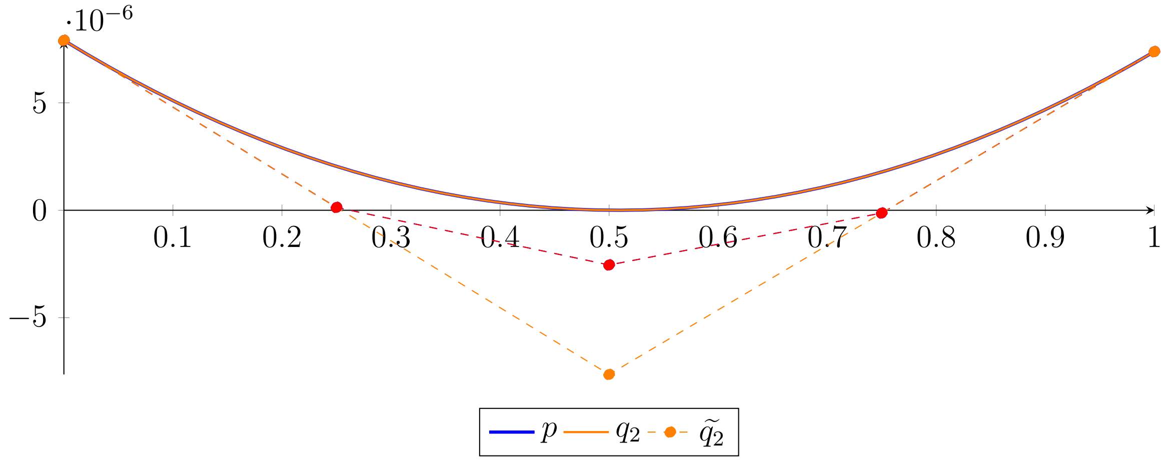
$$\begin{aligned}p &= -1.862993414511891\cdot 10^{-10}X^4 + 1.046428359410323\cdot 10^{-08}X^3 + 3.055842313534119\\&\quad\cdot 10^{-05}X^2 - 3.109078018724982\cdot 10^{-05}X + 7.906738260659747\cdot 10^{-06}\\&= 7.906738260659747\cdot 10^{-06}B_{0,4}(X) + 1.340432138472922\cdot 10^{-07}B_{1,4}(X) - 2.545581310408299\\&\quad\cdot 10^{-06}B_{2,4}(X) - 1.295192412084993\cdot 10^{-07}B_{3,4}(X) + 7.384659193003765\cdot 10^{-06}B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 3.057380019043271\cdot 10^{-05}X^2 - 3.109688842657981\cdot 10^{-05}X + 7.907245506324471\cdot 10^{-06}\\&= 7.907245506324471\cdot 10^{-06}B_{0,2} - 7.641198706965435\cdot 10^{-06}B_{1,2} + 7.384157270177368\cdot 10^{-06}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 2.673714696616243\cdot 10^{-311}X^4 - 5.466261157526541\cdot 10^{-311}X^3 + 3.057380019043271\\&\quad\cdot 10^{-05}X^2 - 3.109688842657981\cdot 10^{-05}X + 7.907245506324471\cdot 10^{-06}\\&= 7.907245506324471\cdot 10^{-06}B_{0,4} + 1.330233996795178\cdot 10^{-07}B_{1,4} - 2.545565341893317\\&\quad\cdot 10^{-06}B_{2,4} - 1.285207183940336\cdot 10^{-07}B_{3,4} + 7.384157270177368\cdot 10^{-06}B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.019814167774439 \cdot 10^{-09}$.
Bounding polynomials M and m:

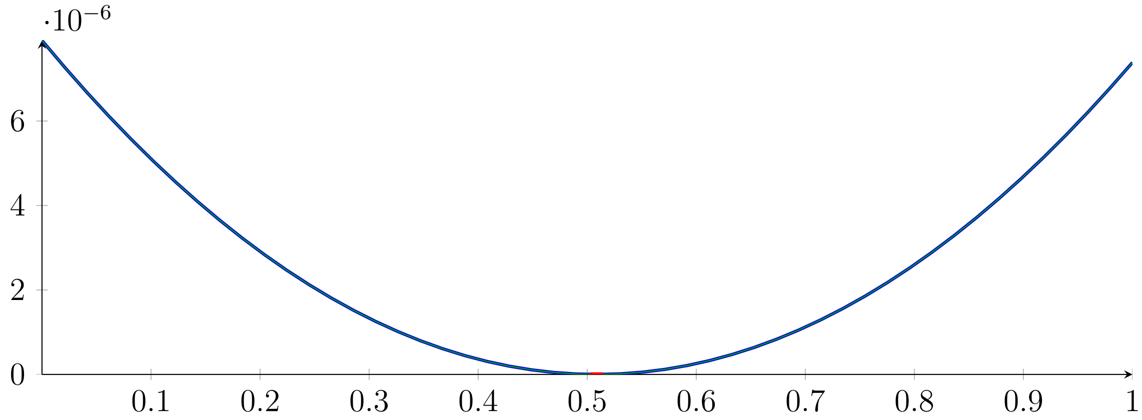
$$M = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.908265320492245 \cdot 10^{-06}$$

$$m = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.906225692156696 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.50281872462556, 0.5142903109829997\}$$

Intersection intervals:



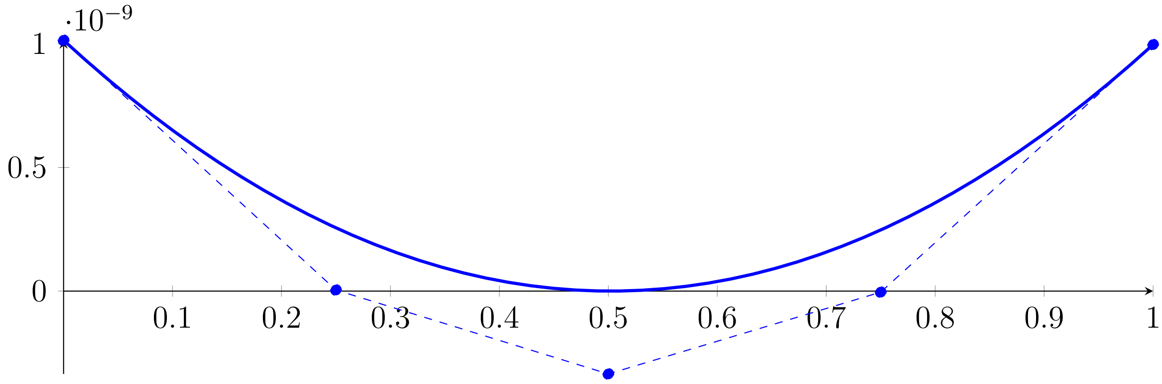
$$[0.50281872462556, 0.5142903109829997]$$

Longest intersection interval: 0.01147158635743977
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999787229673132, 0.4000211044658684]$,

32.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999787229673132, 0.400021104465$

Normalized monomial und Bézier representations and the Bézier polygon:

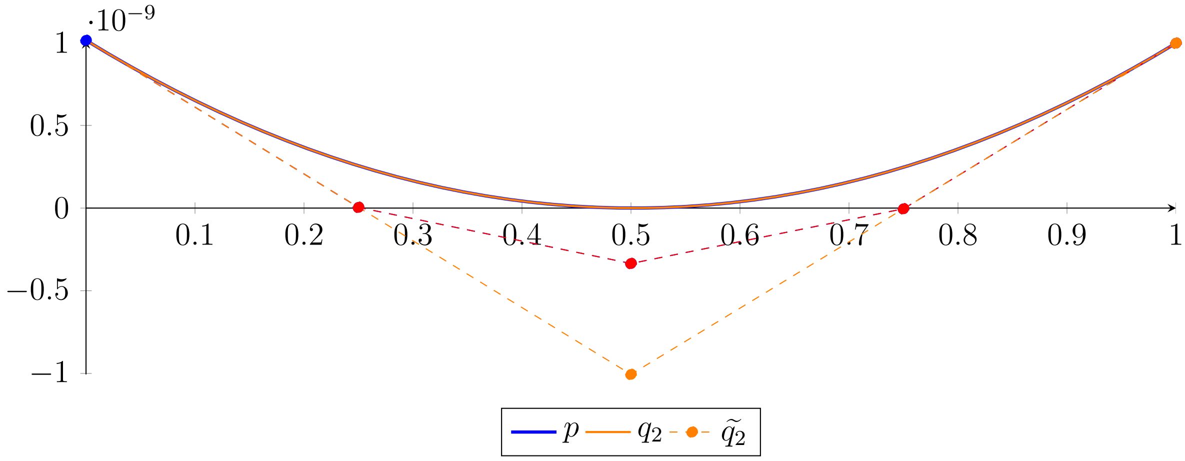
$$p = -3.22630361651822 \cdot 10^{-18} X^4 + 1.523153645441234 \cdot 10^{-14} X^3 + 4.023445841277385 \cdot 10^{-09} X^2 - 4.040789163099954 \cdot 10^{-09} X + 1.014549826650262 \cdot 10^{-09}$$
$$= 1.014549826650262 \cdot 10^{-09} B_{0,4}(X) + 4.352535875273264 \cdot 10^{-12} B_{1,4}(X) - 3.352704480201511 \cdot 10^{-10} B_{2,4}(X) - 4.315317151897678 \cdot 10^{-12} B_{3,4}(X) + 9.972217331378436 \cdot 10^{-10} B_{4,4}(X)$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014550587950544 \cdot 10^{-09} \\ &= 1.014550587950544 \cdot 10^{-09} B_{0,2} - 1.005848561585488 \cdot 10^{-09} B_{1,2} + 9.972209719297413 \cdot 10^{-10} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.491760652125473 \cdot 10^{-315} X^4 - 7.128579610273962 \cdot 10^{-315} X^3 + 4.023468683051261 \\ &\quad \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014550587950544 \cdot 10^{-09} \\ &= 1.014550587950544 \cdot 10^{-09} B_{0,4} + 4.35101318252834 \cdot 10^{-12} B_{1,4} - 3.352704477436108 \\ &\quad \cdot 10^{-10} B_{2,4} - 4.313794827873167 \cdot 10^{-12} B_{3,4} + 9.972209719297413 \cdot 10^{-10} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.522692744924588 \cdot 10^{-15}$.
Bounding polynomials M and m:

$$M = 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014552110643289 \cdot 10^{-09}$$

$$m = 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014549065257799 \cdot 10^{-09}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5015281513143551, 0.5027789820041677\}$$

Intersection intervals:

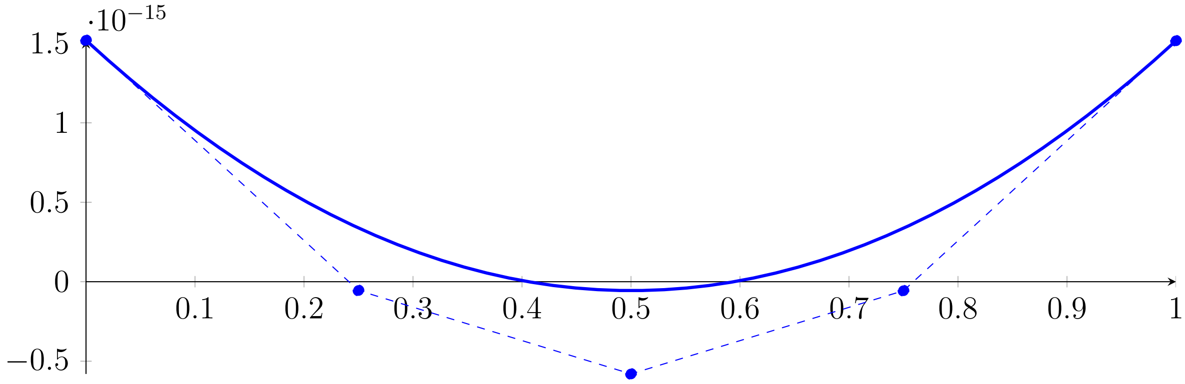
$$[0.5015281513143551, 0.5027789820041677]$$

Longest intersection interval: 0.00125083068981256
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999784819335, 0.4000000314940126]$,

32.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999999784819335, 0.4000000314$

Normalized monomial und Bézier representations and the Bézier polygon:

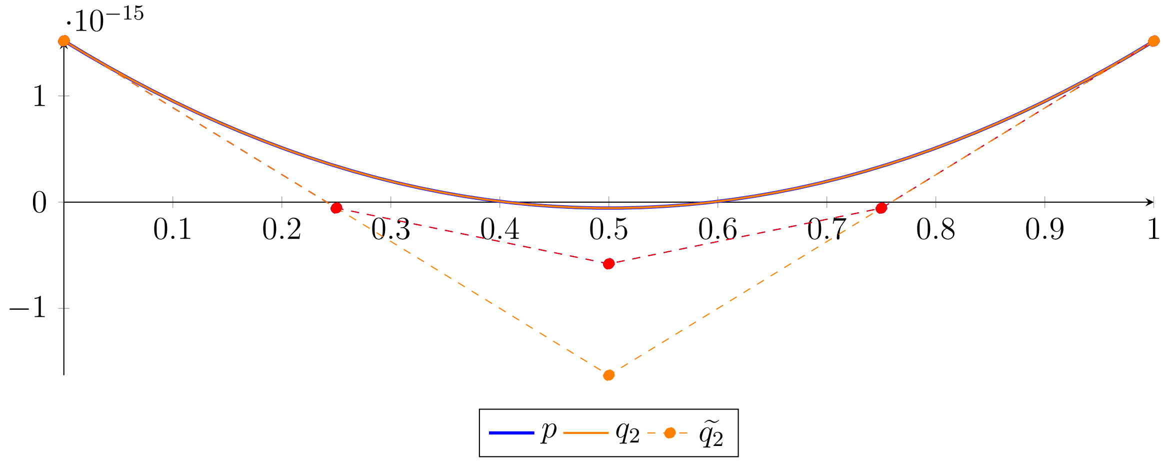
$$\begin{aligned} p = &-7.897676644125011\cdot 10^{-30}X^4 + 2.979577702269594\cdot 10^{-23}X^3 + 6.295028340046794 \\ &\cdot 10^{-15}X^2 - 6.297884694019149\cdot 10^{-15}X + 1.519185582318221\cdot 10^{-15} \\ = &\, 1.519185582318221\cdot 10^{-15}B_{0,4}(X) - 5.52855911865664\cdot 10^{-17}B_{1,4}(X) - 5.805853746835548 \\ &\cdot 10^{-16}B_{2,4}(X) - 5.671376072379998\cdot 10^{-17}B_{3,4}(X) + 1.516329258141634\cdot 10^{-15}B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185583808009\cdot 10^{-15} \\ &= 1.519185583808009\cdot 10^{-15}B_{0,2} - 1.629756772140295\cdot 10^{-15}B_{1,2} + 1.516329256651846\cdot 10^{-15}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &\, 5.424840791336887\cdot 10^{-321}X^4 - 1.106707046684392\cdot 10^{-320}X^3 + 6.295028384740446 \\ &\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185583808009\cdot 10^{-15} \\ = &\, 1.519185583808009\cdot 10^{-15}B_{0,4} - 5.528559416614297\cdot 10^{-17}B_{1,4} - 5.805853746835541 \\ &\cdot 10^{-16}B_{2,4} - 5.671375774422431\cdot 10^{-17}B_{3,4} + 1.516329256651846\cdot 10^{-15}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.979576574030073\cdot 10^{-24}$.

Bounding polynomials M and m:

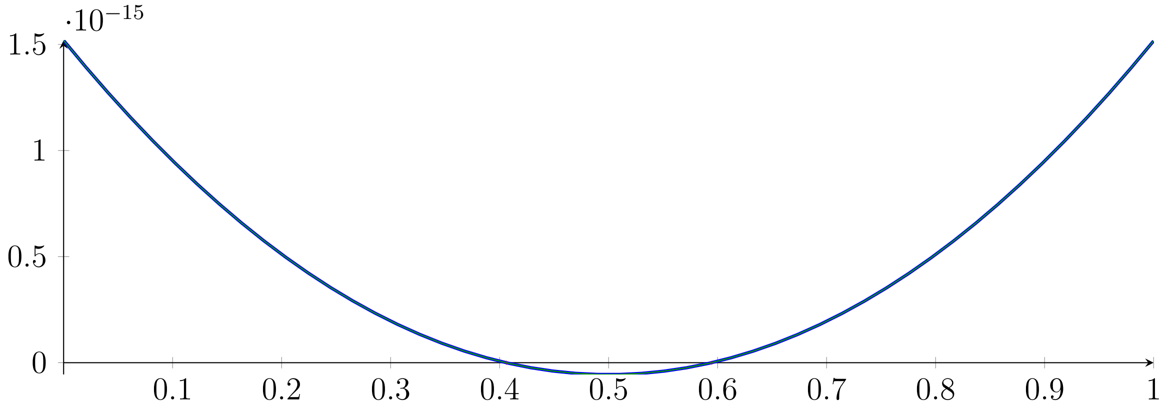
$$M = 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185586787586\cdot 10^{-15}$$

$$m = 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185580828433\cdot 10^{-15}$$

Root of M and m:

$N(M) = \{0.4059087473493063, 0.5945449959829854\}$ $N(m) = \{0.4059087423309477, 0.594545001001344\}$

Intersection intervals:



$$[0.4059087423309477, 0.4059087473493063],\ [0.5945449959829854, 0.594545001001344]$$

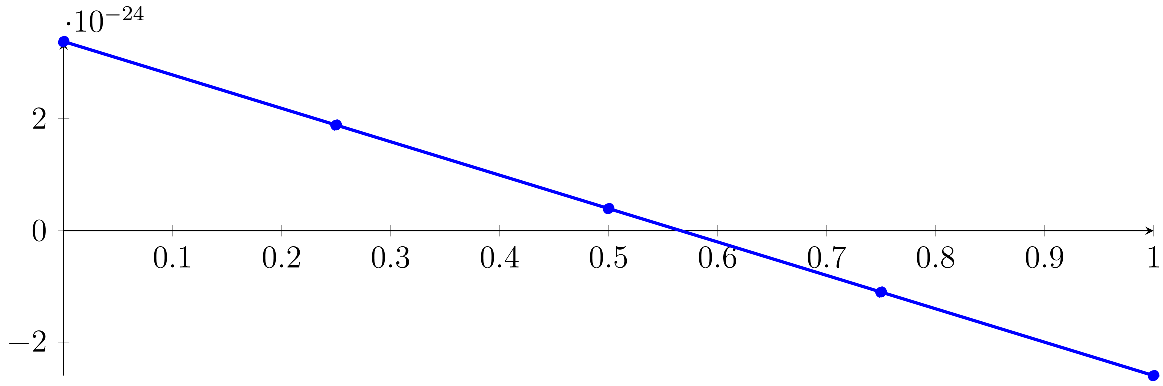
Longest intersection interval: $5.01835859594062 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.4000000000000001]$, interval 2: $[0.400000009999999$

32.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.400000000$

Normalized monomial und Bézier representations and the Bézier polygon:

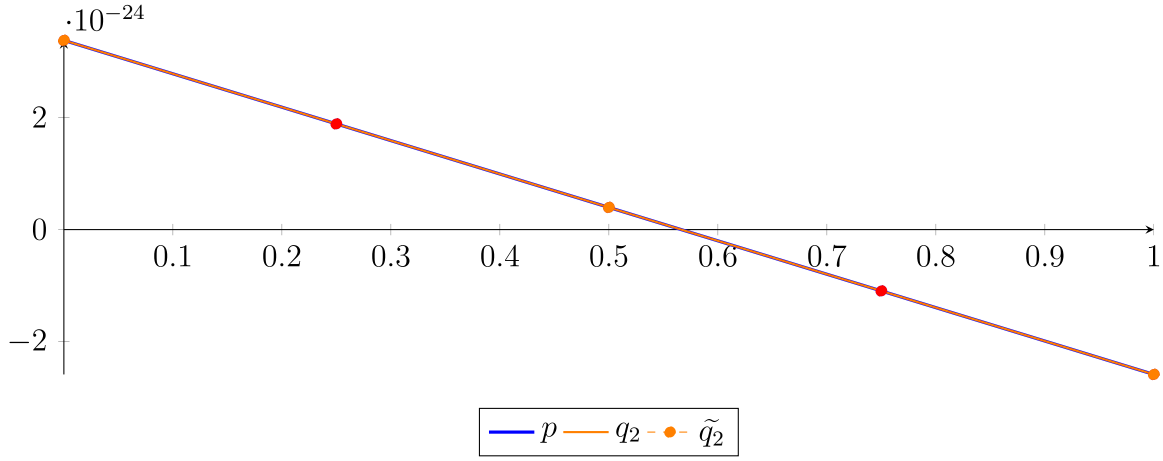
$$\begin{aligned} p = &-5.008943280633806 \cdot 10^{-63} X^4 + 3.765646973599716 \cdot 10^{-48} X^3 + 1.585335098962615 \\ &\cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ = &\ 3.375284612923892 \cdot 10^{-24} B_{0,4}(X) + 1.885496281661092 \cdot 10^{-24} B_{1,4}(X) + 3.95707976820544 \\ &\cdot 10^{-25} B_{2,4}(X) - 1.094080301597753 \cdot 10^{-24} B_{3,4}(X) - 2.583868553593798 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,2} + 3.957079503982923 \cdot 10^{-25} B_{1,2} - 2.583868553593798 \cdot 10^{-24} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= 9.938904969180865 \cdot 10^{-331} X^4 - 2.503131621867773 \cdot 10^{-330} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4} + 1.885496281661092 \cdot 10^{-24} B_{1,4} + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4} - 1.094080301597753 \cdot 10^{-24} B_{3,4} - 2.583868553593798 \cdot 10^{-24} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646973599709 \cdot 10^{-49}$.

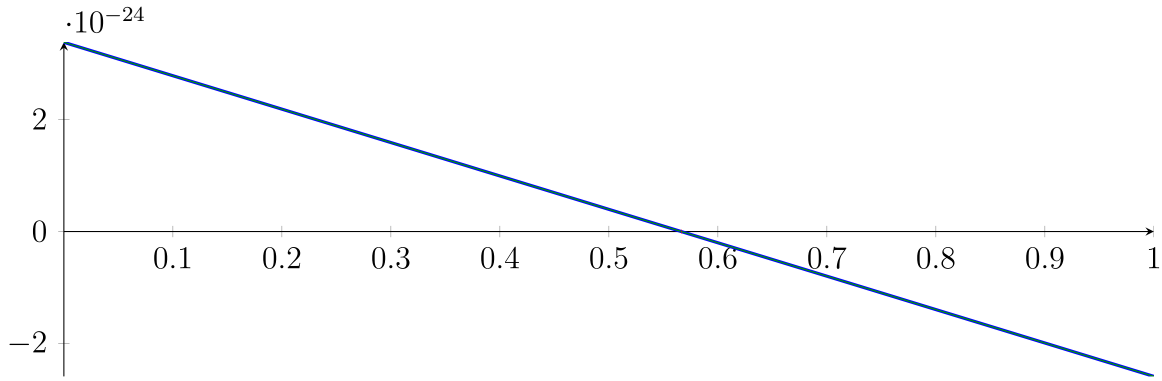
Bounding polynomials M and m:

$$M = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$
$$m = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

Root of M and m:

$$N(M) = \{0.5664033931790926, 37589234.2203029\} \quad N(m) = \{0.5664033931790926, 37589234.2203029\}$$

Intersection intervals:



$$[0.5664033931790926, 0.5664033931790926]$$

Longest intersection interval: $1.263819499744434 \cdot 10^{-25}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

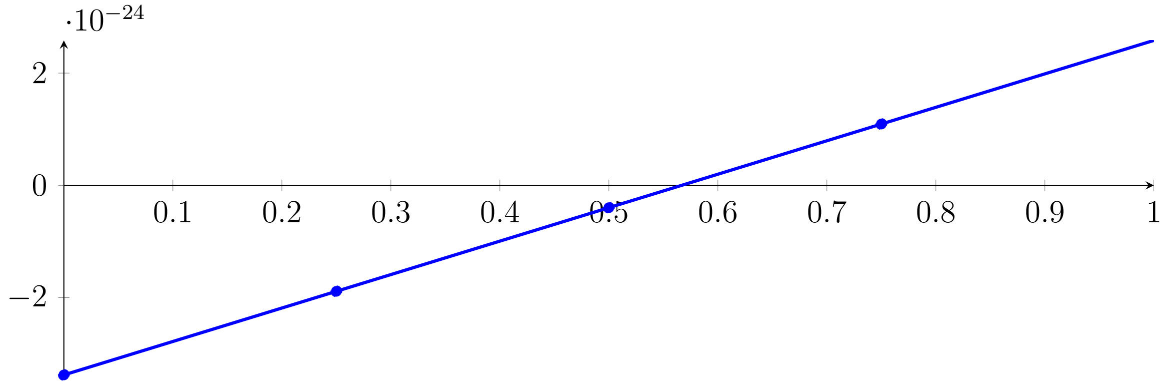
32.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 8!

32.9 Recursion Branch 1 1 1 1 1 1 2 in Interval 2: $[0.4000000099999999, 0.40000001$

Normalized monomial und Bézier representations and the Bézier polygon:

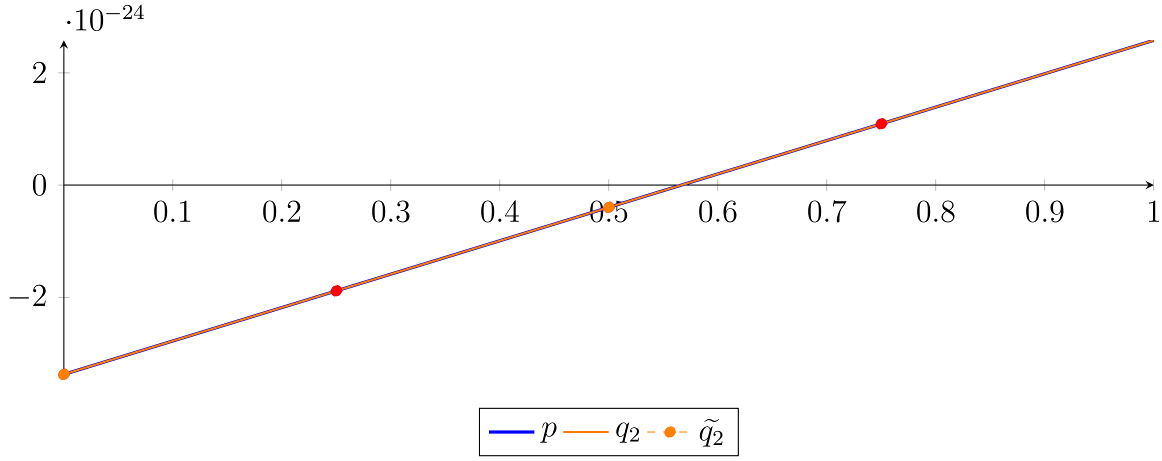
$$\begin{aligned} p = & -5.008943280633806 \cdot 10^{-63} X^4 + 3.76564622047036 \cdot 10^{-48} X^3 + 1.585335103209048 \\ & \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ = & -3.376951785602669 \cdot 10^{-24} B_{0,4}(X) - 1.887163542825797 \cdot 10^{-24} B_{1,4}(X) - 3.973752736266727 \\ & \cdot 10^{-25} B_{2,4}(X) + 1.092413021994703 \cdot 10^{-24} B_{3,4}(X) + 2.582201344038331 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 1.585335103209048\cdot 10^{-31}X^2 + 5.95915297110749\cdot 10^{-24}X - 3.376951785602669\cdot 10^{-24}$$
$$= -3.376951785602669\cdot 10^{-24}B_{0,2} - 3.973753000489244\cdot 10^{-25}B_{1,2} + 2.582201344038331\cdot 10^{-24}B_{2,2}$$

$$\widetilde{q}_2 = -1.030701256063201\cdot 10^{-330}X^4 + 2.355888585287316\cdot 10^{-330}X^3 + 1.585335103209048 \cdot 10^{-31}X^2 + 5.95915297110749\cdot 10^{-24}X - 3.376951785602669\cdot 10^{-24}$$
$$= -3.376951785602669\cdot 10^{-24}B_{0,4} - 1.887163542825797\cdot 10^{-24}B_{1,4} - 3.973752736266727 \cdot 10^{-25}B_{2,4} + 1.092413021994703\cdot 10^{-24}B_{3,4} + 2.582201344038331\cdot 10^{-24}B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646220470353\cdot 10^{-49}$.

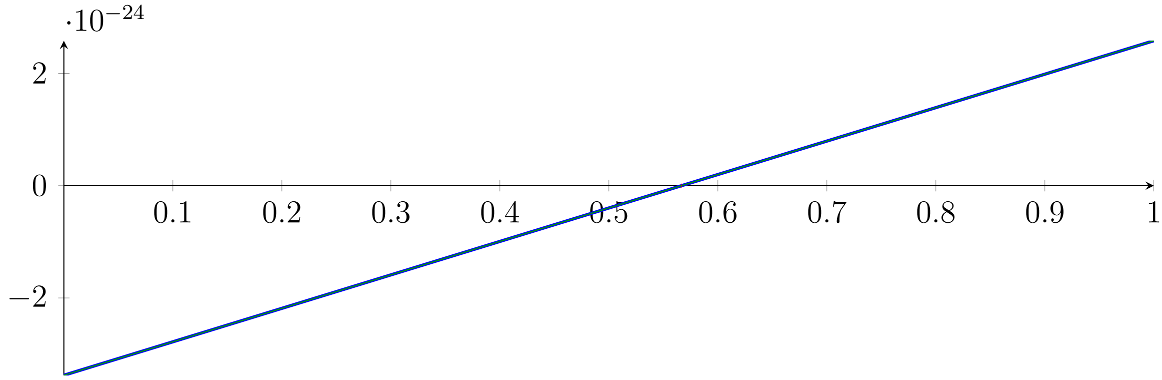
Bounding polynomials M and m:

$$M = 1.585335103209048\cdot 10^{-31}X^2 + 5.95915297110749\cdot 10^{-24}X - 3.376951785602669\cdot 10^{-24}$$
$$m = 1.585335103209048\cdot 10^{-31}X^2 + 5.95915297110749\cdot 10^{-24}X - 3.376951785602669\cdot 10^{-24}$$

Root of M and m:

$$N(M) = \{-37589233.0200927, 0.5666831764624487\} \quad N(m) = \{-37589233.0200927, 0.5666831764624487\}$$

Intersection intervals:



$$[0.5666831764624487, 0.5666831764624487]$$

Longest intersection interval: $1.263819245852043 \cdot 10^{-25}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

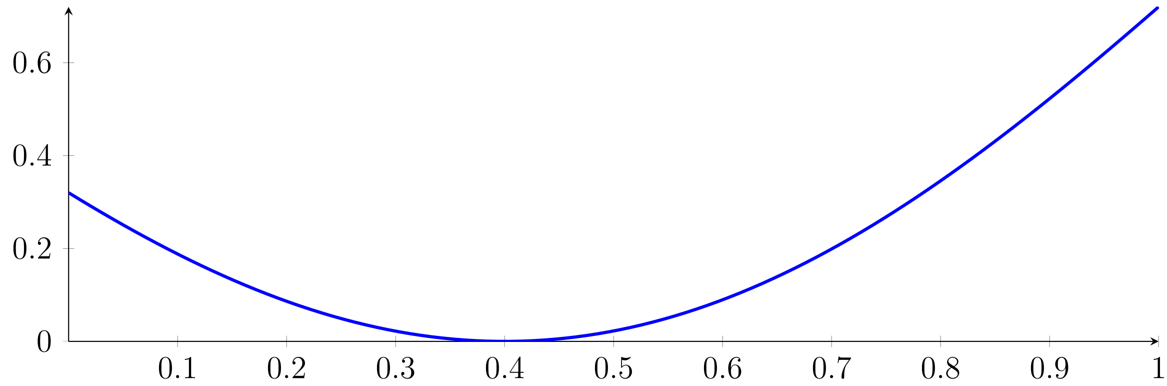
32.10 Recursion Branch 1 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 8!

32.11 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

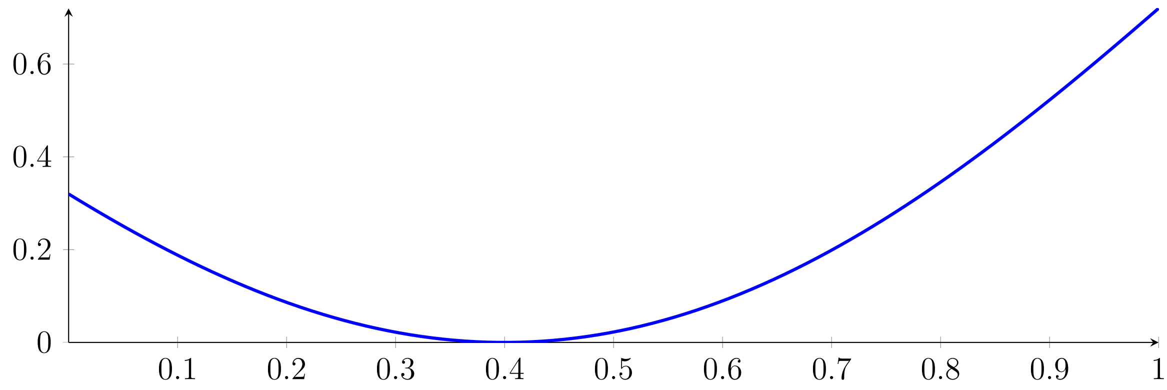
with precision $\varepsilon = 1 \cdot 10^{-16}$.

33 Running `CubeClip` on h_4 with epsilon 16

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `CubeClip` with input polynomial on interval $[0,1]$:

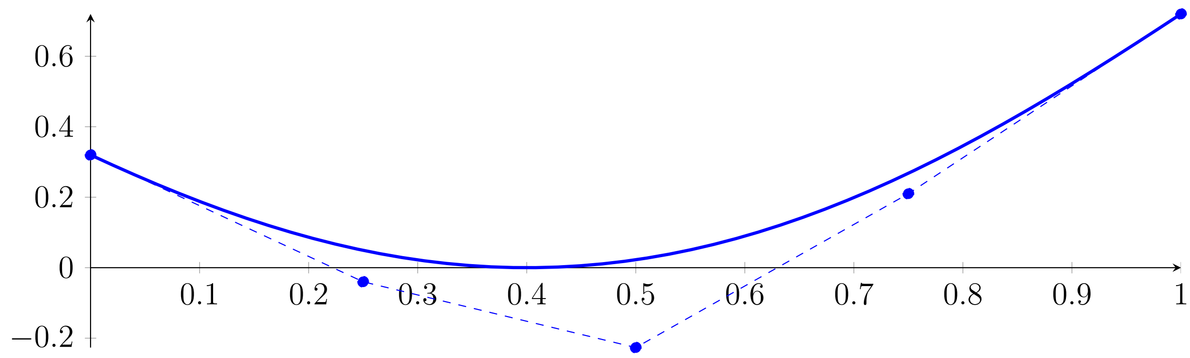
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



33.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

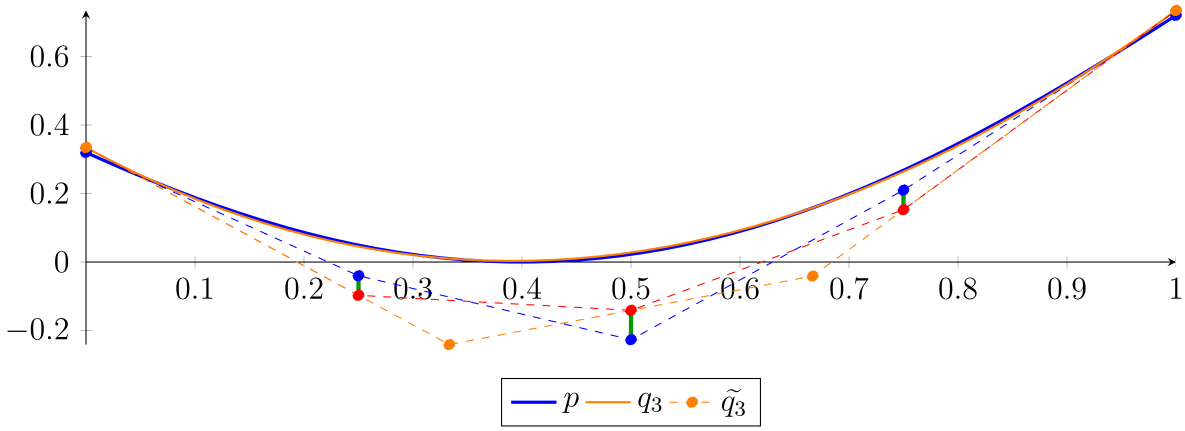
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3} \\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -2.781342323134002 \cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2 \\ &\quad - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4} \\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

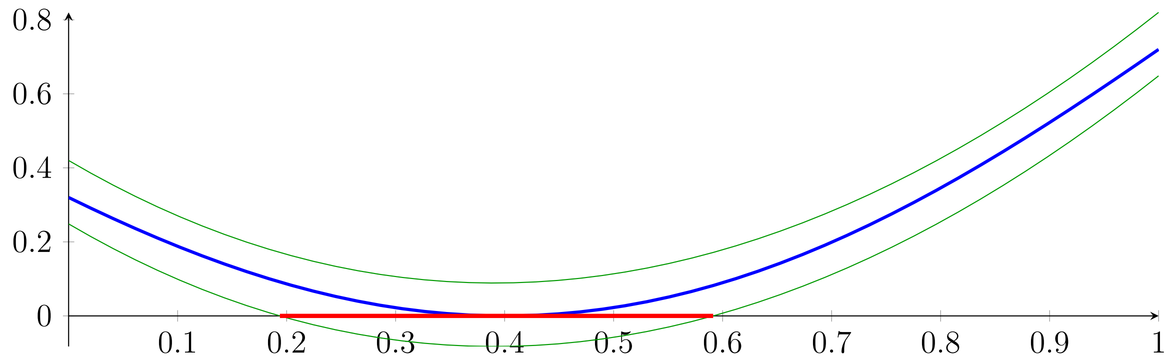
$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$

$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $\quad N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



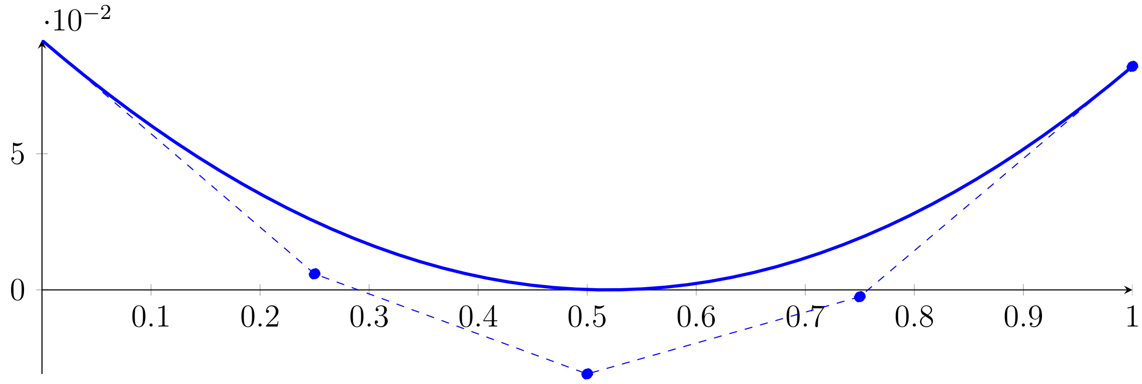
$[0.1938265012447289, 0.5913474001559605]$

Longest intersection interval: 0.3975208989112316
$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

33.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

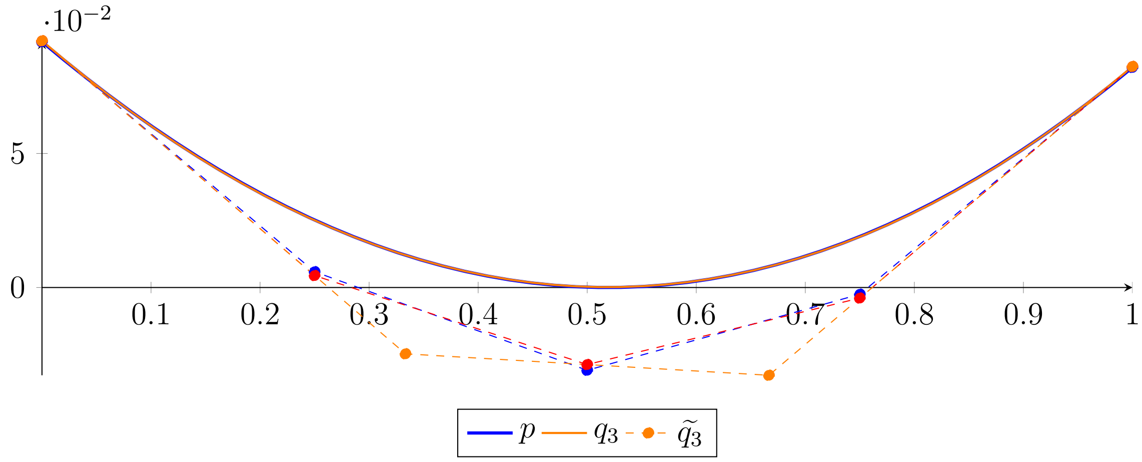
$$\begin{aligned} p = &-0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &+ 0.2941201876709534X^2 - 0.34309913433192X + 0.09165715738594469 \\ = &\ 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &- 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502 \cdot 10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455446609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.
Bounding polynomials M and m:

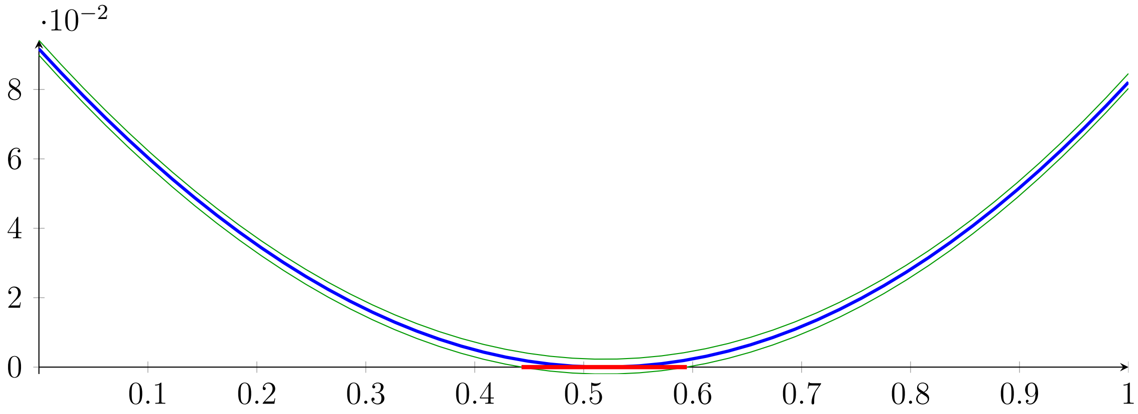
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



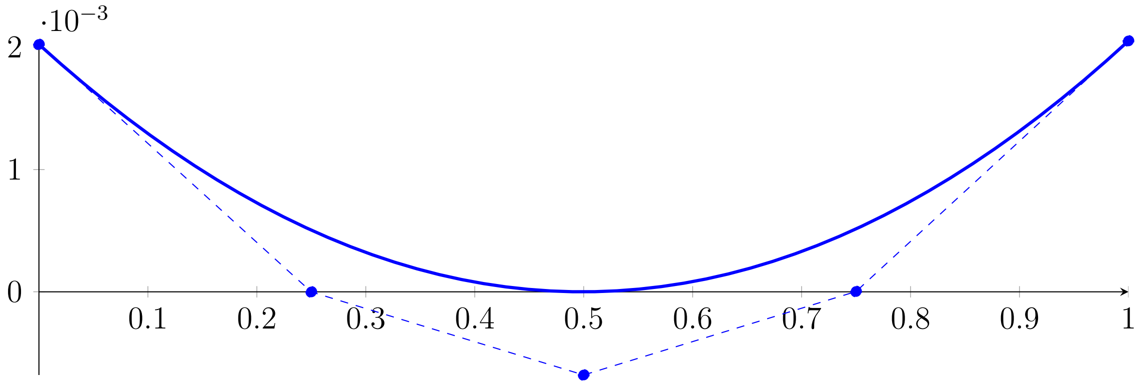
$[0.4429176870393937, 0.5947109255489168]$

Longest intersection interval: 0.1517932385095231
$\implies$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

33.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

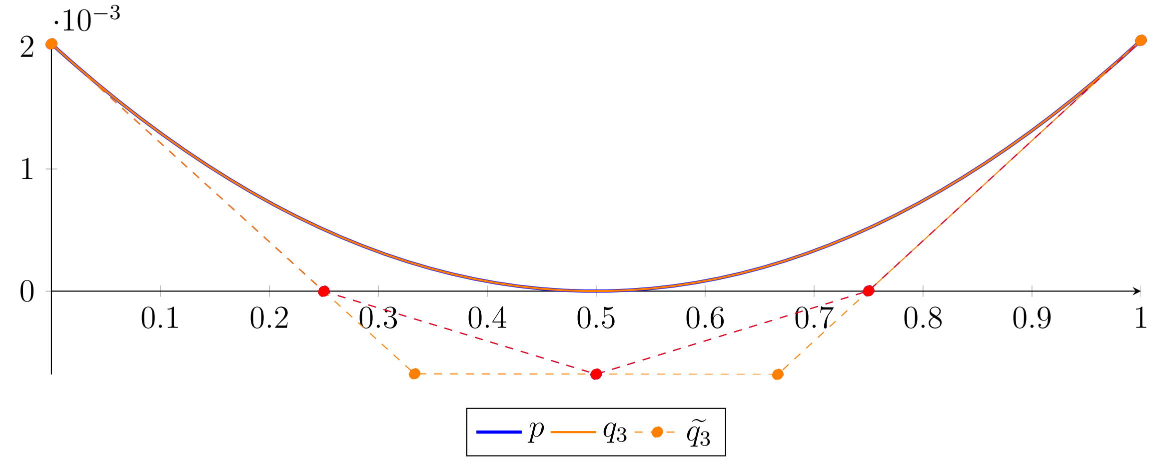
$$\begin{aligned}
p = &-1.325713168422468\cdot 10^{-05}X^4 + 7.039695732710913\cdot 10^{-05}X^3\\
&+ 0.008070351522992233X^2 - 0.008098671960928044X + 0.002023786815863734\\
= &\ 0.002023786815863734B_{0,4}(X) - 8.811743682771854\cdot 10^{-07}B_{1,4}(X)\\
&- 0.0006804905774349 16B_{2,4}(X) + 2.557845995594498\\
&\cdot 10^{-06}B_{3,4}(X) + 0.002052606203570807B_{4,4}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &\ 4.388269395865976\cdot 10^{-05}X^3 + 0.008087396406586236X^2\\
&- 0.008102459712837822X + 0.002023976203459223\\
= &\ 0.002023976203459223B_{0,3} - 0.0006768437008200514B_{1,3}\\
&- 0.0006818648029039136B_{2,3} + 0.002052795591166296B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 = &-6.518771069845317\cdot 10^{-311}X^4 + 4.388269395865976\cdot 10^{-05}X^3\\
&+ 0.008087396406586236X^2 - 0.008102459712837822X + 0.002023976203459223\\
= &\ 0.002023976203459223B_{0,4} - 1.638724750232882\cdot 10^{-06}B_{1,4} - 0.0006793542518619825B_{2,4}\\
&+ 1.800295613638801\cdot 10^{-06}B_{3,4} + 0.002052795591166296B_{4,4}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544 \cdot 10^{-06}$.
Bounding polynomials M and m:

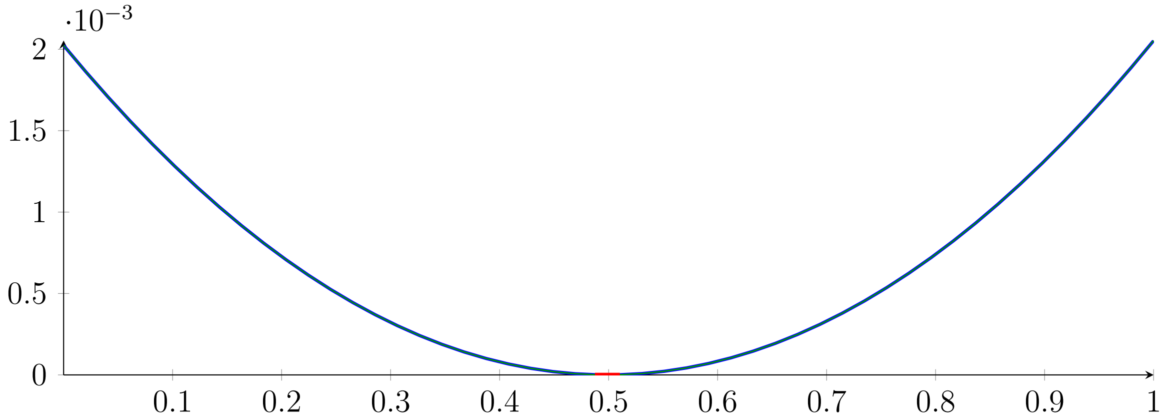
$$M = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002025112529032156$$

$$m = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002022839877886289$$

Root of M and m:

$N(M) = \{-185.2936165687423\}$ $N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649\}$

Intersection intervals:



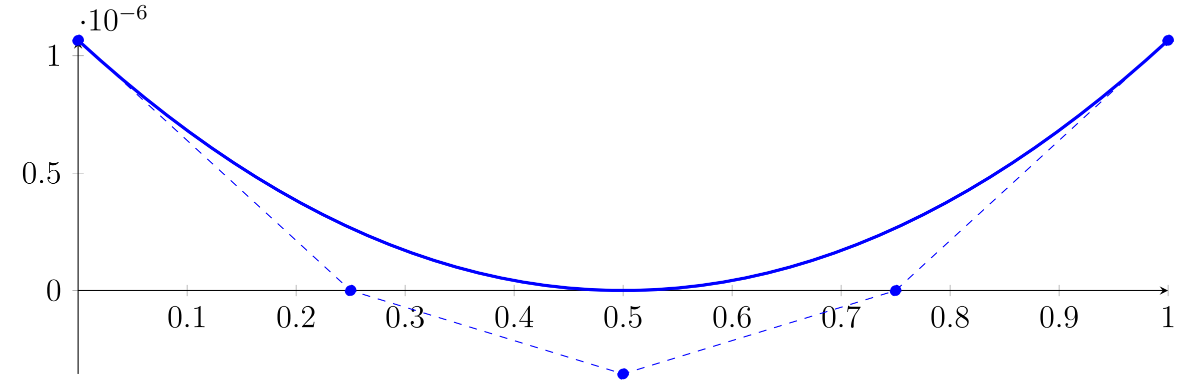
$$[0.4874742490566883, 0.5103358670775649]$$

Longest intersection interval: 0.02286161802087665
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

33.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.40068970704716$

Normalized monomial und Bézier representations and the Bézier polygon:

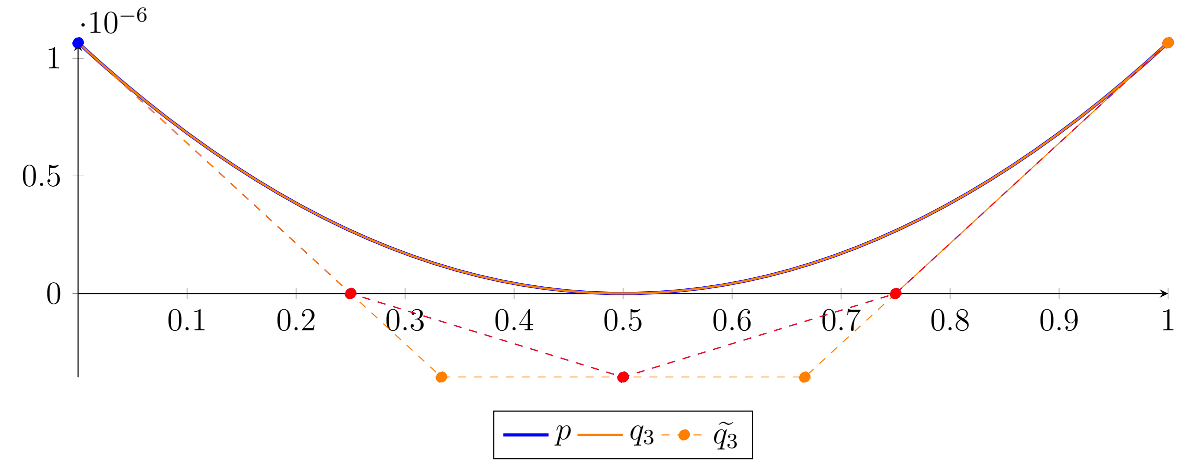
$$p = -3.621407750870066 \cdot 10^{-12} X^4 + 5.322780243378263 \cdot 10^{-10} X^3 + 4.261926231315171 \\ \cdot 10^{-06} X^2 - 4.262596936226815 \cdot 10^{-06} X + 1.065750606194374 \cdot 10^{-06} \\ = 1.065750606194374 \cdot 10^{-06} B_{0,4}(X) + 1.013721376702048 \cdot 10^{-10} B_{1,4}(X) - 3.552268233665051 \\ \cdot 10^{-07} B_{2,4}(X) - 1.009108120675939 \cdot 10^{-10} B_{3,4}(X) + 1.065608557899316 \cdot 10^{-06} B_{4,4}(X)$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06} \\ &= 1.06575065792877\cdot 10^{-06}B_{0,3} - 3.551153323761442\cdot 10^{-07}B_{1,3} \\ &\quad - 3.553376935441088\cdot 10^{-07}B_{2,3} + 1.065608609633713\cdot 10^{-06}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -5.304989477413181\cdot 10^{-314}X^4 + 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085 \\ &\quad \cdot 10^{-06}X^2 - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06} \\ &= 1.06575065792877\cdot 10^{-06}B_{0,4} + 1.011652000844408\cdot 10^{-10}B_{1,4} - 3.552265129601265 \\ &\quad \cdot 10^{-07}B_{2,4} - 1.011177496533579\cdot 10^{-10}B_{3,4} + 1.065608609633713\cdot 10^{-06}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.104063786460057\cdot 10^{-13}$.

Bounding polynomials M and m:

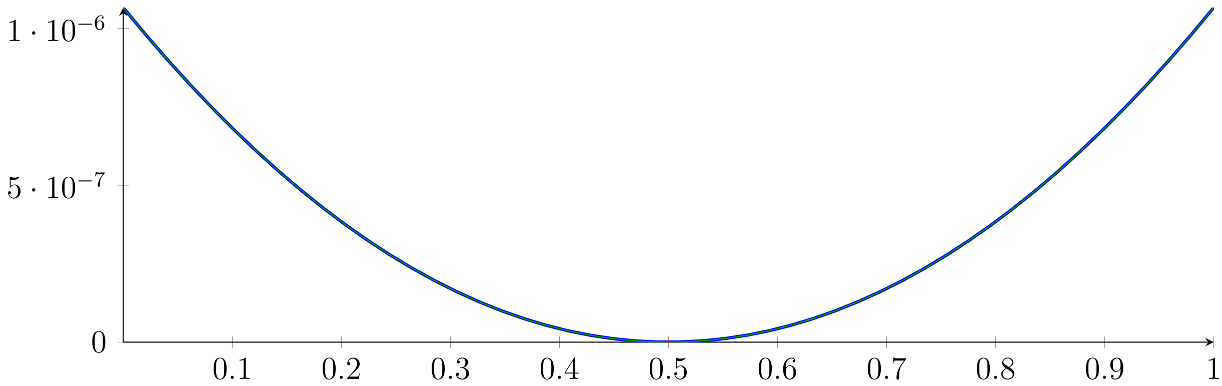
$$\begin{aligned} M &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750968335149\cdot 10^{-06} \end{aligned}$$

$$\begin{aligned} m &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750347522392\cdot 10^{-06} \end{aligned}$$

Root of M and m:

$N(M) = \{-8118.41926893212\}$ $N(m) = \{-8118.419268932102, 0.4997707518163204, 0.5002933638554905\}$

Intersection intervals:



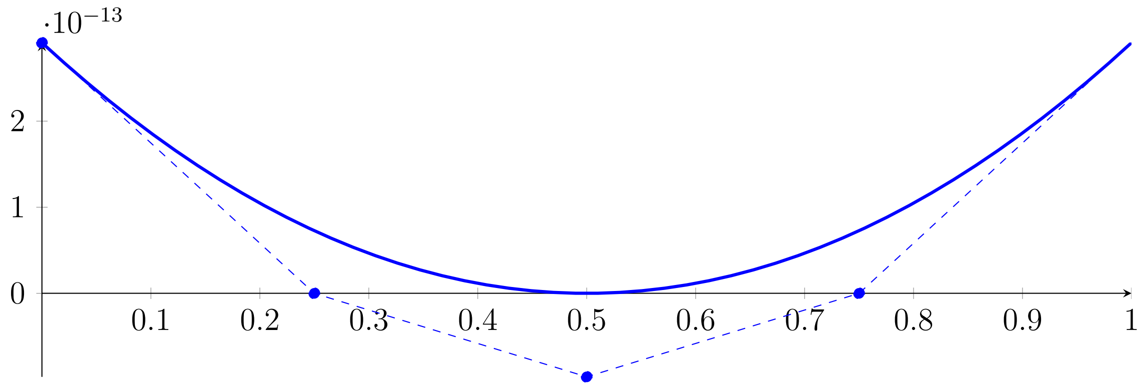
$[0.4997707518163204, 0.5002933638554905]$

Longest intersection interval: 0.0005226120391701048
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996445302967, 0.4000003654697068]$,

33.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999996445302967, 0.400000365469$

Normalized monomial und Bézier representations and the Bézier polygon:

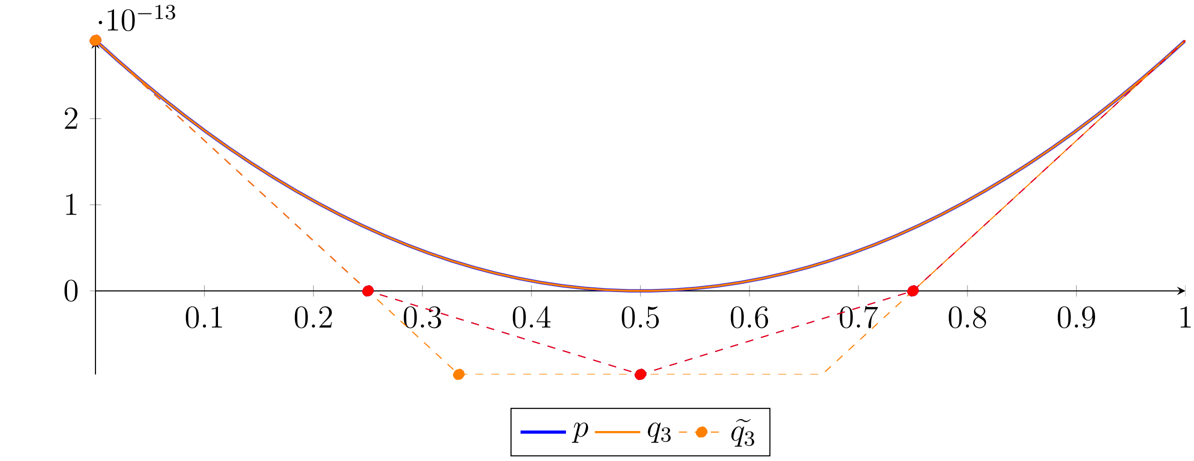
$$\begin{aligned} p = & -2.701438390310769\cdot 10^{-25}X^4 + 7.494271205548112\cdot 10^{-20}X^3 + 1.164248026057075 \\ & \cdot 10^{-12}X^2 - 1.164248076702198\cdot 10^{-12}X + 2.91006022469042\cdot 10^{-13} \\ = & \ 2.91006022469042\cdot 10^{-13}B_{0,4}(X) - 5.599670650739874\cdot 10^{-17}B_{1,4}(X) - 9.707667820587771 \\ & \cdot 10^{-14}B_{2,4}(X) - 5.600329339091629\cdot 10^{-17}B_{3,4}(X) + 2.910060467663608\cdot 10^{-13}B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = & \ 7.494217176780306\cdot 10^{-20}X^3 + 1.164248026057422\cdot 10^{-12}X^2 \\ & - 1.164248076702275\cdot 10^{-12}X + 2.910060224690459\cdot 10^{-13} \\ = & \ 2.910060224690459\cdot 10^{-13}B_{0,3} - 9.707666976504573\cdot 10^{-14}B_{1,3} \\ & - 9.707668664666337\cdot 10^{-14}B_{2,3} + 2.910060467663647\cdot 10^{-13}B_{3,3} \end{aligned}$$

$$\begin{aligned} \tilde{q_3} = & -1.011846442682873\cdot 10^{-320}X^4 + 7.494217176780306\cdot 10^{-20}X^3 + 1.164248026057422 \\ & \cdot 10^{-12}X^2 - 1.164248076702275\cdot 10^{-12}X + 2.910060224690459\cdot 10^{-13} \\ = & \ 2.910060224690459\cdot 10^{-13}B_{0,4} - 5.599670652283553\cdot 10^{-17}B_{1,4} - 9.707667820585455 \\ & \cdot 10^{-14}B_{2,4} - 5.600329340635308\cdot 10^{-17}B_{3,4} + 2.910060467663647\cdot 10^{-13}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.315518620266374 \cdot 10^{-26}$.
Bounding polynomials M and m:

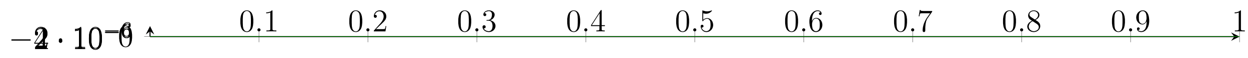
$$M = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ - 1.164248076702275 \cdot 10^{-12} X + 2.91006022469069 \cdot 10^{-13}$$

$$m = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690227 \cdot 10^{-13}$$

Root of M and m:

$N(M) = \{-15535286.38864162, 0.4930646005679288, 0.5069353931872718\}$ $\quad N(m) = \{-15535286.38864162,$

Intersection intervals:



$$[0.4930646005650611, 0.4930646005679288], \; [0.5069353931872718, 0.5069353931901395]$$

Longest intersection interval: $2.867685289107 05 \cdot 10^{-12}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.3999999999999999]$, interval 2: $[0.40000001, 0.400000$

33.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.39999999999$

Found root in interval $[0.3999999999999999, 0.3999999999999999]$ at recursion depth 6!

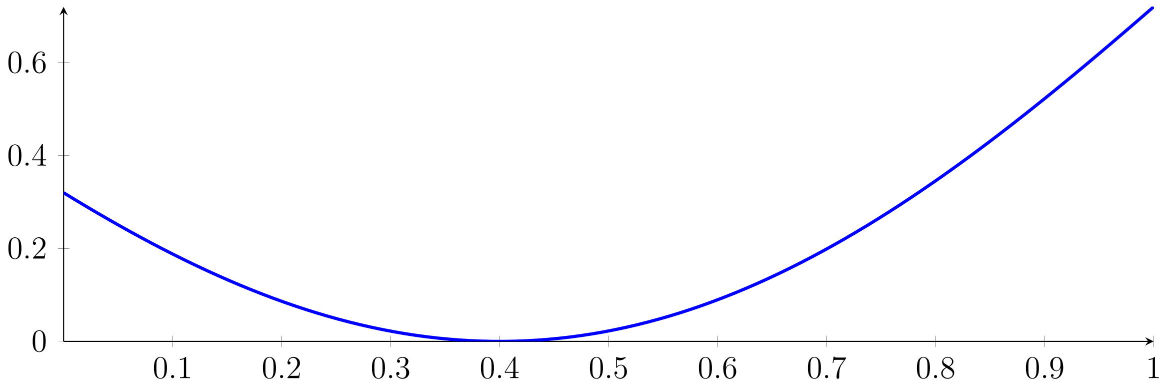
33.7 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 6!

33.8 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.399999999999999, 0.399999999999999], \; [0.40000001, 0.40000001]$$

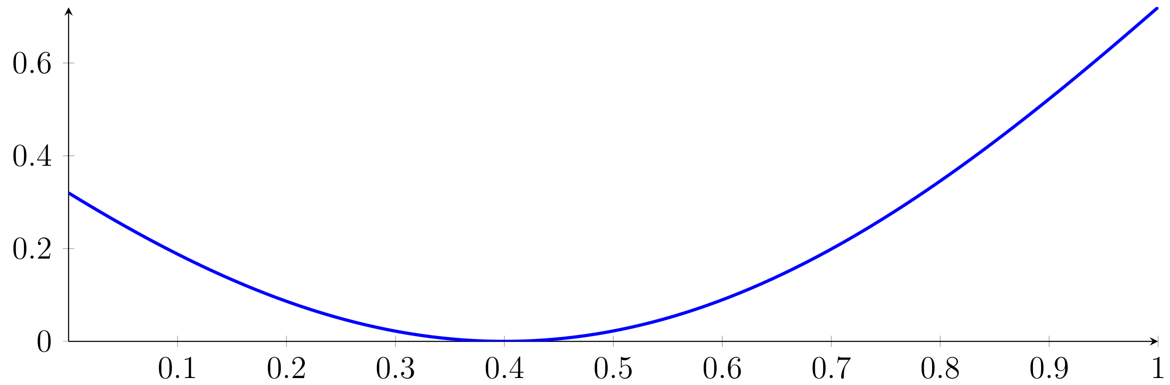
with precision $\varepsilon = 1 \cdot 10^{-16}$.

34 Running BezClip on h_4 with epsilon 32

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called BezClip with input polynomial on interval $[0, 1]$:

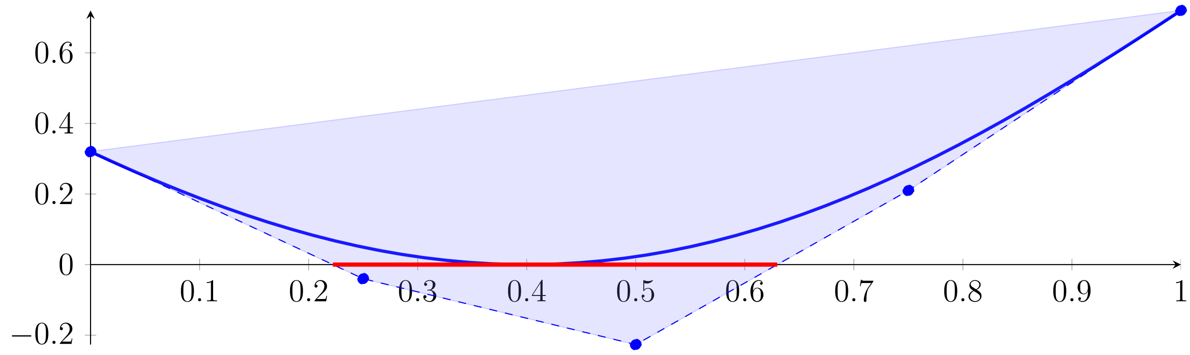
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



34.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

$$[0.222222225308642, 0.6297709955349339]$$

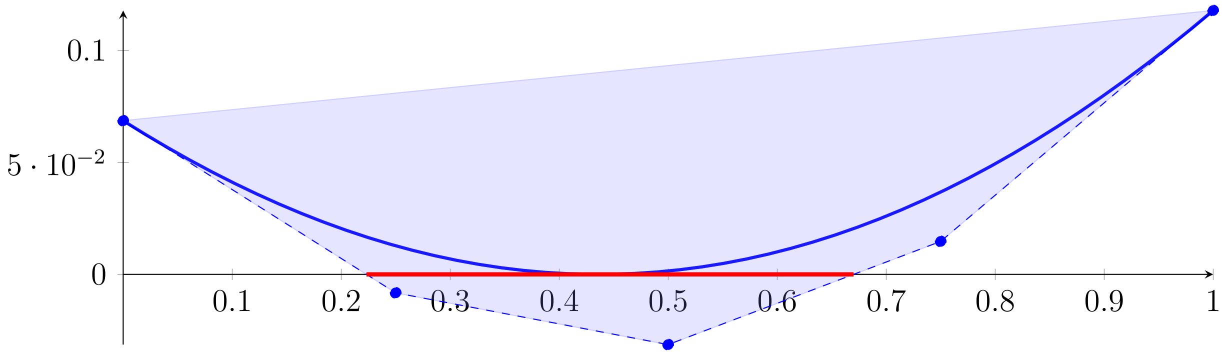
Longest intersection interval: 0.4075487702262920

$\implies$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

34.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &+ 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ = &\ 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &+ 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

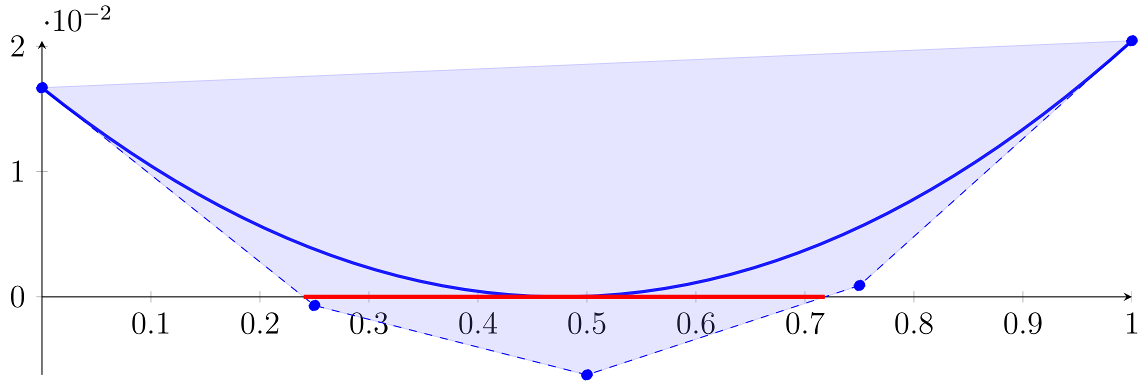
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\Longrightarrow$ Selective recursion: interval 1: $[0.3131782986367004, 0.4953216655933138]$,

34.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.495321665593313$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -0.0011006606529341822X^4 + 0.003307158935864367X^3 \\ & + 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377 \\ = & \ 0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X) \\ & + 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

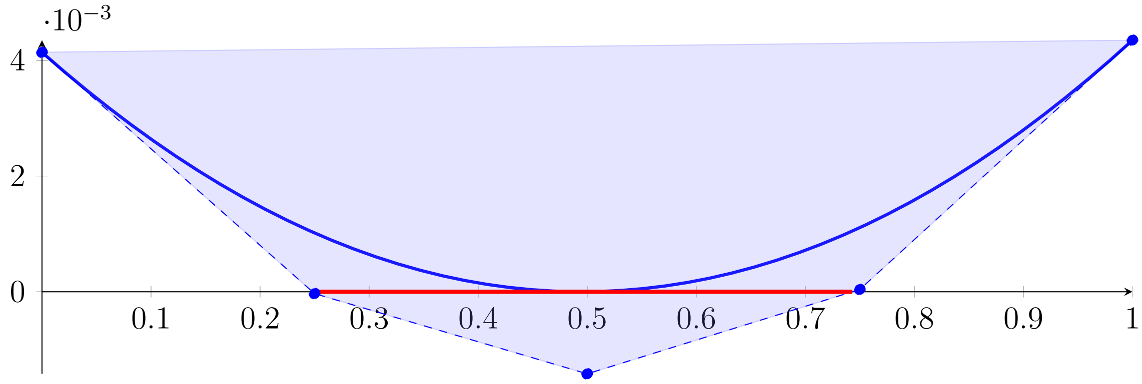
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\Longrightarrow$ Selective recursion: interval 1: $[0.3569093751201798, 0.4439937820951003]$,

34.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.44399378209510$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -5.751241374789061\cdot 10^{-05}X^4 + 0.0002459162030906773X^3 \\
&\quad + 0.01670691422385101X^2 - 0.01668640845758443X + 0.004139787469617577 \\
&= 0.004139787469617577B_{0,4}(X) - 3.181464477852956\cdot 10^{-05}B_{1,4}(X) \\
&\quad - 0.001418931055199467B_{2,4}(X) + 3.991728912743387 \\
&\quad \cdot 10^{-05}B_{3,4}(X) + 0.004348697025226952B_{4,4}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

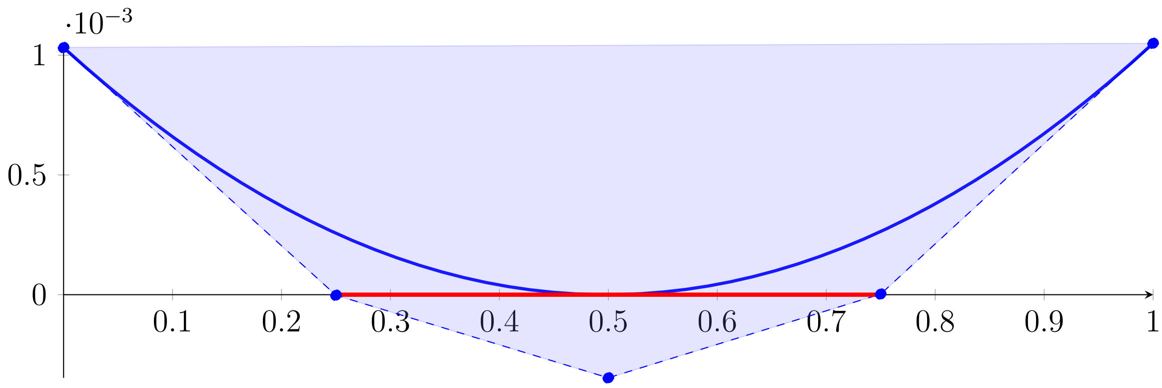
$$[0.2480933797192248, 0.7431594518918532]$$

Longest intersection interval: 0.4950660721726284
$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

34.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -3.454731120985874\cdot 10^{-06}X^4 + 2.291337271769576\cdot 10^{-05}X^3 \\
&\quad + 0.004134358001933594X^2 - 0.004136159738306538X + 0.001031853315293836 \\
&= 0.001031853315293836B_{0,4}(X) - 2.186619282798525\cdot 10^{-06}B_{1,4}(X) \\
&\quad - 0.0003471668868705005B_{2,4}(X) + 2.640855710153786 \\
&\quad \cdot 10^{-06}B_{3,4}(X) + 0.001049510220517602B_{4,4}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

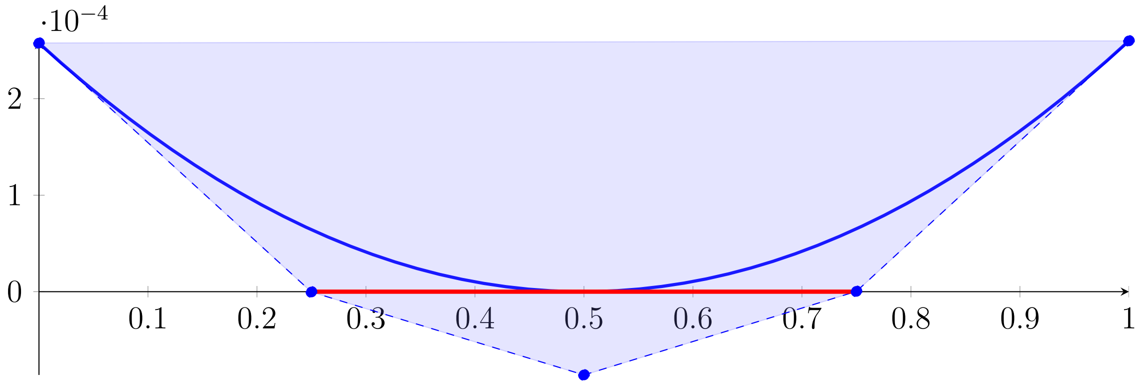
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\Longrightarrow$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

34.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.41076747247$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.135832675829806\cdot 10^{-07}X^4 + 2.413460912088167\cdot 10^{-06}X^3\\ &+ 0.001031922910124016X^2 - 0.001031832710654165X + 0.0002576480709401398\\ = &\ 0.0002576480709401398B_{0,4}(X) - 3.101067234014613\cdot 10^{-07}B_{1,4}(X) - 8.628113269960673\\ &\cdot 10^{-05}B_{2,4}(X) + 3.383582395460159\cdot 10^{-07}B_{3,4}(X) + 0.0002599381480544958B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

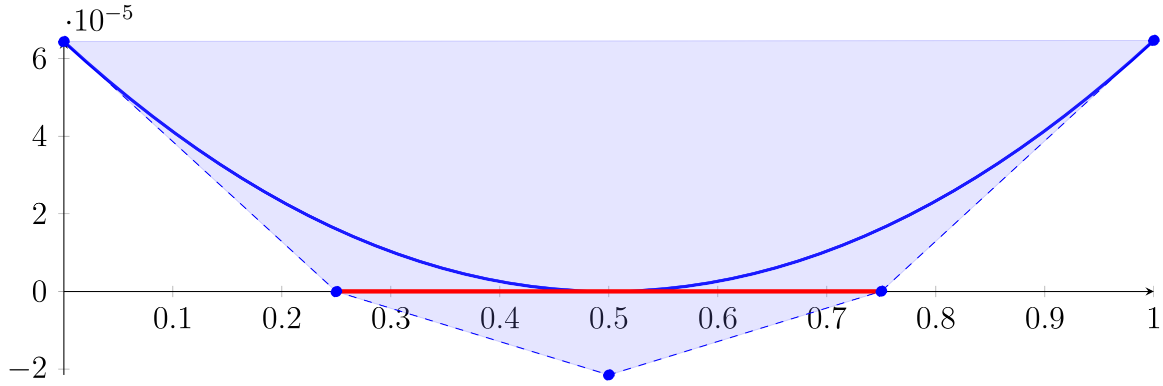
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\Longrightarrow$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

34.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.405372055$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.327690666746824\cdot 10^{-08}X^4 + 2.739027984571277\cdot 10^{-07}X^3\\ &+ 0.0002577144304079\,74X^2 - 0.0002576778286693267X + 6.437695235811399\cdot 10^{-05}\\ = &\ 6.437695235811399\cdot 10^{-05}B_{0,4}(X) - 4.250480921768141\cdot 10^{-08}B_{1,4}(X) - 2.150955690855369\\ &\cdot 10^{-05}B_{2,4}(X) + 4.427175972025921\cdot 10^{-08}B_{3,4}(X) + 6.467417998855097\cdot 10^{-05}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

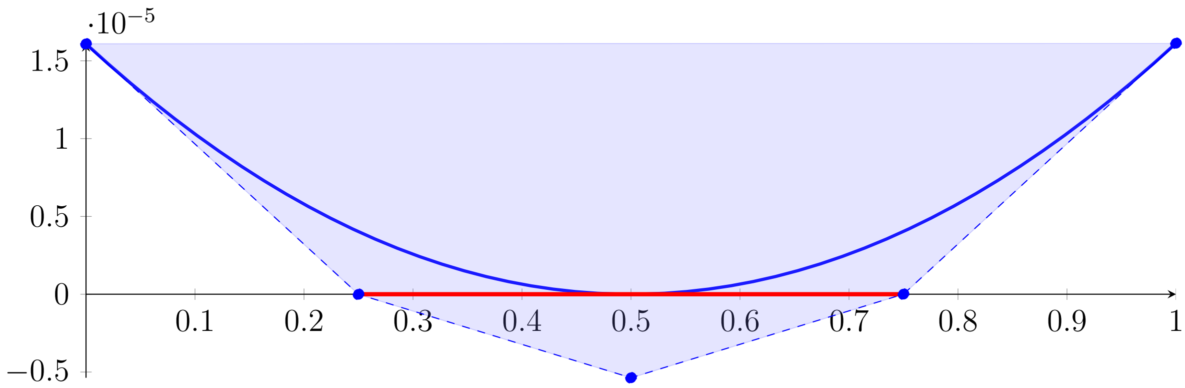
$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\Longrightarrow$ Selective recursion: interval 1: $[0.3973195510833879, 0.4026829657906133]$,

34.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -8.274952589981882 \cdot 10^{-10} X^4 + 3.251124603913595 \cdot 10^{-08} X^3 + 6.438882283687421 \\ &\quad \cdot 10^{-05} X^2 - 6.438267481175771 \cdot 10^{-05} X + 1.609012302857716 \cdot 10^{-05} \\ &= 1.609012302857716 \cdot 10^{-05} B_{0,4}(X) - 5.545674362270907 \cdot 10^{-09} B_{1,4}(X) - 5.369743904489329 \\ &\quad \cdot 10^{-06} B_{2,4}(X) + 5.656149705765249 \cdot 10^{-09} B_{3,4}(X) + 1.61279548044738 \cdot 10^{-05} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499138638713212, 0.7497369428484978\}$$

Intersection intervals with the x axis:

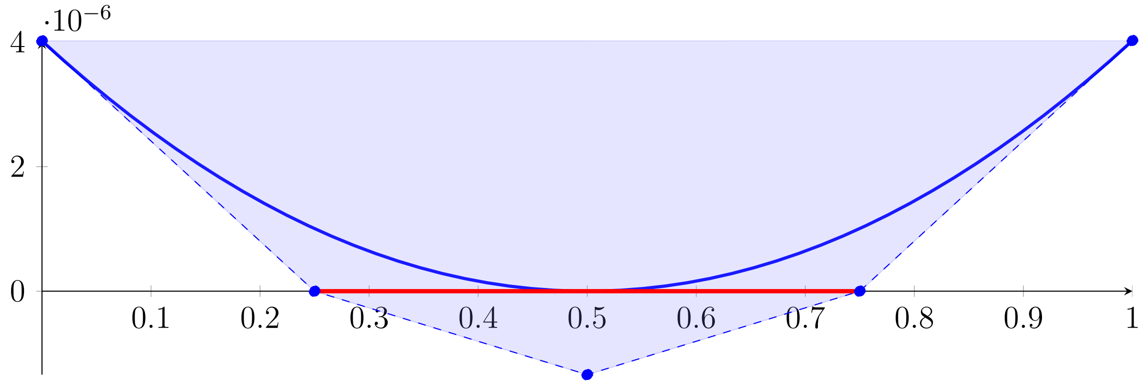
$$[0.2499138638713212, 0.7497369428484978]$$

Longest intersection interval: 0.4998230789771766
$\Longrightarrow$ Selective recursion: interval 1: $[0.3986599427764149, 0.4013407012292117]$,

34.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3986599427764149, 0.40134$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -5.164529187662443 \cdot 10^{-11} X^4 + 3.956301794548619 \cdot 10^{-09} X^3 + 1.609182796552748 \cdot 10^{-05} X^2 - 1.609096222935268 \cdot 10^{-05} X + 4.022033038444256 \cdot 10^{-06}$$
$$= 4.022033038444256 \cdot 10^{-06} B_{0,4}(X) - 7.075188939141538 \cdot 10^{-10} B_{1,4}(X) - 1.341476748644171 \cdot 10^{-06} B_{2,4}(X) + 7.14424642122371 \cdot 10^{-10} B_{3,4}(X) + 4.026803431121726 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499560300444542, 0.7498669294180401\}$$

Intersection intervals with the x axis:

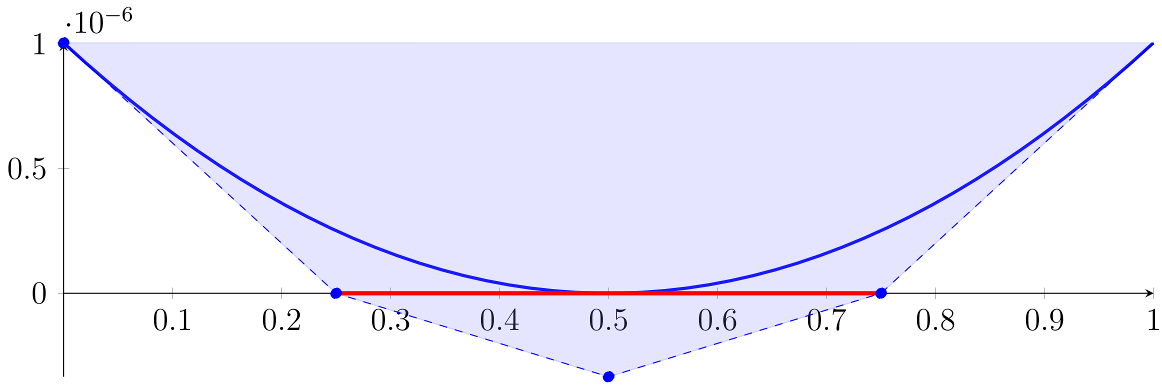
$$[0.2499560300444542, 0.7498669294180401]$$

Longest intersection interval: 0.499910899373586
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993300145167841, 0.4006701548859251]$,

34.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3993300145167841, 0.400$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.225530543299082 \cdot 10^{-12} X^4 + 4.878223136378425 \cdot 10^{-10} X^3 + 4.022259900669466 \cdot 10^{-06} X^2 - 4.022145641341714 \cdot 10^{-06} X + 1.00544708348311 \cdot 10^{-06}$$
$$= 1.00544708348311 \cdot 10^{-06} B_{0,4}(X) - 8.932685231834447 \cdot 10^{-11} B_{1,4}(X) - 3.352490870761692 \cdot 10^{-07} B_{2,4}(X) + 8.975838996701131 \cdot 10^{-11} B_{3,4}(X) + 1.006045939593956 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499777912437083, 0.7499330838112102\}$$

Intersection intervals with the x axis:

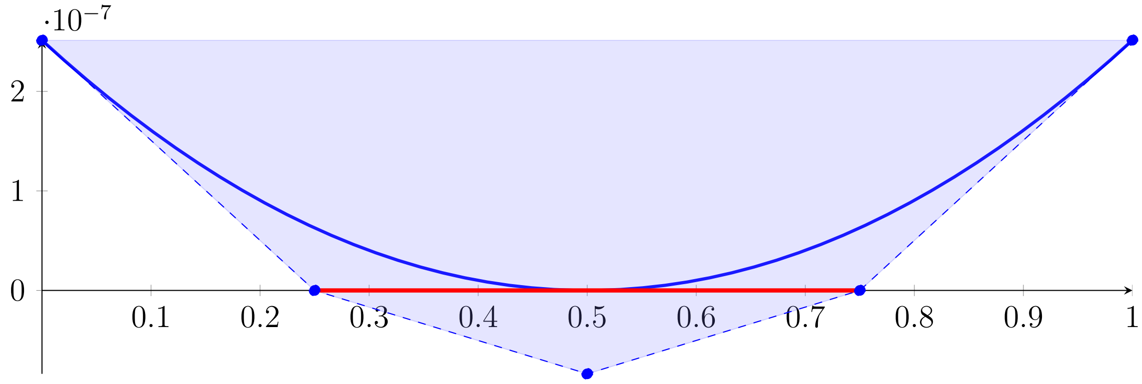
$$[0.2499777912437083, 0.7499330838112102]$$

Longest intersection interval: 0.499955292567502
$\Longrightarrow$ Selective recursion: interval 1: $[0.3996650198462185, 0.4003350301165539]$,

34.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3996650198462185, 0.4$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.015235660316643\cdot 10^{-13}X^4 + 6.055838633908679\cdot 10^{-11}X^3 + 1.005476298211762 \cdot 10^{-06}X^2 - 1.005461639191386\cdot 10^{-06}X + 2.51354188678016\cdot 10^{-07}$$
$$= 2.51354188678016\cdot 10^{-07}B_{0,4}(X) - 1.122111983053638\cdot 10^{-11}B_{1,4}(X) - 8.379724788238336 \cdot 10^{-08}B_{2,4}(X) + 1.124798694225224\cdot 10^{-11}B_{3,4}(X) + 2.51429204561165\cdot 10^{-07}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499888398329751, 0.7499664473546936\}$$

Intersection intervals with the x axis:

$$[0.2499888398329751, 0.7499664473546936]$$

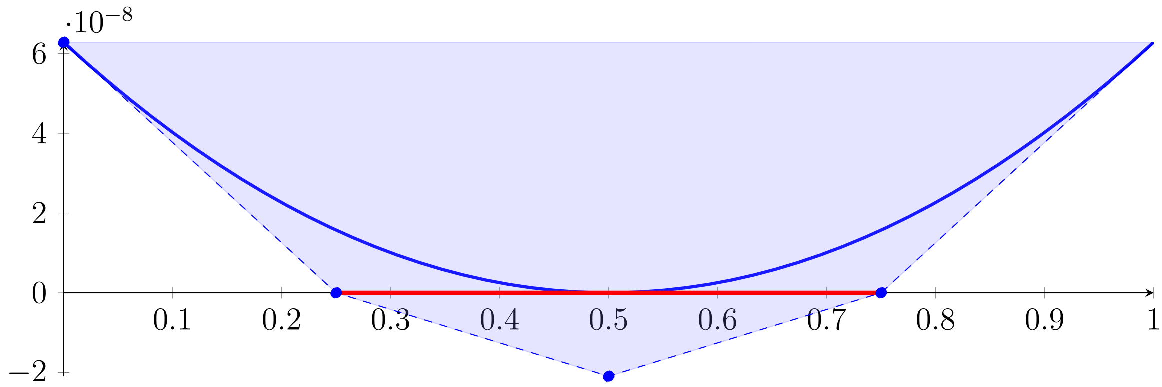
Longest intersection interval: 0.4999776075217186

$\Longrightarrow$ Selective recursion: interval 1: $[0.3998325149363758, 0.4001675050683531]$,

34.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3998325149363758, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.259296672250957\cdot 10^{-14}X^4 + 7.543595361590864\cdot 10^{-12}X^3 + 2.5135789423507 \cdot 10^{-07}X^2 - 2.51356038307676\cdot 10^{-07}X + 6.283760343276371\cdot 10^{-08}$$
$$= 6.283760343276371\cdot 10^{-08}B_{0,4}(X) - 1.406144155297956\cdot 10^{-12}B_{1,4}(X) - 2.094743334856264 \cdot 10^{-08}B_{2,4}(X) + 1.407718382090997\cdot 10^{-12}B_{3,4}(X) + 6.284699036255256\cdot 10^{-08}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499944057673539, 0.7499832005219574\}$$

Intersection intervals with the x axis:

$$[0.2499944057673539, 0.7499832005219574]$$

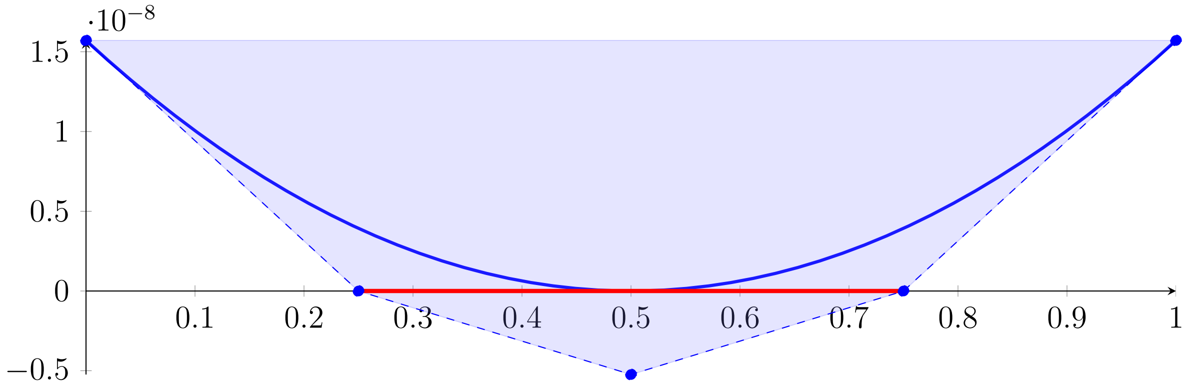
Longest intersection interval: 0.4999887947546035

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999162605953574, 0.4000837519076994]$,

34.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999162605953574

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -7.869898688873272 \cdot 10^{-16} X^4 + 9.413120459510821 \cdot 10^{-13} X^3 + 6.283807021204131 \cdot 10^{-08} X^2 - 6.283783670437638 \cdot 10^{-08} X + 1.570928313186755 \cdot 10^{-08}$$
$$= 1.570928313186755 \cdot 10^{-08} B_{0,4}(X) - 1.760442265467946 \cdot 10^{-13} B_{1,4}(X) - 5.236623518313756 \cdot 10^{-09} B_{2,4}(X) + 1.76037617409066 \cdot 10^{-13} B_{3,4}(X) + 1.571045716458857 \cdot 10^{-08} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499971984359141, 0.7499915961258623\}$$

Intersection intervals with the x axis:

$$[0.2499971984359141, 0.7499915961258623]$$

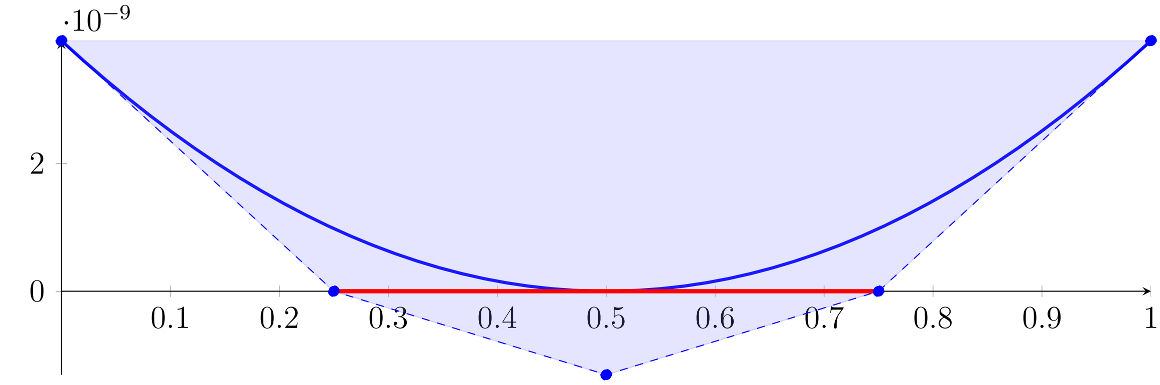
Longest intersection interval: 0.4999943976899482

$\Longrightarrow$ Selective recursion: interval 1: [0.3999581329542053, 0.400041877672038],

34.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999581329542053, 0.400041877672038]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -4.918466236188167 \cdot 10^{-17} X^4 + 1.175616813221855 \cdot 10^{-13} X^3 + 1.570934193292886 \cdot 10^{-08} X^2 - 1.57093126037727 \cdot 10^{-08} X + 3.927306070737634 \cdot 10^{-09}$$
$$= 3.927306070737634 \cdot 10^{-09} B_{0,4}(X) - 2.208020554223638 \cdot 10^{-14} B_{1,4}(X) - 1.309126575660575 \cdot 10^{-09} B_{2,4}(X) + 2.19747928673869 \cdot 10^{-14} B_{3,4}(X) + 3.927452912390452 \cdot 10^{-09} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.249998594451196, 0.749995803609747\}$$

Intersection intervals with the x axis:

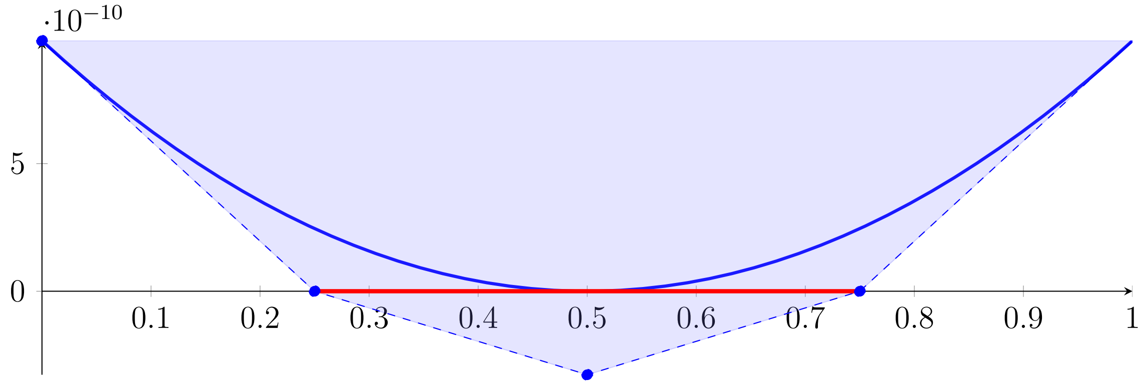
$$[0.249998594451196, 0.749995803609747]$$

Longest intersection interval: 0.499997209158551
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999790690159562, 0.4000209411411543]$,

34.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999790690159562, 0.4000209411411543]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -3.073972764895049 \cdot 10^{-18} X^4 + 1.468881614935617 \cdot 10^{-14} X^3 + 3.92731367890626 \\ &\quad \cdot 10^{-09} X^2 - 3.927309958033017 \cdot 10^{-09} X + 9.818246673938711 \cdot 10^{-10} \\ &= 9.818246673938711 \cdot 10^{-10} B_{0,4}(X) - 2.822114383159451 \cdot 10^{-15} B_{1,4}(X) - 3.272780318049275 \\ &\quad \cdot 10^{-10} B_{2,4}(X) + 2.710526275559223 \cdot 10^{-15} B_{3,4}(X) + 9.818430740092905 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499992814128721, 0.7499979295098023\}$$

Intersection intervals with the x axis:

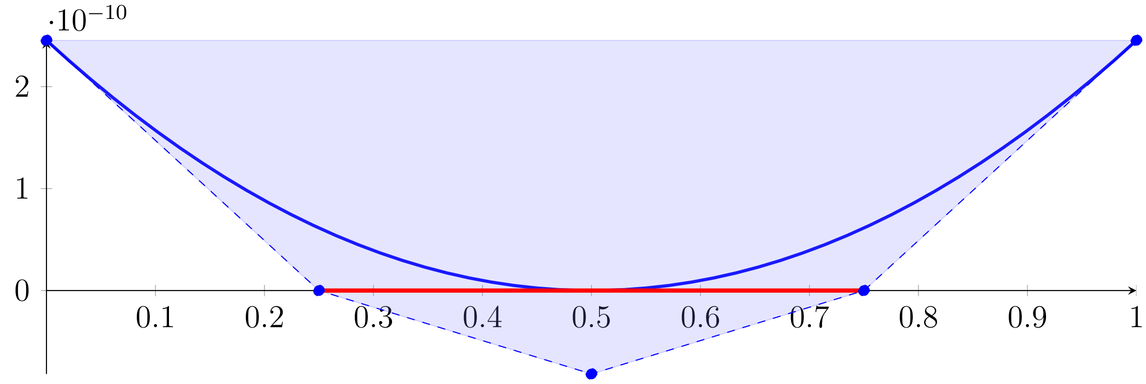
$$[0.2499992814128721, 0.7499979295098023]$$

Longest intersection interval: 0.4999986480969302
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999895370171669, 0.400010473023159]$,

34.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999895370171669, 0.400010473023159]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.921212199577591 \cdot 10^{-19} X^4 + 1.835702882943672 \cdot 10^{-15} X^3 + 9.818258642283608 \\ &\quad \cdot 10^{-10} X^2 - 9.818253497618774 \cdot 10^{-10} X + 2.454559233739064 \cdot 10^{-10} \\ &= 2.454559233739064 \cdot 10^{-10} B_{0,4}(X) - 4.140665629492134 \cdot 10^{-16} B_{1,4}(X) - 8.181910746897216 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.02092399485025 \cdot 10^{-16} B_{3,4}(X) + 2.454582733511515 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995782686167, 0.7499990769537415\}$$

Intersection intervals with the x axis:

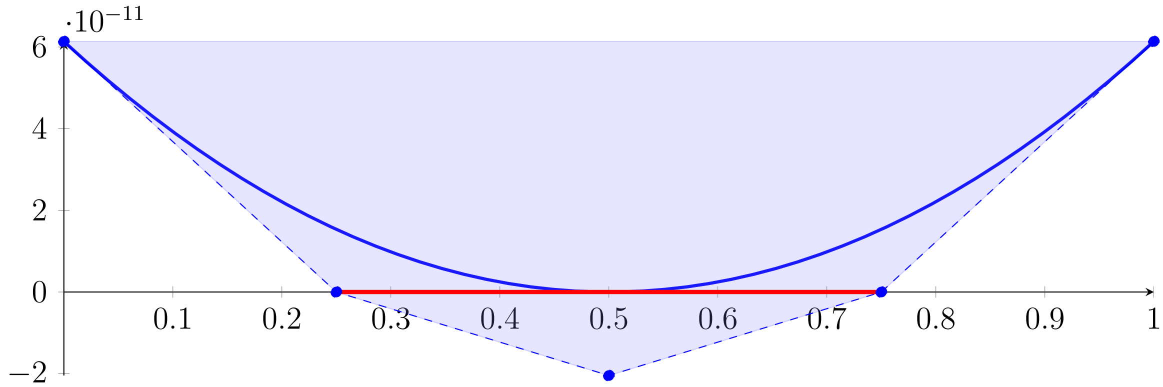
$$[0.2499995782686167, 0.7499990769537415]$$

Longest intersection interval: 0.4999994986851248
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999947710098356, 0.400005239002336]$,

34.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999947710098356, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.200752809081967 \cdot 10^{-20} X^4 + 2.294381551300316 \cdot 10^{-16} X^3 + 2.454563180284348 \\ &\quad \cdot 10^{-10} X^2 - 2.454562046981275 \cdot 10^{-10} X + 6.136393816301913 \cdot 10^{-11} \\ &= 6.136393816301913 \cdot 10^{-11} B_{0,4}(X) - 1.130115127449974 \cdot 10^{-16} B_{1,4}(X) - 2.045477784797216 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 1.013179678980307 \cdot 10^{-18} B_{3,4}(X) + 6.136428091947402 \cdot 10^{-11} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995395858382, 0.7499999876168341\}$$

Intersection intervals with the x axis:

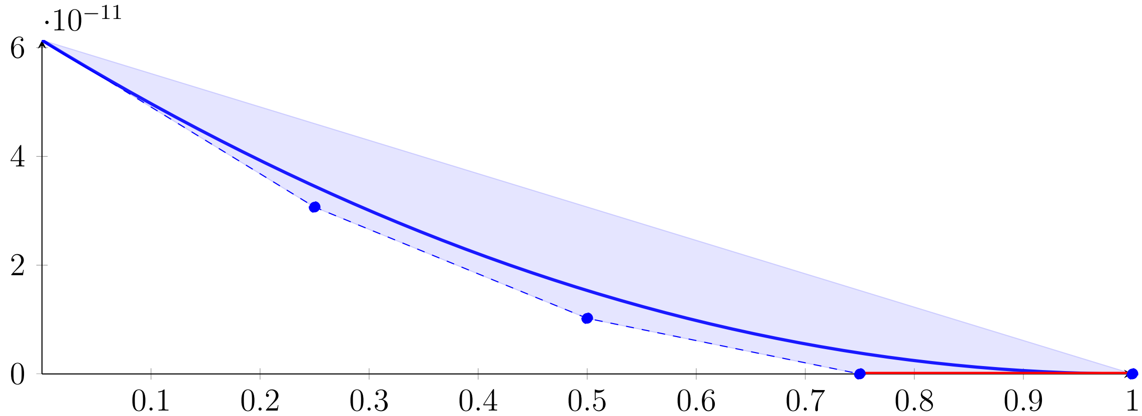
$$[0.2499995395858382, 0.7499999876168341]$$

Longest intersection interval: 0.5000004480309959
$\Longrightarrow$ Bisection: first half $[0.3999947710098356, 0.4000000050060858]$ und second half $[0.4000000050060858, 0.40$

34.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3999947710098356, 0.4000000050060858]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -7.504705056762291 \cdot 10^{-22} X^4 + 2.867976939125394 \cdot 10^{-17} X^3 + 6.13640795071087 \cdot 10^{-11} X^2 - 1.227281023490638 \cdot 10^{-10} X + 6.136393816301913 \cdot 10^{-11}$$
$$= 6.136393816301913 \cdot 10^{-11} B_{0,4}(X) + 3.068191257575319 \cdot 10^{-11} B_{1,4}(X) + 1.022723357300537 \cdot 10^{-11} B_{2,4}(X) - 9.167528198681642 \cdot 10^{-17} B_{3,4}(X) - 5.59999170035142 \cdot 10^{-17} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.7499977590601706, 0.9999990874140811\}$$

Intersection intervals with the x axis:

$$[0.7499977590601706, 0.9999990874140811]$$

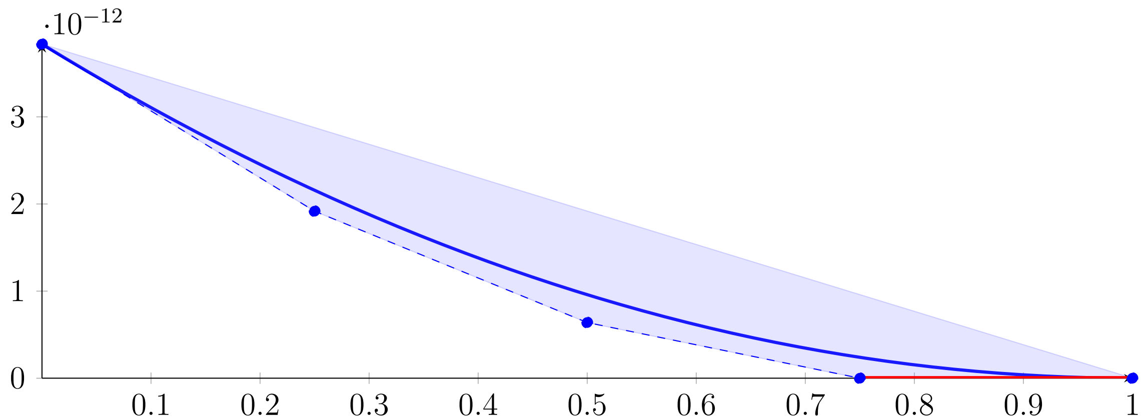
Longest intersection interval: 0.2500013283539104

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999986964952942, 0.4000000050013093]$,

34.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999986964952942, 0.4000000050013093]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.931587718946285 \cdot 10^{-24} X^4 + 4.480933611814181 \cdot 10^{-19} X^3 + 3.835299758875134 \cdot 10^{-12} X^2 - 7.670593186649561 \cdot 10^{-12} X + 3.835236979687871 \cdot 10^{-12}$$
$$= 3.835236979687871 \cdot 10^{-12} B_{0,4}(X) + 1.917588683025481 \cdot 10^{-12} B_{1,4}(X) + 6.391570128422797 \cdot 10^{-13} B_{2,4}(X) - 5.791883839216654 \cdot 10^{-17} B_{3,4}(X) - 5.59999961259787 \cdot 10^{-17} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.7499773476668325, 0.9999853987696839\}$$

Intersection intervals with the x axis:

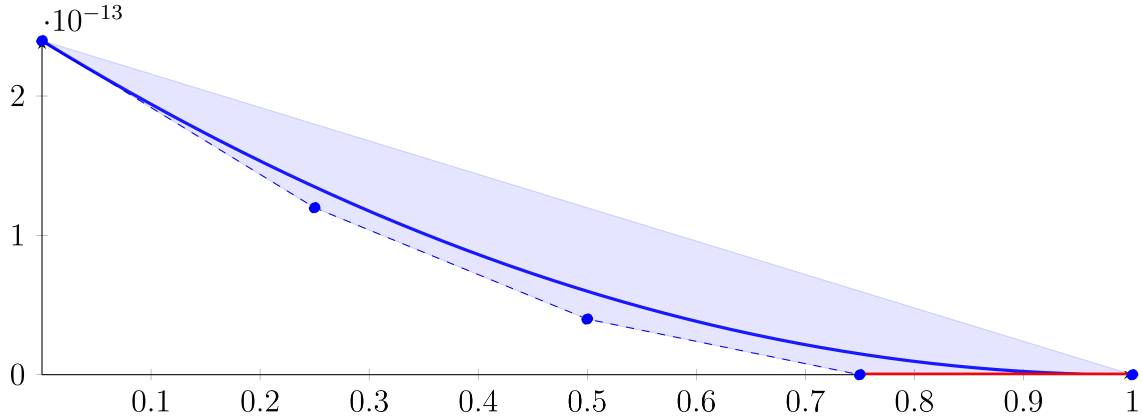
$$[0.7499773476668325, 0.9999853987696839]$$

Longest intersection interval: 0.2500080511028514
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996778451648, 0.4000000049822035]$,

34.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999996778451648, 0.4000000049822035]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.14529897555355\cdot 10^{-26}X^4 + 7.001997796519582\cdot 10^{-21}X^3 + 2.397217373893792\\ &\quad\cdot 10^{-13}X^2 - 4.794695778397391\cdot 10^{-13}X + 2.396918341578483\cdot 10^{-13}\\ &= 2.396918341578483\cdot 10^{-13}B_{0,4}(X) + 1.198244396979135\cdot 10^{-13}B_{1,4}(X) + 3.991066813620863\\ &\quad\cdot 10^{-14}B_{2,4}(X) - 4.947877676695723\cdot 10^{-17}B_{3,4}(X) - 5.599929052523716\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7496904492313635, 0.9997664242061345\}$$

Intersection intervals with the x axis:

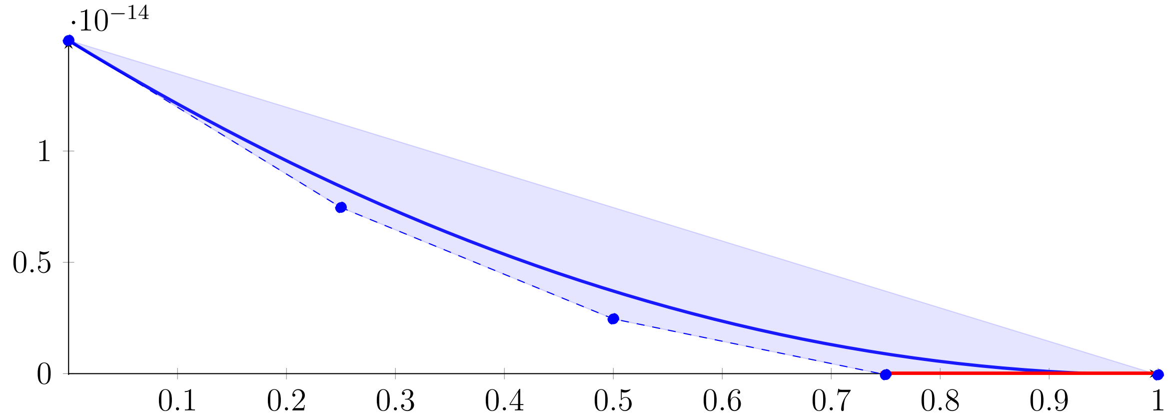
$$[0.7496904492313635, 0.9997664242061345]$$

Longest intersection interval: 0.250075974974771
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999230966783, 0.4000000049057922]$,

34.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999230966783, 0.4000000049057922]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.479264981640815\cdot 10^{-29}X^4 + 1.095054543732688\cdot 10^{-22}X^3 + 1.499171738187779\\ &\quad\cdot 10^{-14}X^2 - 3.001796269578916\cdot 10^{-14}X + 1.497026508467358\cdot 10^{-14}\\ &= 1.497026508467358\cdot 10^{-14}B_{0,4}(X) + 7.465774410726289\cdot 10^{-15}B_{1,4}(X) + 2.459903300425297\\ &\quad\cdot 10^{-15}B_{2,4}(X) - 4.734821885303232\cdot 10^{-17}B_{3,4}(X) - 5.598011973238056\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7452788722542423, 0.9962745104335203\}$$

Intersection intervals with the x axis:

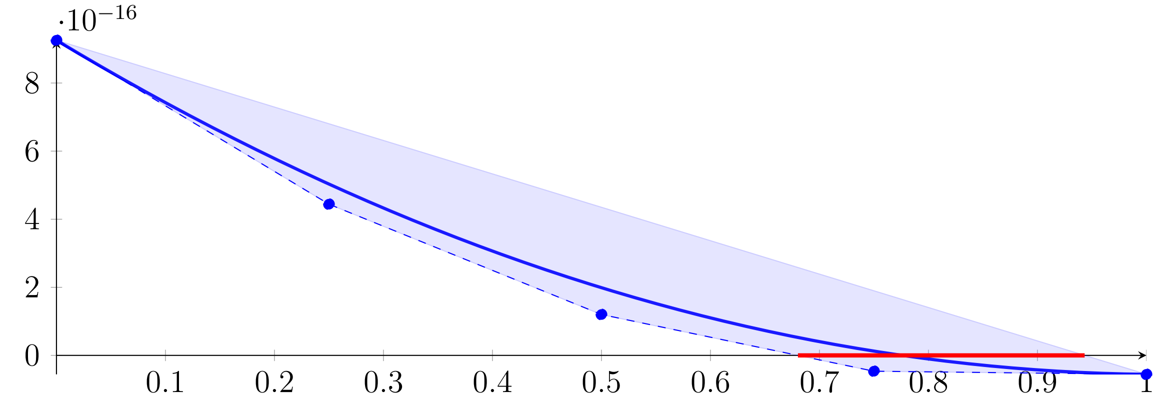
$$[0.7452788722542423, 0.9962745104335203]$$

Longest intersection interval: 0.250995638179278
$\Longrightarrow$ Selective recursion: interval 1: [0.3999999840672825, 0.4000000046010132],

34.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999999840672825, 0.4000000046010132]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.777753131478217 \cdot 10^{-31} X^4 + 1.731544849986219 \cdot 10^{-24} X^3 + 9.444603761111594 \\ &\quad \cdot 10^{-16} X^2 - 1.925623994999771 \cdot 10^{-15} X + 9.25520203786835 \cdot 10^{-16} \\ &= 9.25520203786835 \cdot 10^{-16} B_{0,4}(X) + 4.441142050368923 \cdot 10^{-16} B_{1,4}(X) + 1.201182689721428 \\ &\quad \cdot 10^{-16} B_{2,4}(X) - 4.646760397452725 \cdot 10^{-17} B_{3,4}(X) - 5.564341337023184 \cdot 10^{-17} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.6802647890355577, 0.9432883441688765\}$$

Intersection intervals with the x axis:

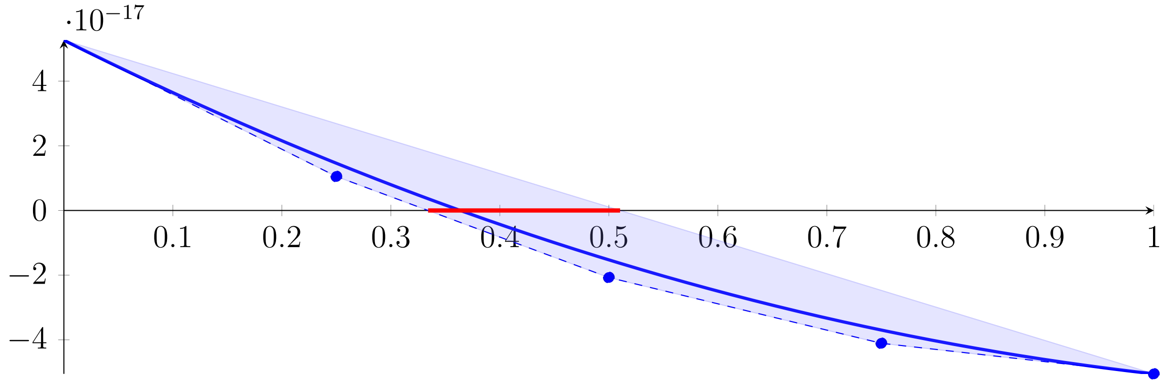
$$[0.6802647890355577, 0.9432883441688765]$$

Longest intersection interval: 0.2630235551333187
$\Longrightarrow$ Selective recursion: interval 1: [0.3999999980356565, 0.4000000034365114],

34.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999980356565, 0.4000000034365114]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.508441684091975\cdot 10^{-34}X^4 + 3.150776186268732\cdot 10^{-26}X^3 + 6.533908238790805 \cdot 10^{-17}X^2 - 1.685080699755733\cdot 10^{-16}X + 5.264466028744181\cdot 10^{-17}$$
$$= 5.264466028744181\cdot 10^{-17}B_{0,4}(X) + 1.05176427935485\cdot 10^{-17}B_{1,4}(X) - 2.071952763569348 \cdot 10^{-17}B_{2,4}(X) - 4.106685099240717\cdot 10^{-17}B_{3,4}(X) - 5.052432726871564\cdot 10^{-17}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3341757003677151, 0.5102760193201083\}$$

Intersection intervals with the x axis:

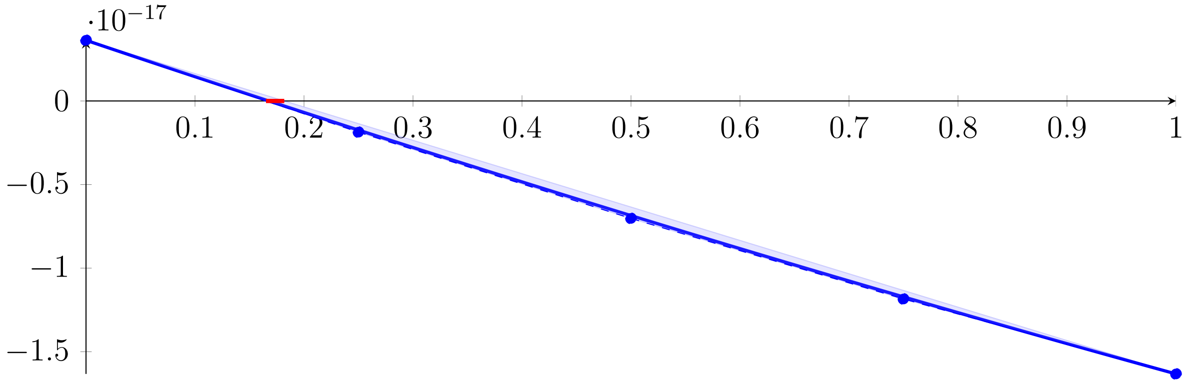
$$[0.3341757003677151, 0.5102760193201083]$$

Longest intersection interval: 0.1761003189523932
$\implies$ Selective recursion: interval 1: $[0.399999999840491, 0.4000000007915832]$,

34.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.399999999840491, 0.4000000007915832]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.182586345704061\cdot 10^{-37}X^4 + 1.720671503885177\cdot 10^{-28}X^3 + 2.026251345992899 \cdot 10^{-18}X^2 - 2.198411775802323\cdot 10^{-17}X + 3.629995386177444\cdot 10^{-18}$$
$$= 3.629995386177444\cdot 10^{-18}B_{0,4}(X) - 1.866034053328363\cdot 10^{-18}B_{1,4}(X) - 7.024354935168688 \cdot 10^{-18}B_{2,4}(X) - 1.184496725930051\cdot 10^{-17}B_{3,4}(X) - 1.632787102568082\cdot 10^{-17}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.1651189929990553, 0.1818829383495937\}$$

Intersection intervals with the x axis:

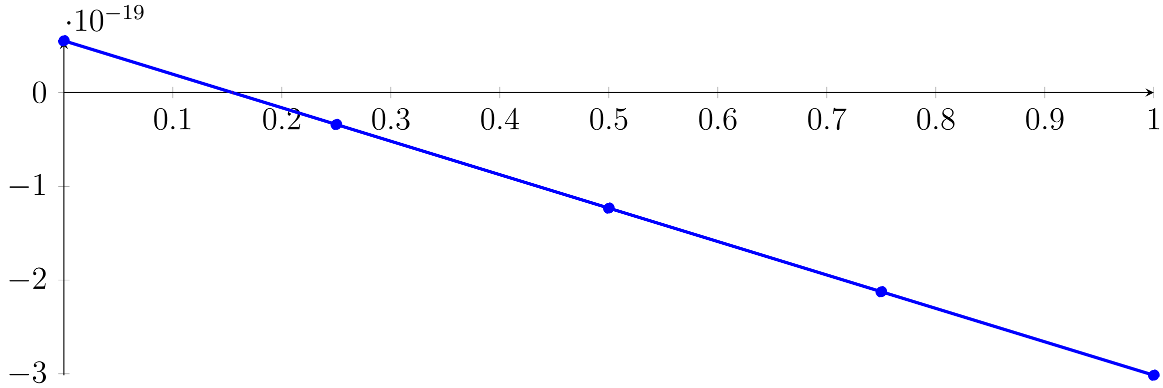
$$[0.1651189929990553, 0.1818829383495937]$$

Longest intersection interval: 0.01676394535053848
$\implies$ Selective recursion: interval 1: $[0.3999999999975344, 0.4000000000134784]$,

34.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999975344, 0.4000000000134784]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -6.462425394283182 \cdot 10^{-44} X^4 + 8.106374699799114 \cdot 10^{-34} X^3 + 5.694371396423764 \\ &\quad \cdot 10^{-22} X^2 - 3.573230357204536 \cdot 10^{-19} X + 5.524428779488534 \cdot 10^{-20} \\ &= 5.524428779488534 \cdot 10^{-20} B_{0,4}(X) - 3.408647113522805 \cdot 10^{-20} B_{1,4}(X) - 1.23322323875401 \\ &\quad \cdot 10^{-19} B_{2,4}(X) - 2.124632704256334 \cdot 10^{-19} B_{3,4}(X) - 3.01509310785925 \cdot 10^{-19} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1546060070924308, 0.1548527835869093\}$$

Intersection intervals with the x axis:

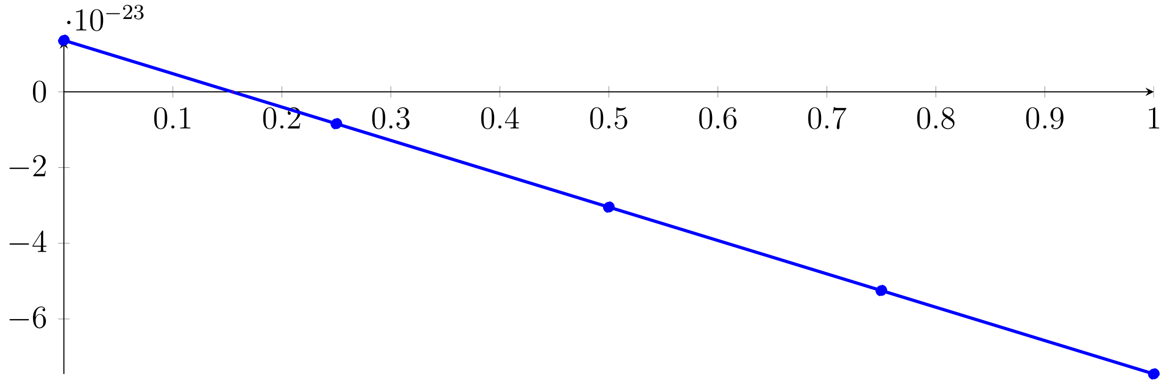
$$[0.1546060070924308, 0.1548527835869093]$$

Longest intersection interval: 0.0002467764944478516
$\implies$ Selective recursion: interval 1: $[0.3999999999999994, 0.4000000000000033]$,

34.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999994, 0.4000000000000033]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.396683605522666 \cdot 10^{-58} X^4 + 1.218254561570257 \cdot 10^{-44} X^3 + 3.467794636018148 \\ &\quad \cdot 10^{-29} X^2 - 8.813547453484276 \cdot 10^{-23} X + 1.36112658736315 \cdot 10^{-23} \\ &= 1.36112658736315 \cdot 10^{-23} B_{0,4}(X) - 8.422602760079189 \cdot 10^{-24} B_{1,4}(X) - 3.045646561413215 \\ &\quad \cdot 10^{-23} B_{2,4}(X) - 5.249032268852739 \cdot 10^{-23} B_{3,4}(X) - 7.45241739832649 \cdot 10^{-23} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544357246099643, 0.1544357853745533\}$$

Intersection intervals with the x axis:

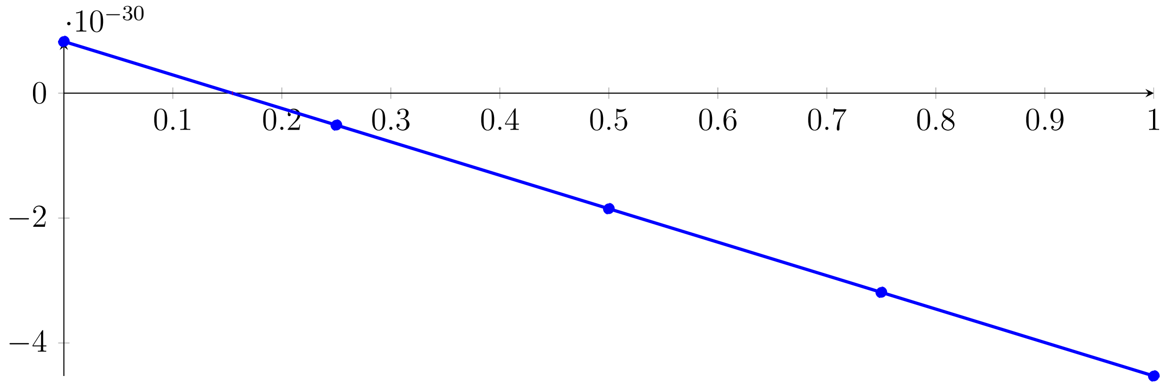
$$[0.1544357246099643, 0.1544357853745533]$$

Longest intersection interval: $6.076458894192547 \cdot 10^{-08}$
$\implies$ Selective recursion: interval 1: $[0.4, 0.4]$,

34.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -3.267480184524245 \cdot 10^{-87} X^4 + 2.733315343334724 \cdot 10^{-66} X^3 + 1.280426044119387 \\ &\quad \cdot 10^{-43} X^2 - 5.355515230460074 \cdot 10^{-30} X + 8.270826503648829 \cdot 10^{-31} \\ &= 8.270826503648829 \cdot 10^{-31} B_{0,4}(X) - 5.117961572501356 \cdot 10^{-31} B_{1,4}(X) - 1.850674964865133 \\ &\quad \cdot 10^{-30} B_{2,4}(X) - 3.189553772480109 \cdot 10^{-30} B_{3,4}(X) - 4.528432580095063 \cdot 10^{-30} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138334, 0.1544356826138371\}$$

Intersection intervals with the x axis:

$$[0.1544356826138334, 0.1544356826138371]$$

Longest intersection interval: $3.692333261147783 \cdot 10^{-15}$
$\implies$ Selective recursion: interval 1: $[0.4, 0.4]$,

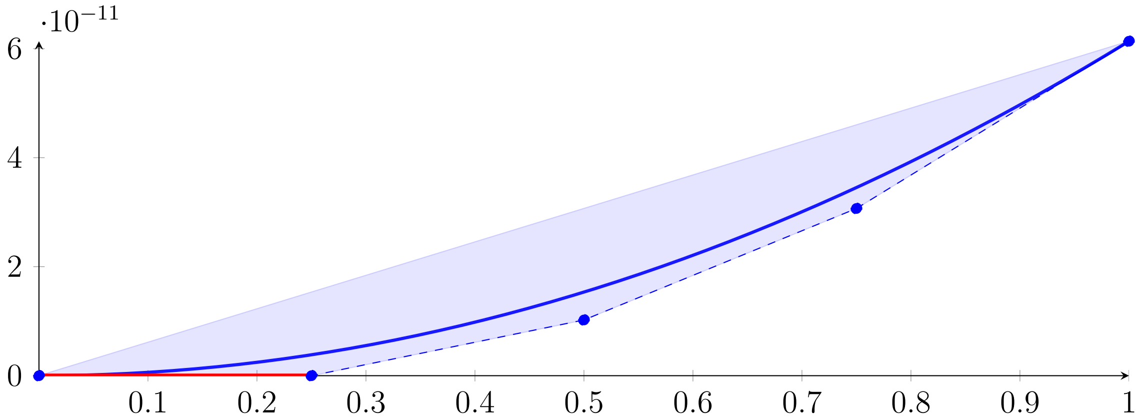
34.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 28!

34.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.4000000050060858, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -7.504705056762291 \cdot 10^{-22} X^4 + 2.867676750923124 \cdot 10^{-17} X^3 + 6.136416554191405 \\ &\quad \cdot 10^{-11} X^2 + 1.427014599332089 \cdot 10^{-16} X - 5.59999170035142 \cdot 10^{-17} \\ &= -5.59999170035142 \cdot 10^{-17} B_{0,4}(X) - 2.032455202021197 \cdot 10^{-17} B_{1,4}(X) + 1.02273762744653 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.068214096632685 \cdot 10^{-11} B_{3,4}(X) + 6.136428091947402 \cdot 10^{-11} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{9.125808216110345e-07, 0.2500004968163653\}$$

Intersection intervals with the x axis:

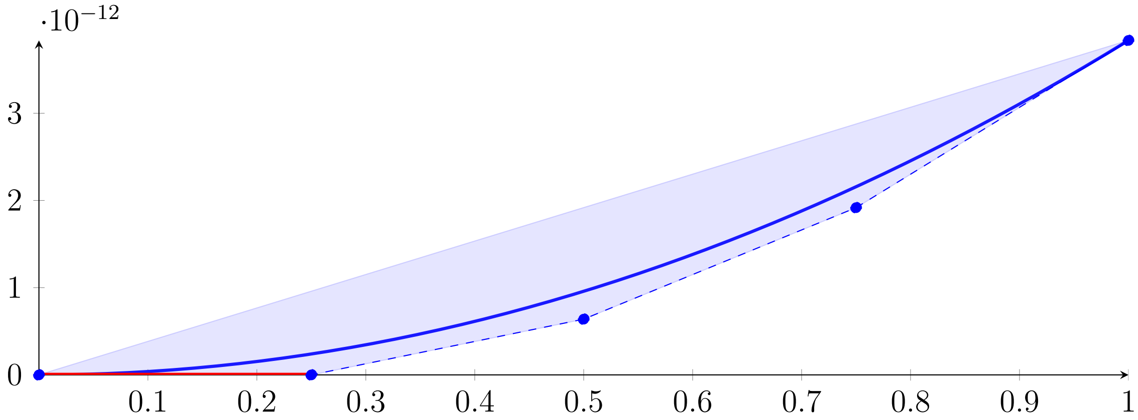
$$[9.125808216110345e-07, 0.2500004968163653]$$

Longest intersection interval: 0.2499995842355437
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050108623, 0.4000013135077487]$,

34.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.4000000050108623, 0.4000013135077487]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -2.931505911661308 \cdot 10^{-24} X^4 + 4.480722567712799 \cdot 10^{-19} X^3 + 3.835247589865681 \\ &\quad \cdot 10^{-12} X^2 + 6.367513939156292 \cdot 10^{-17} X - 5.59997356725911 \cdot 10^{-17} \\ &= -5.59997356725911 \cdot 10^{-17} B_{0,4}(X) - 4.008095082470037 \cdot 10^{-17} B_{1,4}(X) + 6.391837694783034 \\ &\quad \cdot 10^{-13} B_{2,4}(X) + 1.917615663569776 \cdot 10^{-12} B_{3,4}(X) + 3.835255713338726 \cdot 10^{-12} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{1.460109108777016e-05, 0.2500156756317829\}$$

Intersection intervals with the x axis:

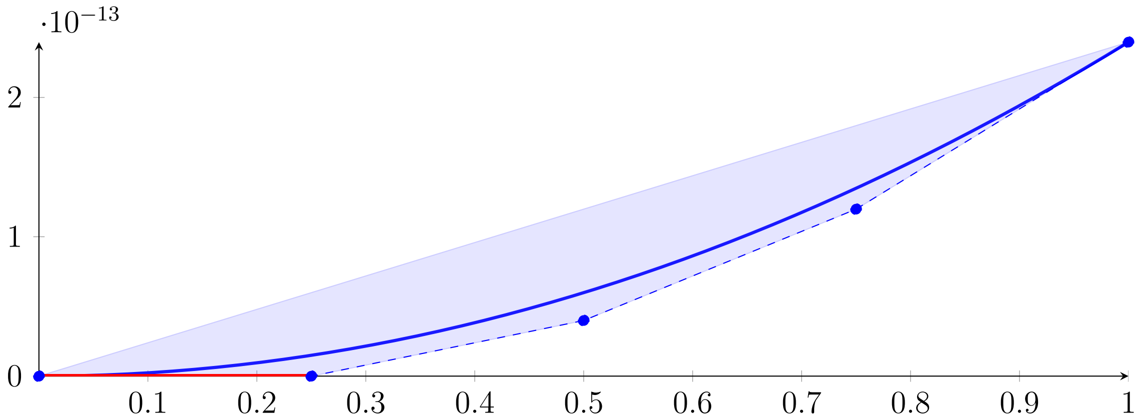
$$[1.460109108777016e-05, 0.2500156756317829]$$

Longest intersection interval: 0.2500010745406951
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050299677, 0.4000003321555954]$,

34.31 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.4000000050299677, 0.4000003321555954]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.145139184509963\cdot 10^{-26}X^4 + 7.00121928574046\cdot 10^{-21}X^3 + 2.397050349370657 \\ &\quad \cdot 10^{-13}X^2 + 4.391837331735849\cdot 10^{-17}X - 5.599798830250992\cdot 10^{-17} \\ &= -5.599798830250992\cdot 10^{-17}B_{0,4}(X) - 4.50183949731703\cdot 10^{-17}B_{1,4}(X) + 3.991680035453379 \\ &\quad \cdot 10^{-14}B_{2,4}(X) + 1.198294600105232\cdot 10^{-13}B_{3,4}(X) + 2.396929623232884\cdot 10^{-13}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0002335692644077866, 0.2502816337968459\}$$

Intersection intervals with the x axis:

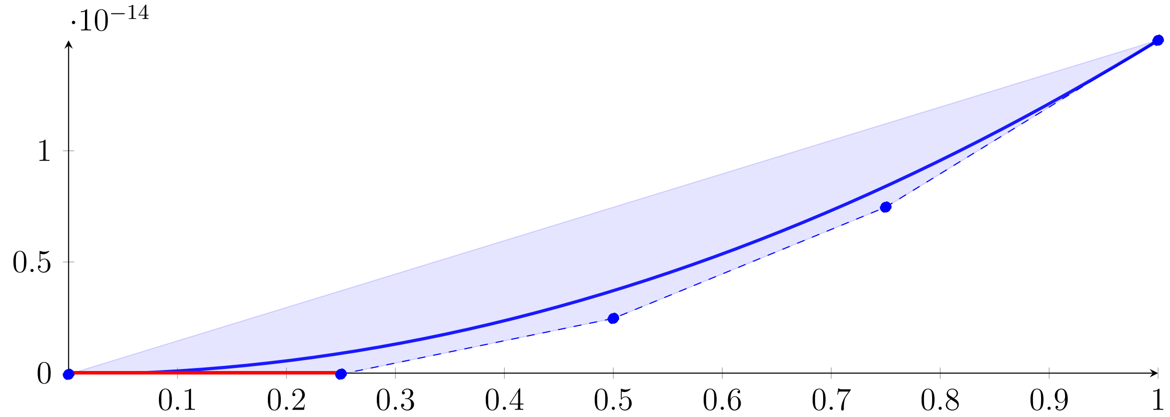
$$[0.0002335692644077866, 0.2502816337968459]$$

Longest intersection interval: 0.2500480645324381
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000051063742, 0.4000000869035043]$,

34.32 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.4000000051063742, 0.4000000869035043]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.476640967897802\cdot10^{-29}X^4 + 1.09457158991016\cdot10^{-22}X^3 + 1.49873258928534\\&\quad\cdot10^{-14}X^2 + 3.898095063625474\cdot10^{-17}X - 5.597465330775527\cdot10^{-17}\\&= -5.597465330775527\cdot10^{-17}B_{0,4}(X) - 4.622941564869158\cdot10^{-17}B_{1,4}(X) + 2.461403470819272\\&\quad\cdot10^{-15}B_{2,4}(X) + 7.466924033460424\cdot10^{-15}B_{3,4}(X) + 1.497033229963901\cdot10^{-14}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.003725110466798912, 0.2546088699723713\}$$

Intersection intervals with the x axis:

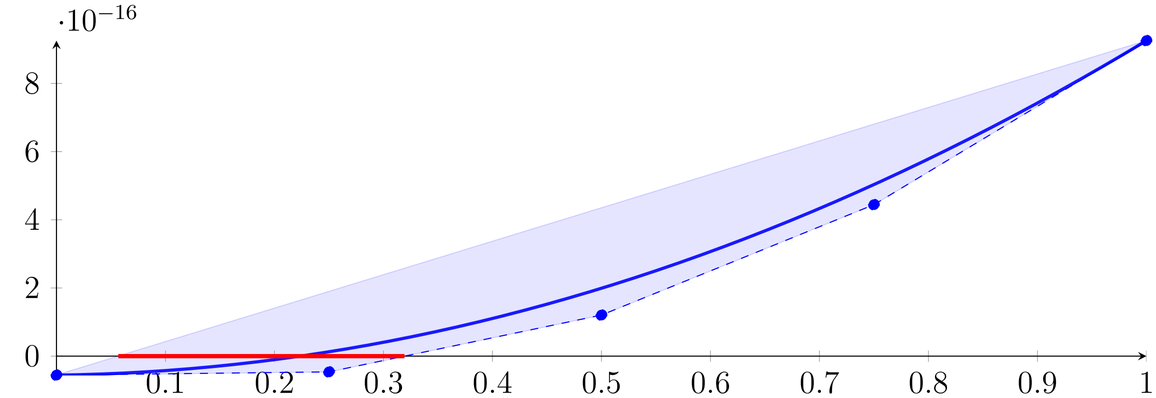
$$[0.003725110466798912, 0.2546088699723713]$$

Longest intersection interval: 0.2508837595055724
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000054110776, 0.4000000259326491]$,

34.33 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.4000000054110776, 0.4000000259326491]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.773546014711518\cdot10^{-31}X^4 + 1.728469879701229\cdot10^{-24}X^3 + 9.433421698048765\\&\quad\cdot10^{-16}X^2 + 3.779308932689389\cdot10^{-17}X - 5.562147411226954\cdot10^{-17}\\&= -5.562147411226954\cdot10^{-17}B_{0,4}(X) - 4.617320178054607\cdot10^{-17}B_{1,4}(X) + 1.204987655186568\\&\quad\cdot10^{-16}B_{2,4}(X) + 4.443944282174566\cdot10^{-16}B_{3,4}(X) + 9.255137867479705\cdot10^{-16}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.05669093378980359, 0.3192576000162909\}$$

Intersection intervals with the x axis:

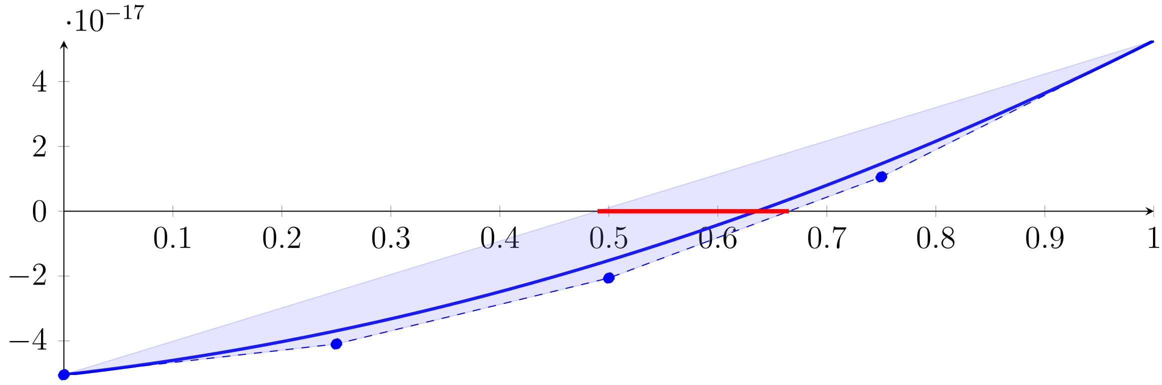
$$[0.05669093378980359, 0.3192576000162909]$$

Longest intersection interval: 0.2625666662264874
$\implies$ Selective recursion: interval 1: $[0.4000000065744646, 0.4000000119627452]$,

34.34 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.4000000065744646, 0.4000000119627452]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.42948070355458\cdot 10^{-34}X^4 + 3.128819977296419\cdot 10^{-26}X^3 + 6.503519235890246\\ &\quad \cdot 10^{-17}X^2 + 3.800678391173011\cdot 10^{-17}X - 5.044717705924284\cdot 10^{-17}\\ &= -5.044717705924284\cdot 10^{-17}B_{0,4}(X) - 4.094548108131031\cdot 10^{-17}B_{1,4}(X) - 2.060458637689404\\ &\quad \cdot 10^{-17}B_{2,4}(X) + 1.057550706182803\cdot 10^{-17}B_{3,4}(X) + 5.259479924267794\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4895788965792814, 0.6652062590622766\}$$

Intersection intervals with the x axis:

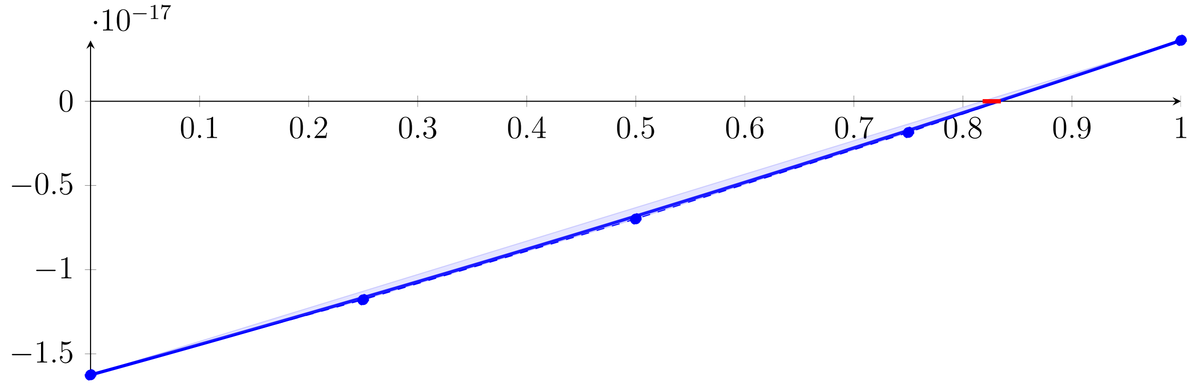
$$[0.4895788965792814, 0.6652062590622766]$$

Longest intersection interval: 0.1756273624829952
$\implies$ Selective recursion: interval 1: $[0.4000000092124531, 0.4000000101587826]$,

34.35 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.4000000092124531, 0.4000000101587826]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.01991079982412\cdot 10^{-37}X^4 + 1.694950778648659\cdot 10^{-28}X^3 + 2.006008588115633\\ &\quad \cdot 10^{-18}X^2 + 1.785893168307927\cdot 10^{-17}X - 1.625173531873033\cdot 10^{-17}\\ &= -1.625173531873033\cdot 10^{-17}B_{0,4}(X) - 1.178700239796051\cdot 10^{-17}B_{1,4}(X) - 6.987934712504758\\ &\quad \cdot 10^{-18}B_{2,4}(X) - 1.85453226232069\cdot 10^{-18}B_{3,4}(X) + 3.613204952634064\cdot 10^{-18}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8181114615359527, 0.8347943211886801\}$$

Intersection intervals with the x axis:

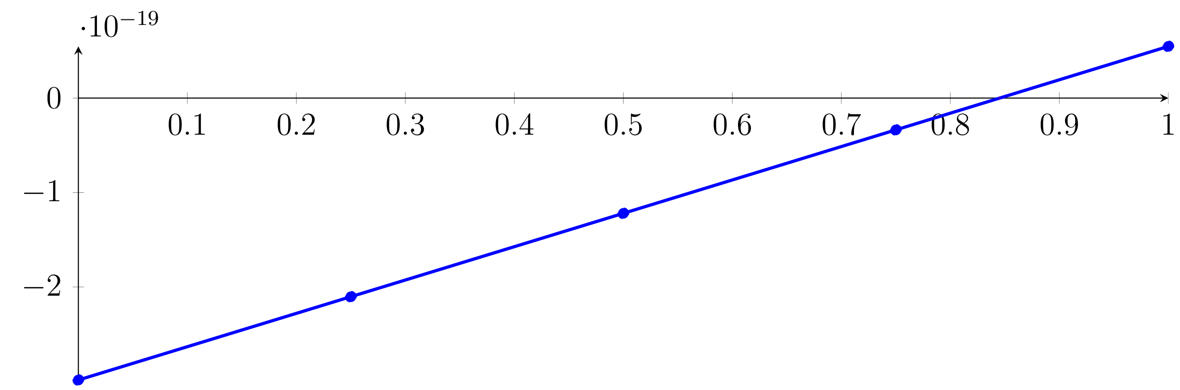
$$[0.8181114615359527, 0.8347943211886801]$$

Longest intersection interval: 0.01668285965272735
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099866561, 0.4000000100024436]$,

34.36 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.4000000099866561, 0.4000000100024436]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -6.212287164586666 \cdot 10^{-44} X^4 + 7.869888381434003 \cdot 10^{-34} X^3 + 5.583079095636332 \\ &\quad \cdot 10^{-22} X^2 + 3.526958212875592 \cdot 10^{-19} X - 2.985043046684015 \cdot 10^{-19} \\ &= -2.985043046684015 \cdot 10^{-19} B_{0,4}(X) - 2.103303493465117 \cdot 10^{-19} B_{1,4}(X) - 1.220633427063613 \\ &\quad \cdot 10^{-19} B_{2,4}(X) - 3.370328474795011 \cdot 10^{-20} B_{3,4}(X) + 5.474982452872209 \cdot 10^{-20} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845012924114553, 0.8452574901650142\}$$

Intersection intervals with the x axis:

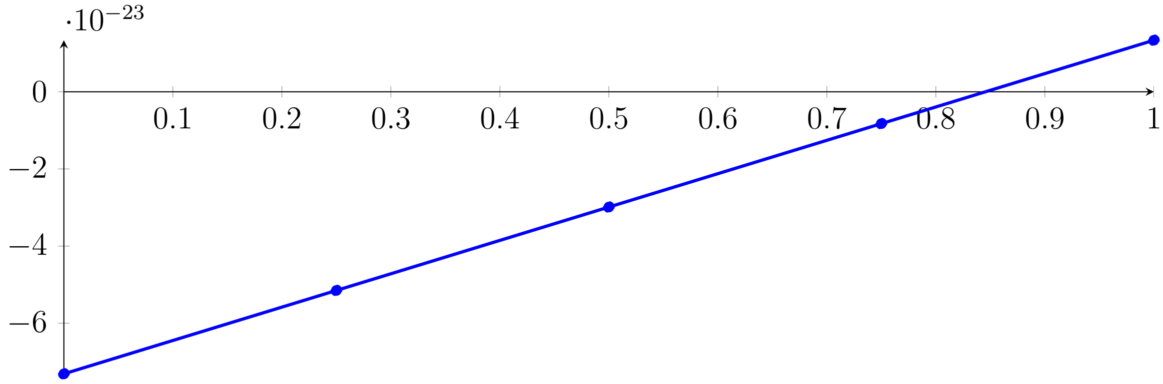
$$[0.845012924114553, 0.8452574901650142]$$

Longest intersection interval: 0.0002445660504611815
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099999968, 0.4000000100000006]$,

34.37 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.4000000099999968, 0.4000000100000006]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.22247151470982\cdot 10^{-58}X^4 + 1.151216705335136\cdot 10^{-44}X^3 + 3.33938214525302\\ &\quad\cdot 10^{-29}X^2 + 8.648818549690798\cdot 10^{-23}X - 7.311939957361979\cdot 10^{-23}\\ &= -7.311939957361979\cdot 10^{-23}B_{0,4}(X) - 5.149735319939279\cdot 10^{-23}B_{1,4}(X) - 2.987530125952889\\ &\quad\cdot 10^{-23}B_{2,4}(X) - 8.253243754028074\cdot 10^{-24}B_{3,4}(X) + 1.336881931710965\cdot 10^{-23}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454261229035133, 0.8454261825857513\}$$

Intersection intervals with the x axis:

$$[0.8454261229035133, 0.8454261825857513]$$

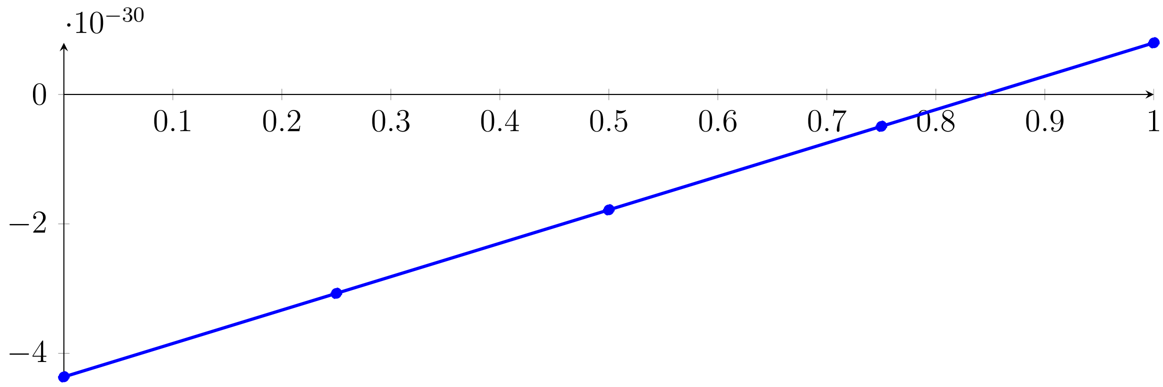
Longest intersection interval: $5.968223795299727\cdot 10^{-08}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

34.38 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.819788940081592\cdot 10^{-87}X^4 + 2.447329147319284\cdot 10^{-66}X^3 + 1.189477744066025\\ &\quad\cdot 10^{-43}X^2 + 5.161811836848388\cdot 10^{-30}X - 4.363931089282542\cdot 10^{-30}\\ &= -4.363931089282542\cdot 10^{-30}B_{0,4}(X) - 3.073478130070445\cdot 10^{-30}B_{1,4}(X) - 1.783025170858328\\ &\quad\cdot 10^{-30}B_{2,4}(X) - 4.925722116461918\cdot 10^{-31}B_{3,4}(X) + 7.978807475659646\cdot 10^{-31}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454262238173519, 0.8454262238173555\}$$

Intersection intervals with the x axis:

$$[0.8454262238173519, 0.8454262238173555]$$

Longest intersection interval: $3.561967626812098 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

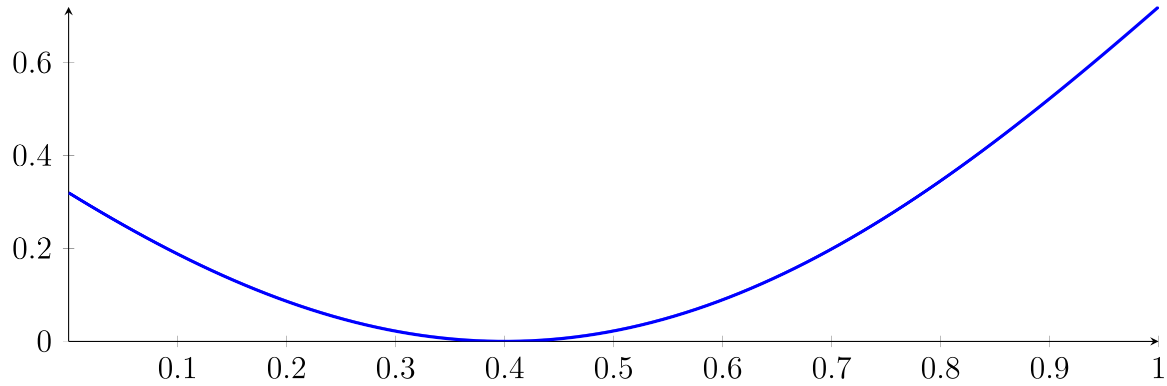
34.39 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 28!

34.40 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

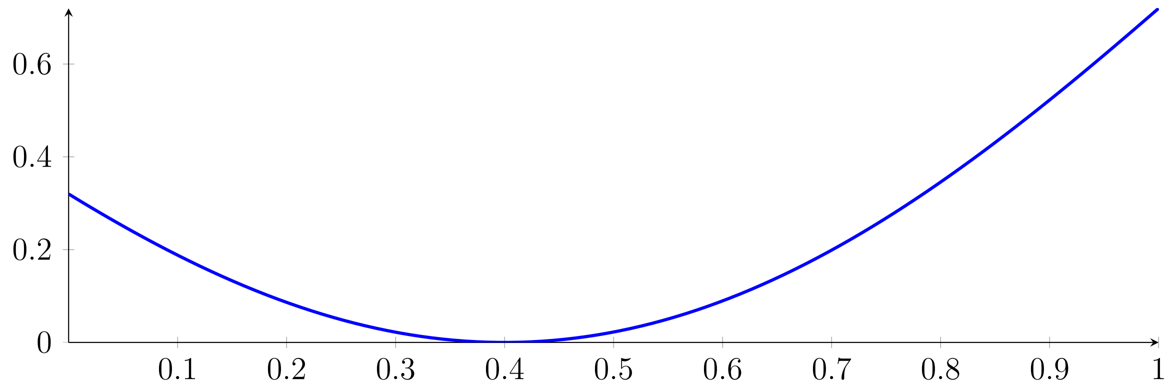
with precision $\varepsilon = 1 \cdot 10^{-32}$.

35 Running QuadClip on h_4 with epsilon 32

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called QuadClip with input polynomial on interval $[0, 1]$:

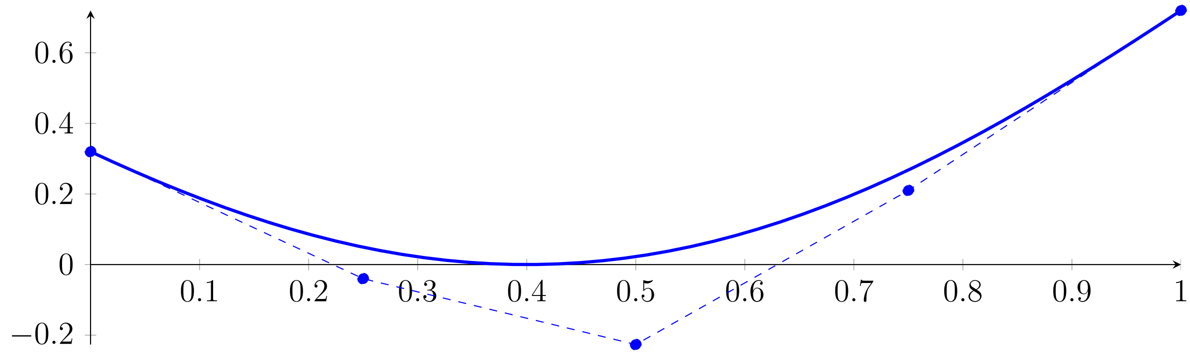
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



35.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

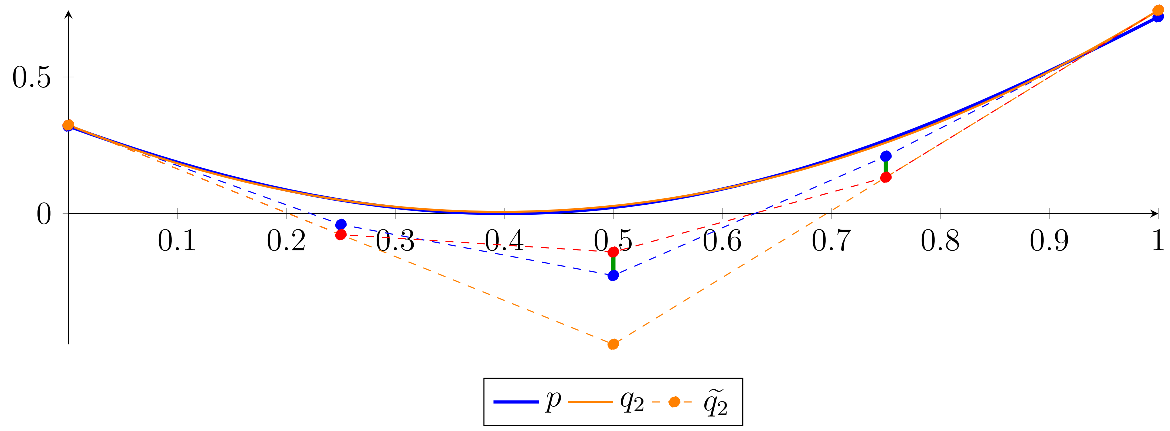
$$\begin{aligned}
p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\
&= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\
&\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\
&= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 &= 1.810653852360235\cdot 10^{-306}X^4 - 3.682497235829418\cdot 10^{-306}X^3 \\
&\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\
&= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\
&\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

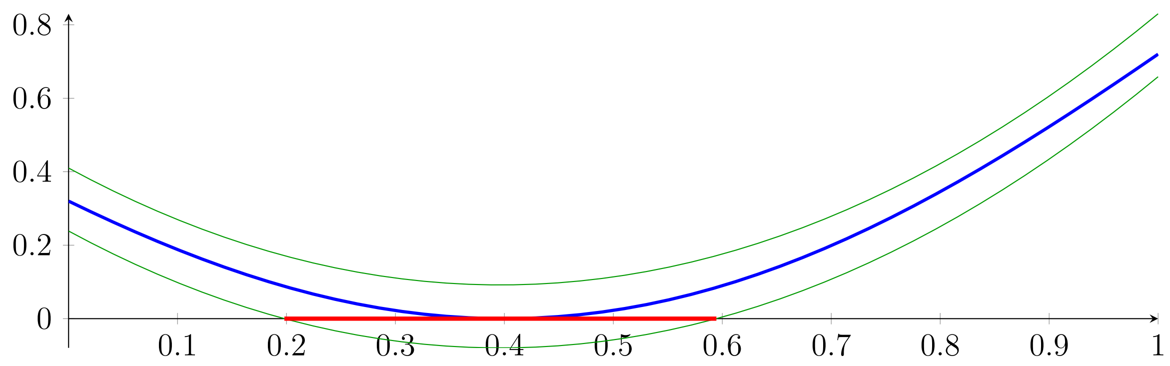
$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$

$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



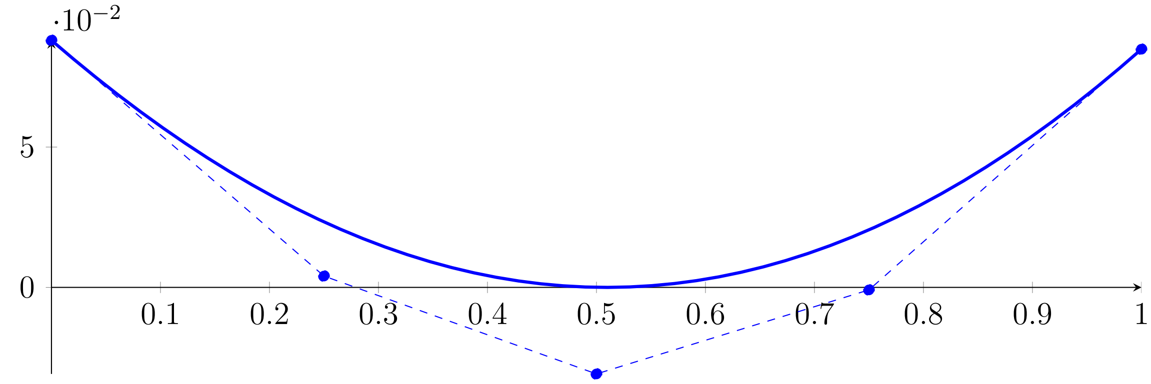
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

35.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

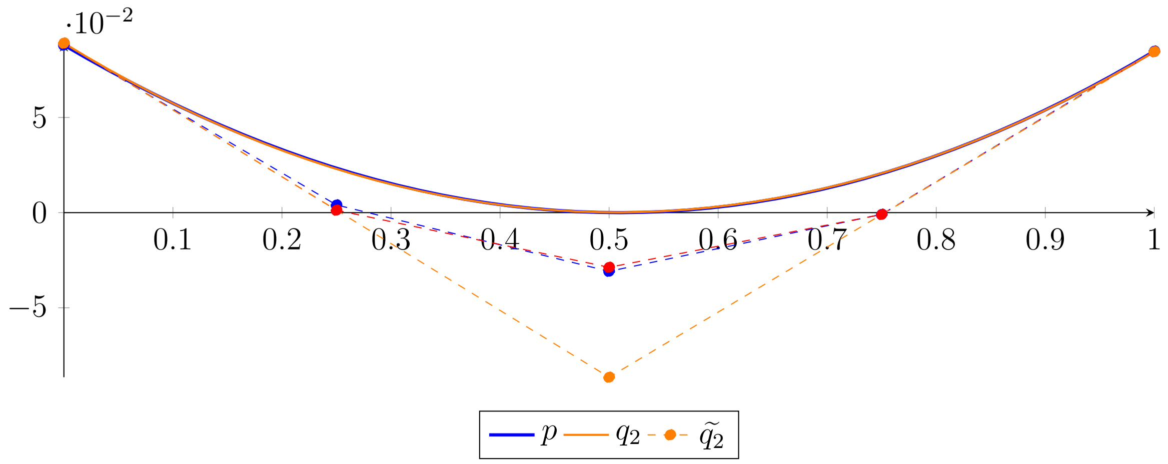
$$\begin{aligned} p &= -0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &\quad + 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ &= 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &\quad - 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402 \cdot 10^{-307}X^4 - 6.258020227051504 \cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

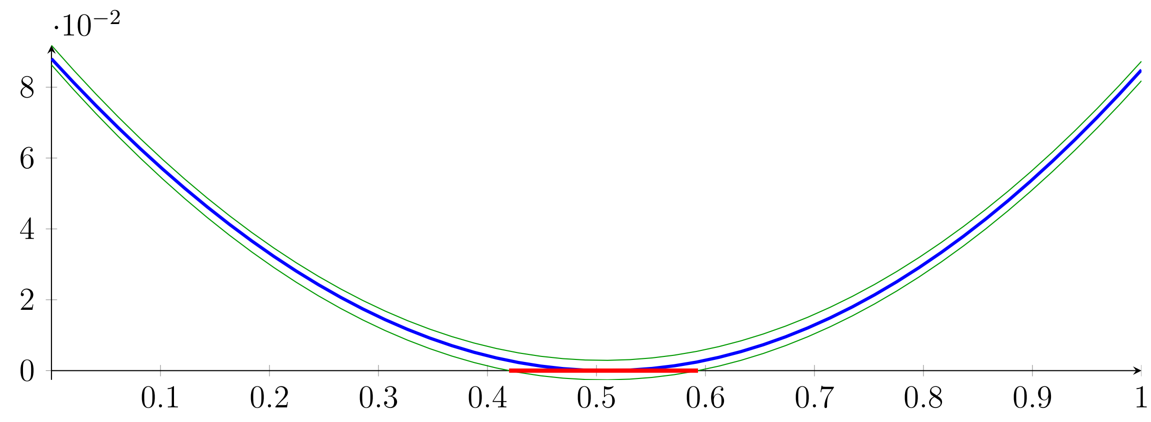
Bounding polynomials M and m:

$$M = 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423$$
$$m = 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



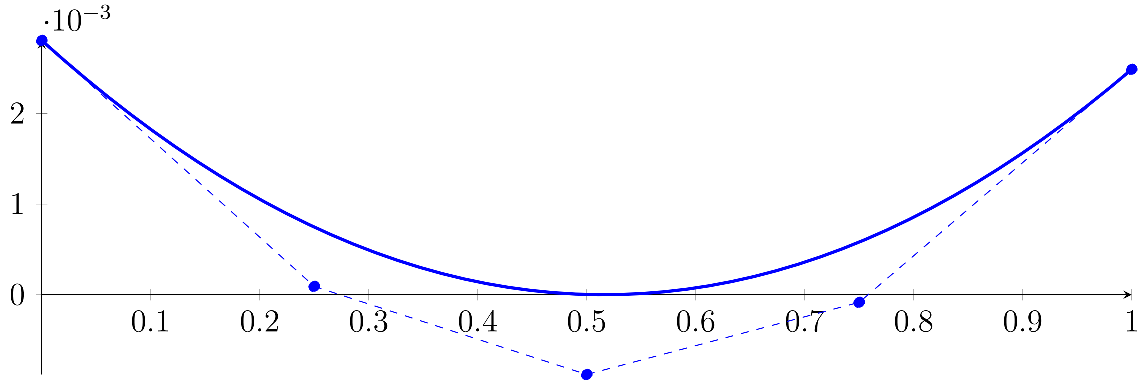
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

35.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

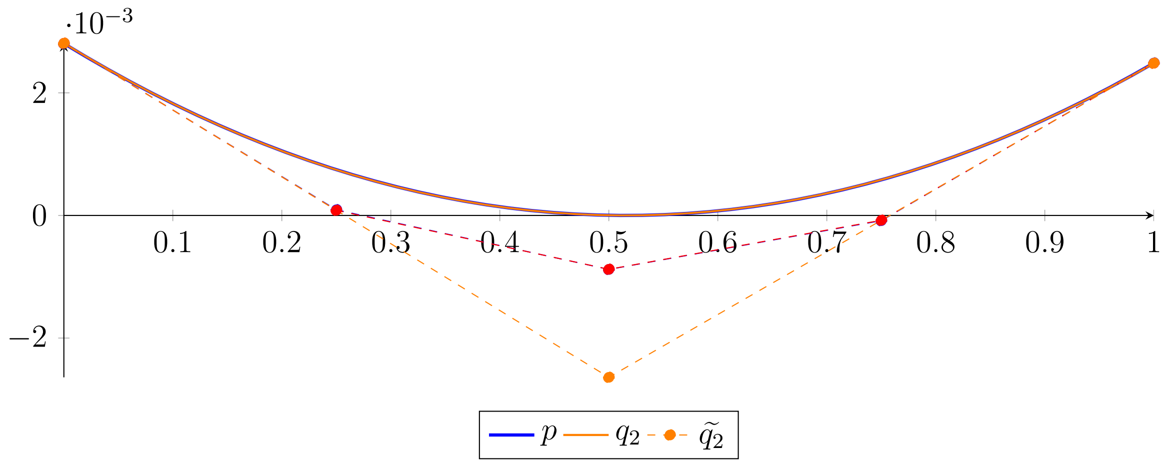
$$\begin{aligned}p &= -2.231982021693433 \cdot 10^{-05} X^4 + 0.0001110015522974976 X^3 \\ &\quad + 0.01044647623937015 X^2 - 0.01085434180562325 X + 0.002805735592529265 \\ &= 0.002805735592529265 B_{0,4}(X) + 9.215014112345361 \cdot 10^{-05} B_{1,4}(X) \\ &\quad - 0.0008803559370539994 B_{2,4}(X) - 8.403225392871914 \\ &\quad \cdot 10^{-05} B_{3,4}(X) + 0.002486551758356734 B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,2} - 0.002640895479419014 B_{1,2} + 0.002483552517338081 B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q_2} &= 9.278384156079834 \cdot 10^{-309} X^4 - 1.903481152394832 \cdot 10^{-308} X^3 \\ &\quad + 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,4} + 8.423853163898018 \cdot 10^{-05} B_{1,4} - 0.0008784428096068336 B_{2,4} \\ &\quad - 7.867148104046678 \cdot 10^{-05} B_{3,4} + 0.002483552517338081 B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429 \cdot 10^{-06}$.
Bounding polynomials M and m:

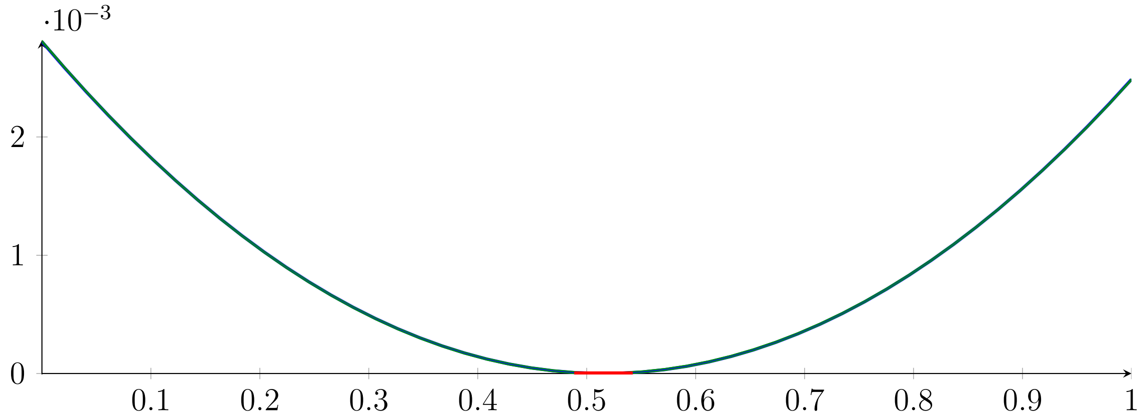
$$M = 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



$$[0.4885305113706042, 0.542280720323693]$$

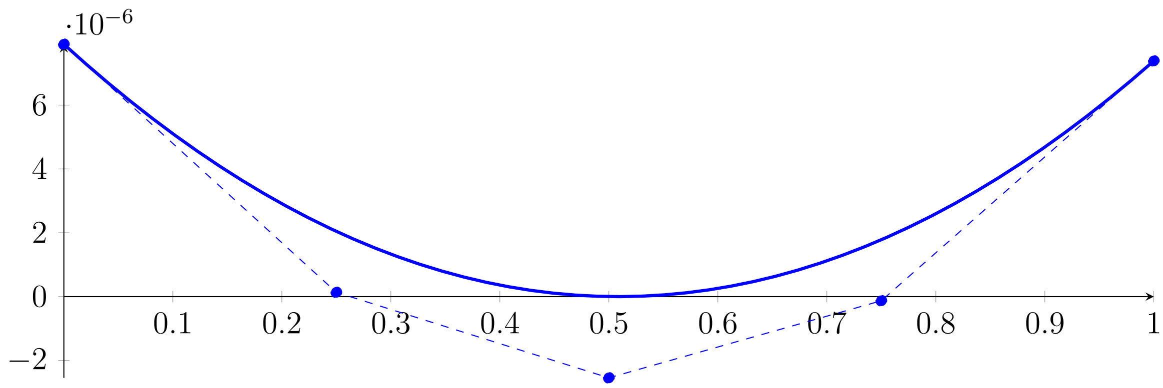
Longest intersection interval: 0.05375020895308878

$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

35.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.40181554717343$

Normalized monomial und Bézier representations and the Bézier polygon:

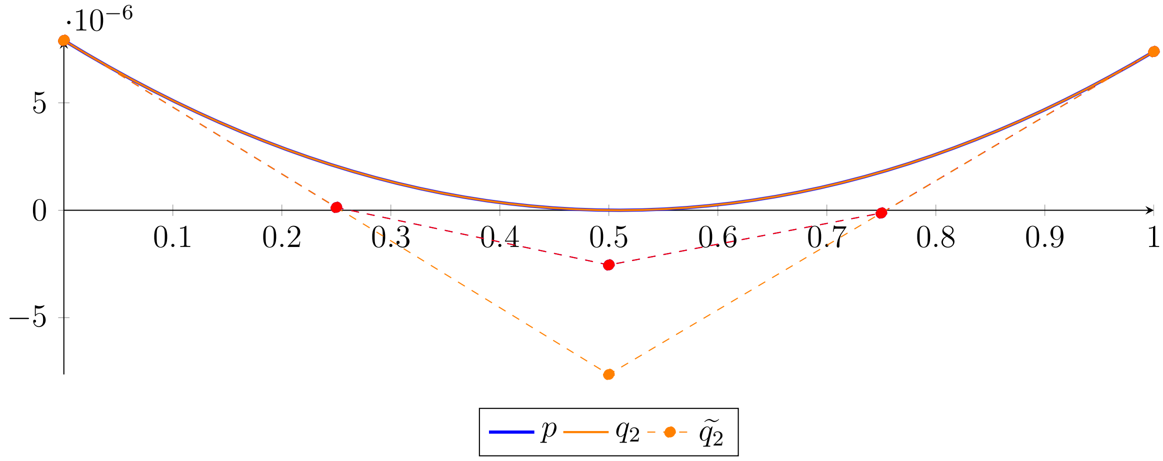
$$\begin{aligned} p &= -1.862993414511891\cdot 10^{-10}X^4 + 1.046428359410323\cdot 10^{-08}X^3 + 3.055842313534119 \\ &\quad \cdot 10^{-05}X^2 - 3.109078018724982\cdot 10^{-05}X + 7.906738260659747\cdot 10^{-06} \\ &= 7.906738260659747\cdot 10^{-06}B_{0,4}(X) + 1.340432138472922\cdot 10^{-07}B_{1,4}(X) - 2.545581310408299 \\ &\quad \cdot 10^{-06}B_{2,4}(X) - 1.295192412084993\cdot 10^{-07}B_{3,4}(X) + 7.384659193003765\cdot 10^{-06}B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 3.057380019043271\cdot 10^{-05}X^2 - 3.109688842657981\cdot 10^{-05}X + 7.907245506324471\cdot 10^{-06} \\ &= 7.907245506324471\cdot 10^{-06}B_{0,2} - 7.641198706965435\cdot 10^{-06}B_{1,2} + 7.384157270177368\cdot 10^{-06}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= 2.673714696616243\cdot 10^{-311}X^4 - 5.466261157526541\cdot 10^{-311}X^3 + 3.057380019043271 \\ &\quad \cdot 10^{-05}X^2 - 3.109688842657981\cdot 10^{-05}X + 7.907245506324471\cdot 10^{-06} \\ &= 7.907245506324471\cdot 10^{-06}B_{0,4} + 1.330233996795178\cdot 10^{-07}B_{1,4} - 2.545565341893317 \\ &\quad \cdot 10^{-06}B_{2,4} - 1.285207183940336\cdot 10^{-07}B_{3,4} + 7.384157270177368\cdot 10^{-06}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.019814167774439 \cdot 10^{-09}$.
Bounding polynomials M and m:

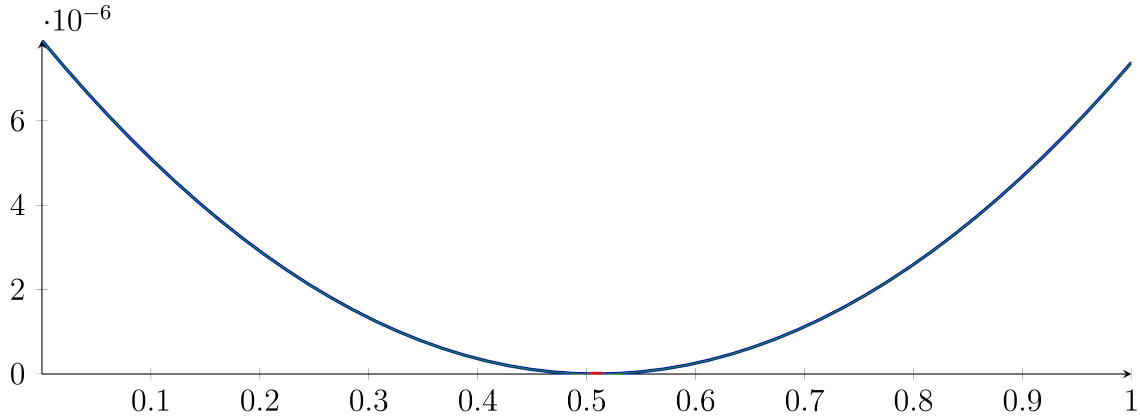
$$M = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.908265320492245 \cdot 10^{-06}$$

$$m = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.906225692156696 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.50281872462556, 0.5142903109829997\}$$

Intersection intervals:



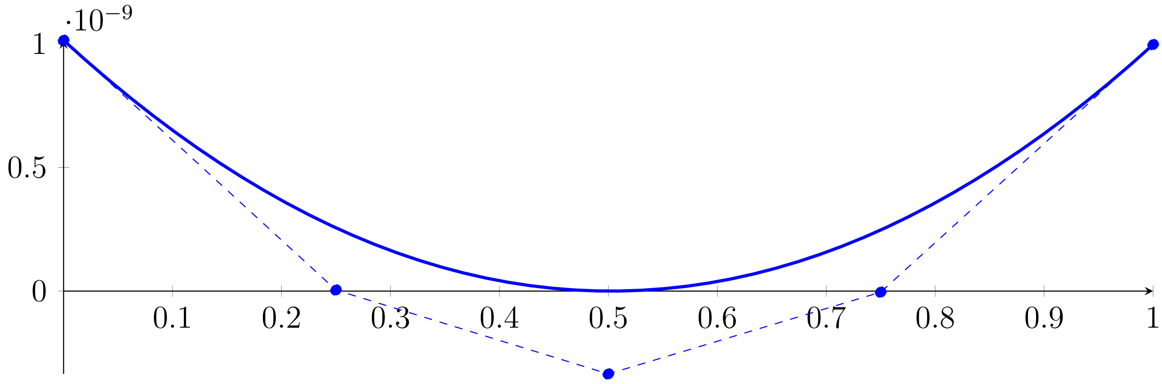
$$[0.50281872462556, 0.5142903109829997]$$

Longest intersection interval: 0.01147158635743977
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999787229673132, 0.4000211044658684]$,

35.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999787229673132, 0.400021104465$

Normalized monomial und Bézier representations and the Bézier polygon:

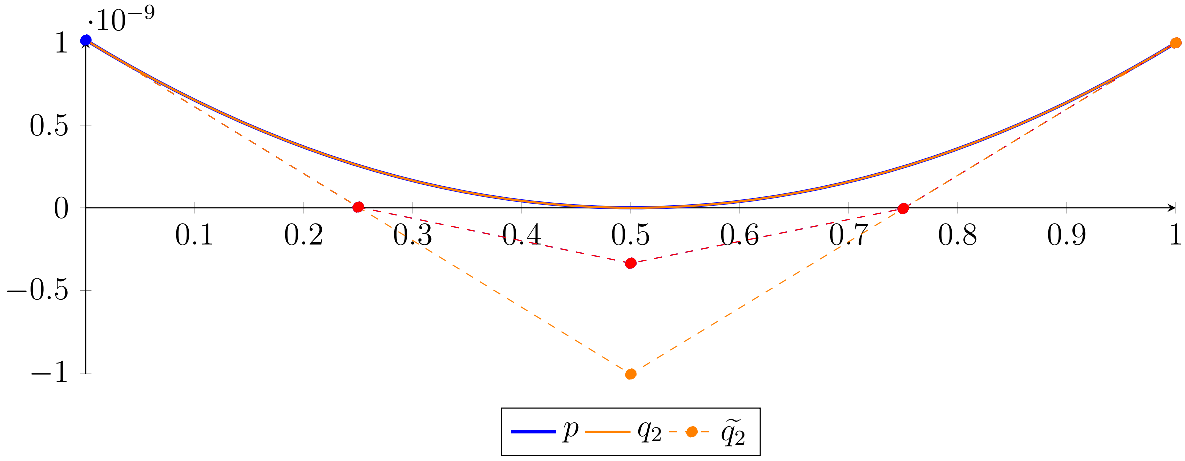
$$\begin{aligned} p = & -3.226303616518 22 \cdot 10^{-18} X^4 + 1.523153645441234 \cdot 10^{-14} X^3 + 4.023445841277385 \\ & \cdot 10^{-09} X^2 - 4.040789163099954 \cdot 10^{-09} X + 1.014549826650262 \cdot 10^{-09} \\ = & \, 1.014549826650262 \cdot 10^{-09} B_{0,4}(X) + 4.352535875273264 \cdot 10^{-12} B_{1,4}(X) - 3.352704480201511 \\ & \cdot 10^{-10} B_{2,4}(X) - 4.315317151897678 \cdot 10^{-12} B_{3,4}(X) + 9.972217331378436 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 4.023468683051261\cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014550587950544\cdot 10^{-09} \\
&= 1.014550587950544\cdot 10^{-09}B_{0,2} - 1.005848561585488\cdot 10^{-09}B_{1,2} + 9.972209719297413\cdot 10^{-10}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 &= 3.491760652125473\cdot 10^{-315}X^4 - 7.128579610273962\cdot 10^{-315}X^3 + 4.023468683051261 \\
&\quad \cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014550587950544\cdot 10^{-09} \\
&= 1.014550587950544\cdot 10^{-09}B_{0,4} + 4.35101318252834\cdot 10^{-12}B_{1,4} - 3.352704477436108 \\
&\quad \cdot 10^{-10}B_{2,4} - 4.313794827873167\cdot 10^{-12}B_{3,4} + 9.972209719297413\cdot 10^{-10}B_{4,4}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 1.522692744924588\cdot 10^{-15}$.
Bounding polynomials M and m:

$$M = 4.023468683051261\cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014552110643289\cdot 10^{-09}$$

$$m = 4.023468683051261\cdot 10^{-09}X^2 - 4.040798299072064\cdot 10^{-09}X + 1.014549065257799\cdot 10^{-09}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.5015281513143551, 0.5027789820041677\}$$

Intersection intervals:

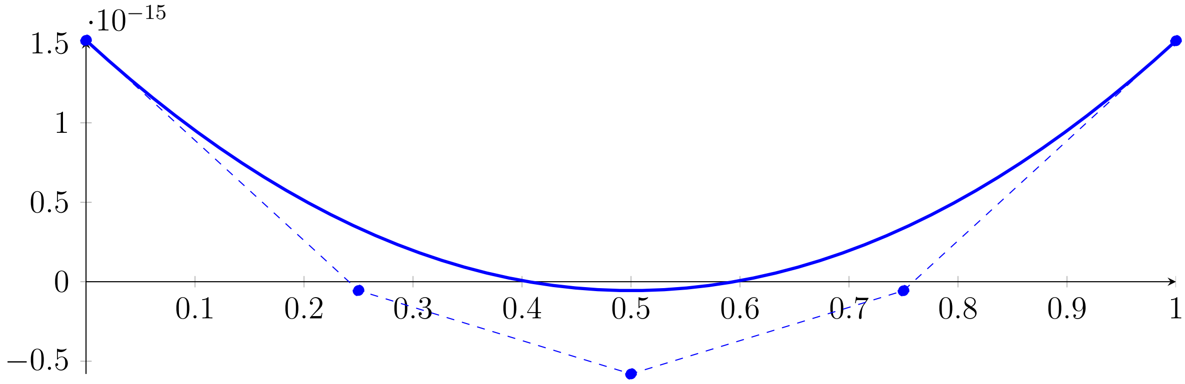
$$[0.5015281513143551, 0.5027789820041677]$$

Longest intersection interval: 0.00125083068981256
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999784819335, 0.4000000314940126]$,

35.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999997784819335, 0.4000000314$

Normalized monomial und Bézier representations and the Bézier polygon:

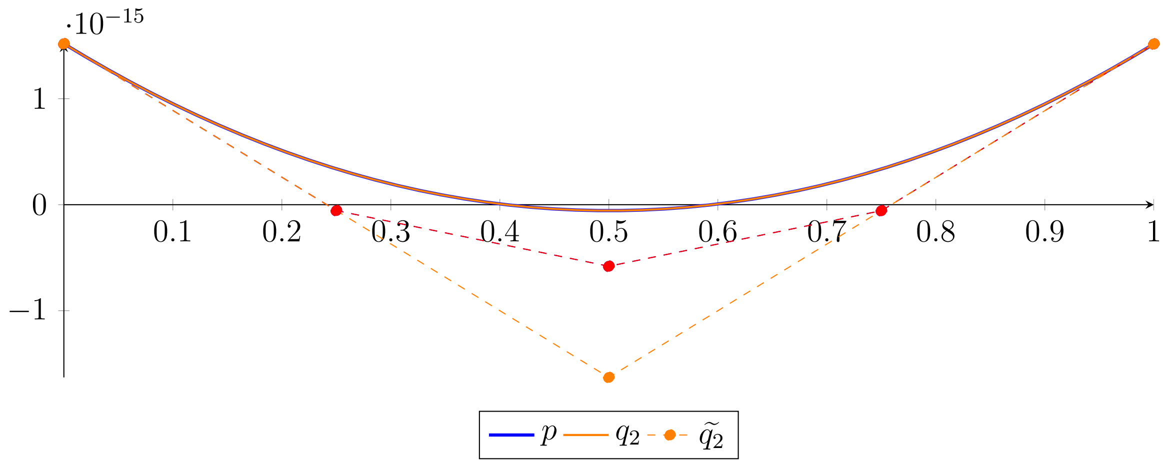
$$\begin{aligned}p &= -7.897676644125011\cdot10^{-30}X^4 + 2.979577702269594\cdot10^{-23}X^3 + 6.295028340046794\\&\quad\cdot10^{-15}X^2 - 6.297884694019149\cdot10^{-15}X + 1.519185582318221\cdot10^{-15}\\&= 1.519185582318221\cdot10^{-15}B_{0,4}(X) - 5.52855911865664\cdot10^{-17}B_{1,4}(X) - 5.805853746835548\\&\quad\cdot10^{-16}B_{2,4}(X) - 5.671376072379998\cdot10^{-17}B_{3,4}(X) + 1.516329258141634\cdot10^{-15}B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185583808009\cdot10^{-15}\\&= 1.519185583808009\cdot10^{-15}B_{0,2} - 1.629756772140295\cdot10^{-15}B_{1,2} + 1.516329256651846\cdot10^{-15}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 5.424840791336887\cdot10^{-321}X^4 - 1.106707046684392\cdot10^{-320}X^3 + 6.295028384740446\\&\quad\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185583808009\cdot10^{-15}\\&= 1.519185583808009\cdot10^{-15}B_{0,4} - 5.528559416614297\cdot10^{-17}B_{1,4} - 5.805853746835541\\&\quad\cdot10^{-16}B_{2,4} - 5.671375774422431\cdot10^{-17}B_{3,4} + 1.516329256651846\cdot10^{-15}B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.979576574030073\cdot10^{-24}$.

Bounding polynomials M and m:

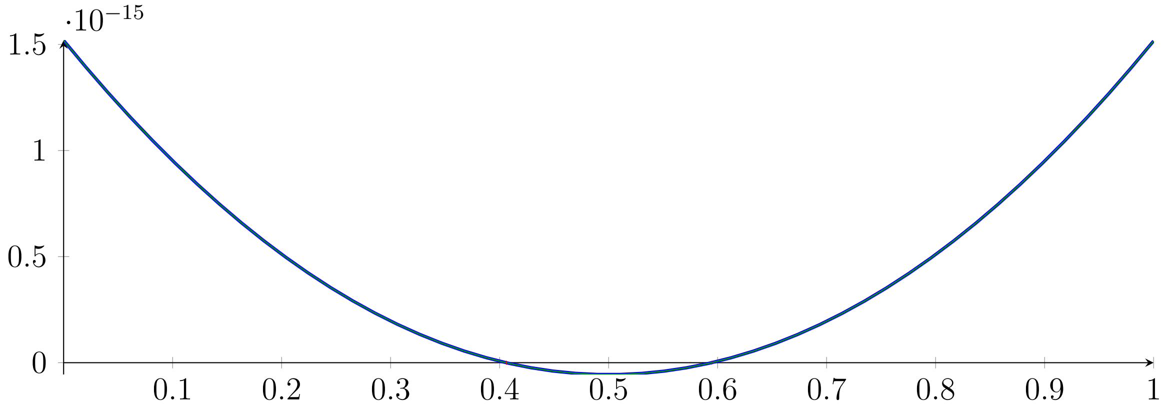
$$M = 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185586787586\cdot10^{-15}$$

$$m = 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185580828433\cdot10^{-15}$$

Root of M and m:

$N(M) = \{0.4059087473493063, 0.5945449959829854\}$ $\quad N(m) = \{0.4059087423309477, 0.594545001001344\}$

Intersection intervals:



$$[0.4059087423309477, 0.4059087473493063],\ [0.5945449959829854, 0.594545001001344]$$

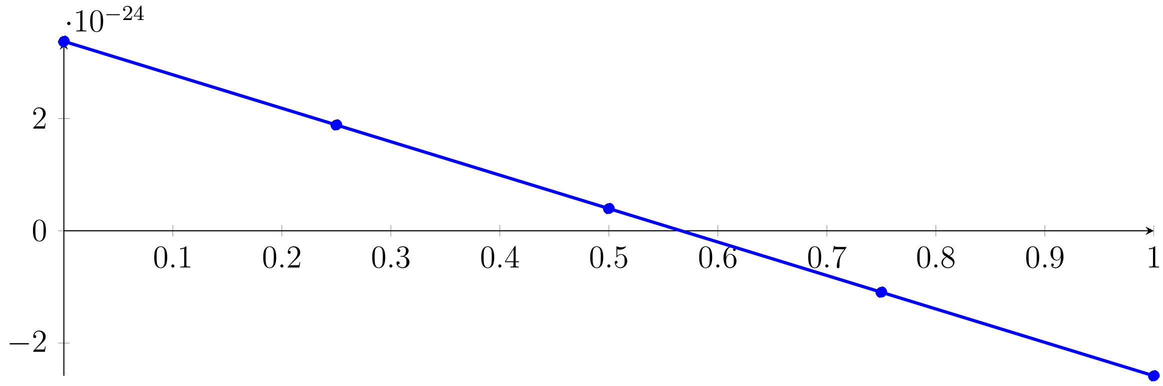
Longest intersection interval: $5.01835859594062 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.4000000000000001]$, interval 2: $[0.400000009999999$

35.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.40000000$

Normalized monomial und Bézier representations and the Bézier polygon:

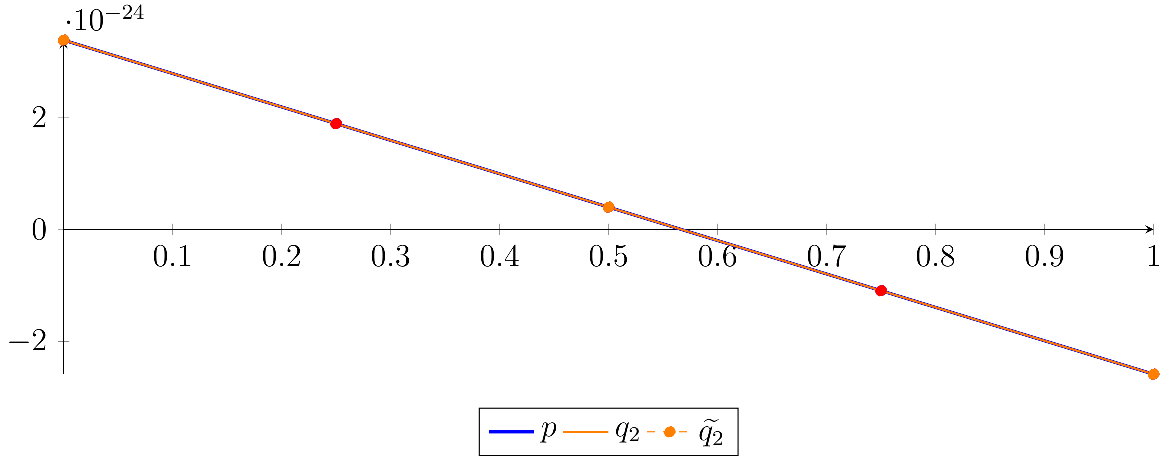
$$\begin{aligned} p &= -5.008943280633806 \cdot 10^{-63} X^4 + 3.765646973599716 \cdot 10^{-48} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4}(X) + 1.885496281661092 \cdot 10^{-24} B_{1,4}(X) + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4}(X) - 1.094080301597753 \cdot 10^{-24} B_{3,4}(X) - 2.583868553593798 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,2} + 3.957079503982923 \cdot 10^{-25} B_{1,2} - 2.583868553593798 \cdot 10^{-24} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= 9.938904969180865 \cdot 10^{-331} X^4 - 2.503131621867773 \cdot 10^{-330} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4} + 1.885496281661092 \cdot 10^{-24} B_{1,4} + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4} - 1.094080301597753 \cdot 10^{-24} B_{3,4} - 2.583868553593798 \cdot 10^{-24} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646973599709 \cdot 10^{-49}$.

Bounding polynomials M and m:

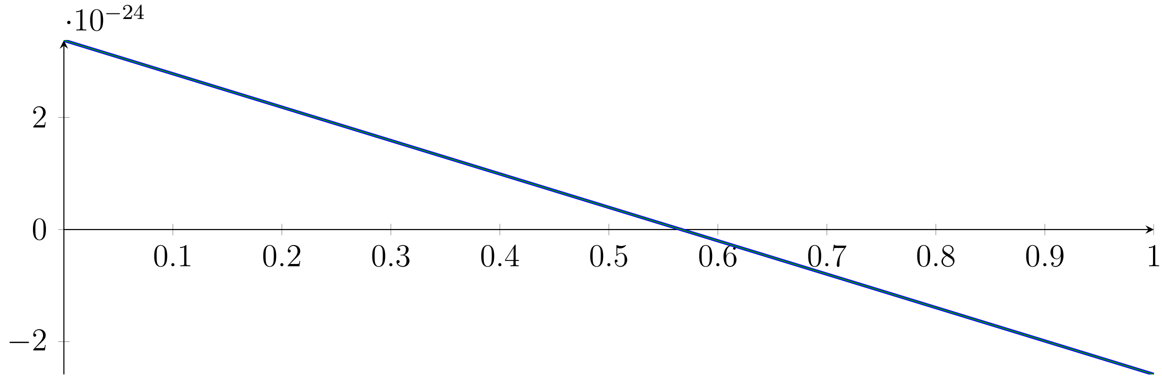
$$M = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

$$m = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

Root of M and m:

$$N(M) = \{0.5664033931790926, 37589234.2203029\} \quad N(m) = \{0.5664033931790926, 37589234.2203029\}$$

Intersection intervals:



$$[0.5664033931790926, 0.5664033931790926]$$

Longest intersection interval: $1.263819499744344 \cdot 10^{-25}$
$\implies$ Selective recursion: interval 1: $[0.4, 0.4]$,

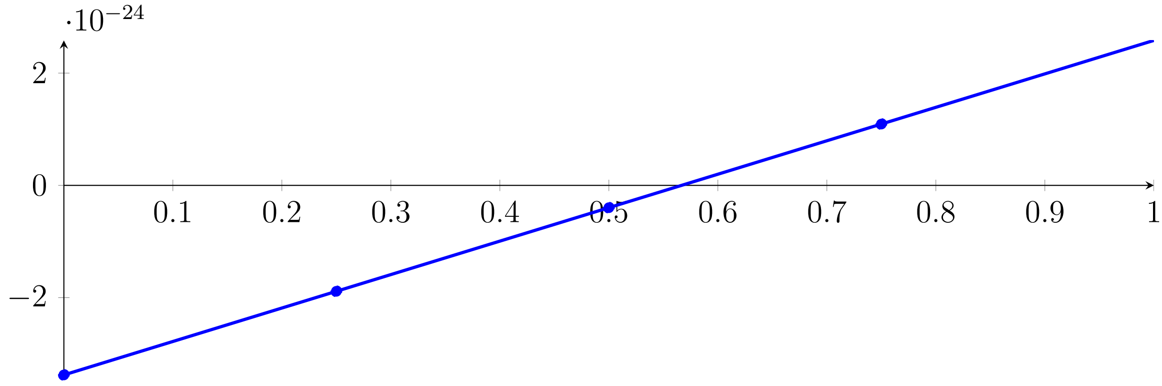
35.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 8!

35.9 Recursion Branch 1 1 1 1 1 1 2 in Interval 2: $[0.4000000099999999, 0.40000001$

Normalized monomial und Bézier representations and the Bézier polygon:

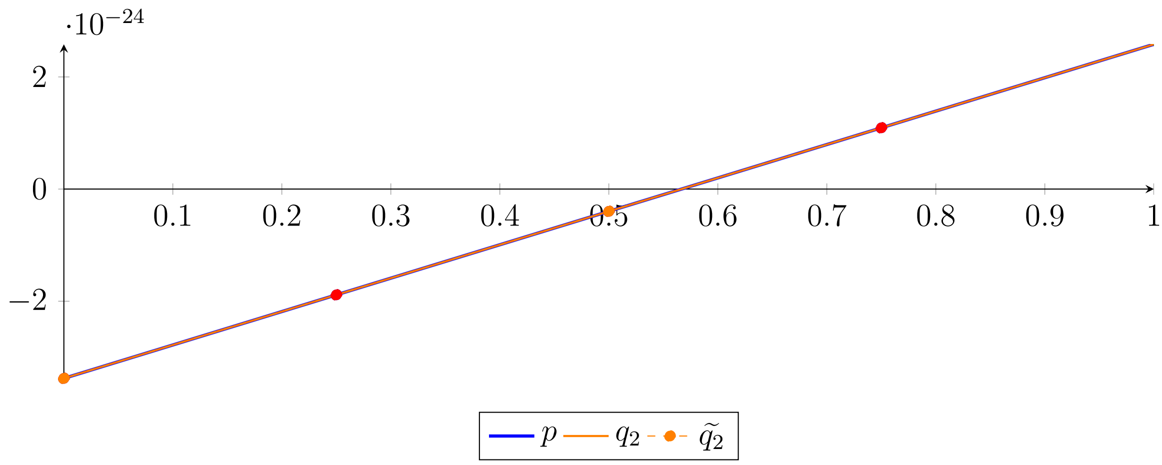
$$\begin{aligned} p = &-5.008943280633806 \cdot 10^{-63} X^4 + 3.76564622047036 \cdot 10^{-48} X^3 + 1.585335103209048 \\ &\cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ = &-3.376951785602669 \cdot 10^{-24} B_{0,4}(X) - 1.887163542825797 \cdot 10^{-24} B_{1,4}(X) - 3.973752736266727 \\ &\cdot 10^{-25} B_{2,4}(X) + 1.092413021994703 \cdot 10^{-24} B_{3,4}(X) + 2.582201344038331 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$
$$= -3.376951785602669 \cdot 10^{-24} B_{0,2} - 3.973753000489244 \cdot 10^{-25} B_{1,2} + 2.582201344038331 \cdot 10^{-24} B_{2,2}$$

$$\widetilde{q}_2 = -1.030701256063201 \cdot 10^{-330} X^4 + 2.355888585287316 \cdot 10^{-330} X^3 + 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$
$$= -3.376951785602669 \cdot 10^{-24} B_{0,4} - 1.887163542825797 \cdot 10^{-24} B_{1,4} - 3.973752736266727 \cdot 10^{-25} B_{2,4} + 1.092413021994703 \cdot 10^{-24} B_{3,4} + 2.582201344038331 \cdot 10^{-24} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646220470353 \cdot 10^{-49}$.

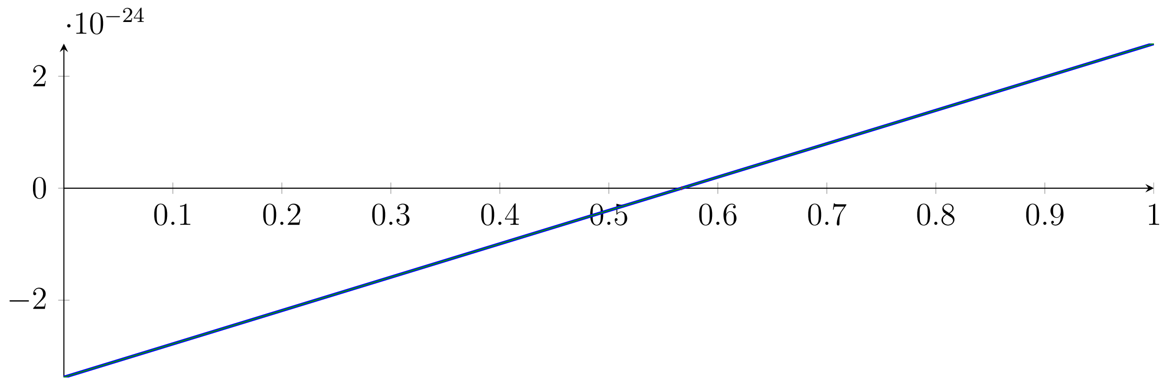
Bounding polynomials M and m:

$$M = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$
$$m = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$

Root of M and m:

$$N(M) = \{-37589233.0200927, 0.5666831764624487\} \quad N(m) = \{-37589233.0200927, 0.5666831764624487\}$$

Intersection intervals:



$$[0.5666831764624487, 0.5666831764624487]$$

Longest intersection interval: $1.263819245852043 \cdot 10^{-25}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

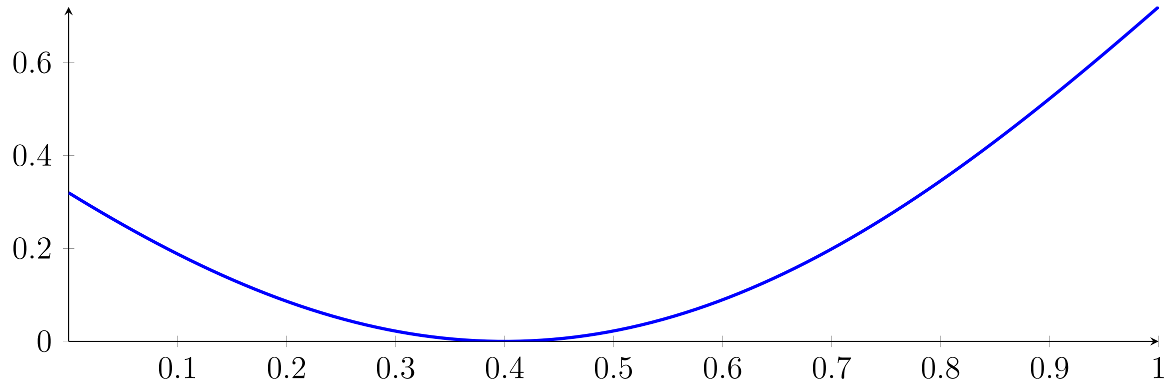
35.10 Recursion Branch 1 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 8!

35.11 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

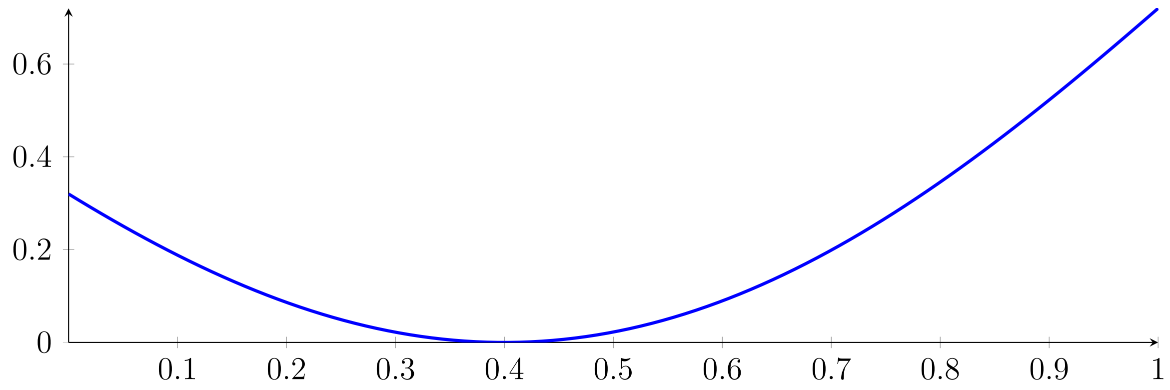
with precision $\varepsilon = 1{\cdot}10^{-32}$.

36 Running CubeClip on h_4 with epsilon 32

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called CubeClip with input polynomial on interval $[0, 1]$:

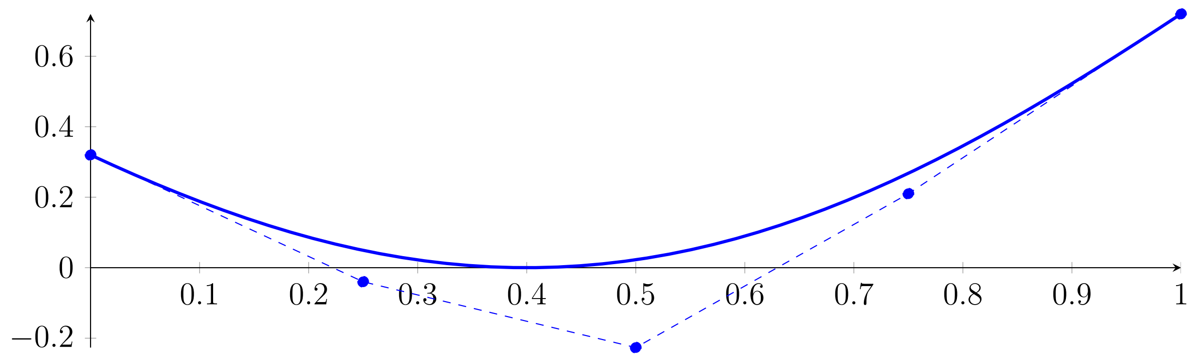
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



36.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

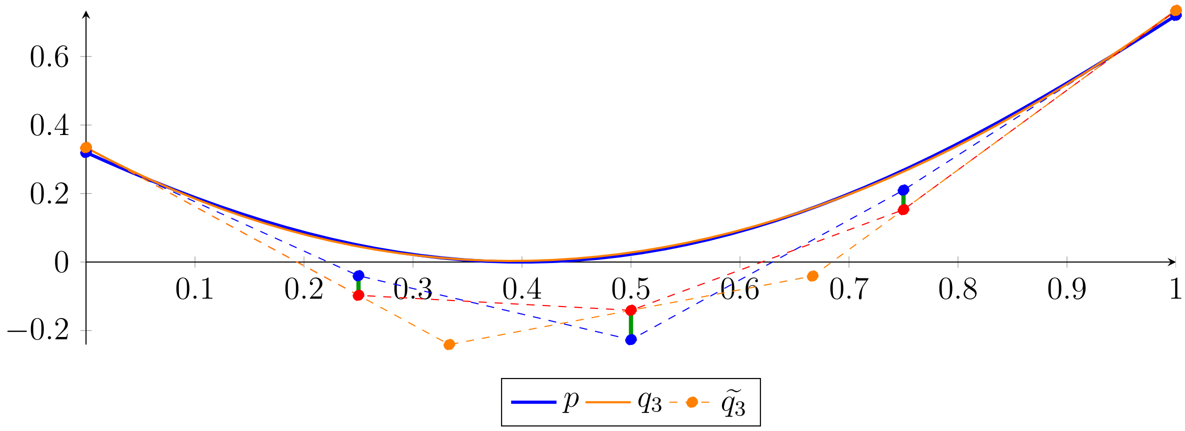
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3} \\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -2.781342323134002 \cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2 \\ &\quad - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4} \\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.

Bounding polynomials M and m:

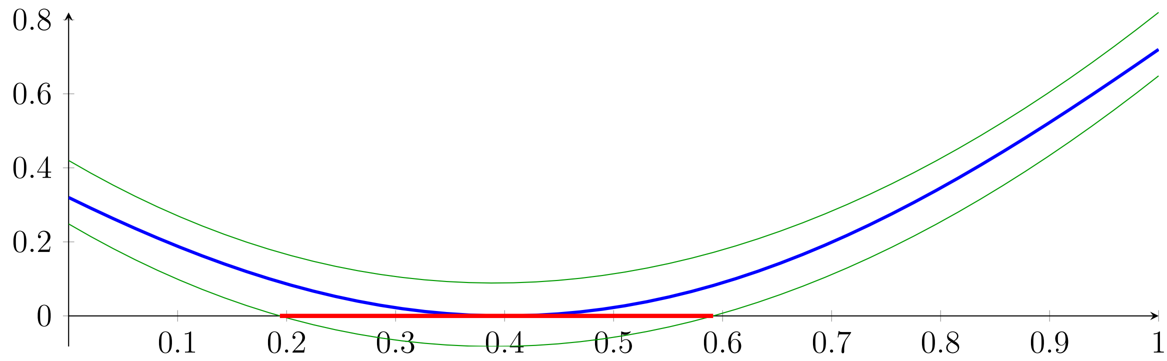
$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$

$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



$[0.1938265012447289, 0.5913474001559605]$

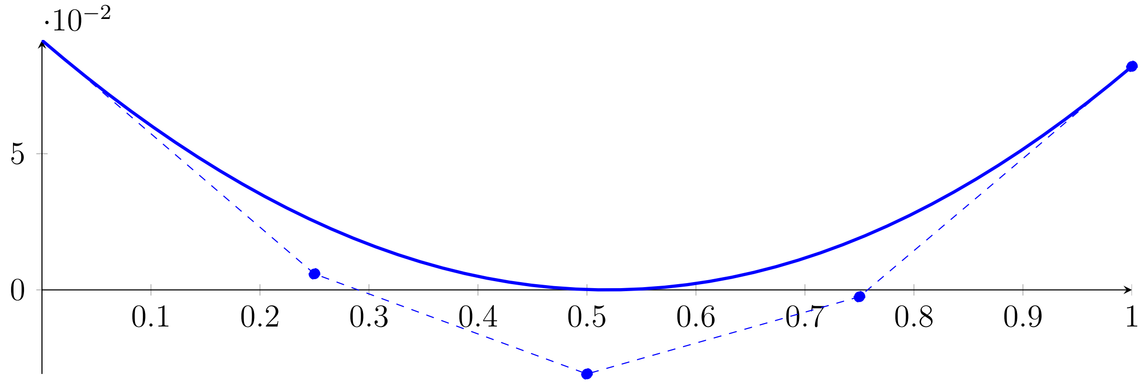
Longest intersection interval: 0.3975208989112316

$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

36.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

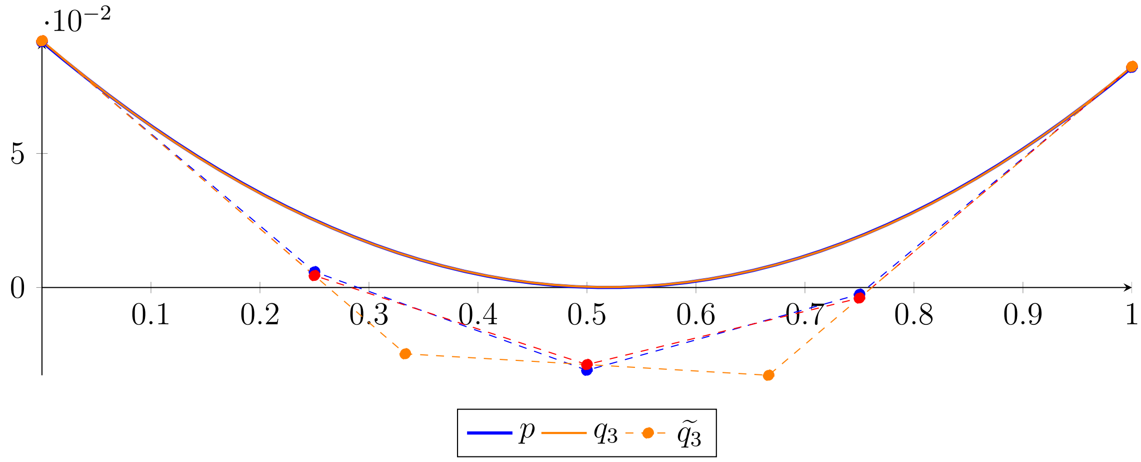
$$\begin{aligned} p = &-0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &+ 0.2941201876709534X^2 - 0.343099134331 92X + 0.09165715738594469 \\ = &\ 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &- 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502\cdot 10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455446609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.

Bounding polynomials M and m:

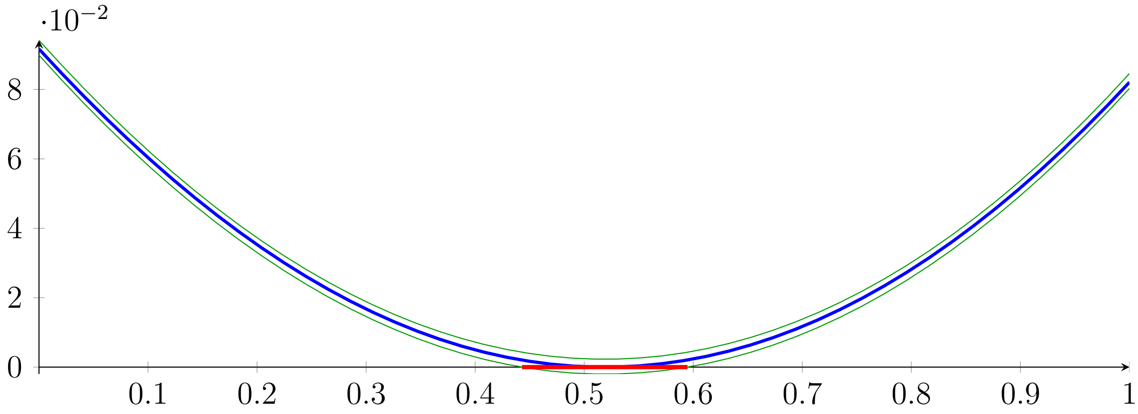
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



$[0.4429176870393937, 0.5947109255489168]$

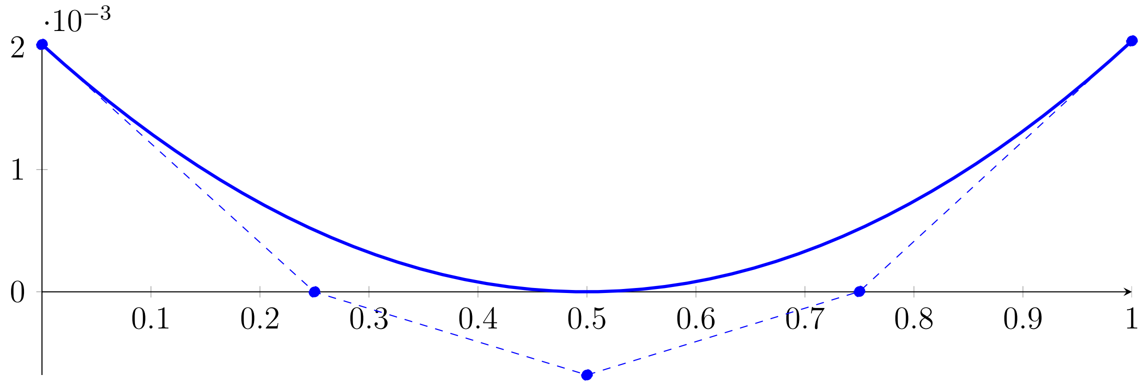
Longest intersection interval: 0.1517932385095231

$\Longrightarrow$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

36.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

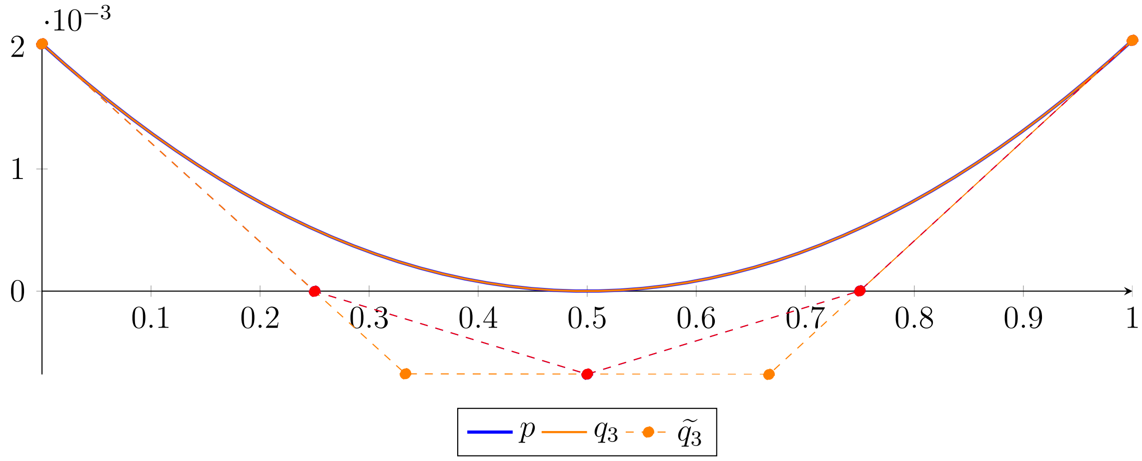
$$
\begin{aligned}
p = &-1.325713168422468 \cdot 10^{-05} X^4 + 7.039695732710913 \cdot 10^{-05} X^3 \\
&+ 0.008070351522992233 X^2 - 0.008098671960928044 X + 0.002023786815863734 \\
= &\ 0.002023786815863734 B_{0,4}(X) - 8.811743682771854 \cdot 10^{-07} B_{1,4}(X) \\
&- 0.0006804905774349 16 B_{2,4}(X) + 2.557845995594498 \\
&\cdot 10^{-06} B_{3,4}(X) + 0.002052606203570807 B_{4,4}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = &\ 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\
&- 0.008102459712837822 X + 0.002023976203459223 \\
= &\ 0.002023976203459223 B_{0,3} - 0.0006768437008200514 B_{1,3} \\
&- 0.0006818648029039136 B_{2,3} + 0.002052795591166296 B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 = &-6.518771069845317 \cdot 10^{-311} X^4 + 4.388269395865976 \cdot 10^{-05} X^3 \\
&+ 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002023976203459223 \\
= &\ 0.002023976203459223 B_{0,4} - 1.638724750232882 \cdot 10^{-06} B_{1,4} - 0.0006793542518619825 B_{2,4} \\
&+ 1.800295613638801 \cdot 10^{-06} B_{3,4} + 0.002052795591166296 B_{4,4}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544 \cdot 10^{-06}$.
Bounding polynomials M and m:

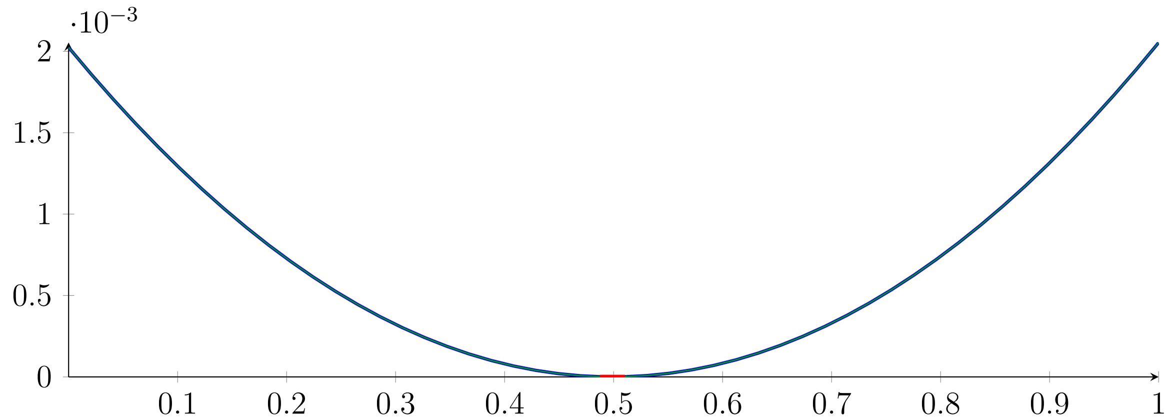
$$M = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002025112529032156$$

$$m = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002022839877886289$$

Root of M and m:

$N(M) = \{-185.2936165687423\}$ $N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649\}$

Intersection intervals:



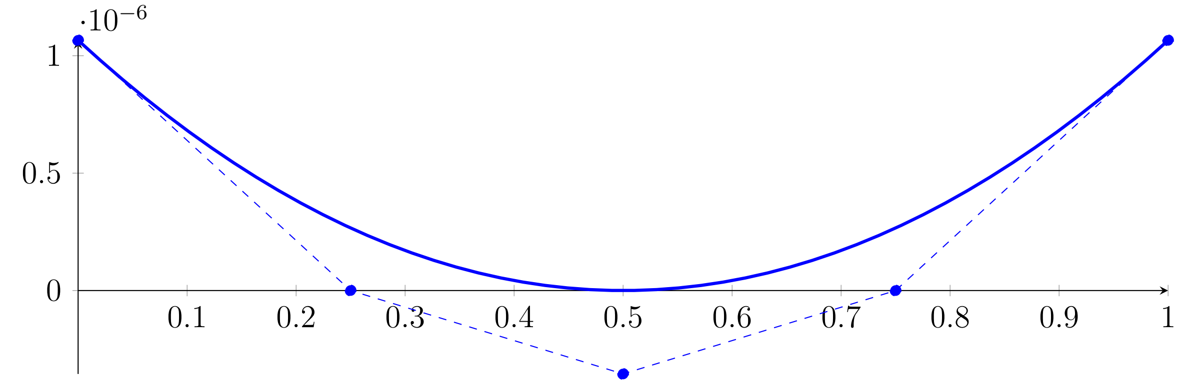
$$[0.4874742490566883, 0.5103358670775649]$$

Longest intersection interval: 0.02286161802087665
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

36.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.40068970704716$

Normalized monomial und Bézier representations and the Bézier polygon:

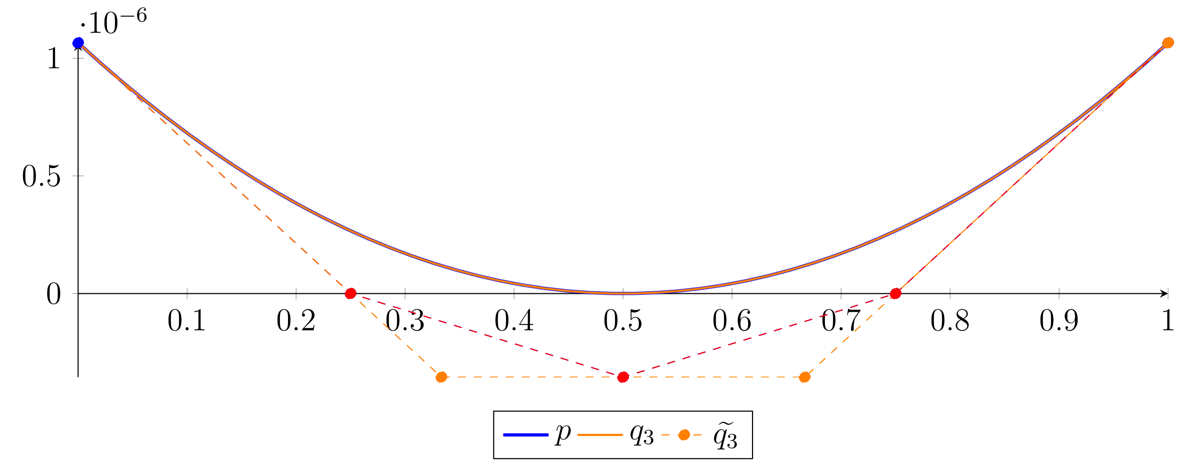
$$p = -3.621407750870066 \cdot 10^{-12} X^4 + 5.322780243378263 \cdot 10^{-10} X^3 + 4.261926231315171 \\ \cdot 10^{-06} X^2 - 4.262596936226815 \cdot 10^{-06} X + 1.065750606194374 \cdot 10^{-06} \\ = 1.065750606194374 \cdot 10^{-06} B_{0,4}(X) + 1.013721376702048 \cdot 10^{-10} B_{1,4}(X) - 3.552268233665051 \\ \cdot 10^{-07} B_{2,4}(X) - 1.009108120675939 \cdot 10^{-10} B_{3,4}(X) + 1.065608557899316 \cdot 10^{-06} B_{4,4}(X)$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2\\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06}\\ &= 1.06575065792877\cdot 10^{-06}B_{0,3} - 3.551153323761442\cdot 10^{-07}B_{1,3}\\ &\quad - 3.553376935441088\cdot 10^{-07}B_{2,3} + 1.065608609633713\cdot 10^{-06}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -5.304989477413181\cdot 10^{-314}X^4 + 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\\ &\quad \cdot 10^{-06}X^2 - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06}\\ &= 1.06575065792877\cdot 10^{-06}B_{0,4} + 1.011652000844408\cdot 10^{-10}B_{1,4} - 3.552265129601265\\ &\quad \cdot 10^{-07}B_{2,4} - 1.011177496533579\cdot 10^{-10}B_{3,4} + 1.065608609633713\cdot 10^{-06}B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.104063786460057\cdot 10^{-13}$.
Bounding polynomials M and m:

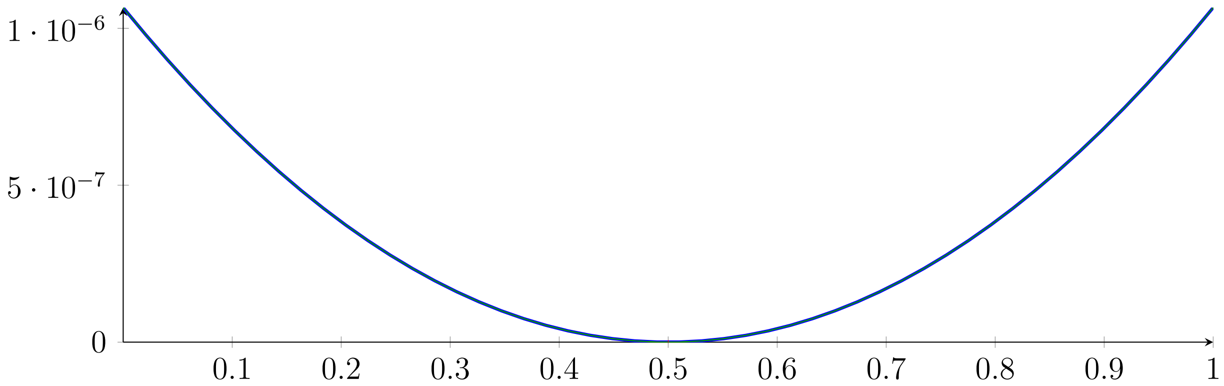
$$\begin{aligned}M &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2\\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750968335149\cdot 10^{-06}\end{aligned}$$

$$\begin{aligned}m &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2\\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750347522392\cdot 10^{-06}\end{aligned}$$

Root of M and m:

$N(M) = \{-8118.41926893212\}$ $\quad N(m) = \{-8118.419268932102, 0.4997707518163204, 0.5002933638554905\}$

Intersection intervals:



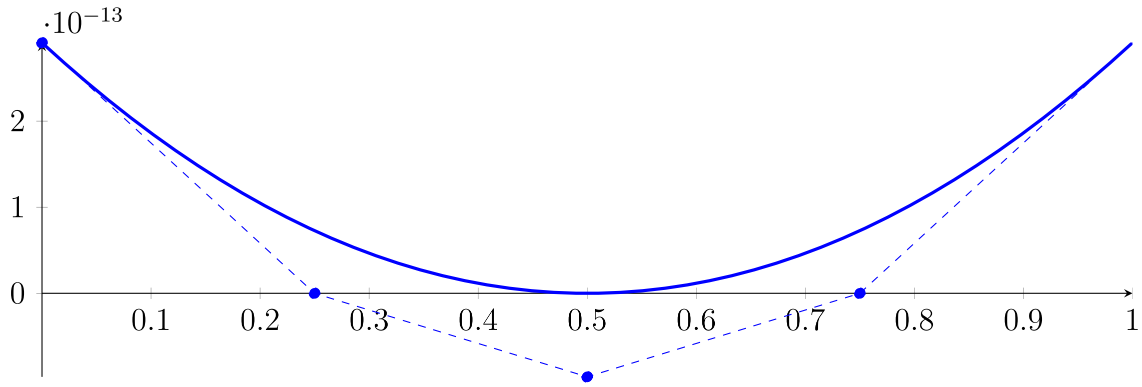
$$[0.4997707518163204, 0.5002933638554905]$$

Longest intersection interval: 0.0005226120391701048
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996445302967, 0.4000003654697068]$,

36.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999996445302967, 0.400000365469$

Normalized monomial und Bézier representations and the Bézier polygon:

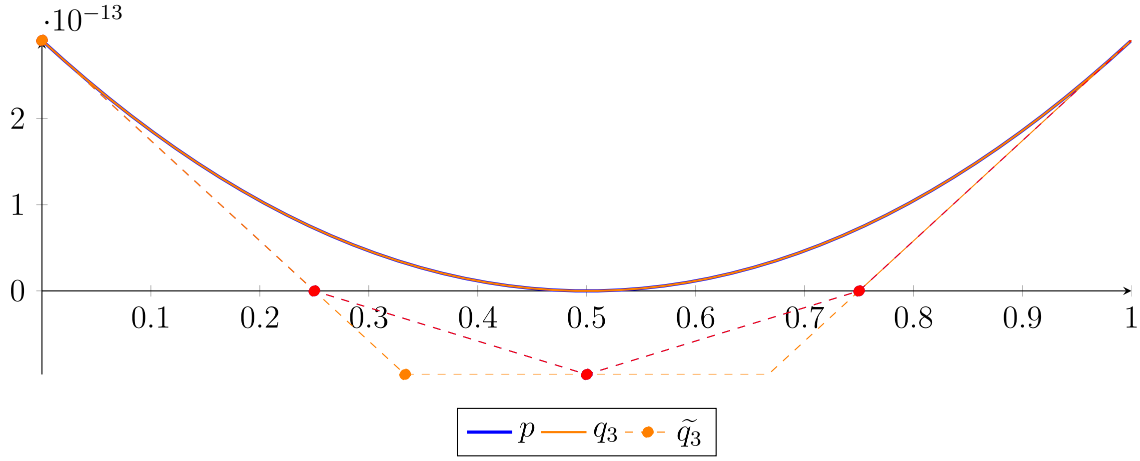
$$\begin{aligned}p &= -2.701438390310769\cdot10^{-25}X^4 + 7.494271205548112\cdot10^{-20}X^3 + 1.164248026057075\\&\quad\cdot10^{-12}X^2 - 1.164248076702198\cdot10^{-12}X + 2.91006022469042\cdot10^{-13}\\&= 2.91006022469042\cdot10^{-13}B_{0,4}(X) - 5.599670650739874\cdot10^{-17}B_{1,4}(X) - 9.707667820587771\\&\quad\cdot10^{-14}B_{2,4}(X) - 5.600329339091629\cdot10^{-17}B_{3,4}(X) + 2.910060467663608\cdot10^{-13}B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 7.494217176780306\cdot10^{-20}X^3 + 1.164248026057422\cdot10^{-12}X^2\\&\quad - 1.164248076702275\cdot10^{-12}X + 2.910060224690459\cdot10^{-13}\\&= 2.910060224690459\cdot10^{-13}B_{0,3} - 9.707666976504573\cdot10^{-14}B_{1,3}\\&\quad - 9.707668664666337\cdot10^{-14}B_{2,3} + 2.910060467663647\cdot10^{-13}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= -1.011846442682873\cdot10^{-320}X^4 + 7.494217176780306\cdot10^{-20}X^3 + 1.164248026057422\\&\quad\cdot10^{-12}X^2 - 1.164248076702275\cdot10^{-12}X + 2.910060224690459\cdot10^{-13}\\&= 2.910060224690459\cdot10^{-13}B_{0,4} - 5.599670652283553\cdot10^{-17}B_{1,4} - 9.707667820585455\\&\quad\cdot10^{-14}B_{2,4} - 5.600329340635308\cdot10^{-17}B_{3,4} + 2.910060467663647\cdot10^{-13}B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.315518620266374 \cdot 10^{-26}$.

Bounding polynomials M and m:

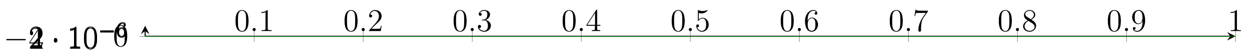
$$M = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ - 1.164248076702275 \cdot 10^{-12} X + 2.91006022469069 \cdot 10^{-13}$$

$$m = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690227 \cdot 10^{-13}$$

Root of M and m:

$N(M) = \{-15535286.38864162, 0.4930646005679288, 0.5069353931872718\}$ $N(m) = \{-15535286.38864162,$

Intersection intervals:



$$[0.4930646005650611, 0.4930646005679288], [0.5069353931872718, 0.5069353931901395]$$

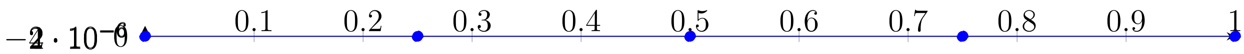
Longest intersection interval: $2.86768528910705 \cdot 10^{-12}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999999999, 0.399999999999999]$, interval 2: $[0.40000001, 0.400000$

36.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.399999999999999, 0.39999999999$

Normalized monomial und Bézier representations and the Bézier polygon:

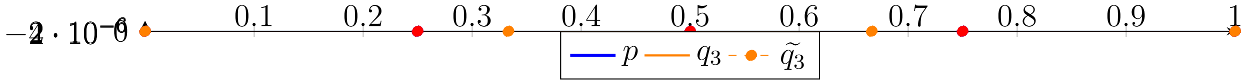
$$p = -1.826926271922849 \cdot 10^{-71} X^4 + 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \\ \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \\ = 2.367301365424222 \cdot 10^{-23} B_{0,4}(X) + 2.366143605869363 \cdot 10^{-23} B_{1,4}(X) + 2.364985846314663 \\ \cdot 10^{-23} B_{2,4}(X) + 2.363828086760123 \cdot 10^{-23} B_{3,4}(X) + 2.362670327205742 \cdot 10^{-23} B_{4,4}(X)$$



Degree reduction and raising:

$$q_3 = 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 \\ - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \\ = 2.367301365424222 \cdot 10^{-23} B_{0,3} + 2.365757686017743 \cdot 10^{-23} B_{1,3} \\ + 2.364214006611583 \cdot 10^{-23} B_{2,3} + 2.362670327205742 \cdot 10^{-23} B_{3,3}$$

$$\widetilde{q}_3 = 2.944860731609145 \cdot 10^{-331} X^4 + 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \\ \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \\ = 2.367301365424222 \cdot 10^{-23} B_{0,4} + 2.366143605869363 \cdot 10^{-23} B_{1,4} + 2.364985846314663 \\ \cdot 10^{-23} B_{2,4} + 2.363828086760123 \cdot 10^{-23} B_{3,4} + 2.362670327205742 \cdot 10^{-23} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 1.565936804505299 \cdot 10^{-72}$.
Bounding polynomials M and m:

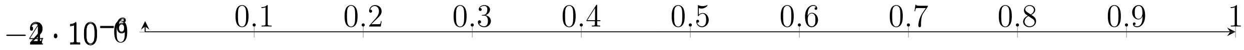
$$\begin{aligned} M = & \ 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 \\ & - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \end{aligned}$$

$$\begin{aligned} m = & \ 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 \\ & - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \end{aligned}$$

Root of M and m:

$N(M) = \{-5.417360610380978 \cdot 10^{18}, 0.4688611824150995, 4836929866.893817\}$ $N(m) = \{-5.417360610380$

Intersection intervals:



$$[0.4688611824150995, 0.4688611824150995]$$

Longest intersection interval: $6.762790809710196 \cdot 10^{-47}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999999999, 0.399999999999999]$,

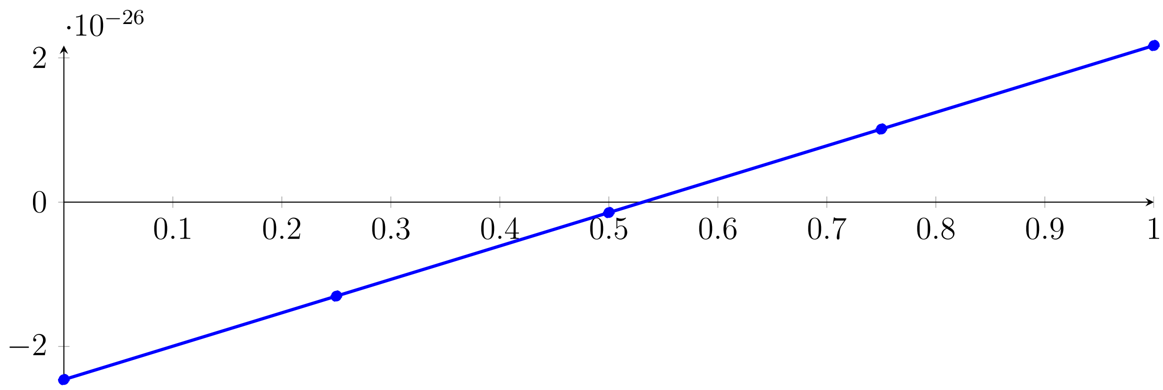
36.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.399999999999999, 0.399999999$

Reached interval $[0.399999999999999, 0.399999999999999]$ **without sign change** at depth 7!
p(0) = 2.365130051369057e-23 - p(1) 2.365130051369057e-23

36.8 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

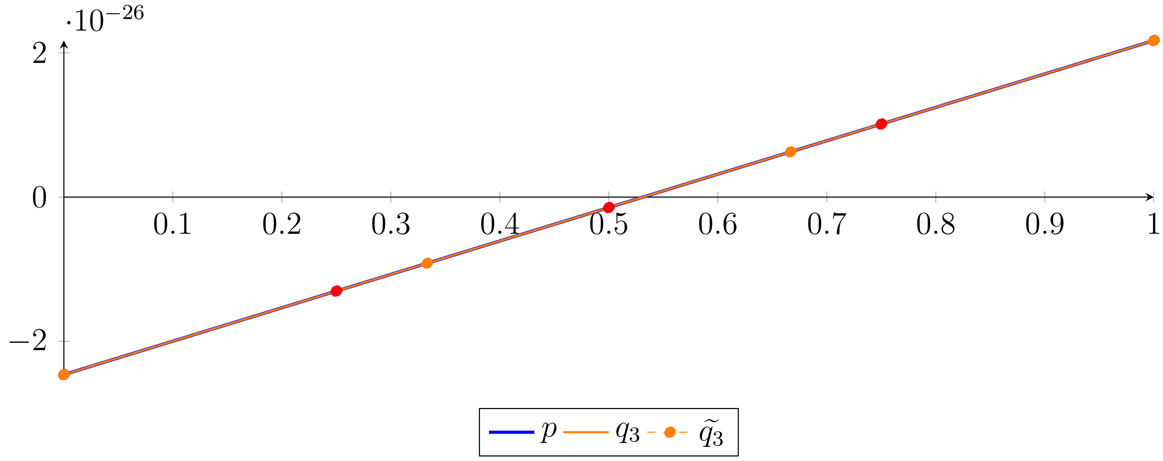
$$\begin{aligned} p = & -1.826926265398112 \cdot 10^{-71} X^4 + 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \\ & \cdot 10^{-36} X^2 + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ = & -2.459960157279009 \cdot 10^{-26} B_{0,4}(X) - 1.302200847385123 \cdot 10^{-26} B_{1,4}(X) - 1.444415373316643 \\ & \cdot 10^{-27} B_{2,4}(X) + 1.013317772881366 \cdot 10^{-26} B_{3,4}(X) + 2.171077083253969 \cdot 10^{-26} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= 1.767342394171994\cdot 10^{-54}X^3 + 9.574333011756007\cdot 10^{-36}X^2 \\
&\quad + 4.631037239575544\cdot 10^{-26}X - 2.459960157279009\cdot 10^{-26} \\
&= -2.459960157279009\cdot 10^{-26}B_{0,3} - 9.162810774204939\cdot 10^{-27}B_{1,3} \\
&\quad + 6.273980027571653\cdot 10^{-27}B_{2,3} + 2.171077083253969\cdot 10^{-26}B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -2.875840558212056\cdot 10^{-334}X^4 + 1.767342394171994\cdot 10^{-54}X^3 + 9.574333011756007 \\
&\quad \cdot 10^{-36}X^2 + 4.631037239575544\cdot 10^{-26}X - 2.459960157279009\cdot 10^{-26} \\
&= -2.459960157279009\cdot 10^{-26}B_{0,4} - 1.302200847385123\cdot 10^{-26}B_{1,4} - 1.444415373316643 \\
&\quad \cdot 10^{-27}B_{2,4} + 1.013317772881366\cdot 10^{-26}B_{3,4} + 2.171077083253969\cdot 10^{-26}B_{4,4}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.565936798912668\cdot 10^{-72}$.

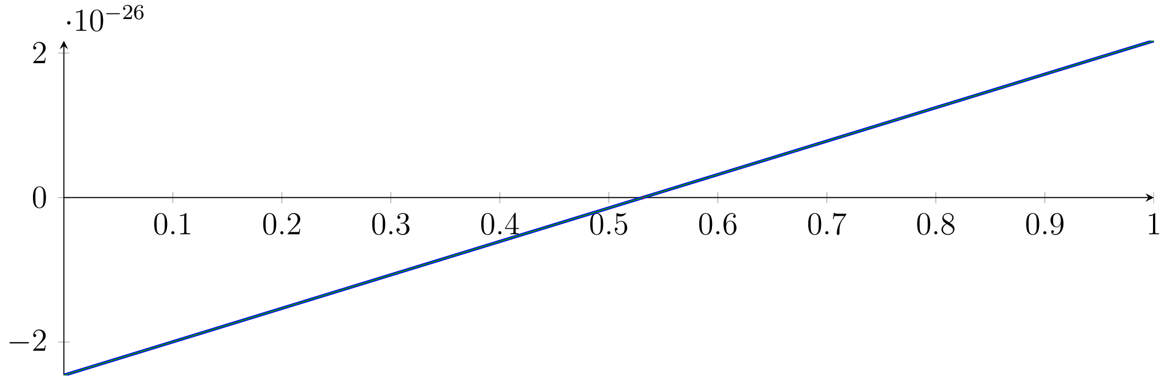
Bounding polynomials M and m:

$$\begin{aligned}
M &= 1.767342394171994\cdot 10^{-54}X^3 + 9.574333011756007\cdot 10^{-36}X^2 \\
&\quad + 4.631037239575544\cdot 10^{-26}X - 2.459960157279009\cdot 10^{-26} \\
m &= 1.767342394171994\cdot 10^{-54}X^3 + 9.574333011756007\cdot 10^{-36}X^2 \\
&\quad + 4.631037239575544\cdot 10^{-26}X - 2.459960157279009\cdot 10^{-26}
\end{aligned}$$

Root of M and m:

$N(M) = \{-5.417361703527237\cdot 10^{18}, -4836929870.212556, 0.5311898889490086\}$ $N(m) = \{-5.41736170352$

Intersection intervals:



$[0.5311898889490086, 0.5311898889490086]$

Longest intersection interval: $6.76279078555737\cdot 10^{-47}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

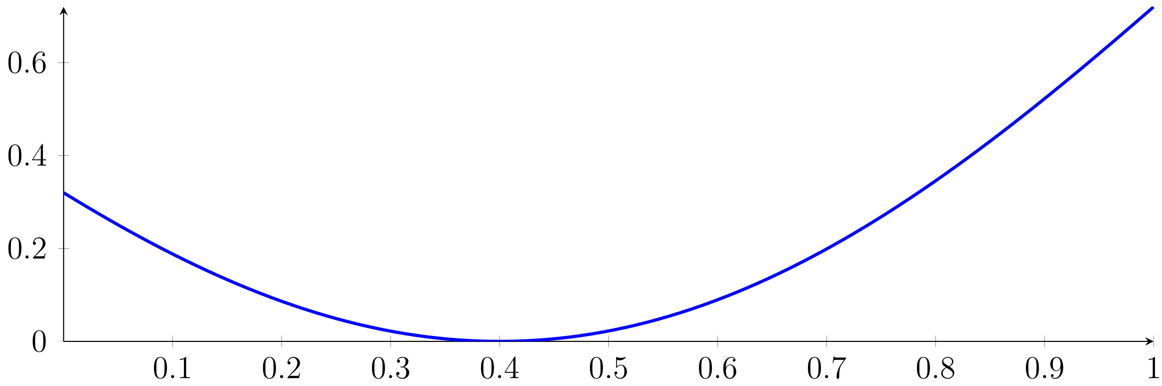
36.9 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 7!

36.10 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.40000001, 0.40000001]$$

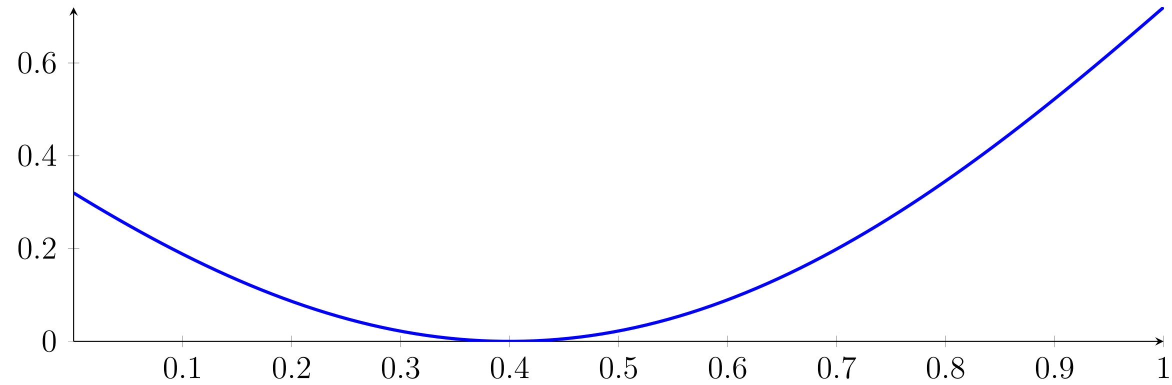
with precision $\varepsilon = 1 \cdot 10^{-32}$.

37 Running `BezClip` on h_4 with epsilon 64

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

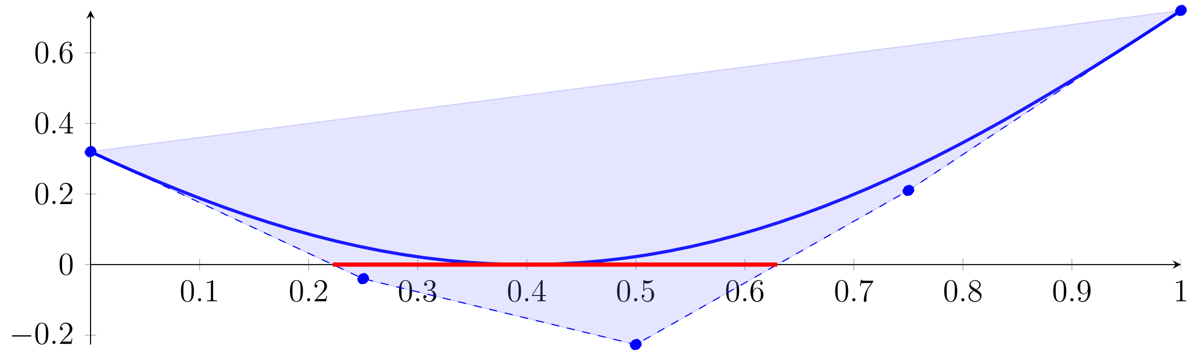
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



37.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

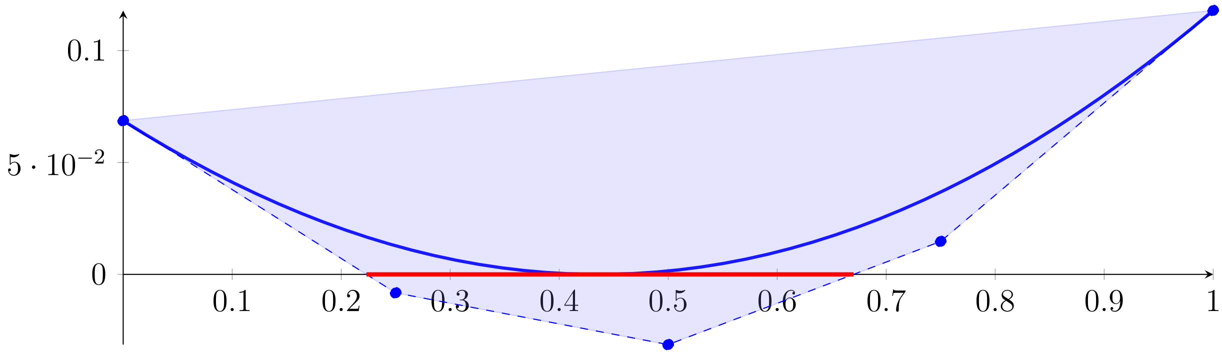
$$[0.222222225308642, 0.6297709955349339]$$

Longest intersection interval: 0.407548770226292
$\Longrightarrow$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

37.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &+ 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ = &\ 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &+ 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

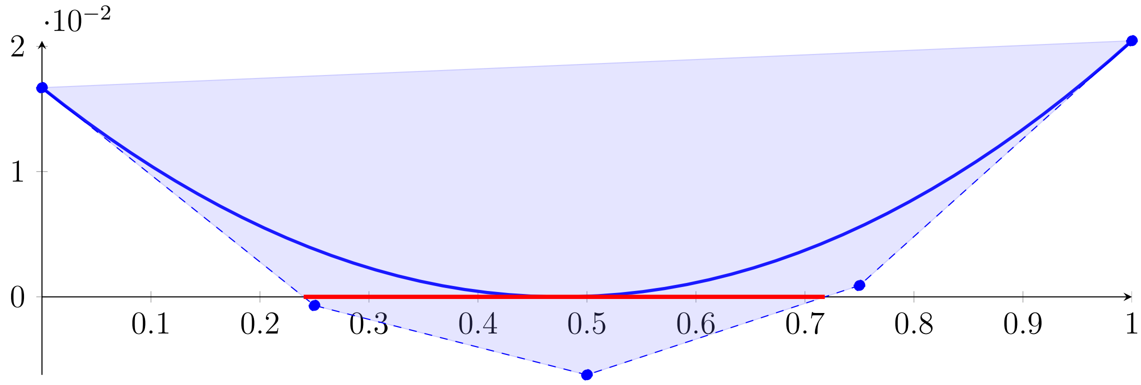
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\Longrightarrow$ Selective recursion: interval 1: $[0.3131782986367004, 0.4953216655933138]$,

37.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.495321665593313$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-0.0011006606529341\,82X^4 + 0.003307158935864367X^3\\ &+ 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377\\ = &\,0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X)\\ &+ 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

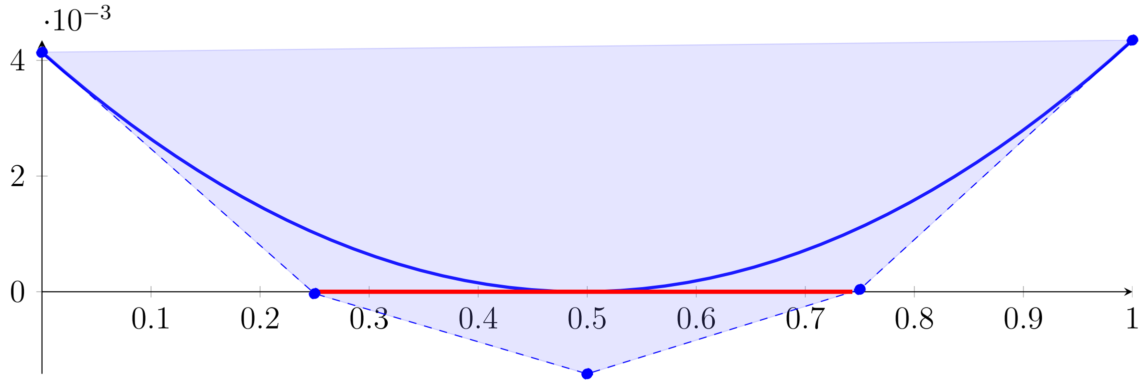
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\Longrightarrow$ Selective recursion: interval 1: $[0.3569093751201798, 0.4439937820951003]$,

37.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.44399378209510$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-5.751241374789061 \cdot 10^{-05} X^4 + 0.0002459162030906773 X^3 \\ &+ 0.01670691422385101 X^2 - 0.01668640845758443 X + 0.004139787469617577 \\ = &\; 0.004139787469617577 B_{0,4}(X) - 3.181464477852956 \cdot 10^{-05} B_{1,4}(X) \\ &- 0.001418931055199467 B_{2,4}(X) + 3.991728912743387 \\ &\cdot 10^{-05} B_{3,4}(X) + 0.004348697025226952 B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

$$[0.2480933797192248, 0.7431594518918532]$$

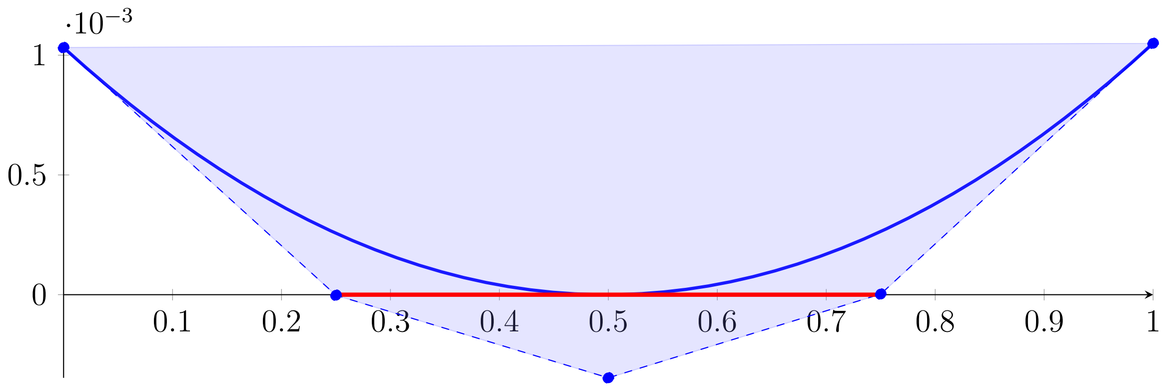
Longest intersection interval: 0.4950660721726284

$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

37.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-3.454731120985874 \cdot 10^{-06} X^4 + 2.291337271769576 \cdot 10^{-05} X^3 \\ &+ 0.004134358001933594 X^2 - 0.004136159738306538 X + 0.001031853315293836 \\ = &\; 0.001031853315293836 B_{0,4}(X) - 2.186619282798525 \cdot 10^{-06} B_{1,4}(X) \\ &- 0.0003471668868705005 B_{2,4}(X) + 2.640855710153786 \\ &\cdot 10^{-06} B_{3,4}(X) + 0.001049510220517602 B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

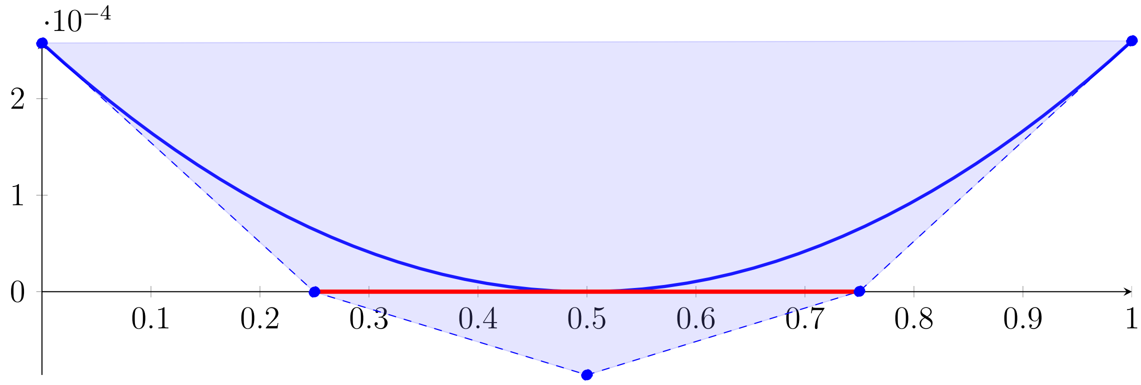
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\implies$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

37.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.4107674724$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-2.135832675829806\cdot 10^{-07}X^4 + 2.413460912088167\cdot 10^{-06}X^3 \\ &+ 0.001031922910124016X^2 - 0.001031832710654165X + 0.0002576480709401398 \\ = &\; 0.0002576480709401398B_{0,4}(X) - 3.101067234014613\cdot 10^{-07}B_{1,4}(X) - 8.628113269960673 \\ &\cdot 10^{-05}B_{2,4}(X) + 3.383582395460159\cdot 10^{-07}B_{3,4}(X) + 0.0002599381480544958B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

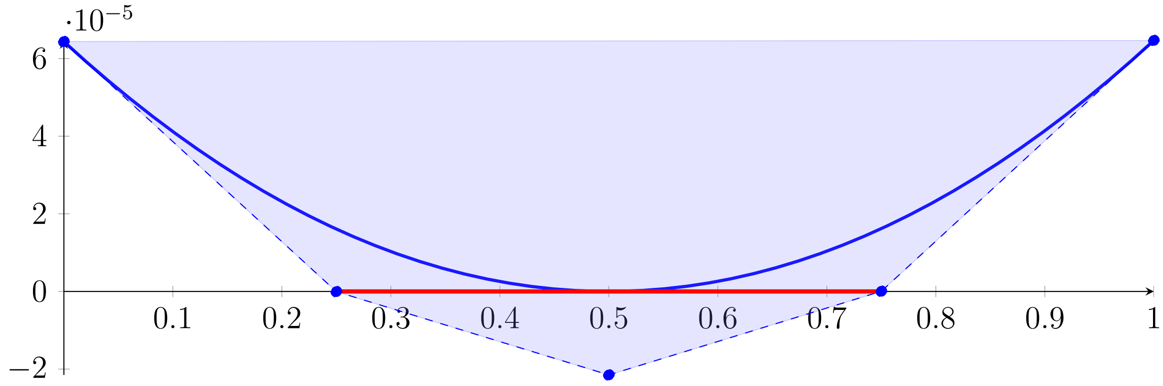
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\implies$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

37.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.405372055$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.327690666746824\cdot 10^{-08}X^4 + 2.739027984571277\cdot 10^{-07}X^3 \\ &+ 0.0002577144304079747X^2 - 0.0002576778286693267X + 6.437695235811399\cdot 10^{-05} \\ = &\; 6.437695235811399\cdot 10^{-05}B_{0,4}(X) - 4.250480921768141\cdot 10^{-08}B_{1,4}(X) - 2.150955690855369 \\ &\cdot 10^{-05}B_{2,4}(X) + 4.427175972025921\cdot 10^{-08}B_{3,4}(X) + 6.467417998855097\cdot 10^{-05}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

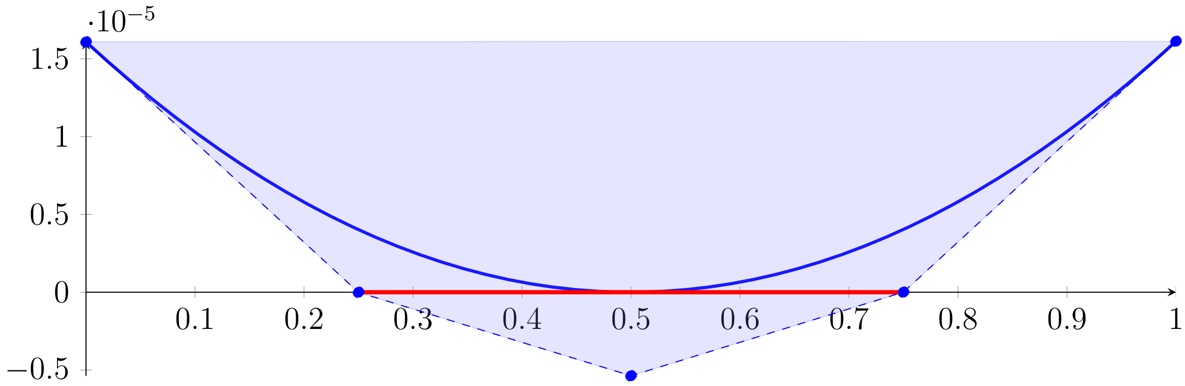
$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\Longrightarrow$ Selective recursion: interval 1: $[0.3973195510833879, 0.4026829657906133]$,

37.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-8.274952589981882\cdot 10^{-10}X^4 + 3.251124603913595\cdot 10^{-08}X^3 + 6.438882283687421 \\ &\cdot 10^{-05}X^2 - 6.438267481175771\cdot 10^{-05}X + 1.609012302857716\cdot 10^{-05} \\ = &\ 1.609012302857716\cdot 10^{-05}B_{0,4}(X) - 5.545674362270907\cdot 10^{-09}B_{1,4}(X) - 5.369743904489329 \\ &\cdot 10^{-06}B_{2,4}(X) + 5.656149705765249\cdot 10^{-09}B_{3,4}(X) + 1.61279548044738\cdot 10^{-05}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499138638713212, 0.7497369428484978\}$$

Intersection intervals with the x axis:

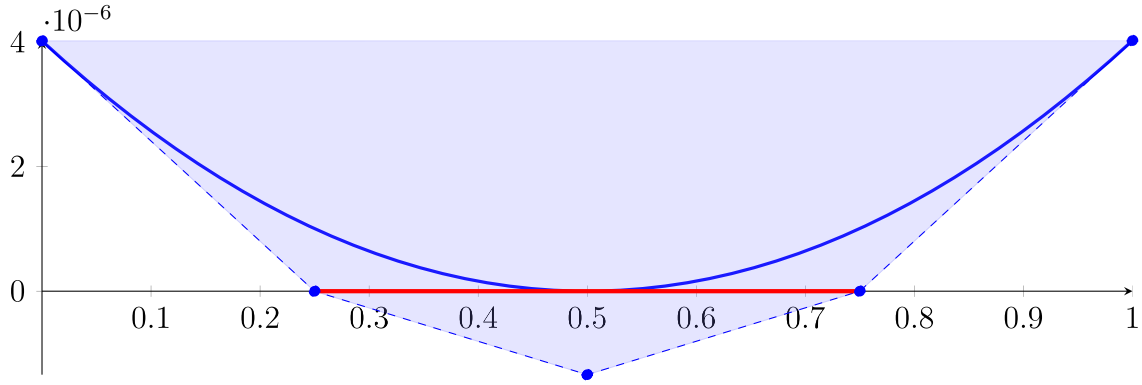
$$[0.2499138638713212, 0.7497369428484978]$$

Longest intersection interval: 0.4998230789771766
$\Longrightarrow$ Selective recursion: interval 1: $[0.3986599427764149, 0.4013407012292117]$,

37.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3986599427764149, 0.40134$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -5.164529187662443 \cdot 10^{-11} X^4 + 3.956301794548619 \cdot 10^{-09} X^3 + 1.609182796552748 \cdot 10^{-05} X^2 - 1.609096222935268 \cdot 10^{-05} X + 4.022033038444256 \cdot 10^{-06}$$
$$= 4.022033038444256 \cdot 10^{-06} B_{0,4}(X) - 7.075188939141538 \cdot 10^{-10} B_{1,4}(X) - 1.341476748644171 \cdot 10^{-06} B_{2,4}(X) + 7.14424642122371 \cdot 10^{-10} B_{3,4}(X) + 4.026803431121726 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499560300444542, 0.7498669294180401\}$$

Intersection intervals with the x axis:

$$[0.2499560300444542, 0.7498669294180401]$$

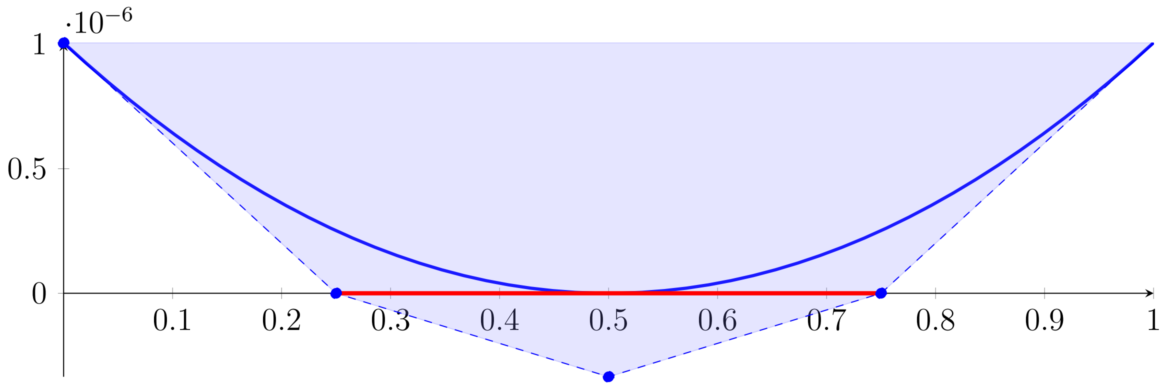
Longest intersection interval: 0.499910899373586

$\Longrightarrow$ Selective recursion: interval 1: $[0.3993300145167841, 0.4006701548859251]$,

37.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3993300145167841, 0.400$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.225530543299082 \cdot 10^{-12} X^4 + 4.878223136378425 \cdot 10^{-10} X^3 + 4.022259900669466 \cdot 10^{-06} X^2 - 4.022145641341714 \cdot 10^{-06} X + 1.00544708348311 \cdot 10^{-06}$$
$$= 1.00544708348311 \cdot 10^{-06} B_{0,4}(X) - 8.932685231834447 \cdot 10^{-11} B_{1,4}(X) - 3.352490870761692 \cdot 10^{-07} B_{2,4}(X) + 8.975838996701131 \cdot 10^{-11} B_{3,4}(X) + 1.006045939593956 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499777912437083, 0.7499330838112102\}$$

Intersection intervals with the x axis:

$$[0.2499777912437083, 0.7499330838112102]$$

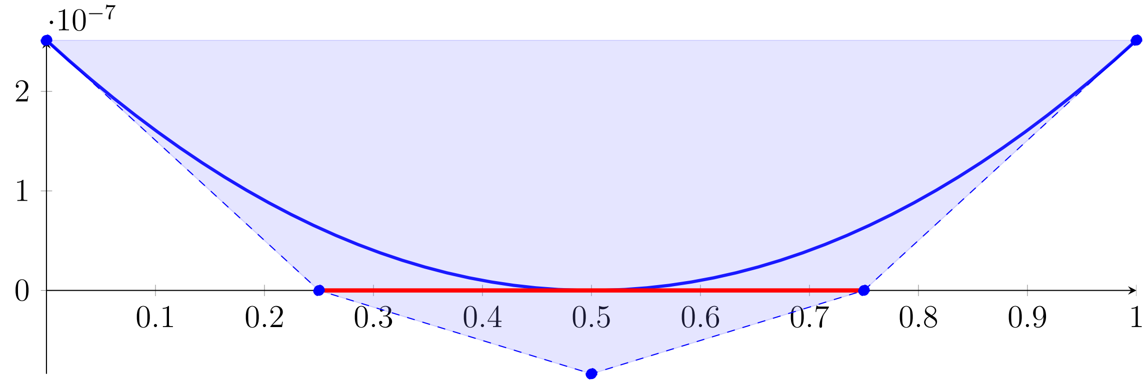
Longest intersection interval: 0.499955292567502

$\Longrightarrow$ Selective recursion: interval 1: $[0.3996650198462185, 0.4003350301165539]$,

37.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3996650198462185, 0.4

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.015235660316643 \cdot 10^{-13} X^4 + 6.055838633908679 \cdot 10^{-11} X^3 + 1.005476298211762 \cdot 10^{-06} X^2 - 1.005461639191386 \cdot 10^{-06} X + 2.51354188678016 \cdot 10^{-07}$$
$$= 2.51354188678016 \cdot 10^{-07} B_{0,4}(X) - 1.122111983053638 \cdot 10^{-11} B_{1,4}(X) - 8.379724788238336 \cdot 10^{-08} B_{2,4}(X) + 1.124798694225224 \cdot 10^{-11} B_{3,4}(X) + 2.51429204561165 \cdot 10^{-07} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499888398329751, 0.7499664473546936\}$$

Intersection intervals with the x axis:

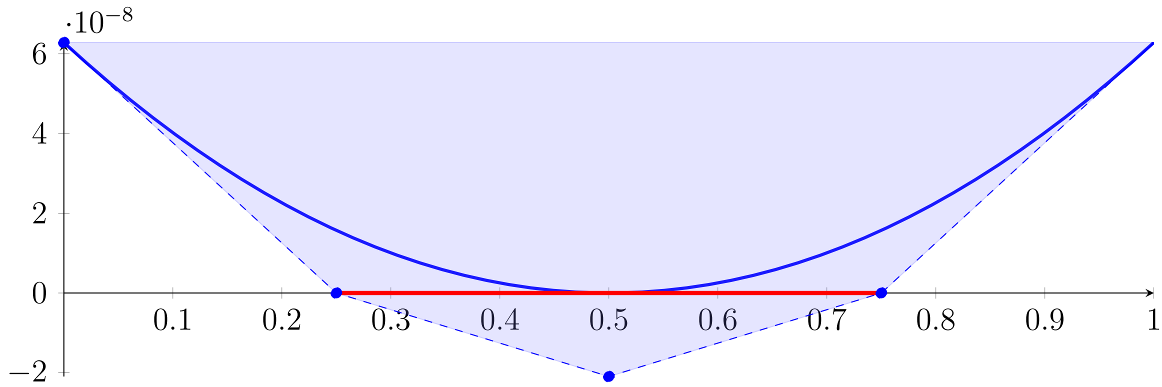
$$[0.2499888398329751, 0.7499664473546936]$$

Longest intersection interval: 0.4999776075217186
$\Longrightarrow$ Selective recursion: interval 1: [0.3998325149363758, 0.4001675050683531],

37.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3998325149363758, 0

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.259296672250957 \cdot 10^{-14} X^4 + 7.543595361590864 \cdot 10^{-12} X^3 + 2.5135789423507 \cdot 10^{-07} X^2 - 2.51356038307676 \cdot 10^{-07} X + 6.283760343276371 \cdot 10^{-08}$$
$$= 6.283760343276371 \cdot 10^{-08} B_{0,4}(X) - 1.406144155297956 \cdot 10^{-12} B_{1,4}(X) - 2.094743334856264 \cdot 10^{-08} B_{2,4}(X) + 1.407718382090997 \cdot 10^{-12} B_{3,4}(X) + 6.284699036255256 \cdot 10^{-08} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499944057673539, 0.7499832005219574\}$$

Intersection intervals with the x axis:

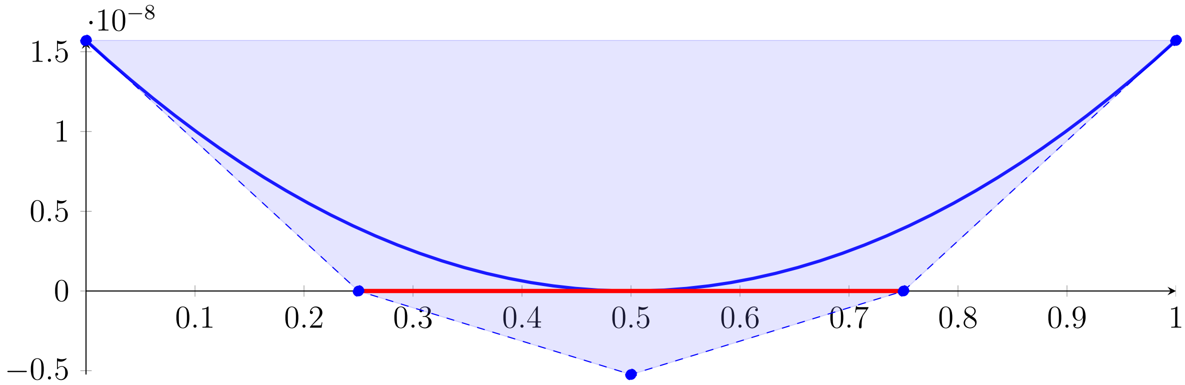
$$[0.2499944057673539, 0.7499832005219574]$$

Longest intersection interval: 0.4999887947546035
$\Longrightarrow$ Selective recursion: interval 1: [0.3999162605953574, 0.4000837519076994],

37.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999162605953574

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p &= -7.869898688873272\cdot 10^{-16}X^4 + 9.413120459510821\cdot 10^{-13}X^3 + 6.283807021204131\\
&\quad \cdot 10^{-08}X^2 - 6.283783670437638\cdot 10^{-08}X + 1.570928313186755\cdot 10^{-08}\\
&= 1.570928313186755\cdot 10^{-08}B_{0,4}(X) - 1.760442265467946\cdot 10^{-13}B_{1,4}(X) - 5.236623518313756\\
&\quad \cdot 10^{-09}B_{2,4}(X) + 1.76037617409066\cdot 10^{-13}B_{3,4}(X) + 1.571045716458857\cdot 10^{-08}B_{4,4}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.2499971984359141, 0.7499915961258623\}$$

Intersection intervals with the x axis:

$$[0.2499971984359141, 0.7499915961258623]$$

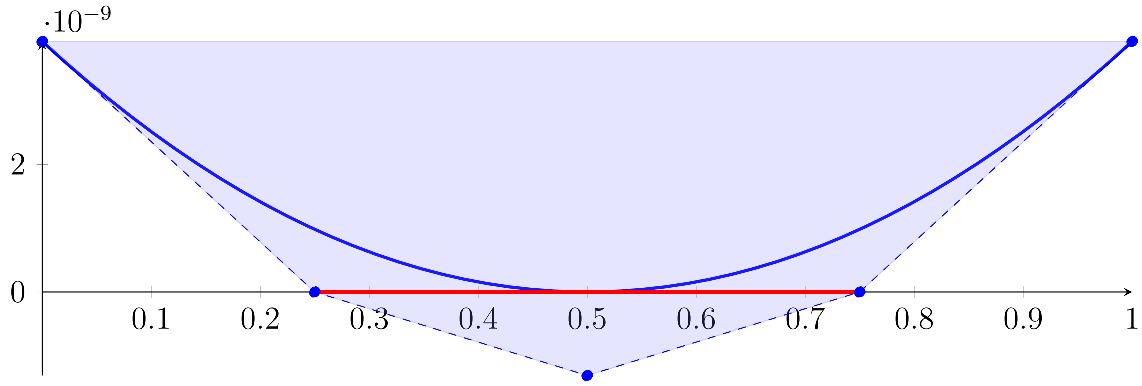
Longest intersection interval: 0.4999943976899482

$\Longrightarrow$ Selective recursion: interval 1: [0.3999581329542053, 0.400041877672038],

37.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999581329542053, 0.400041877672038]

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p &= -4.918466236188167\cdot 10^{-17}X^4 + 1.175616813221855\cdot 10^{-13}X^3 + 1.570934193292886\\
&\quad \cdot 10^{-08}X^2 - 1.57093126037727\cdot 10^{-08}X + 3.927306070737634\cdot 10^{-09}\\
&= 3.927306070737634\cdot 10^{-09}B_{0,4}(X) - 2.208020554223638\cdot 10^{-14}B_{1,4}(X) - 1.309126575660575\\
&\quad \cdot 10^{-09}B_{2,4}(X) + 2.19747928673869\cdot 10^{-14}B_{3,4}(X) + 3.927452912390452\cdot 10^{-09}B_{4,4}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.249998594451196, 0.749995803609747\}$$

Intersection intervals with the x axis:

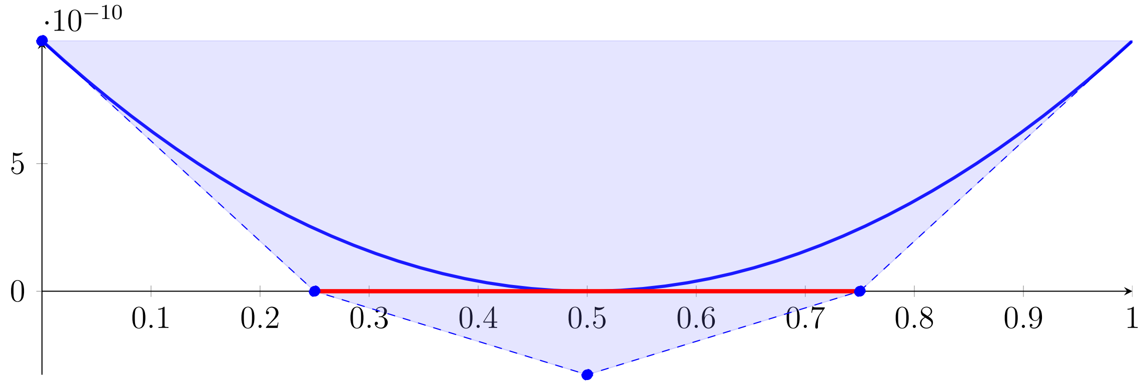
$$[0.249998594451196, 0.749995803609747]$$

Longest intersection interval: 0.499997209158551
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999790690159562, 0.4000209411411543]$,

37.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999790690159562, 0.4000209411411543]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.073972764895049 \cdot 10^{-18} X^4 + 1.468881614935617 \cdot 10^{-14} X^3 + 3.92731367890626 \\ &\quad \cdot 10^{-09} X^2 - 3.927309958033017 \cdot 10^{-09} X + 9.818246673938711 \cdot 10^{-10} \\ &= 9.818246673938711 \cdot 10^{-10} B_{0,4}(X) - 2.822114383159451 \cdot 10^{-15} B_{1,4}(X) - 3.272780318049275 \\ &\quad \cdot 10^{-10} B_{2,4}(X) + 2.710526275559223 \cdot 10^{-15} B_{3,4}(X) + 9.818430740092905 \cdot 10^{-10} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499992814128721, 0.7499979295098023\}$$

Intersection intervals with the x axis:

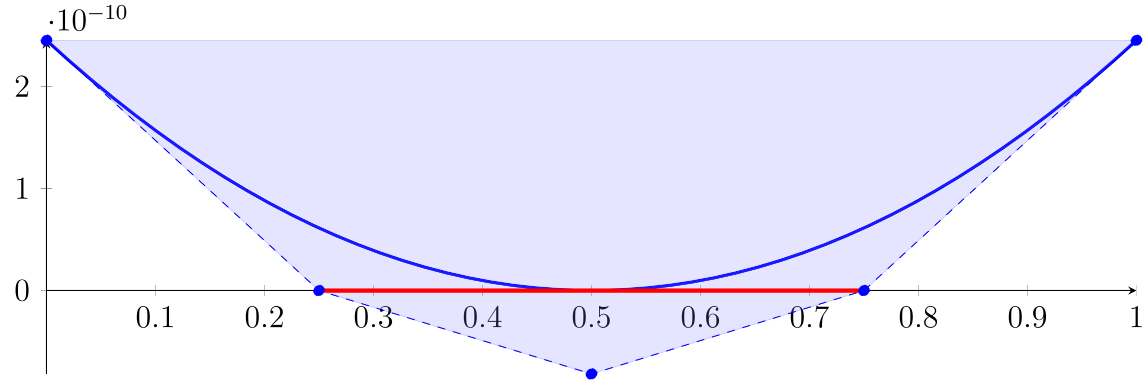
$$[0.2499992814128721, 0.7499979295098023]$$

Longest intersection interval: 0.4999986480969302
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999895370171669, 0.400010473023159]$,

37.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999895370171669, 0.400010473023159]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.921212199577591 \cdot 10^{-19} X^4 + 1.835702882943672 \cdot 10^{-15} X^3 + 9.818258642283608 \\ &\quad \cdot 10^{-10} X^2 - 9.818253497618774 \cdot 10^{-10} X + 2.454559233739064 \cdot 10^{-10} \\ &= 2.454559233739064 \cdot 10^{-10} B_{0,4}(X) - 4.140665629492134 \cdot 10^{-16} B_{1,4}(X) - 8.181910746897216 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.02092399485025 \cdot 10^{-16} B_{3,4}(X) + 2.454582733511515 \cdot 10^{-10} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995782686167, 0.7499990769537415\}$$

Intersection intervals with the x axis:

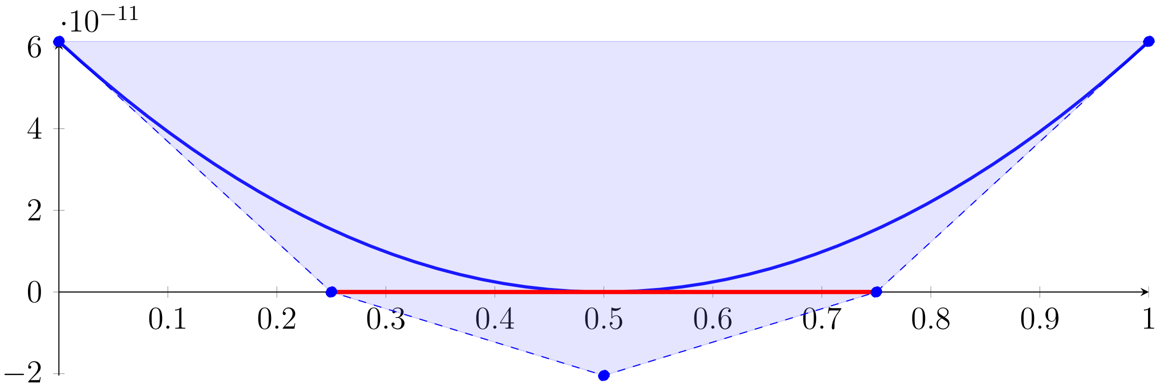
$$[0.2499995782686167, 0.7499990769537415]$$

Longest intersection interval: 0.4999994986851248
$\Longrightarrow$ Selective recursion: interval 1: [0.3999947710098356, 0.400005239002336],

37.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999947710098356, 0.400005239002336]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.200752809081967\cdot 10^{-20}X^4 + 2.294381551300316\cdot 10^{-16}X^3 + 2.454563180284348 \\ &\cdot 10^{-10}X^2 - 2.454562046981275\cdot 10^{-10}X + 6.136393816301913\cdot 10^{-11} \\ = &\ 6.136393816301913\cdot 10^{-11}B_{0,4}(X) - 1.130115127449974\cdot 10^{-16}B_{1,4}(X) - 2.045477784797216 \\ &\cdot 10^{-11}B_{2,4}(X) + 1.013179678980307\cdot 10^{-18}B_{3,4}(X) + 6.136428091947402\cdot 10^{-11}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995395858382, 0.7499999876168341\}$$

Intersection intervals with the x axis:

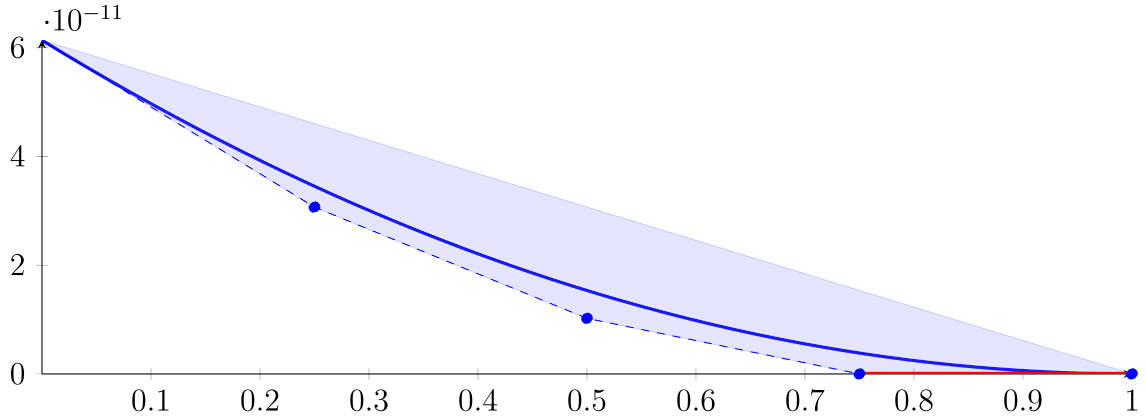
$$[0.2499995395858382, 0.7499999876168341]$$

Longest intersection interval: 0.5000004480309959
$\Longrightarrow$ Bisection: first half [0.3999947710098356, 0.4000000050060858] und second half [0.4000000050060858, 0.40

37.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3999947710098356, 0.4000000050060858]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.504705056762291\cdot 10^{-22}X^4 + 2.867976939125394\cdot 10^{-17}X^3 + 6.13640795071087\\ &\quad \cdot 10^{-11}X^2 - 1.227281023490638\cdot 10^{-10}X + 6.136393816301913\cdot 10^{-11}\\ &= 6.136393816301913\cdot 10^{-11}B_{0,4}(X) + 3.068191257575319\cdot 10^{-11}B_{1,4}(X) + 1.022723357300537\\ &\quad \cdot 10^{-11}B_{2,4}(X) - 9.167528198681642\cdot 10^{-17}B_{3,4}(X) - 5.59999170035142\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7499977590601706, 0.9999990874140811\}$$

Intersection intervals with the x axis:

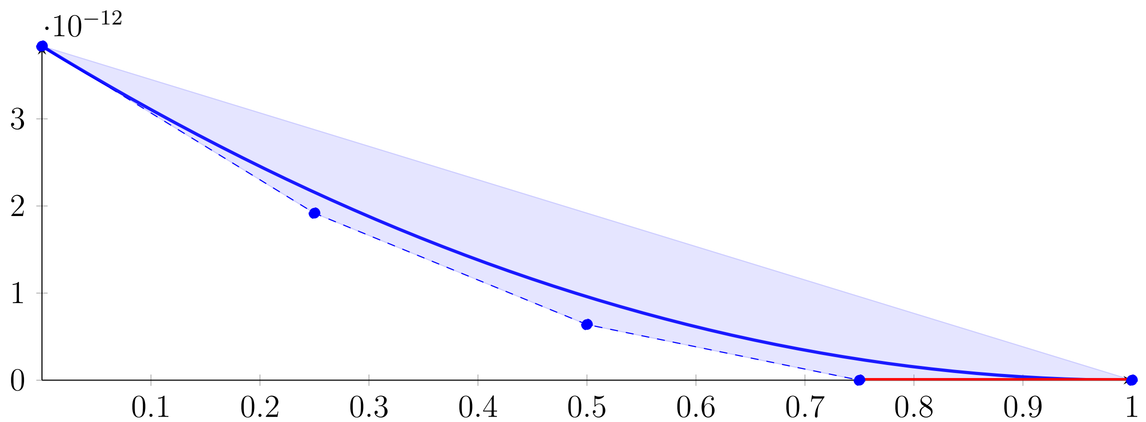
$$[0.7499977590601706, 0.9999990874140811]$$

Longest intersection interval: 0.2500013283539104
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999986964952942, 0.4000000050013093]$,

37.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999986964952942, 0.4000000050013093]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.931587718946285\cdot 10^{-24}X^4 + 4.480933611814181\cdot 10^{-19}X^3 + 3.835299758875134\\ &\quad \cdot 10^{-12}X^2 - 7.670593186649561\cdot 10^{-12}X + 3.835236979687871\cdot 10^{-12}\\ &= 3.835236979687871\cdot 10^{-12}B_{0,4}(X) + 1.917588683025481\cdot 10^{-12}B_{1,4}(X) + 6.391570128422797\\ &\quad \cdot 10^{-13}B_{2,4}(X) - 5.791883839216654\cdot 10^{-17}B_{3,4}(X) - 5.59999961259787\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7499773476668325, 0.9999853987696839\}$$

Intersection intervals with the x axis:

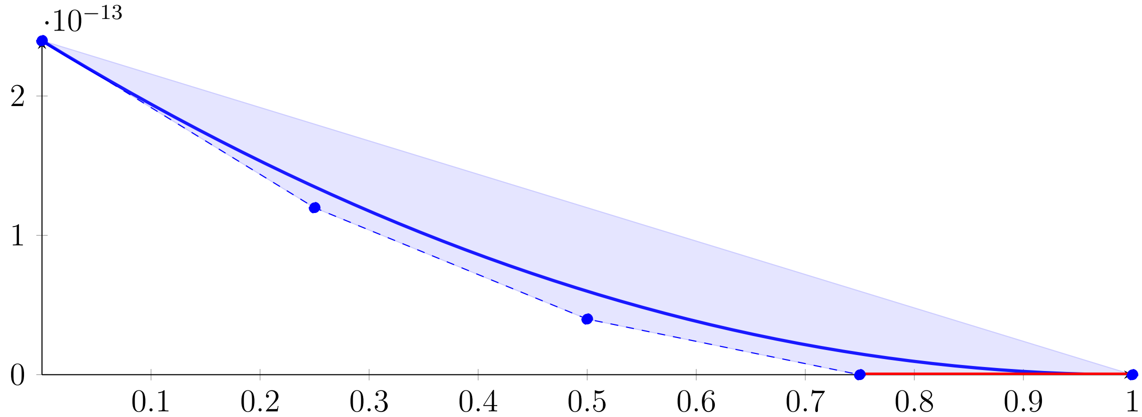
$$[0.7499773476668325, 0.9999853987696839]$$

Longest intersection interval: 0.2500080511028514
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996778451648, 0.4000000049822035]$,

37.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999996778451648, 0.4000000049822035]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.14529897555355\cdot 10^{-26}X^4 + 7.001997796519582\cdot 10^{-21}X^3 + 2.397217373893792\\&\quad \cdot 10^{-13}X^2 - 4.794695778397391\cdot 10^{-13}X + 2.396918341578483\cdot 10^{-13}\\&= 2.396918341578483\cdot 10^{-13}B_{0,4}(X) + 1.198244396979135\cdot 10^{-13}B_{1,4}(X) + 3.991066813620863\\&\quad \cdot 10^{-14}B_{2,4}(X) - 4.947877676695723\cdot 10^{-17}B_{3,4}(X) - 5.599929052523716\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7496904492313635, 0.9997664242061345\}$$

Intersection intervals with the x axis:

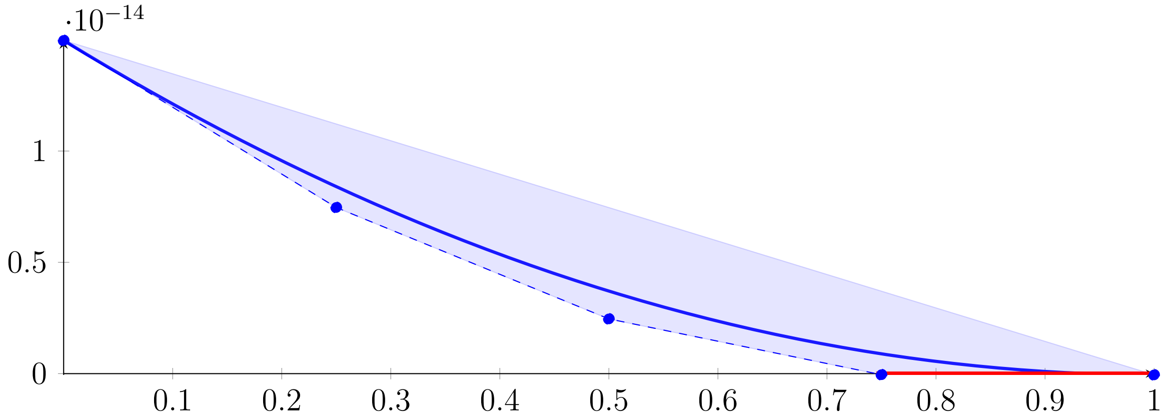
$$[0.7496904492313635, 0.9997664242061345]$$

Longest intersection interval: 0.250075974974771
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999230966783, 0.4000000049057922]$,

37.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999230966783, 0.4000000049057922]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.479264981640815\cdot 10^{-29}X^4 + 1.095054543732688\cdot 10^{-22}X^3 + 1.499171738187779\\&\quad \cdot 10^{-14}X^2 - 3.001796269578916\cdot 10^{-14}X + 1.497026508467358\cdot 10^{-14}\\&= 1.497026508467358\cdot 10^{-14}B_{0,4}(X) + 7.465774410726289\cdot 10^{-15}B_{1,4}(X) + 2.459903300425297\\&\quad \cdot 10^{-15}B_{2,4}(X) - 4.734821885303232\cdot 10^{-17}B_{3,4}(X) - 5.598011973238056\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7452788722542423, 0.9962745104335203\}$$

Intersection intervals with the x axis:

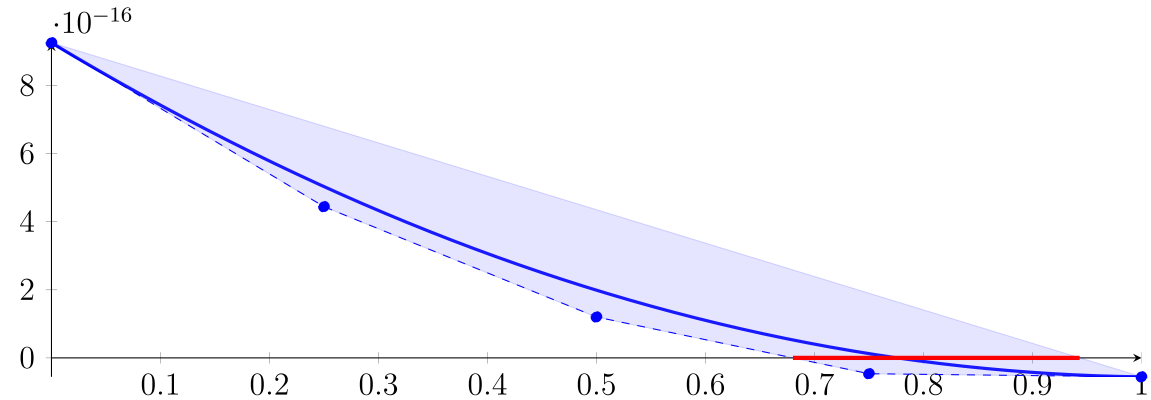
$$[0.7452788722542423, 0.9962745104335203]$$

Longest intersection interval: 0.250995638179278
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999840672825, 0.4000000046010132]$,

37.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999840672825, 0.4000000046010132]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.777753131478217 \cdot 10^{-31} X^4 + 1.731544849986219 \cdot 10^{-24} X^3 + 9.444603761111594 \\ &\cdot 10^{-16} X^2 - 1.925623994999771 \cdot 10^{-15} X + 9.25520203786835 \cdot 10^{-16} \\ = &\ 9.25520203786835 \cdot 10^{-16} B_{0,4}(X) + 4.441142050368923 \cdot 10^{-16} B_{1,4}(X) + 1.201182689721428 \\ &\cdot 10^{-16} B_{2,4}(X) - 4.646760397452725 \cdot 10^{-17} B_{3,4}(X) - 5.564341337023184 \cdot 10^{-17} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.6802647890355577, 0.9432883441688765\}$$

Intersection intervals with the x axis:

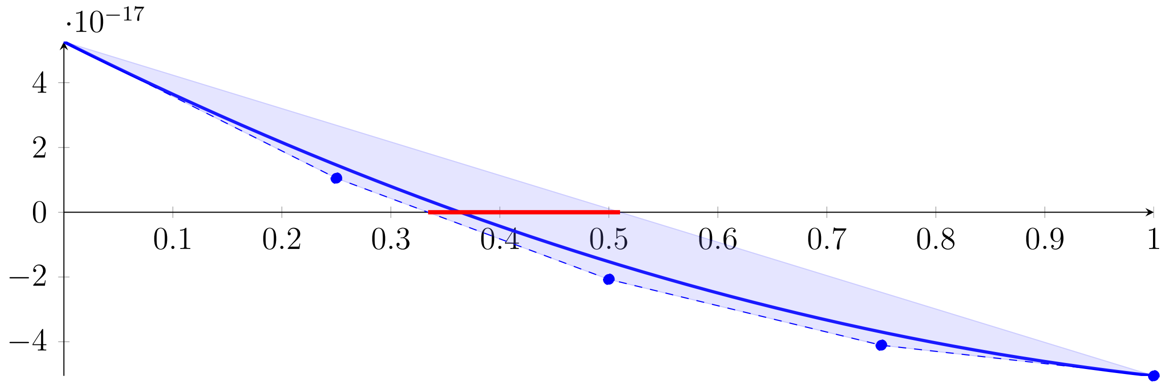
$$[0.6802647890355577, 0.9432883441688765]$$

Longest intersection interval: 0.2630235551333187
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999980356565, 0.4000000034365114]$,

37.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999980356565, 0.4000000034365114]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.508441684091975\cdot 10^{-34}X^4 + 3.150776186268732\cdot 10^{-26}X^3 + 6.533908238790805\\ &\quad\cdot 10^{-17}X^2 - 1.685080699755733\cdot 10^{-16}X + 5.264466028744181\cdot 10^{-17}\\ &= 5.264466028744181\cdot 10^{-17}B_{0,4}(X) + 1.05176427935485\cdot 10^{-17}B_{1,4}(X) - 2.071952763569348\\ &\quad\cdot 10^{-17}B_{2,4}(X) - 4.106685099240717\cdot 10^{-17}B_{3,4}(X) - 5.052432726871564\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3341757003677151, 0.5102760193201083\}$$

Intersection intervals with the x axis:

$$[0.3341757003677151, 0.5102760193201083]$$

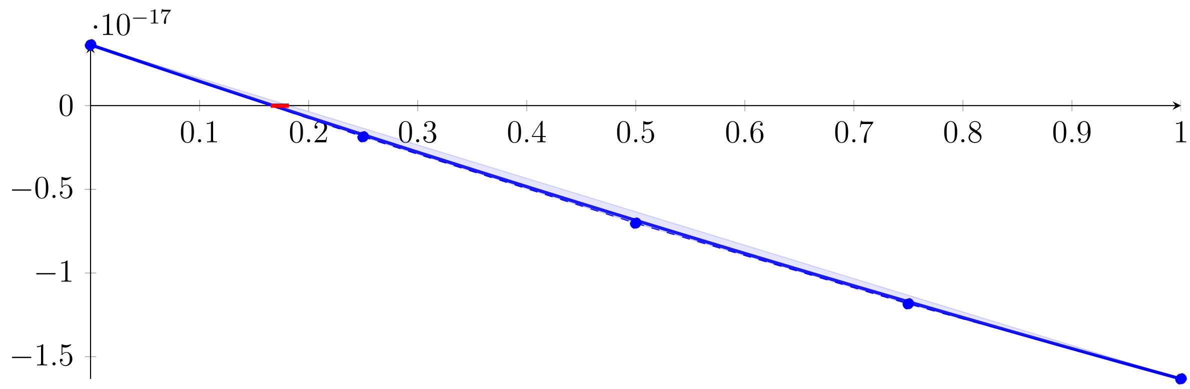
Longest intersection interval: 0.1761003189523932

$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999840491, 0.4000000007915832]$,

37.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.399999999840491, 0.4000000007915832]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.182586345704061\cdot 10^{-37}X^4 + 1.720671503885177\cdot 10^{-28}X^3 + 2.026251345992899\\ &\quad\cdot 10^{-18}X^2 - 2.198411775802323\cdot 10^{-17}X + 3.629995386177444\cdot 10^{-18}\\ &= 3.629995386177444\cdot 10^{-18}B_{0,4}(X) - 1.866034053328363\cdot 10^{-18}B_{1,4}(X) - 7.024354935168688\\ &\quad\cdot 10^{-18}B_{2,4}(X) - 1.184496725930051\cdot 10^{-17}B_{3,4}(X) - 1.632787102568082\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1651189929990553, 0.1818829383495937\}$$

Intersection intervals with the x axis:

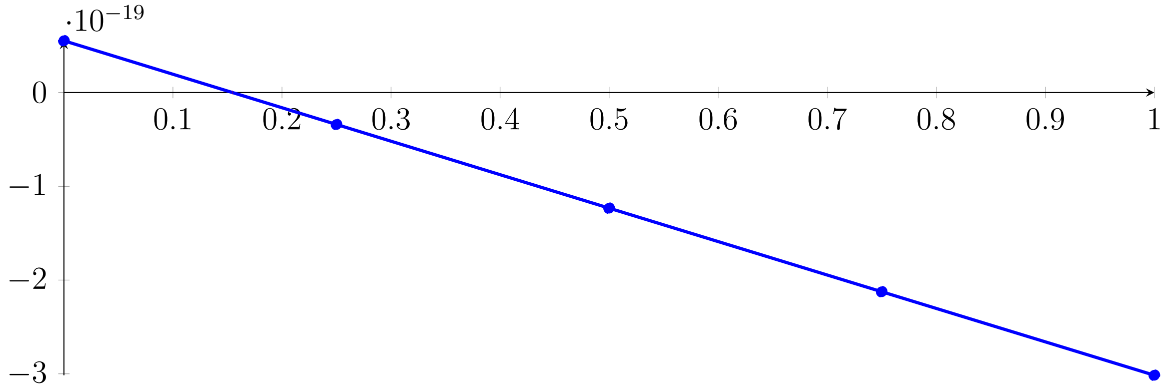
$$[0.1651189929990553, 0.1818829383495937]$$

Longest intersection interval: 0.01676394535053848
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999975344, 0.4000000000134784]$,

37.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999975344, 0.4000000000134784]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -6.462425394283182\cdot 10^{-44}X^4 + 8.106374699799114\cdot 10^{-34}X^3 + 5.694371396423764\\ &\quad\cdot 10^{-22}X^2 - 3.573230357204536\cdot 10^{-19}X + 5.524428779488534\cdot 10^{-20}\\ &= 5.524428779488534\cdot 10^{-20}B_{0,4}(X) - 3.408647113522805\cdot 10^{-20}B_{1,4}(X) - 1.23322323875401\\ &\quad\cdot 10^{-19}B_{2,4}(X) - 2.124632704256334\cdot 10^{-19}B_{3,4}(X) - 3.01509310785925\cdot 10^{-19}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1546060070924308, 0.1548527835869093\}$$

Intersection intervals with the x axis:

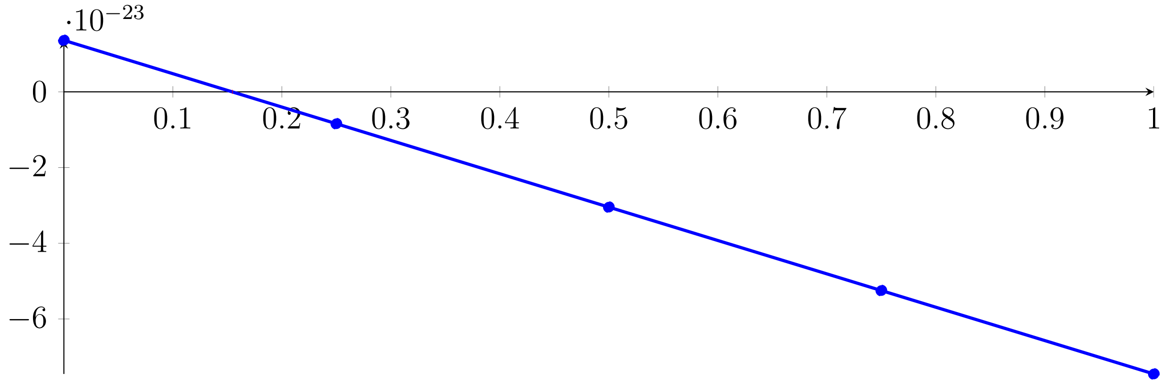
$$[0.1546060070924308, 0.1548527835869093]$$

Longest intersection interval: 0.000246776494478516
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999994, 0.4000000000000033]$,

37.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999994, 0.4000000000000033]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.396683605522666\cdot 10^{-58}X^4 + 1.218254561570257\cdot 10^{-44}X^3 + 3.467794636018148\\ &\quad\cdot 10^{-29}X^2 - 8.813547453484276\cdot 10^{-23}X + 1.36112658736315\cdot 10^{-23}\\ &= 1.36112658736315\cdot 10^{-23}B_{0,4}(X) - 8.422602760079189\cdot 10^{-24}B_{1,4}(X) - 3.045646561413215\\ &\quad\cdot 10^{-23}B_{2,4}(X) - 5.249032268852739\cdot 10^{-23}B_{3,4}(X) - 7.45241739832649\cdot 10^{-23}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544357246099643, 0.1544357853745533\}$$

Intersection intervals with the x axis:

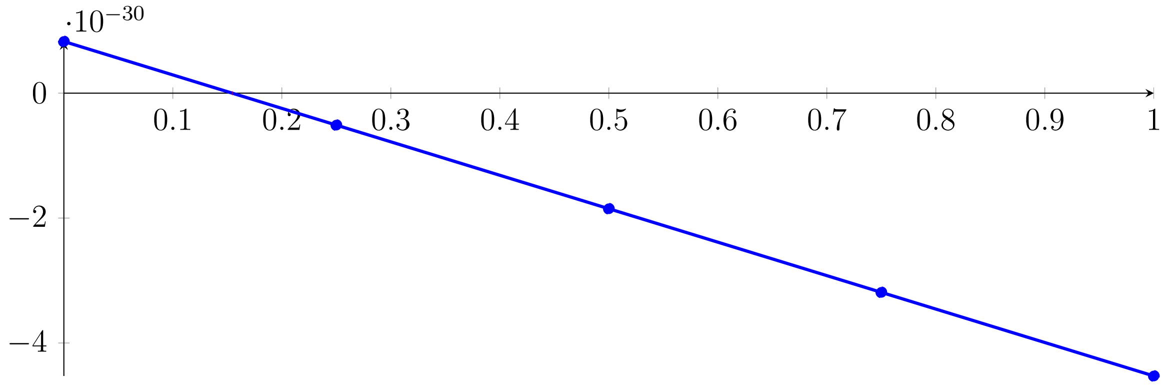
$$[0.1544357246099643, 0.1544357853745533]$$

Longest intersection interval: $6.076458894192547 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

37.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.267480184524245 \cdot 10^{-87} X^4 + 2.733315343334724 \cdot 10^{-66} X^3 + 1.280426044119387 \\ &\quad \cdot 10^{-43} X^2 - 5.355515230460074 \cdot 10^{-30} X + 8.270826503648829 \cdot 10^{-31} \\ &= 8.270826503648829 \cdot 10^{-31} B_{0,4}(X) - 5.117961572501356 \cdot 10^{-31} B_{1,4}(X) - 1.850674964865133 \\ &\quad \cdot 10^{-30} B_{2,4}(X) - 3.189553772480109 \cdot 10^{-30} B_{3,4}(X) - 4.528432580095063 \cdot 10^{-30} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138334, 0.1544356826138371\}$$

Intersection intervals with the x axis:

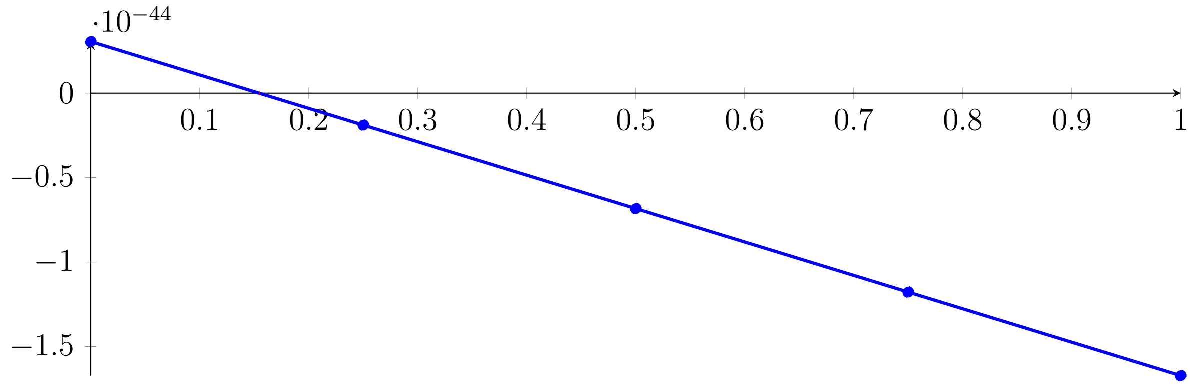
$$[0.1544356826138334, 0.1544356826138371]$$

Longest intersection interval: $3.692333261147783 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

37.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -6.073185304909619 \cdot 10^{-145} X^4 + 1.375917570883003 \cdot 10^{-109} X^3 + 1.745646428447031 \cdot 10^{-72} X^2 - 1.977434701601112 \cdot 10^{-44} X + 3.053864779660447 \cdot 10^{-45}$$
$$= 3.053864779660447 \cdot 10^{-45} B_{0,4}(X) - 1.889721974342333 \cdot 10^{-45} B_{1,4}(X) - 6.833308728345114 \cdot 10^{-45} B_{2,4}(X) - 1.177689548234789 \cdot 10^{-44} B_{3,4}(X) - 1.672048223635067 \cdot 10^{-44} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138309, 0.1544356826138309\}$$

Intersection intervals with the x axis:

$$[0.1544356826138309, 0.1544356826138309]$$

Longest intersection interval: $1.363332491137777 \cdot 10^{-29}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

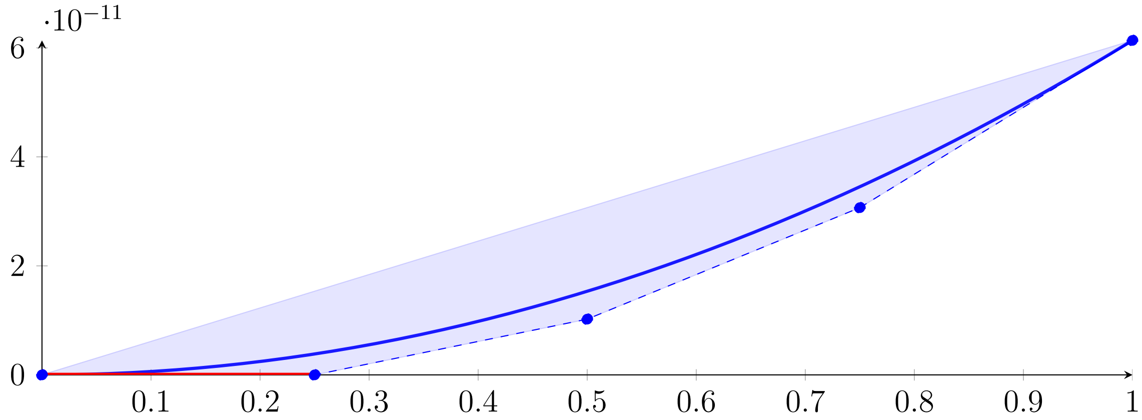
37.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 29!

37.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.4000000050060858, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -7.504705056762291 \cdot 10^{-22} X^4 + 2.867676750923124 \cdot 10^{-17} X^3 + 6.136416554191405 \cdot 10^{-11} X^2 + 1.427014599332089 \cdot 10^{-16} X - 5.59999170035142 \cdot 10^{-17}$$
$$= -5.59999170035142 \cdot 10^{-17} B_{0,4}(X) - 2.032455202021197 \cdot 10^{-17} B_{1,4}(X) + 1.02273762744653 \cdot 10^{-11} B_{2,4}(X) + 3.068214096632685 \cdot 10^{-11} B_{3,4}(X) + 6.136428091947402 \cdot 10^{-11} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{9.125808216110345e-07, 0.2500004968163653\}$$

Intersection intervals with the x axis:

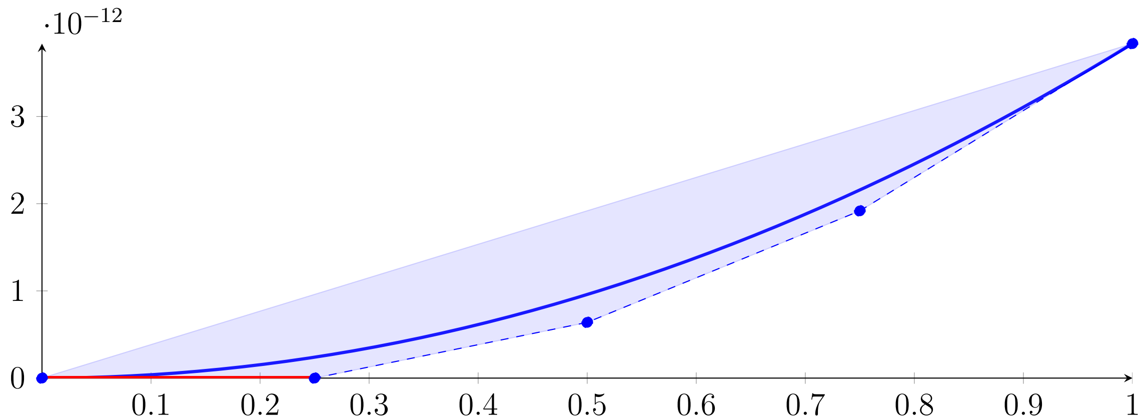
$$[9.125808216110345e-07, 0.2500004968163653]$$

Longest intersection interval: 0.2499995842355437
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050108623, 0.4000013135077487]$,

37.31 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.4000000050108623, 0.4000013135077487]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.931505911661308\cdot 10^{-24}X^4 + 4.480722567712799\cdot 10^{-19}X^3 + 3.835247589865681\\&\quad\cdot 10^{-12}X^2 + 6.367513939156292\cdot 10^{-17}X - 5.59997356725911\cdot 10^{-17}\\&= -5.59997356725911\cdot 10^{-17}B_{0,4}(X) - 4.008095082470037\cdot 10^{-17}B_{1,4}(X) + 6.391837694783034\\&\quad\cdot 10^{-13}B_{2,4}(X) + 1.917615663569776\cdot 10^{-12}B_{3,4}(X) + 3.835255713338726\cdot 10^{-12}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{1.460109108777016e-05, 0.2500156756317829\}$$

Intersection intervals with the x axis:

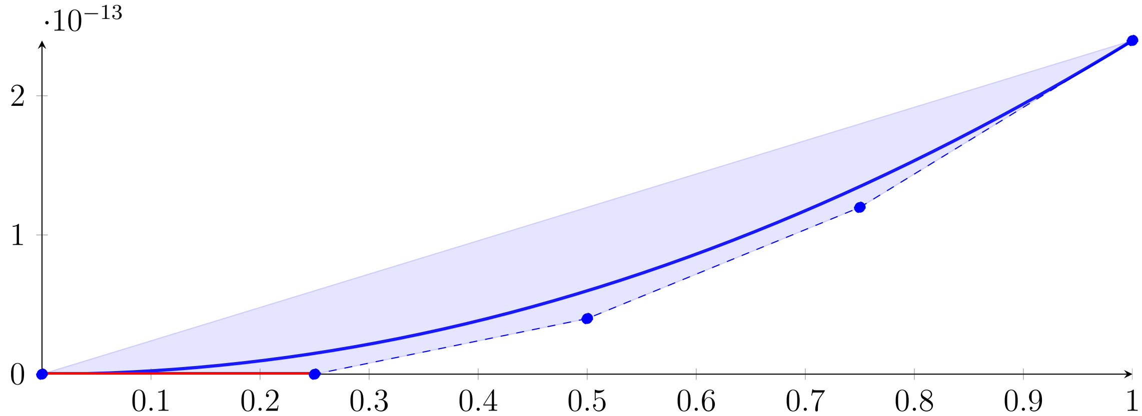
$$[1.460109108777016e-05, 0.2500156756317829]$$

Longest intersection interval: 0.2500010745406951
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050299677, 0.4000003321555954]$,

37.32 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.4000000050299677, 0.4000003321555954]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.14513918450963\cdot 10^{-26}X^4 + 7.00121928574046\cdot 10^{-21}X^3 + 2.397050349370657\\ &\quad \cdot 10^{-13}X^2 + 4.391837331735849\cdot 10^{-17}X - 5.599798830250992\cdot 10^{-17}\\ &= -5.599798830250992\cdot 10^{-17}B_{0,4}(X) - 4.50183949731703\cdot 10^{-17}B_{1,4}(X) + 3.991680035453379\\ &\quad \cdot 10^{-14}B_{2,4}(X) + 1.198294600105232\cdot 10^{-13}B_{3,4}(X) + 2.396929623232884\cdot 10^{-13}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.0002335692644077866, 0.2502816337968459\}$$

Intersection intervals with the x axis:

$$[0.0002335692644077866, 0.2502816337968459]$$

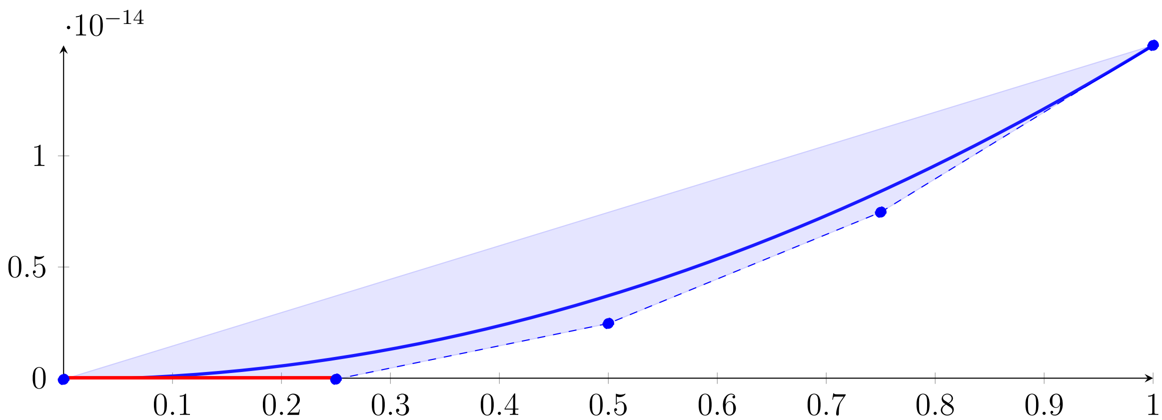
Longest intersection interval: 0.2500480645324381

$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000051063742, 0.4000000869035043]$,

37.33 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.4000000051063742, 0.4000000869035043]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.476640967897802\cdot 10^{-29}X^4 + 1.09457158991016\cdot 10^{-22}X^3 + 1.49873258928534\\ &\quad \cdot 10^{-14}X^2 + 3.898095063625474\cdot 10^{-17}X - 5.597465330775527\cdot 10^{-17}\\ &= -5.597465330775527\cdot 10^{-17}B_{0,4}(X) - 4.622941564869158\cdot 10^{-17}B_{1,4}(X) + 2.461403470819272\\ &\quad \cdot 10^{-15}B_{2,4}(X) + 7.466924033460424\cdot 10^{-15}B_{3,4}(X) + 1.497033229963901\cdot 10^{-14}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.003725110466798912, 0.2546088699723713\}$$

Intersection intervals with the x axis:

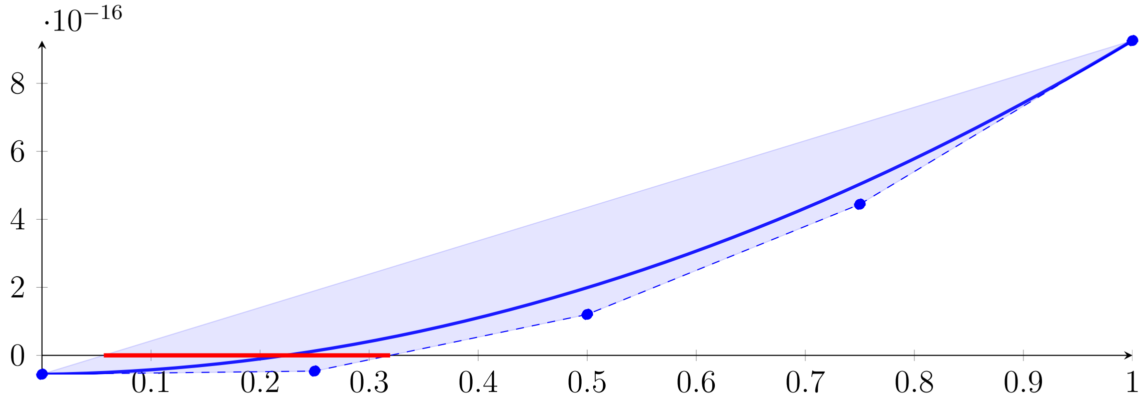
$$[0.003725110466798912, 0.2546088699723713]$$

Longest intersection interval: 0.2508837595055724
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000054110776, 0.4000000259326491]$,

37.34 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.4000000054110776, 0.4000000259326491]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.773546014711518\cdot 10^{-31}X^4 + 1.728469879701229\cdot 10^{-24}X^3 + 9.433421698048765\\ &\quad \cdot 10^{-16}X^2 + 3.779308932689389\cdot 10^{-17}X - 5.562147411226954\cdot 10^{-17}\\ &= -5.562147411226954\cdot 10^{-17}B_{0,4}(X) - 4.617320178054607\cdot 10^{-17}B_{1,4}(X) + 1.204987655186568\\ &\quad \cdot 10^{-16}B_{2,4}(X) + 4.443944282174566\cdot 10^{-16}B_{3,4}(X) + 9.255137867479705\cdot 10^{-16}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.05669093378980359, 0.3192576000162909\}$$

Intersection intervals with the x axis:

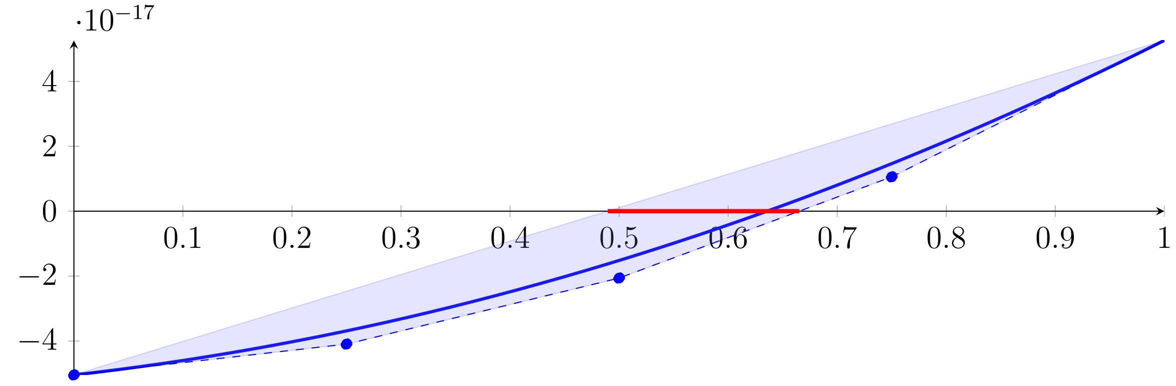
$$[0.05669093378980359, 0.3192576000162909]$$

Longest intersection interval: 0.2625666662264874
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000065744646, 0.4000000119627452]$,

37.35 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.4000000065744646, 0.4000000119627452]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.42948070355458\cdot 10^{-34}X^4 + 3.128819977296419\cdot 10^{-26}X^3 + 6.503519235890246\\ &\quad \cdot 10^{-17}X^2 + 3.800678391173011\cdot 10^{-17}X - 5.044717705924284\cdot 10^{-17}\\ &= -5.044717705924284\cdot 10^{-17}B_{0,4}(X) - 4.094548108131031\cdot 10^{-17}B_{1,4}(X) - 2.060458637689404\\ &\quad \cdot 10^{-17}B_{2,4}(X) + 1.057550706182803\cdot 10^{-17}B_{3,4}(X) + 5.259479924267794\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4895788965792814, 0.6652062590622766\}$$

Intersection intervals with the x axis:

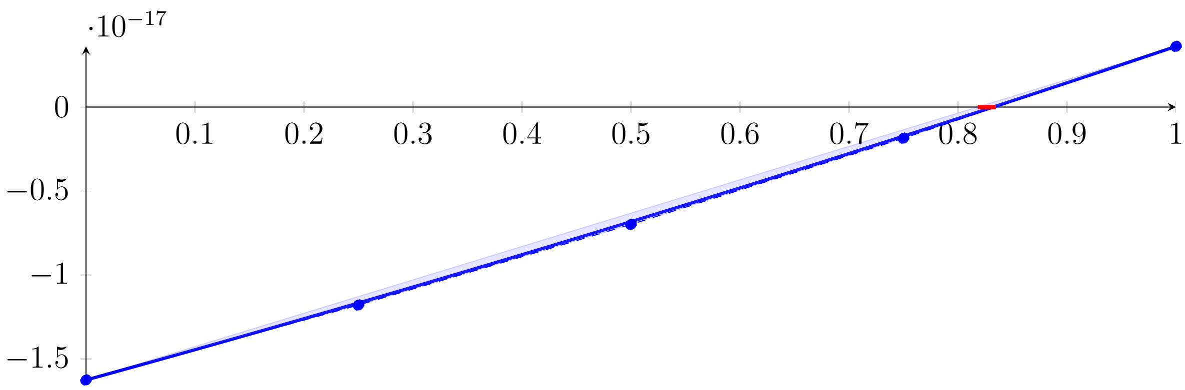
$$[0.4895788965792814, 0.6652062590622766]$$

Longest intersection interval: 0.1756273624829952
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000092124531, 0.4000000101587826]$,

37.36 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.4000000092124531, 0.4000000101587826]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -8.01991079982412 \cdot 10^{-37} X^4 + 1.694950778648659 \cdot 10^{-28} X^3 + 2.006008588115633 \\ &\quad \cdot 10^{-18} X^2 + 1.785893168307927 \cdot 10^{-17} X - 1.625173531873033 \cdot 10^{-17} \\ &= -1.625173531873033 \cdot 10^{-17} B_{0,4}(X) - 1.178700239796051 \cdot 10^{-17} B_{1,4}(X) - 6.987934712504758 \\ &\quad \cdot 10^{-18} B_{2,4}(X) - 1.85453226232069 \cdot 10^{-18} B_{3,4}(X) + 3.613204952634064 \cdot 10^{-18} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8181114615359527, 0.8347943211886801\}$$

Intersection intervals with the x axis:

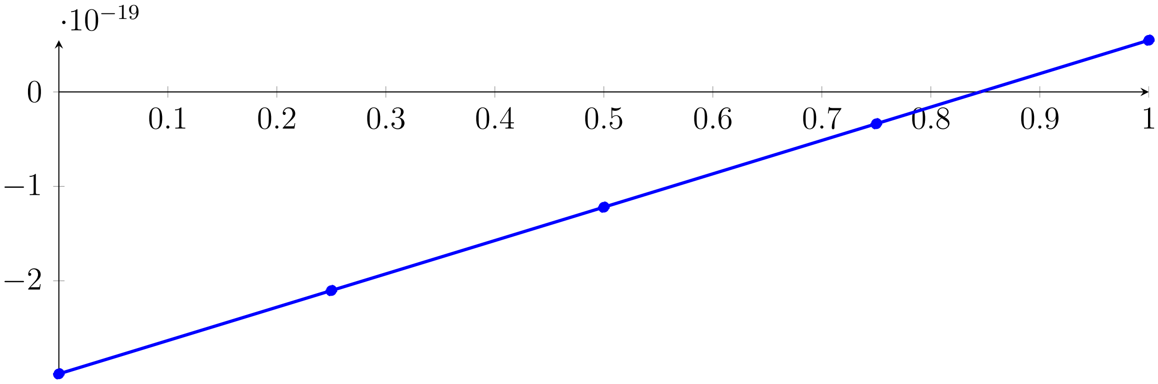
$$[0.8181114615359527, 0.8347943211886801]$$

Longest intersection interval: 0.01668285965272735
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099866561, 0.4000000100024436]$,

37.37 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.4000000099866561, 0.4000000100024436]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -6.212287164586666\cdot10^{-44}X^4 + 7.869888381434003\cdot10^{-34}X^3 + 5.583079095636332\\&\quad\cdot10^{-22}X^2 + 3.526958212875592\cdot10^{-19}X - 2.985043046684015\cdot10^{-19}\\&= -2.985043046684015\cdot10^{-19}B_{0,4}(X) - 2.103303493465117\cdot10^{-19}B_{1,4}(X) - 1.220633427063613\\&\quad\cdot10^{-19}B_{2,4}(X) - 3.370328474795011\cdot10^{-20}B_{3,4}(X) + 5.474982452872209\cdot10^{-20}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845012924114553, 0.8452574901650142\}$$

Intersection intervals with the x axis:

$$[0.845012924114553, 0.8452574901650142]$$

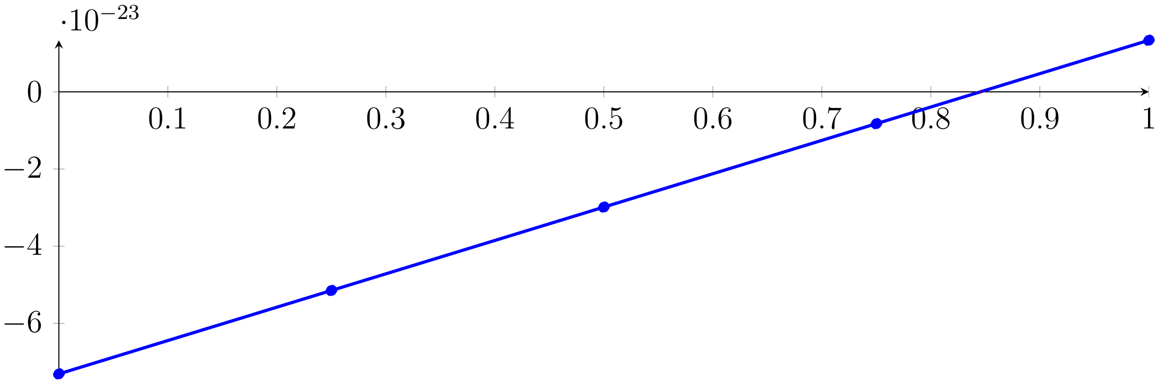
Longest intersection interval: 0.0002445660504611815

$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099999968, 0.4000000100000006]$,

37.38 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.4000000099999968, 0.4000000100000006]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.22247151470982\cdot10^{-58}X^4 + 1.151216705335136\cdot10^{-44}X^3 + 3.33938214525302\\&\quad\cdot10^{-29}X^2 + 8.648818549690798\cdot10^{-23}X - 7.311939957361979\cdot10^{-23}\\&= -7.311939957361979\cdot10^{-23}B_{0,4}(X) - 5.149735319939279\cdot10^{-23}B_{1,4}(X) - 2.987530125952889\\&\quad\cdot10^{-23}B_{2,4}(X) - 8.253243754028074\cdot10^{-24}B_{3,4}(X) + 1.336881931710965\cdot10^{-23}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454261229035133, 0.8454261825857513\}$$

Intersection intervals with the x axis:

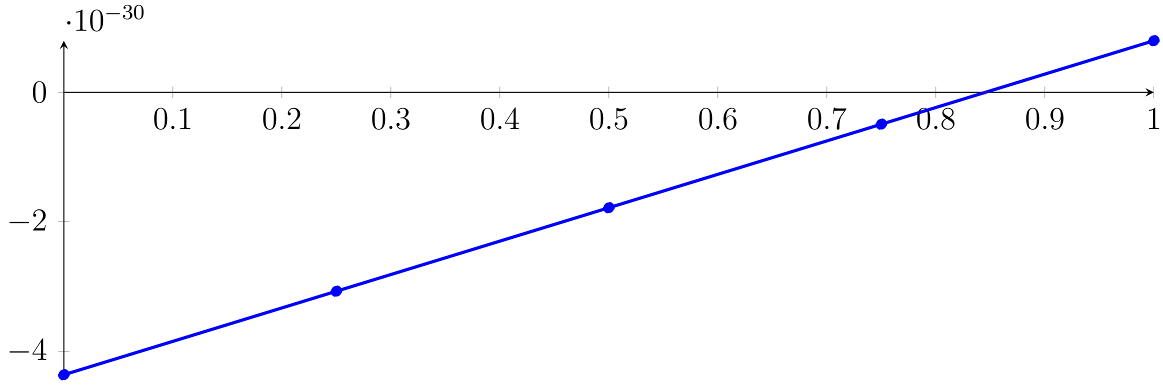
$$[0.8454261229035133, 0.8454261825857513]$$

Longest intersection interval: $5.968223795299727 \cdot 10^{-08}$
$\implies$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

37.39 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -2.819788940081592 \cdot 10^{-87} X^4 + 2.447329147319284 \cdot 10^{-66} X^3 + 1.189477744066025 \\ &\quad \cdot 10^{-43} X^2 + 5.161811836848388 \cdot 10^{-30} X - 4.363931089282542 \cdot 10^{-30} \\ &= -4.363931089282542 \cdot 10^{-30} B_{0,4}(X) - 3.073478130070445 \cdot 10^{-30} B_{1,4}(X) - 1.783025170858328 \\ &\quad \cdot 10^{-30} B_{2,4}(X) - 4.925722116461918 \cdot 10^{-31} B_{3,4}(X) + 7.978807475659646 \cdot 10^{-31} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454262238173519, 0.8454262238173555\}$$

Intersection intervals with the x axis:

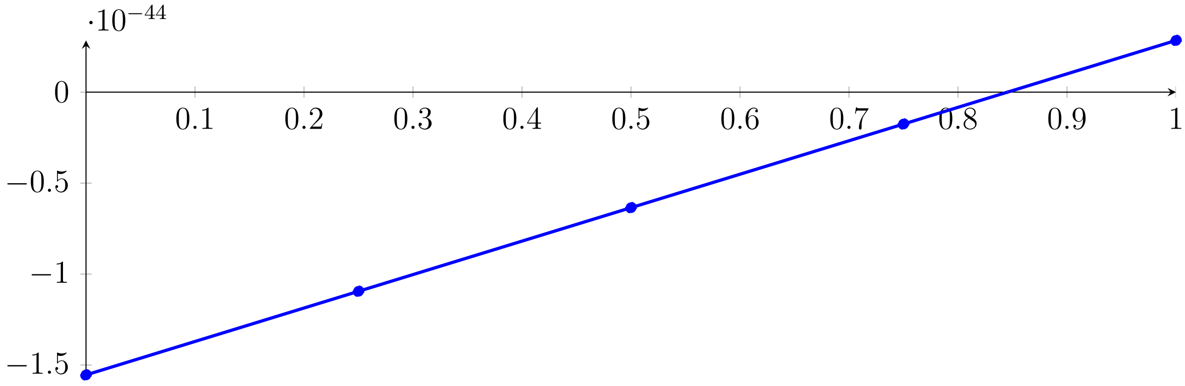
$$[0.8454262238173519, 0.8454262238173555]$$

Longest intersection interval: $3.561967626812098 \cdot 10^{-15}$
$\implies$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

37.40 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -4.539170279710935 \cdot 10^{-145} X^4 + 1.106018233553298 \cdot 10^{-109} X^3 + 1.509163373423153 \\ &\quad \cdot 10^{-72} X^2 + 1.838620665855016 \cdot 10^{-44} X - 1.554418126566363 \cdot 10^{-44} \\ &= -1.554418126566363 \cdot 10^{-44} B_{0,4}(X) - 1.094762960102609 \cdot 10^{-44} B_{1,4}(X) - 6.351077936388546 \\ &\quad \cdot 10^{-45} B_{2,4}(X) - 1.754526271751005 \cdot 10^{-45} B_{3,4}(X) + 2.842025392886536 \cdot 10^{-45} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845426223817358, 0.845426223817358\}$$

Intersection intervals with the x axis:

$$[0.845426223817358, 0.845426223817358]$$

Longest intersection interval: $1.268761337445701 \cdot 10^{-29}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

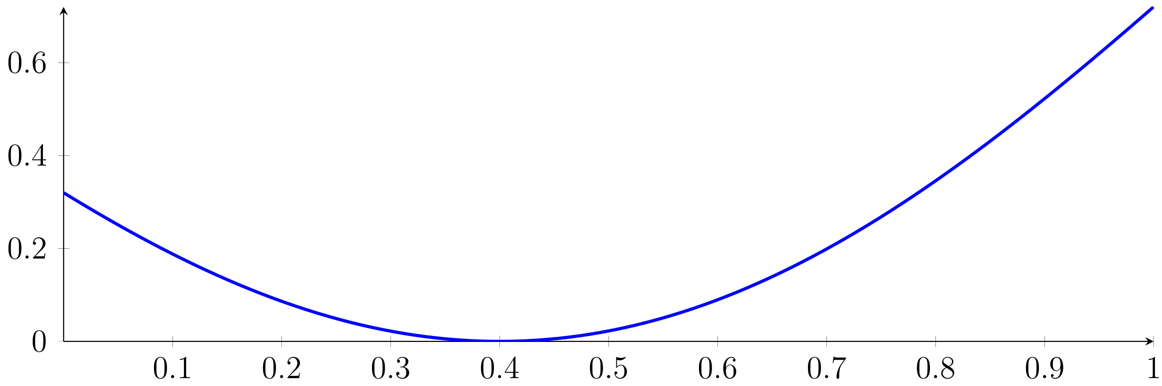
37.41 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 29!

37.42 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4], \ [0.40000001, 0.40000001]$$

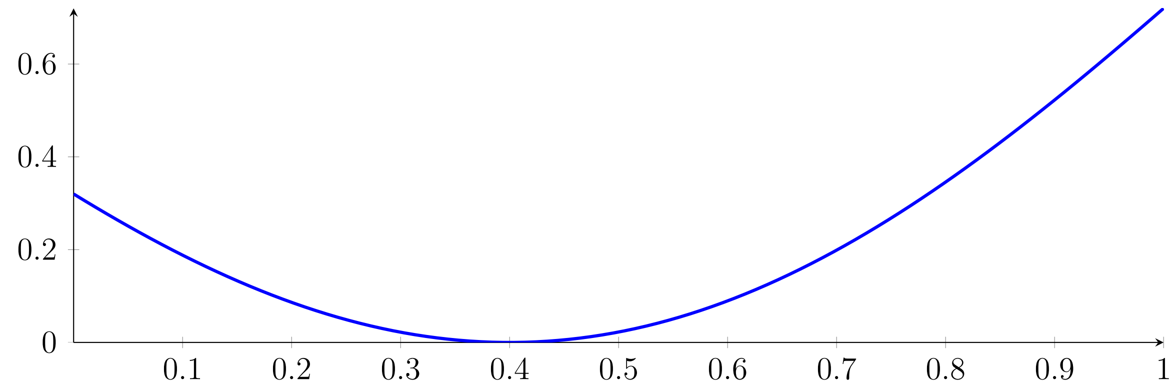
with precision $\varepsilon = 1 \cdot 10^{-64}$.

38 Running QuadClip on h_4 with epsilon 64

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called QuadClip with input polynomial on interval $[0,1]$:

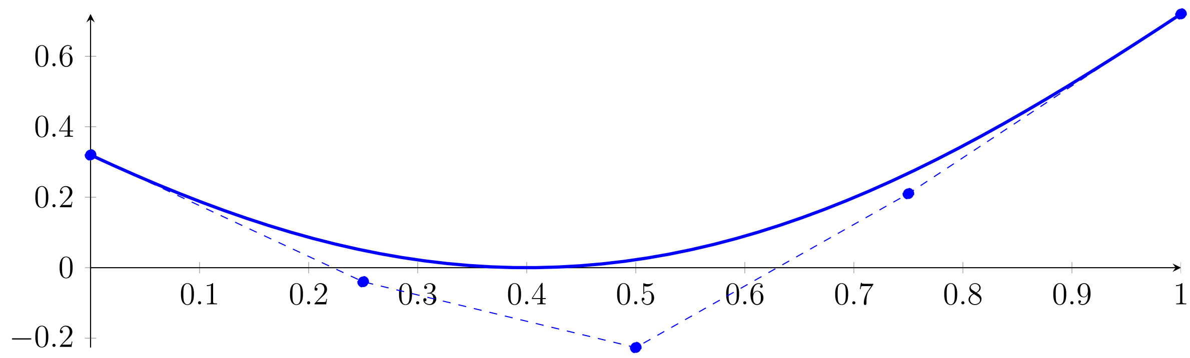
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



38.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

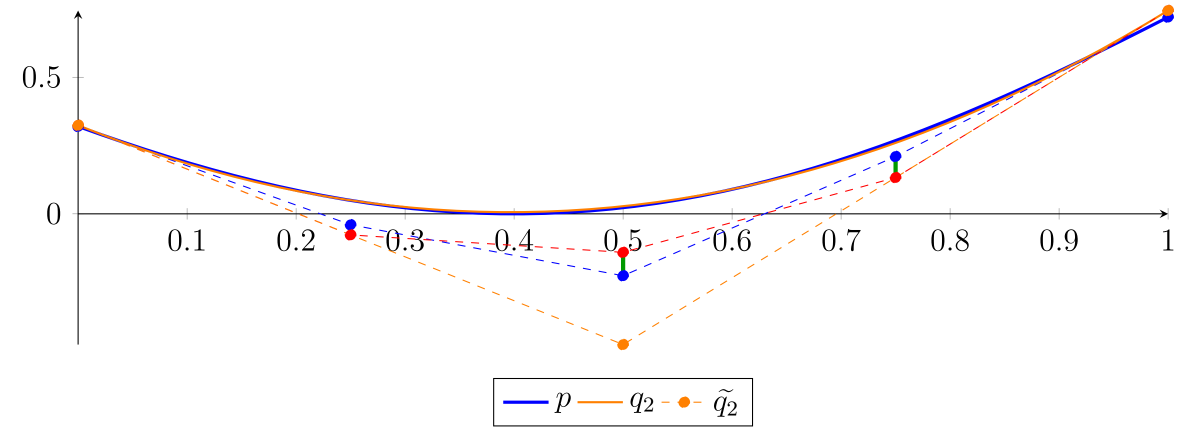
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.810653852360235 \cdot 10^{-306}X^4 - 3.682497235829418 \cdot 10^{-306}X^3 \\ &\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\ &\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4} \end{aligned}$$



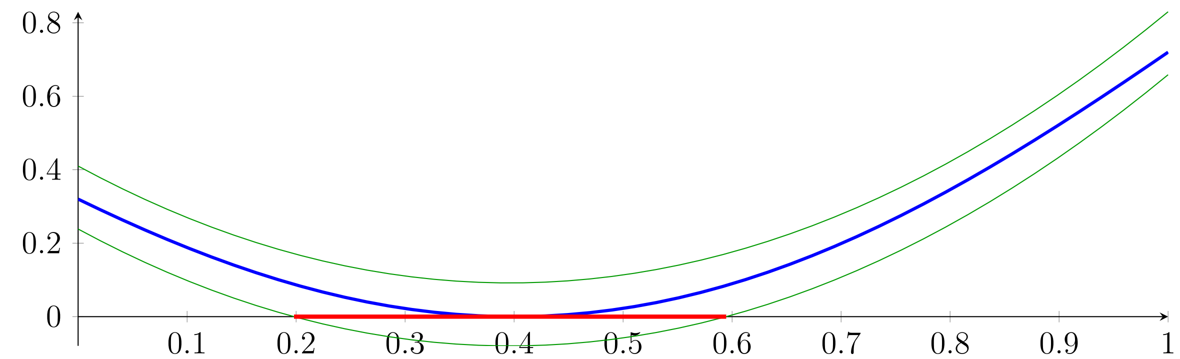
The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$
$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



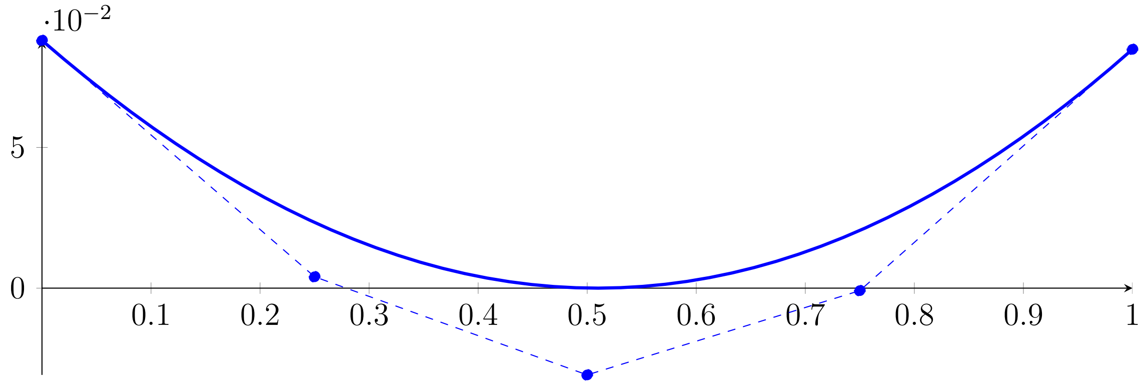
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

38.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

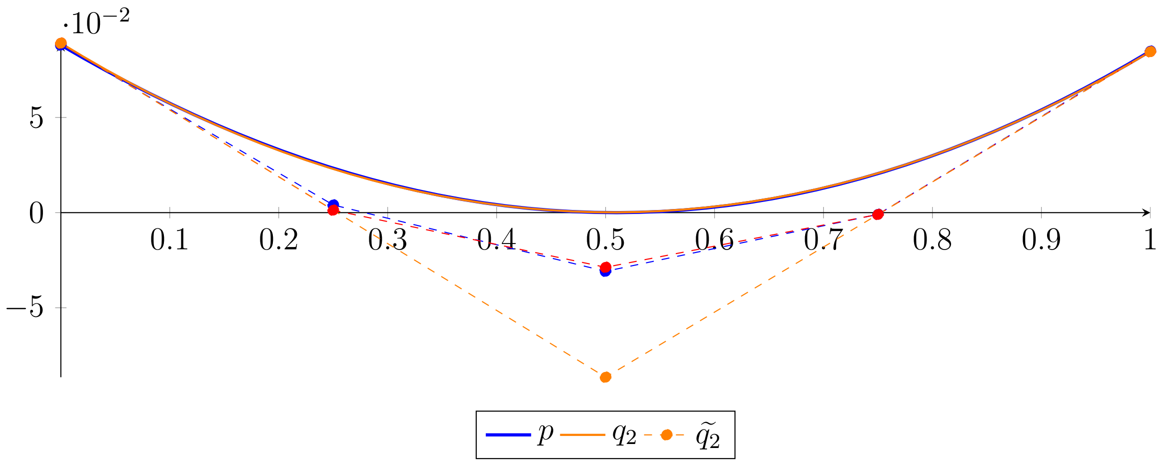
$$\begin{aligned} p = &-0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &+ 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ = &\ 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &- 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402 \cdot 10^{-307}X^4 - 6.258020227051504 \cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

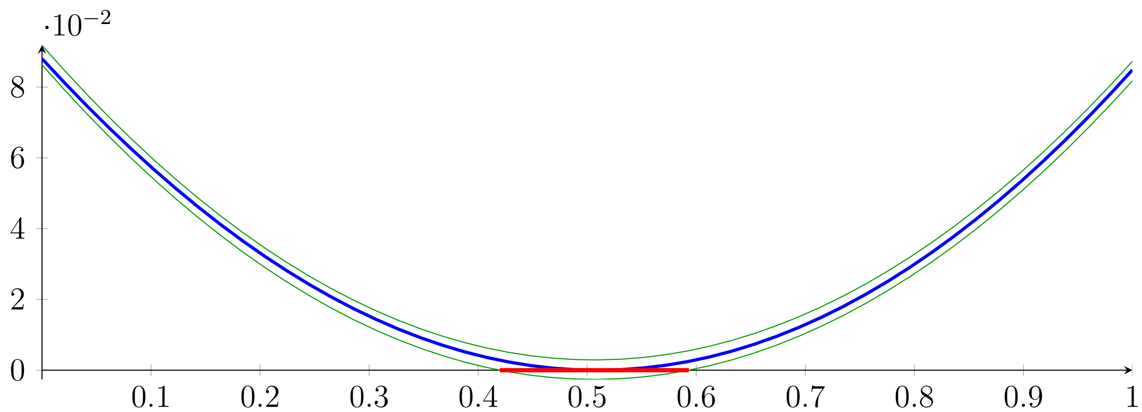
Bounding polynomials M and m:

$$\begin{aligned} M &= 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423 \\ m &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473 \end{aligned}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



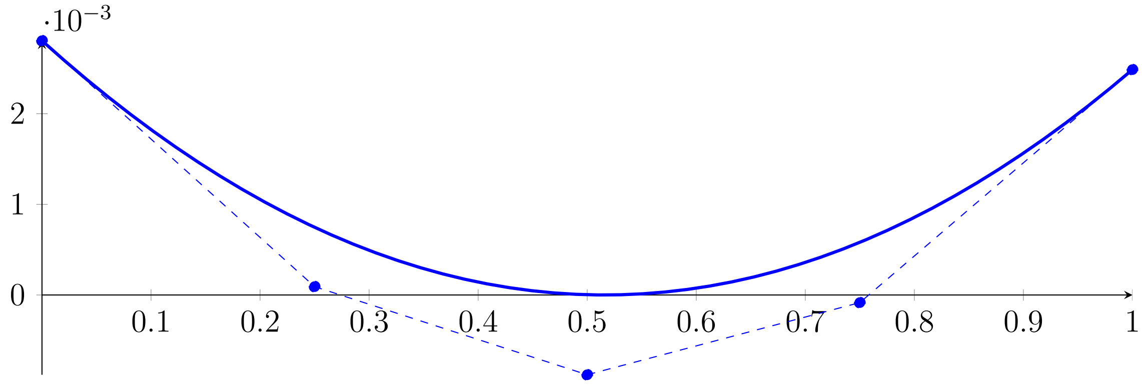
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

38.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

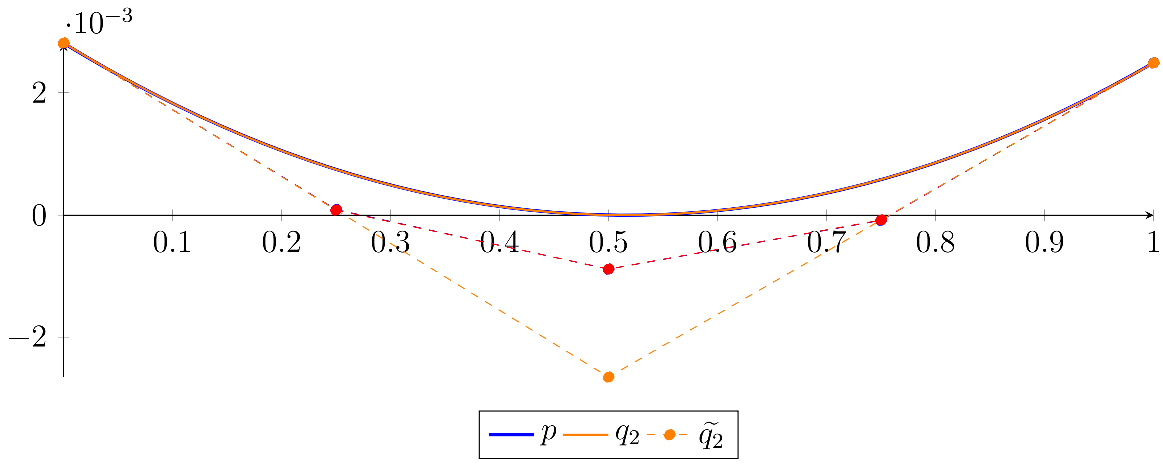
$$\begin{aligned} p = &-2.231982021693433 \cdot 10^{-05} X^4 + 0.0001110015522974976 X^3 \\ &+ 0.01044647623937015 X^2 - 0.01085434180562325 X + 0.002805735592529265 \\ = &\ 0.002805735592529265 B_{0,4}(X) + 9.215014112345361 \cdot 10^{-05} B_{1,4}(X) \\ &- 0.0008803559370539994 B_{2,4}(X) - 8.403225392871914 \\ &\cdot 10^{-05} B_{3,4}(X) + 0.002486551758356734 B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,2} - 0.002640895479419014 B_{1,2} + 0.002483552517338081 B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &\ 9.278384156079834 \cdot 10^{-309} X^4 - 1.903481152394832 \cdot 10^{-308} X^3 \\ &+ 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ = &\ 0.002809372542696975 B_{0,4} + 8.423853163898018 \cdot 10^{-05} B_{1,4} - 0.0008784428096068336 B_{2,4} \\ &- 7.867148104046678 \cdot 10^{-05} B_{3,4} + 0.002483552517338081 B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429 \cdot 10^{-06}$.
Bounding polynomials M and m:

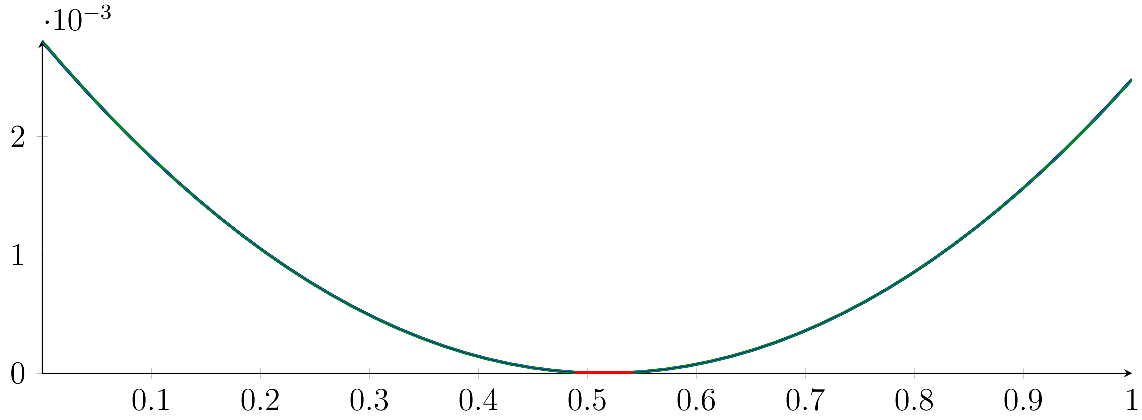
$$M = 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



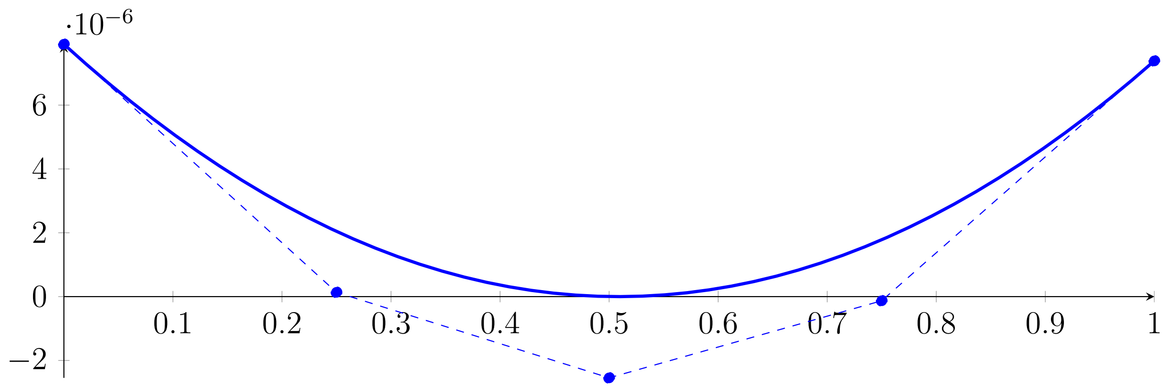
$$[0.4885305113706042, 0.542280720323693]$$

Longest intersection interval: 0.05375020895308878
$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

38.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.401815547173435$

Normalized monomial und Bézier representations and the Bézier polygon:

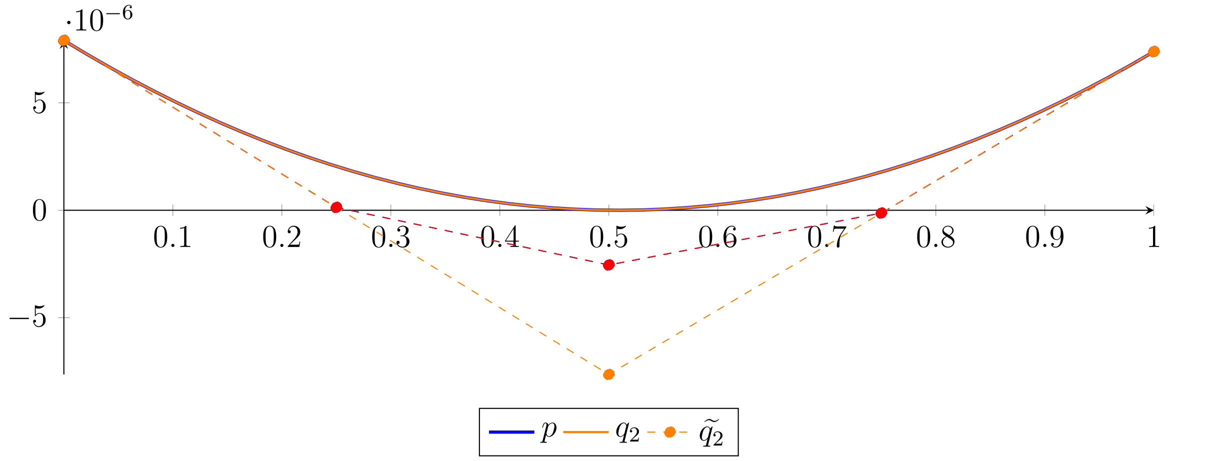
$$\begin{aligned} p &= -1.862993414511891 \cdot 10^{-10} X^4 + 1.046428359410323 \cdot 10^{-08} X^3 + 3.055842313534119 \\ &\quad \cdot 10^{-05} X^2 - 3.109078018724982 \cdot 10^{-05} X + 7.906738260659747 \cdot 10^{-06} \\ &= 7.906738260659747 \cdot 10^{-06} B_{0,4}(X) + 1.340432138472922 \cdot 10^{-07} B_{1,4}(X) - 2.545581310408299 \\ &\quad \cdot 10^{-06} B_{2,4}(X) - 1.295192412084993 \cdot 10^{-07} B_{3,4}(X) + 7.384659193003765 \cdot 10^{-06} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06} \\ &= 7.907245506324471 \cdot 10^{-06} B_{0,2} - 7.641198706965435 \cdot 10^{-06} B_{1,2} + 7.384157270177368 \cdot 10^{-06} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 2.673714696616243 \cdot 10^{-311} X^4 - 5.466261157526541 \cdot 10^{-311} X^3 + 3.057380019043271 \\ &\quad \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06} \\ &= 7.907245506324471 \cdot 10^{-06} B_{0,4} + 1.330233996795178 \cdot 10^{-07} B_{1,4} - 2.545565341893317 \\ &\quad \cdot 10^{-06} B_{2,4} - 1.285207183940336 \cdot 10^{-07} B_{3,4} + 7.384157270177368 \cdot 10^{-06} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.019814167774439 \cdot 10^{-09}$.

Bounding polynomials M and m:

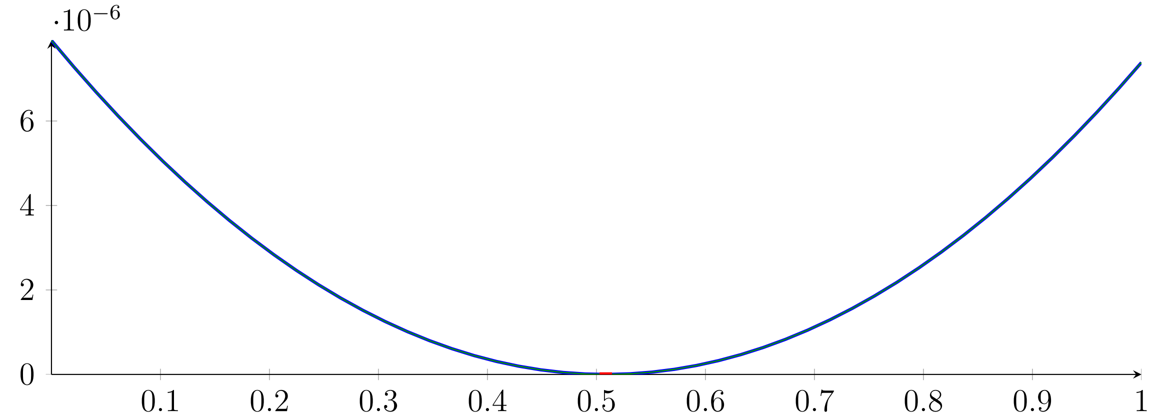
$$M = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.908265320492245 \cdot 10^{-06}$$

$$m = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.906225692156696 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.50281872462556, 0.5142903109829997\}$$

Intersection intervals:



$$[0.50281872462556, 0.5142903109829997]$$

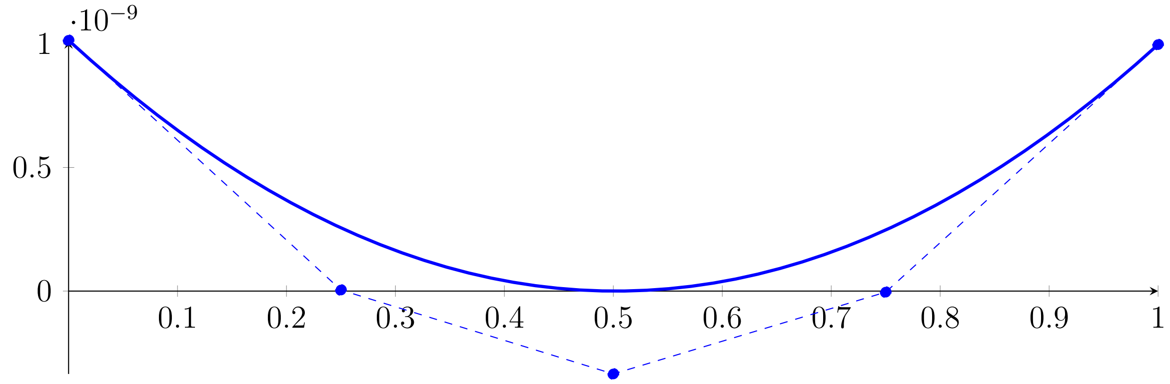
Longest intersection interval: 0.01147158635743977

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999787229673132, 0.4000211044658684]$,

38.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999787229673132, 0.400021104465$

Normalized monomial und Bézier representations and the Bézier polygon:

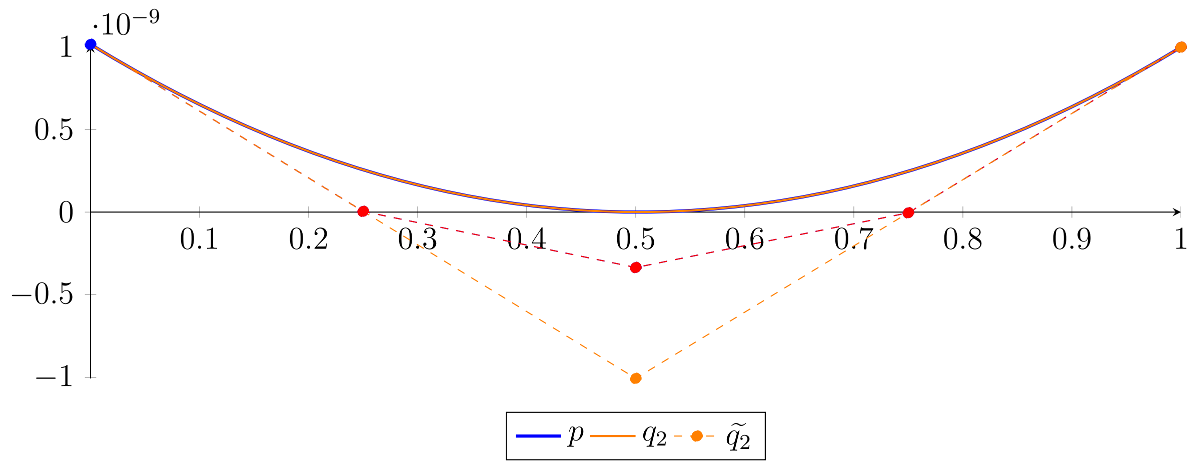
$$\begin{aligned} p = &-3.22630361651822 \cdot 10^{-18} X^4 + 1.523153645441234 \cdot 10^{-14} X^3 + 4.023445841277385 \\ &\cdot 10^{-09} X^2 - 4.040789163099954 \cdot 10^{-09} X + 1.014549826650262 \cdot 10^{-09} \\ = &\ 1.014549826650262 \cdot 10^{-09} B_{0,4}(X) + 4.352535875273264 \cdot 10^{-12} B_{1,4}(X) - 3.352704480201511 \\ &\cdot 10^{-10} B_{2,4}(X) - 4.315317151897678 \cdot 10^{-12} B_{3,4}(X) + 9.972217331378436 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014550587950544 \cdot 10^{-09} \\ &= 1.014550587950544 \cdot 10^{-09} B_{0,2} - 1.005848561585488 \cdot 10^{-09} B_{1,2} + 9.972209719297413 \cdot 10^{-10} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.491760652125473 \cdot 10^{-315} X^4 - 7.128579610273962 \cdot 10^{-315} X^3 + 4.023468683051261 \\ &\quad \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014550587950544 \cdot 10^{-09} \\ &= 1.014550587950544 \cdot 10^{-09} B_{0,4} + 4.35101318252834 \cdot 10^{-12} B_{1,4} - 3.352704477436108 \\ &\quad \cdot 10^{-10} B_{2,4} - 4.313794827873167 \cdot 10^{-12} B_{3,4} + 9.972209719297413 \cdot 10^{-10} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.522692744924588 \cdot 10^{-15}$.
Bounding polynomials M and m:

$$M = 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014552110643289 \cdot 10^{-09}$$

$$m = 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014549065257799 \cdot 10^{-09}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.5015281513143551, 0.5027789820041677\}$$

Intersection intervals:

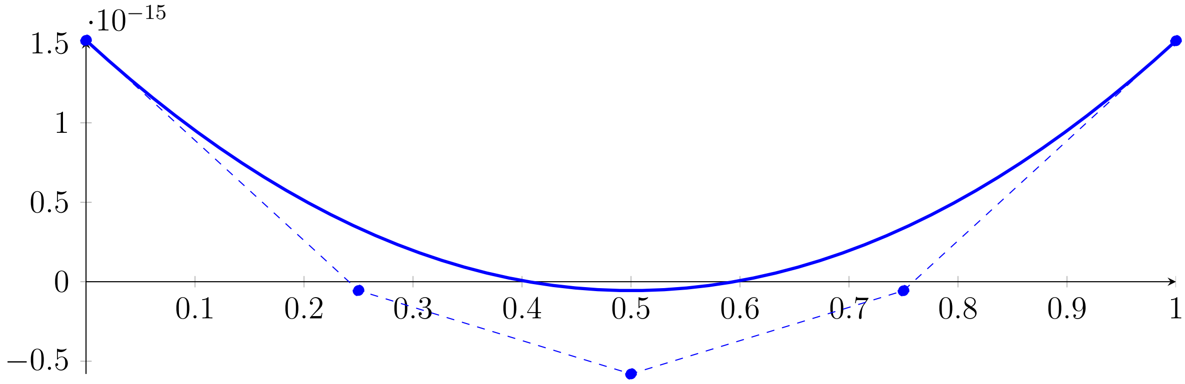
$$[0.5015281513143551, 0.5027789820041677]$$

Longest intersection interval: 0.00125083068981256
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999784819335, 0.4000000314940126]$,

38.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999999784819335, 0.4000000314$

Normalized monomial und Bézier representations and the Bézier polygon:

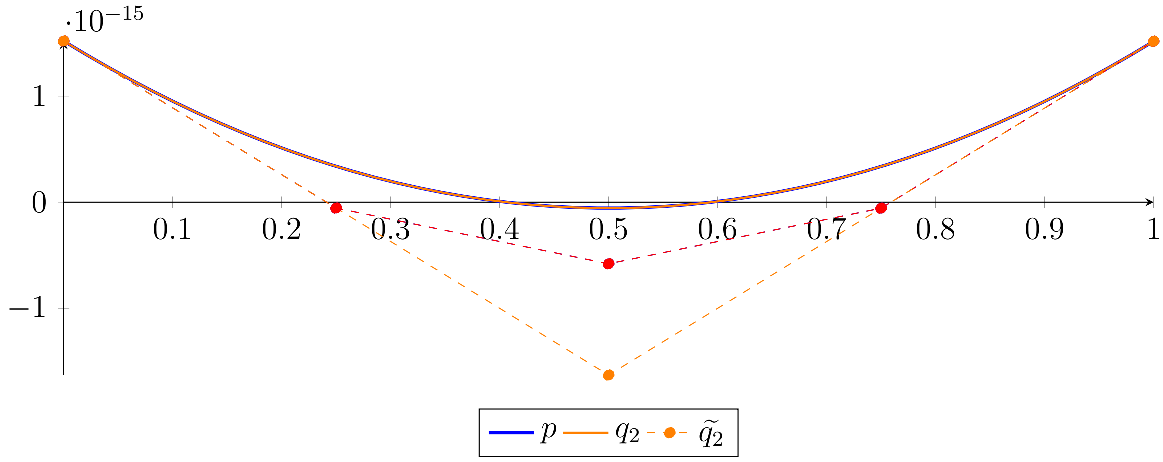
$$p = -7.897676644125011\cdot10^{-30}X^4 + 2.979577702269594\cdot10^{-23}X^3 + 6.295028340046794\cdot10^{-15}X^2 - 6.297884694019149\cdot10^{-15}X + 1.519185582318221\cdot10^{-15}$$
$$= 1.519185582318221\cdot10^{-15}B_{0,4}(X) - 5.52855911865664\cdot10^{-17}B_{1,4}(X) - 5.805853746835548\cdot10^{-16}B_{2,4}(X) - 5.671376072379998\cdot10^{-17}B_{3,4}(X) + 1.516329258141634\cdot10^{-15}B_{4,4}(X)$$



Degree reduction and raising:

$$q_2 = 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185583808009\cdot10^{-15}$$
$$= 1.519185583808009\cdot10^{-15}B_{0,2} - 1.629756772140295\cdot10^{-15}B_{1,2} + 1.516329256651846\cdot10^{-15}B_{2,2}$$

$$\widetilde{q}_2 = 5.424840791336887\cdot10^{-321}X^4 - 1.106707046684392\cdot10^{-320}X^3 + 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185583808009\cdot10^{-15}$$
$$= 1.519185583808009\cdot10^{-15}B_{0,4} - 5.528559416614297\cdot10^{-17}B_{1,4} - 5.805853746835541\cdot10^{-16}B_{2,4} - 5.671375774422431\cdot10^{-17}B_{3,4} + 1.516329256651846\cdot10^{-15}B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 2.979576574030073\cdot10^{-24}$.

Bounding polynomials M and m:

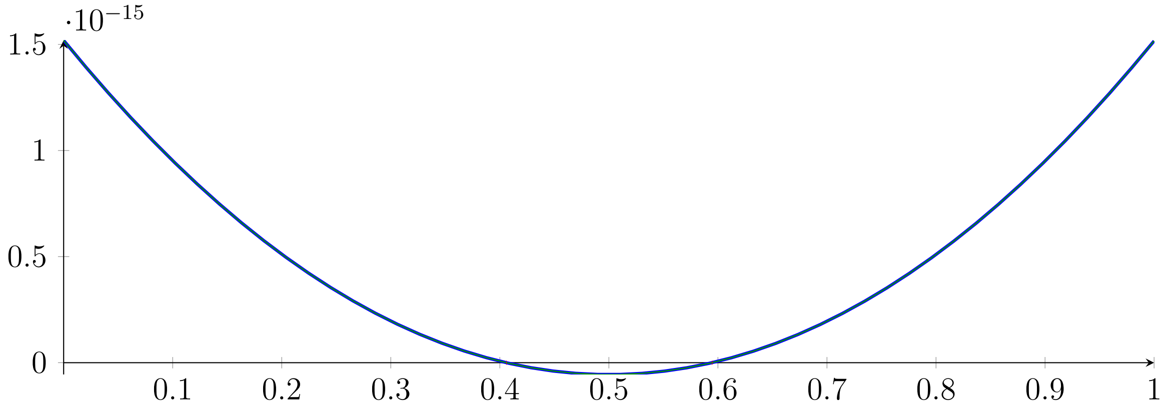
$$M = 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185586787586\cdot10^{-15}$$

$$m = 6.295028384740446\cdot10^{-15}X^2 - 6.297884711896608\cdot10^{-15}X + 1.519185580828433\cdot10^{-15}$$

Root of M and m:

$N(M) = \{0.4059087473493063, 0.5945449959829854\}$ $N(m) = \{0.4059087423309477, 0.594545001001344\}$

Intersection intervals:



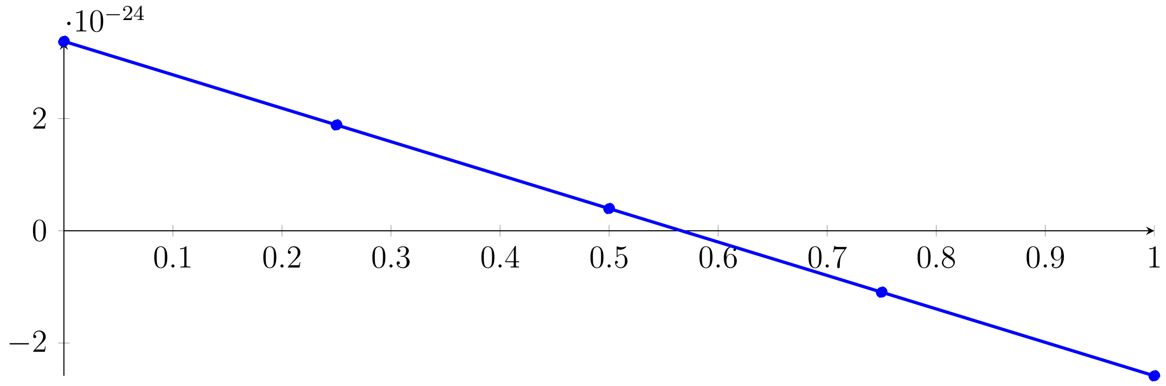
$$[0.4059087423309477, 0.4059087473493063],\ [0.5945449959829854, 0.594545001001344]$$

Longest intersection interval: $5.01835859594062 \cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.4000000000000001]$, interval 2: $[0.400000009999999$

38.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.400000000$

Normalized monomial und Bézier representations and the Bézier polygon:

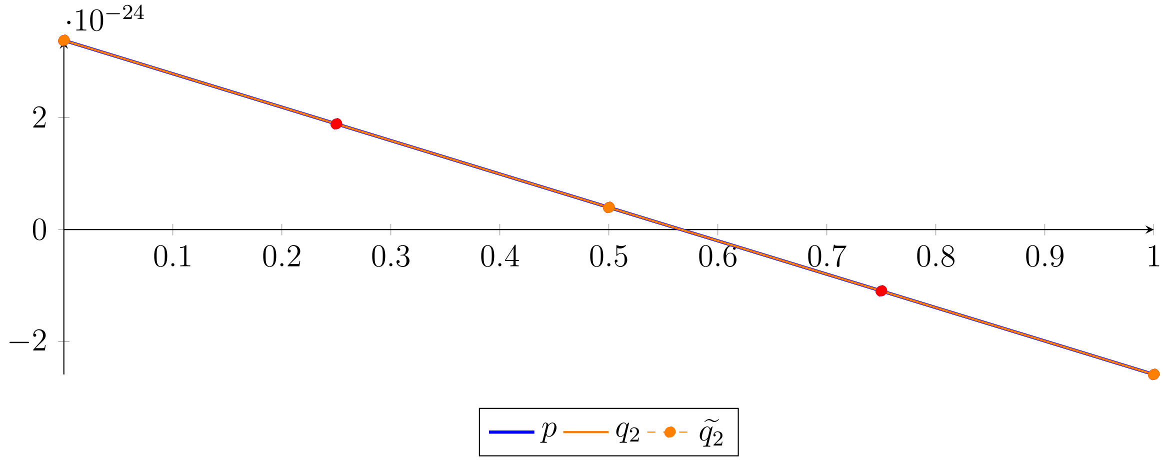
$$\begin{aligned} p &= -5.008943280633806 \cdot 10^{-63} X^4 + 3.765646973599716 \cdot 10^{-48} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4}(X) + 1.885496281661092 \cdot 10^{-24} B_{1,4}(X) + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4}(X) - 1.094080301597753 \cdot 10^{-24} B_{3,4}(X) - 2.583868553593798 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,2} + 3.957079503982923 \cdot 10^{-25} B_{1,2} - 2.583868553593798 \cdot 10^{-24} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= 9.938904969180865 \cdot 10^{-331} X^4 - 2.503131621867773 \cdot 10^{-330} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4} + 1.885496281661092 \cdot 10^{-24} B_{1,4} + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4} - 1.094080301597753 \cdot 10^{-24} B_{3,4} - 2.583868553593798 \cdot 10^{-24} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646973599709 \cdot 10^{-49}$.
Bounding polynomials M and m:

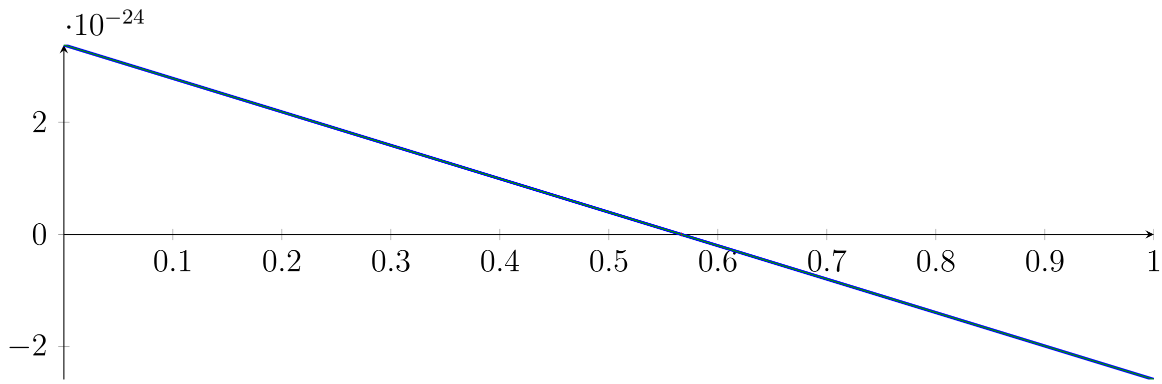
$$M = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

$$m = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

Root of M and m:

$N(M) = \{0.5664033931790926, 37589234.2203029\}$ $N(m) = \{0.5664033931790926, 37589234.2203029\}$

Intersection intervals:



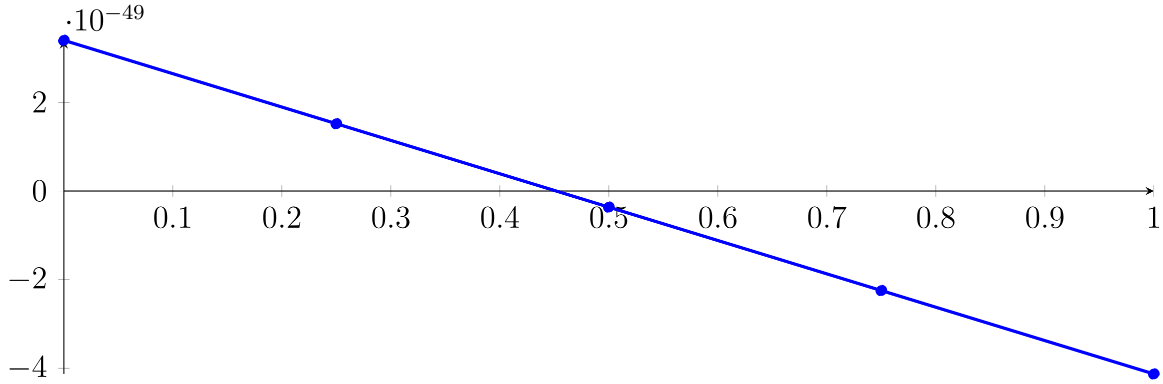
$$[0.5664033931790926, 0.5664033931790926]$$

Longest intersection interval: $1.263819499744334 \cdot 10^{-25}$
$\implies$ Selective recursion: interval 1: $[0.4, 0.4]$,

38.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

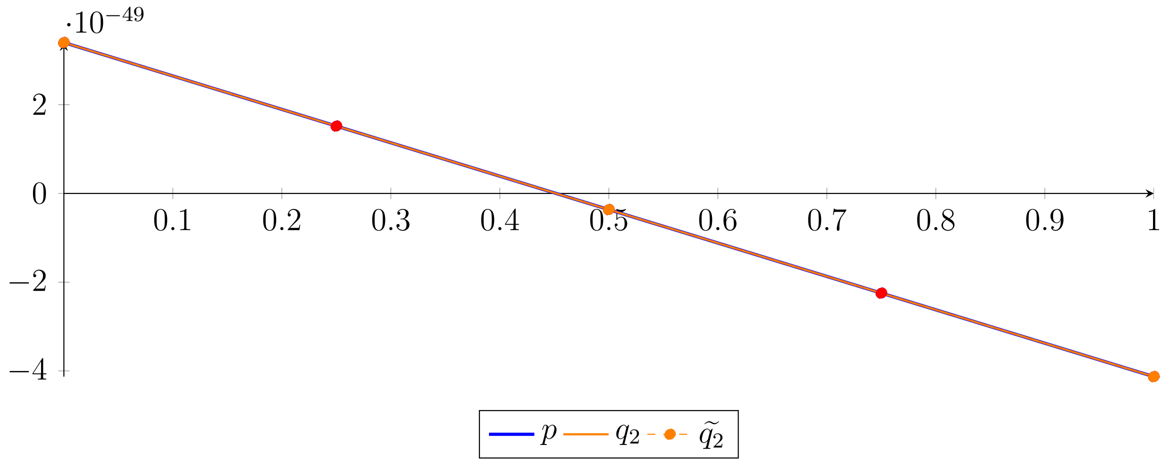
$$\begin{aligned} p &= -1.277868961417556 \cdot 10^{-162} X^4 + 7.601420513547777 \cdot 10^{-123} X^3 + 2.532160202151322 \\ &\quad \cdot 10^{-81} X^2 - 7.531293947199418 \cdot 10^{-49} X + 3.401595176598994 \cdot 10^{-49} \\ &= 3.401595176598994 \cdot 10^{-49} B_{0,4}(X) + 1.51877168979914 \cdot 10^{-49} B_{1,4}(X) - 3.640517970007151 \\ &\quad \cdot 10^{-50} B_{2,4}(X) - 2.24687528380057 \cdot 10^{-49} B_{3,4}(X) - 4.129698770600424 \cdot 10^{-49} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 2.532160202151322 \cdot 10^{-81} X^2 - 7.531293947199418 \cdot 10^{-49} X + 3.401595176598994 \cdot 10^{-49} \\
&= 3.401595176598994 \cdot 10^{-49} B_{0,2} - 3.640517970007151 \cdot 10^{-50} B_{1,2} - 4.129698770600424 \cdot 10^{-49} B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 &= -2.854607465456027 \cdot 10^{-356} X^4 + 3.044914629819762 \cdot 10^{-356} X^3 + 2.532160202151322 \\
&\quad \cdot 10^{-81} X^2 - 7.531293947199418 \cdot 10^{-49} X + 3.401595176598994 \cdot 10^{-49} \\
&= 3.401595176598994 \cdot 10^{-49} B_{0,4} + 1.51877168979914 \cdot 10^{-49} B_{1,4} - 3.640517970007151 \\
&\quad \cdot 10^{-50} B_{2,4} - 2.24687528380057 \cdot 10^{-49} B_{3,4} - 4.129698770600424 \cdot 10^{-49} B_{4,4}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 7.601420513547777 \cdot 10^{-124}$.

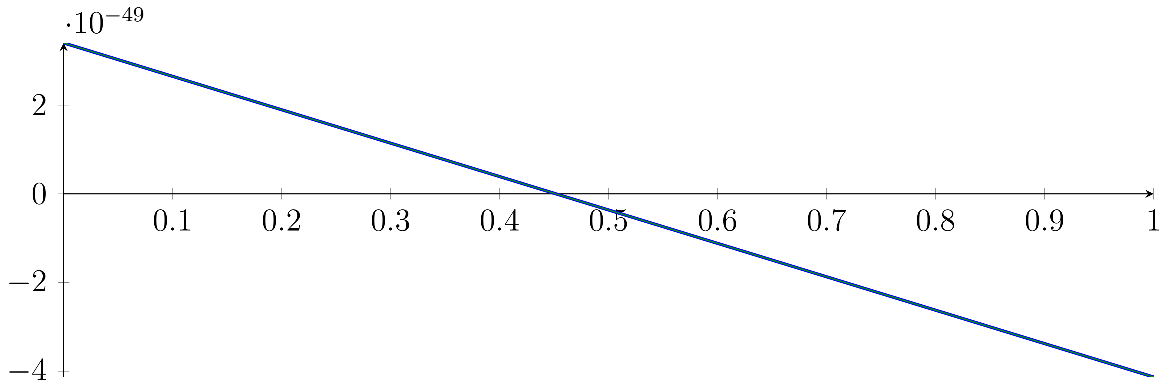
Bounding polynomials M and m:

$$
\begin{aligned}
M &= 2.532160202151322 \cdot 10^{-81} X^2 - 7.531293947199418 \cdot 10^{-49} X + 3.401595176598994 \cdot 10^{-49} \\
m &= 2.532160202151322 \cdot 10^{-81} X^2 - 7.531293947199418 \cdot 10^{-49} X + 3.401595176598994 \cdot 10^{-49}
\end{aligned}
$$

Root of M and m:

$N(M) = \{0.4516614542530117, 2.974256502728712 \cdot 10^{32}\}$ $\quad N(m) = \{0.4516614542530117, 2.97425650272871$

Intersection intervals:



$$[0.4516614542530117, 0.4516614542530117]$$

Longest intersection interval: $2.018622713929374 \cdot 10^{-75}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

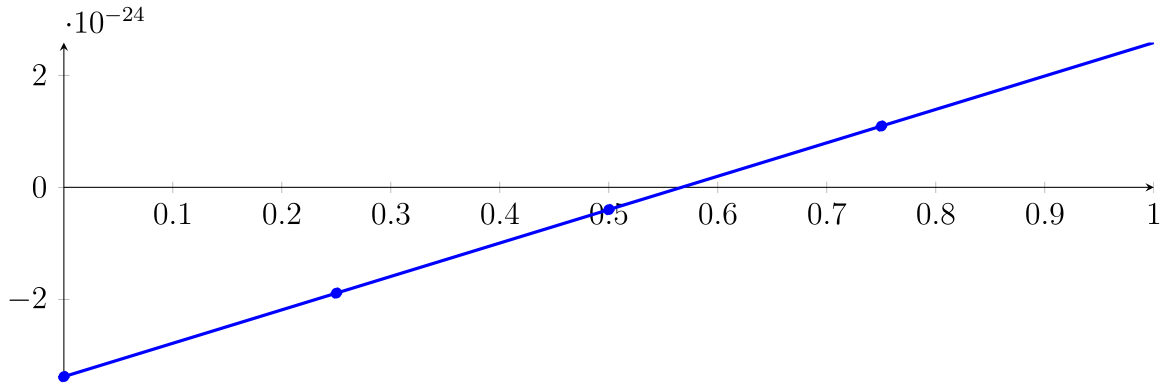
38.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 9!

38.10 Recursion Branch 1 1 1 1 1 1 2 in Interval 2: $[0.4000000099999999, 0.4000000$

Normalized monomial und Bézier representations and the Bézier polygon:

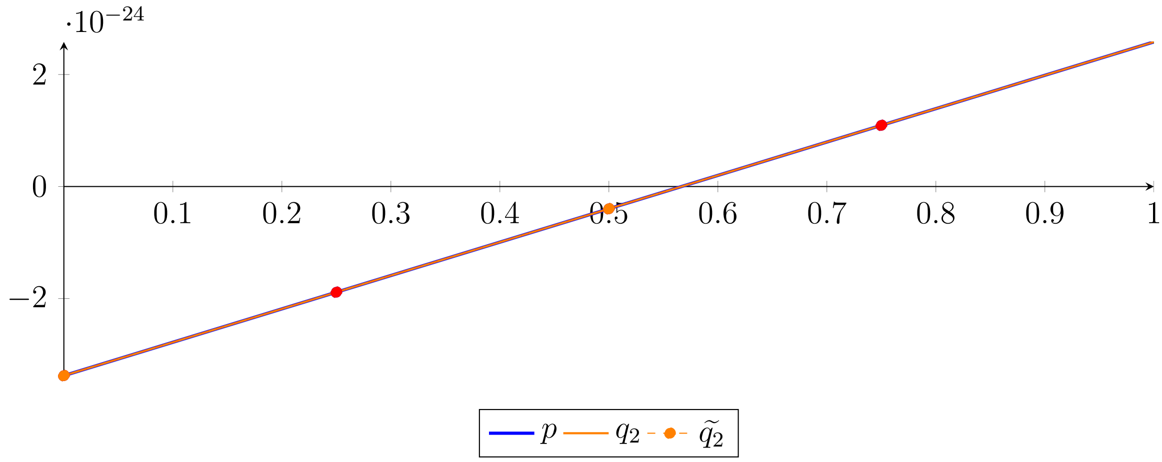
$$\begin{aligned} p &= -5.008943280633806 \cdot 10^{-63} X^4 + 3.76564622047036 \cdot 10^{-48} X^3 + 1.585335103209048 \\ &\quad \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ &= -3.376951785602669 \cdot 10^{-24} B_{0,4}(X) - 1.887163542825797 \cdot 10^{-24} B_{1,4}(X) - 3.973752736266727 \\ &\quad \cdot 10^{-25} B_{2,4}(X) + 1.092413021994703 \cdot 10^{-24} B_{3,4}(X) + 2.582201344038331 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ &= -3.376951785602669 \cdot 10^{-24} B_{0,2} - 3.973753000489244 \cdot 10^{-25} B_{1,2} + 2.582201344038331 \cdot 10^{-24} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -1.030701256063201 \cdot 10^{-330} X^4 + 2.355888585287316 \cdot 10^{-330} X^3 + 1.585335103209048 \\ &\quad \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ &= -3.376951785602669 \cdot 10^{-24} B_{0,4} - 1.887163542825797 \cdot 10^{-24} B_{1,4} - 3.973752736266727 \\ &\quad \cdot 10^{-25} B_{2,4} + 1.092413021994703 \cdot 10^{-24} B_{3,4} + 2.582201344038331 \cdot 10^{-24} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646220470353 \cdot 10^{-49}$.

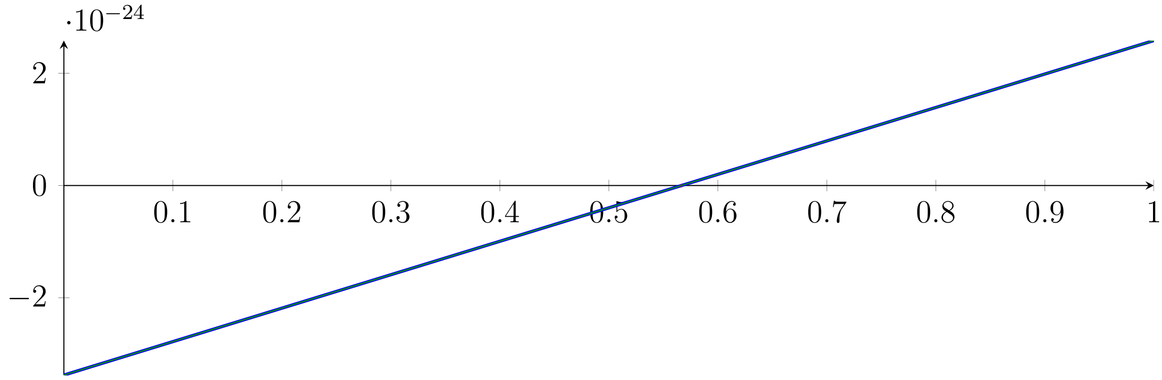
Bounding polynomials M and m:

$$M = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$
$$m = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$

Root of M and m:

$N(M) = \{-37589233.0200927, 0.5666831764624487\}$ $N(m) = \{-37589233.0200927, 0.5666831764624487\}$

Intersection intervals:



$[0.5666831764624487, 0.5666831764624487]$

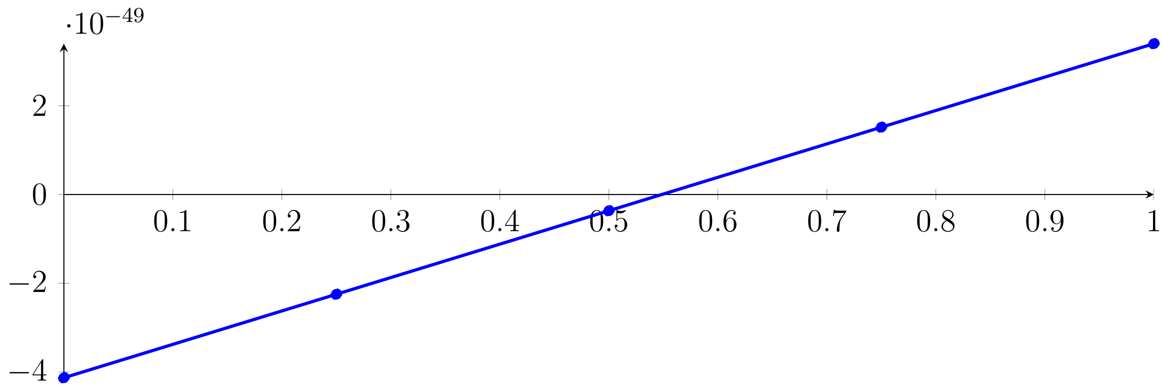
Longest intersection interval: $1.263819245852043 \cdot 10^{-25}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

38.11 Recursion Branch 1 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

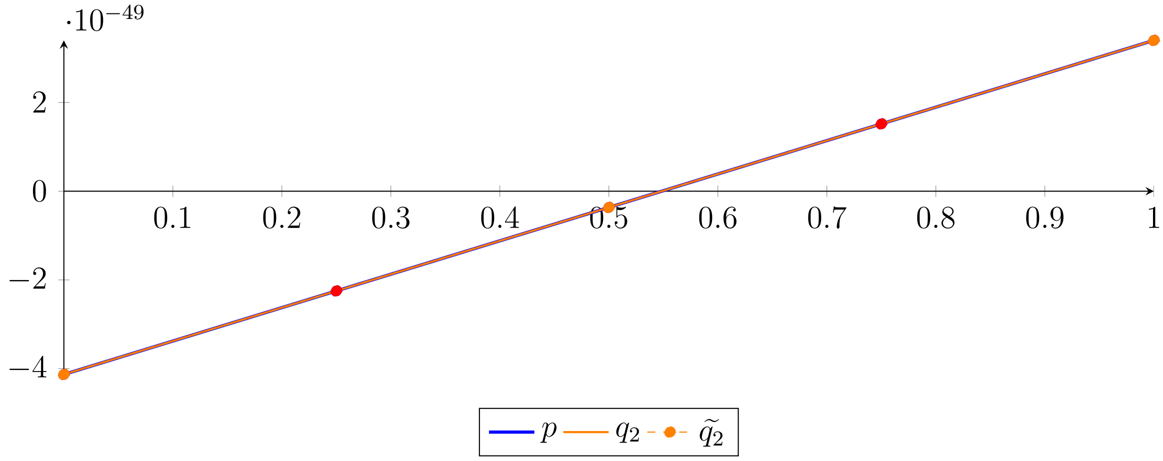
$$\begin{aligned} p &= -1.277867934558931 \cdot 10^{-162} X^4 + 7.601414412052567 \cdot 10^{-123} X^3 + 2.53215919154825 \\ &\quad \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49} \\ &= -4.131138335916633 \cdot 10^{-49} B_{0,4}(X) - 2.248315225681457 \cdot 10^{-49} B_{1,4}(X) - 3.654921154462805 \\ &\quad \cdot 10^{-50} B_{2,4}(X) + 1.517330994788896 \cdot 10^{-49} B_{3,4}(X) + 3.400154105024072 \cdot 10^{-49} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.53215919154825 \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49} \\ &= -4.131138335916633 \cdot 10^{-49} B_{0,2} - 3.654921154462805 \cdot 10^{-50} B_{1,2} + 3.400154105024072 \cdot 10^{-49} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -1.103781553309664 \cdot 10^{-355} X^4 + 2.588177435346798 \cdot 10^{-355} X^3 + 2.53215919154825 \\ &\quad \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49} \\ &= -4.131138335916633 \cdot 10^{-49} B_{0,4} - 2.248315225681457 \cdot 10^{-49} B_{1,4} - 3.654921154462805 \\ &\quad \cdot 10^{-50} B_{2,4} + 1.517330994788896 \cdot 10^{-49} B_{3,4} + 3.400154105024072 \cdot 10^{-49} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.601414412052567 \cdot 10^{-124}$.
Bounding polynomials M and m:

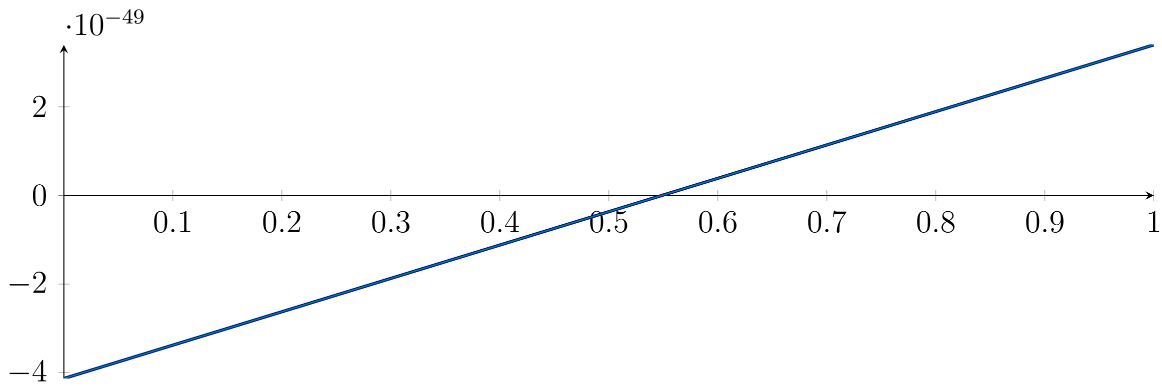
$$M = 2.53215919154825 \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49}$$

$$m = 2.53215919154825 \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49}$$

Root of M and m:

$N(M) = \{-2.974257094924515 \cdot 10^{32}, 0.5485297999397071\}$ $N(m) = \{-2.974257094924515 \cdot 10^{32}, 0.54852979$

Intersection intervals:



$$[0.5485297999397071, 0.5485297999397071]$$

Longest intersection interval: $2.018621497349027 \cdot 10^{-75}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

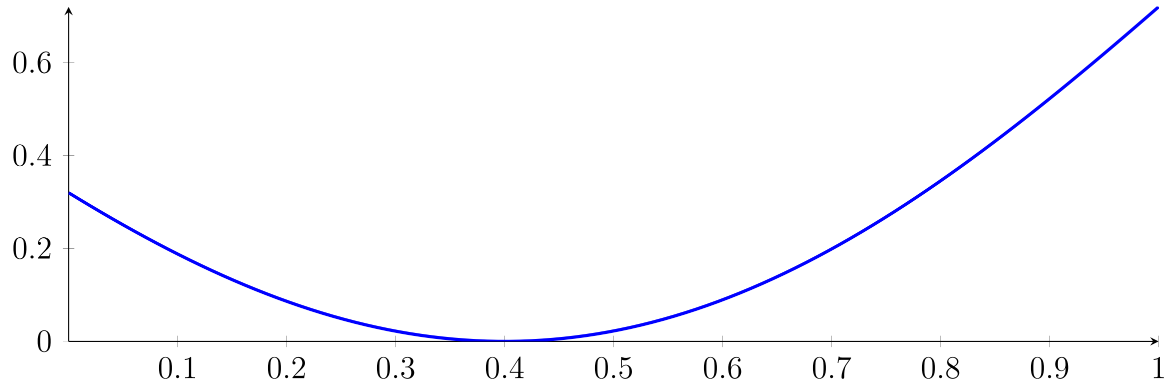
38.12 Recursion Branch 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 9!

38.13 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

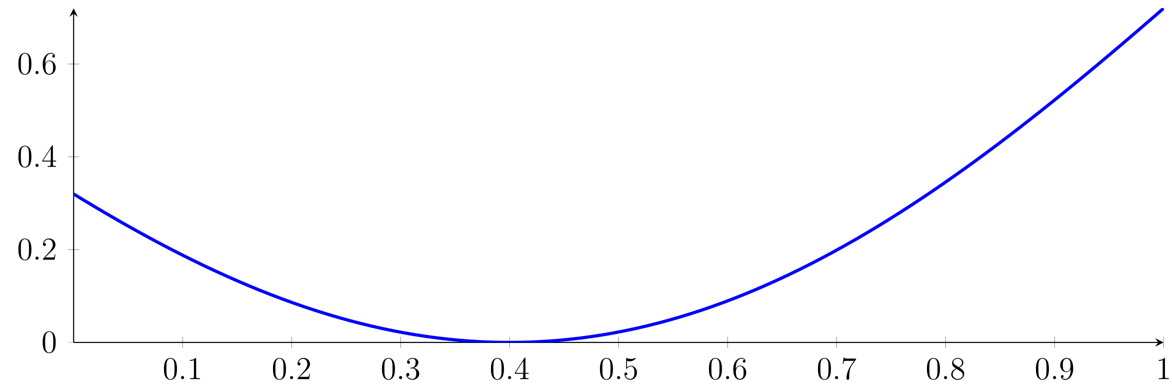
with precision $\varepsilon = 1 \cdot 10^{-64}$.

39 Running CubeClip on h_4 with epsilon 64

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called CubeClip with input polynomial on interval $[0, 1]$:

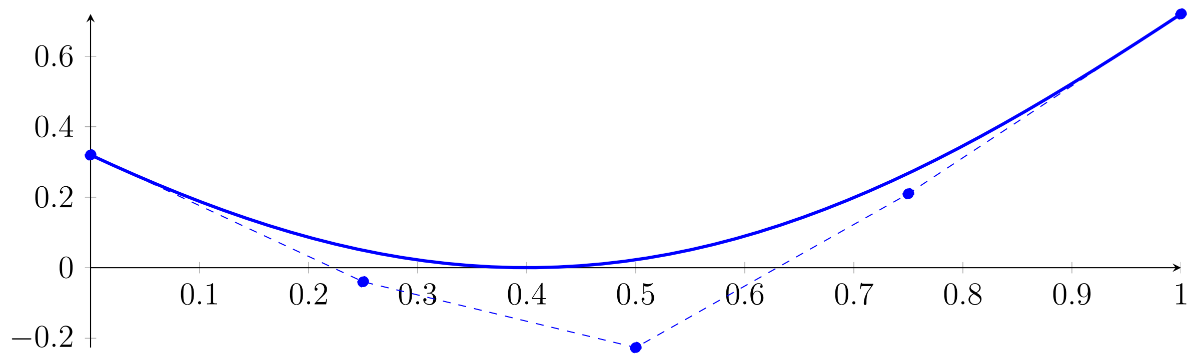
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



39.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

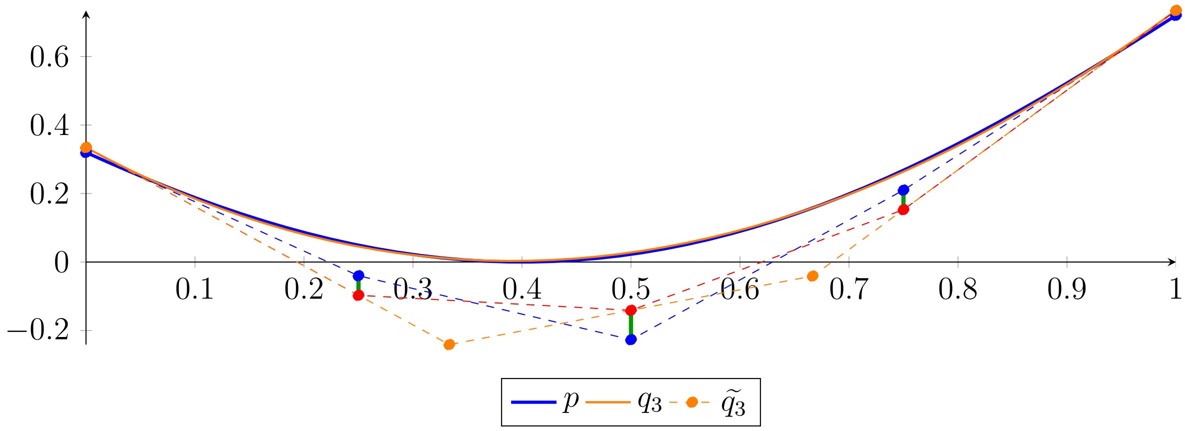
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.03999999599999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3} \\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -2.781342323134002 \cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2 \\ &\quad - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4} \\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4} \end{aligned}$$



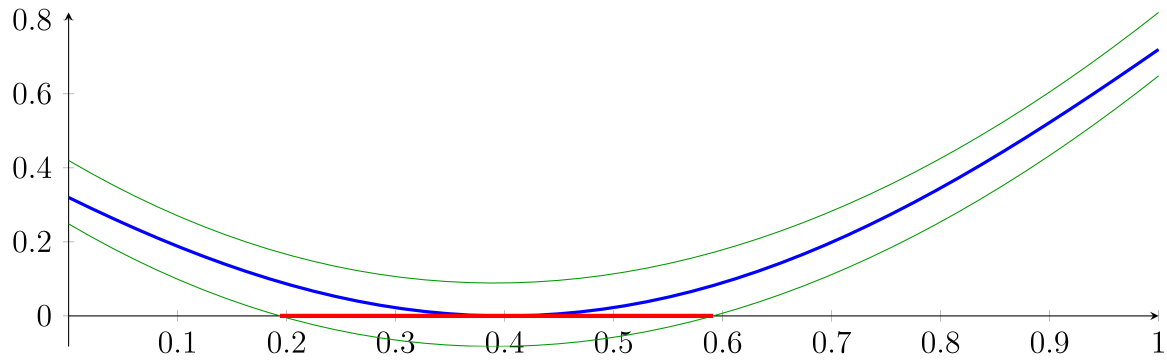
The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$
$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $\quad N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



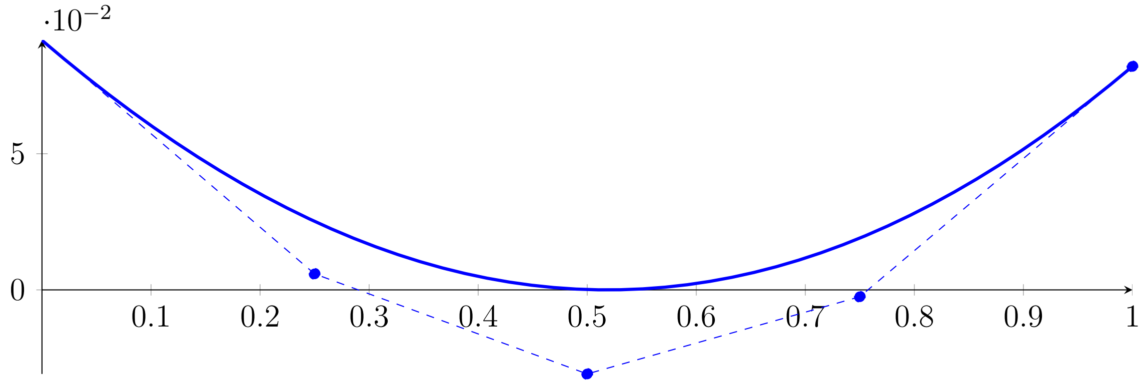
$$[0.1938265012447289, 0.5913474001559605]$$

Longest intersection interval: 0.3975208989112316
$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

39.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

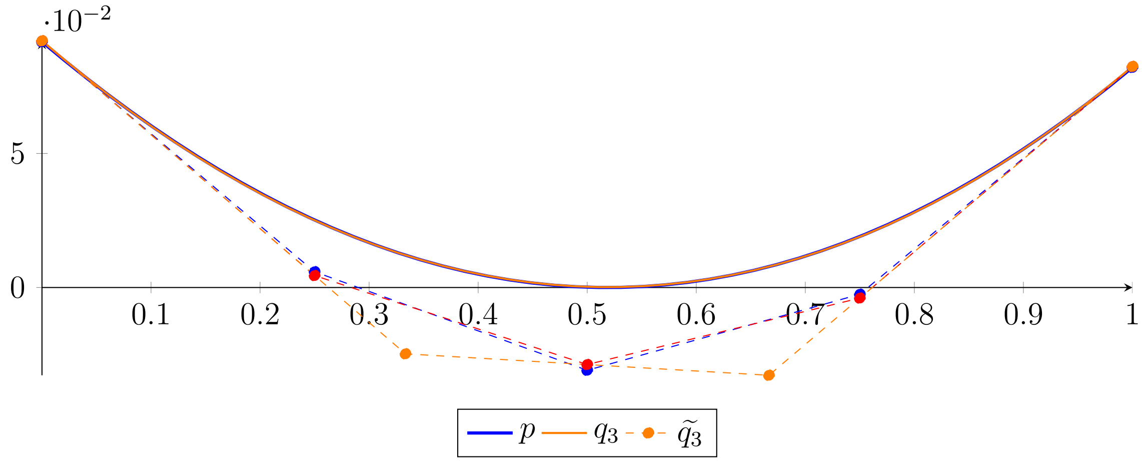
$$\begin{aligned} p = &-0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &+ 0.2941201876709534X^2 - 0.34309913433192X + 0.09165715738594469 \\ = &\ 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &- 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502\cdot 10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455446609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.

Bounding polynomials M and m:

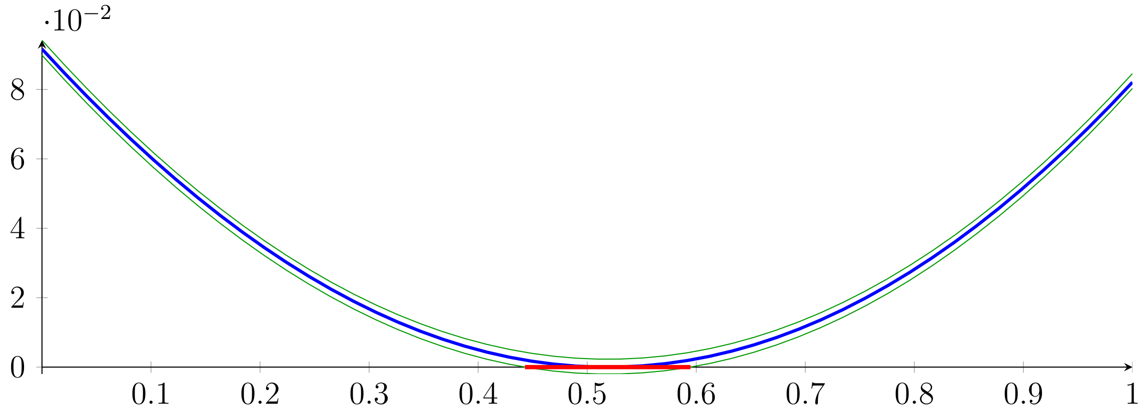
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



$[0.4429176870393937, 0.5947109255489168]$

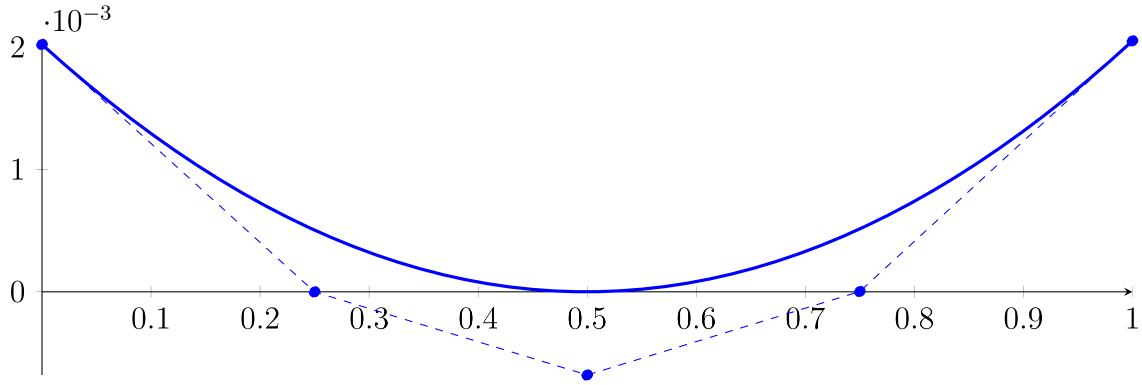
Longest intersection interval: 0.1517932385095231

$\Longrightarrow$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

39.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

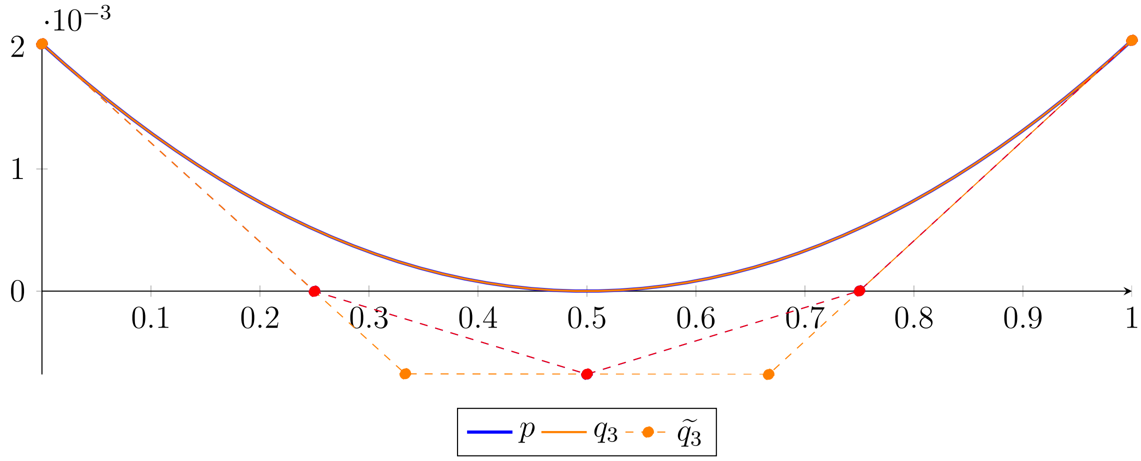
$$\begin{aligned} p = & -1.325713168422468 \cdot 10^{-05} X^4 + 7.039695732710913 \cdot 10^{-05} X^3 \\ & + 0.008070351522992233 X^2 - 0.008098671960928044 X + 0.002023786815863734 \\ = & \ 0.002023786815863734 B_{0,4}(X) - 8.811743682771854 \cdot 10^{-07} B_{1,4}(X) \\ & - 0.0006804905774349916 B_{2,4}(X) + 2.557845995594498 \\ & \cdot 10^{-06} B_{3,4}(X) + 0.002052606203570807 B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = & \ 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ & - 0.008102459712837822 X + 0.002023976203459223 \\ = & \ 0.002023976203459223 B_{0,3} - 0.0006768437008200514 B_{1,3} \\ & - 0.0006818648029039136 B_{2,3} + 0.002052795591166296 B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} = & -6.518771069845317 \cdot 10^{-311} X^4 + 4.388269395865976 \cdot 10^{-05} X^3 \\ & + 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002023976203459223 \\ = & \ 0.002023976203459223 B_{0,4} - 1.638724750232882 \cdot 10^{-06} B_{1,4} - 0.0006793542518619825 B_{2,4} \\ & + 1.800295613638801 \cdot 10^{-06} B_{3,4} + 0.002052795591166296 B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544{\cdot}10^{-06}$.
Bounding polynomials M and m:

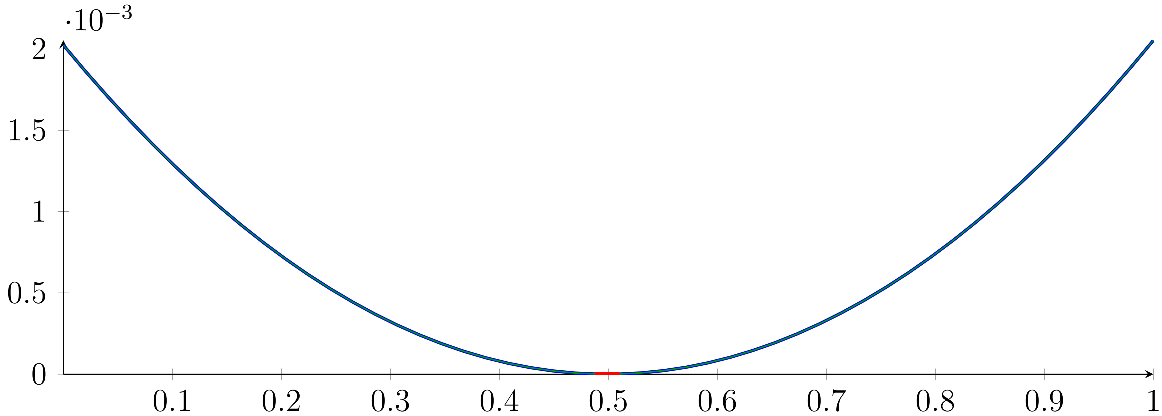
$$M = 4.388269395865976{\cdot}10^{-05}X^3 + 0.008087396406586236X^2 \\ - 0.008102459712837822X + 0.002025112529032156$$

$$m = 4.388269395865976{\cdot}10^{-05}X^3 + 0.008087396406586236X^2 \\ - 0.008102459712837822X + 0.002022839877886289$$

Root of M and m:

$N(M) = \{-185.2936165687423\}$ $\quad N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649$

Intersection intervals:



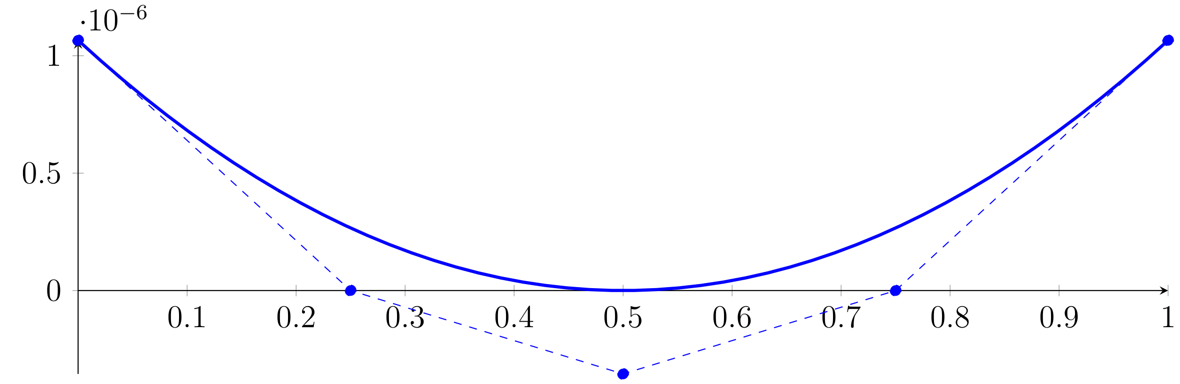
$$[0.4874742490566883, 0.5103358670775649]$$

Longest intersection interval: 0.02286161802087665
$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

39.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.40068970704716$

Normalized monomial und Bézier representations and the Bézier polygon:

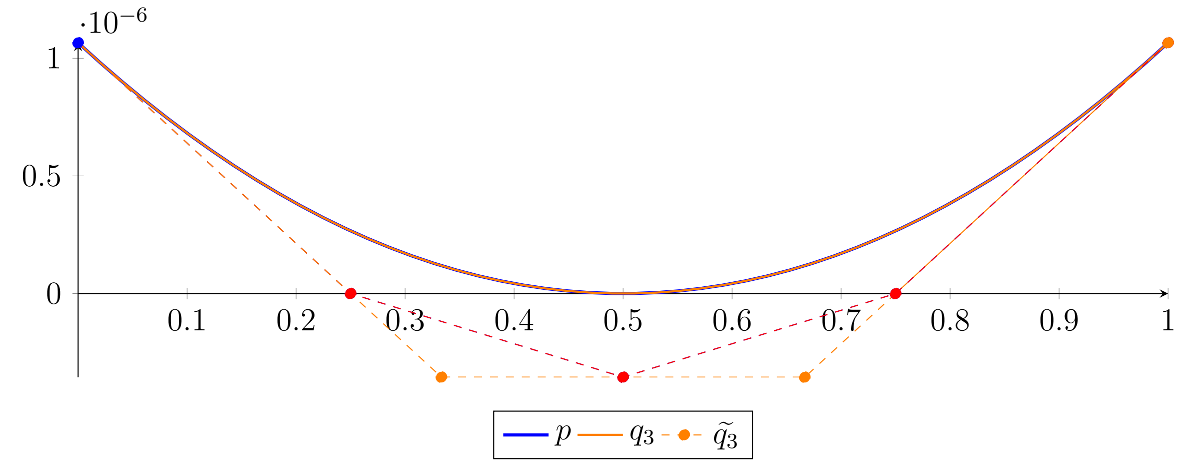
$$p = -3.621407750870066{\cdot}10^{-12}X^4 + 5.322780243378263{\cdot}10^{-10}X^3 + 4.261926231315171 \\ {\cdot}10^{-06}X^2 - 4.262596936226815{\cdot}10^{-06}X + 1.065750606194374{\cdot}10^{-06} \\ = 1.065750606194374{\cdot}10^{-06}B_{0,4}(X) + 1.013721376702048{\cdot}10^{-10}B_{1,4}(X) - 3.552268233665051 \\ {\cdot}10^{-07}B_{2,4}(X) - 1.009108120675939{\cdot}10^{-10}B_{3,4}(X) + 1.065608557899316{\cdot}10^{-06}B_{4,4}(X)$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \cdot 10^{-06} X^2 \\
&\quad - 4.262597970914744 \cdot 10^{-06} X + 1.06575065792877 \cdot 10^{-06} \\
&= 1.06575065792877 \cdot 10^{-06} B_{0,3} - 3.551153323761442 \cdot 10^{-07} B_{1,3} \\
&\quad - 3.553376935441088 \cdot 10^{-07} B_{2,3} + 1.065608609633713 \cdot 10^{-06} B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q_3} &= -5.304989477413181 \cdot 10^{-314} X^4 + 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \\
&\quad \cdot 10^{-06} X^2 - 4.262597970914744 \cdot 10^{-06} X + 1.06575065792877 \cdot 10^{-06} \\
&= 1.06575065792877 \cdot 10^{-06} B_{0,4} + 1.011652000844408 \cdot 10^{-10} B_{1,4} - 3.552265129601265 \\
&\quad \cdot 10^{-07} B_{2,4} - 1.011177496533579 \cdot 10^{-10} B_{3,4} + 1.065608609633713 \cdot 10^{-06} B_{4,4}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.104063786460057 \cdot 10^{-13}$.
Bounding polynomials M and m:

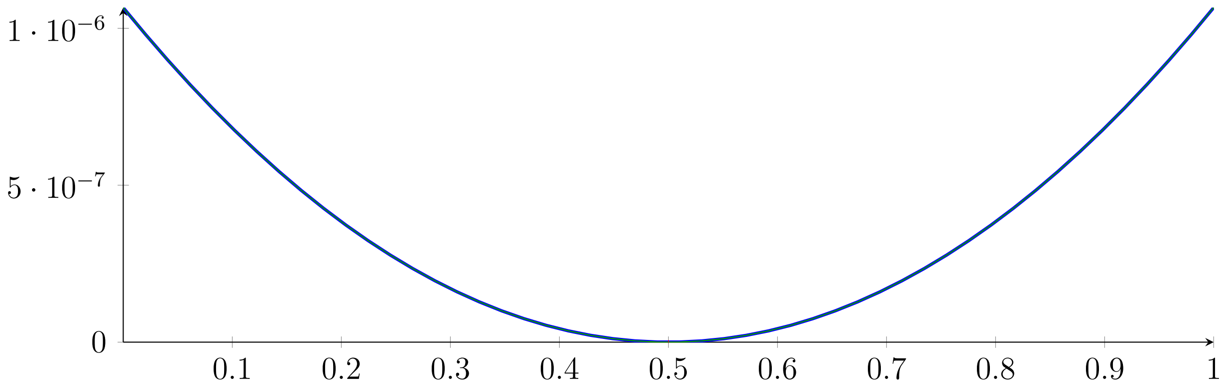
$$
\begin{aligned}
M &= 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \cdot 10^{-06} X^2 \\
&\quad - 4.262597970914744 \cdot 10^{-06} X + 1.065750968335149 \cdot 10^{-06}
\end{aligned}
$$

$$
\begin{aligned}
m &= 5.250352088360861 \cdot 10^{-10} X^3 + 4.26193088741085 \cdot 10^{-06} X^2 \\
&\quad - 4.262597970914744 \cdot 10^{-06} X + 1.065750347522392 \cdot 10^{-06}
\end{aligned}
$$

Root of M and m:

$N(M) = \{-8118.41926893212\}$ $\quad N(m) = \{-8118.419268932102, 0.4997707518163204, 0.5002933638554905\}$

Intersection intervals:



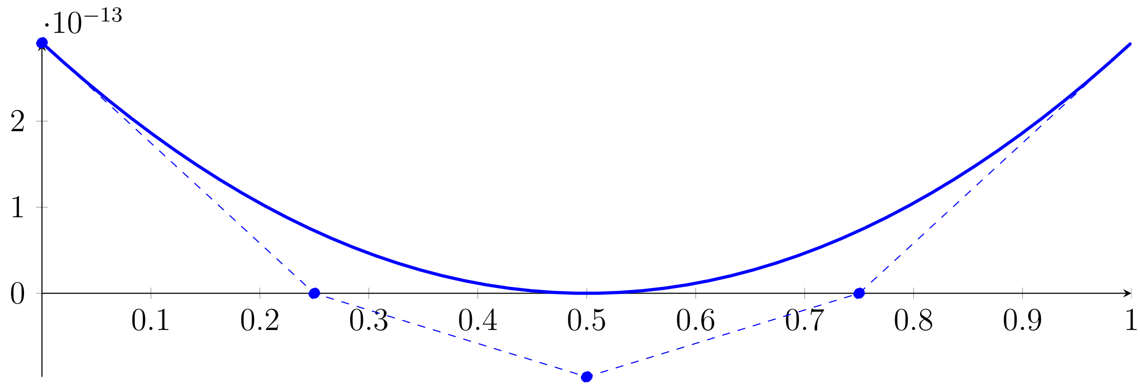
$$[0.4997707518163204, 0.5002933638554905]$$

Longest intersection interval: 0.0005226120391701048
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996445302967, 0.4000003654697068]$,

39.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999996445302967, 0.400000365469$

Normalized monomial und Bézier representations and the Bézier polygon:

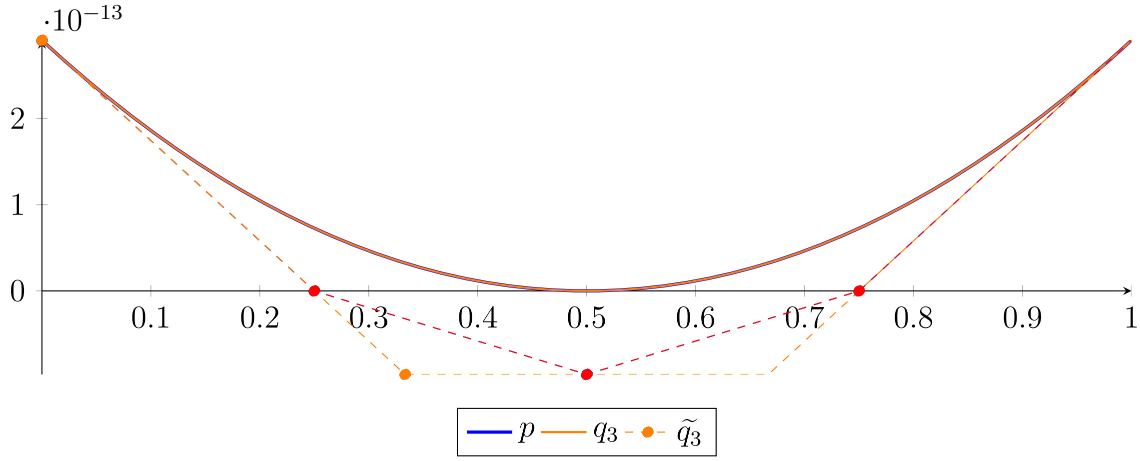
$$\begin{aligned}p &= -2.701438390310769\cdot10^{-25}X^4 + 7.494271205548112\cdot10^{-20}X^3 + 1.164248026057075\\&\quad\cdot10^{-12}X^2 - 1.164248076702198\cdot10^{-12}X + 2.91006022469042\cdot10^{-13}\\&= 2.91006022469042\cdot10^{-13}B_{0,4}(X) - 5.599670650739874\cdot10^{-17}B_{1,4}(X) - 9.707667820587771\\&\quad\cdot10^{-14}B_{2,4}(X) - 5.600329339091629\cdot10^{-17}B_{3,4}(X) + 2.910060467663608\cdot10^{-13}B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 7.494217176780306\cdot10^{-20}X^3 + 1.164248026057422\cdot10^{-12}X^2\\&\quad - 1.164248076702275\cdot10^{-12}X + 2.910060224690459\cdot10^{-13}\\&= 2.910060224690459\cdot10^{-13}B_{0,3} - 9.707666976504573\cdot10^{-14}B_{1,3}\\&\quad - 9.707668664666337\cdot10^{-14}B_{2,3} + 2.910060467663647\cdot10^{-13}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= -1.011846442682873\cdot10^{-320}X^4 + 7.494217176780306\cdot10^{-20}X^3 + 1.164248026057422\\&\quad\cdot10^{-12}X^2 - 1.164248076702275\cdot10^{-12}X + 2.910060224690459\cdot10^{-13}\\&= 2.910060224690459\cdot10^{-13}B_{0,4} - 5.599670652283553\cdot10^{-17}B_{1,4} - 9.707667820585455\\&\quad\cdot10^{-14}B_{2,4} - 5.600329340635308\cdot10^{-17}B_{3,4} + 2.910060467663647\cdot10^{-13}B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.315518620266374 \cdot 10^{-26}$.

Bounding polynomials M and m:

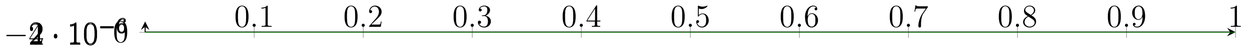
$$M = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 - 1.164248076702275 \cdot 10^{-12} X + 2.91006022469069 \cdot 10^{-13}$$

$$m = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690227 \cdot 10^{-13}$$

Root of M and m:

$N(M) = \{-15535286.38864162, 0.4930646005679288, 0.5069353931872718\}$ $N(m) = \{-15535286.38864162,$

Intersection intervals:



$[0.4930646005650611, 0.4930646005679288]$, $[0.5069353931872718, 0.5069353931901395]$

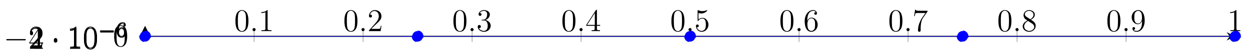
Longest intersection interval: $2.86768528910705 \cdot 10^{-12}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999999999, 0.399999999999999]$, interval 2: $[0.40000001, 0.400000$

39.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.399999999999999, 0.39999999999$

Normalized monomial und Bézier representations and the Bézier polygon:

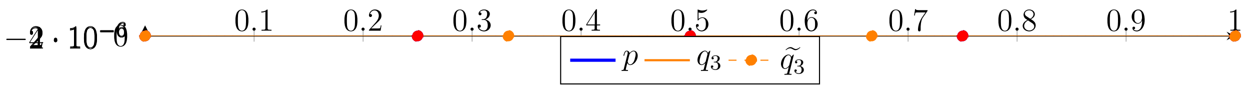
$$\begin{aligned} p &= -1.826926271922849 \cdot 10^{-71} X^4 + 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \\ &= 2.367301365424222 \cdot 10^{-23} B_{0,4}(X) + 2.366143605869363 \cdot 10^{-23} B_{1,4}(X) + 2.364985846314663 \cdot 10^{-23} B_{2,4}(X) + 2.363828086760123 \cdot 10^{-23} B_{3,4}(X) + 2.362670327205742 \cdot 10^{-23} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \\ &= 2.367301365424222 \cdot 10^{-23} B_{0,3} + 2.365757686017743 \cdot 10^{-23} B_{1,3} + 2.364214006611583 \cdot 10^{-23} B_{2,3} + 2.362670327205742 \cdot 10^{-23} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 2.944860731609145 \cdot 10^{-331} X^4 + 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23} \\ &= 2.367301365424222 \cdot 10^{-23} B_{0,4} + 2.366143605869363 \cdot 10^{-23} B_{1,4} + 2.364985846314663 \cdot 10^{-23} B_{2,4} + 2.363828086760123 \cdot 10^{-23} B_{3,4} + 2.362670327205742 \cdot 10^{-23} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.565936804505299 \cdot 10^{-72}$.
Bounding polynomials M and m:

$$M = 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$

$$m = 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$

Root of M and m:

$N(M) = \{-5.417360610380978 \cdot 10^{18}, 0.4688611824150995, 4836929866.893817\}$ $N(m) = \{-5.417360610380$

Intersection intervals:

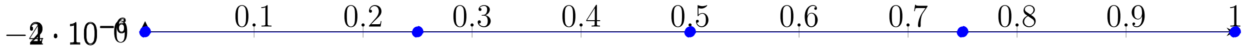
$$[0.4688611824150995, 0.4688611824150995]$$

Longest intersection interval: $6.762790809710196 \cdot 10^{-47}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999999999, 0.399999999999999]$,

39.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.399999999999999, 0.3999999999$

Normalized monomial und Bézier representations and the Bézier polygon:

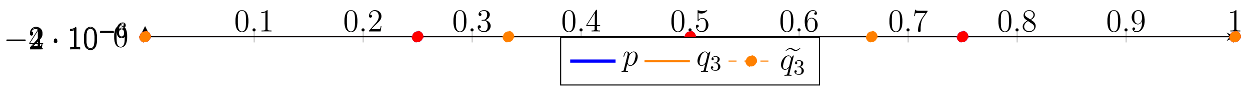
$$p = -3.821420564473856 \cdot 10^{-256} X^4 + 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$
$$= 2.365130051369057 \cdot 10^{-23} B_{0,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{1,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{2,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{3,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{4,4}(X)$$



Degree reduction and raising:

$$q_3 = 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$
$$= 2.365130051369057 \cdot 10^{-23} B_{0,3} + 2.365130051369057 \cdot 10^{-23} B_{1,3} + 2.365130051369057 \cdot 10^{-23} B_{2,3} + 2.365130051369057 \cdot 10^{-23} B_{3,3}$$

$$\tilde{q}_3 = 5.88972146321829 \cdot 10^{-331} X^4 + 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$
$$= 2.365130051369057 \cdot 10^{-23} B_{0,4} + 2.365130051369057 \cdot 10^{-23} B_{1,4} + 2.365130051369057 \cdot 10^{-23} B_{2,4} + 2.365130051369057 \cdot 10^{-23} B_{3,4} + 2.365130051369057 \cdot 10^{-23} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 3.275503340977591 \cdot 10^{-257}$.

Bounding polynomials M and m:

$$M = 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$

$$m = 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$

Root of M and m:

$N(M) = \{-8.010539972052051 \cdot 10^{64}, 9.723286868432262 \cdot 10^{30}, 7.15226894121871 \cdot 10^{55}\}$ $\quad N(m) = \{-8.010539$

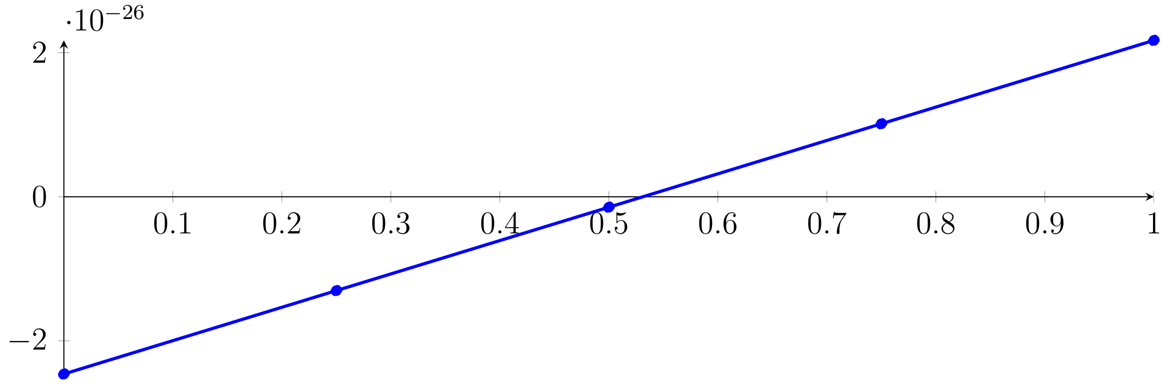
Intersection intervals:

No intersection intervals with the x axis.

39.8 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

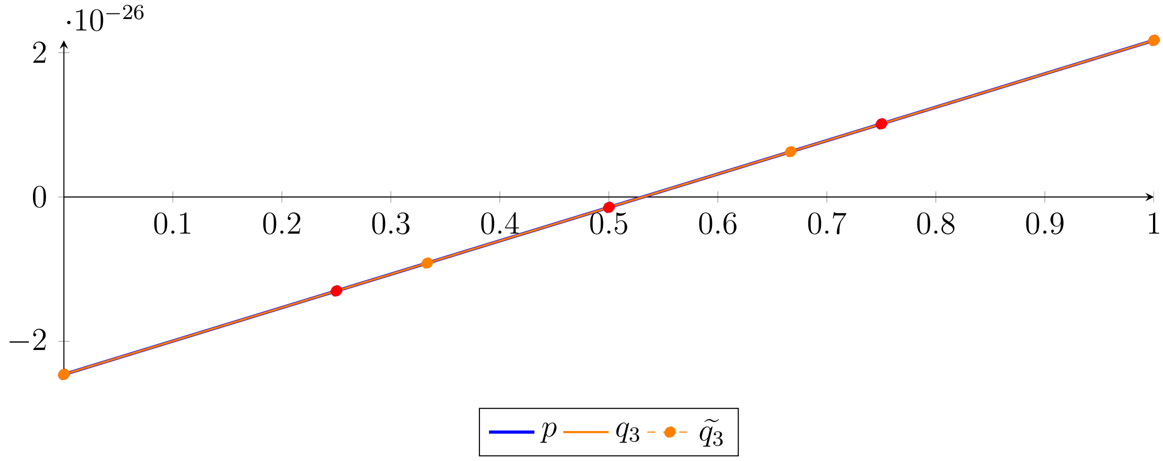
$$\begin{aligned} p &= -1.826926265398112 \cdot 10^{-71} X^4 + 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \\ &\quad \cdot 10^{-36} X^2 + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ &= -2.459960157279009 \cdot 10^{-26} B_{0,4}(X) - 1.302200847385123 \cdot 10^{-26} B_{1,4}(X) - 1.444415373316643 \\ &\quad \cdot 10^{-27} B_{2,4}(X) + 1.013317772881366 \cdot 10^{-26} B_{3,4}(X) + 2.171077083253969 \cdot 10^{-26} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \cdot 10^{-36} X^2 \\ &\quad + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ &= -2.459960157279009 \cdot 10^{-26} B_{0,3} - 9.162810774204939 \cdot 10^{-27} B_{1,3} \\ &\quad + 6.273980027571653 \cdot 10^{-27} B_{2,3} + 2.171077083253969 \cdot 10^{-26} B_{3,3} \end{aligned}$$

$$\begin{aligned} \tilde{q}_3 &= -2.875840558212056 \cdot 10^{-334} X^4 + 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \\ &\quad \cdot 10^{-36} X^2 + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ &= -2.459960157279009 \cdot 10^{-26} B_{0,4} - 1.302200847385123 \cdot 10^{-26} B_{1,4} - 1.444415373316643 \\ &\quad \cdot 10^{-27} B_{2,4} + 1.013317772881366 \cdot 10^{-26} B_{3,4} + 2.171077083253969 \cdot 10^{-26} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.565936798912668 \cdot 10^{-72}$.
Bounding polynomials M and m:

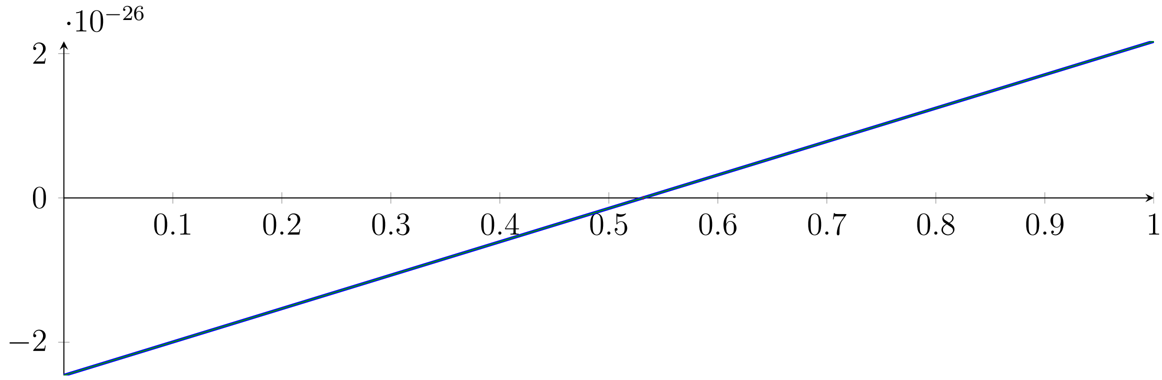
$$\begin{aligned} M = {} & 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \cdot 10^{-36} X^2 \\ & + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \end{aligned}$$

$$\begin{aligned} m = {} & 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \cdot 10^{-36} X^2 \\ & + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \end{aligned}$$

Root of M and m:

$N(M) = \{-5.417361703527237 \cdot 10^{18}, -4836929870.212556, 0.5311898889490086\}$ $\quad N(m) = \{-5.41736170352$

Intersection intervals:



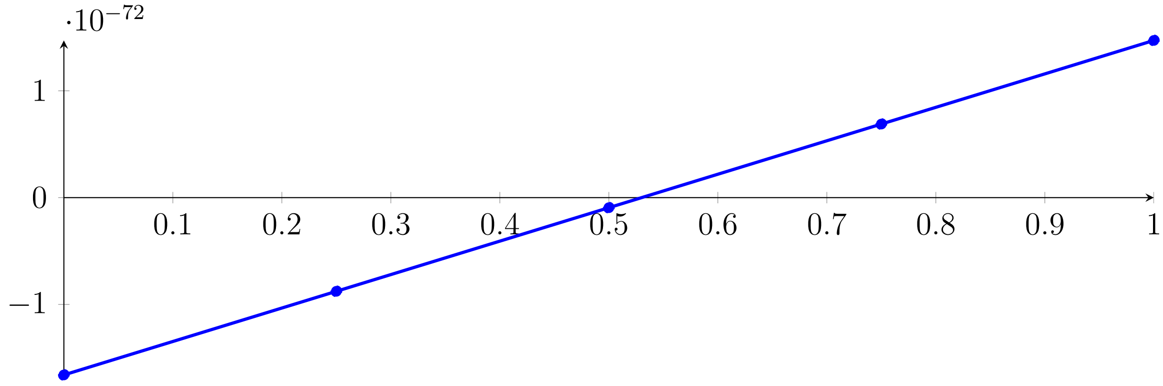
$$[0.5311898889490086, 0.5311898889490086]$$

Longest intersection interval: $6.76279078555737 \cdot 10^{-47}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

39.9 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

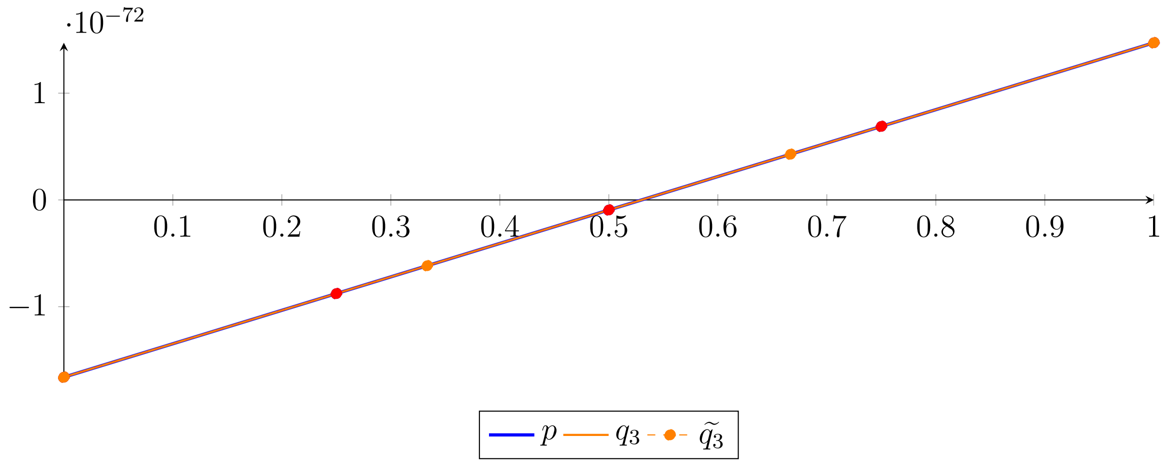
$$\begin{aligned} p = {} & -3.8214204962342 \cdot 10^{-256} X^4 + 5.466364055464 \cdot 10^{-193} X^3 + 4.378853679946907 \\ & \cdot 10^{-128} X^2 + 3.131873597825335 \cdot 10^{-72} X - 1.66001674389741 \cdot 10^{-72} \\ = {} & -1.66001674389741 \cdot 10^{-72} B_{0,4}(X) - 8.770483444410761 \cdot 10^{-73} B_{1,4}(X) - 9.407994498474221 \\ & \cdot 10^{-74} B_{2,4}(X) + 6.888884544715917 \cdot 10^{-73} B_{3,4}(X) + 1.471856853927926 \cdot 10^{-72} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 5.466364055464\cdot 10^{-193}X^3 + 4.378853679946907\cdot 10^{-128}X^2 \\ &\quad + 3.131873597825335\cdot 10^{-72}X - 1.66001674389741\cdot 10^{-72} \\ &= -1.66001674389741\cdot 10^{-72}B_{0,3} - 6.160588779556315\cdot 10^{-73}B_{1,3} \\ &\quad + 4.27898987986147\cdot 10^{-73}B_{2,3} + 1.471856853927926\cdot 10^{-72}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.77804152299898\cdot 10^{-380}X^4 + 5.466364055464\cdot 10^{-193}X^3 + 4.378853679946907 \\ &\quad \cdot 10^{-128}X^2 + 3.131873597825335\cdot 10^{-72}X - 1.66001674389741\cdot 10^{-72} \\ &= -1.66001674389741\cdot 10^{-72}B_{0,4} - 8.770483444410761\cdot 10^{-73}B_{1,4} - 9.407994498474221 \\ &\quad \cdot 10^{-74}B_{2,4} + 6.888884544715917\cdot 10^{-73}B_{3,4} + 1.471856853927926\cdot 10^{-72}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.275503282486457\cdot 10^{-257}$.

Bounding polynomials M and m:

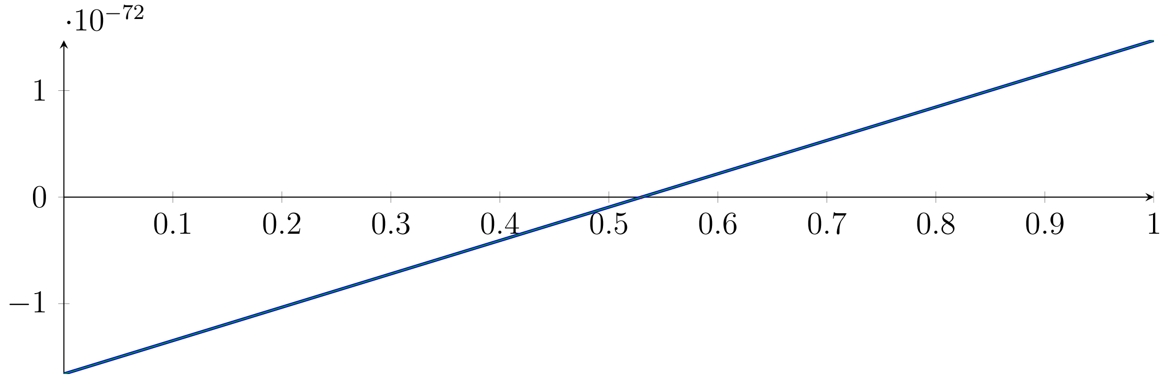
$$\begin{aligned} M &= 5.466364055464\cdot 10^{-193}X^3 + 4.378853679946907\cdot 10^{-128}X^2 \\ &\quad + 3.131873597825335\cdot 10^{-72}X - 1.66001674389741\cdot 10^{-72} \end{aligned}$$

$$\begin{aligned} m &= 5.466364055464\cdot 10^{-193}X^3 + 4.378853679946907\cdot 10^{-128}X^2 \\ &\quad + 3.131873597825335\cdot 10^{-72}X - 1.66001674389741\cdot 10^{-72} \end{aligned}$$

Root of M and m:

$N(M) = \{-8.010541617074072\cdot 10^{64}, -7.152268973148635\cdot 10^{55}, 0.5300395089540229\}$ $\quad N(m) = \{-8.010541$

Intersection intervals:



$$[0.5300395089540229, 0.5300395089540229]$$

Longest intersection interval: $2.091721252582386 \cdot 10^{-185}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

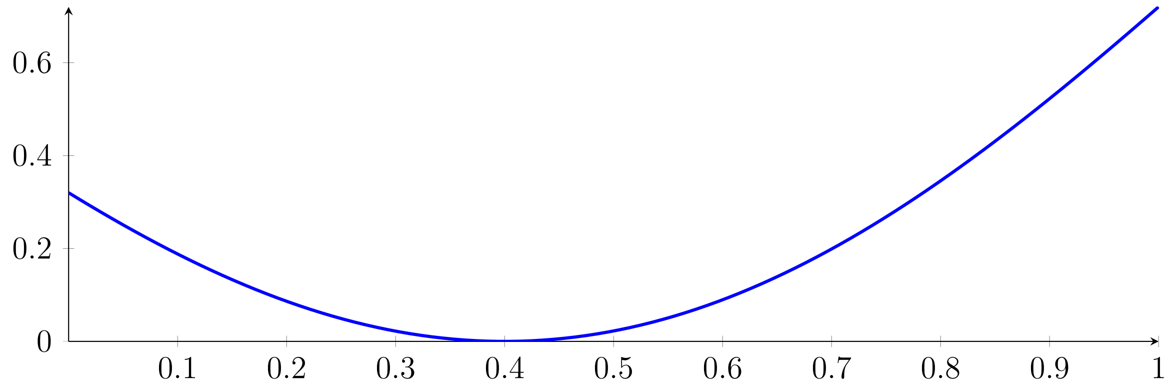
39.10 Recursion Branch 1 1 1 1 1 2 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 8!

39.11 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.40000001, 0.40000001]$$

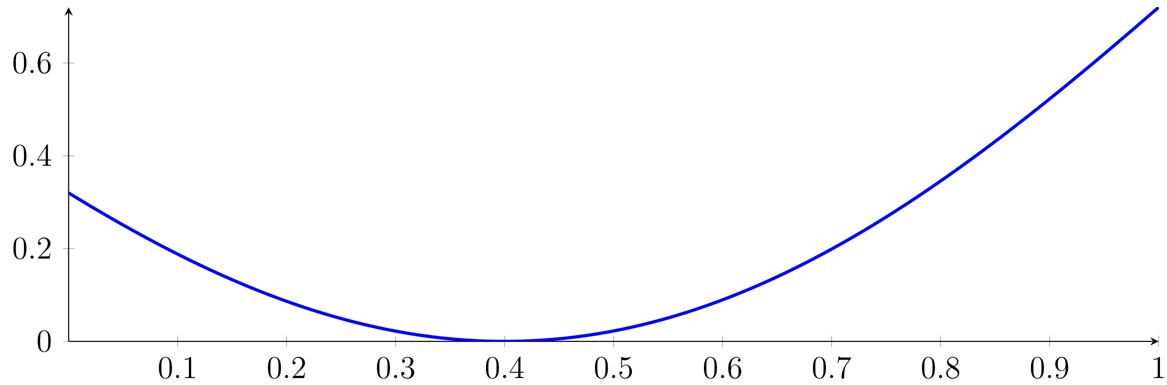
with precision $\varepsilon = 1 \cdot 10^{-64}$.

40 Running `BezClip` on h_4 with epsilon 128

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `BezClip` with input polynomial on interval $[0, 1]$**:**

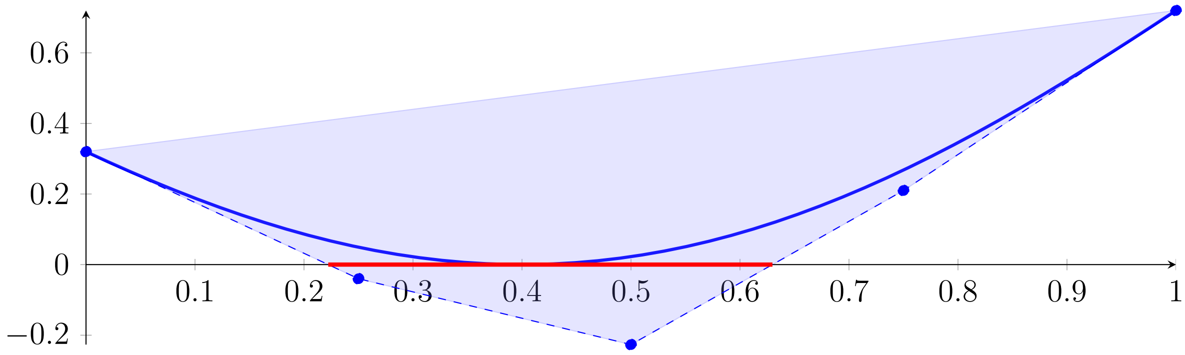
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



40.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.222222225308642, 0.6297709955349339\}$$

Intersection intervals with the x axis:

$$[0.222222225308642, 0.6297709955349339]$$

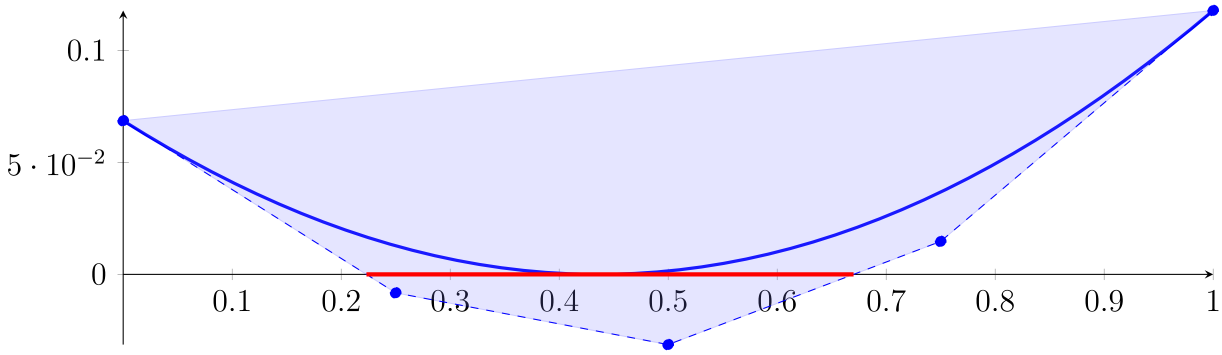
Longest intersection interval: 0.4075487702262921

$\Longrightarrow$ Selective recursion: interval 1: $[0.222222225308642, 0.6297709955349339]$,

40.2 Recursion Branch 1 1 in Interval 1: $[0.222222225308642, 0.6297709955349339]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-0.02758788125352538X^4 + 0.06167513415248929X^3 \\ &+ 0.3228414107731209X^2 - 0.3077021203429291X + 0.06867245999925484 \\ = &\ 0.06867245999925484B_{0,4}(X) - 0.008253070086477433B_{1,4}(X) - 0.03137169837668956B_{2,4}(X) \\ &+ 0.01473535866674078B_{3,4}(X) + 0.1178990033284105B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2231783775903802, 0.6701024766509123\}$$

Intersection intervals with the x axis:

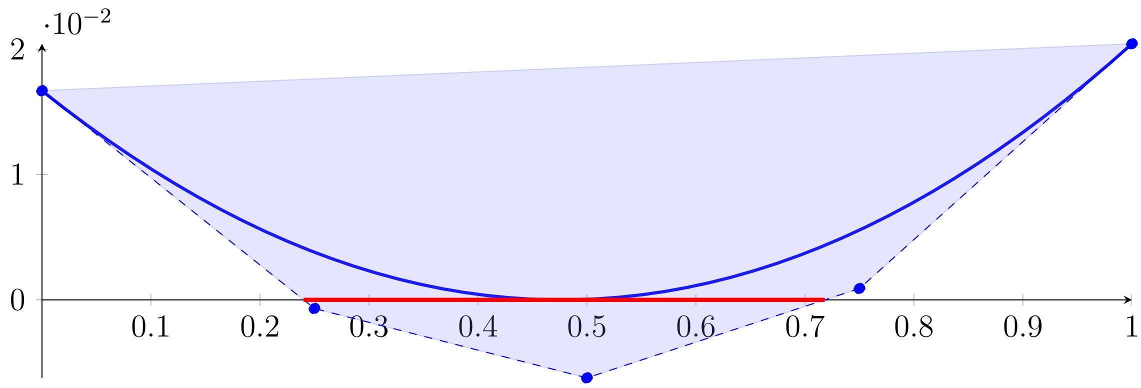
$$[0.2231783775903802, 0.6701024766509123]$$

Longest intersection interval: 0.4469240990605321
$\Longrightarrow$ Selective recursion: interval 1: $[0.3131782986367004, 0.4953216655933138]$,

40.3 Recursion Branch 1 1 1 in Interval 1: $[0.3131782986367004, 0.495321665593313$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-0.001100660652934182X^4 + 0.003307158935864367X^3 \\
&+ 0.07108595776490571X^2 - 0.06954608757725321X + 0.01669742536214377 \\
= &\ 0.01669742536214377B_{0,4}(X) - 0.0006890965321695315B_{1,4}(X) - 0.006227958798998549B_{2,4}(X) \\
&+ 0.0009076282956228099B_{3,4}(X) + 0.02044379383272645B_{4,4}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2400915126044428, 0.7182006440539784\}$$

Intersection intervals with the x axis:

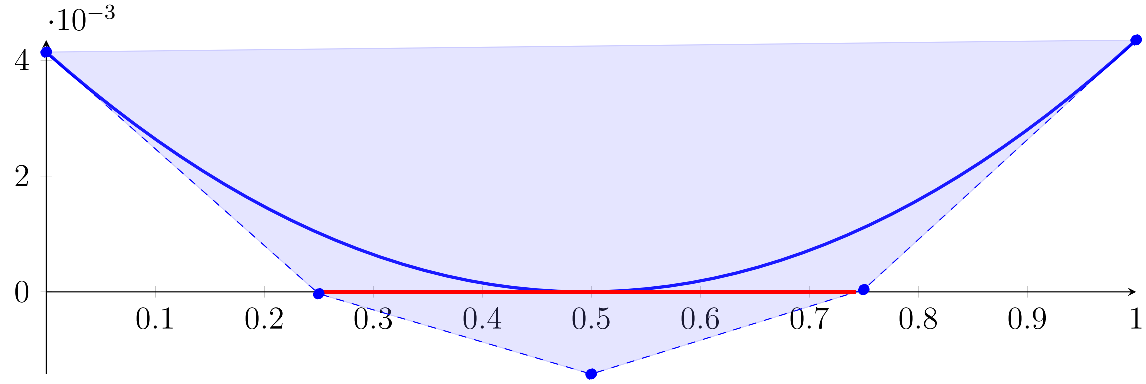
$$[0.2400915126044428, 0.7182006440539784]$$

Longest intersection interval: 0.4781091314495356
$\Longrightarrow$ Selective recursion: interval 1: $[0.3569093751201798, 0.4439937820951003]$,

40.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3569093751201798, 0.44399378209510$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -5.751241374789061\cdot 10^{-05}X^4 + 0.0002459162030906773X^3\\
&\quad + 0.01670691422385101X^2 - 0.01668640845758443X + 0.004139787469617577\\
&= 0.004139787469617577B_{0,4}(X) - 3.181464477852956\cdot 10^{-05}B_{1,4}(X)\\
&\quad - 0.001418931055199467B_{2,4}(X) + 3.991728912743387\\
&\quad \cdot 10^{-05}B_{3,4}(X) + 0.004348697025226952B_{4,4}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2480933797192248, 0.7431594518918532\}$$

Intersection intervals with the x axis:

$$[0.2480933797192248, 0.7431594518918532]$$

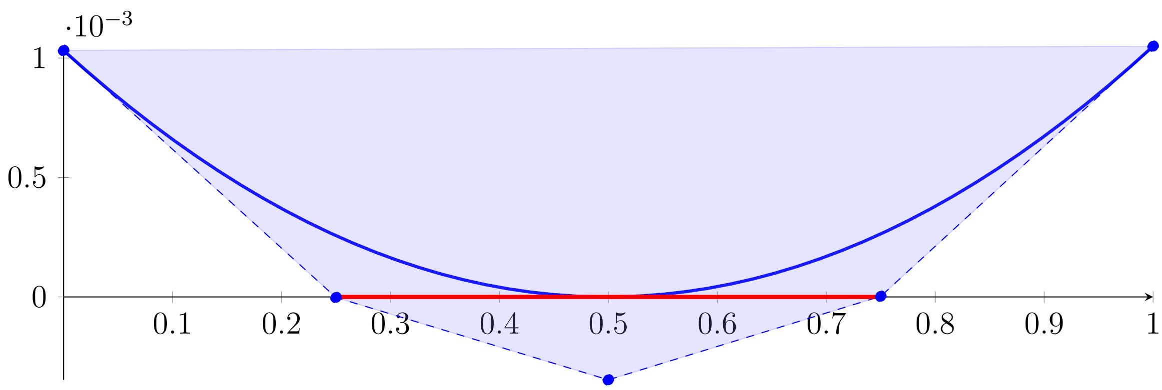
Longest intersection interval: 0.4950660721726284

$\Longrightarrow$ Selective recursion: interval 1: $[0.3785144399674323, 0.4216269752759888]$,

40.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3785144399674323, 0.421626975275$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -3.454731120985874\cdot 10^{-06}X^4 + 2.291337271769576\cdot 10^{-05}X^3\\
&\quad + 0.004134358001933594X^2 - 0.004136159738306538X + 0.001031853315293836\\
&= 0.001031853315293836B_{0,4}(X) - 2.186619282798525\cdot 10^{-06}B_{1,4}(X)\\
&\quad - 0.0003471668868705005B_{2,4}(X) + 2.640855710153786\\
&\quad \cdot 10^{-06}B_{3,4}(X) + 0.001049510220517602B_{4,4}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2494713407070459, 0.748112637751618\}$$

Intersection intervals with the x axis:

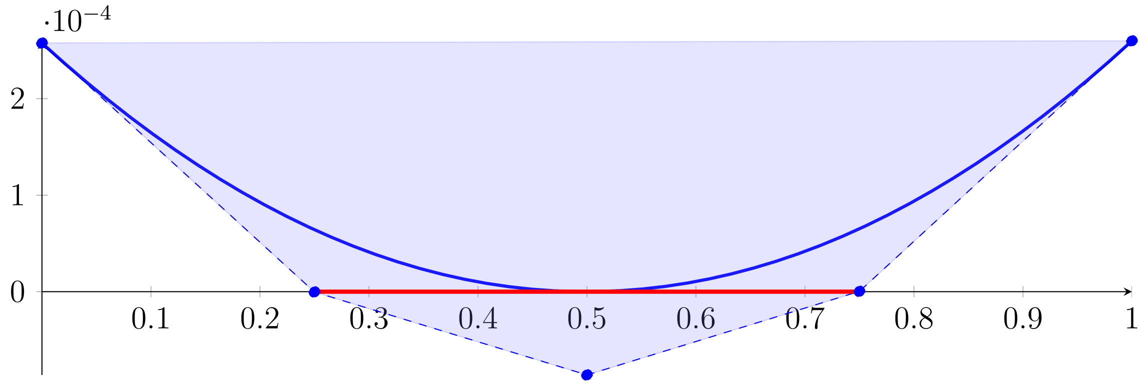
$$[0.2494713407070459, 0.748112637751618]$$

Longest intersection interval: 0.4986412970445722
$\Longrightarrow$ Selective recursion: interval 1: $[0.3892697819521377, 0.4107674724772763]$,

40.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3892697819521377, 0.41076747247$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.135832675829806 \cdot 10^{-07} X^4 + 2.413460912088167 \cdot 10^{-06} X^3 \\ &\quad + 0.001031922910124016 X^2 - 0.001031832710654165 X + 0.0002576480709401398 \\ &= 0.0002576480709401398 B_{0,4}(X) - 3.101067234014613 \cdot 10^{-07} B_{1,4}(X) - 8.628113269960673 \\ &\quad \cdot 10^{-05} B_{2,4}(X) + 3.383582395460159 \cdot 10^{-07} B_{3,4}(X) + 0.0002599381480544958 B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2496994602708371, 0.7490234350378955\}$$

Intersection intervals with the x axis:

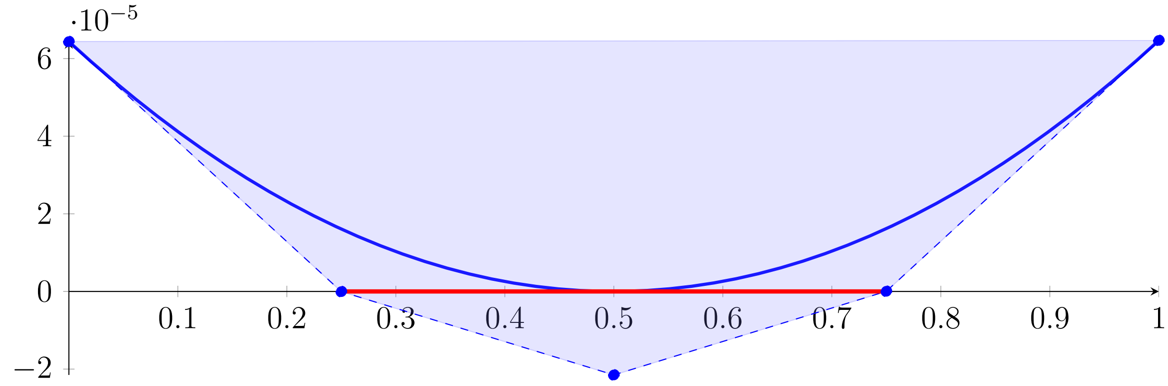
$$[0.2496994602708371, 0.7490234350378955]$$

Longest intersection interval: 0.4993239747670584
$\Longrightarrow$ Selective recursion: interval 1: $[0.3946377436733343, 0.4053720559546586]$,

40.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3946377436733343, 0.40537205$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.327690666746824 \cdot 10^{-08} X^4 + 2.739027984571277 \cdot 10^{-07} X^3 \\ &\quad + 0.000257714430407974 X^2 - 0.0002576778286693267 X + 6.437695235811399 \cdot 10^{-05} \\ &= 6.437695235811399 \cdot 10^{-05} B_{0,4}(X) - 4.250480921768141 \cdot 10^{-08} B_{1,4}(X) - 2.150955690855369 \\ &\quad \cdot 10^{-05} B_{2,4}(X) + 4.427175972025921 \cdot 10^{-08} B_{3,4}(X) + 6.467417998855097 \cdot 10^{-05} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2498350466959568, 0.7494864977308483\}$$

Intersection intervals with the x axis:

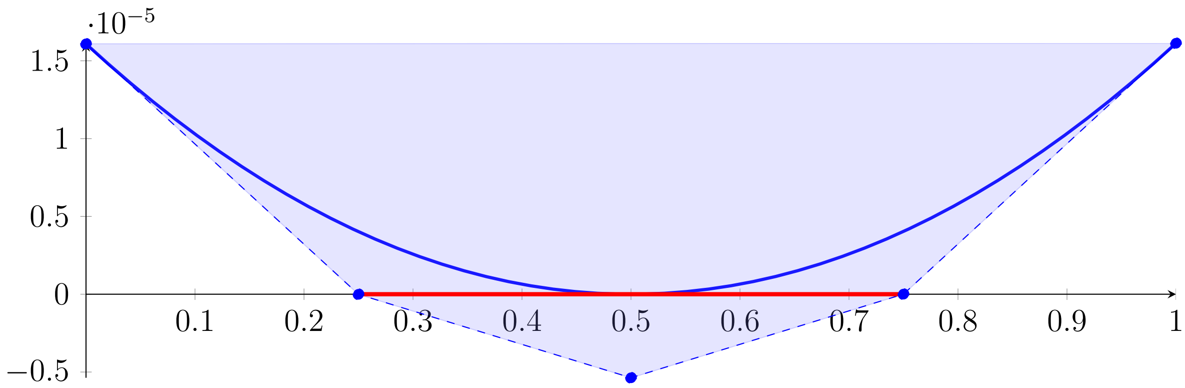
$$[0.2498350466959568, 0.7494864977308483]$$

Longest intersection interval: 0.4996514510348915
$\Longrightarrow$ Selective recursion: interval 1: $[0.3973195510833879, 0.4026829657906133]$,

40.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.3973195510833879, 0.4026829$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -8.274952589981882 \cdot 10^{-10} X^4 + 3.251124603913595 \cdot 10^{-08} X^3 + 6.438882283687421 \\ & \cdot 10^{-05} X^2 - 6.438267481175771 \cdot 10^{-05} X + 1.609012302857716 \cdot 10^{-05} \\ = & \ 1.609012302857716 \cdot 10^{-05} B_{0,4}(X) - 5.545674362270907 \cdot 10^{-09} B_{1,4}(X) - 5.369743904489329 \\ & \cdot 10^{-06} B_{2,4}(X) + 5.656149705765249 \cdot 10^{-09} B_{3,4}(X) + 1.61279548044738 \cdot 10^{-05} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499138638713212, 0.7497369428484978\}$$

Intersection intervals with the x axis:

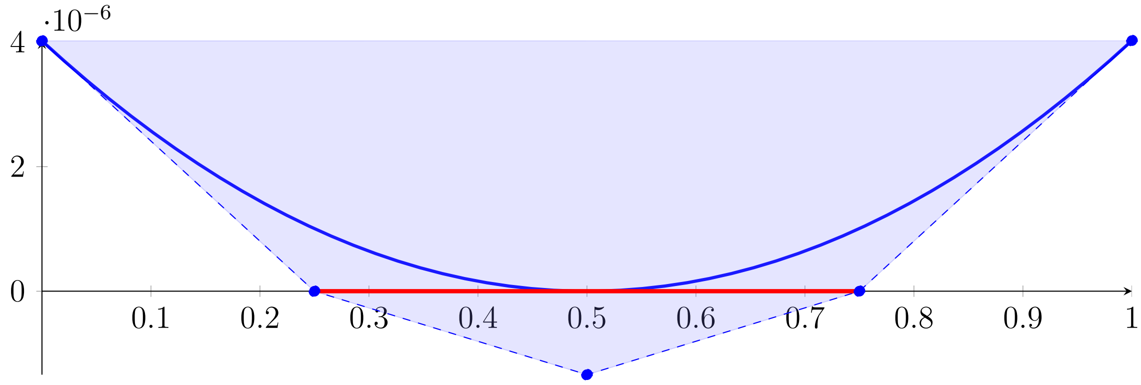
$$[0.2499138638713212, 0.7497369428484978]$$

Longest intersection interval: 0.4998230789771766
$\Longrightarrow$ Selective recursion: interval 1: $[0.3986599427764149, 0.4013407012292117]$,

40.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3986599427764149, 0.40134$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -5.164529187662443 \cdot 10^{-11} X^4 + 3.956301794548619 \cdot 10^{-09} X^3 + 1.609182796552748 \cdot 10^{-05} X^2 - 1.609096222935268 \cdot 10^{-05} X + 4.022033038444256 \cdot 10^{-06}$$
$$= 4.022033038444256 \cdot 10^{-06} B_{0,4}(X) - 7.075188939141538 \cdot 10^{-10} B_{1,4}(X) - 1.341476748644171 \cdot 10^{-06} B_{2,4}(X) + 7.14424642122371 \cdot 10^{-10} B_{3,4}(X) + 4.026803431121726 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499560300444542, 0.7498669294180401\}$$

Intersection intervals with the x axis:

$$[0.2499560300444542, 0.7498669294180401]$$

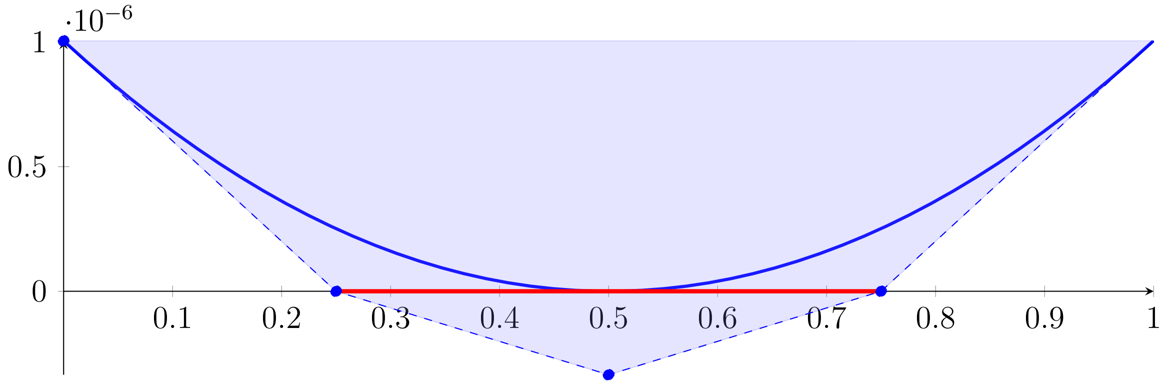
Longest intersection interval: 0.499910899373586

$\Longrightarrow$ Selective recursion: interval 1: $[0.3993300145167841, 0.4006701548859251]$,

40.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3993300145167841, 0.400$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.225530543299082 \cdot 10^{-12} X^4 + 4.878223136378425 \cdot 10^{-10} X^3 + 4.022259900669466 \cdot 10^{-06} X^2 - 4.022145641341714 \cdot 10^{-06} X + 1.00544708348311 \cdot 10^{-06}$$
$$= 1.00544708348311 \cdot 10^{-06} B_{0,4}(X) - 8.932685231834447 \cdot 10^{-11} B_{1,4}(X) - 3.352490870761692 \cdot 10^{-07} B_{2,4}(X) + 8.975838996701131 \cdot 10^{-11} B_{3,4}(X) + 1.006045939593956 \cdot 10^{-06} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499777912437083, 0.7499330838112102\}$$

Intersection intervals with the x axis:

$$[0.2499777912437083, 0.7499330838112102]$$

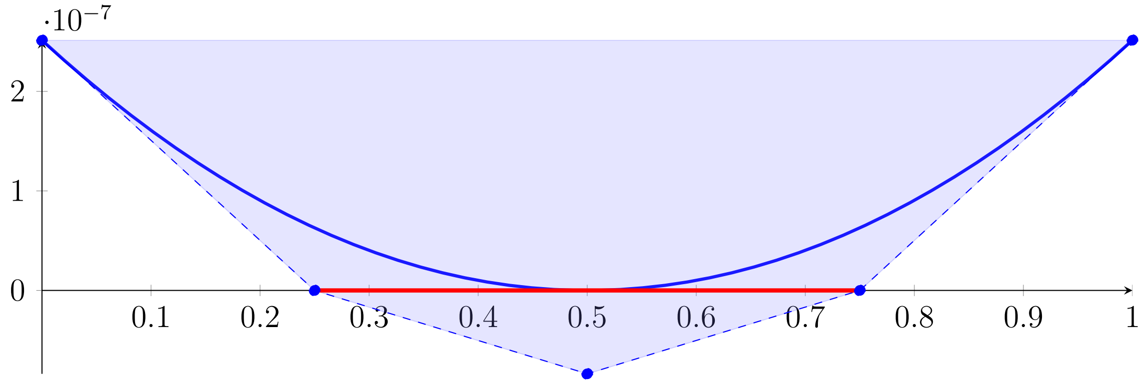
Longest intersection interval: 0.499955292567502

$\Longrightarrow$ Selective recursion: interval 1: $[0.3996650198462185, 0.4003350301165539]$,

40.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3996650198462185, 0.4$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.015235660316643 \cdot 10^{-13} X^4 + 6.055838633908679 \cdot 10^{-11} X^3 + 1.005476298211762 \cdot 10^{-06} X^2 - 1.005461639191386 \cdot 10^{-06} X + 2.51354188678016 \cdot 10^{-07}$$
$$= 2.51354188678016 \cdot 10^{-07} B_{0,4}(X) - 1.122111983053638 \cdot 10^{-11} B_{1,4}(X) - 8.379724788238336 \cdot 10^{-08} B_{2,4}(X) + 1.124798694225224 \cdot 10^{-11} B_{3,4}(X) + 2.51429204561165 \cdot 10^{-07} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499888398329751, 0.7499664473546936\}$$

Intersection intervals with the x axis:

$$[0.2499888398329751, 0.7499664473546936]$$

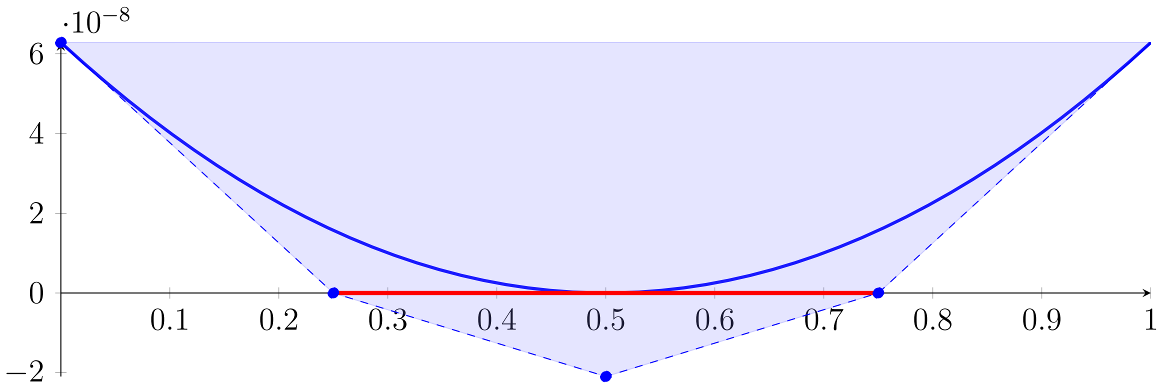
Longest intersection interval: 0.4999776075217186

$\Longrightarrow$ Selective recursion: interval 1: $[0.3998325149363758, 0.4001675050683531]$,

40.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3998325149363758, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.259296672250957 \cdot 10^{-14} X^4 + 7.543595361590864 \cdot 10^{-12} X^3 + 2.5135789423507 \cdot 10^{-07} X^2 - 2.51356038307676 \cdot 10^{-07} X + 6.283760343276371 \cdot 10^{-08}$$
$$= 6.283760343276371 \cdot 10^{-08} B_{0,4}(X) - 1.406144155297956 \cdot 10^{-12} B_{1,4}(X) - 2.094743334856264 \cdot 10^{-08} B_{2,4}(X) + 1.407718382090997 \cdot 10^{-12} B_{3,4}(X) + 6.284699036255256 \cdot 10^{-08} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.2499944057673539, 0.7499832005219574\}$$

Intersection intervals with the x axis:

$$[0.2499944057673539, 0.7499832005219574]$$

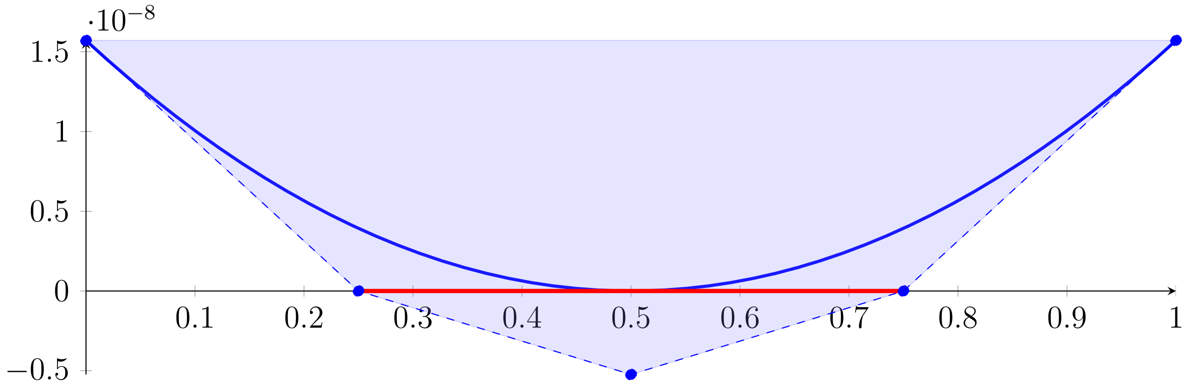
Longest intersection interval: 0.4999887947546035

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999162605953574, 0.4000837519076994]$,

40.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999162605953574

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.869898688873272\cdot10^{-16}X^4 + 9.413120459510821\cdot10^{-13}X^3 + 6.283807021204131\\&\quad\cdot10^{-08}X^2 - 6.283783670437638\cdot10^{-08}X + 1.570928313186755\cdot10^{-08}\\&= 1.570928313186755\cdot10^{-08}B_{0,4}(X) - 1.760442265467946\cdot10^{-13}B_{1,4}(X) - 5.236623518313756\\&\quad\cdot10^{-09}B_{2,4}(X) + 1.76037617409066\cdot10^{-13}B_{3,4}(X) + 1.571045716458857\cdot10^{-08}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499971984359141, 0.7499915961258623\}$$

Intersection intervals with the x axis:

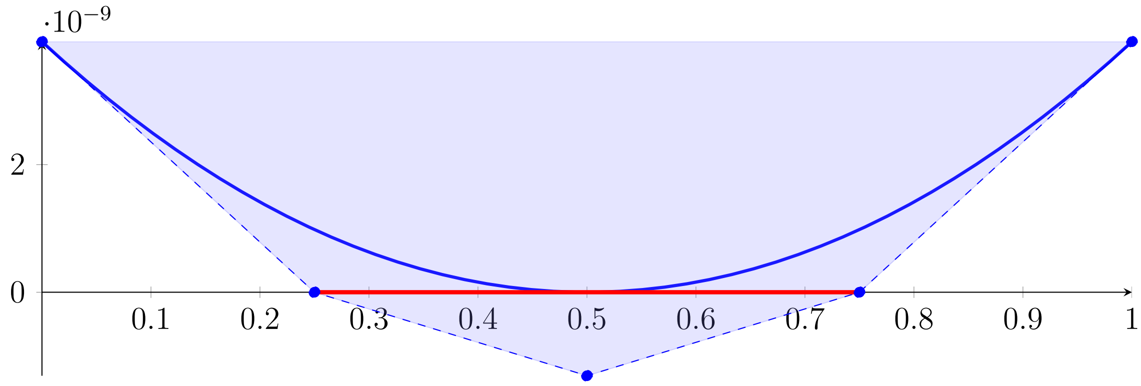
$$[0.2499971984359141, 0.7499915961258623]$$

Longest intersection interval: 0.4999943976899482
$\Longrightarrow$ Selective recursion: interval 1: [0.3999581329542053, 0.400041877672038],

40.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999581329542053, 0.400041877672038]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.918466236188167\cdot10^{-17}X^4 + 1.175616813221855\cdot10^{-13}X^3 + 1.570934193292886\\&\quad\cdot10^{-08}X^2 - 1.57093126037727\cdot10^{-08}X + 3.927306070737634\cdot10^{-09}\\&= 3.927306070737634\cdot10^{-09}B_{0,4}(X) - 2.208020554223638\cdot10^{-14}B_{1,4}(X) - 1.309126575660575\\&\quad\cdot10^{-09}B_{2,4}(X) + 2.19747928673869\cdot10^{-14}B_{3,4}(X) + 3.927452912390452\cdot10^{-09}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.249998594451196, 0.749995803609747\}$$

Intersection intervals with the x axis:

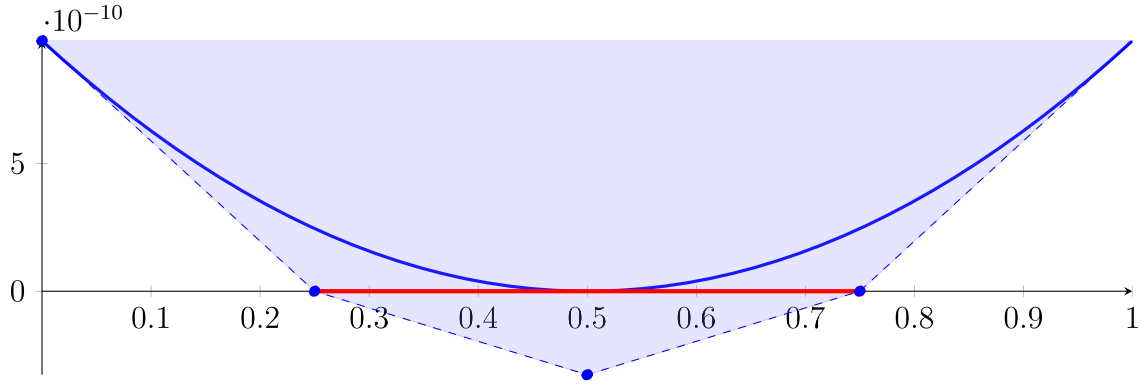
$$[0.249998594451196, 0.749995803609747]$$

Longest intersection interval: 0.499997209158551
$\Longrightarrow$ Selective recursion: interval 1: [0.3999790690159562, 0.4000209411411543],

40.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999790690159562, 0.4000209411411543]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.073972764895049\cdot10^{-18}X^4 + 1.468881614935617\cdot10^{-14}X^3 + 3.92731367890626\\&\quad\cdot10^{-09}X^2 - 3.927309958033017\cdot10^{-09}X + 9.818246673938711\cdot10^{-10}\\&= 9.818246673938711\cdot10^{-10}B_{0,4}(X) - 2.822114383159451\cdot10^{-15}B_{1,4}(X) - 3.272780318049275\\&\quad\cdot10^{-10}B_{2,4}(X) + 2.710526275559223\cdot10^{-15}B_{3,4}(X) + 9.818430740092905\cdot10^{-10}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499992814128721, 0.7499979295098023\}$$

Intersection intervals with the x axis:

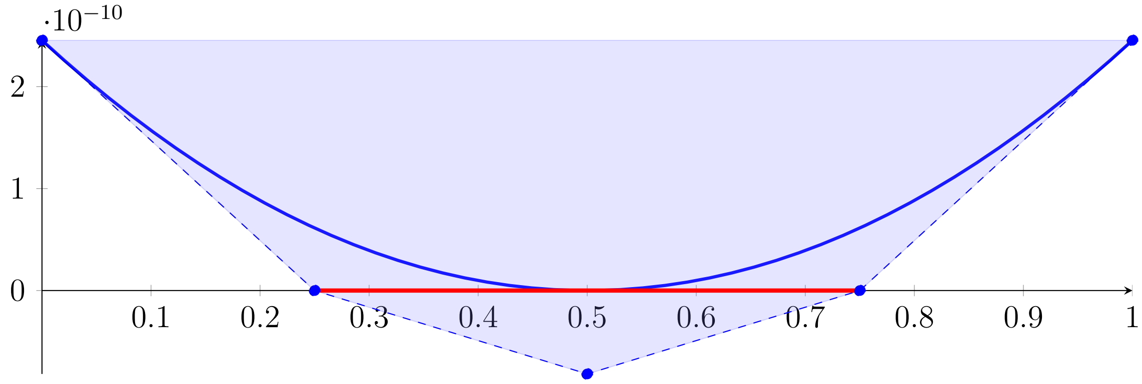
$$[0.2499992814128721, 0.7499979295098023]$$

Longest intersection interval: 0.4999986480969302
$\Longrightarrow$ Selective recursion: interval 1: [0.3999895370171669, 0.400010473023159],

40.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999895370171669, 0.400010473023159]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.921212199577591\cdot10^{-19}X^4 + 1.835702882943672\cdot10^{-15}X^3 + 9.818258642283608\\&\quad\cdot10^{-10}X^2 - 9.818253497618774\cdot10^{-10}X + 2.454559233739064\cdot10^{-10}\\&= 2.454559233739064\cdot10^{-10}B_{0,4}(X) - 4.140665629492134\cdot10^{-16}B_{1,4}(X) - 8.181910746897216\\&\quad\cdot10^{-11}B_{2,4}(X) + 3.02092399485025\cdot10^{-16}B_{3,4}(X) + 2.454582733511515\cdot10^{-10}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995782686167, 0.7499990769537415\}$$

Intersection intervals with the x axis:

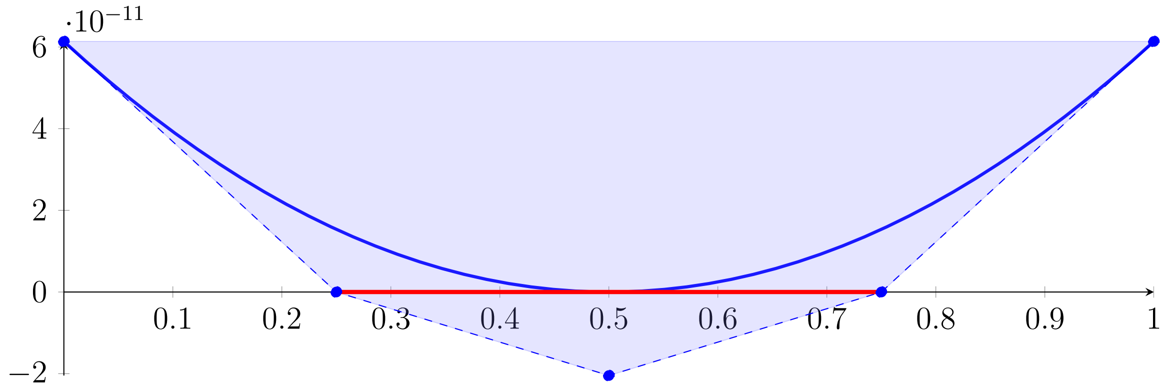
$$[0.2499995782686167, 0.7499990769537415]$$

Longest intersection interval: 0.4999994986851248
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999947710098356, 0.400005239002336]$,

40.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999947710098356, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.200752809081967\cdot 10^{-20}X^4 + 2.294381551300316\cdot 10^{-16}X^3 + 2.454563180284348 \\ &\quad \cdot 10^{-10}X^2 - 2.454562046981275\cdot 10^{-10}X + 6.136393816301913\cdot 10^{-11} \\ &= 6.136393816301913\cdot 10^{-11}B_{0,4}(X) - 1.130115127449974\cdot 10^{-16}B_{1,4}(X) - 2.045477784797216 \\ &\quad \cdot 10^{-11}B_{2,4}(X) + 1.013179678980307\cdot 10^{-18}B_{3,4}(X) + 6.136428091947402\cdot 10^{-11}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2499995395858382, 0.7499999876168341\}$$

Intersection intervals with the x axis:

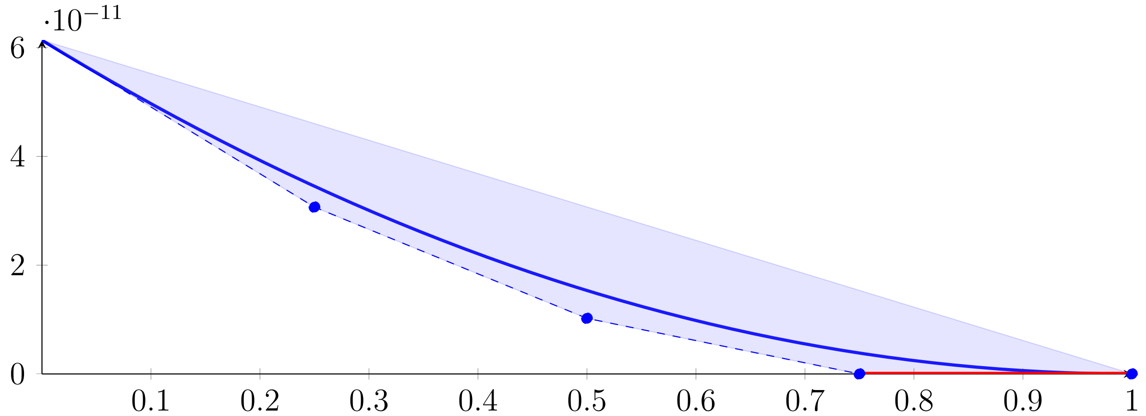
$$[0.2499995395858382, 0.7499999876168341]$$

Longest intersection interval: 0.5000004480309959
$\Longrightarrow$ Bisection: first half $[0.3999947710098356, 0.4000000050060858]$ und second half $[0.4000000050060858, 0.40$

40.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half [0.3999947710098356, 0.4000000050060858]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.504705056762291\cdot 10^{-22}X^4 + 2.867976939125394\cdot 10^{-17}X^3 + 6.13640795071087\\&\quad\cdot 10^{-11}X^2 - 1.227281023490638\cdot 10^{-10}X + 6.136393816301913\cdot 10^{-11}\\&= 6.136393816301913\cdot 10^{-11}B_{0,4}(X) + 3.068191257575319\cdot 10^{-11}B_{1,4}(X) + 1.022723357300537\\&\quad\cdot 10^{-11}B_{2,4}(X) - 9.167528198681642\cdot 10^{-17}B_{3,4}(X) - 5.59999170035142\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7499977590601706, 0.9999990874140811\}$$

Intersection intervals with the x axis:

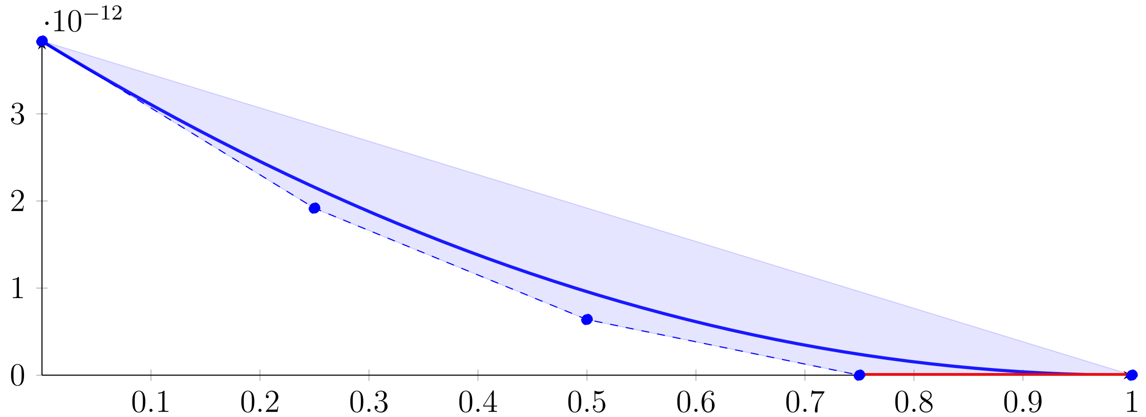
$$[0.7499977590601706, 0.9999990874140811]$$

Longest intersection interval: 0.2500013283539104
$\Longrightarrow$ Selective recursion: interval 1: [0.3999986964952942, 0.4000000050013093],

40.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3999986964952942, 0.4000000050013093]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.931587718946285\cdot 10^{-24}X^4 + 4.480933611814181\cdot 10^{-19}X^3 + 3.835299758875134\\&\quad\cdot 10^{-12}X^2 - 7.670593186649561\cdot 10^{-12}X + 3.835236979687871\cdot 10^{-12}\\&= 3.835236979687871\cdot 10^{-12}B_{0,4}(X) + 1.917588683025481\cdot 10^{-12}B_{1,4}(X) + 6.391570128422797\\&\quad\cdot 10^{-13}B_{2,4}(X) - 5.791883839216654\cdot 10^{-17}B_{3,4}(X) - 5.59999961259787\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7499773476668325, 0.9999853987696839\}$$

Intersection intervals with the x axis:

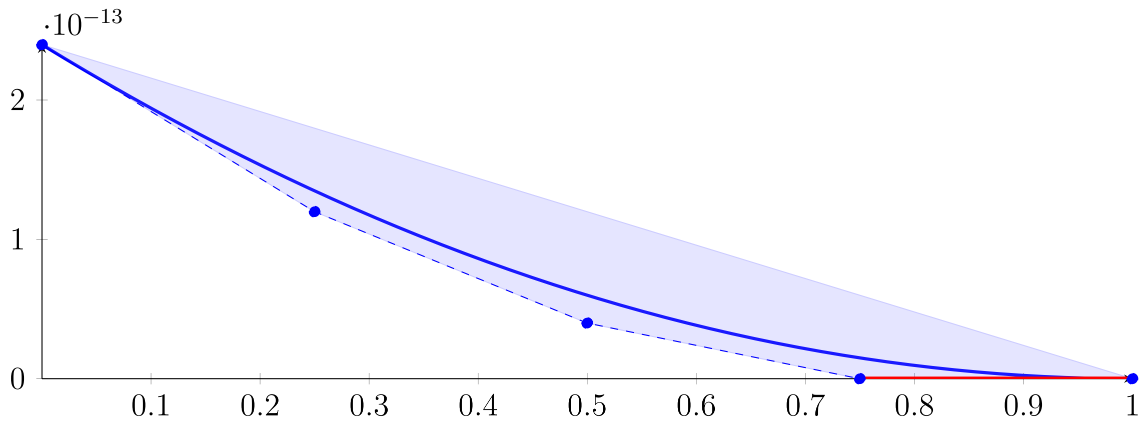
$$[0.7499773476668325, 0.9999853987696839]$$

Longest intersection interval: 0.2500080511028514
$\implies$ Selective recursion: interval 1: $[0.3999996778451648, 0.4000000049822035]$,

40.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999996778451648, 0.4000000049822035]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.14529897555355\cdot 10^{-26}X^4 + 7.001997796519582\cdot 10^{-21}X^3 + 2.397217373893792 \\ &\quad \cdot 10^{-13}X^2 - 4.794695778397391\cdot 10^{-13}X + 2.396918341578483\cdot 10^{-13} \\ &= 2.396918341578483\cdot 10^{-13}B_{0,4}(X) + 1.198244396979135\cdot 10^{-13}B_{1,4}(X) + 3.991066813620863 \\ &\quad \cdot 10^{-14}B_{2,4}(X) - 4.947877676695723\cdot 10^{-17}B_{3,4}(X) - 5.599929052523716\cdot 10^{-17}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7496904492313635, 0.9997664242061345\}$$

Intersection intervals with the x axis:

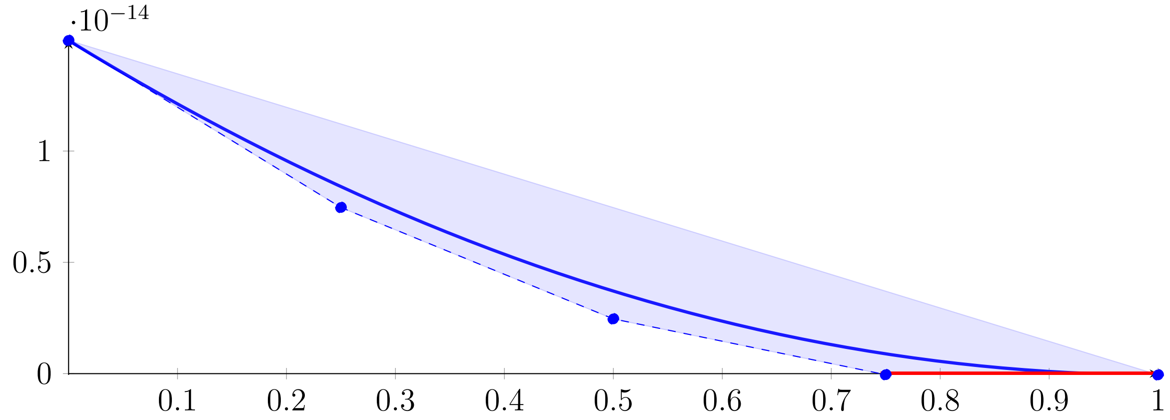
$$[0.7496904492313635, 0.9997664242061345]$$

Longest intersection interval: 0.250075974974771
$\implies$ Selective recursion: interval 1: $[0.3999999230966783, 0.4000000049057922]$,

40.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999230966783, 0.4000000049057922]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -4.479264981640815\cdot 10^{-29}X^4 + 1.095054543732688\cdot 10^{-22}X^3 + 1.499171738187779 \\ &\quad \cdot 10^{-14}X^2 - 3.001796269578916\cdot 10^{-14}X + 1.497026508467358\cdot 10^{-14} \\ &= 1.497026508467358\cdot 10^{-14}B_{0,4}(X) + 7.465774410726289\cdot 10^{-15}B_{1,4}(X) + 2.459903300425297 \\ &\quad \cdot 10^{-15}B_{2,4}(X) - 4.734821885303232\cdot 10^{-17}B_{3,4}(X) - 5.598011973238056\cdot 10^{-17}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.7452788722542423, 0.9962745104335203\}$$

Intersection intervals with the x axis:

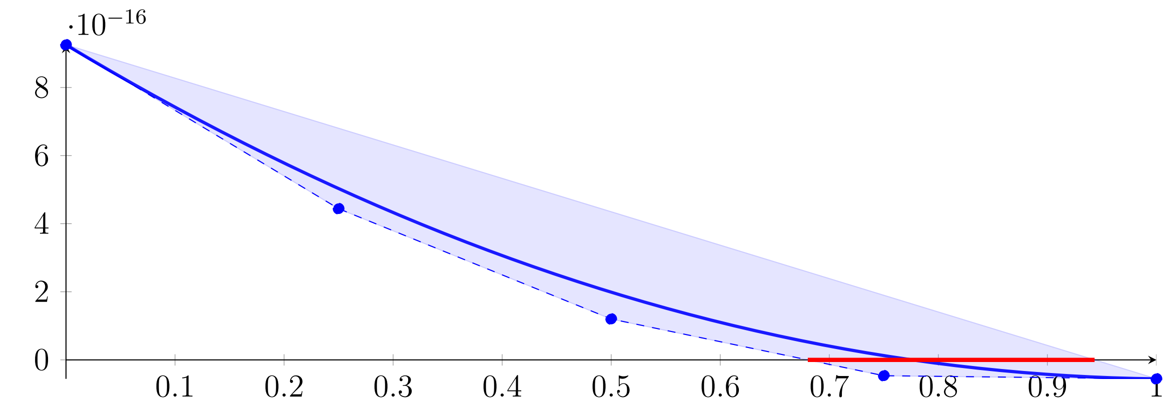
$$[0.7452788722542423, 0.9962745104335203]$$

Longest intersection interval: 0.250995638179278
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999840672825, 0.4000000046010132]$,

40.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999840672825, 0.4000000046010132]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.777753131478217\cdot 10^{-31}X^4 + 1.731544849986219\cdot 10^{-24}X^3 + 9.444603761111594\\&\cdot 10^{-16}X^2 - 1.925623994999771\cdot 10^{-15}X + 9.25520203786835\cdot 10^{-16}\\ = &\, 9.25520203786835\cdot 10^{-16}B_{0,4}(X) + 4.441142050368923\cdot 10^{-16}B_{1,4}(X) + 1.201182689721428\\&\cdot 10^{-16}B_{2,4}(X) - 4.646760397452725\cdot 10^{-17}B_{3,4}(X) - 5.564341337023184\cdot 10^{-17}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.6802647890355577, 0.9432883441688765\}$$

Intersection intervals with the x axis:

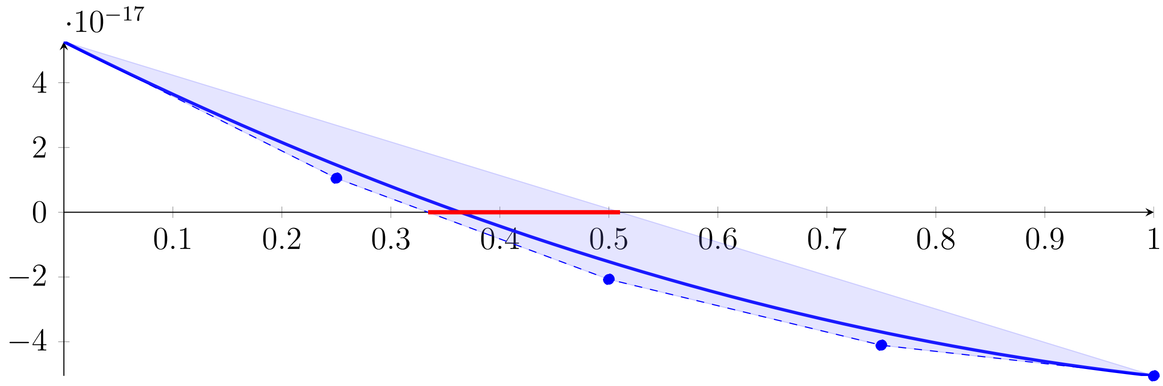
$$[0.6802647890355577, 0.9432883441688765]$$

Longest intersection interval: 0.2630235551333187
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999980356565, 0.4000000034365114]$,

40.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999980356565, 0.4000000034365114]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.508441684091975 \cdot 10^{-34} X^4 + 3.150776186268732 \cdot 10^{-26} X^3 + 6.533908238790805 \cdot 10^{-17} X^2 - 1.685080699755733 \cdot 10^{-16} X + 5.264466028744181 \cdot 10^{-17}$$
$$= 5.264466028744181 \cdot 10^{-17} B_{0,4}(X) + 1.05176427935485 \cdot 10^{-17} B_{1,4}(X) - 2.071952763569348 \cdot 10^{-17} B_{2,4}(X) - 4.106685099240717 \cdot 10^{-17} B_{3,4}(X) - 5.052432726871564 \cdot 10^{-17} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3341757003677151, 0.5102760193201083\}$$

Intersection intervals with the x axis:

$$[0.3341757003677151, 0.5102760193201083]$$

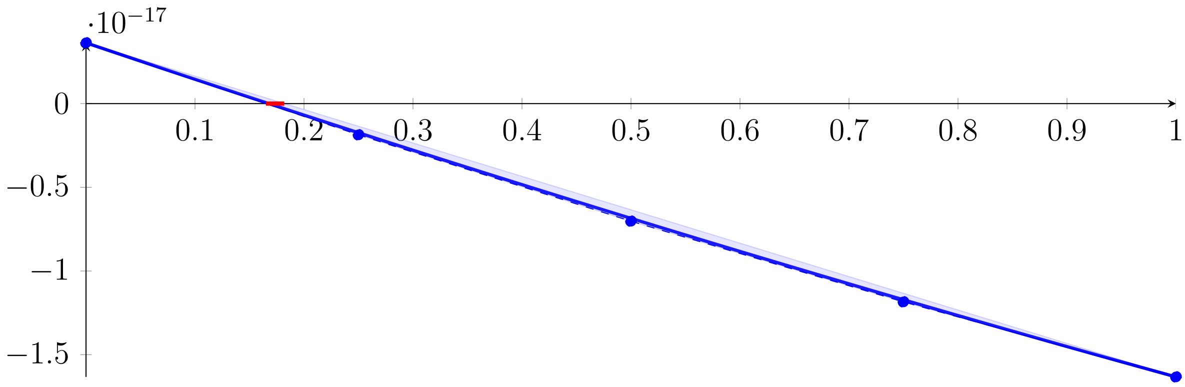
Longest intersection interval: 0.1761003189523932

$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999840491, 0.4000000007915832]$,

40.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.399999999840491, 0.4000000007915832]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.182586345704061 \cdot 10^{-37} X^4 + 1.720671503885177 \cdot 10^{-28} X^3 + 2.026251345992899 \cdot 10^{-18} X^2 - 2.198411775802323 \cdot 10^{-17} X + 3.629995386177444 \cdot 10^{-18}$$
$$= 3.629995386177444 \cdot 10^{-18} B_{0,4}(X) - 1.866034053328363 \cdot 10^{-18} B_{1,4}(X) - 7.024354935168688 \cdot 10^{-18} B_{2,4}(X) - 1.184496725930051 \cdot 10^{-17} B_{3,4}(X) - 1.632787102568082 \cdot 10^{-17} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.1651189929990553, 0.1818829383495937\}$$

Intersection intervals with the x axis:

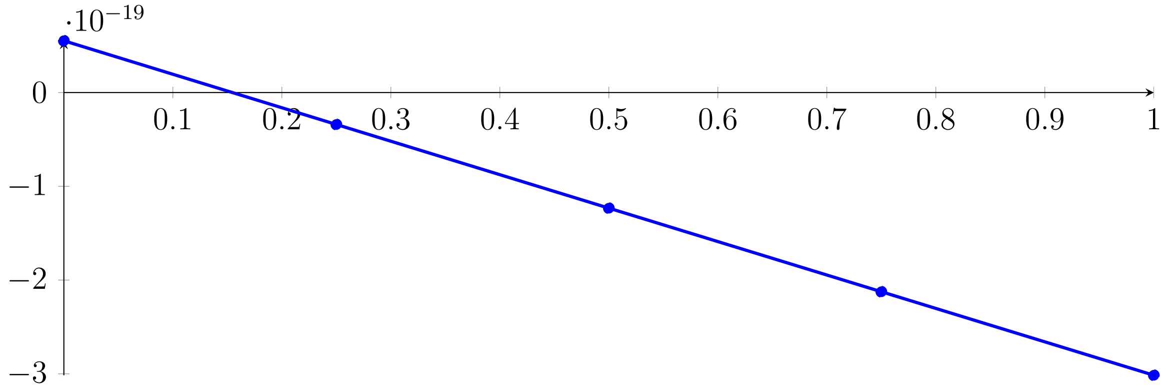
$$[0.1651189929990553, 0.1818829383495937]$$

Longest intersection interval: 0.01676394535053848
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999975344, 0.4000000000134784]$,

40.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999975344, 0.4000000000134784]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -6.462425394283182\cdot 10^{-44}X^4 + 8.106374699799114\cdot 10^{-34}X^3 + 5.694371396423764\\&\quad\cdot 10^{-22}X^2 - 3.573230357204536\cdot 10^{-19}X + 5.524428779488534\cdot 10^{-20}\\&= 5.524428779488534\cdot 10^{-20}B_{0,4}(X) - 3.408647113522805\cdot 10^{-20}B_{1,4}(X) - 1.23322323875401\\&\quad\cdot 10^{-19}B_{2,4}(X) - 2.124632704256334\cdot 10^{-19}B_{3,4}(X) - 3.01509310785925\cdot 10^{-19}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1546060070924308, 0.1548527835869093\}$$

Intersection intervals with the x axis:

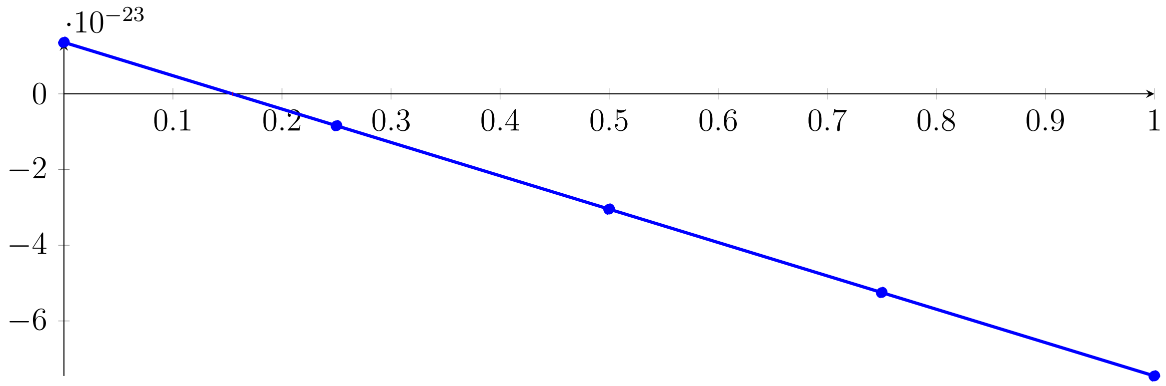
$$[0.1546060070924308, 0.1548527835869093]$$

Longest intersection interval: 0.0002467764944478516
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999994, 0.4000000000000033]$,

40.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999994, 0.4000000000000033]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.396683605522666\cdot 10^{-58}X^4 + 1.218254561570257\cdot 10^{-44}X^3 + 3.467794636018148\\&\quad\cdot 10^{-29}X^2 - 8.813547453484276\cdot 10^{-23}X + 1.36112658736315\cdot 10^{-23}\\&= 1.36112658736315\cdot 10^{-23}B_{0,4}(X) - 8.422602760079189\cdot 10^{-24}B_{1,4}(X) - 3.045646561413215\\&\quad\cdot 10^{-23}B_{2,4}(X) - 5.249032268852739\cdot 10^{-23}B_{3,4}(X) - 7.45241739832649\cdot 10^{-23}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544357246099643, 0.1544357853745533\}$$

Intersection intervals with the x axis:

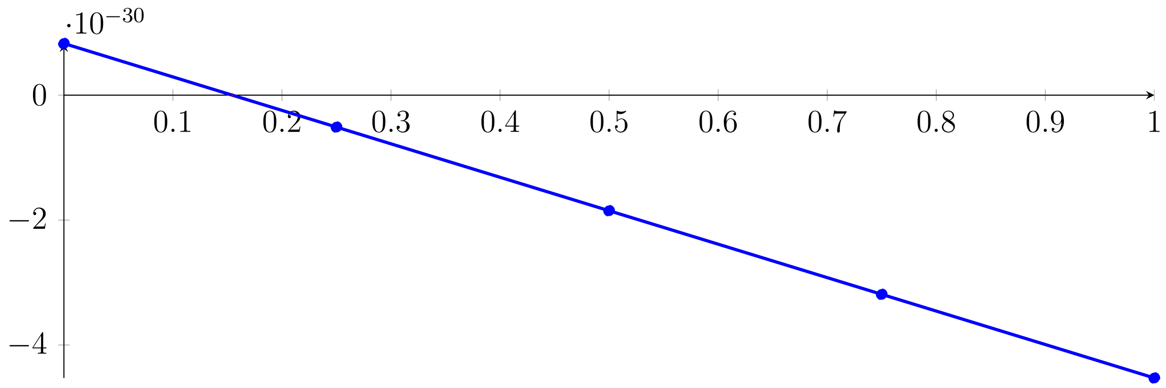
$$[0.1544357246099643, 0.1544357853745533]$$

Longest intersection interval: $6.076458894192547 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

40.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.267480184524245 \cdot 10^{-87} X^4 + 2.733315343334724 \cdot 10^{-66} X^3 + 1.280426044119387 \\ &\quad \cdot 10^{-43} X^2 - 5.355515230460074 \cdot 10^{-30} X + 8.270826503648829 \cdot 10^{-31} \\ &= 8.270826503648829 \cdot 10^{-31} B_{0,4}(X) - 5.117961572501356 \cdot 10^{-31} B_{1,4}(X) - 1.850674964865133 \\ &\quad \cdot 10^{-30} B_{2,4}(X) - 3.189553772480109 \cdot 10^{-30} B_{3,4}(X) - 4.528432580095063 \cdot 10^{-30} B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138334, 0.1544356826138371\}$$

Intersection intervals with the x axis:

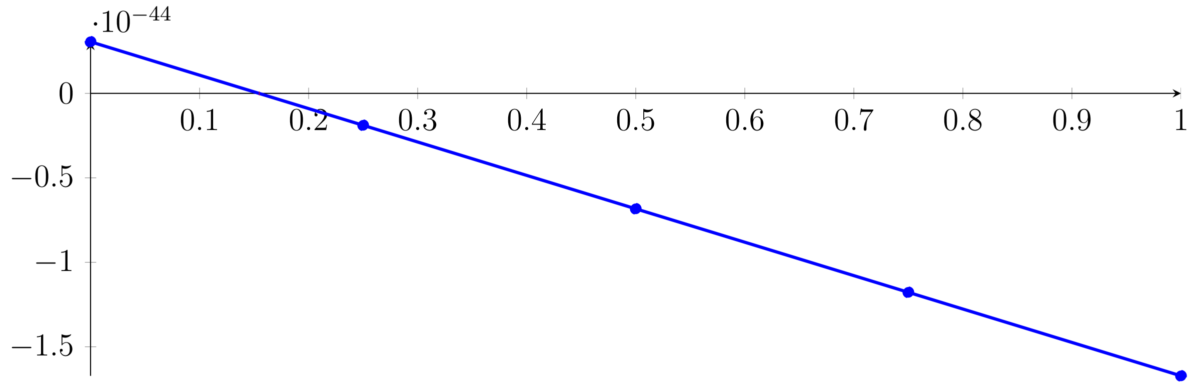
$$[0.1544356826138334, 0.1544356826138371]$$

Longest intersection interval: $3.692333261147783 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

40.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -6.073185304909619 \cdot 10^{-145} X^4 + 1.375917570883003 \cdot 10^{-109} X^3 + 1.745646428447031 \cdot 10^{-72} X^2 - 1.977434701601112 \cdot 10^{-44} X + 3.053864779660447 \cdot 10^{-45}$$
$$= 3.053864779660447 \cdot 10^{-45} B_{0,4}(X) - 1.889721974342333 \cdot 10^{-45} B_{1,4}(X) - 6.833308728345114 \cdot 10^{-45} B_{2,4}(X) - 1.177689548234789 \cdot 10^{-44} B_{3,4}(X) - 1.672048223635067 \cdot 10^{-44} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138309, 0.1544356826138309\}$$

Intersection intervals with the x axis:

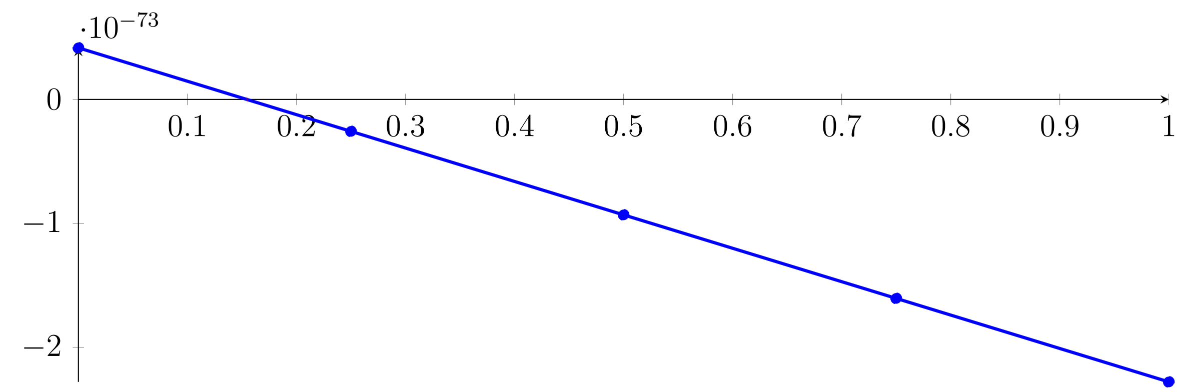
$$[0.1544356826138309, 0.1544356826138309]$$

Longest intersection interval: $1.363332491137777 \cdot 10^{-29}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

40.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.098087868071396 \cdot 10^{-260} X^4 + 3.486565045006964 \cdot 10^{-196} X^3 + 3.244590215733902 \cdot 10^{-130} X^2 - 2.695900977796131 \cdot 10^{-73} X + 4.163433077652396 \cdot 10^{-74}$$
$$= 4.163433077652396 \cdot 10^{-74} B_{0,4}(X) - 2.576319366837932 \cdot 10^{-74} B_{1,4}(X) - 9.31607181132826 \cdot 10^{-74} B_{2,4}(X) - 1.605582425581859 \cdot 10^{-73} B_{3,4}(X) - 2.279557670030892 \cdot 10^{-73} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138309, 0.1544356826138309\}$$

Intersection intervals with the x axis:

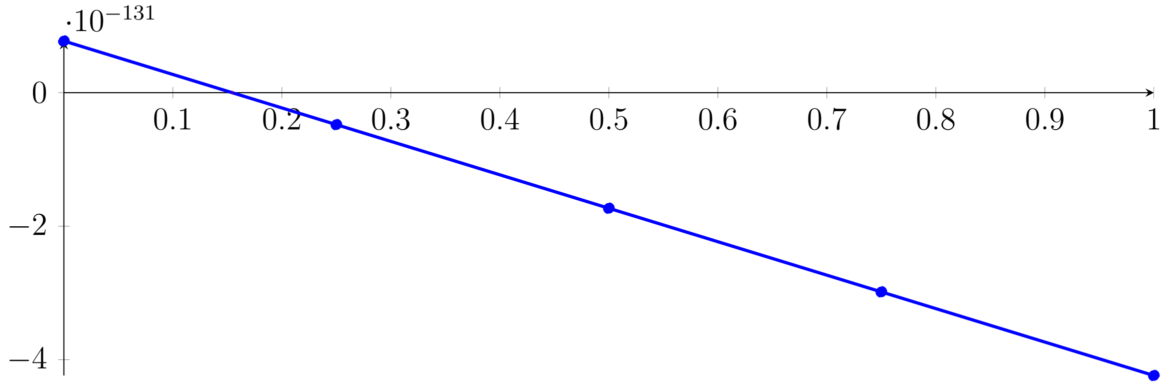
$$[0.1544356826138309, 0.1544356826138309]$$

Longest intersection interval: $1.858675481391937 \cdot 10^{-58}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

40.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -7.523459237765257 \cdot 10^{-439} X^4 + 2.238764861319437 \cdot 10^{-369} X^3 + 1.120900322766582 \\ &\quad \cdot 10^{-245} X^2 - 5.010805047690219 \cdot 10^{-131} X + 7.738470979848683 \cdot 10^{-132} \\ &= 7.738470979848683 \cdot 10^{-132} B_{0,4}(X) - 4.788541639376865 \cdot 10^{-132} B_{1,4}(X) - 1.731555425860241 \\ &\quad \cdot 10^{-131} B_{2,4}(X) - 2.984256687782796 \cdot 10^{-131} B_{3,4}(X) - 4.236957949705351 \cdot 10^{-131} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1544356826138309, 0.1544356826138309\}$$

Intersection intervals with the x axis:

$$[0.1544356826138309, 0.1544356826138309]$$

Longest intersection interval: $3.45467454512755 \cdot 10^{-116}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

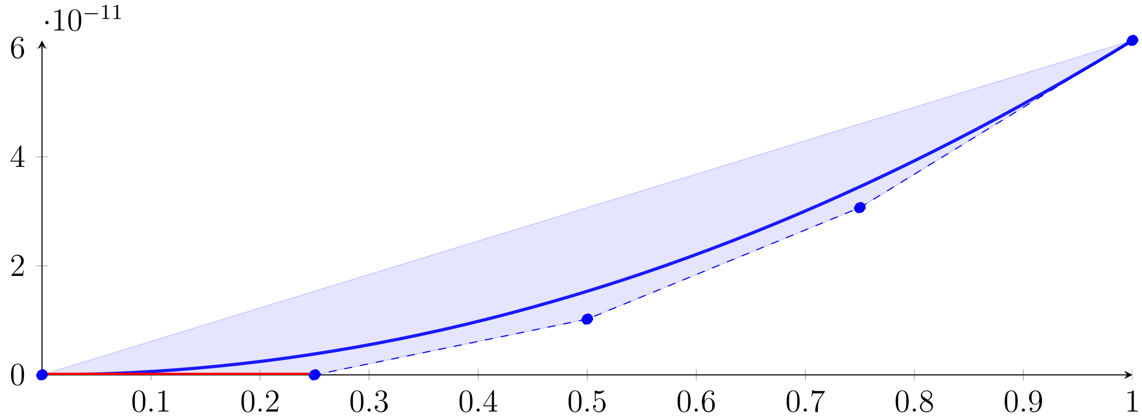
40.31 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 31!

40.32 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.4000000050060858, 0.400005239002336]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -7.504705056762291 \cdot 10^{-22} X^4 + 2.867676750923124 \cdot 10^{-17} X^3 + 6.136416554191405 \\ &\quad \cdot 10^{-11} X^2 + 1.427014599332089 \cdot 10^{-16} X - 5.59999170035142 \cdot 10^{-17} \\ &= -5.59999170035142 \cdot 10^{-17} B_{0,4}(X) - 2.032455202021197 \cdot 10^{-17} B_{1,4}(X) + 1.02273762744653 \\ &\quad \cdot 10^{-11} B_{2,4}(X) + 3.068214096632685 \cdot 10^{-11} B_{3,4}(X) + 6.136428091947402 \cdot 10^{-11} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{9.125808216110345e-07, 0.2500004968163653\}$$

Intersection intervals with the x axis:

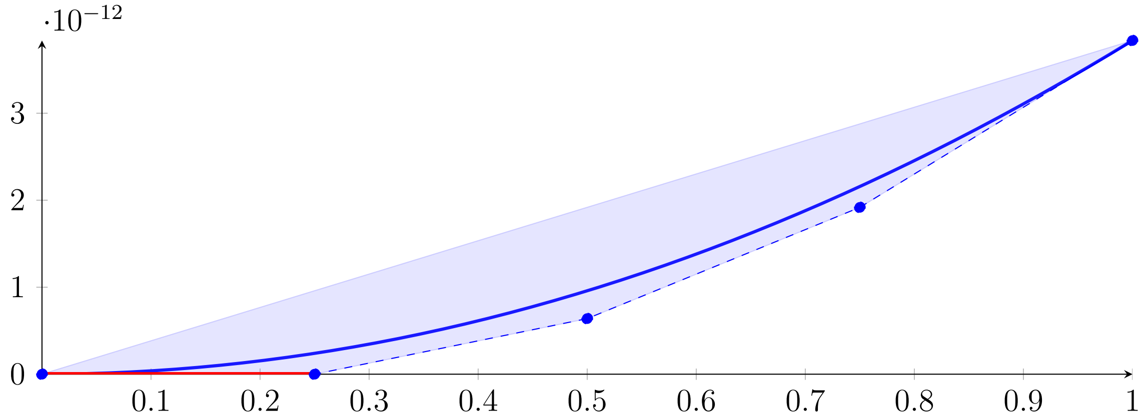
$$[9.125808216110345e-07, 0.2500004968163653]$$

Longest intersection interval: 0.2499995842355437
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050108623, 0.4000013135077487]$,

40.33 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.4000000050108623, 0.4000013135077487]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-2.931505911661308 \cdot 10^{-24} X^4 + 4.480722567712799 \cdot 10^{-19} X^3 + 3.835247589865681 \\ &\cdot 10^{-12} X^2 + 6.367513939156292 \cdot 10^{-17} X - 5.59997356725911 \cdot 10^{-17} \\ = &-5.59997356725911 \cdot 10^{-17} B_{0,4}(X) - 4.008095082470037 \cdot 10^{-17} B_{1,4}(X) + 6.391837694783034 \\ &\cdot 10^{-13} B_{2,4}(X) + 1.917615663569776 \cdot 10^{-12} B_{3,4}(X) + 3.835255713338726 \cdot 10^{-12} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{1.460109108777016e-05, 0.2500156756317829\}$$

Intersection intervals with the x axis:

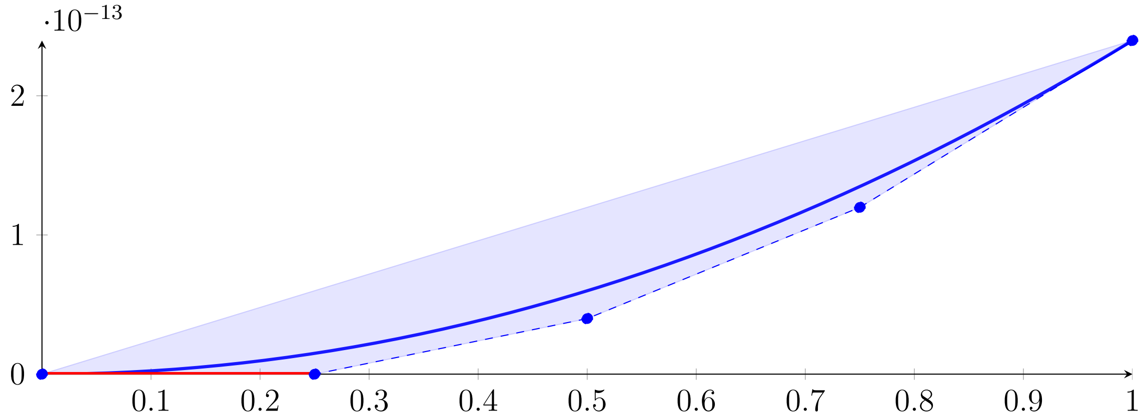
$$[1.460109108777016e-05, 0.2500156756317829]$$

Longest intersection interval: 0.2500010745406951
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000050299677, 0.4000003321555954]$,

40.34 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: [0.4000000050299677, 0.4000003321555954]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.14513918450963\cdot 10^{-26}X^4 + 7.00121928574046\cdot 10^{-21}X^3 + 2.397050349370657 \cdot 10^{-13}X^2 + 4.391837331735849\cdot 10^{-17}X - 5.599798830250992\cdot 10^{-17}$$
$$= -5.599798830250992\cdot 10^{-17}B_{0,4}(X) - 4.50183949731703\cdot 10^{-17}B_{1,4}(X) + 3.991680035453379 \cdot 10^{-14}B_{2,4}(X) + 1.198294600105232\cdot 10^{-13}B_{3,4}(X) + 2.396929623232884\cdot 10^{-13}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.0002335692644077866, 0.2502816337968459\}$$

Intersection intervals with the x axis:

$$[0.0002335692644077866, 0.2502816337968459]$$

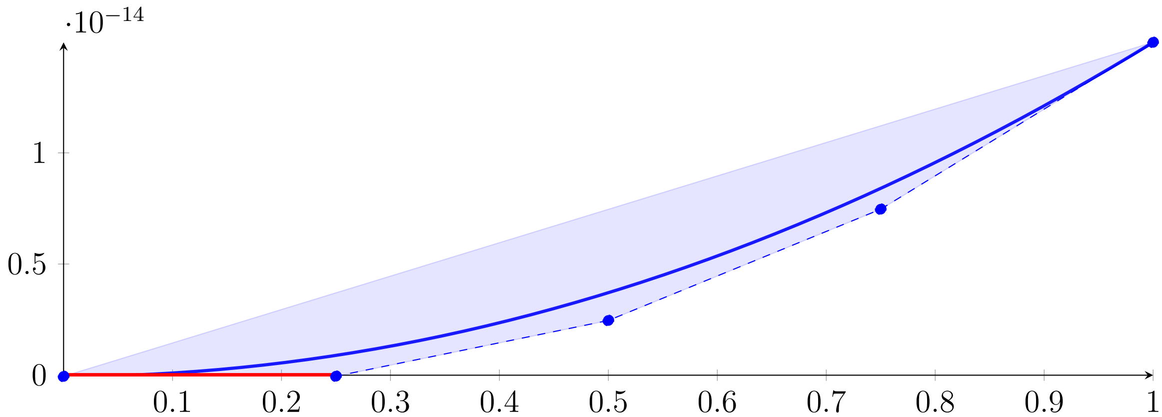
Longest intersection interval: 0.2500480645324381

$\Longrightarrow$ Selective recursion: interval 1: [0.4000000051063742, 0.4000000869035043],

40.35 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: [0.4000000051063742, 0.4000000869035043]

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -4.476640967897802\cdot 10^{-29}X^4 + 1.09457158991016\cdot 10^{-22}X^3 + 1.49873258928534 \cdot 10^{-14}X^2 + 3.898095063625474\cdot 10^{-17}X - 5.597465330775527\cdot 10^{-17}$$
$$= -5.597465330775527\cdot 10^{-17}B_{0,4}(X) - 4.622941564869158\cdot 10^{-17}B_{1,4}(X) + 2.461403470819272 \cdot 10^{-15}B_{2,4}(X) + 7.466924033460424\cdot 10^{-15}B_{3,4}(X) + 1.497033229963901\cdot 10^{-14}B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.003725110466798912, 0.2546088699723713\}$$

Intersection intervals with the x axis:

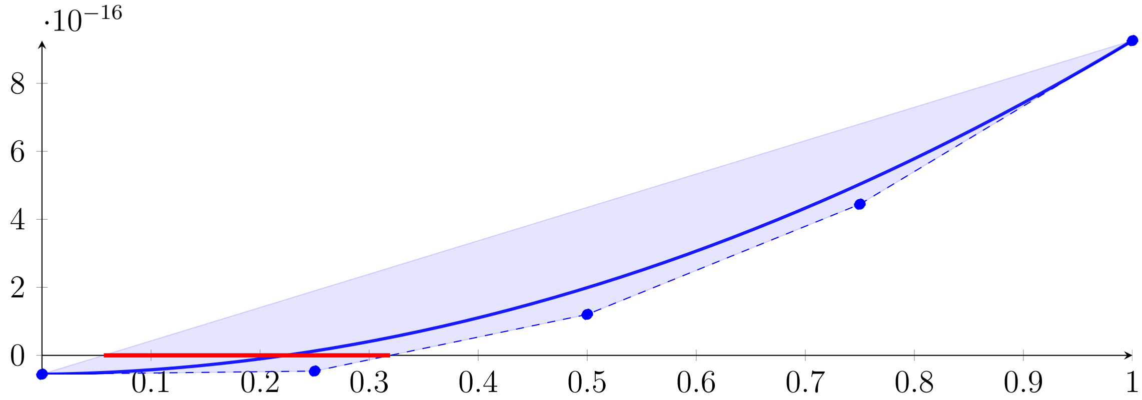
$$[0.003725110466798912, 0.2546088699723713]$$

Longest intersection interval: 0.2508837595055724
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000054110776, 0.4000000259326491]$,

40.36 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.4000000054110776, 0.4000000259326491]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.773546014711518\cdot 10^{-31}X^4 + 1.728469879701229\cdot 10^{-24}X^3 + 9.433421698048765\\ &\cdot 10^{-16}X^2 + 3.779308932689389\cdot 10^{-17}X - 5.562147411226954\cdot 10^{-17}\\ = &-5.562147411226954\cdot 10^{-17}B_{0,4}(X) - 4.617320178054607\cdot 10^{-17}B_{1,4}(X) + 1.204987655186568\\ &\cdot 10^{-16}B_{2,4}(X) + 4.443944282174566\cdot 10^{-16}B_{3,4}(X) + 9.255137867479705\cdot 10^{-16}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.05669093378980359, 0.3192576000162909\}$$

Intersection intervals with the x axis:

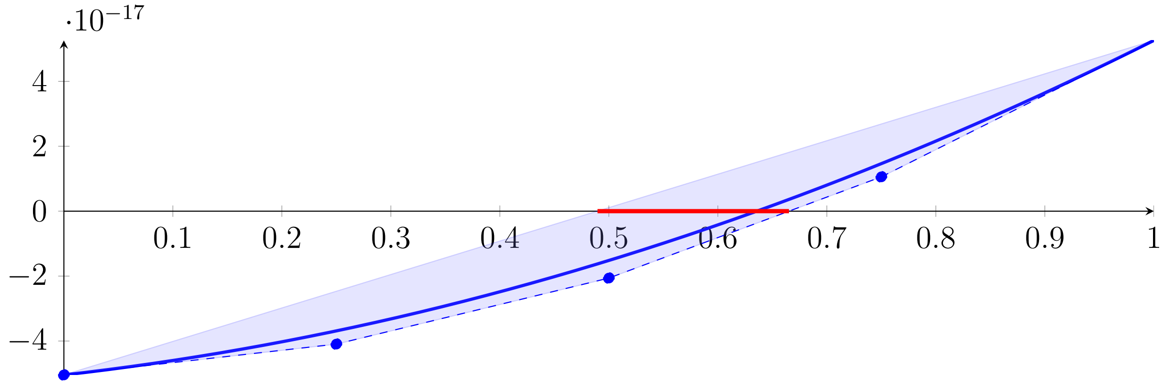
$$[0.05669093378980359, 0.3192576000162909]$$

Longest intersection interval: 0.2625666662264874
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000065744646, 0.4000000119627452]$,

40.37 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.4000000065744646, 0.4000000119627452]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-8.42948070355458\cdot 10^{-34}X^4 + 3.128819977296419\cdot 10^{-26}X^3 + 6.503519235890246\\ &\cdot 10^{-17}X^2 + 3.800678391173011\cdot 10^{-17}X - 5.044717705924284\cdot 10^{-17}\\ = &-5.044717705924284\cdot 10^{-17}B_{0,4}(X) - 4.094548108131031\cdot 10^{-17}B_{1,4}(X) - 2.060458637689404\\ &\cdot 10^{-17}B_{2,4}(X) + 1.057550706182803\cdot 10^{-17}B_{3,4}(X) + 5.259479924267794\cdot 10^{-17}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4895788965792814, 0.6652062590622766\}$$

Intersection intervals with the x axis:

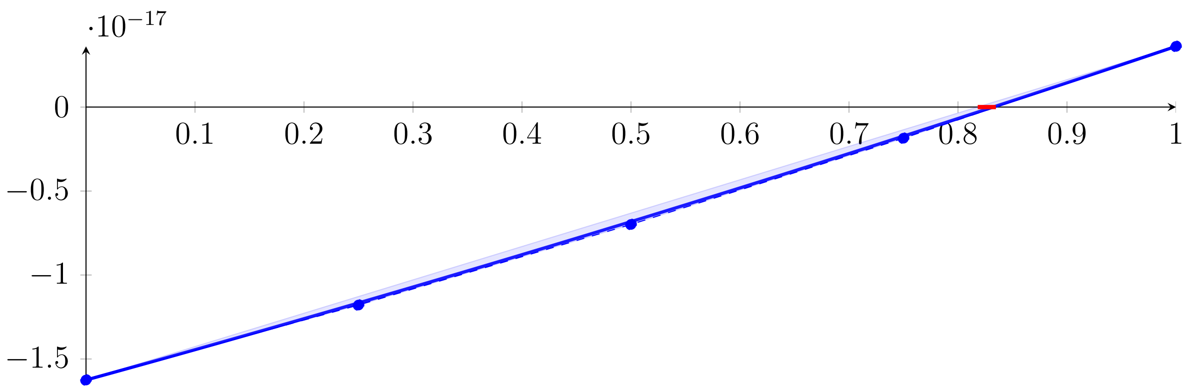
$$[0.4895788965792814, 0.6652062590622766]$$

Longest intersection interval: 0.1756273624829952
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000092124531, 0.4000000101587826]$,

40.38 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.4000000092124531, 0.4000000101587826]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -8.01991079982412\cdot 10^{-37}X^4 + 1.694950778648659\cdot 10^{-28}X^3 + 2.006008588115633 \\ &\quad \cdot 10^{-18}X^2 + 1.785893168307927\cdot 10^{-17}X - 1.625173531873033\cdot 10^{-17} \\ &= -1.625173531873033\cdot 10^{-17}B_{0,4}(X) - 1.178700239796051\cdot 10^{-17}B_{1,4}(X) - 6.987934712504758 \\ &\quad \cdot 10^{-18}B_{2,4}(X) - 1.85453226232069\cdot 10^{-18}B_{3,4}(X) + 3.613204952634064\cdot 10^{-18}B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8181114615359527, 0.8347943211886801\}$$

Intersection intervals with the x axis:

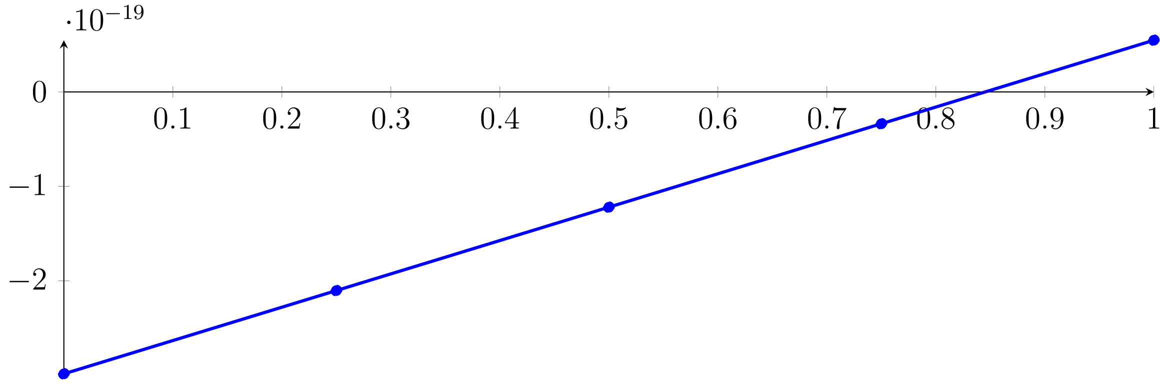
$$[0.8181114615359527, 0.8347943211886801]$$

Longest intersection interval: 0.01668285965272735
$\Longrightarrow$ Selective recursion: interval 1: $[0.4000000099866561, 0.4000000100024436]$,

40.39 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 in Interval 1: [0.4000000099866561, 0.4000000100024436]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -6.212287164586666\cdot10^{-44}X^4 + 7.869888381434003\cdot10^{-34}X^3 + 5.583079095636332\\&\quad\cdot10^{-22}X^2 + 3.526958212875592\cdot10^{-19}X - 2.985043046684015\cdot10^{-19}\\&= -2.985043046684015\cdot10^{-19}B_{0,4}(X) - 2.103303493465117\cdot10^{-19}B_{1,4}(X) - 1.220633427063613\\&\quad\cdot10^{-19}B_{2,4}(X) - 3.370328474795011\cdot10^{-20}B_{3,4}(X) + 5.474982452872209\cdot10^{-20}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845012924114553, 0.8452574901650142\}$$

Intersection intervals with the x axis:

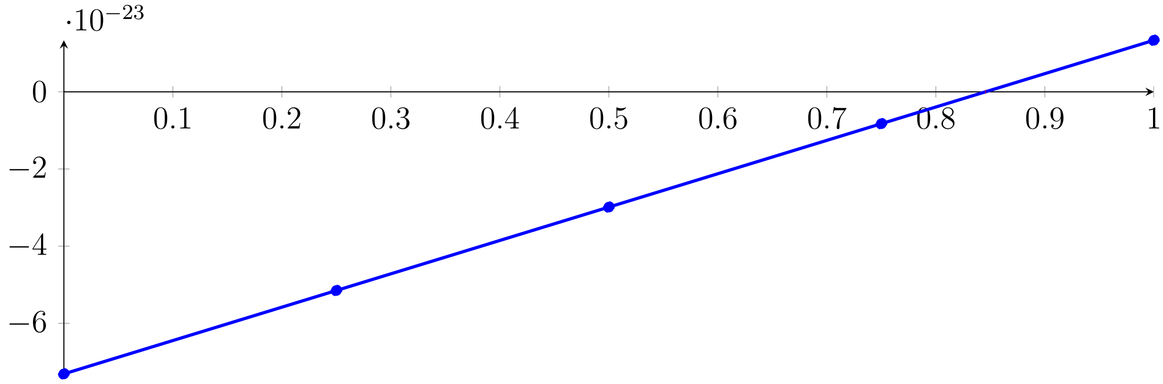
$$[0.845012924114553, 0.8452574901650142]$$

Longest intersection interval: 0.0002445660504611815
$\Longrightarrow$ Selective recursion: interval 1: [0.4000000099999968, 0.4000000100000006],

40.40 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 in Interval 1: [0.4000000099999968, 0.4000000100000006]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.22247151470982\cdot10^{-58}X^4 + 1.151216705335136\cdot10^{-44}X^3 + 3.33938214525302\\&\quad\cdot10^{-29}X^2 + 8.648818549690798\cdot10^{-23}X - 7.311939957361979\cdot10^{-23}\\&= -7.311939957361979\cdot10^{-23}B_{0,4}(X) - 5.149735319939279\cdot10^{-23}B_{1,4}(X) - 2.987530125952889\\&\quad\cdot10^{-23}B_{2,4}(X) - 8.253243754028074\cdot10^{-24}B_{3,4}(X) + 1.336881931710965\cdot10^{-23}B_{4,4}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454261229035133, 0.8454261825857513\}$$

Intersection intervals with the x axis:

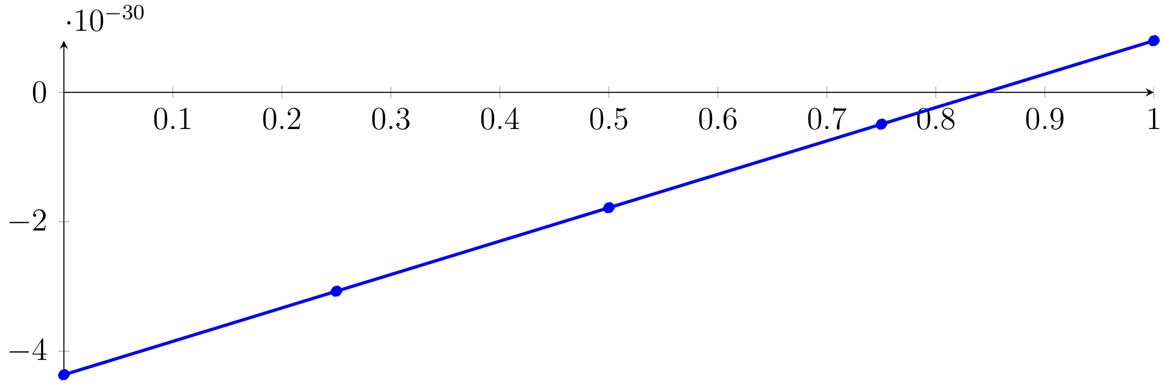
$$[0.8454261229035133, 0.8454261825857513]$$

Longest intersection interval: $5.968223795299727 \cdot 10^{-08}$
$\implies$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

40.41 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -2.819788940081592 \cdot 10^{-87} X^4 + 2.447329147319284 \cdot 10^{-66} X^3 + 1.189477744066025 \\ & \cdot 10^{-43} X^2 + 5.161811836848388 \cdot 10^{-30} X - 4.363931089282542 \cdot 10^{-30} \\ = & -4.363931089282542 \cdot 10^{-30} B_{0,4}(X) - 3.073478130070445 \cdot 10^{-30} B_{1,4}(X) - 1.783025170858328 \\ & \cdot 10^{-30} B_{2,4}(X) - 4.925722116461918 \cdot 10^{-31} B_{3,4}(X) + 7.978807475659646 \cdot 10^{-31} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.8454262238173519, 0.8454262238173555\}$$

Intersection intervals with the x axis:

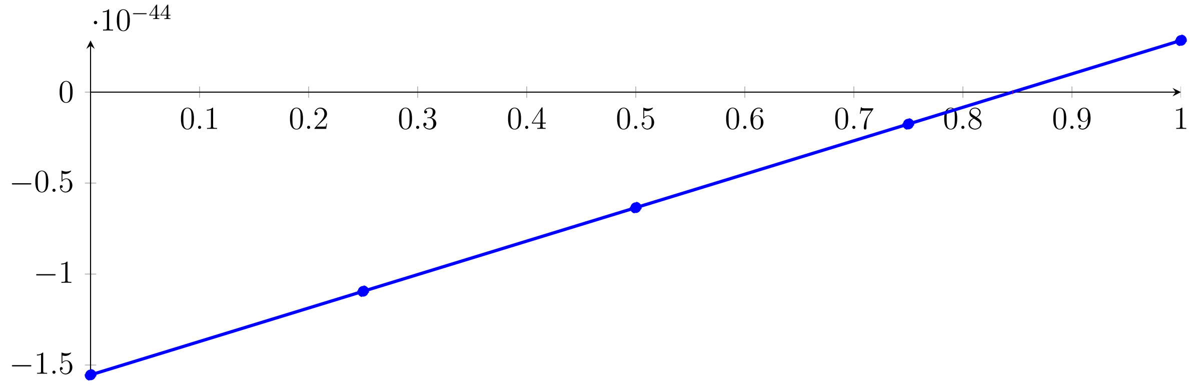
$$[0.8454262238173519, 0.8454262238173555]$$

Longest intersection interval: $3.561967626812098 \cdot 10^{-15}$
$\implies$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

40.42 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -4.539170279710935 \cdot 10^{-145} X^4 + 1.106018233553298 \cdot 10^{-109} X^3 + 1.509163373423153 \\ & \cdot 10^{-72} X^2 + 1.838620665855016 \cdot 10^{-44} X - 1.554418126566363 \cdot 10^{-44} \\ = & -1.554418126566363 \cdot 10^{-44} B_{0,4}(X) - 1.094762960102609 \cdot 10^{-44} B_{1,4}(X) - 6.351077936388546 \\ & \cdot 10^{-45} B_{2,4}(X) - 1.754526271751005 \cdot 10^{-45} B_{3,4}(X) + 2.842025392886536 \cdot 10^{-45} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845426223817358, 0.845426223817358\}$$

Intersection intervals with the x axis:

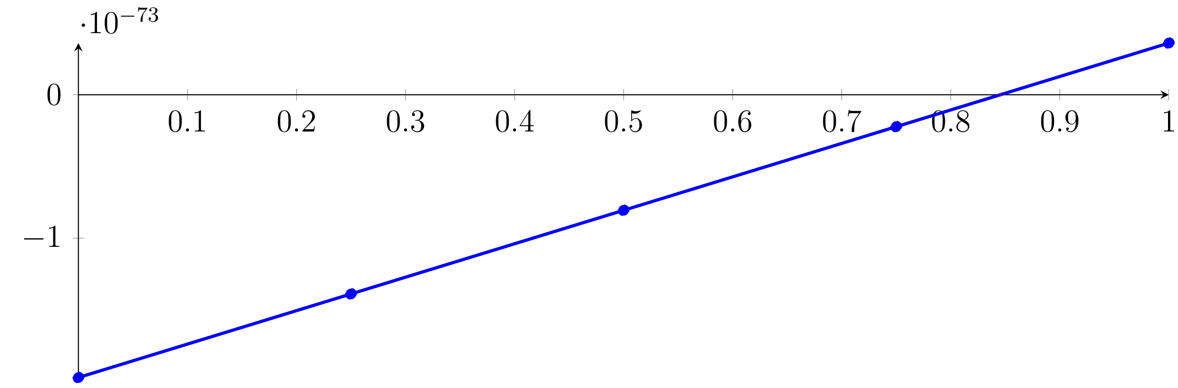
$$[0.845426223817358, 0.845426223817358]$$

Longest intersection interval: $1.268761337445701 \cdot 10^{-29}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

40.43 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1.176240744607327 \cdot 10^{-260} X^4 + 2.258926472033365 \cdot 10^{-196} X^3 + 2.429383786317006 \\ &\quad \cdot 10^{-130} X^2 + 2.332770815065515 \cdot 10^{-73} X - 1.972185621212179 \cdot 10^{-73} \\ &= -1.972185621212179 \cdot 10^{-73} B_{0,4}(X) - 1.3889929174458 \cdot 10^{-73} B_{1,4}(X) - 8.058002136794211 \\ &\quad \cdot 10^{-74} B_{2,4}(X) - 2.226075099130424 \cdot 10^{-74} B_{3,4}(X) + 3.605851938533364 \cdot 10^{-74} B_{4,4}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.845426223817358, 0.845426223817358\}$$

Intersection intervals with the x axis:

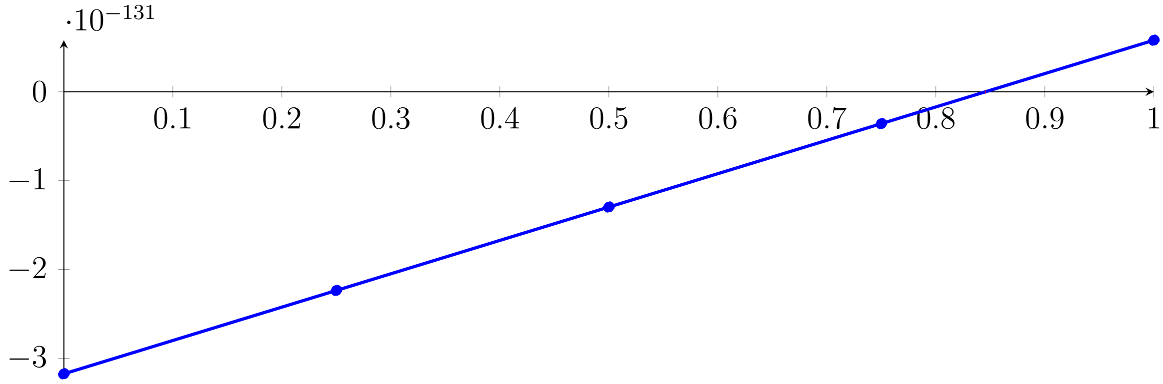
$$[0.845426223817358, 0.845426223817358]$$

Longest intersection interval: $1.609755331397003 \cdot 10^{-58}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

40.44 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = 5.015639491843504 \cdot 10^{-439} X^4 + 9.422837708557983 \cdot 10^{-370} X^3 + 6.295291909464248 \cdot 10^{-246} X^2 + 3.755190256479044 \cdot 10^{-131} X - 3.174736318250814 \cdot 10^{-131}$$
$$= -3.174736318250814 \cdot 10^{-131} B_{0,4}(X) - 2.235938754131053 \cdot 10^{-131} B_{1,4}(X) - 1.297141190011292 \cdot 10^{-131} B_{2,4}(X) - 3.583436258915311 \cdot 10^{-132} B_{3,4}(X) + 5.8045393822823 \cdot 10^{-132} B_{4,4}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.845426223817358, 0.845426223817358\}$$

Intersection intervals with the x axis:

$$[0.845426223817358, 0.845426223817358]$$

Longest intersection interval: $2.591312226961074 \cdot 10^{-116}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

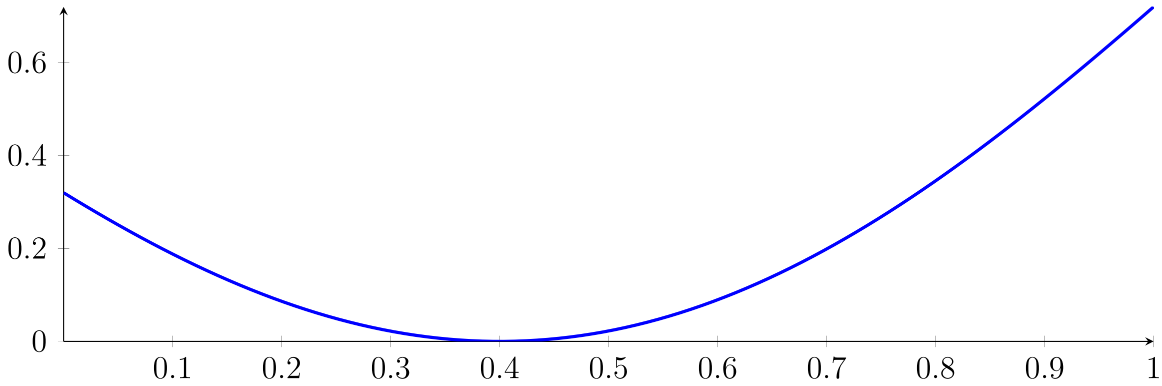
40.45 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 31!

40.46 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

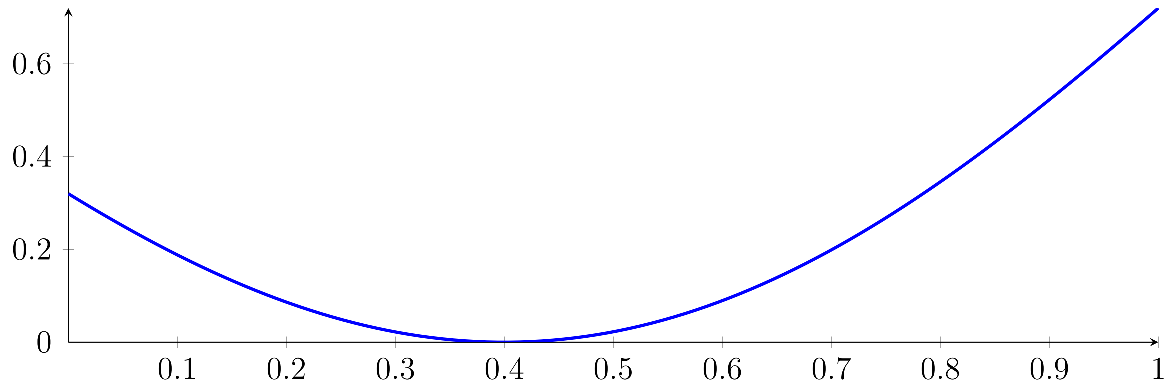
with precision $\varepsilon = 1 \cdot 10^{-128}$.

41 Running `QuadClip` on h_4 with epsilon 128

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

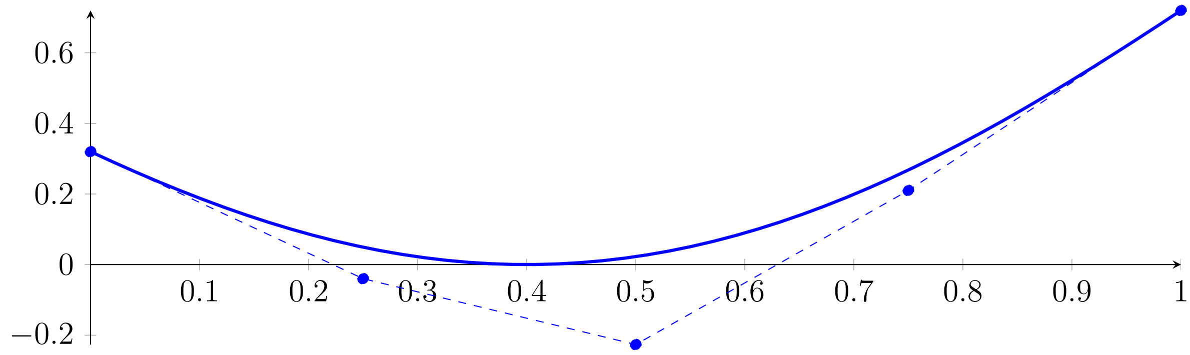
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



41.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

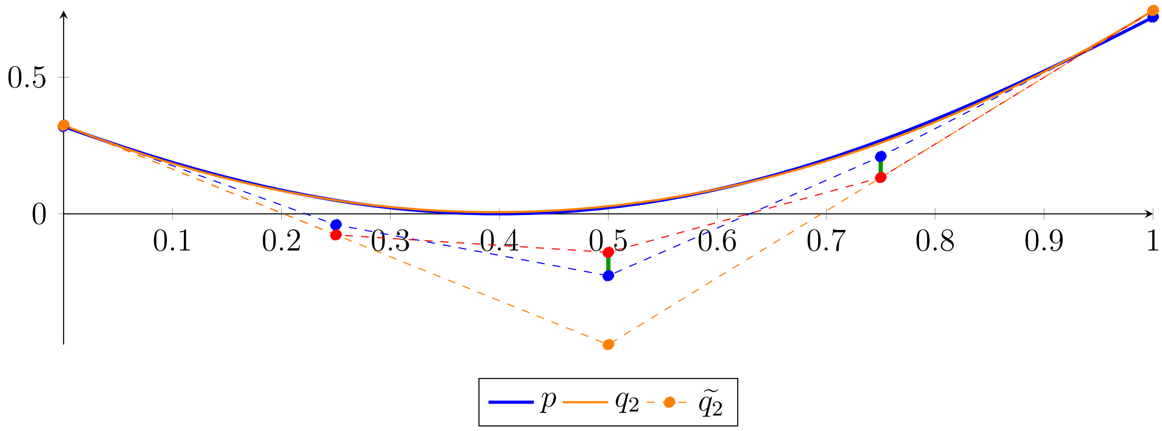
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,2} - 0.4785714310714286B_{1,2} + 0.7442857017857142B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.810653852360235 \cdot 10^{-306}X^4 - 3.682497235829418 \cdot 10^{-306}X^3 \\ &\quad + 2.025714286714286X^2 - 1.605714307714286X + 0.3242857227857143 \\ &= 0.3242857227857143B_{0,4} - 0.07714285414285713B_{1,4} - 0.1409523832857143B_{2,4} \\ &\quad + 0.1328571353571428B_{3,4} + 0.7442857017857142B_{4,4} \end{aligned}$$



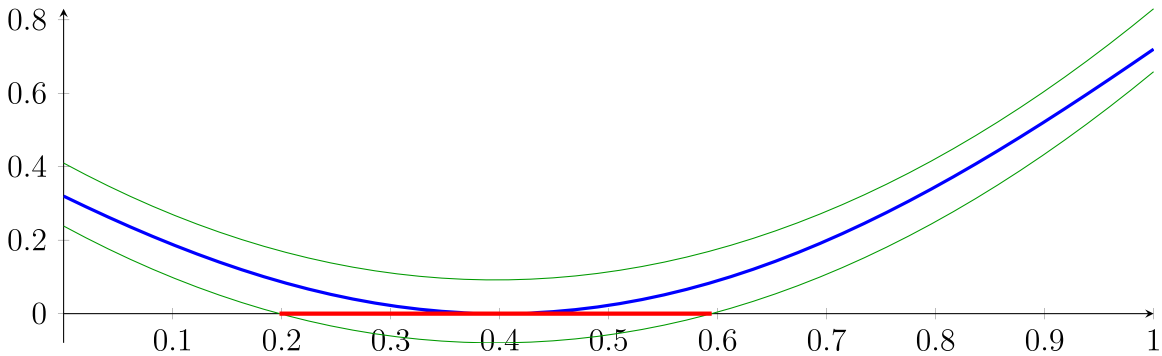
The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

$$M = 2.025714286714286X^2 - 1.605714307714286X + 0.4100000085$$
$$m = 2.025714286714286X^2 - 1.605714307714286X + 0.2385714370714286$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.1980698378643502, 0.594595898979891\}$$

Intersection intervals:



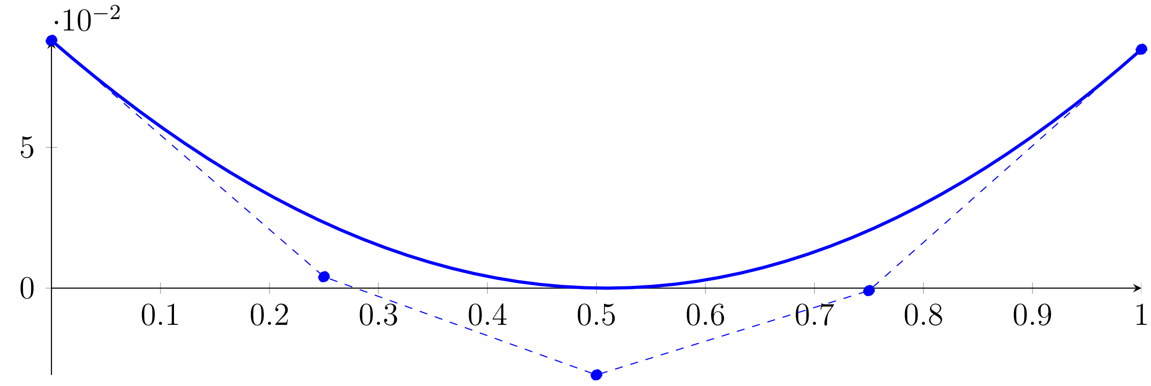
$$[0.1980698378643502, 0.594595898979891]$$

Longest intersection interval: 0.3965260611155408
$\Longrightarrow$ Selective recursion: interval 1: $[0.1980698378643502, 0.594595898979891]$,

41.2 Recursion Branch 1 1 in Interval 1: $[0.1980698378643502, 0.594595898979891]$

Normalized monomial und Bézier representations and the Bézier polygon:

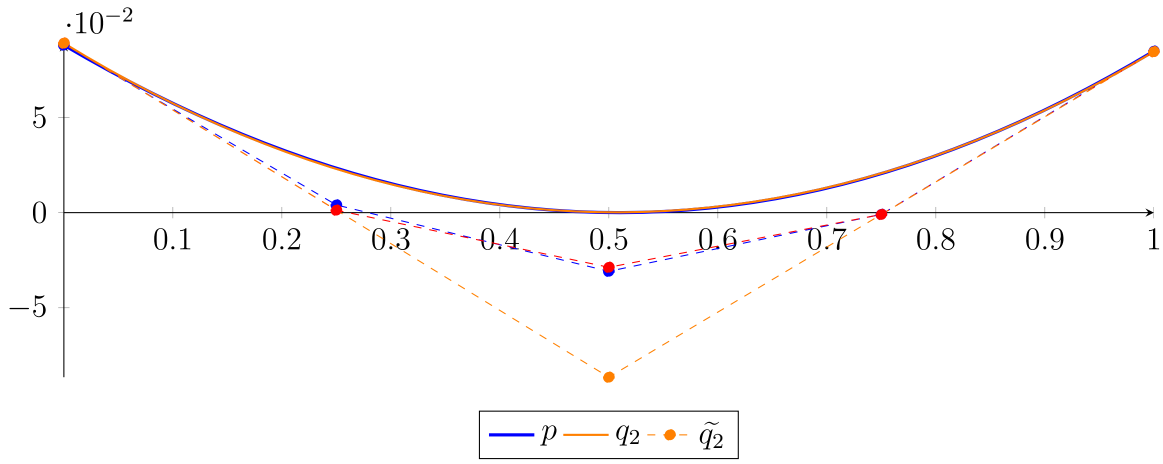
$$\begin{aligned} p = &-0.02472219023355085X^4 + 0.06282830881955585X^3 \\ &+ 0.294683913242138X^2 - 0.3359552082586349X + 0.08802833733717818 \\ = &\ 0.08802833733717818B_{0,4}(X) + 0.004039535272519469B_{1,4}(X) - 0.03083528125178292B_{2,4}(X) \\ &- 0.0008890350308400162B_{3,4}(X) + 0.08486316090668629B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,2} - 0.08647381619599504B_{1,2} + 0.08454713863525717B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 3.059476555447402\cdot 10^{-307}X^4 - 6.258020227051504\cdot 10^{-307}X^3 \\ &\quad + 0.3465454789282417X^2 - 0.351049048193979X + 0.08905070790099448 \\ &= 0.08905070790099448B_{0,4} + 0.001288445852499719B_{1,4} - 0.02871623637462142B_{2,4} \\ &\quad - 0.0009633387803689349B_{3,4} + 0.08454713863525717B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00275108942001975$.

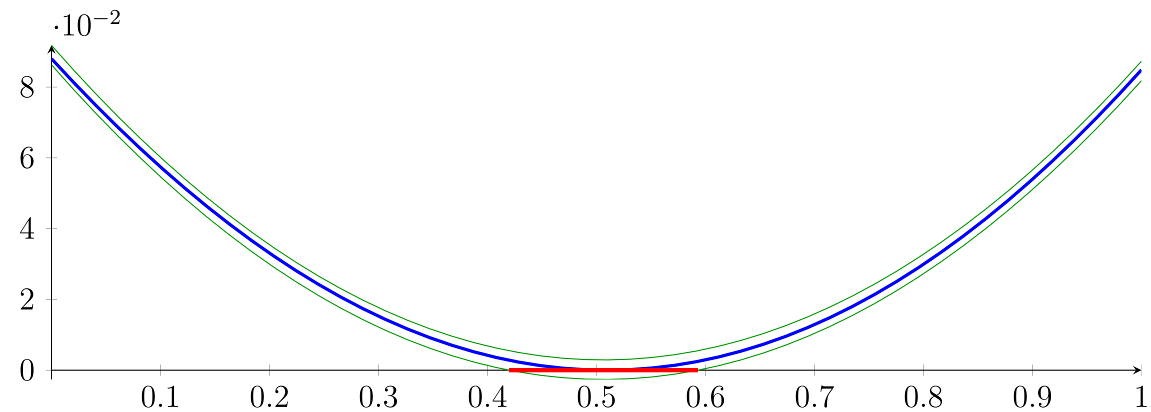
Bounding polynomials M and m:

$$\begin{aligned} M &= 0.3465454789282417X^2 - 0.351049048193979X + 0.09180179732101423 \\ m &= 0.3465454789282417X^2 - 0.351049048193979X + 0.08629961848097473 \end{aligned}$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4198273760664465, 0.5931682321280838\}$$

Intersection intervals:



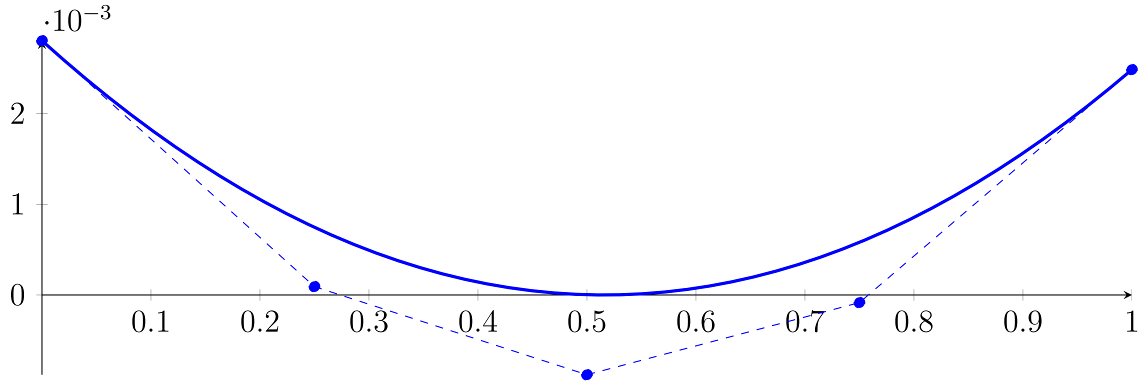
$$[0.4198273760664465, 0.5931682321280838]$$

Longest intersection interval: 0.1733408560616373
$\Longrightarrow$ Selective recursion: interval 1: $[0.3645423336444511, 0.4332765005289681]$,

41.3 Recursion Branch 1 1 1 in Interval 1: $[0.3645423336444511, 0.433276500528968$

Normalized monomial und Bézier representations and the Bézier polygon:

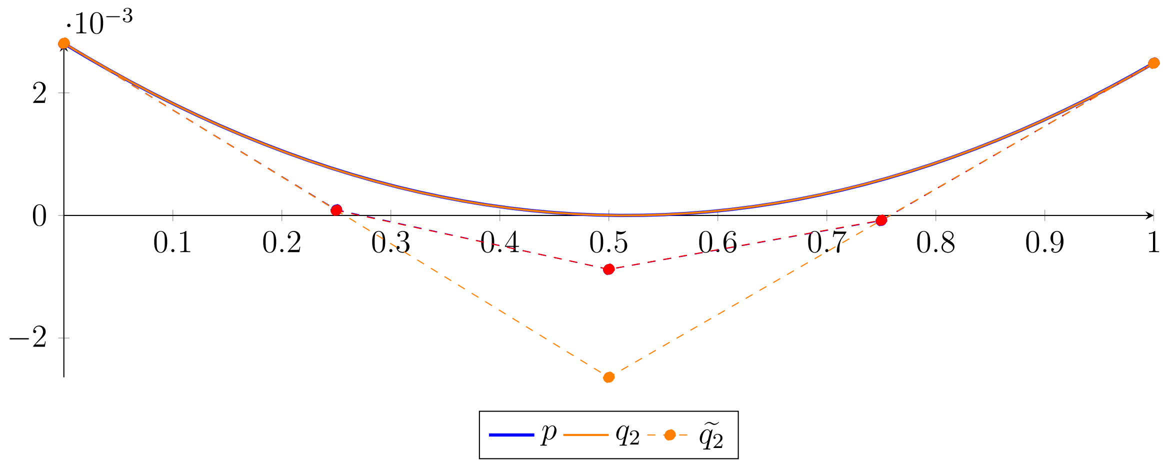
$$\begin{aligned}p &= -2.231982021693433 \cdot 10^{-05} X^4 + 0.0001110015522974976 X^3 \\ &\quad + 0.01044647623937015 X^2 - 0.01085434180562325 X + 0.002805735592529265 \\ &= 0.002805735592529265 B_{0,4}(X) + 9.215014112345361 \cdot 10^{-05} B_{1,4}(X) \\ &\quad - 0.0008803559370539994 B_{2,4}(X) - 8.403225392871914 \\ &\quad \cdot 10^{-05} B_{3,4}(X) + 0.002486551758356734 B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,2} - 0.002640895479419014 B_{1,2} + 0.002483552517338081 B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q_2} &= 9.278384156079834 \cdot 10^{-309} X^4 - 1.903481152394832 \cdot 10^{-308} X^3 \\ &\quad + 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002809372542696975 \\ &= 0.002809372542696975 B_{0,4} + 8.423853163898018 \cdot 10^{-05} B_{1,4} - 0.0008784428096068336 B_{2,4} \\ &\quad - 7.867148104046678 \cdot 10^{-05} B_{3,4} + 0.002483552517338081 B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.911609484473429 \cdot 10^{-06}$.
Bounding polynomials M and m:

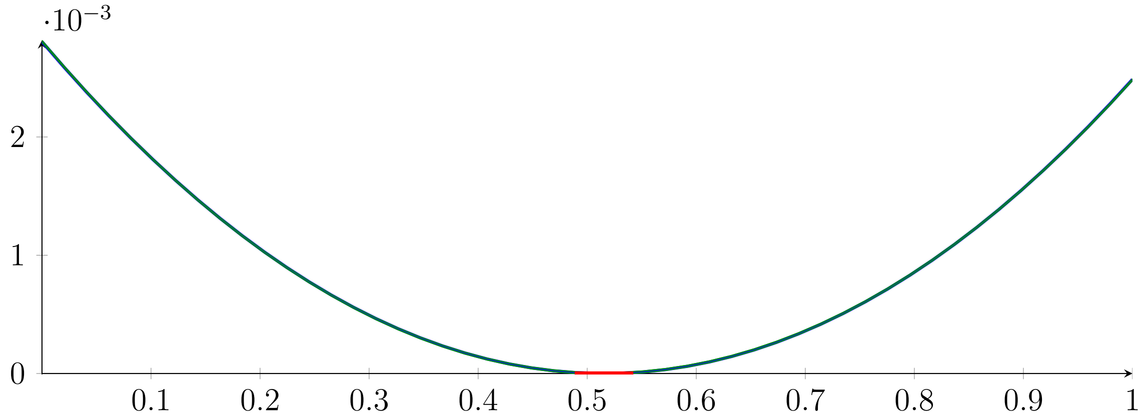
$$M = 0.01057471601887308 X^2 - 0.01090053604423198 X + 0.002817284152181448$$

$$m = 0.01057471601887308X^2 - 0.01090053604423198X + 0.002801460933212501$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.4885305113706042, 0.542280720323693\}$$

Intersection intervals:



$$[0.4885305113706042, 0.542280720323693]$$

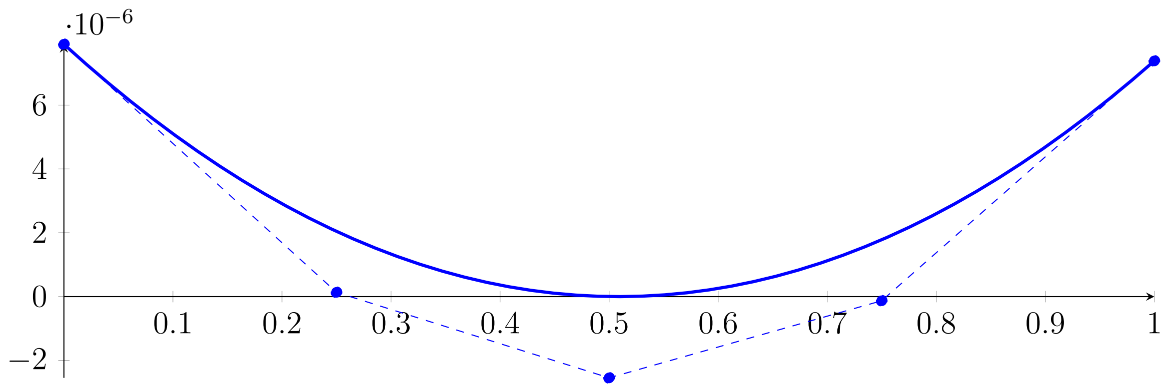
Longest intersection interval: 0.05375020895308878

$\Longrightarrow$ Selective recursion: interval 1: $[0.3981210713411766, 0.4018155471734359]$,

41.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3981210713411766, 0.40181554717343$

Normalized monomial und Bézier representations and the Bézier polygon:

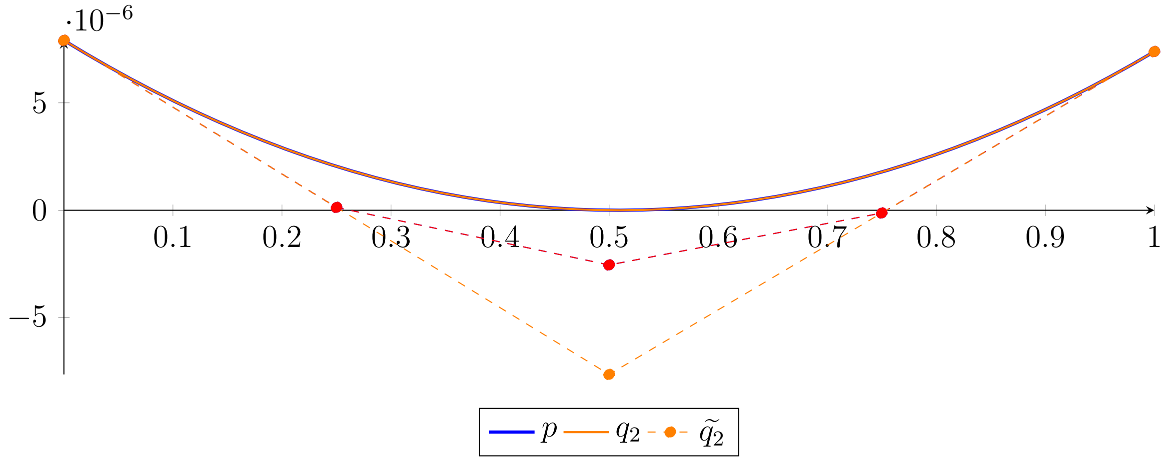
$$p = -1.862993414511891 \cdot 10^{-10} X^4 + 1.046428359410323 \cdot 10^{-08} X^3 + 3.055842313534119 \cdot 10^{-05} X^2 - 3.109078018724982 \cdot 10^{-05} X + 7.906738260659747 \cdot 10^{-06}$$
$$= 7.906738260659747 \cdot 10^{-06} B_{0,4}(X) + 1.340432138472922 \cdot 10^{-07} B_{1,4}(X) - 2.545581310408299 \cdot 10^{-06} B_{2,4}(X) - 1.295192412084993 \cdot 10^{-07} B_{3,4}(X) + 7.384659193003765 \cdot 10^{-06} B_{4,4}(X)$$



Degree reduction and raising:

$$q_2 = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06}$$
$$= 7.907245506324471 \cdot 10^{-06} B_{0,2} - 7.641198706965435 \cdot 10^{-06} B_{1,2} + 7.384157270177368 \cdot 10^{-06} B_{2,2}$$

$$\widetilde{q}_2 = 2.673714696616243 \cdot 10^{-311} X^4 - 5.466261157526541 \cdot 10^{-311} X^3 + 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.907245506324471 \cdot 10^{-06}$$
$$= 7.907245506324471 \cdot 10^{-06} B_{0,4} + 1.330233996795178 \cdot 10^{-07} B_{1,4} - 2.545565341893317 \cdot 10^{-06} B_{2,4} - 1.285207183940336 \cdot 10^{-07} B_{3,4} + 7.384157270177368 \cdot 10^{-06} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 1.019814167774439 \cdot 10^{-09}$.

Bounding polynomials M and m:

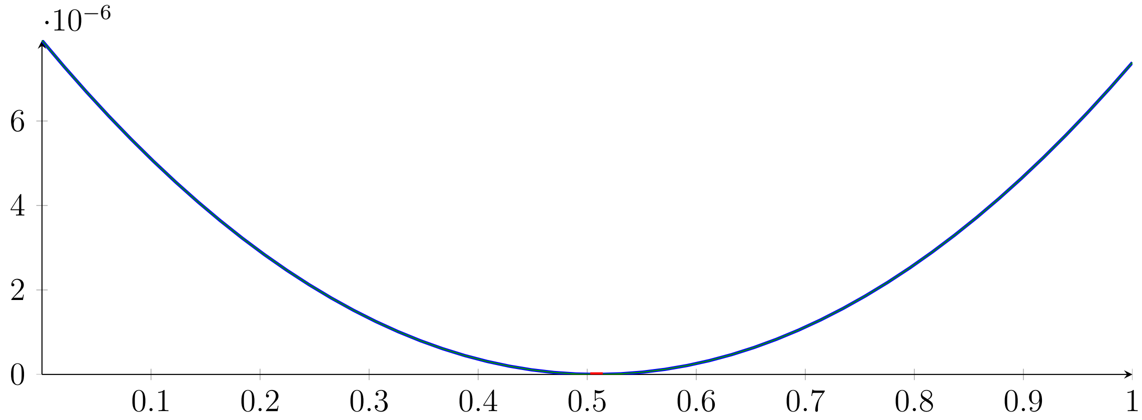
$$M = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.908265320492245 \cdot 10^{-06}$$

$$m = 3.057380019043271 \cdot 10^{-05} X^2 - 3.109688842657981 \cdot 10^{-05} X + 7.906225692156696 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.50281872462556, 0.5142903109829997\}$$

Intersection intervals:



$$[0.50281872462556, 0.5142903109829997]$$

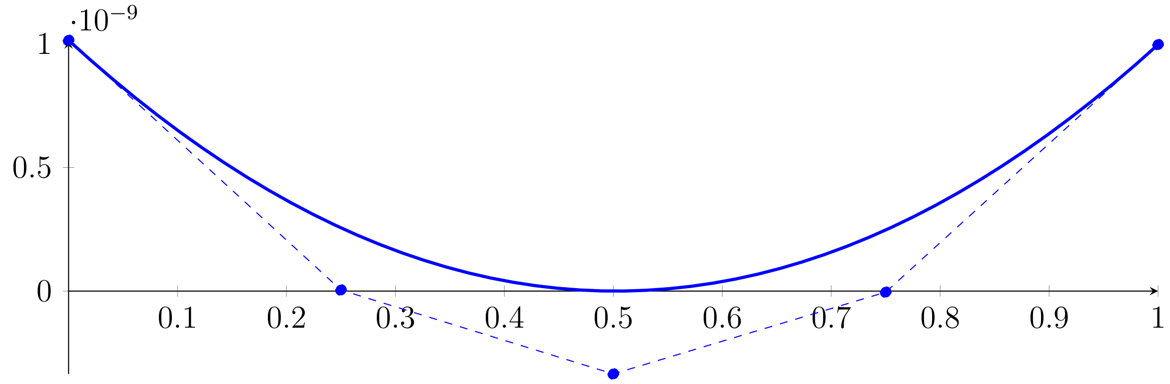
Longest intersection interval: 0.01147158635743977

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999787229673132, 0.4000211044658684]$,

41.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999787229673132, 0.400021104465$

Normalized monomial und Bézier representations and the Bézier polygon:

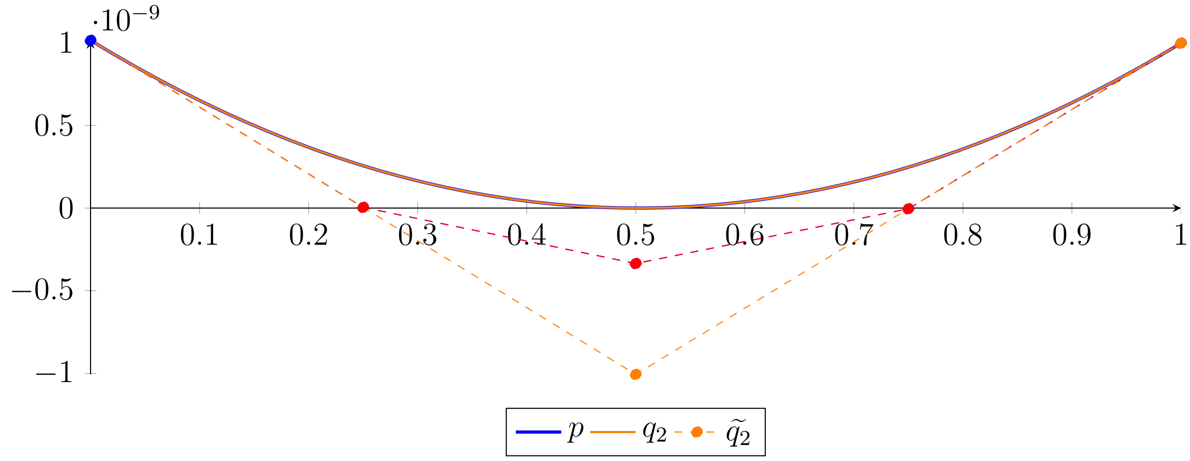
$$\begin{aligned} p = & -3.22630361651822 \cdot 10^{-18} X^4 + 1.523153645441234 \cdot 10^{-14} X^3 + 4.023445841277385 \\ & \cdot 10^{-09} X^2 - 4.040789163099954 \cdot 10^{-09} X + 1.014549826650262 \cdot 10^{-09} \\ = & \; 1.014549826650262 \cdot 10^{-09} B_{0,4}(X) + 4.352535875273264 \cdot 10^{-12} B_{1,4}(X) - 3.352704480201511 \\ & \cdot 10^{-10} B_{2,4}(X) - 4.315317151897678 \cdot 10^{-12} B_{3,4}(X) + 9.972217331378436 \cdot 10^{-10} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014550587950544 \cdot 10^{-09} \\ &= 1.014550587950544 \cdot 10^{-09} B_{0,2} - 1.005848561585488 \cdot 10^{-09} B_{1,2} + 9.972209719297413 \cdot 10^{-10} B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q_2} &= 3.491760652125473 \cdot 10^{-315} X^4 - 7.128579610273962 \cdot 10^{-315} X^3 + 4.023468683051261 \\ &\quad \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014550587950544 \cdot 10^{-09} \\ &= 1.014550587950544 \cdot 10^{-09} B_{0,4} + 4.35101318252834 \cdot 10^{-12} B_{1,4} - 3.352704477436108 \\ &\quad \cdot 10^{-10} B_{2,4} - 4.313794827873167 \cdot 10^{-12} B_{3,4} + 9.972209719297413 \cdot 10^{-10} B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.522692744924588 \cdot 10^{-15}$.
Bounding polynomials M and m:

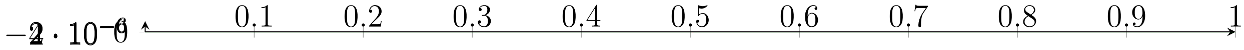
$$M = 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014552110643289 \cdot 10^{-09}$$

$$m = 4.023468683051261 \cdot 10^{-09} X^2 - 4.040798299072064 \cdot 10^{-09} X + 1.014549065257799 \cdot 10^{-09}$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.5015281513143551, 0.5027789820041677\}$$

Intersection intervals:



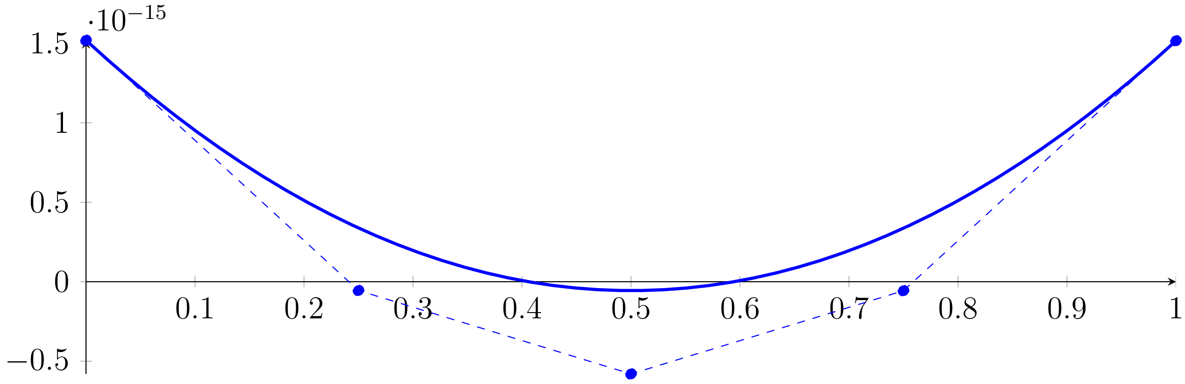
$$[0.5015281513143551, 0.5027789820041677]$$

Longest intersection interval: 0.00125083068981256
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999784819335, 0.4000000314940126]$,

41.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.3999999784819335, 0.4000000314$

Normalized monomial und Bézier representations and the Bézier polygon:

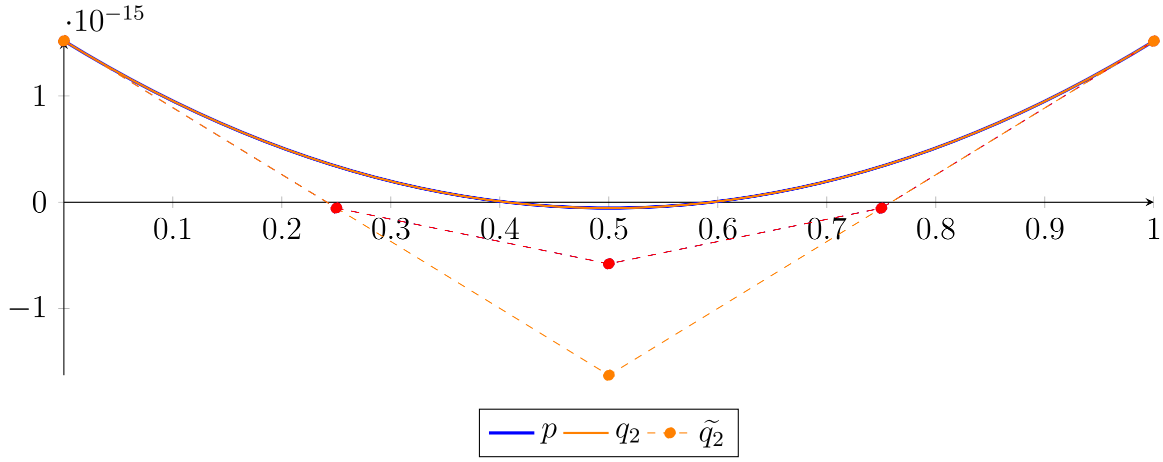
$$\begin{aligned} p &= -7.897676644125011\cdot 10^{-30}X^4 + 2.979577702269594\cdot 10^{-23}X^3 + 6.295028340046794 \\ &\quad \cdot 10^{-15}X^2 - 6.297884694019149\cdot 10^{-15}X + 1.519185582318221\cdot 10^{-15} \\ &= 1.519185582318221\cdot 10^{-15}B_{0,4}(X) - 5.52855911865664\cdot 10^{-17}B_{1,4}(X) - 5.805853746835548 \\ &\quad \cdot 10^{-16}B_{2,4}(X) - 5.671376072379998\cdot 10^{-17}B_{3,4}(X) + 1.516329258141634\cdot 10^{-15}B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185583808009\cdot 10^{-15} \\ &= 1.519185583808009\cdot 10^{-15}B_{0,2} - 1.629756772140295\cdot 10^{-15}B_{1,2} + 1.516329256651846\cdot 10^{-15}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 5.424840791336887\cdot 10^{-321}X^4 - 1.106707046684392\cdot 10^{-320}X^3 + 6.295028384740446 \\ &\quad \cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185583808009\cdot 10^{-15} \\ &= 1.519185583808009\cdot 10^{-15}B_{0,4} - 5.528559416614297\cdot 10^{-17}B_{1,4} - 5.805853746835541 \\ &\quad \cdot 10^{-16}B_{2,4} - 5.671375774422431\cdot 10^{-17}B_{3,4} + 1.516329256651846\cdot 10^{-15}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.979576574030073\cdot 10^{-24}$.
Bounding polynomials M and m:

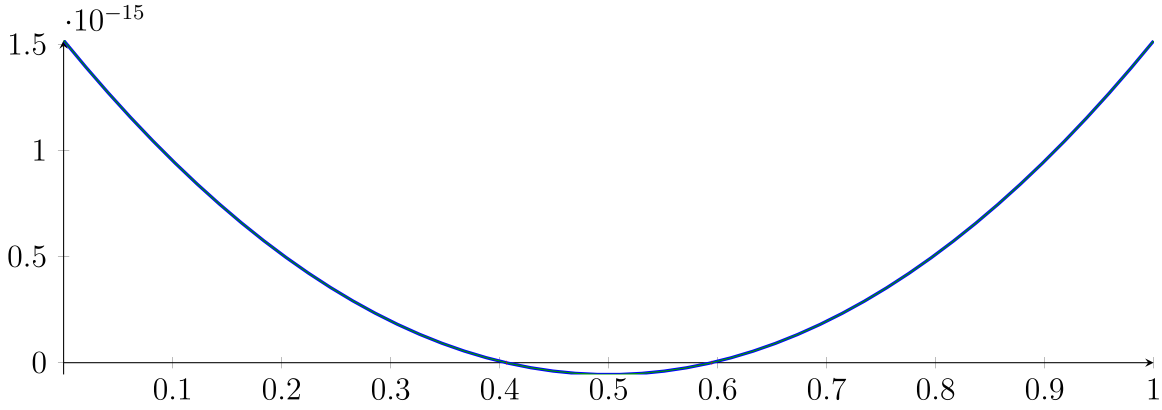
$$M = 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185586787586\cdot 10^{-15}$$

$$m = 6.295028384740446\cdot 10^{-15}X^2 - 6.297884711896608\cdot 10^{-15}X + 1.519185580828433\cdot 10^{-15}$$

Root of M and m:

$N(M) = \{0.4059087473493063, 0.5945449959829854\}$ $\quad N(m) = \{0.4059087423309477, 0.594545001001344\}$

Intersection intervals:



$$[0.4059087423309477, 0.4059087473493063],\ [0.5945449959829854, 0.594545001001344]$$

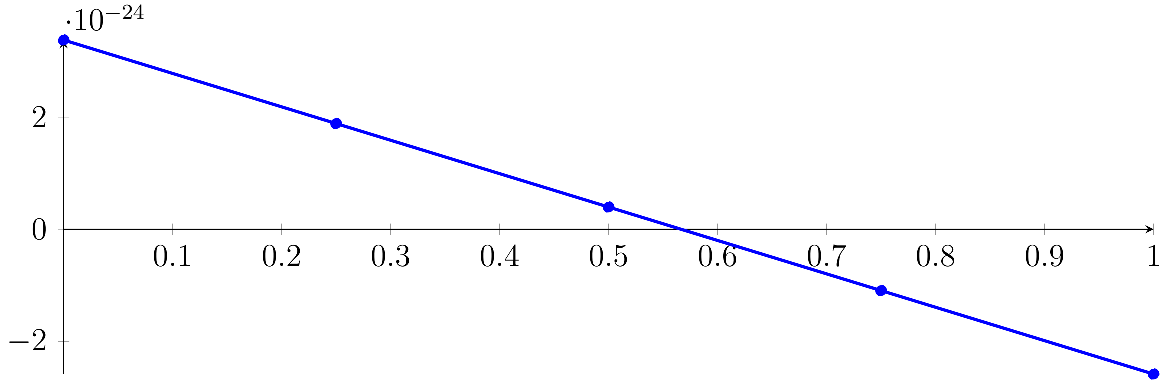
Longest intersection interval: $5.01835859594062 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3999999999999999, 0.4000000000000001]$, interval 2: $[0.400000009999999$

41.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.3999999999999999, 0.40000000$

Normalized monomial und Bézier representations and the Bézier polygon:

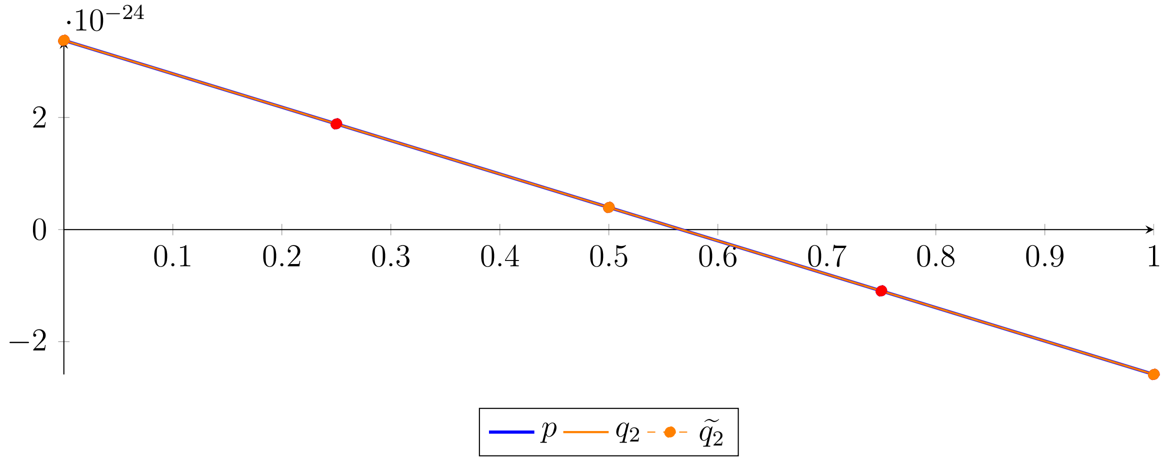
$$\begin{aligned} p &= -5.008943280633806 \cdot 10^{-63} X^4 + 3.765646973599716 \cdot 10^{-48} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4}(X) + 1.885496281661092 \cdot 10^{-24} B_{1,4}(X) + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4}(X) - 1.094080301597753 \cdot 10^{-24} B_{3,4}(X) - 2.583868553593798 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,2} + 3.957079503982923 \cdot 10^{-25} B_{1,2} - 2.583868553593798 \cdot 10^{-24} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= 9.938904969180865 \cdot 10^{-331} X^4 - 2.503131621867773 \cdot 10^{-330} X^3 + 1.585335098962615 \\ &\quad \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24} \\ &= 3.375284612923892 \cdot 10^{-24} B_{0,4} + 1.885496281661092 \cdot 10^{-24} B_{1,4} + 3.95707976820544 \\ &\quad \cdot 10^{-25} B_{2,4} - 1.094080301597753 \cdot 10^{-24} B_{3,4} - 2.583868553593798 \cdot 10^{-24} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646973599709 \cdot 10^{-49}$.
Bounding polynomials M and m:

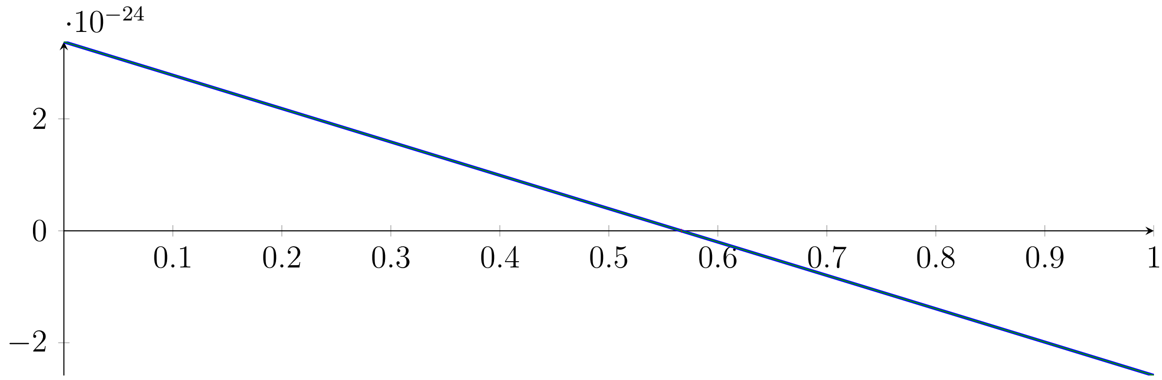
$$M = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

$$m = 1.585335098962615 \cdot 10^{-31} X^2 - 5.9591533250512 \cdot 10^{-24} X + 3.375284612923892 \cdot 10^{-24}$$

Root of M and m:

$N(M) = \{0.5664033931790926, 37589234.2203029\}$ $N(m) = \{0.5664033931790926, 37589234.2203029\}$

Intersection intervals:



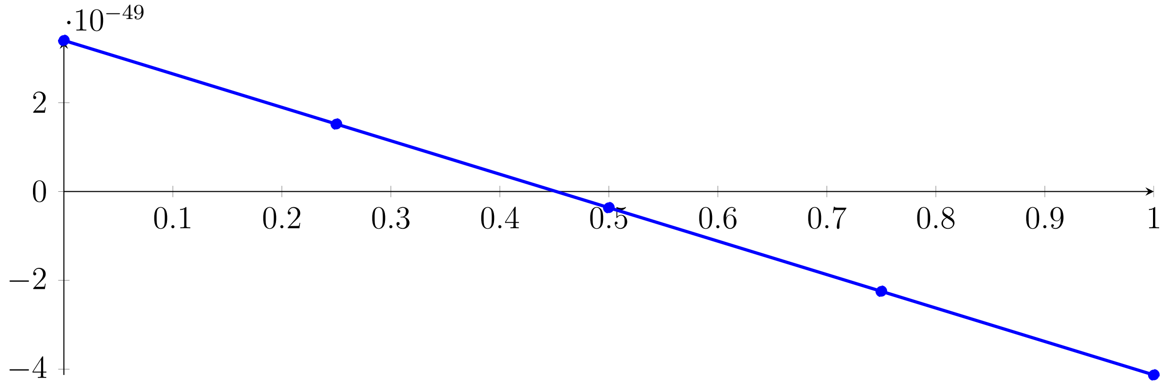
$$[0.5664033931790926, 0.5664033931790926]$$

Longest intersection interval: $1.263819499744434 \cdot 10^{-25}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

41.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

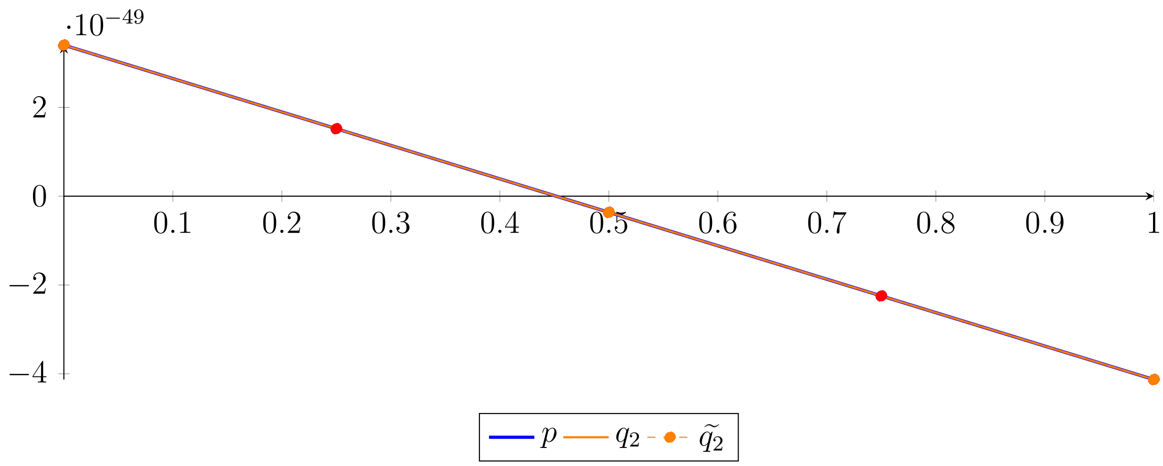
$$\begin{aligned} p &= -1.277868961417556 \cdot 10^{-162} X^4 + 7.601420513547777 \cdot 10^{-123} X^3 + 2.532160202151322 \\ &\quad \cdot 10^{-81} X^2 - 7.531293947199418 \cdot 10^{-49} X + 3.401595176598994 \cdot 10^{-49} \\ &= 3.401595176598994 \cdot 10^{-49} B_{0,4}(X) + 1.51877168979914 \cdot 10^{-49} B_{1,4}(X) - 3.640517970007151 \\ &\quad \cdot 10^{-50} B_{2,4}(X) - 2.24687528380057 \cdot 10^{-49} B_{3,4}(X) - 4.129698770600424 \cdot 10^{-49} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.532160202151322\cdot 10^{-81}X^2 - 7.531293947199418\cdot 10^{-49}X + 3.401595176598994\cdot 10^{-49} \\ &= 3.401595176598994\cdot 10^{-49}B_{0,2} - 3.640517970007151\cdot 10^{-50}B_{1,2} - 4.129698770600424\cdot 10^{-49}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} &= -2.854607465456027\cdot 10^{-356}X^4 + 3.044914629819762\cdot 10^{-356}X^3 + 2.532160202151322 \\ &\quad \cdot 10^{-81}X^2 - 7.531293947199418\cdot 10^{-49}X + 3.401595176598994\cdot 10^{-49} \\ &= 3.401595176598994\cdot 10^{-49}B_{0,4} + 1.51877168979914\cdot 10^{-49}B_{1,4} - 3.640517970007151 \\ &\quad \cdot 10^{-50}B_{2,4} - 2.24687528380057\cdot 10^{-49}B_{3,4} - 4.129698770600424\cdot 10^{-49}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.601420513547777\cdot 10^{-124}$.

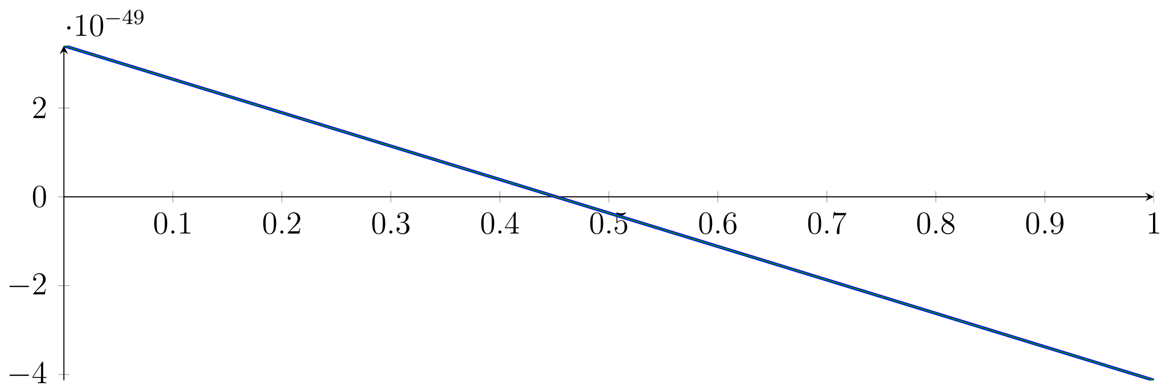
Bounding polynomials M and m:

$$\begin{aligned} M &= 2.532160202151322\cdot 10^{-81}X^2 - 7.531293947199418\cdot 10^{-49}X + 3.401595176598994\cdot 10^{-49} \\ m &= 2.532160202151322\cdot 10^{-81}X^2 - 7.531293947199418\cdot 10^{-49}X + 3.401595176598994\cdot 10^{-49} \end{aligned}$$

Root of M and m:

$N(M) = \{0.4516614542530117, 2.974256502728712\cdot 10^{32}\}$ $\quad N(m) = \{0.4516614542530117, 2.97425650272871$

Intersection intervals:



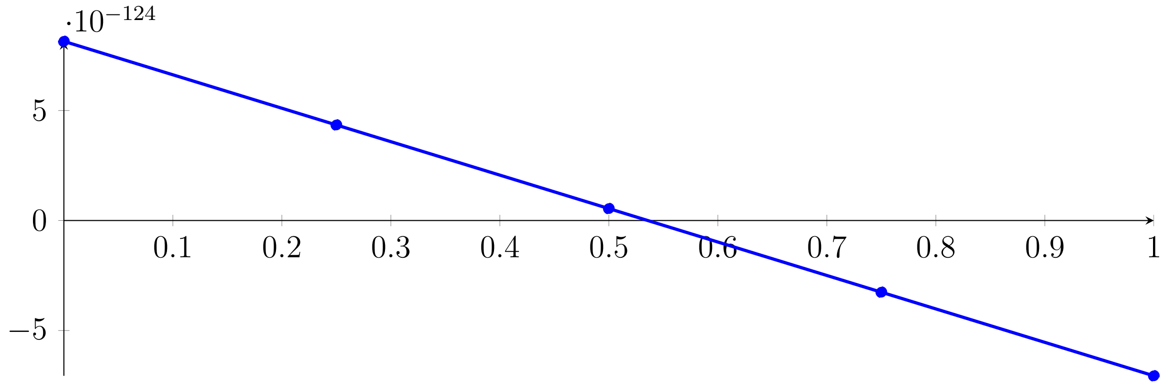
$$[0.4516614542530117, 0.4516614542530117]$$

Longest intersection interval: $2.018622713929374 \cdot 10^{-75}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

41.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Normalized monomial und Bézier representations and the Bézier polygon:

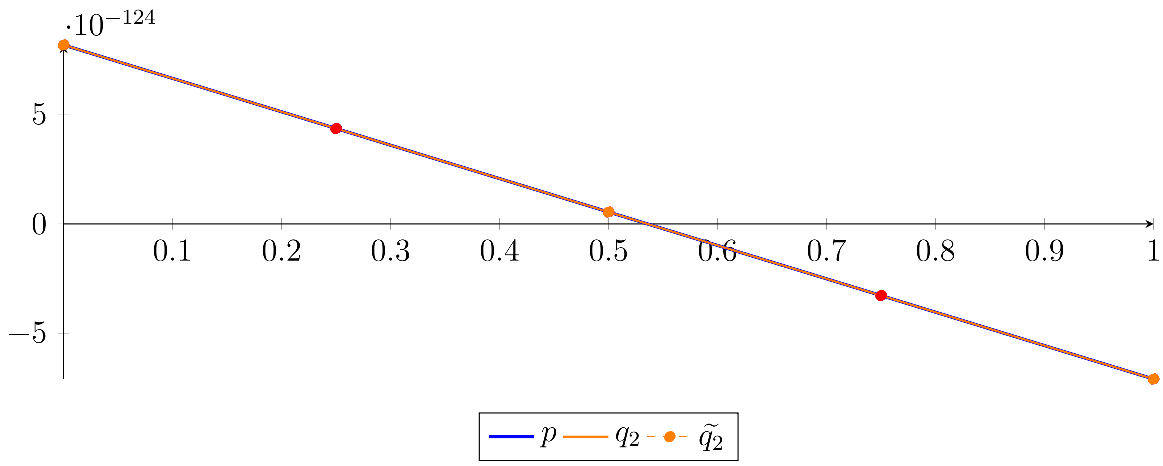
$$\begin{aligned} p &= -1.682969342655774 \cdot 10^{-431} X^4 + 6.252593944348401 \cdot 10^{-347} X^3 + 1.031814175589671 \\ &\quad \cdot 10^{-230} X^2 - 1.520284102709555 \cdot 10^{-123} X + 8.143997237532866 \cdot 10^{-124} \\ &= 8.143997237532866 \cdot 10^{-124} B_{0,4}(X) + 4.343286980758977 \cdot 10^{-124} B_{1,4}(X) + 5.425767239850886 \\ &\quad \cdot 10^{-125} B_{2,4}(X) - 3.2581335327888 \cdot 10^{-124} B_{3,4}(X) - 7.058843789562688 \cdot 10^{-124} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.031814175589671 \cdot 10^{-230} X^2 - 1.520284102709555 \cdot 10^{-123} X + 8.143997237532866 \cdot 10^{-124} \\ &= 8.143997237532866 \cdot 10^{-124} B_{0,2} + 5.425767239850886 \cdot 10^{-125} B_{1,2} - 7.058843789562688 \cdot 10^{-124} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.514672408390197 \cdot 10^{-430} X^4 - 3.702532553842703 \cdot 10^{-430} X^3 + 1.031814175589671 \\ &\quad \cdot 10^{-230} X^2 - 1.520284102709555 \cdot 10^{-123} X + 8.143997237532866 \cdot 10^{-124} \\ &= 8.143997237532866 \cdot 10^{-124} B_{0,4} + 4.343286980758977 \cdot 10^{-124} B_{1,4} + 5.425767239850886 \\ &\quad \cdot 10^{-125} B_{2,4} - 3.2581335327888 \cdot 10^{-124} B_{3,4} - 7.058843789562688 \cdot 10^{-124} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.252593944348401 \cdot 10^{-348}$.
Bounding polynomials M and m:

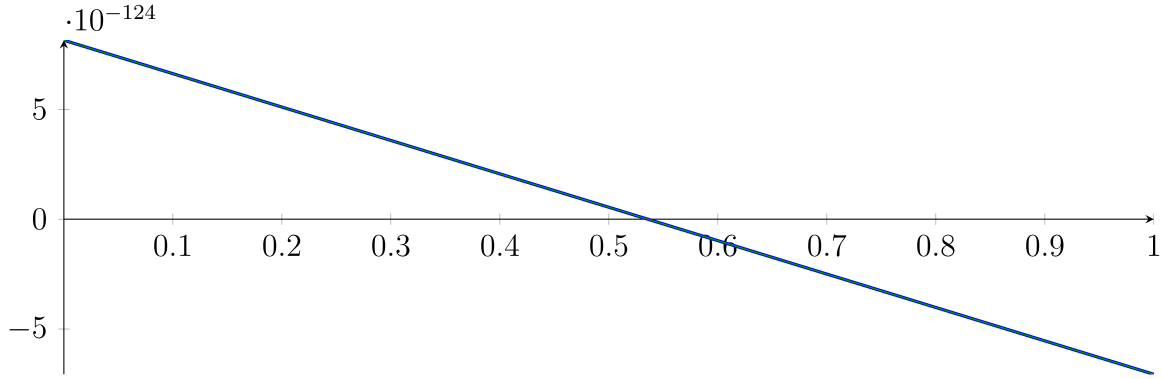
$$M = 1.031814175589671 \cdot 10^{-230} X^2 - 1.520284102709555 \cdot 10^{-123} X + 8.143997237532866 \cdot 10^{-124}$$

$$m = 1.031814175589671 \cdot 10^{-230} X^2 - 1.520284102709555 \cdot 10^{-123} X + 8.143997237532866 \cdot 10^{-124}$$

Root of M and m:

$N(M) = \{0.5356891664536958, 1.473408815924368 \cdot 10^{107}\}$ $\quad N(m) = \{0.5356891664536958, 1.47340881592436$

Intersection intervals:



$[0.5356891664536958, 0.5356891664536958]$

Longest intersection interval: 0

$\Longrightarrow$ Selective recursion: interval 1: $[0.4, 0.4]$,

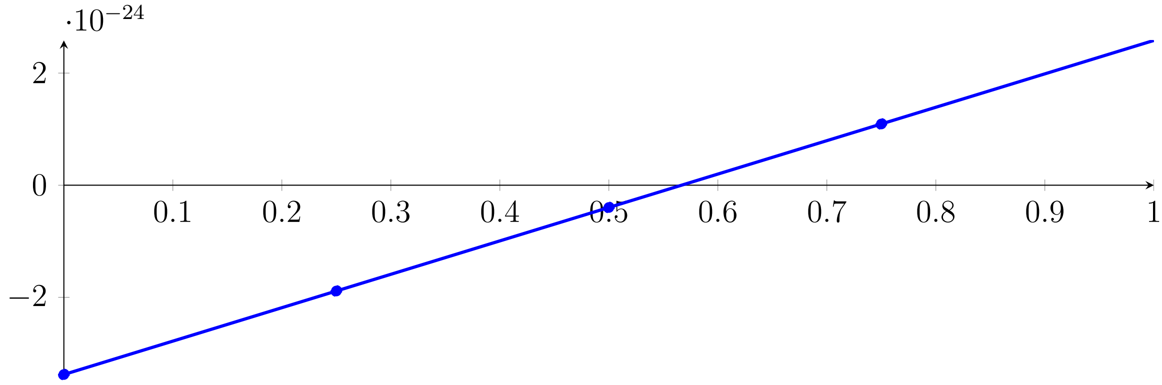
41.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4, 0.4]$

Found root in interval $[0.4, 0.4]$ at recursion depth 10!

41.11 Recursion Branch 1 1 1 1 1 1 2 in Interval 2: $[0.4000000099999999, 0.40000001$

Normalized monomial und Bézier representations and the Bézier polygon:

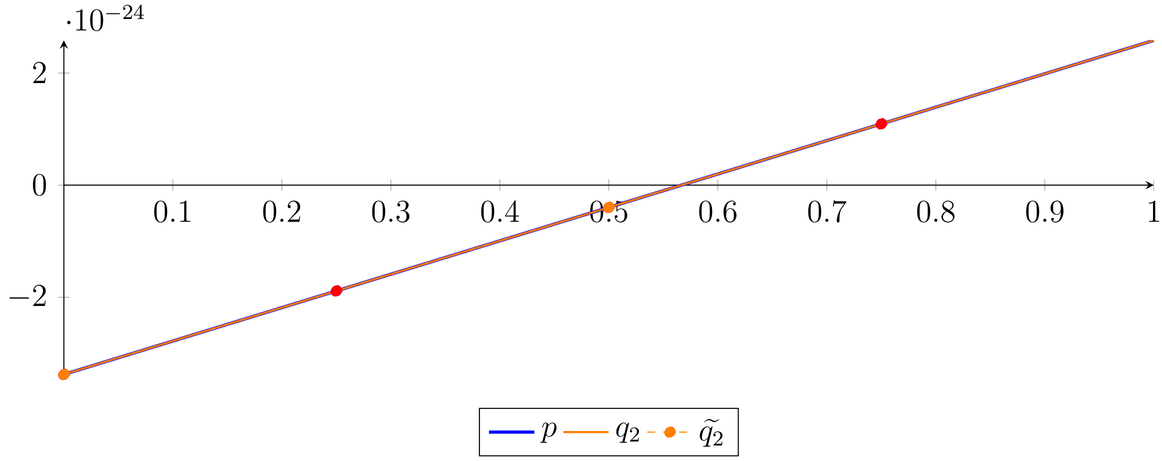
$$\begin{aligned} p &= -5.008943280633806 \cdot 10^{-63} X^4 + 3.76564622047036 \cdot 10^{-48} X^3 + 1.585335103209048 \\ &\quad \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ &= -3.376951785602669 \cdot 10^{-24} B_{0,4}(X) - 1.887163542825797 \cdot 10^{-24} B_{1,4}(X) - 3.973752736266727 \\ &\quad \cdot 10^{-25} B_{2,4}(X) + 1.092413021994703 \cdot 10^{-24} B_{3,4}(X) + 2.582201344038331 \cdot 10^{-24} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ &= -3.376951785602669 \cdot 10^{-24} B_{0,2} - 3.973753000489244 \cdot 10^{-25} B_{1,2} + 2.582201344038331 \cdot 10^{-24} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -1.030701256063201 \cdot 10^{-330} X^4 + 2.355888585287316 \cdot 10^{-330} X^3 + 1.585335103209048 \\ &\quad \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24} \\ &= -3.376951785602669 \cdot 10^{-24} B_{0,4} - 1.887163542825797 \cdot 10^{-24} B_{1,4} - 3.973752736266727 \\ &\quad \cdot 10^{-25} B_{2,4} + 1.092413021994703 \cdot 10^{-24} B_{3,4} + 2.582201344038331 \cdot 10^{-24} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.765646220470353 \cdot 10^{-49}$.
Bounding polynomials M and m:

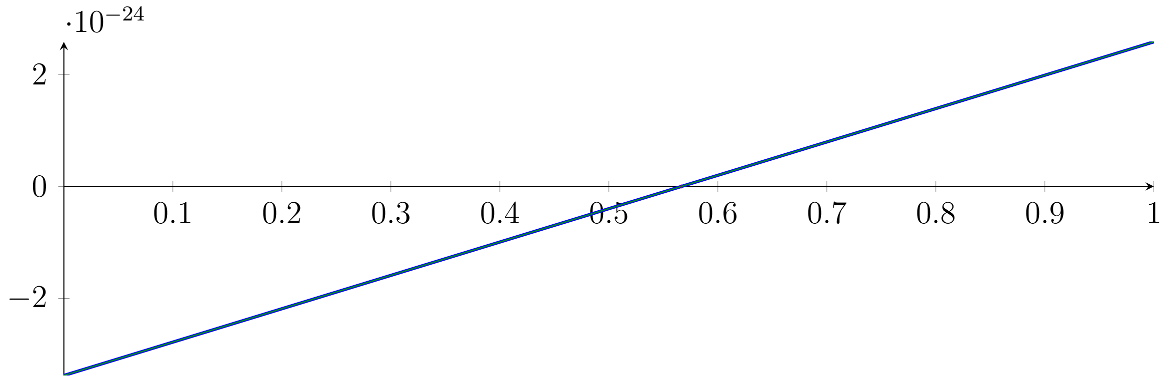
$$M = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$

$$m = 1.585335103209048 \cdot 10^{-31} X^2 + 5.95915297110749 \cdot 10^{-24} X - 3.376951785602669 \cdot 10^{-24}$$

Root of M and m:

$N(M) = \{-37589233.0200927, 0.5666831764624487\}$ $\quad N(m) = \{-37589233.0200927, 0.5666831764624487\}$

Intersection intervals:



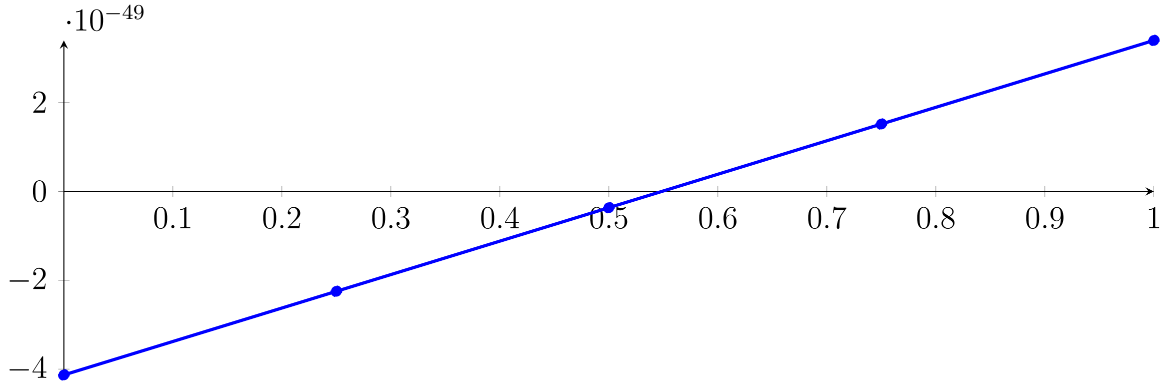
$$[0.5666831764624487, 0.5666831764624487]$$

Longest intersection interval: $1.263819245852043 \cdot 10^{-25}$
$\implies$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

41.12 Recursion Branch 1 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

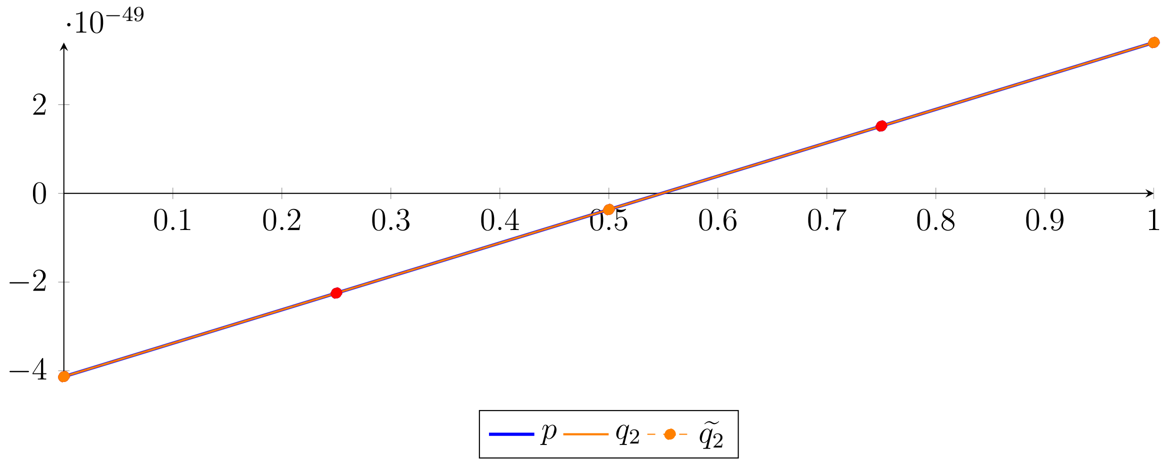
$$\begin{aligned} p = &-1.277867934558931 \cdot 10^{-162} X^4 + 7.601414412052567 \cdot 10^{-123} X^3 + 2.53215919154825 \\ &\cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49} \\ = &-4.131138335916633 \cdot 10^{-49} B_{0,4}(X) - 2.248315225681457 \cdot 10^{-49} B_{1,4}(X) - 3.654921154462805 \\ &\cdot 10^{-50} B_{2,4}(X) + 1.517330994788896 \cdot 10^{-49} B_{3,4}(X) + 3.400154105024072 \cdot 10^{-49} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 2.53215919154825 \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49} \\ &= -4.131138335916633 \cdot 10^{-49} B_{0,2} - 3.654921154462805 \cdot 10^{-50} B_{1,2} + 3.400154105024072 \cdot 10^{-49} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -1.103781553309664 \cdot 10^{-355} X^4 + 2.588177435346798 \cdot 10^{-355} X^3 + 2.53215919154825 \\ &\quad \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49} \\ &= -4.131138335916633 \cdot 10^{-49} B_{0,4} - 2.248315225681457 \cdot 10^{-49} B_{1,4} - 3.654921154462805 \\ &\quad \cdot 10^{-50} B_{2,4} + 1.517330994788896 \cdot 10^{-49} B_{3,4} + 3.400154105024072 \cdot 10^{-49} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.601414412052567 \cdot 10^{-124}$.

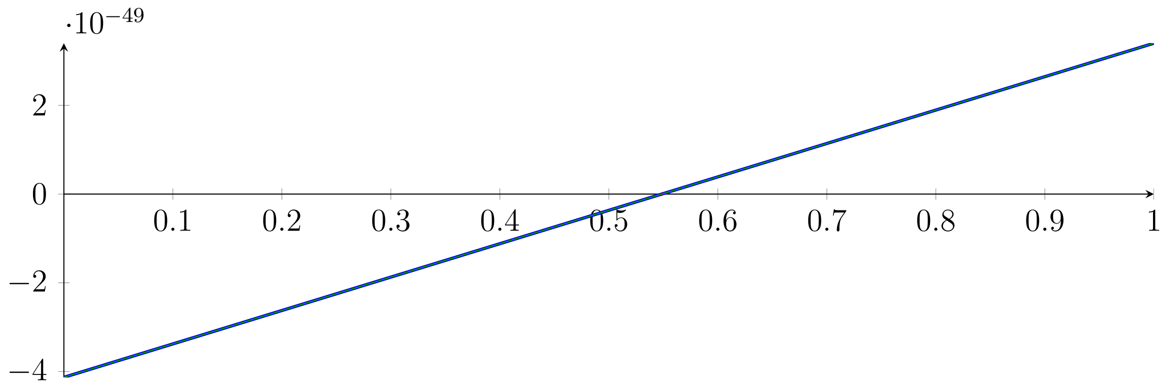
Bounding polynomials M and m:

$$M = 2.53215919154825 \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49}$$
$$m = 2.53215919154825 \cdot 10^{-81} X^2 + 7.531292440940705 \cdot 10^{-49} X - 4.131138335916633 \cdot 10^{-49}$$

Root of M and m:

$N(M) = \{-2.974257094924515 \cdot 10^{32}, 0.5485297999397071\}$ $\quad N(m) = \{-2.974257094924515 \cdot 10^{32}, 0.54852979$

Intersection intervals:



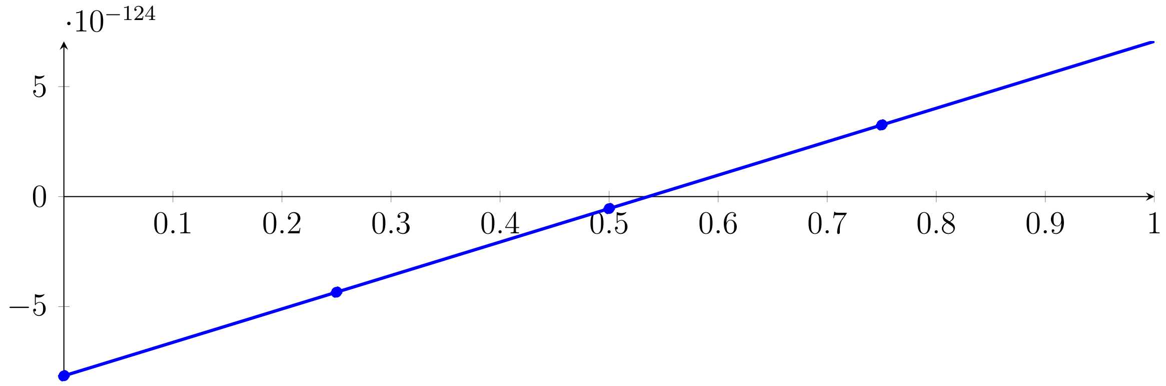
$$[0.5485297999397071, 0.5485297999397071]$$

Longest intersection interval: $2.018621497349027 \cdot 10^{-75}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

41.13 Recursion Branch 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

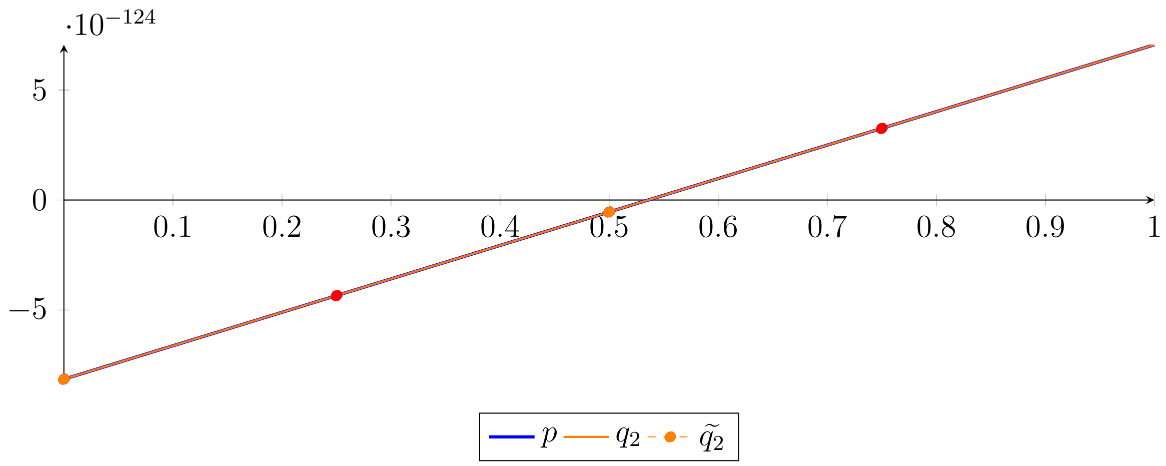
$$\begin{aligned} p = & \ 6.252577620632575 \cdot 10^{-347} X^3 + 1.031812520081924 \cdot 10^{-230} X^2 \\ & + 1.520282882410513 \cdot 10^{-123} X - 8.146069091053342 \cdot 10^{-124} \\ = & -8.146069091053342 \cdot 10^{-124} B_{0,4}(X) - 4.345361885027058 \cdot 10^{-124} B_{1,4}(X) - 5.446546790007743 \\ & \cdot 10^{-125} B_{2,4}(X) + 3.256052527025509 \cdot 10^{-124} B_{3,4}(X) + 7.056759733051793 \cdot 10^{-124} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & \ 1.031812520081924 \cdot 10^{-230} X^2 + 1.520282882410513 \cdot 10^{-123} X - 8.146069091053342 \cdot 10^{-124} \\ = & -8.146069091053342 \cdot 10^{-124} B_{0,2} - 5.446546790007743 \cdot 10^{-125} B_{1,2} + 7.056759733051793 \cdot 10^{-124} B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & -1.682969342655774 \cdot 10^{-430} X^4 + 4.375720290905013 \cdot 10^{-430} X^3 + 1.031812520081924 \\ & \cdot 10^{-230} X^2 + 1.520282882410513 \cdot 10^{-123} X - 8.146069091053342 \cdot 10^{-124} \\ = & -8.146069091053342 \cdot 10^{-124} B_{0,4} - 4.345361885027058 \cdot 10^{-124} B_{1,4} - 5.446546790007743 \\ & \cdot 10^{-125} B_{2,4} + 3.256052527025509 \cdot 10^{-124} B_{3,4} + 7.056759733051793 \cdot 10^{-124} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.252577620632575 \cdot 10^{-348}$.
Bounding polynomials M and m:

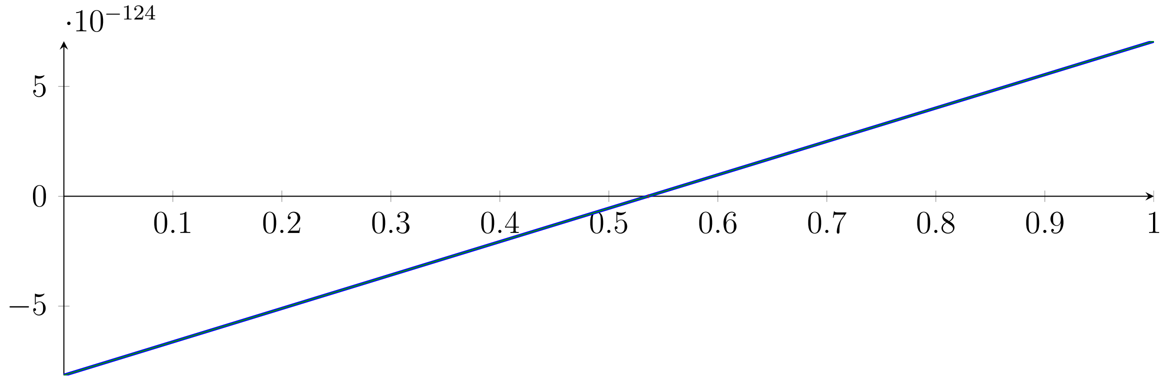
$$M = 1.031812520081924 \cdot 10^{-230} X^2 + 1.520282882410513 \cdot 10^{-123} X - 8.146069091053342 \cdot 10^{-124}$$

$$m = 1.031812520081924\cdot 10^{-230}X^2 + 1.520282882410513\cdot 10^{-123}X - 8.146069091053342\cdot 10^{-124}$$

Root of M and m:

$N(M) = \{-1.473409997283039\cdot 10^{107}, 0.5358258772299789\}$ $\quad N(m) = \{-1.473409997283039\cdot 10^{107}, 0.5358258$

Intersection intervals:



$$[0.5358258772299789, 0.5358258772299789]$$

Longest intersection interval: 0
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

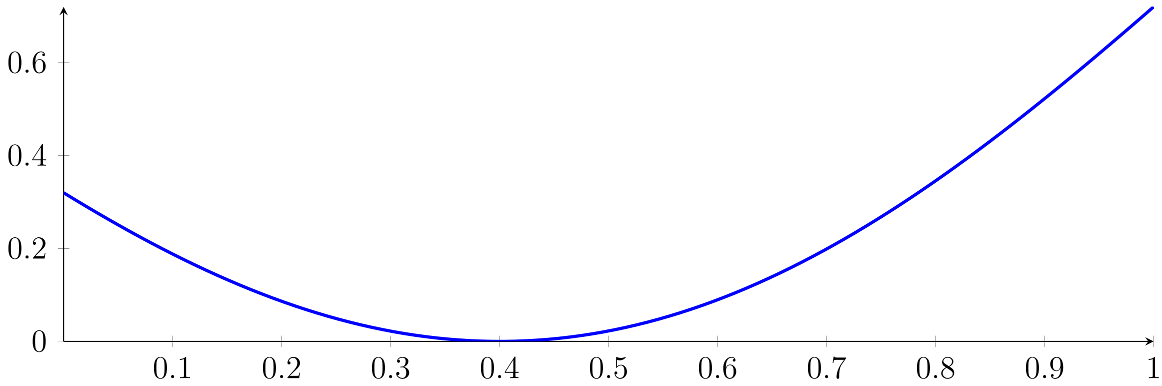
41.14 Recursion Branch 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 10!

41.15 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.4, 0.4],\ [0.40000001, 0.40000001]$$

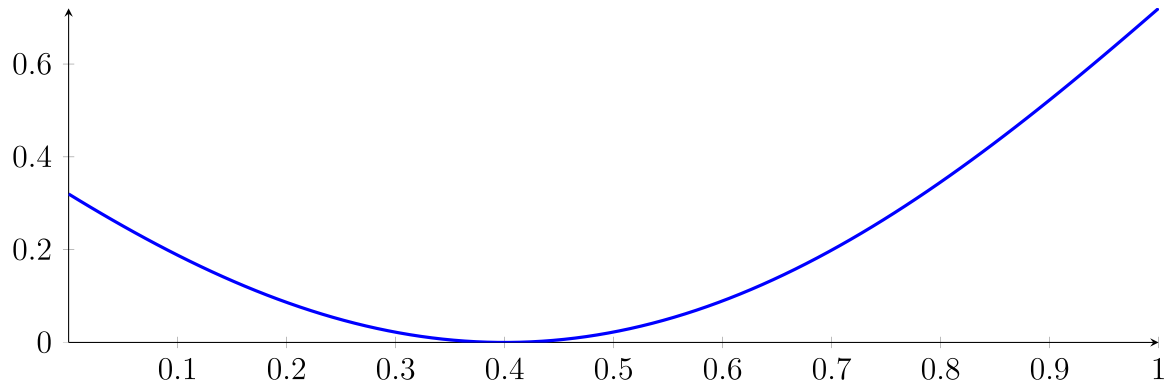
with precision $\varepsilon = 1 \cdot 10^{-128}$.

42 Running CubeClip on h_4 with epsilon 128

$$-1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$

Called CubeClip with input polynomial on interval $[0, 1]$:

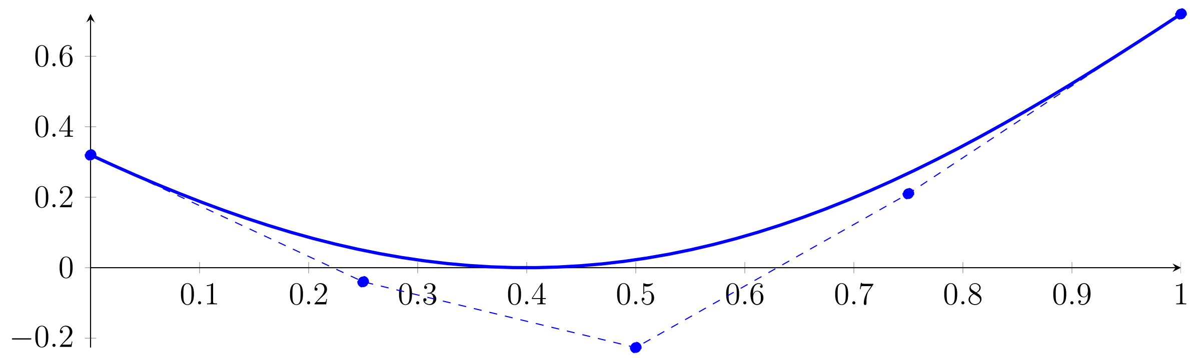
$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



42.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

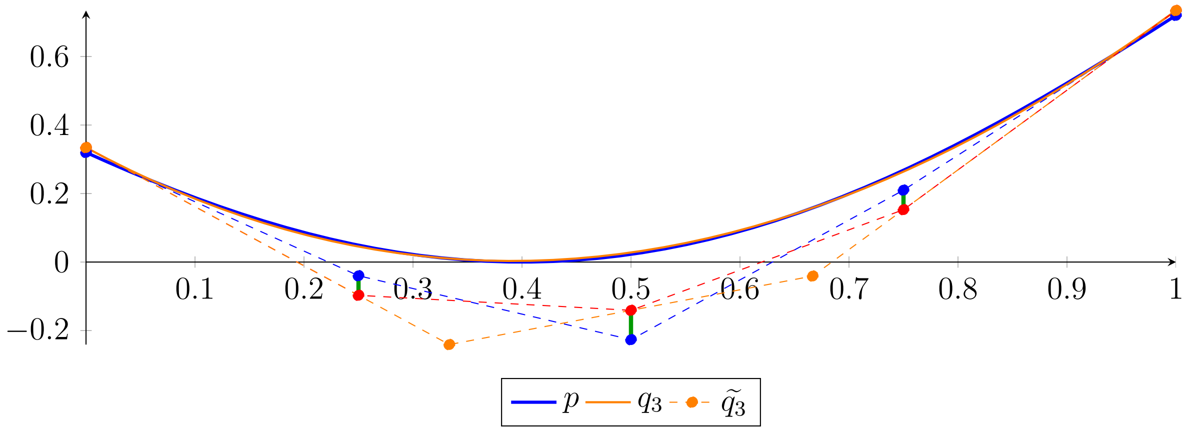
$$\begin{aligned} p &= -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008 \\ &= 0.320000008B_{0,4}(X) - 0.0399999959999998B_{1,4}(X) \\ &\quad - 0.226666669B_{2,4}(X) + 0.2099999915B_{3,4}(X) + 0.719999988B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,3} - 0.2409523782857143B_{1,3} \\ &\quad - 0.04095238828571431B_{2,3} + 0.7342857022857142B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= -2.781342323134002 \cdot 10^{-308}X^4 - 0.19999999X^3 + 2.325714271714286X^2 \\ &\quad - 1.725714301714286X + 0.3342857222857143 \\ &= 0.3342857222857143B_{0,4} - 0.09714285314285713B_{1,4} \\ &\quad - 0.1409523832857143B_{2,4} + 0.1528571343571428B_{3,4} + 0.7342857022857142B_{4,4} \end{aligned}$$



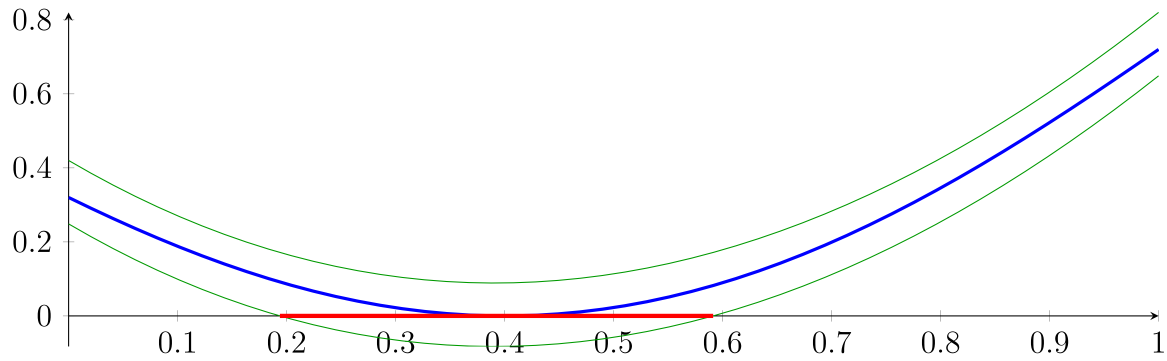
The maximum difference of the Bézier coefficients is $\delta = 0.08571428571428571$.
Bounding polynomials M and m:

$$M = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.420000008$$
$$m = -0.19999999X^3 + 2.325714271714286X^2 - 1.725714301714286X + 0.2485714365714286$$

Root of M and m:

$N(M) = \{10.85123700584596\}$ $\quad N(m) = \{0.1938265012447289, 0.5913474001559605, 10.84339803859934\}$

Intersection intervals:



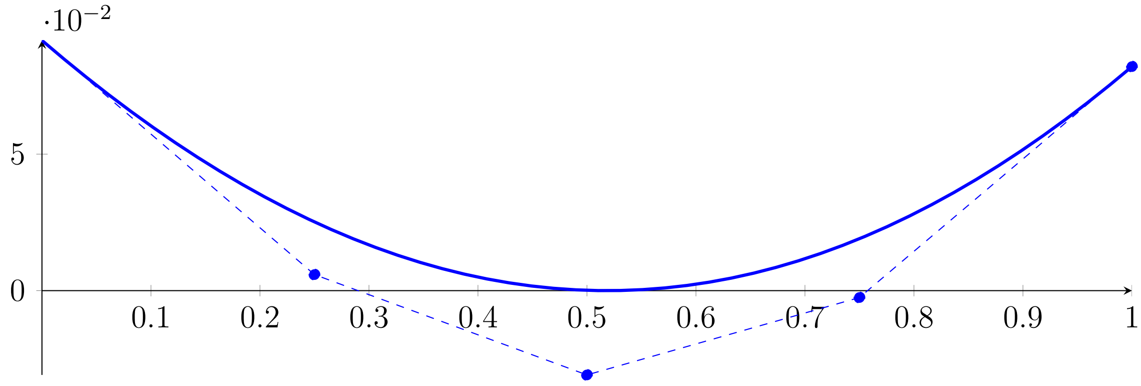
$[0.1938265012447289, 0.5913474001559605]$

Longest intersection interval: 0.3975208989112316
$\Longrightarrow$ Selective recursion: interval 1: $[0.1938265012447289, 0.5913474001559605]$,

42.2 Recursion Branch 1 1 in Interval 1: $[0.1938265012447289, 0.5913474001559605]$

Normalized monomial und Bézier representations and the Bézier polygon:

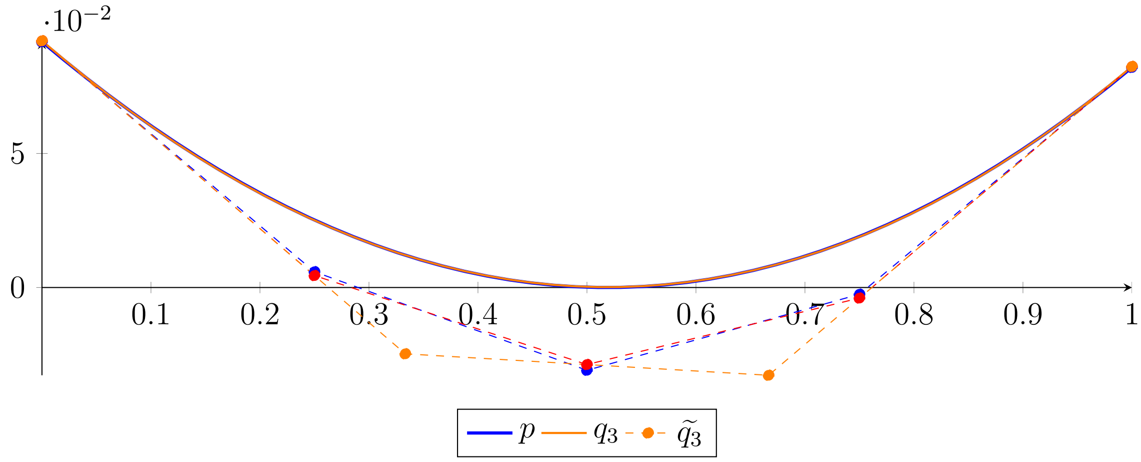
$$\begin{aligned} p = &-0.02497122588530867X^4 + 0.06436860434957161X^3 \\ &+ 0.2941201876709534X^2 - 0.34309913433192X + 0.09165715738594469 \\ = &\ 0.09165715738594469B_{0,4}(X) + 0.005882373802964686B_{1,4}(X) - 0.03087237850152309B_{2,4}(X) \\ &- 0.002514948440125733B_{3,4}(X) + 0.08207558918924098B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,3} - 0.02473070091541078B_{1,3} \\ &\quad - 0.03273327450729677B_{2,3} + 0.08243232098760253B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.476677903917502 \cdot 10^{-309}X^4 + 0.01442615257895426X^3 \\ &\quad + 0.3262260495234931X^2 - 0.3502337702991511X + 0.09201388918430624 \\ &= 0.09201388918430624B_{0,4} + 0.004455446609518477B_{1,4} - 0.02873198771135377B_{2,4} \\ &\quad - 0.003941875633571942B_{3,4} + 0.08243232098760253B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.002140390790169315$.

Bounding polynomials M and m:

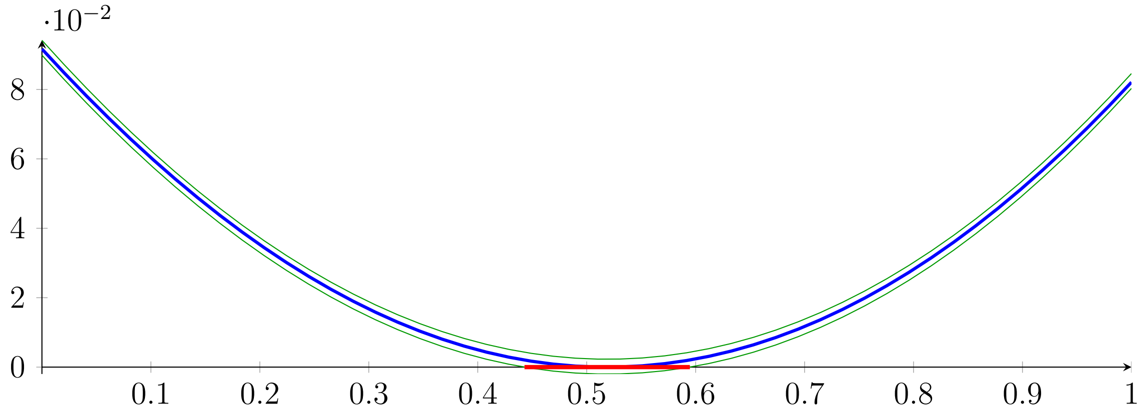
$$\begin{aligned} M &= 0.01442615257895426X^3 + 0.3262260495234931X^2 \\ &\quad - 0.3502337702991511X + 0.09415427997447556 \end{aligned}$$

$$m = 0.01442615257895426X^3 + 0.3262260495234931X^2 - 0.3502337702991511X + 0.08987349839413693$$

Root of M and m:

$$N(M) = \{-23.65165374934574\} \quad N(m) = \{-23.65114581601865, 0.4429176870393937, 0.5947109255489168\}$$

Intersection intervals:



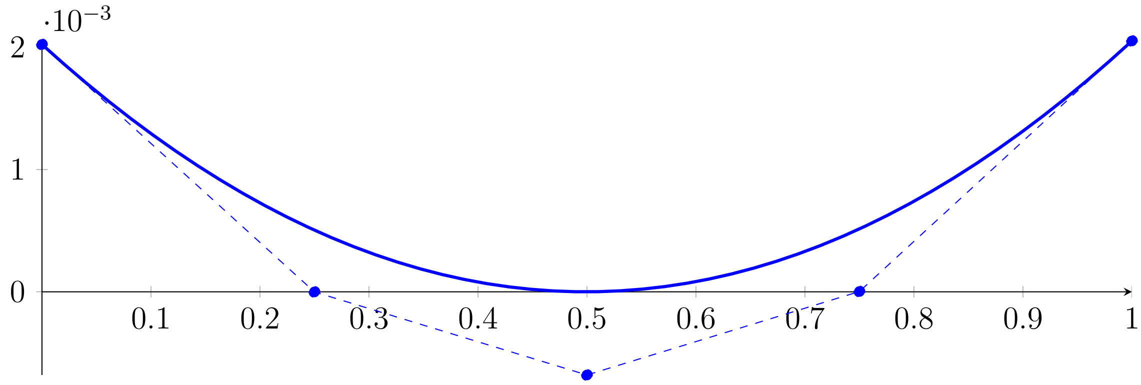
$$[0.4429176870393937, 0.5947109255489168]$$

Longest intersection interval: 0.1517932385095231
$\Longrightarrow$ Selective recursion: interval 1: $[0.3698955383403123, 0.4302365229612649]$,

42.3 Recursion Branch 1 1 1 in Interval 1: $[0.3698955383403123, 0.430236522961264$

Normalized monomial und Bézier representations and the Bézier polygon:

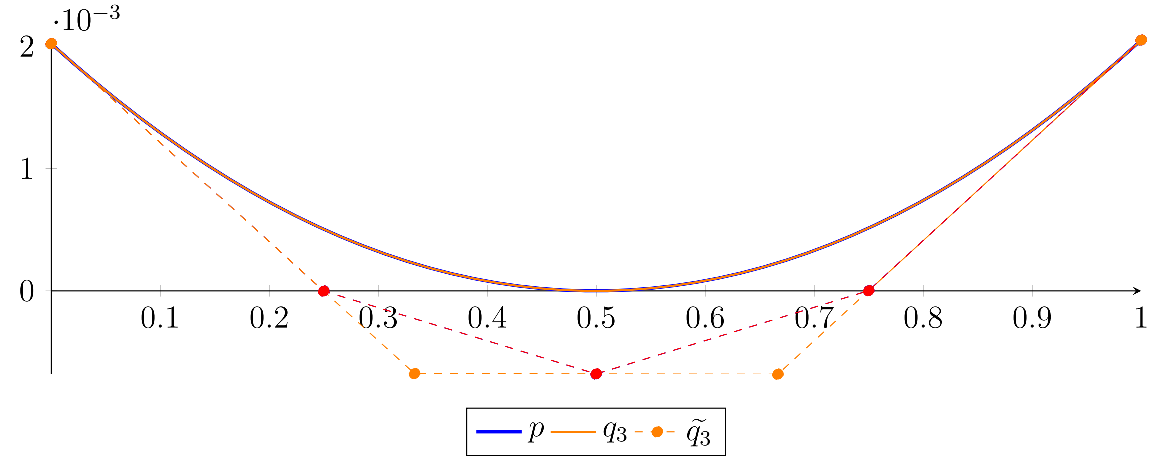
$$\begin{aligned}p = &-1.325713168422468 \cdot 10^{-05} X^4 + 7.039695732710913 \cdot 10^{-05} X^3\\ &+ 0.008070351522992233 X^2 - 0.008098671960928044 X + 0.002023786815863734\\ = &\ 0.002023786815863734 B_{0,4}(X) - 8.811743682771854 \cdot 10^{-07} B_{1,4}(X)\\ &- 0.0006804905774349 16 B_{2,4}(X) + 2.557845995594498\\ &\cdot 10^{-06} B_{3,4}(X) + 0.002052606203570807 B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 = &\ 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2\\ &- 0.008102459712837822 X + 0.002023976203459223\\ = &\ 0.002023976203459223 B_{0,3} - 0.0006768437008200514 B_{1,3}\\ &- 0.0006818648029039136 B_{2,3} + 0.002052795591166296 B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} = &-6.518771069845317 \cdot 10^{-311} X^4 + 4.388269395865976 \cdot 10^{-05} X^3\\ &+ 0.008087396406586236 X^2 - 0.008102459712837822 X + 0.002023976203459223\\ = &\ 0.002023976203459223 B_{0,4} - 1.638724750232882 \cdot 10^{-06} B_{1,4} - 0.0006793542518619825 B_{2,4}\\ &+ 1.800295613638801 \cdot 10^{-06} B_{3,4} + 0.002052795591166296 B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.136325572933544 \cdot 10^{-06}$.

Bounding polynomials M and m:

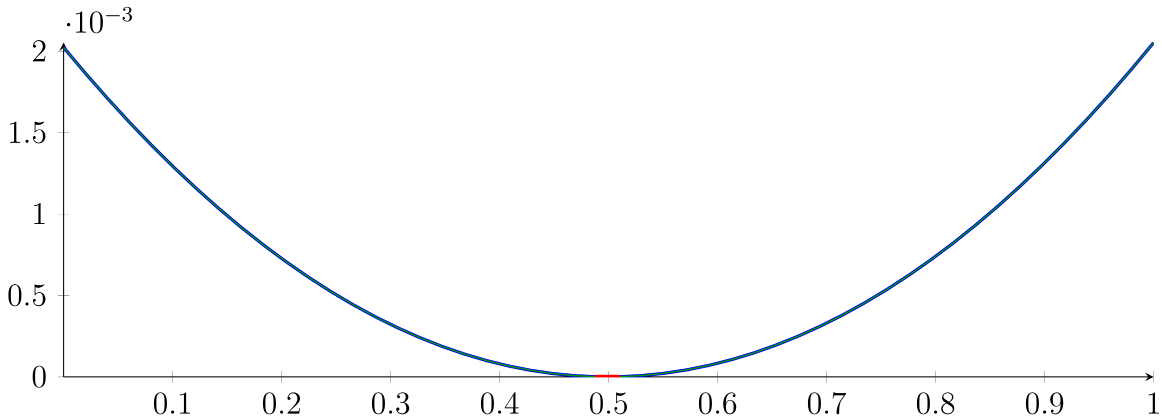
$$M = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002025112529032156$$

$$m = 4.388269395865976 \cdot 10^{-05} X^3 + 0.008087396406586236 X^2 \\ - 0.008102459712837822 X + 0.002022839877886289$$

Root of M and m:

$N(M) = \{-185.2936165687423\} \quad N(m) = \{-185.2936150684252, 0.4874742490566883, 0.5103358670775649\}$

Intersection intervals:



$$[0.4874742490566883, 0.5103358670775649]$$

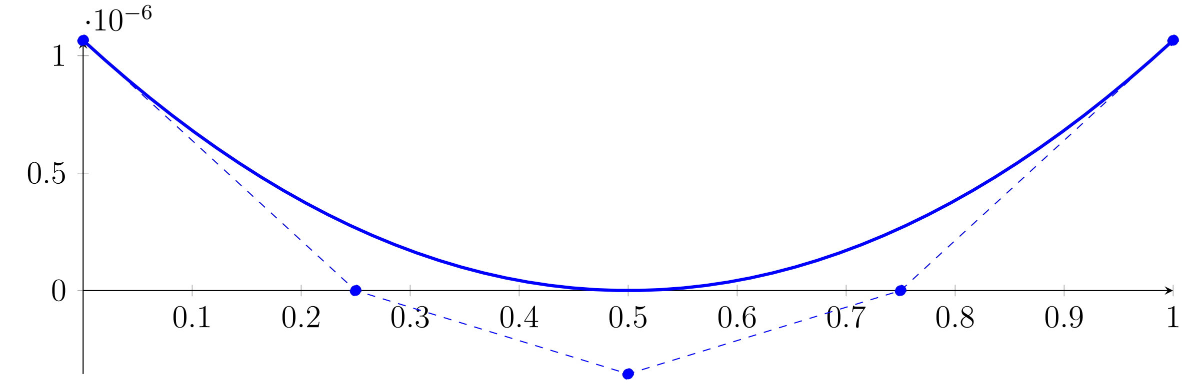
Longest intersection interval: 0.02286161802087665

$\Longrightarrow$ Selective recursion: interval 1: $[0.3993102145057523, 0.4006897070471601]$,

42.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3993102145057523, 0.4006897070471601]$

Normalized monomial und Bézier representations and the Bézier polygon:

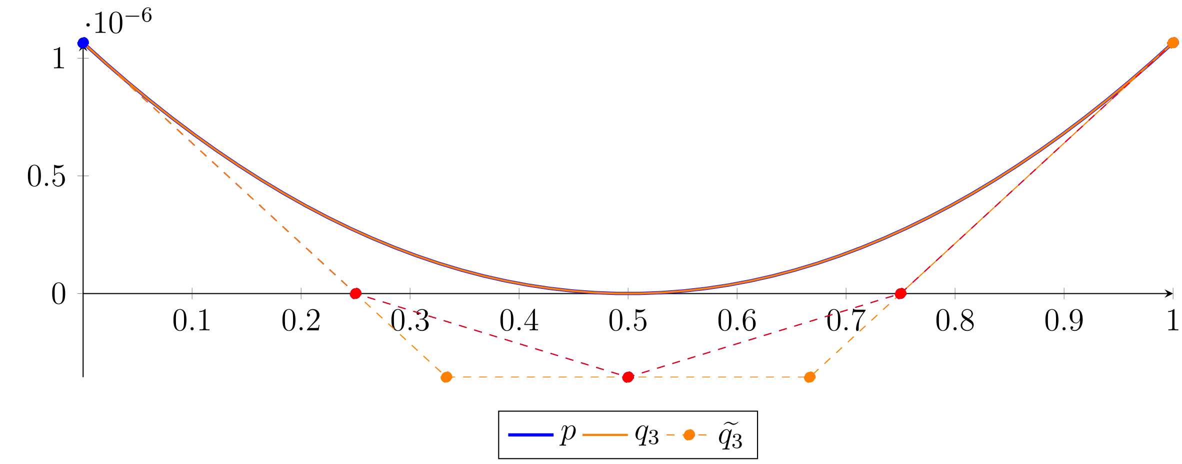
$$p = -3.621407750870066 \cdot 10^{-12} X^4 + 5.322780243378263 \cdot 10^{-10} X^3 + 4.261926231315171 \\ \cdot 10^{-06} X^2 - 4.262596936226815 \cdot 10^{-06} X + 1.065750606194374 \cdot 10^{-06} \\ = 1.065750606194374 \cdot 10^{-06} B_{0,4}(X) + 1.013721376702048 \cdot 10^{-10} B_{1,4}(X) - 3.552268233665051 \\ \cdot 10^{-07} B_{2,4}(X) - 1.009108120675939 \cdot 10^{-10} B_{3,4}(X) + 1.065608557899316 \cdot 10^{-06} B_{4,4}(X)$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06} \\ &= 1.06575065792877\cdot 10^{-06}B_{0,3} - 3.551153323761442\cdot 10^{-07}B_{1,3} \\ &\quad - 3.553376935441088\cdot 10^{-07}B_{2,3} + 1.065608609633713\cdot 10^{-06}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -5.304989477413181\cdot 10^{-314}X^4 + 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085 \\ &\quad \cdot 10^{-06}X^2 - 4.262597970914744\cdot 10^{-06}X + 1.06575065792877\cdot 10^{-06} \\ &= 1.06575065792877\cdot 10^{-06}B_{0,4} + 1.011652000844408\cdot 10^{-10}B_{1,4} - 3.552265129601265 \\ &\quad \cdot 10^{-07}B_{2,4} - 1.011177496533579\cdot 10^{-10}B_{3,4} + 1.065608609633713\cdot 10^{-06}B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.104063786460057\cdot 10^{-13}$.
Bounding polynomials M and m:

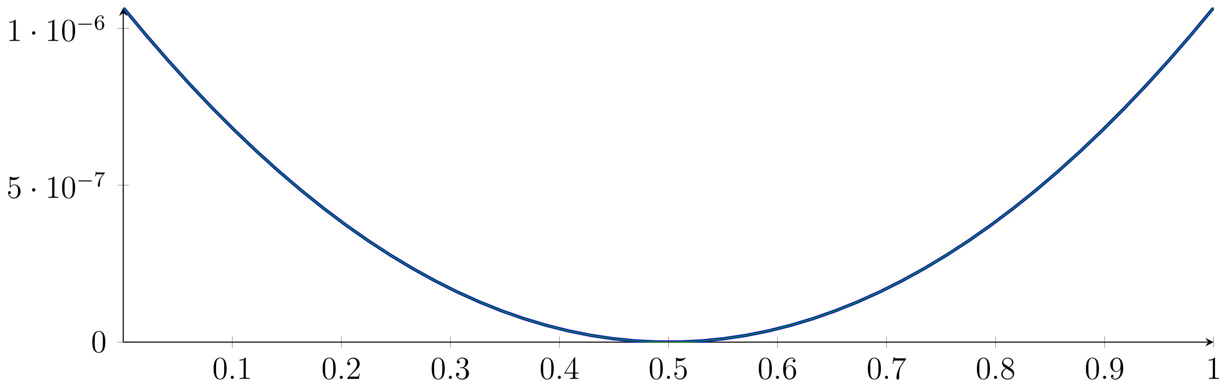
$$\begin{aligned} M &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750968335149\cdot 10^{-06} \end{aligned}$$

$$\begin{aligned} m &= 5.250352088360861\cdot 10^{-10}X^3 + 4.26193088741085\cdot 10^{-06}X^2 \\ &\quad - 4.262597970914744\cdot 10^{-06}X + 1.065750347522392\cdot 10^{-06} \end{aligned}$$

Root of M and m:

$N(M) = \{-8118.41926893212\}$ $\quad N(m) = \{-8118.419268932102, 0.4997707518163204, 0.5002933638554905\}$

Intersection intervals:



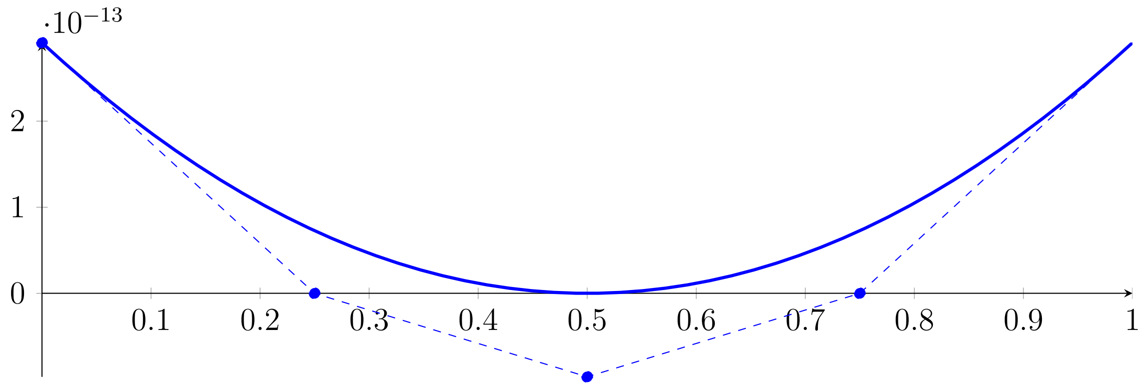
$$[0.4997707518163204, 0.5002933638554905]$$

Longest intersection interval: 0.0005226120391701048
$\Longrightarrow$ Selective recursion: interval 1: $[0.3999996445302967, 0.4000003654697068]$,

42.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3999996445302967, 0.400000365469$

Normalized monomial und Bézier representations and the Bézier polygon:

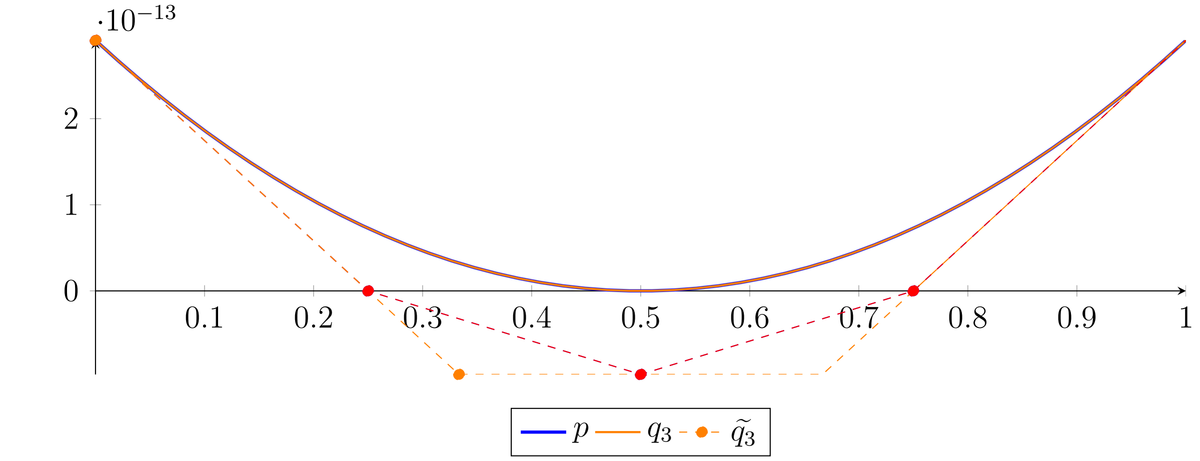
$$\begin{aligned}p &= -2.701438390310769 \cdot 10^{-25} X^4 + 7.494271205548112 \cdot 10^{-20} X^3 + 1.164248026057075 \\ &\quad \cdot 10^{-12} X^2 - 1.164248076702198 \cdot 10^{-12} X + 2.91006022469042 \cdot 10^{-13} \\ &= 2.91006022469042 \cdot 10^{-13} B_{0,4}(X) - 5.599670650739874 \cdot 10^{-17} B_{1,4}(X) - 9.707667820587771 \\ &\quad \cdot 10^{-14} B_{2,4}(X) - 5.600329339091629 \cdot 10^{-17} B_{3,4}(X) + 2.910060467663608 \cdot 10^{-13} B_{4,4}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 \\ &\quad - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690459 \cdot 10^{-13} \\ &= 2.910060224690459 \cdot 10^{-13} B_{0,3} - 9.707666976504573 \cdot 10^{-14} B_{1,3} \\ &\quad - 9.707668664666337 \cdot 10^{-14} B_{2,3} + 2.910060467663647 \cdot 10^{-13} B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= -1.011846442682873 \cdot 10^{-320} X^4 + 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \\ &\quad \cdot 10^{-12} X^2 - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690459 \cdot 10^{-13} \\ &= 2.910060224690459 \cdot 10^{-13} B_{0,4} - 5.599670652283553 \cdot 10^{-17} B_{1,4} - 9.707667820585455 \\ &\quad \cdot 10^{-14} B_{2,4} - 5.600329340635308 \cdot 10^{-17} B_{3,4} + 2.910060467663647 \cdot 10^{-13} B_{4,4}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.315518620266374 \cdot 10^{-26}$.

Bounding polynomials M and m:

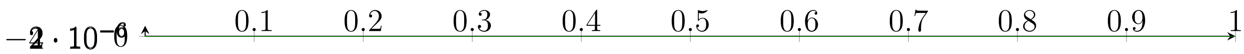
$$M = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 - 1.164248076702275 \cdot 10^{-12} X + 2.91006022469069 \cdot 10^{-13}$$

$$m = 7.494217176780306 \cdot 10^{-20} X^3 + 1.164248026057422 \cdot 10^{-12} X^2 - 1.164248076702275 \cdot 10^{-12} X + 2.910060224690227 \cdot 10^{-13}$$

Root of M and m:

$N(M) = \{-15535286.38864162, 0.4930646005679288, 0.5069353931872718\}$ $N(m) = \{-15535286.38864162,$

Intersection intervals:



$$[0.4930646005650611, 0.4930646005679288], \; [0.5069353931872718, 0.5069353931901395]$$

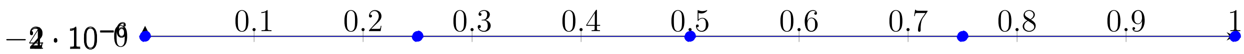
Longest intersection interval: $2.86768528910705 \cdot 10^{-12}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999999999, 0.399999999999999]$, interval 2: $[0.40000001, 0.400000$

42.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.399999999999999, 0.39999999999$

Normalized monomial und Bézier representations and the Bézier polygon:

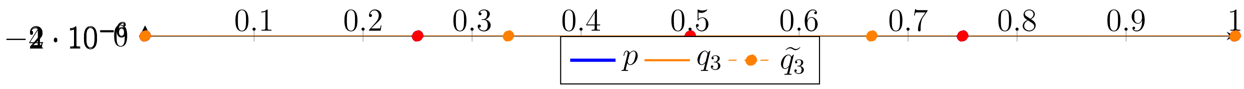
$$p = -1.826926271922849 \cdot 10^{-71} X^4 + 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$
$$= 2.367301365424222 \cdot 10^{-23} B_{0,4}(X) + 2.366143605869363 \cdot 10^{-23} B_{1,4}(X) + 2.364985846314663 \cdot 10^{-23} B_{2,4}(X) + 2.363828086760123 \cdot 10^{-23} B_{3,4}(X) + 2.362670327205742 \cdot 10^{-23} B_{4,4}(X)$$



Degree reduction and raising:

$$q_3 = 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$
$$= 2.367301365424222 \cdot 10^{-23} B_{0,3} + 2.365757686017743 \cdot 10^{-23} B_{1,3} + 2.364214006611583 \cdot 10^{-23} B_{2,3} + 2.362670327205742 \cdot 10^{-23} B_{3,3}$$

$$\widetilde{q}_3 = 2.944860731609145 \cdot 10^{-331} X^4 + 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$
$$= 2.367301365424222 \cdot 10^{-23} B_{0,4} + 2.366143605869363 \cdot 10^{-23} B_{1,4} + 2.364985846314663 \cdot 10^{-23} B_{2,4} + 2.363828086760123 \cdot 10^{-23} B_{3,4} + 2.362670327205742 \cdot 10^{-23} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 1.565936804505299 \cdot 10^{-72}$.
Bounding polynomials M and m:

$$M = 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$

$$m = 1.767342752374517 \cdot 10^{-54} X^3 + 9.574333003207495 \cdot 10^{-36} X^2 - 4.631038219437312 \cdot 10^{-26} X + 2.367301365424222 \cdot 10^{-23}$$

Root of M and m:

$N(M) = \{-5.417360610380978 \cdot 10^{18}, 0.4688611824150995, 4836929866.893817\}$ $N(m) = \{-5.417360610380$

Intersection intervals:

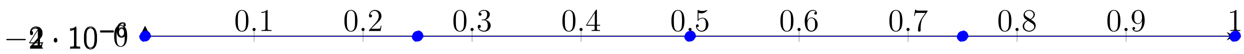
$$[0.4688611824150995, 0.4688611824150995]$$

Longest intersection interval: $6.762790809710196 \cdot 10^{-47}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.399999999999999, 0.399999999999999]$,

42.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.399999999999999, 0.39999999999$

Normalized monomial und Bézier representations and the Bézier polygon:

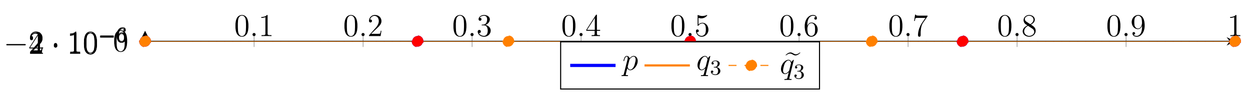
$$p = -3.821420564473856 \cdot 10^{-256} X^4 + 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$
$$= 2.365130051369057 \cdot 10^{-23} B_{0,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{1,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{2,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{3,4}(X) + 2.365130051369057 \cdot 10^{-23} B_{4,4}(X)$$



Degree reduction and raising:

$$q_3 = 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$
$$= 2.365130051369057 \cdot 10^{-23} B_{0,3} + 2.365130051369057 \cdot 10^{-23} B_{1,3} + 2.365130051369057 \cdot 10^{-23} B_{2,3} + 2.365130051369057 \cdot 10^{-23} B_{3,3}$$

$$\widetilde{q}_3 = 5.88972146321829 \cdot 10^{-331} X^4 + 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$
$$= 2.365130051369057 \cdot 10^{-23} B_{0,4} + 2.365130051369057 \cdot 10^{-23} B_{1,4} + 2.365130051369057 \cdot 10^{-23} B_{2,4} + 2.365130051369057 \cdot 10^{-23} B_{3,4} + 2.365130051369057 \cdot 10^{-23} B_{4,4}$$



The maximum difference of the Bézier coefficients is $\delta = 3.275503340977591 \cdot 10^{-257}$.

Bounding polynomials M and m:

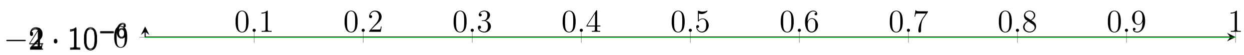
$$M = 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$

$$m = 5.466365221947342 \cdot 10^{-193} X^3 + 4.378853707314745 \cdot 10^{-128} X^2 - 3.131874270375564 \cdot 10^{-72} X + 2.365130051369057 \cdot 10^{-23}$$

Root of M and m:

$N(M) = \{-8.010539972052051 \cdot 10^{64}, 9.723286868432262 \cdot 10^{30}, 7.15226894121871 \cdot 10^{55}\}$ $\quad N(m) = \{-8.010539$

Intersection intervals:

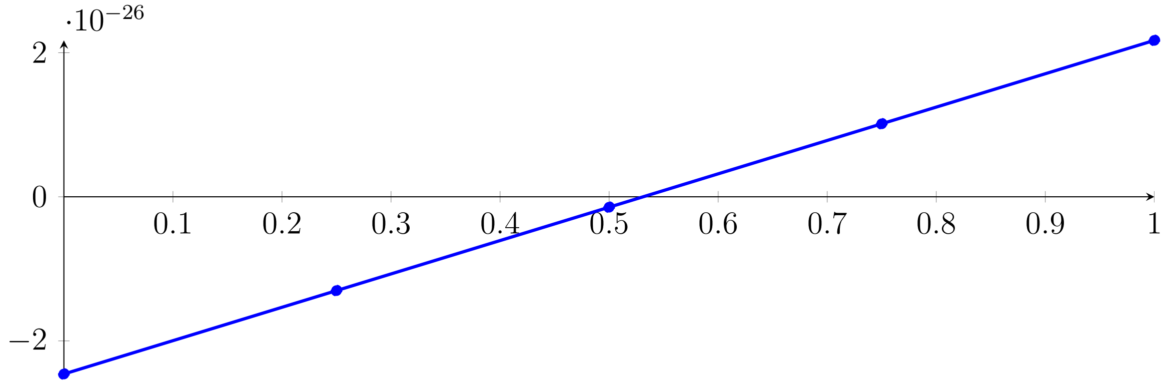


No intersection intervals with the x axis.

42.8 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

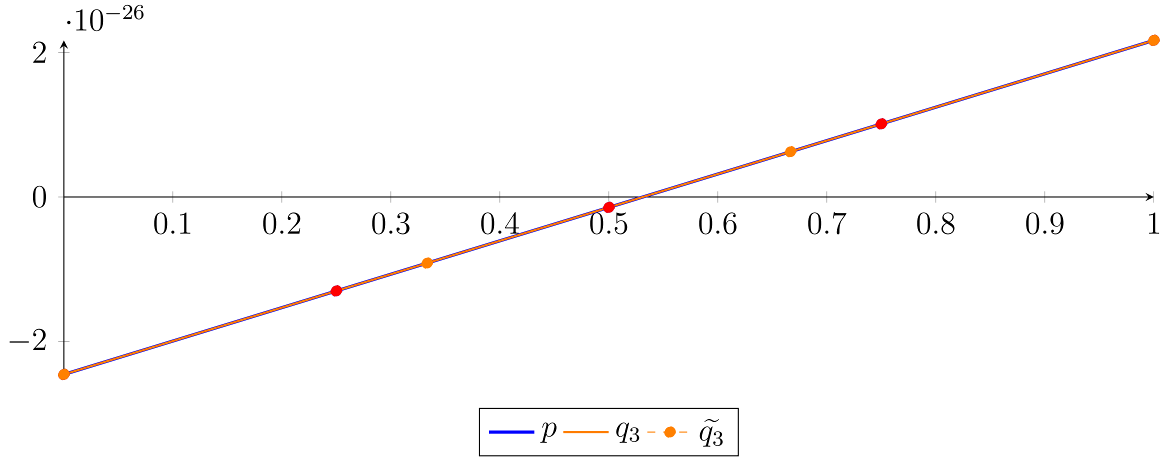
$$\begin{aligned} p &= -1.826926265398112 \cdot 10^{-71} X^4 + 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \\ &\quad \cdot 10^{-36} X^2 + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ &= -2.459960157279009 \cdot 10^{-26} B_{0,4}(X) - 1.302200847385123 \cdot 10^{-26} B_{1,4}(X) - 1.444415373316643 \\ &\quad \cdot 10^{-27} B_{2,4}(X) + 1.013317772881366 \cdot 10^{-26} B_{3,4}(X) + 2.171077083253969 \cdot 10^{-26} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \cdot 10^{-36} X^2 \\ &\quad + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ &= -2.459960157279009 \cdot 10^{-26} B_{0,3} - 9.162810774204939 \cdot 10^{-27} B_{1,3} \\ &\quad + 6.273980027571653 \cdot 10^{-27} B_{2,3} + 2.171077083253969 \cdot 10^{-26} B_{3,3} \end{aligned}$$

$$\begin{aligned} \tilde{q}_3 &= -2.875840558212056 \cdot 10^{-334} X^4 + 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \\ &\quad \cdot 10^{-36} X^2 + 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \\ &= -2.459960157279009 \cdot 10^{-26} B_{0,4} - 1.302200847385123 \cdot 10^{-26} B_{1,4} - 1.444415373316643 \\ &\quad \cdot 10^{-27} B_{2,4} + 1.013317772881366 \cdot 10^{-26} B_{3,4} + 2.171077083253969 \cdot 10^{-26} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.565936798912668 \cdot 10^{-72}$.
Bounding polynomials M and m:

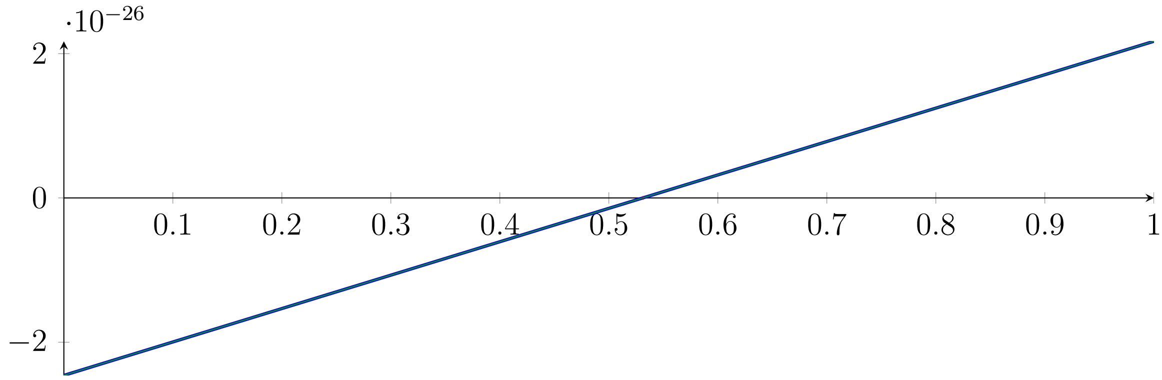
$$\begin{aligned} M = &\ 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \cdot 10^{-36} X^2 \\ &+ 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \end{aligned}$$

$$\begin{aligned} m = &\ 1.767342394171994 \cdot 10^{-54} X^3 + 9.574333011756007 \cdot 10^{-36} X^2 \\ &+ 4.631037239575544 \cdot 10^{-26} X - 2.459960157279009 \cdot 10^{-26} \end{aligned}$$

Root of M and m:

$N(M) = \{-5.417361703527237 \cdot 10^{18}, -4836929870.212556, 0.5311898889490086\}$ $N(m) = \{-5.41736170352$

Intersection intervals:



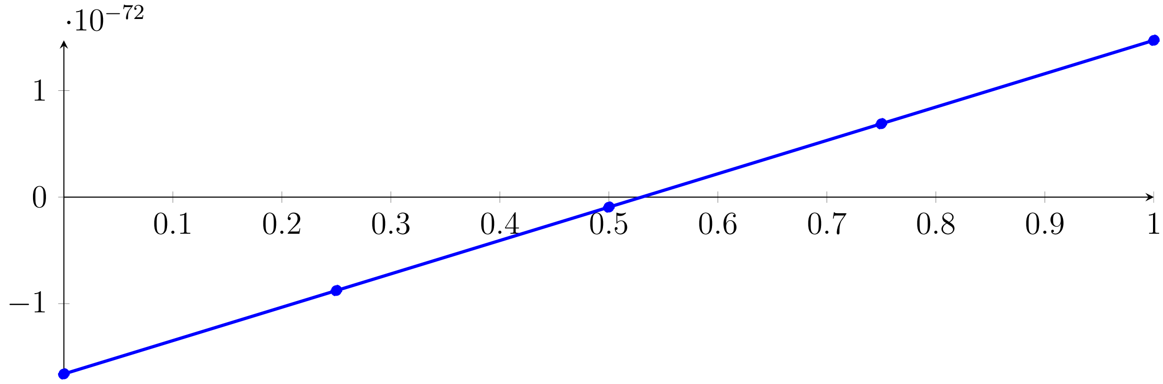
$$[0.5311898889490086, 0.5311898889490086]$$

Longest intersection interval: $6.76279078555737 \cdot 10^{-47}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

42.9 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.40000001, 0.40000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

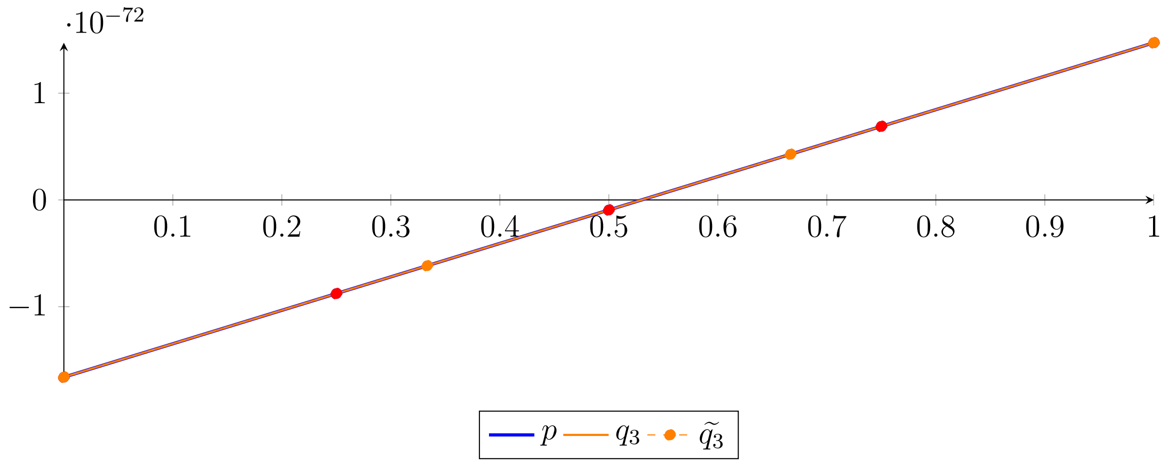
$$\begin{aligned} p = &-3.8214204962342 \cdot 10^{-256} X^4 + 5.466364055464 \cdot 10^{-193} X^3 + 4.378853679946907 \\ &\cdot 10^{-128} X^2 + 3.131873597825335 \cdot 10^{-72} X - 1.66001674389741 \cdot 10^{-72} \\ = &-1.66001674389741 \cdot 10^{-72} B_{0,4}(X) - 8.770483444410761 \cdot 10^{-73} B_{1,4}(X) - 9.407994498474221 \\ &\cdot 10^{-74} B_{2,4}(X) + 6.888884544715917 \cdot 10^{-73} B_{3,4}(X) + 1.471856853927926 \cdot 10^{-72} B_{4,4}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 5.466364055464 \cdot 10^{-193} X^3 + 4.378853679946907 \cdot 10^{-128} X^2 \\ &\quad + 3.131873597825335 \cdot 10^{-72} X - 1.66001674389741 \cdot 10^{-72} \\ &= -1.66001674389741 \cdot 10^{-72} B_{0,3} - 6.160588779556315 \cdot 10^{-73} B_{1,3} \\ &\quad + 4.27898987986147 \cdot 10^{-73} B_{2,3} + 1.471856853927926 \cdot 10^{-72} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.77804152299898 \cdot 10^{-380} X^4 + 5.466364055464 \cdot 10^{-193} X^3 + 4.378853679946907 \\ &\quad \cdot 10^{-128} X^2 + 3.131873597825335 \cdot 10^{-72} X - 1.66001674389741 \cdot 10^{-72} \\ &= -1.66001674389741 \cdot 10^{-72} B_{0,4} - 8.770483444410761 \cdot 10^{-73} B_{1,4} - 9.407994498474221 \\ &\quad \cdot 10^{-74} B_{2,4} + 6.888884544715917 \cdot 10^{-73} B_{3,4} + 1.471856853927926 \cdot 10^{-72} B_{4,4} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.275503282486457 \cdot 10^{-257}$.

Bounding polynomials M and m:

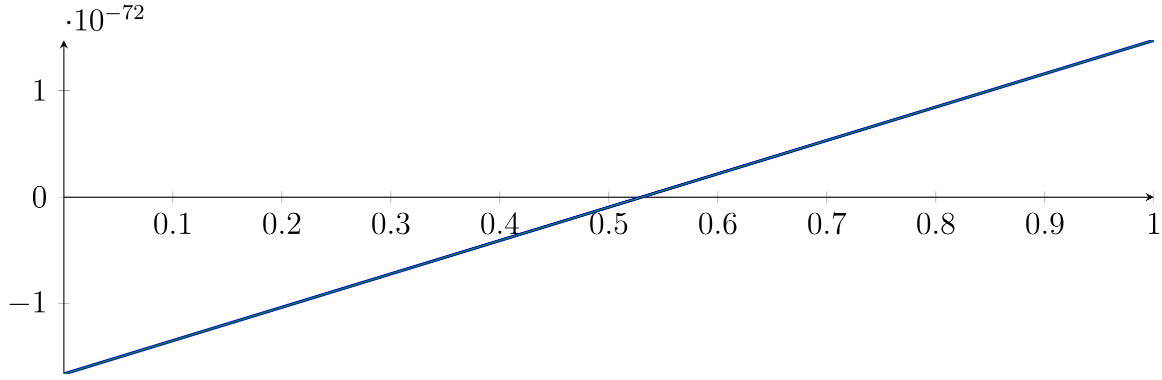
$$\begin{aligned} M &= 5.466364055464 \cdot 10^{-193} X^3 + 4.378853679946907 \cdot 10^{-128} X^2 \\ &\quad + 3.131873597825335 \cdot 10^{-72} X - 1.66001674389741 \cdot 10^{-72} \end{aligned}$$

$$\begin{aligned} m &= 5.466364055464 \cdot 10^{-193} X^3 + 4.378853679946907 \cdot 10^{-128} X^2 \\ &\quad + 3.131873597825335 \cdot 10^{-72} X - 1.66001674389741 \cdot 10^{-72} \end{aligned}$$

Root of M and m:

$N(M) = \{-8.010541617074072 \cdot 10^{64}, -7.152268973148635 \cdot 10^{55}, 0.5300395089540229\}$ $\quad N(m) = \{-8.010541$

Intersection intervals:



$[0.5300395089540229, 0.5300395089540229]$

Longest intersection interval: $2.091721252582386 \cdot 10^{-185}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.40000001, 0.40000001]$,

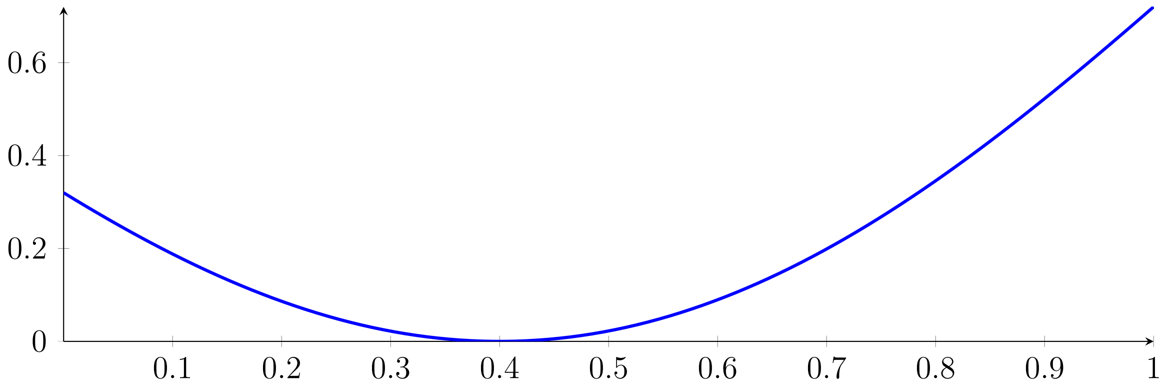
42.10 Recursion Branch 1 1 1 1 1 2 1 1 in Interval 1: $[0.40000001, 0.40000001]$

Found root in interval $[0.40000001, 0.40000001]$ at recursion depth 8!

42.11 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^4 + 1.80000001X^3 + 1.039999986X^2 - 1.440000016X + 0.320000008$$



Result: Root Intervals

$$[0.40000001, 0.40000001]$$

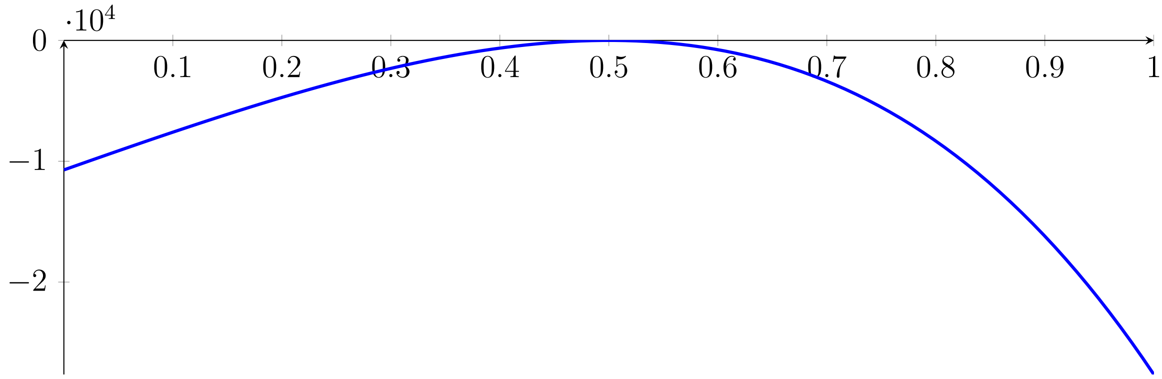
with precision $\varepsilon = 1{\cdot}10^{-128}$.

43 Running BezClip on h_8 with epsilon 2

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called BezClip with input polynomial on interval $[0,1]$:

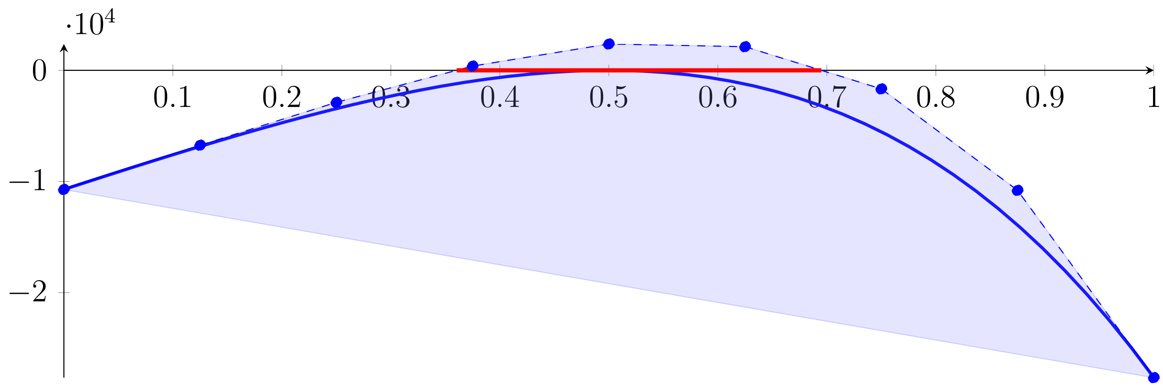
$$\begin{aligned} p = & -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ & - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



43.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ & - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = & -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ & + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ & - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

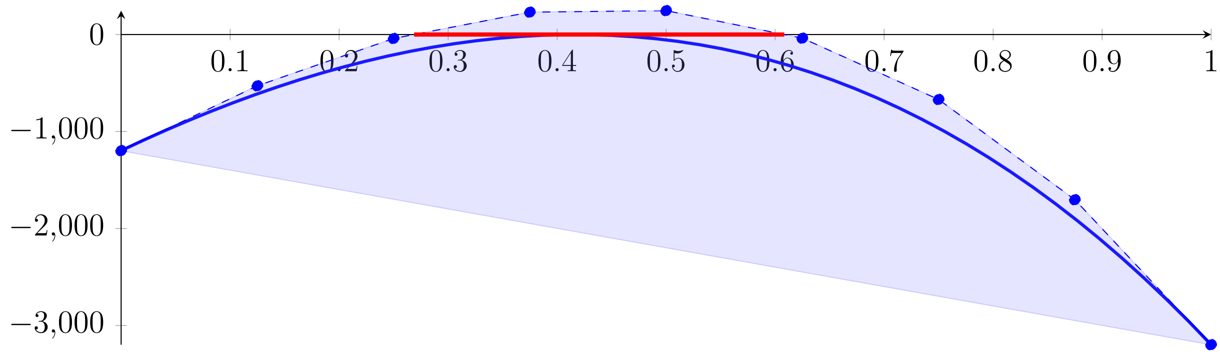
$$[0.3603640692588709, 0.6949437907012195]$$

Longest intersection interval: 0.3345797214423486
$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

43.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6 \\
& - 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3 \\
& - 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098 \\
= & -1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X) \\
& + 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X) \\
& - 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

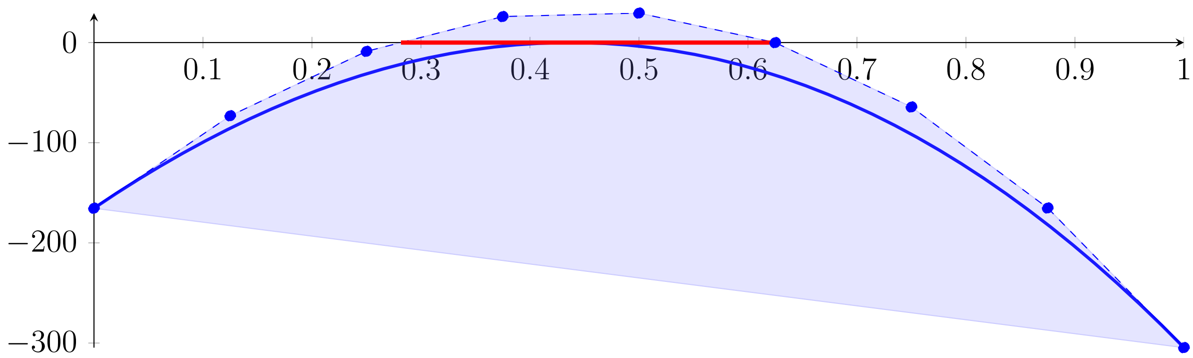
$$[0.2687909235445741, 0.6084084634355219]$$

Longest intersection interval: 0.3396175398909478
$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

43.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -2.779187295559603 \cdot 10^{-08}X^8 - 9.441522673365855 \cdot 10^{-06}X^7 - 0.001328615938017263X^6 \\
& - 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3 \\
& - 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987 \\
= & -165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X) \\
& + 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X) \\
& - 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

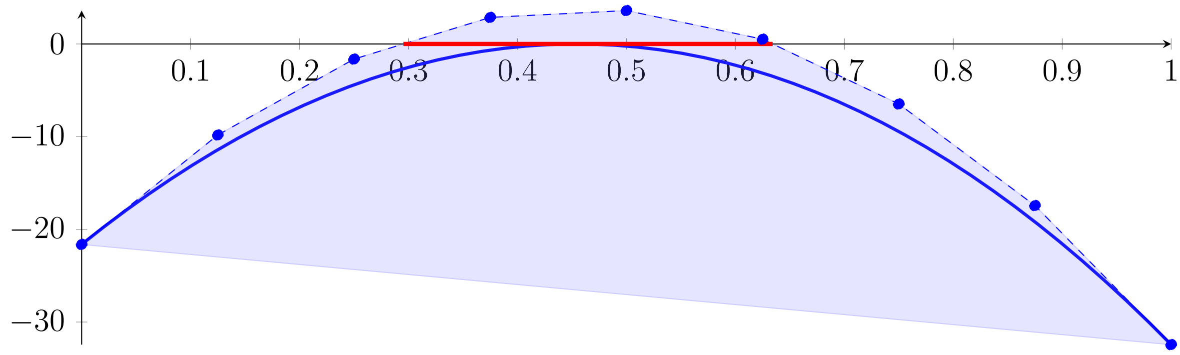
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\Longrightarrow$ Selective recursion: interval 1: [0.4822990094256734, 0.5212832145711177],

43.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4822990094256734, 0.52128321457111$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-5.334693469973702\cdot 10^{-12}X^8 - 5.317476896558791\cdot 10^{-09}X^7 - 2.197128069269196\cdot 10^{-06}X^6\\
&- 0.000481521724173462X^5 - 0.05899466848037266X^4 - 3.82353929224187X^3\\
&- 101.3899724153744X^2 + 94.46052284159652X - 21.62667247491518\\
= &-21.62667247491518B_{0,8}(X) - 9.819107119715613B_{1,8}(X) - 1.632612207922276B_{2,8}(X)\\
&+ 2.86453477310337B_{3,8}(X) + 3.603213555021573B_{4,8}(X) + 0.5134524899120698B_{5,8}(X)\\
&- 6.475580126796985B_{6,8}(X) - 17.435584812533364B_{7,8}(X) - 32.43913973359036B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

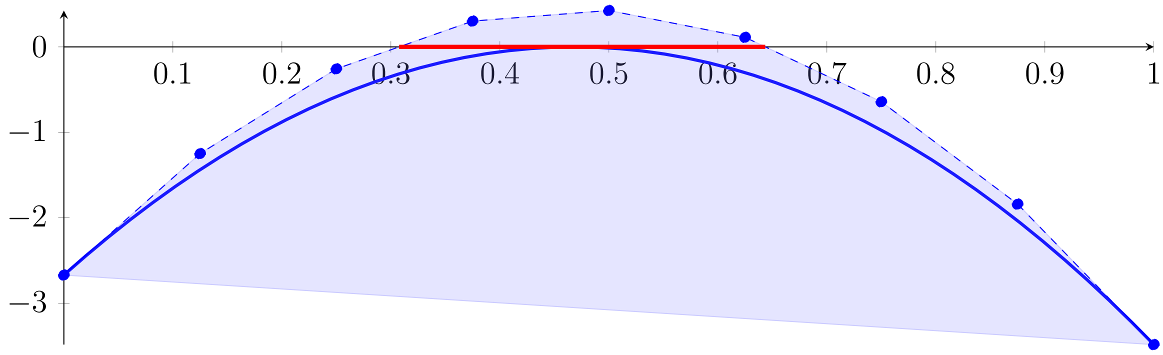
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\Longrightarrow$ Selective recursion: interval 1: [0.4938141292321744, 0.5070221367077007],

43.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4938141292321744, 0.507022136707$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-9.261865042605128\cdot 10^{-16}X^8 - 2.731330938941274\cdot 10^{-12}X^7 - 3.339777336661422\\
&\cdot 10^{-09}X^6 - 2.167033369334019\cdot 10^{-06}X^5 - 0.0007867416616259877X^4\\
&- 0.1514273448103067X^3 - 12.0308548973337X^2 + 11.3691349552019X - 2.67015106585462\\
= &-2.67015106585462B_{0,8}(X) - 1.249009196454382B_{1,8}(X) - 0.2575407162446335B_{2,8}(X)\\
&+ 0.3015503150458701B_{3,8}(X) + 0.4255485985217787B_{4,8}(X) + 0.1117275574241232B_{5,8}(X)\\
&- 0.6426507016859872B_{6,8}(X) - 1.840335427863286B_{7,8}(X) - 3.484087264834231B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\Longrightarrow$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

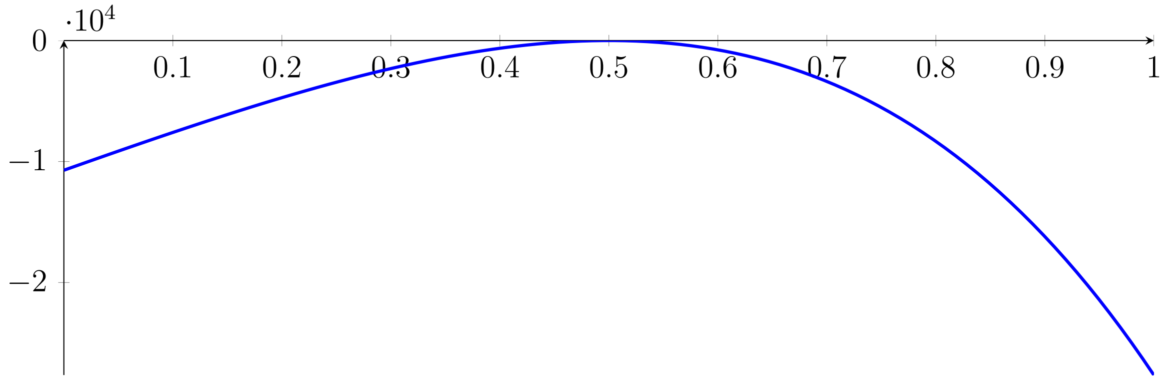
43.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Reached interval $[0.4978766511941703, 0.5023136561973824]$ **without sign change** at depth 6!

43.7 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

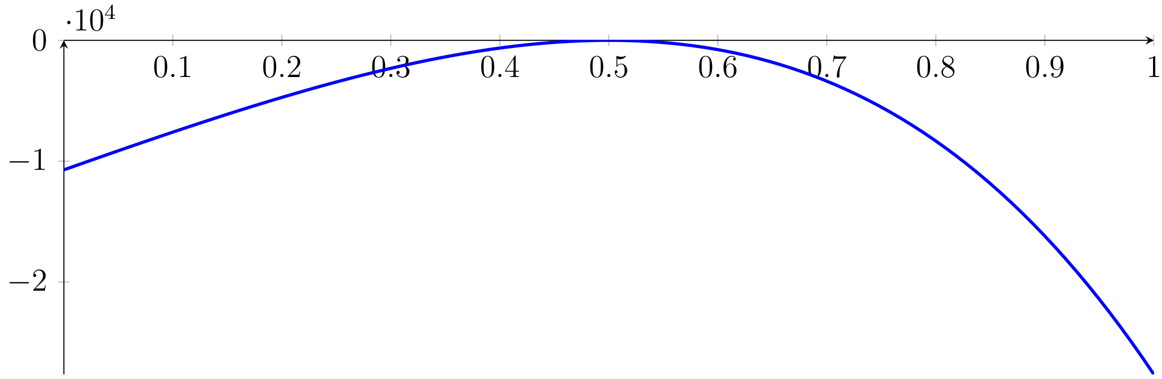
with precision $\varepsilon = 0.01$.

44 Running `QuadClip` on h_8 with epsilon 2

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

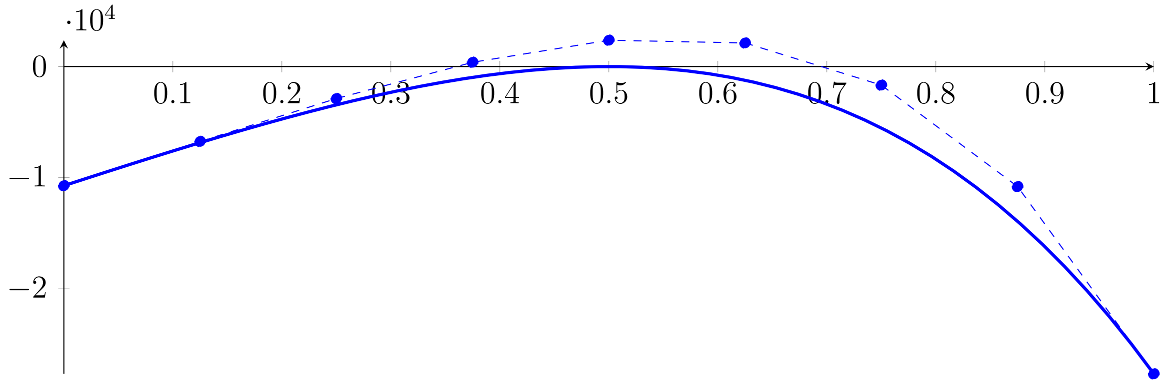
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



44.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

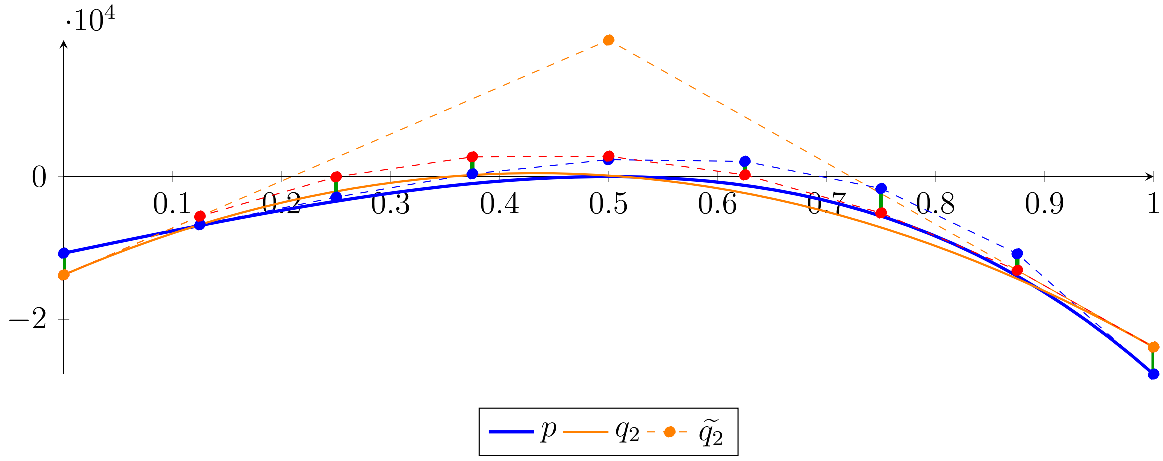
$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 8.415084934481114 \cdot 10^{-299}X^8 - 3.337656926892332 \cdot 10^{-298}X^7 + 5.61048922956057 \cdot 10^{-298}X^6 \\ &\quad - 5.189733160056009 \cdot 10^{-298}X^5 + 2.787623795639522 \cdot 10^{-298}X^4 - 8.054794139884735 \\ &\quad \cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8} \\ &\quad + 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8} \\ &\quad - 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.
Bounding polynomials M and m:

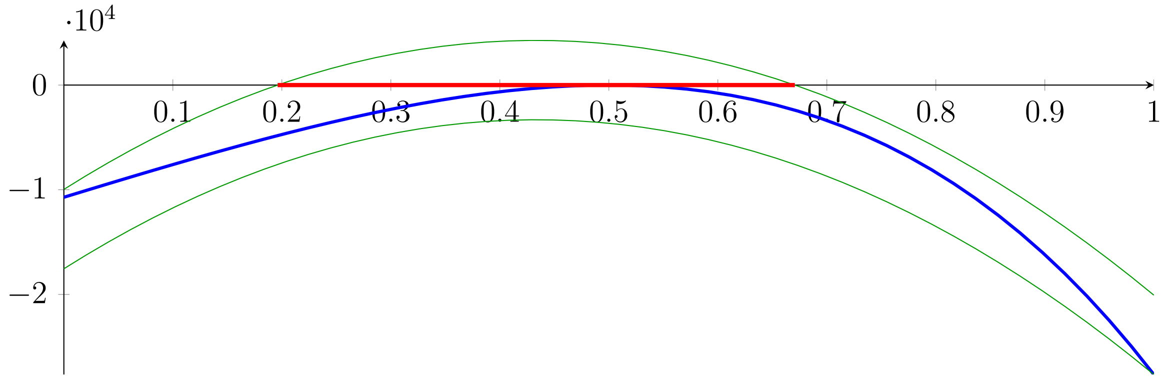
$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$

$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



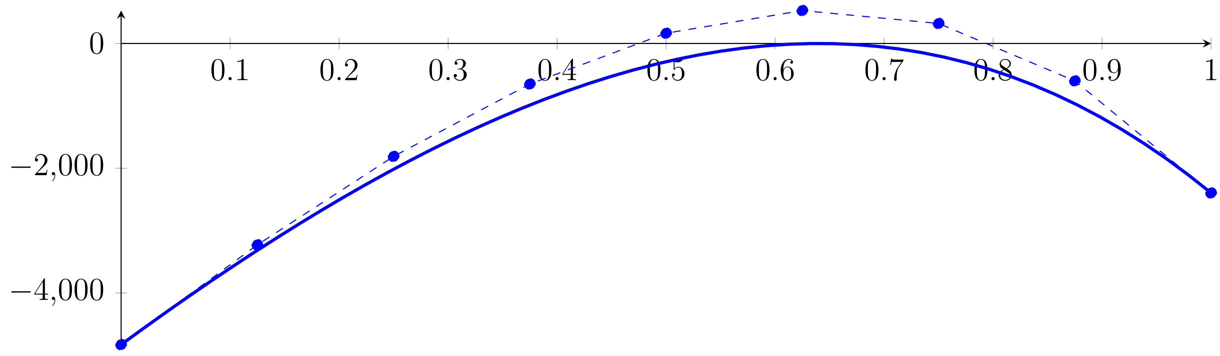
$$[0.196099166583006, 0.6706514347613278]$$

Longest intersection interval: 0.4745522681783218
$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

44.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

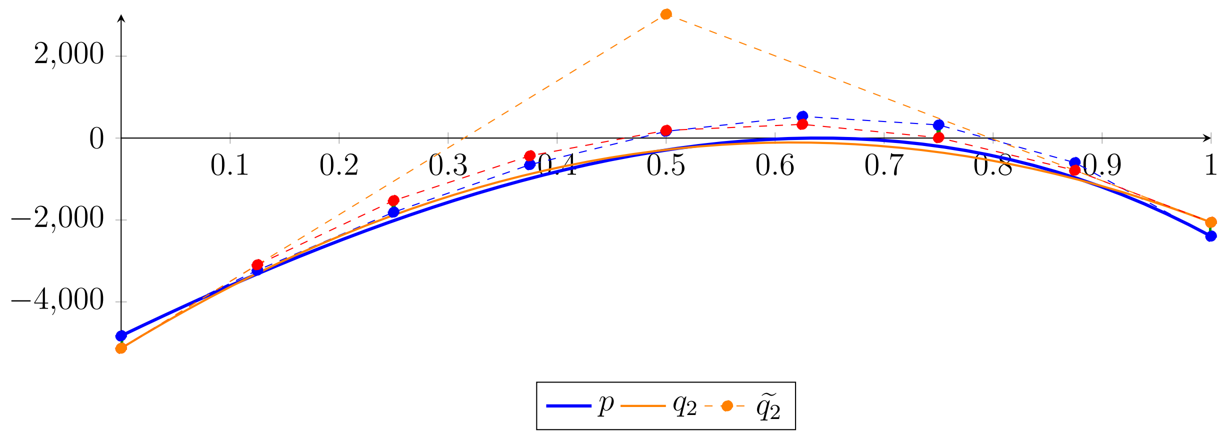
$$\begin{aligned}p &= -0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6\\&\quad - 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3\\&\quad - 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684\\&= -4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X)\\&\quad - 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X)\\&\quad + 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.476670942620757\cdot 10^{-299}X^8 - 6.381627458836932\cdot 10^{-299}X^7 + 1.1779791924066\cdot 10^{-298}X^6 \\ &\quad - 1.190713137013755\cdot 10^{-298}X^5 + 6.850874958113058\cdot 10^{-299}X^4 - 2.05172173630317 \\ &\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\ &\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\ &\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

Bounding polynomials M and m:

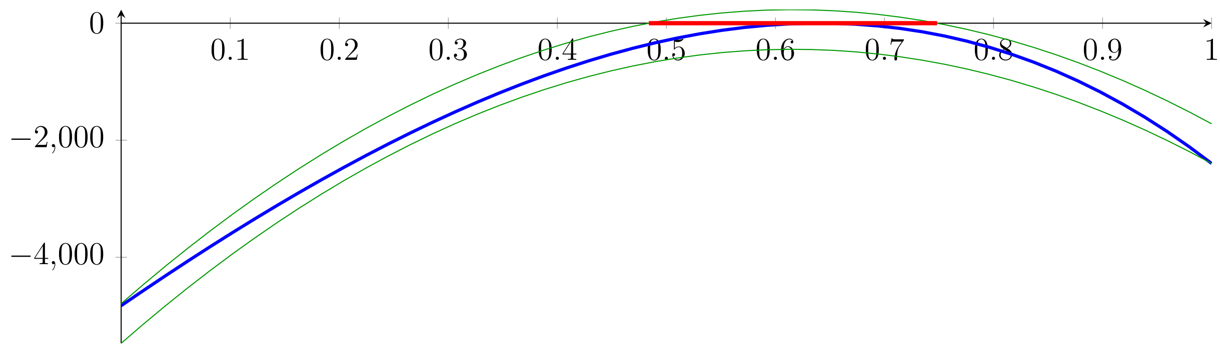
$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$

$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.4839311609916354, 0.7482816274420809]$$

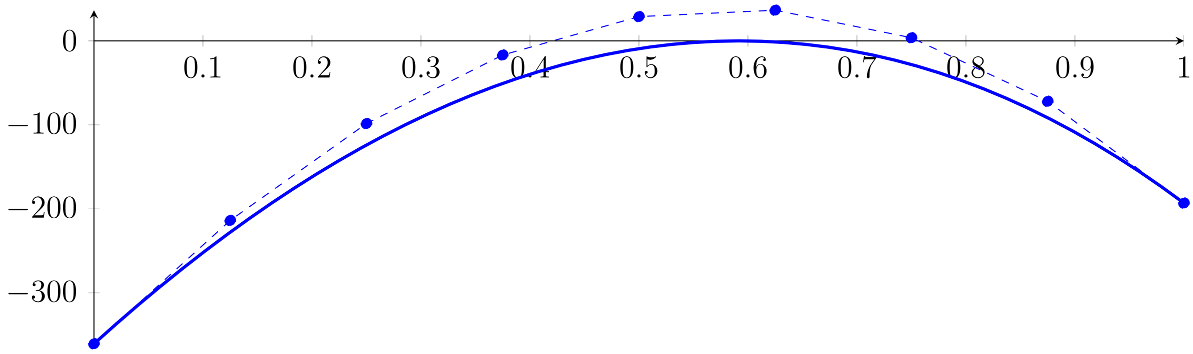
Longest intersection interval: 0.2643504664504455

$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

44.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.551197910121811$

Normalized monomial und Bézier representations and the Bézier polygon:

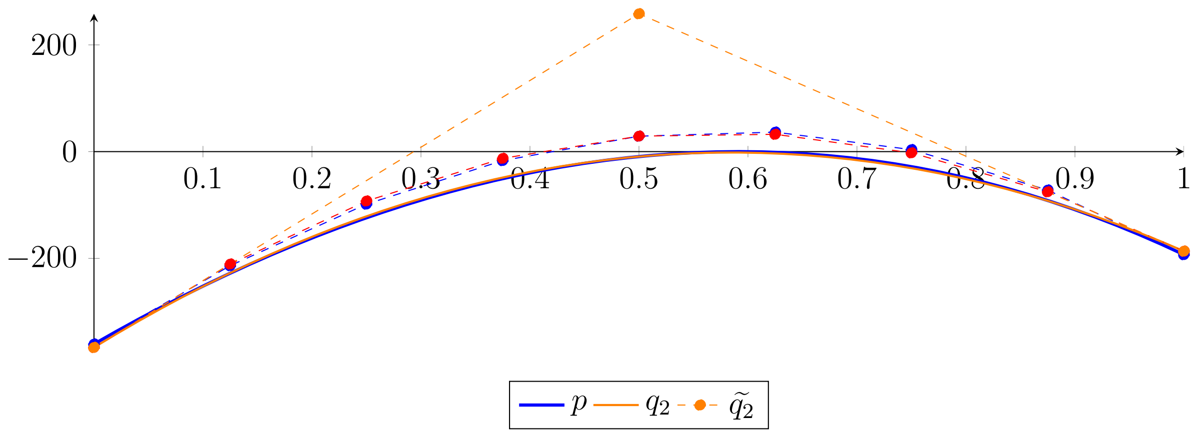
$$\begin{aligned}p &= -6.133566475094875\cdot 10^{-08}X^8 - 1.877794231289375\cdot 10^{-05}X^7 - 0.002379939031153127X^6\\&\quad - 0.1596298583101837X^5 - 5.958684697205818X^4 - 116.3336704708797X^3\\&\quad - 885.1606719876373X^2 + 1175.121582406799X - 360.5751654247078\\&= -360.5751654247078B_{0,8}(X) - 213.6849676238579B_{1,8}(X) - 98.40765096542357B_{2,8}(X)\\&\quad - 16.82060242209915B_{3,8}(X) + 28.91366696631814B_{4,8}(X) + 36.54467155974404B_{5,8}(X)\\&\quad + 3.731015586800578B_{6,8}(X) - 71.9626297312559B_{7,8}(X) - 193.0686388102505B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984\\&= -366.9199822310984B_{0,2} + 258.3517672110292B_{1,2} - 186.5418852681748B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 1.223655337688364\cdot 10^{-300}X^8 - 5.205587704887534\cdot 10^{-300}X^7 + 9.409306890019087\cdot 10^{-300}X^6\\&\quad - 9.282190734559383\cdot 10^{-300}X^5 + 5.21941805196838\cdot 10^{-300}X^4 - 1.541981168110934\\&\quad \cdot 10^{-300}X^3 - 1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984\\&= -366.9199822310984B_{0,8} - 210.6020448705665B_{1,8} - 92.50430043579644B_{2,8}\\&\quad - 12.62674892678823B_{3,8} + 29.03060965645814B_{4,8} + 32.46777531394266B_{5,8}\\&\quad - 2.315251954334665B_{6,8} - 75.31847214837383B_{7,8} - 186.5418852681748B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.

Bounding polynomials M and m:

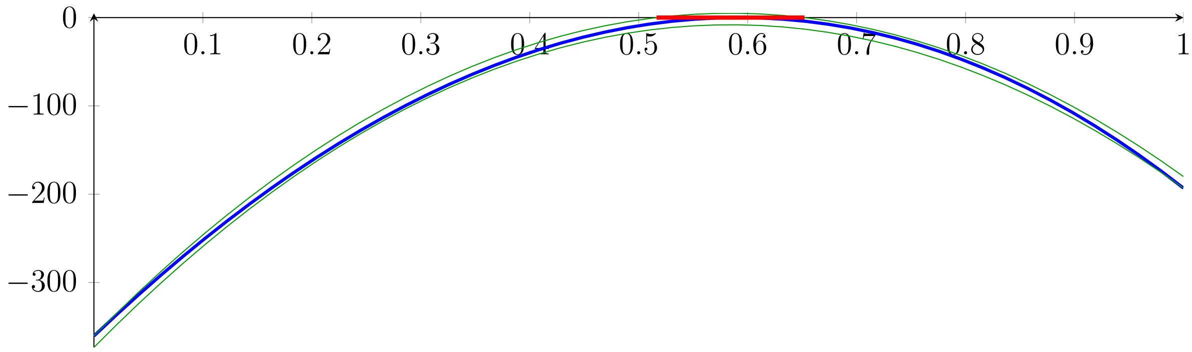
$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$

$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5163481231478299, 0.652203482649816]$$

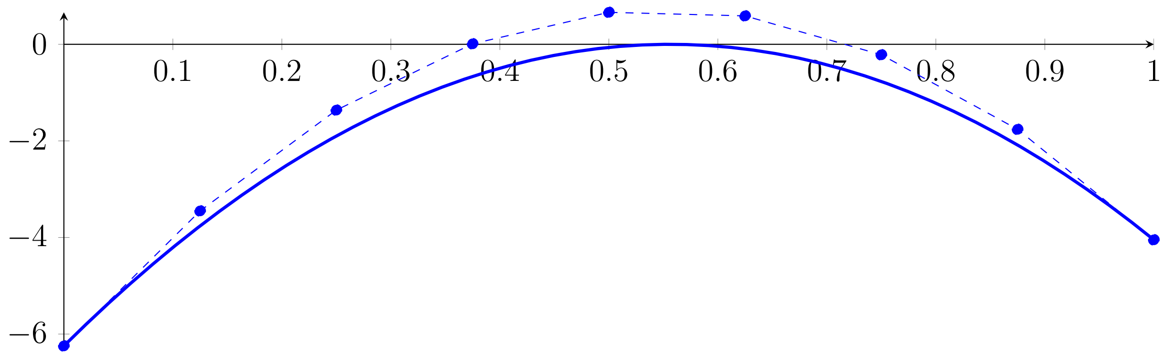
Longest intersection interval: 0.1358553595019861

$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

44.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.50756749315642$

Normalized monomial und Bézier representations and the Bézier polygon:

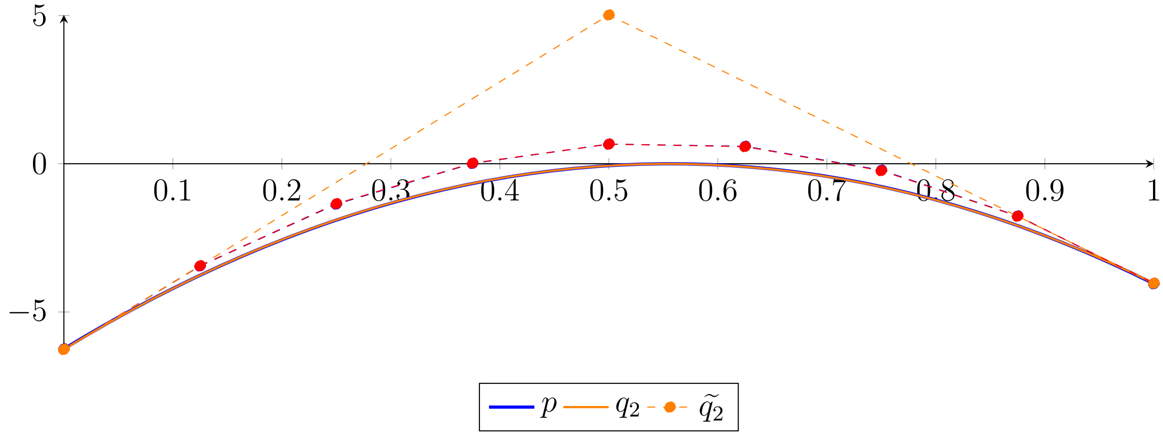
$$\begin{aligned} p = &-7.117496868854 28\cdot 10^{-15}X^8 - 1.625571369884896\cdot 10^{-11}X^7 - 1.539287417434116\cdot 10^{-08}X^6 \\ &- 7.733623372922873\cdot 10^{-06}X^5 - 0.002173482359120639X^4 - 0.3236423633498969X^3 \\ &- 19.84316397394663X^2 + 22.36613094108362X - 6.245496834711843 \\ = &-6.245496834711843B_{0,8}(X) - 3.449730467076391B_{1,8}(X) - 1.36264852708189B_{2,8}(X) \\ &+ 0.009969657354697074B_{3,8}(X) + 0.662313708568421B_{4,8}(X) + 0.5885420610459267B_{5,8}(X) \\ &- 0.217218177224708B_{6,8}(X) - 1.760871363955914B_{7,8}(X) - 4.048353462316386B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = {} & 2.334129178460356{\cdot}10^{-302}X^8 - 9.797943630323215{\cdot}10^{-302}X^7 + 1.744532667551292{\cdot}10^{-301}X^6 \\ & - 1.695264192146682{\cdot}10^{-301}X^5 + 9.412284928871313{\cdot}10^{-302}X^4 - 2.760729238908791 \\ & {\cdot}10^{-302}X^3 - 20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ = {} & -6.261866081804285B_{0,8} - 3.441577100979207B_{1,8} - 1.347444096092966B_{2,8} \\ & + 0.02053293285443696B_{3,8} + 0.6623539858630032B_{4,8} + 0.5780190629327322B_{5,8} \\ & - 0.232471835936376B_{6,8} - 1.769118710744321B_{7,8} - 4.031921561491104B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.
Bounding polynomials M and m:

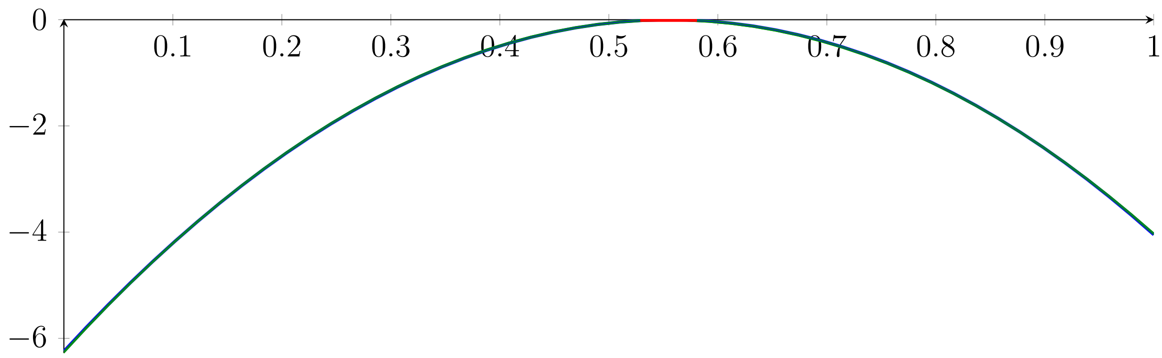
$$M = -20.33236732628744X^2 + 22.56231184660062X - 6.245434180979003$$

$$m = -20.33236732628744X^2 + 22.56231184660062X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

Longest intersection interval: 0.05205162765602134
$\Longrightarrow$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

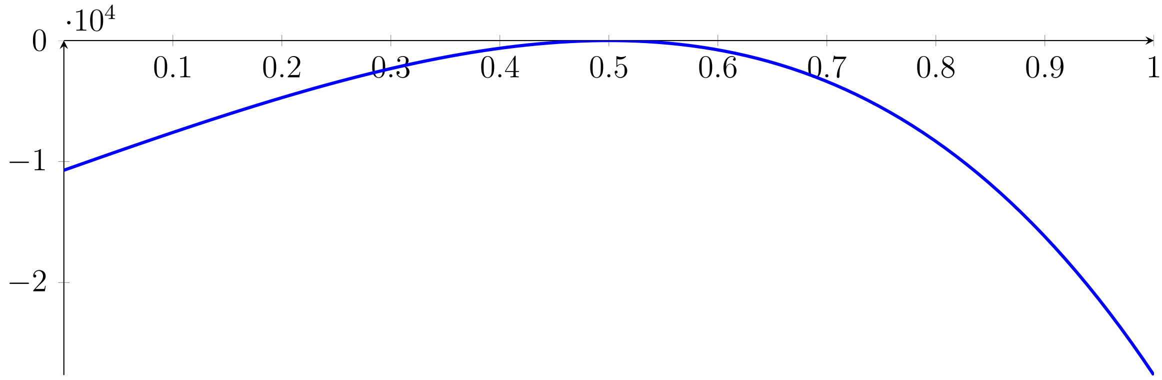
44.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Reached interval $[0.4995371223473506, 0.5004242277517611]$ **without sign change** at depth 5!
p(0) = -0.01503353559864481 - p(1) -0.01263551669178453

44.6 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

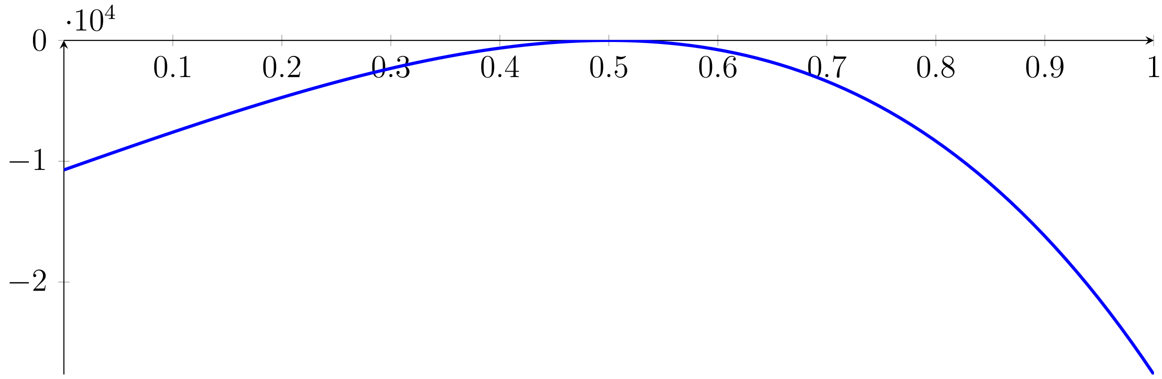
with precision $\varepsilon = 0.01$.

45 Running CubeClip on h_8 with epsilon 2

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called CubeClip with input polynomial on interval $[0, 1]$:

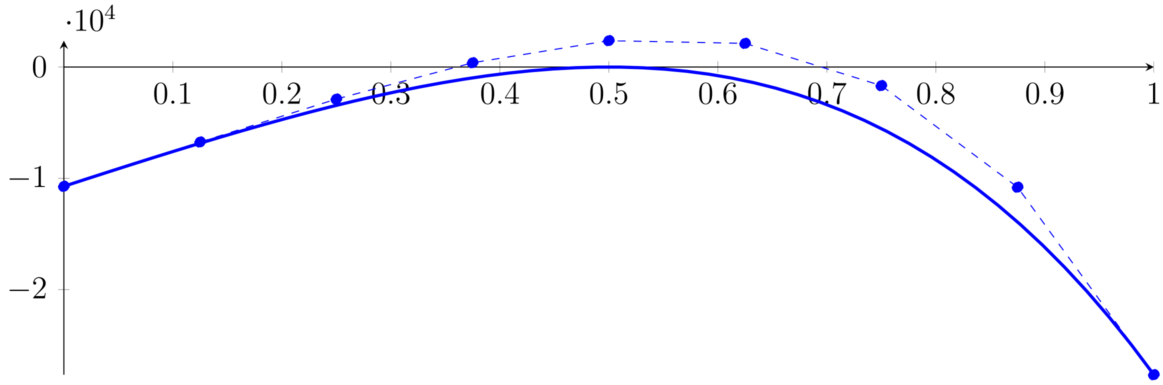
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



45.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

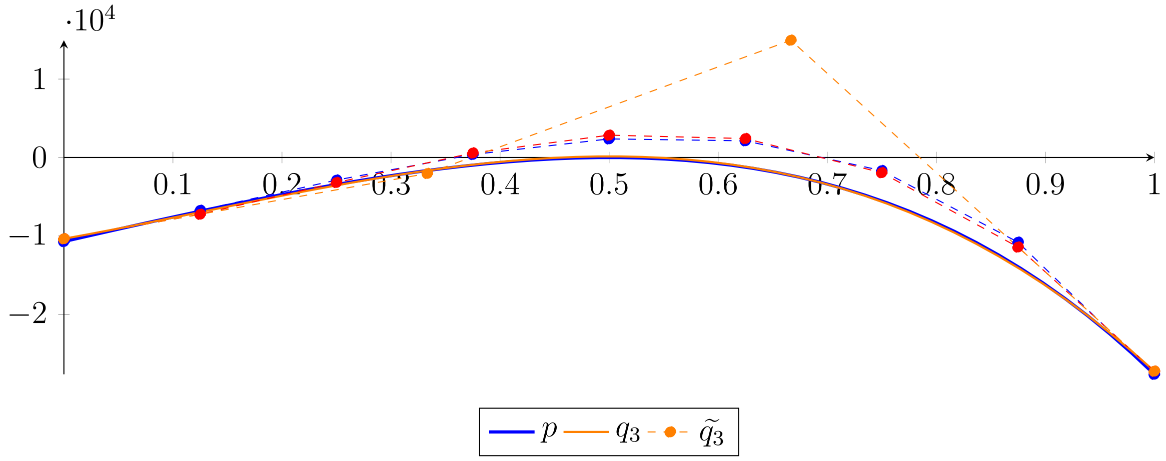
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ = &-10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &+ 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q_3} &= -5.703152099307112 \cdot 10^{-299} X^8 + 1.997854926822688 \cdot 10^{-298} X^7 - 2.620079279971631 \\
&\quad \cdot 10^{-298} X^6 + 1.512761996617561 \cdot 10^{-298} X^5 - 2.636105166170616 \cdot 10^{-299} X^4 \\
&\quad - 67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10364.86272786554 \\
&= -10364.86272786554 B_{0,8} - 7243.22568371353 B_{1,8} - 3192.719831416224 B_{2,8} \\
&\quad + 574.7343076805747 B_{3,8} + 2847.216212231063 B_{4,8} + 2412.80536088944 B_{5,8} \\
&\quad - 1940.418767690097 B_{6,8} - 11424.37669485335 B_{7,8} - 27250.98894194612 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

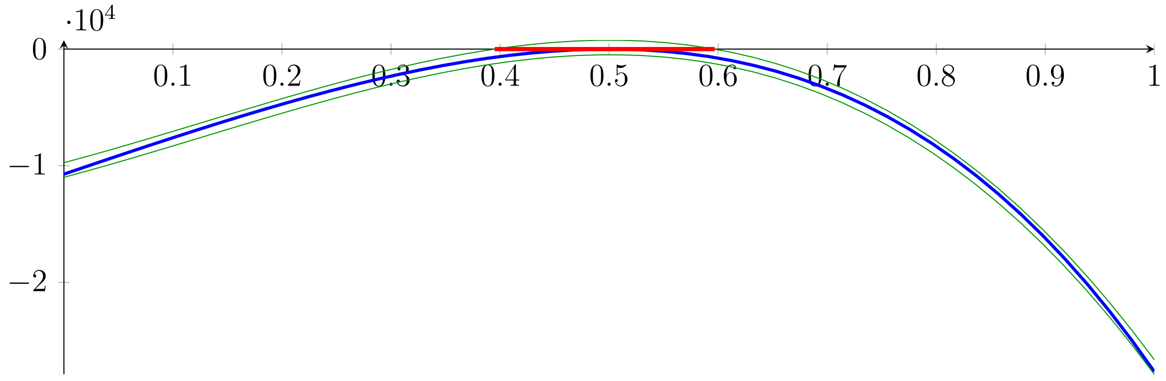
$$M = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 9740.484261812251$$

$$m = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $\quad N(m) = \{-0.623487965792376$

Intersection intervals:



$$[0.3950604114095972, 0.5968461976869074]$$

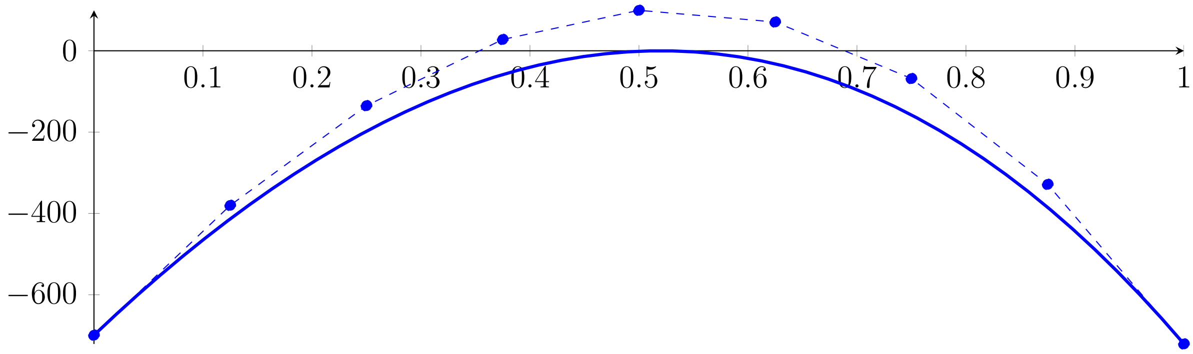
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

45.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

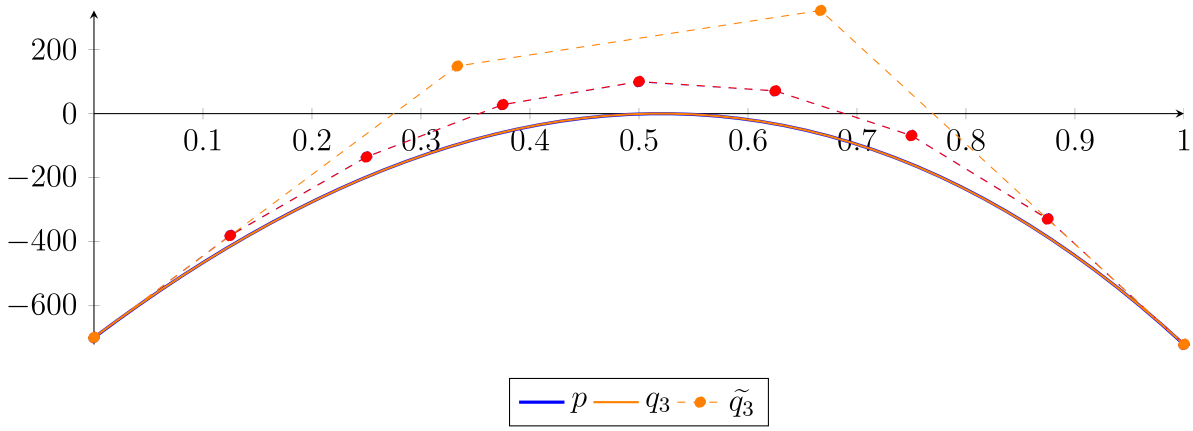
$$\begin{aligned}p = &-2.748682461629333\cdot 10^{-06}X^8 - 0.0005198138726602274X^7 - 0.04066637698425721X^6\\&- 1.681520506726176X^5 - 38.59639865464246X^4 - 460.2831920090805X^3\\&- 2074.780222477586X^2 + 2553.796807278985X - 699.3475151603227\\=&-699.3475151603227B_{0,8}(X) - 380.1229142504495B_{1,8}(X) - 134.9976070004902B_{2,8}(X)\\&+ 27.8090638751075B_{3,8}(X) + 99.52637853825791B_{4,8}(X) + 70.80221287533186B_{5,8}(X)\\&- 68.32844102535583B_{6,8}(X) - 328.4764717433994B_{7,8}(X) - 720.9332304689123B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\&= -698.7407882409323B_{0,3} + 148.5035278252892B_{1,3}\\&\quad + 322.0745267793334B_{2,3} - 720.3121660793493B_{3,3}\end{aligned}$$

$$\begin{aligned}\tilde{q}_3 &= -2.24002635483637\cdot 10^{-300}X^8 + 8.593407551274415\cdot 10^{-300}X^7 - 1.289726574039331\\&\quad \cdot 10^{-299}X^6 + 9.445830854385802\cdot 10^{-300}X^5 - 3.303106122422159\cdot 10^{-300}X^4\\&\quad - 542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\&= -698.7407882409323B_{0,8} - 381.0241697160992B_{1,8} - 135.4868351675709B_{2,8}\\&\quad + 28.18756585642868B_{3,8} + 100.3153838076753B_{4,8} + 71.21296913794494B_{5,8}\\&\quad - 68.80332770098655B_{6,8} - 329.4171562573433B_{7,8} - 720.3121660793493B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

Bounding polynomials M and m:

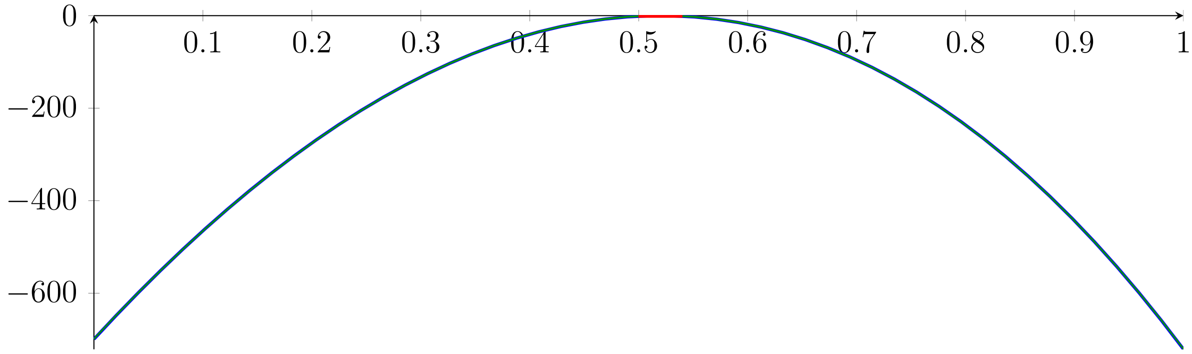
$$M = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 697.8001037269884$$

$$m = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 699.6814727548762$$

Root of M and m:

$$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\} \quad N(m) = \{-4.76690070753073\}$$

Intersection intervals:



$[0.4997746332555131, 0.5401382492675976]$

Longest intersection interval: 0.04036361601208447
$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

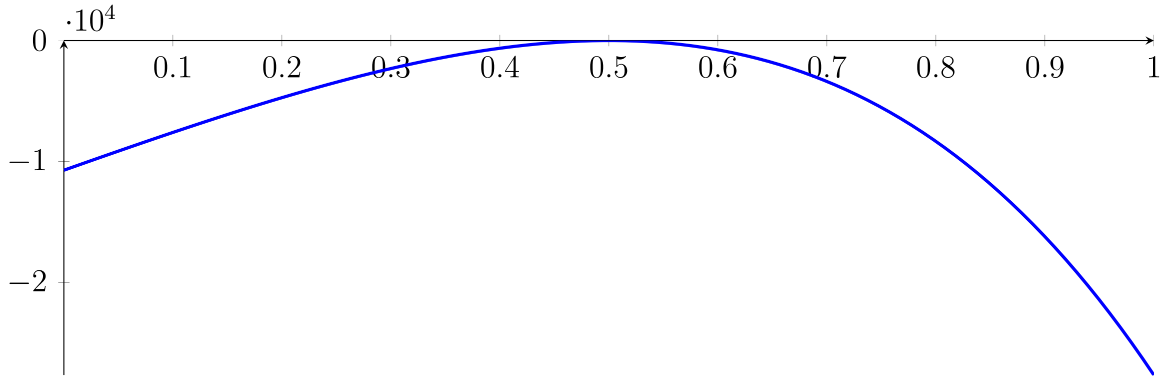
45.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Reached interval $[0.4959078287425153, 0.5040526327365092]$ **without sign change** at depth 3!
p(0) = -1.170857231885768 - p(1) -1.157189508205582

45.4 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

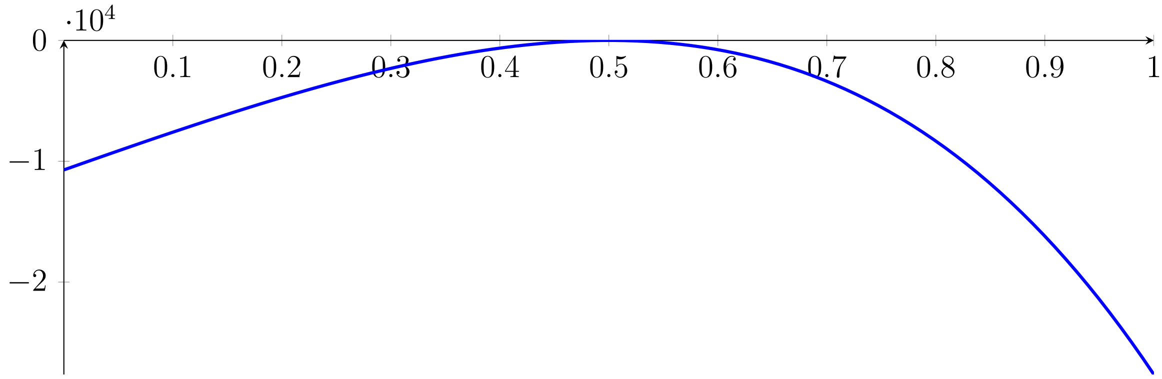
with precision $\varepsilon = 0.01$.

46 Running BezClip on h_8 with epsilon 4

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

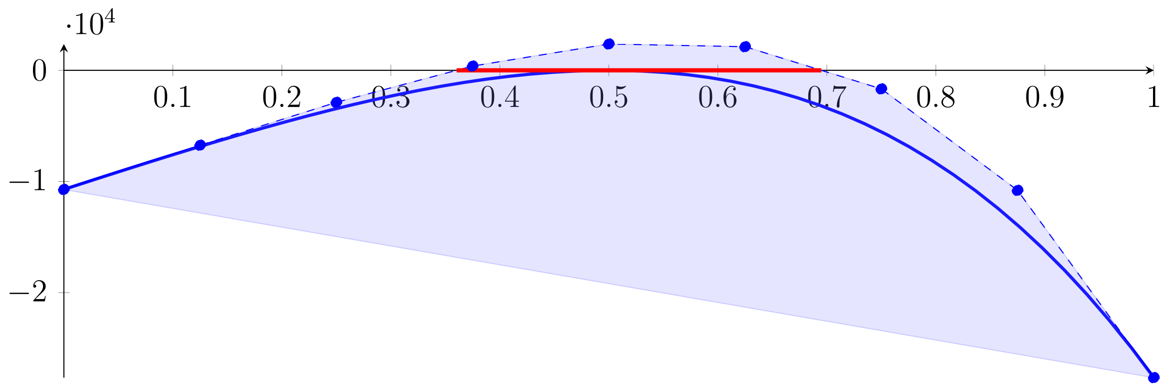
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



46.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

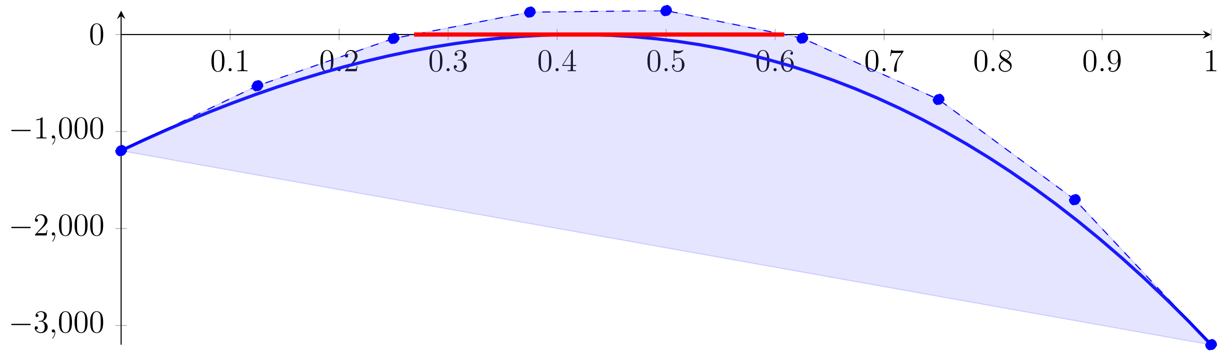
$$[0.3603640692588709, 0.6949437907012195]$$

Longest intersection interval: 0.3345797214423486
$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

46.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6\\&- 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3\\&- 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098\\= &-1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X)\\&+ 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X)\\&- 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

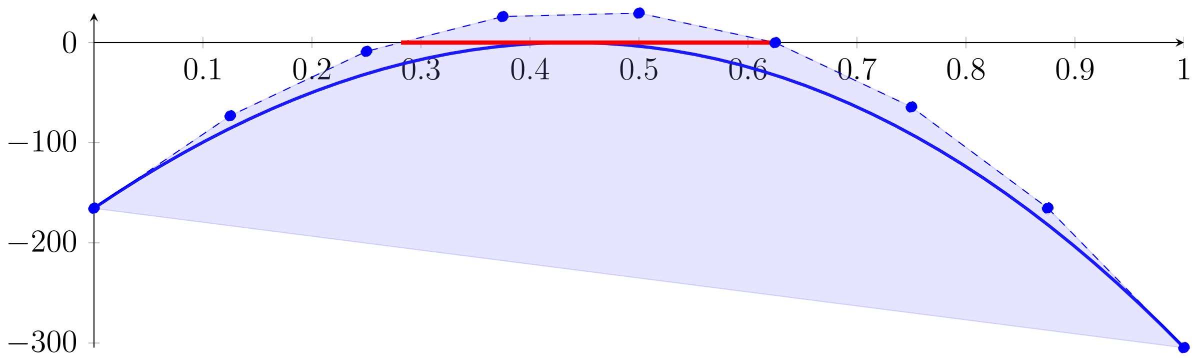
$$[0.2687909235445741, 0.6084084634355219]$$

Longest intersection interval: 0.3396175398909478
$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

46.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.779187295559603\cdot 10^{-08}X^8 - 9.441522673365855\cdot 10^{-06}X^7 - 0.001328615938017263X^6\\&- 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3\\&- 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987\\= &-165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X)\\&+ 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X)\\&- 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

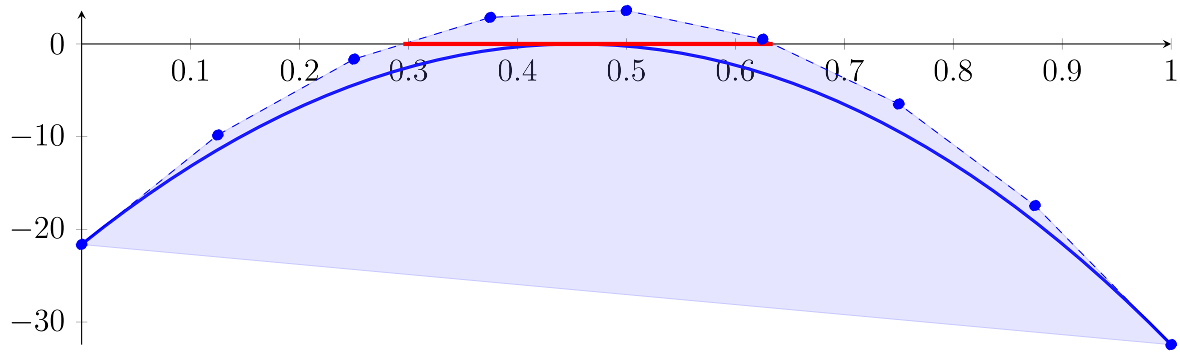
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\Longrightarrow$ Selective recursion: interval 1: [0.4822990094256734, 0.5212832145711177],

46.4 Recursion Branch 1 1 1 1 in Interval 1: [0.4822990094256734, 0.52128321457111

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -5.334693469973702\cdot 10^{-12}X^8 - 5.317476896558791\cdot 10^{-09}X^7 - 2.197128069269196\cdot 10^{-06}X^6\\&\quad - 0.000481521724173462X^5 - 0.05899466848037266X^4 - 3.82353929224187X^3\\&\quad - 101.3899724153744X^2 + 94.46052284159652X - 21.62667247491518\\&= -21.62667247491518B_{0,8}(X) - 9.819107119715613B_{1,8}(X) - 1.632612207922276B_{2,8}(X)\\&\quad + 2.86453477310337B_{3,8}(X) + 3.603213555021573B_{4,8}(X) + 0.5134524899120698B_{5,8}(X)\\&\quad - 6.475580126796985B_{6,8}(X) - 17.43558481253364B_{7,8}(X) - 32.43913973359036B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

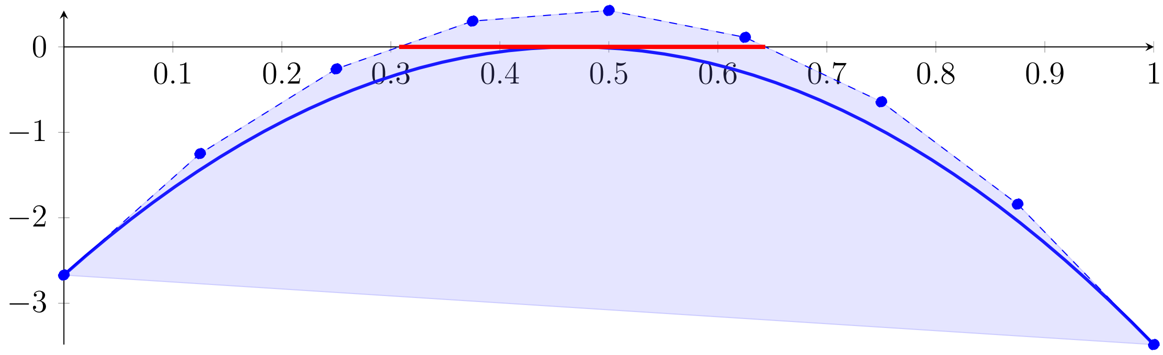
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\Longrightarrow$ Selective recursion: interval 1: [0.4938141292321744, 0.5070221367077007],

46.5 Recursion Branch 1 1 1 1 1 in Interval 1: [0.4938141292321744, 0.507022136707

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -9.261865042605128\cdot 10^{-16}X^8 - 2.731330938941274\cdot 10^{-12}X^7 - 3.339777336661422\\&\quad \cdot 10^{-09}X^6 - 2.167033369334019\cdot 10^{-06}X^5 - 0.0007867416616259877X^4\\&\quad - 0.1514273448103067X^3 - 12.0308548973337X^2 + 11.3691349552019X - 2.67015106585462\\&= -2.67015106585462B_{0,8}(X) - 1.249009196454382B_{1,8}(X) - 0.2575407162446335B_{2,8}(X)\\&\quad + 0.3015503150458701B_{3,8}(X) + 0.4255485985217787B_{4,8}(X) + 0.1117275574241232B_{5,8}(X)\\&\quad - 0.6426507016859872B_{6,8}(X) - 1.840335427863286B_{7,8}(X) - 3.484087264834231B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

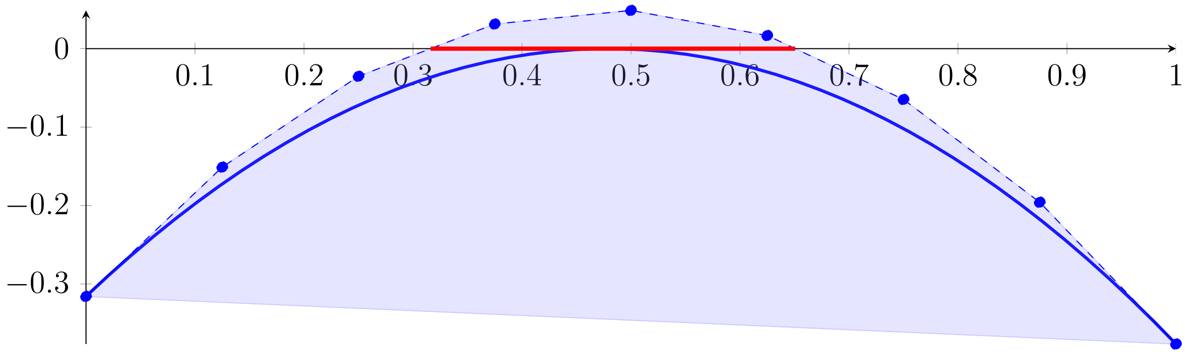
$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\implies$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

46.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.502170909909546 \cdot 10^{-19} X^8 - 1.319790000383349 \cdot 10^{-15} X^7 - 4.808366303277033 \cdot 10^{-12} X^6 \\
& - 9.297436410097437 \cdot 10^{-09} X^5 - 1.006192361000331 \cdot 10^{-05} X^4 - 0.005777437133356019 X^3 \\
& - 1.373512346650201 X^2 + 1.318590421802199 X - 0.3158295498425207 \\
= & -0.3158295498425207 B_{0,8}(X) - 0.1510057471172458 B_{1,8}(X) - 0.03523595677233526 B_{2,8}(X) \\
& + 0.03137665267197247 B_{3,8}(X) + 0.04872876895367301 B_{4,8}(X) + 0.01671693590297049 B_{5,8}(X) \\
& - 0.06476244672391982 B_{6,8}(X) - 0.1958131234111408 B_{7,8}(X) - 0.376538983049735 B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3161210337394805, 0.6506459600024226\}$$

Intersection intervals with the x axis:

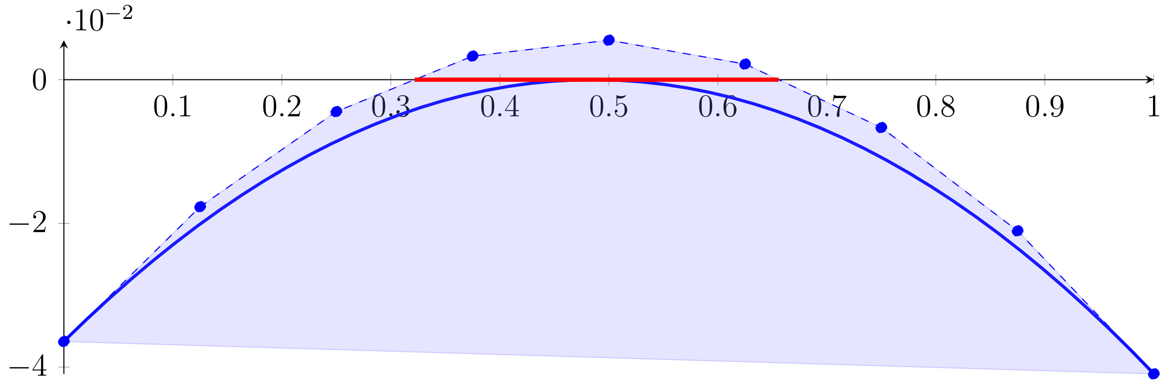
$$[0.3161210337394805, 0.6506459600024226]$$

Longest intersection interval: 0.3345249262629421
$\implies$ Selective recursion: interval 1: $[0.499279281802493, 0.5007635705740208]$,

46.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.499279281802493, 0.500763570$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -2.355847730899963 \cdot 10^{-23} X^8 - 6.189124375986835 \cdot 10^{-19} X^7 - 6.742674354644286 \cdot 10^{-15} X^6 \\
& - 3.898805488577673 \cdot 10^{-11} X^5 - 1.261912061383275 \cdot 10^{-07} X^4 - 0.000216758797755982 X^3 \\
& - 0.154319370565961 X^2 + 0.1500226550818807 X - 0.0364365303427455 \\
= & -0.0364365303427455 B_{0,8}(X) - 0.01768369845751041 B_{1,8}(X) - 0.004442272663916792 B_{2,8}(X) \\
& + 0.003283876345218297 B_{3,8}(X) + 0.005490876074346265 B_{4,8}(X) \\
& + 0.002174852224490793 B_{5,8}(X) - 0.006668071307448625 B_{6,8}(X) \\
& - 0.02104177242939338 B_{7,8}(X) - 0.04095013085478269 B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3218707447051634, 0.6557428337562166\}$$

Intersection intervals with the x axis:

$$[0.3218707447051634, 0.6557428337562166]$$

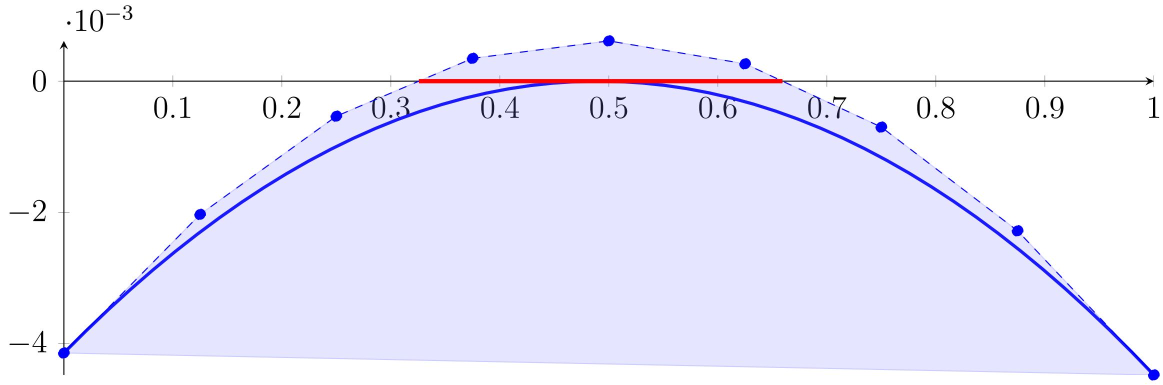
Longest intersection interval: 0.3338720890510532

$\Longrightarrow$ Selective recursion: interval 1: $[0.4997570309347421, 0.5002525935276471]$,

46.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4997570309347421, 0.5002525$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -3.637375463877353 \cdot 10^{-27} X^8 - 2.862414855005092 \cdot 10^{-22} X^7 - 9.341200314035287 \cdot 10^{-18} X^6 \\
& - 1.617995026179791 \cdot 10^{-13} X^5 - 1.568792391901099 \cdot 10^{-09} X^4 - 8.073141350089629 \\
& \cdot 10^{-06} X^3 - 0.01722540857155108 X^2 + 0.01689843085777645 X - 0.004143462623424353 \\
= & -0.004143462623424353 B_{0,8}(X) - 0.002031158766202297 B_{1,8}(X) \\
& - 0.0005340480722499233 B_{2,8}(X) + 0.0003477252951943749 B_{3,8}(X) \\
& + 0.0006140171504808826 B_{4,8}(X) + 0.0002646832855456765 B_{5,8}(X) \\
& - 0.0007004205300922658 B_{6,8}(X) - 0.002281438549333955 B_{7,8}(X) \\
& - 0.004478515047503282 B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3257065380923488, 0.6592817116222256\}$$

Intersection intervals with the x axis:

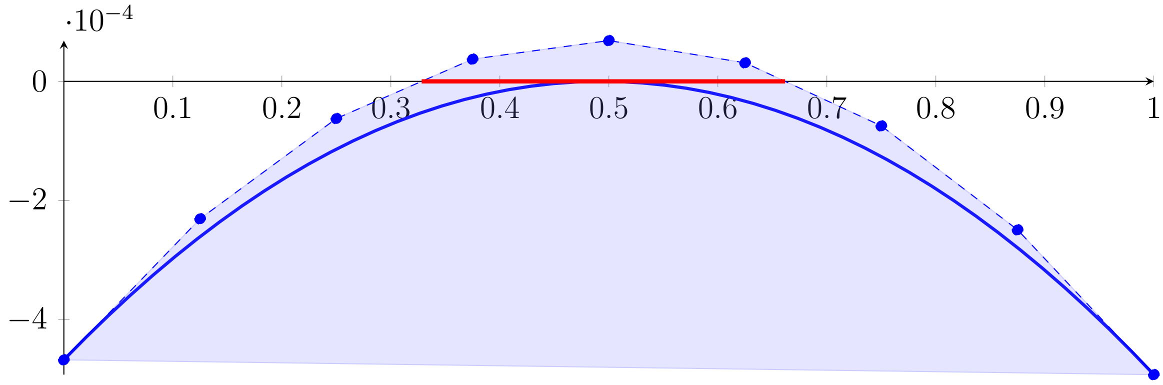
$$[0.3257065380923488, 0.6592817116222256]$$

Longest intersection interval: 0.3335751735298769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999184389112853, 0.5000837462892085]$,

46.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999184389112853, 0.50008$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-5.57619429667528\cdot 10^{-31}X^8 - 1.315536799505273\cdot 10^{-25}X^7 - 1.287049795843581\cdot 10^{-20}X^6\\ &- 6.683358895768893\cdot 10^{-16}X^5 - 1.942733814710911\cdot 10^{-11}X^4 - 2.997323807488328\\ &\cdot 10^{-07}X^3 - 0.001917590365692151X^2 + 0.001893040758741533X - 0.0004671653183669499\\ = &-0.0004671653183669499B_{0,8}(X) - 0.0002305352235242582B_{1,8}(X) - 6.239049888485767\\ &\cdot 10^{-05}B_{2,8}(X) + 3.726350318730984\cdot 10^{-05}B_{3,8}(X) + 6.842143005076897\\ &\cdot 10^{-05}B_{4,8}(X) + 3.107792878649904\cdot 10^{-05}B_{5,8}(X) - 7.47723538020779\\ &\cdot 10^{-05}B_{6,8}(X) - 0.000249134771189109B_{7,8}(X) - 0.0004920146771263226B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3282588977707033, 0.661700337526842\}$$

Intersection intervals with the x axis:

$$[0.3282588977707033, 0.661700337526842]$$

Longest intersection interval: 0.3334414397561387
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999727025289557, 0.5000278228590528]$,

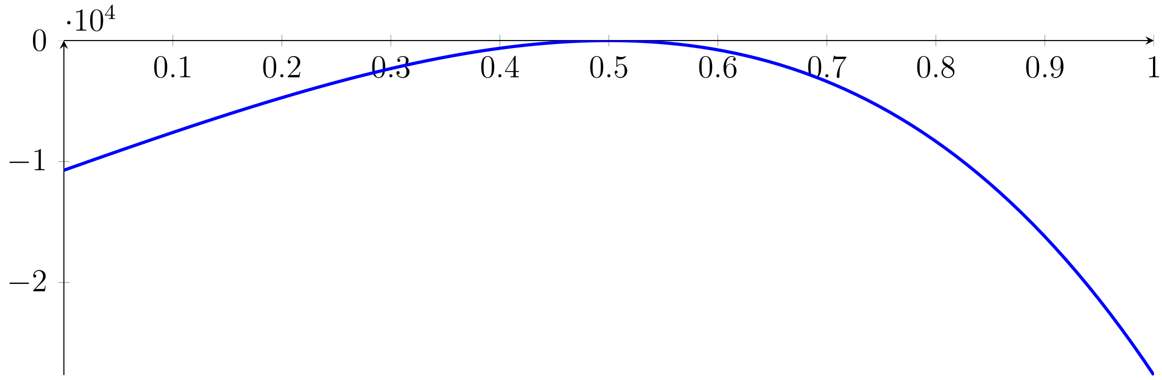
46.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999727025289557, 0.500$

Found root in interval $[0.4999727025289557, 0.5000278228590528]$ at recursion depth 10!

46.11 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.4999727025289557, 0.5000278228590528]$$

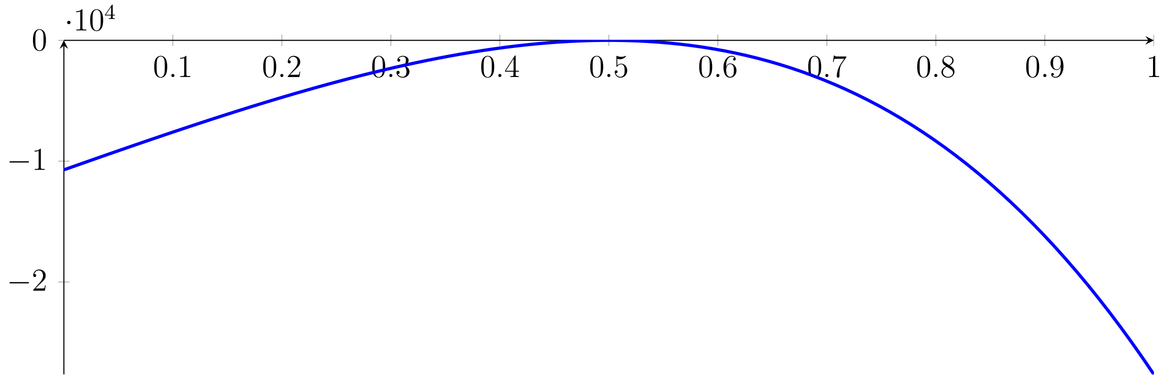
with precision $\varepsilon = 0.0001$.

47 Running QuadClip on h_8 with epsilon 4

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called QuadClip with input polynomial on interval $[0, 1]$:

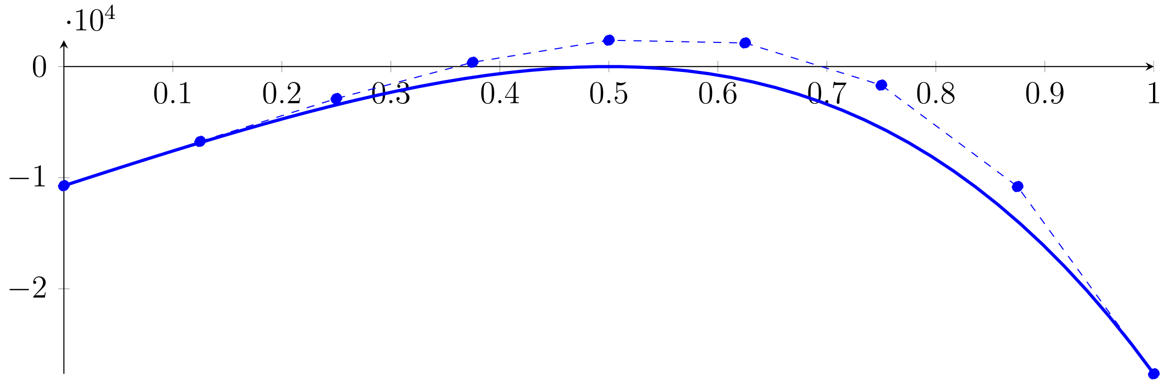
$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



47.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

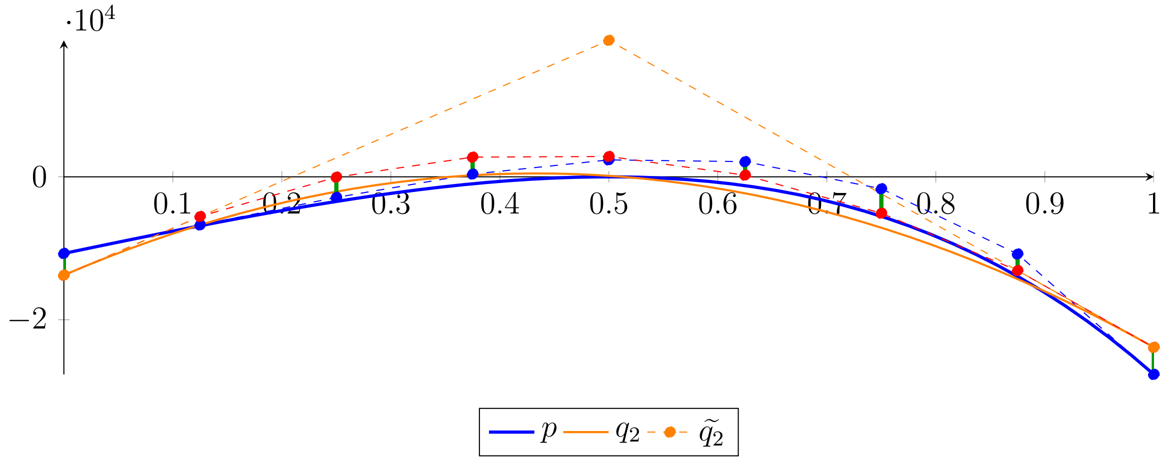
$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 8.415084934481114 \cdot 10^{-299}X^8 - 3.337656926892332 \cdot 10^{-298}X^7 + 5.61048922956057 \cdot 10^{-298}X^6 \\ &\quad - 5.189733160056009 \cdot 10^{-298}X^5 + 2.787623795639522 \cdot 10^{-298}X^4 - 8.054794139884735 \\ &\quad \cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8} \\ &\quad + 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8} \\ &\quad - 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.
Bounding polynomials M and m:

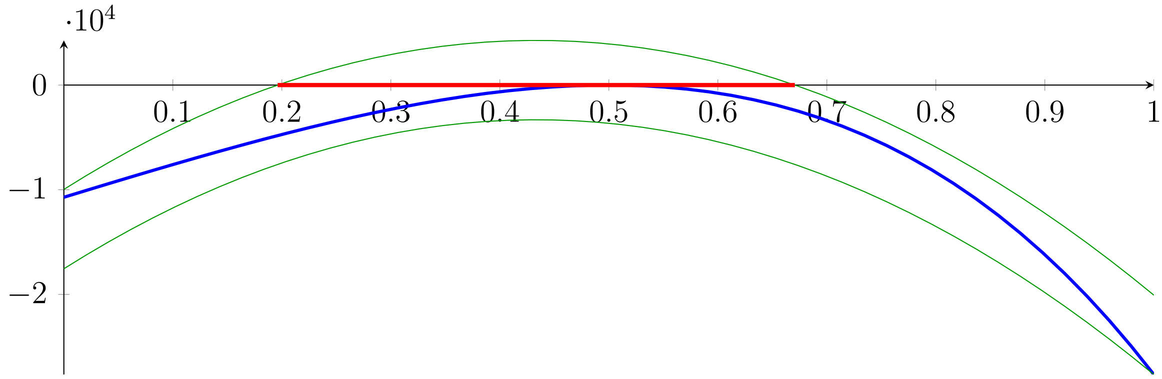
$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$

$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



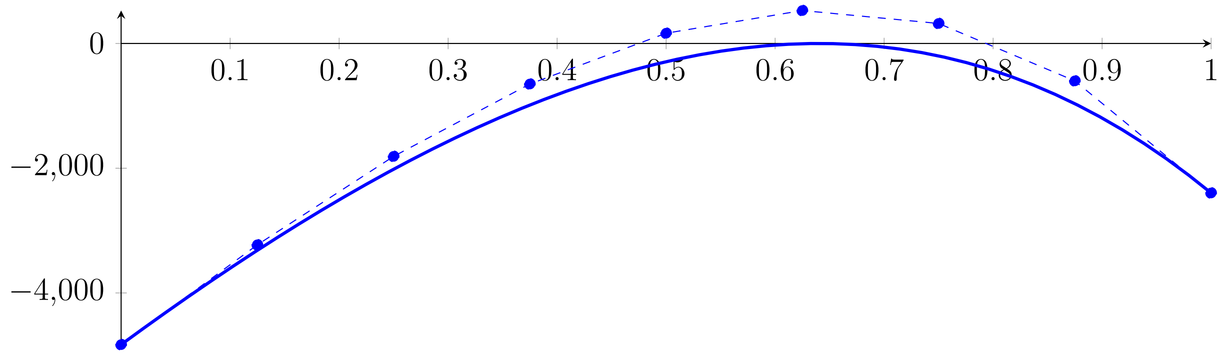
$$[0.196099166583006, 0.6706514347613278]$$

Longest intersection interval: 0.4745522681783218
$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

47.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

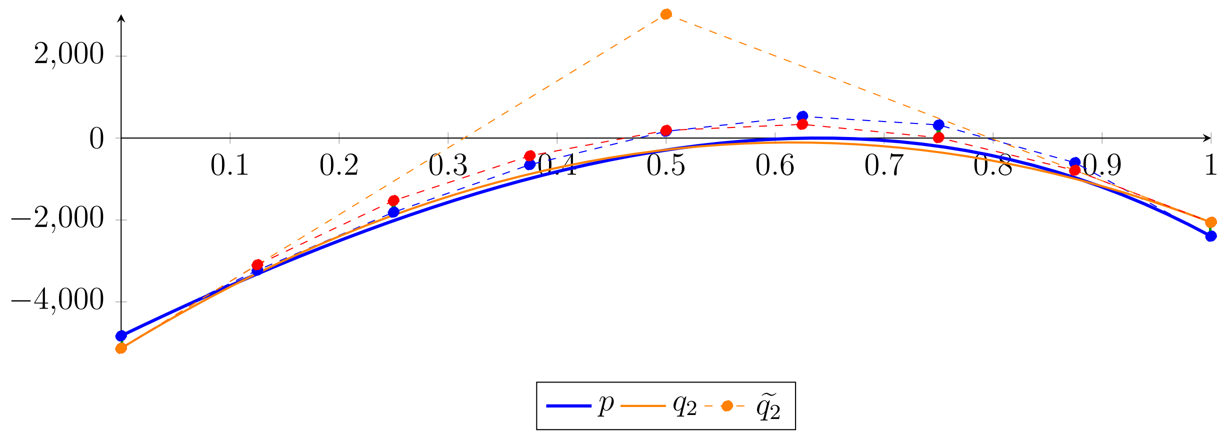
$$\begin{aligned} p = &-0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6 \\ &- 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3 \\ &- 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684 \\ = &-4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X) \\ &- 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X) \\ &+ 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\
&= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 &= 1.476670942620757\cdot 10^{-299}X^8 - 6.381627458836932\cdot 10^{-299}X^7 + 1.1779791924066\cdot 10^{-298}X^6 \\
&\quad - 1.190713137013755\cdot 10^{-298}X^5 + 6.850874958113058\cdot 10^{-299}X^4 - 2.05172173630317 \\
&\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\
&= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\
&\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\
&\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

Bounding polynomials M and m:

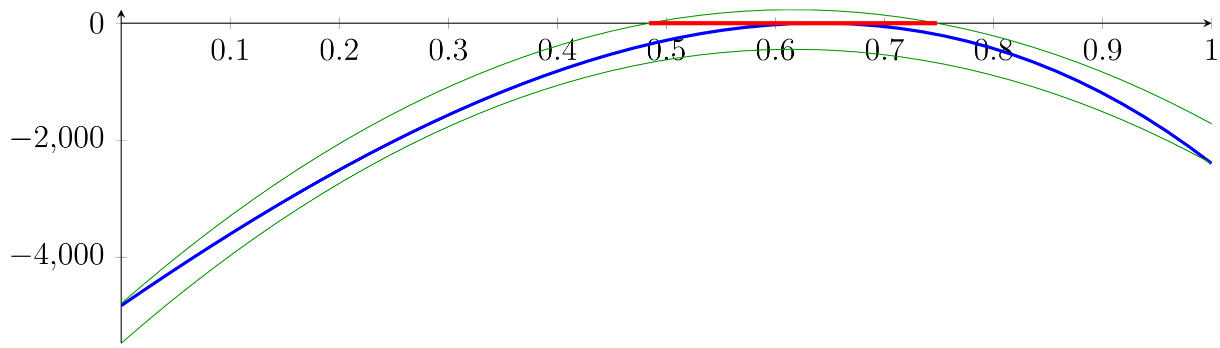
$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$

$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.4839311609916354, 0.7482816274420809]$$

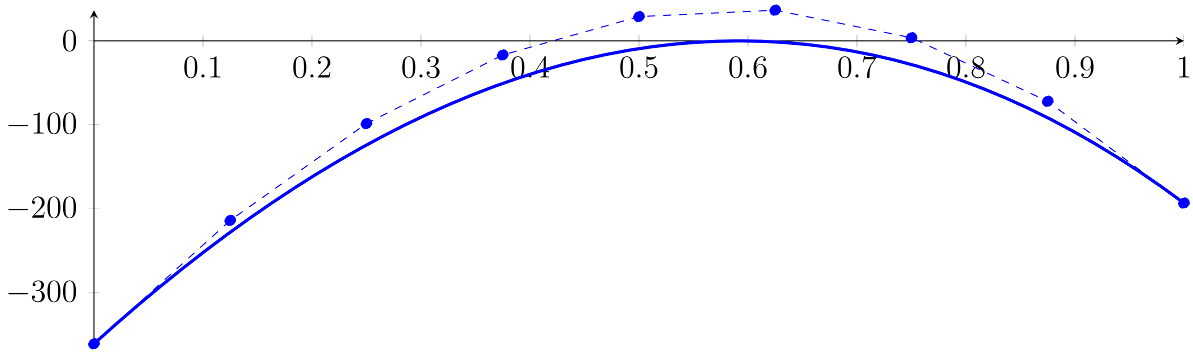
Longest intersection interval: 0.2643504664504455

$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

47.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.5511979101218114]$

Normalized monomial und Bézier representations and the Bézier polygon:

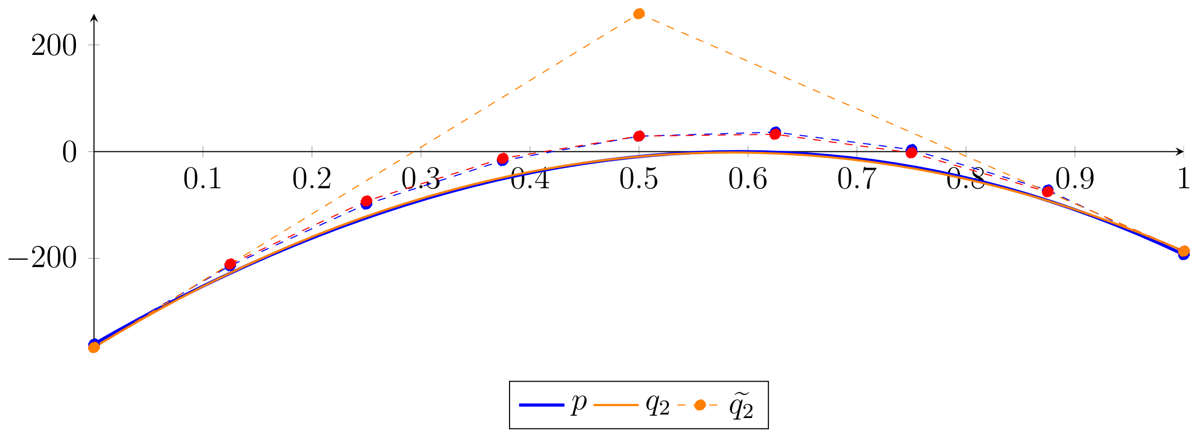
$$\begin{aligned}
p &= -6.133566475094875\cdot 10^{-08}X^8 - 1.877794231289375\cdot 10^{-05}X^7 - 0.002379939031153127X^6 \\
&\quad - 0.1596298583101837X^5 - 5.958684697205818X^4 - 116.3336704708797X^3 \\
&\quad - 885.1606719876373X^2 + 1175.121582406799X - 360.5751654247078 \\
&= -360.5751654247078B_{0,8}(X) - 213.6849676238579B_{1,8}(X) - 98.40765096542357B_{2,8}(X) \\
&\quad - 16.82060242209915B_{3,8}(X) + 28.91366696631814B_{4,8}(X) + 36.54467155974404B_{5,8}(X) \\
&\quad + 3.731015586800578B_{6,8}(X) - 71.9626297312559B_{7,8}(X) - 193.0686388102505B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= -1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984 \\
&= -366.9199822310984B_{0,2} + 258.3517672110292B_{1,2} - 186.5418852681748B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 &= 1.223655337688364\cdot 10^{-300}X^8 - 5.205587704887534\cdot 10^{-300}X^7 + 9.409306890019087\cdot 10^{-300}X^6 \\
&\quad - 9.282190734559383\cdot 10^{-300}X^5 + 5.21941805196838\cdot 10^{-300}X^4 - 1.541981168110934 \\
&\quad \cdot 10^{-300}X^3 - 1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984 \\
&= -366.9199822310984B_{0,8} - 210.6020448705665B_{1,8} - 92.50430043579644B_{2,8} \\
&\quad - 12.62674892678823B_{3,8} + 29.03060965645814B_{4,8} + 32.46777531394266B_{5,8} \\
&\quad - 2.315251954334665B_{6,8} - 75.31847214837383B_{7,8} - 186.5418852681748B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.
Bounding polynomials M and m:

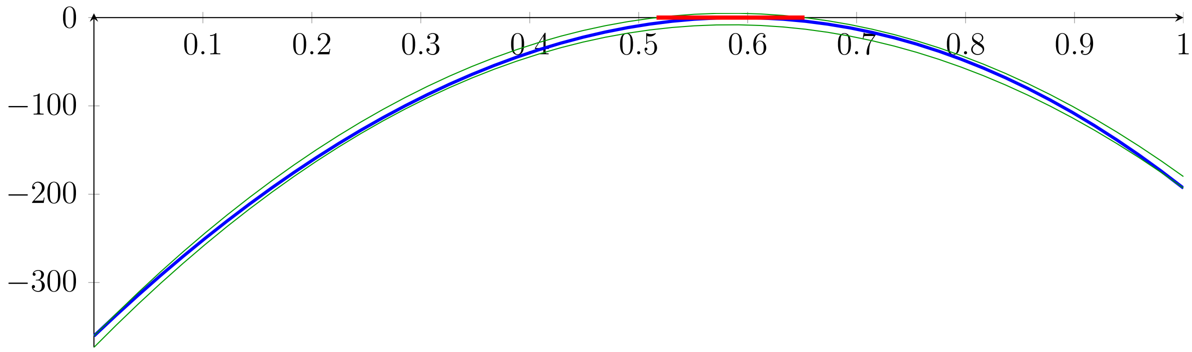
$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$

$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



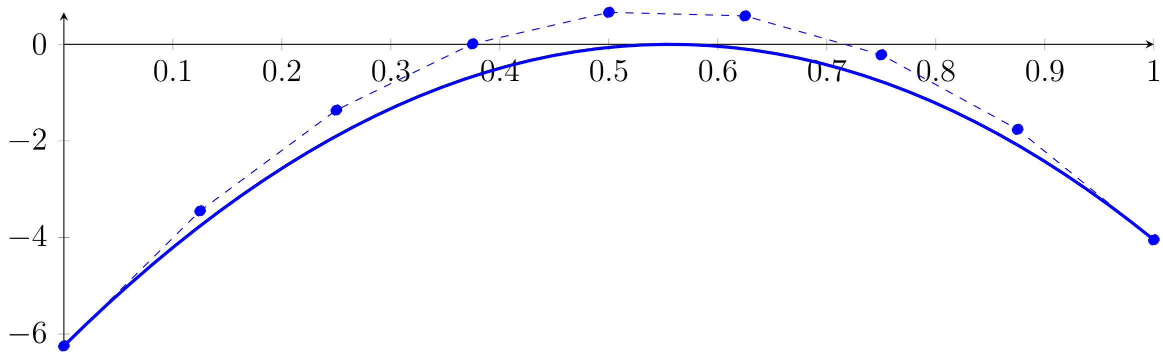
$$[0.5163481231478299, 0.652203482649816]$$

Longest intersection interval: 0.1358553595019861
$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

47.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.507567493156426$

Normalized monomial und Bézier representations and the Bézier polygon:

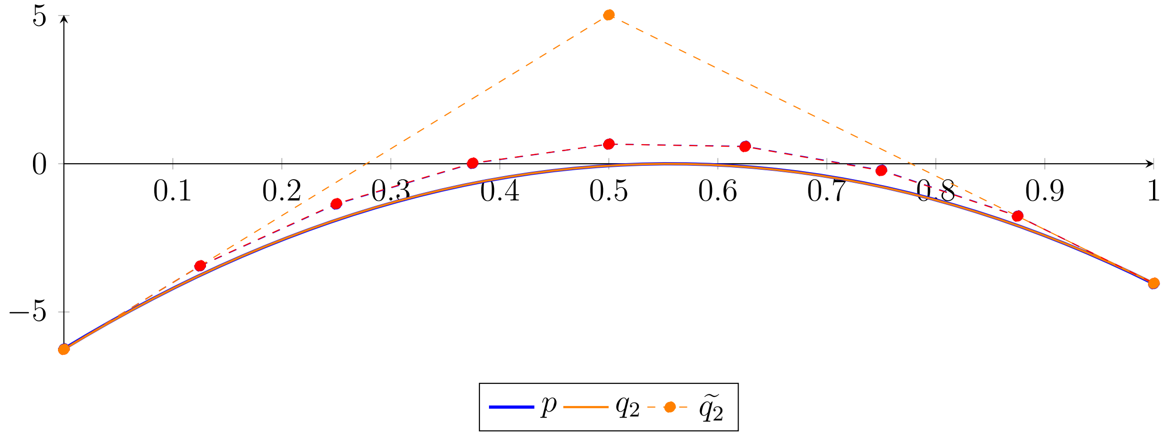
$$\begin{aligned}p = &-7.117496868854 28\cdot 10^{-15}X^8 - 1.625571369884896\cdot 10^{-11}X^7 - 1.539287417434116\cdot 10^{-08}X^6\\&- 7.733623372922873\cdot 10^{-06}X^5 - 0.002173482359120639X^4 - 0.3236423633498969X^3\\&- 19.84316397394663X^2 + 22.36613094108362X - 6.245496834711843\\= &-6.245496834711843B_{0,8}(X) - 3.449730467076391B_{1,8}(X) - 1.36264852708189B_{2,8}(X)\\&+ 0.009969657354697074B_{3,8}(X) + 0.662313708568421B_{4,8}(X) + 0.5885420610459267B_{5,8}(X)\\&- 0.217218177224708B_{6,8}(X) - 1.760871363955914B_{7,8}(X) - 4.048353462316386B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285\\&= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2}\end{aligned}$$

$$\widetilde{q_2} = 2.334129178460356 \cdot 10^{-302} X^8 - 9.797943630323215 \cdot 10^{-302} X^7 + 1.744532667551292 \cdot 10^{-301} X^6 - 1.695264192146682 \cdot 10^{-301} X^5 + 9.412284928871313 \cdot 10^{-302} X^4 - 2.760729238908791 \cdot 10^{-302} X^3 - 20.33236732628744 X^2 + 22.56231184660062 X - 6.261866081804285$$
$$= -6.261866081804285 B_{0,8} - 3.441577100979207 B_{1,8} - 1.347444096092966 B_{2,8} + 0.02053293285443696 B_{3,8} + 0.6623539858630032 B_{4,8} + 0.5780190629327322 B_{5,8} - 0.232471835936376 B_{6,8} - 1.769118710744321 B_{7,8} - 4.031921561491104 B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.

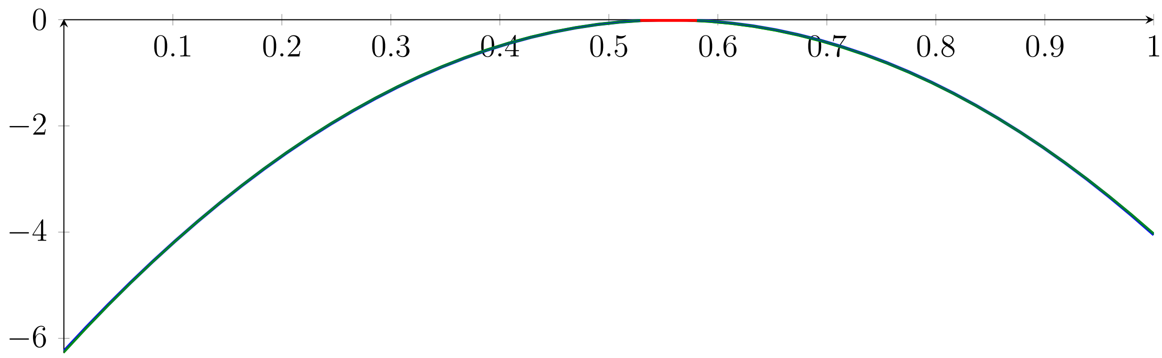
Bounding polynomials M and m:

$$M = -20.33236732628744 X^2 + 22.56231184660062 X - 6.245434180979003$$
$$m = -20.33236732628744 X^2 + 22.56231184660062 X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

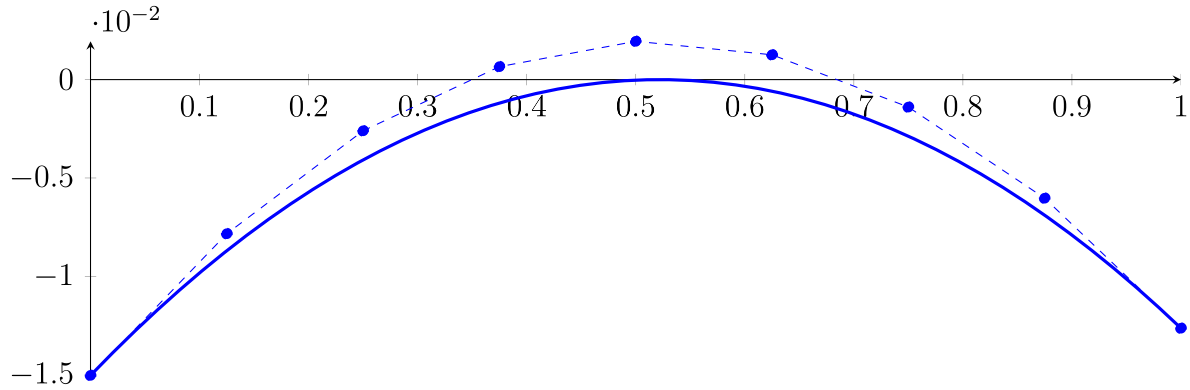
Longest intersection interval: 0.05205162765602134

$\Longrightarrow$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

47.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Normalized monomial und Bézier representations and the Bézier polygon:

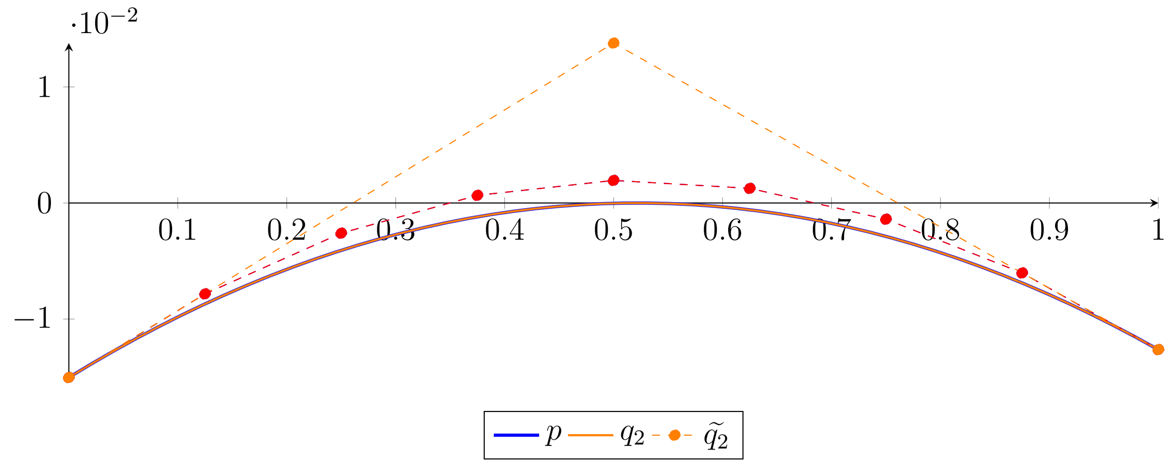
$$\begin{aligned}p &= -3.835321724591002\cdot10^{-25}X^8 - 1.685970393664051\cdot10^{-20}X^7 - 3.073417757909172\cdot10^{-16}X^6\\&\quad - 2.973678167232232\cdot10^{-12}X^5 - 1.610545215791654\cdot10^{-08}X^4 - 4.629380451575473\\&\quad \cdot10^{-05}X^3 - 0.05516351610802166X^2 + 0.05760784492782384X - 0.01503353559864481\\&= -0.01503353559864481B_{0,8}(X) - 0.007832554982666833B_{1,8}(X) - 0.002601699941975341B_{2,8}(X)\\&\quad + 0.0006582028483490249B_{3,8}(X) + 0.001946326483147738B_{4,8}(X)\\&\quad + 0.001261843827131283B_{5,8}(X) - 0.001396072485173959B_{6,8}(X)\\&\quad - 0.006028250049478829B_{7,8}(X) - 0.01263551669178453B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657\\&= -0.01503585166965657B_{0,2} + 0.01378196629969571B_{1,2} - 0.01263320016040455B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 6.306459041947834\cdot10^{-305}X^8 - 2.608238530297949\cdot10^{-304}X^7 + 4.573872253581115\cdot10^{-304}X^6\\&\quad - 4.384946099604334\cdot10^{-304}X^5 + 2.410615466544519\cdot10^{-304}X^4 - 7.036239809064398\\&\quad \cdot10^{-305}X^3 - 0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657\\&= -0.01503585166965657B_{0,8} - 0.007831397177318504B_{1,8} - 0.002599549271746596B_{2,8}\\&\quad + 0.0006596920470591496B_{3,8} + 0.001946326779098733B_{4,8} + 0.001260354924372155B_{5,8}\\&\quad - 0.001398223517120585B_{6,8} - 0.006029408545379487B_{7,8} - 0.01263320016040455B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.316531379978639\cdot10^{-06}$.
Bounding polynomials M and m:

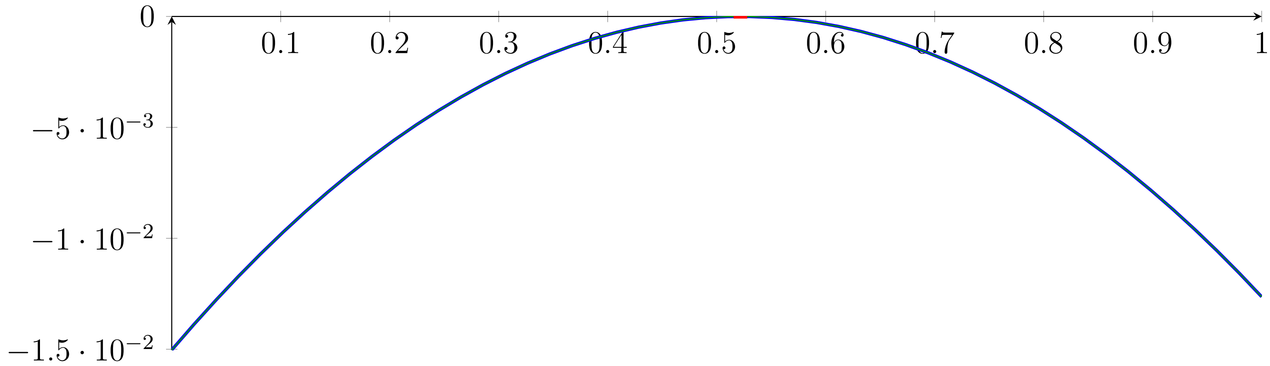
$$M = -0.05523298442945254X^2 + 0.05763563593870456X - 0.0150335351382766$$

$$m = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503816820103655$$

Root of M and m:

$$N(M) = \{0.5154882826278122, 0.5280120194846123\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5154882826278122, 0.5280120194846123]$$

Longest intersection interval: 0.012523736856800011
$\implies$ Selective recursion: interval 1: $[0.4999944147887801, 0.5000055246634291]$,

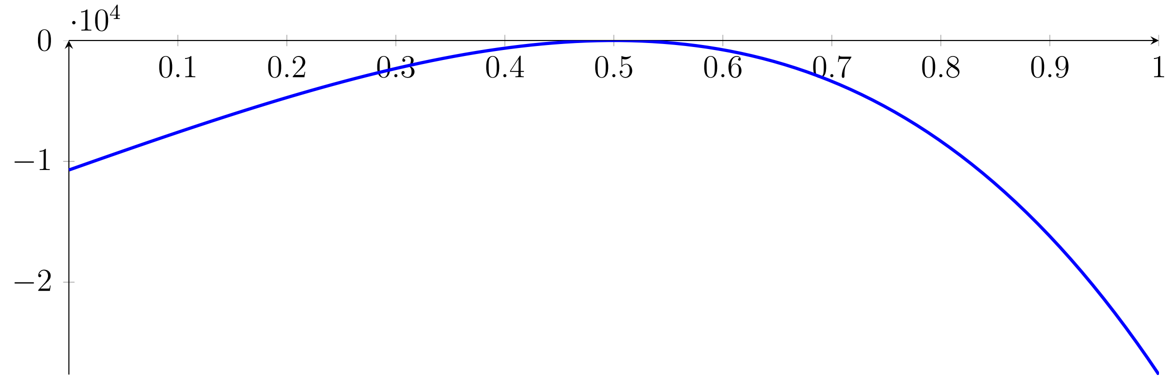
47.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4999944147887801, 0.50000552466$

Found root in interval $[0.4999944147887801, 0.5000055246634291]$ at recursion depth 6!

47.7 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.4999944147887801, 0.5000055246634291]$$

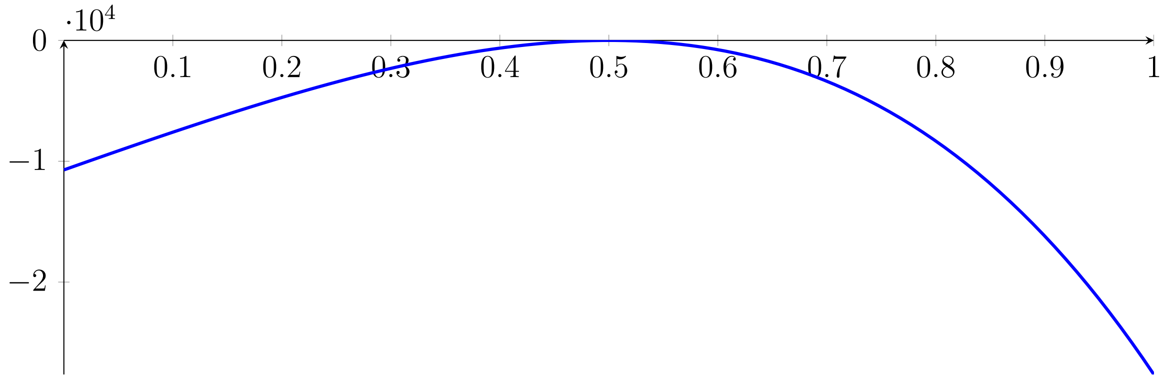
with precision $\varepsilon = 0.0001$.

48 Running `CubeClip` on h_8 with epsilon 4

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

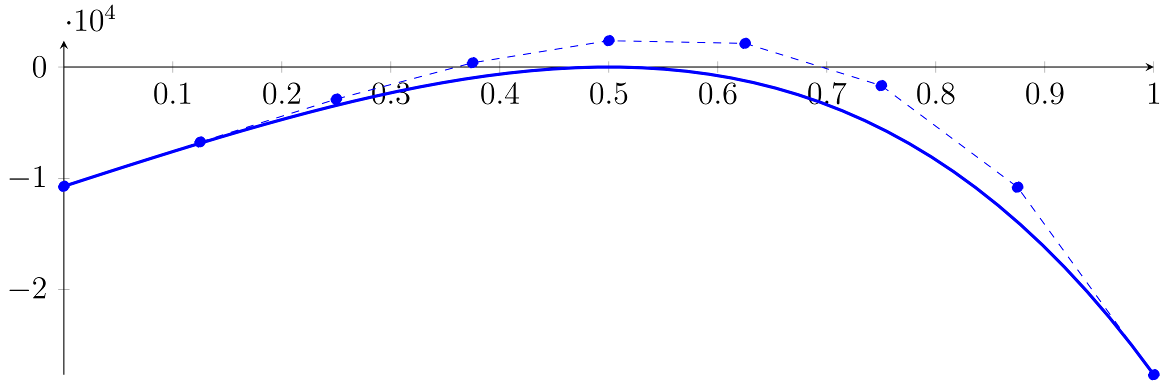
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



48.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

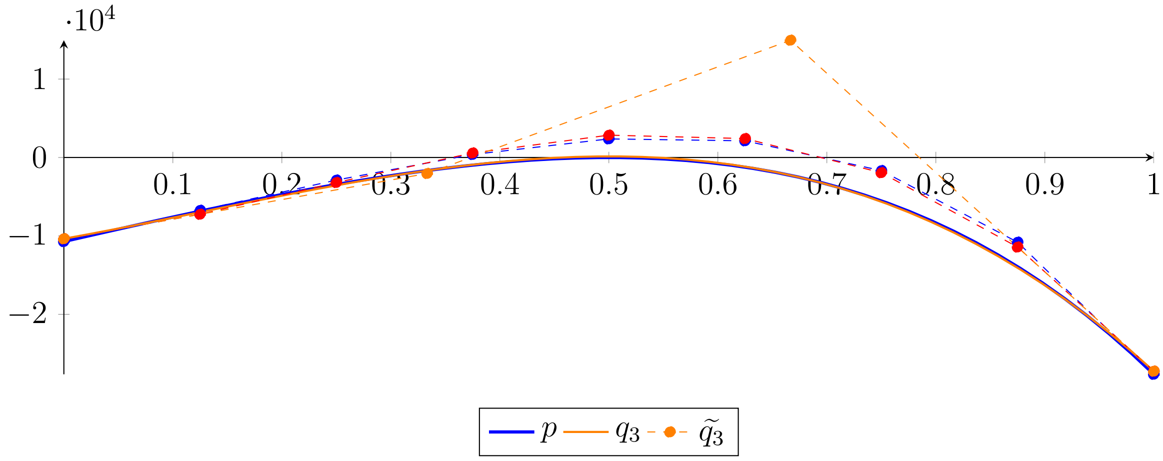
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ = &-10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &+ 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_3 &= -5.703152099307112 \cdot 10^{-299} X^8 + 1.997854926822688 \cdot 10^{-298} X^7 - 2.620079279971631 \\
&\quad \cdot 10^{-298} X^6 + 1.512761996617561 \cdot 10^{-298} X^5 - 2.636105166170616 \cdot 10^{-299} X^4 \\
&\quad - 67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10364.86272786554 \\
&= -10364.86272786554 B_{0,8} - 7243.22568371353 B_{1,8} - 3192.719831416224 B_{2,8} \\
&\quad + 574.7343076805747 B_{3,8} + 2847.216212231063 B_{4,8} + 2412.80536088944 B_{5,8} \\
&\quad - 1940.418767690097 B_{6,8} - 11424.37669485335 B_{7,8} - 27250.98894194612 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

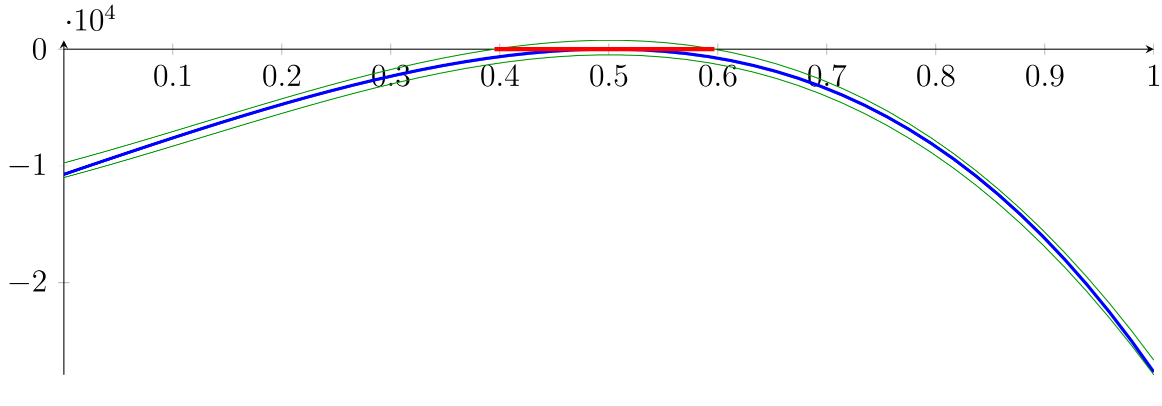
$$M = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 9740.484261812251$$

$$m = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $\quad N(m) = \{-0.623487965792376$

Intersection intervals:



$$[0.3950604114095972, 0.5968461976869074]$$

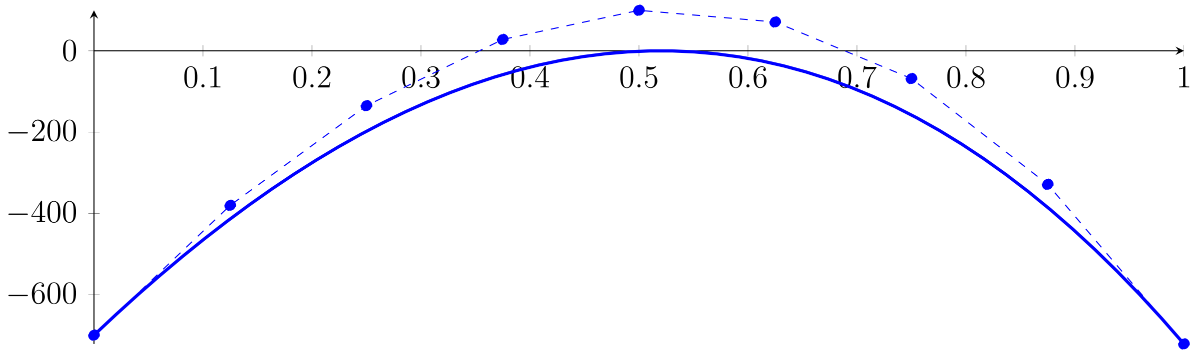
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

48.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

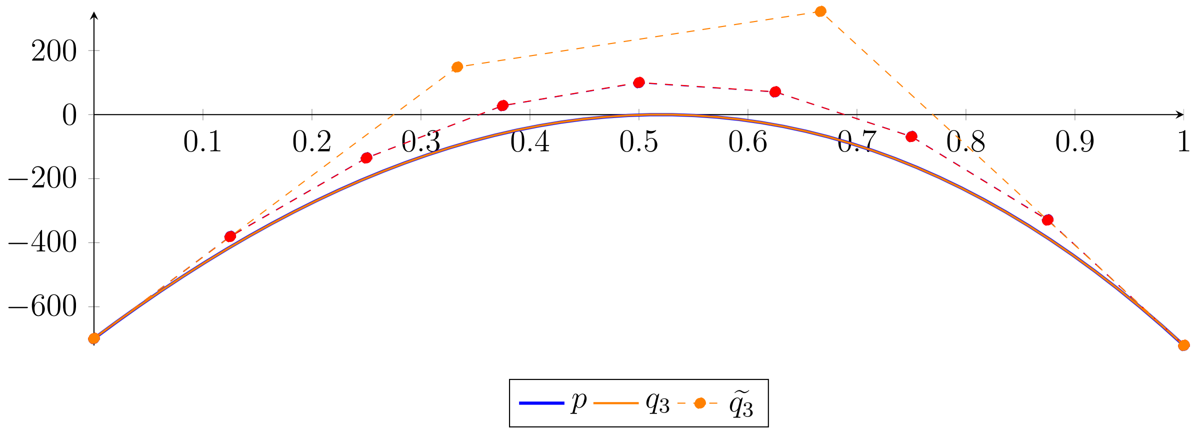
$$\begin{aligned}p = &-2.748682461629333\cdot 10^{-06}X^8 - 0.0005198138726602274X^7 - 0.04066637698425721X^6\\ &- 1.681520506726176X^5 - 38.59639865464246X^4 - 460.2831920090805X^3\\ &- 2074.780222477586X^2 + 2553.796807278985X - 699.3475151603227\\ = &-699.3475151603227B_{0,8}(X) - 380.1229142504495B_{1,8}(X) - 134.9976070004902B_{2,8}(X)\\ &+ 27.8090638751075B_{3,8}(X) + 99.52637853825791B_{4,8}(X) + 70.80221287533186B_{5,8}(X)\\ &- 68.32844102535583B_{6,8}(X) - 328.4764717433994B_{7,8}(X) - 720.9332304689123B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\ &= -698.7407882409323B_{0,3} + 148.5035278252892B_{1,3}\\ &\quad + 322.0745267793334B_{2,3} - 720.3121660793493B_{3,3}\end{aligned}$$

$$\begin{aligned}\tilde{q}_3 &= -2.24002635483637\cdot 10^{-300}X^8 + 8.593407551274415\cdot 10^{-300}X^7 - 1.289726574039331\\ &\quad \cdot 10^{-299}X^6 + 9.445830854385802\cdot 10^{-300}X^5 - 3.303106122422159\cdot 10^{-300}X^4\\ &\quad - 542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\ &= -698.7407882409323B_{0,8} - 381.0241697160992B_{1,8} - 135.4868351675709B_{2,8}\\ &\quad + 28.18756585642868B_{3,8} + 100.3153838076753B_{4,8} + 71.21296913794494B_{5,8}\\ &\quad - 68.80332770098655B_{6,8} - 329.4171562573433B_{7,8} - 720.3121660793493B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

Bounding polynomials M and m:

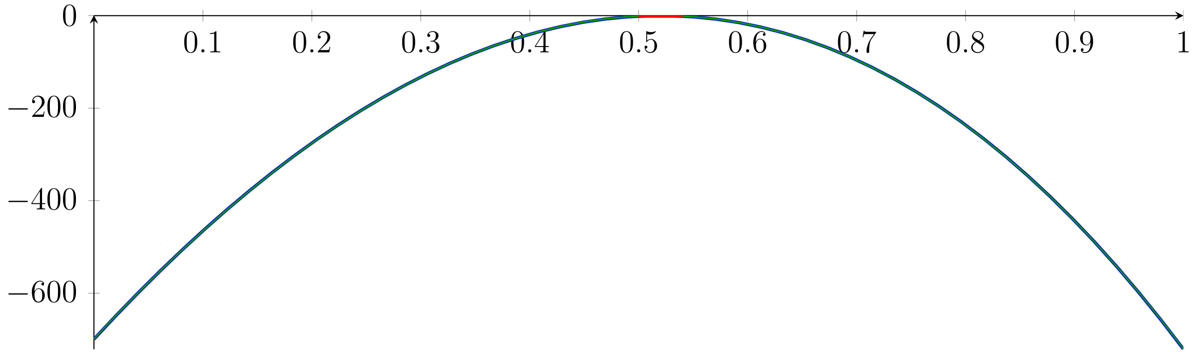
$$M = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 697.8001037269884$$

$$m = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 699.6814727548762$$

Root of M and m:

$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\}$ $N(m) = \{-4.76690070753073\}$

Intersection intervals:



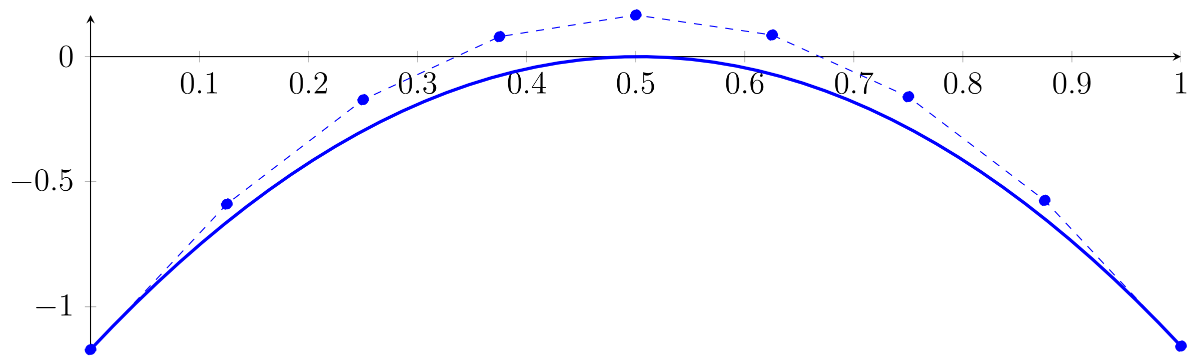
$[0.4997746332555131, 0.5401382492675976]$

Longest intersection interval: 0.04036361601208447
$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

48.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Normalized monomial und Bézier representations and the Bézier polygon:

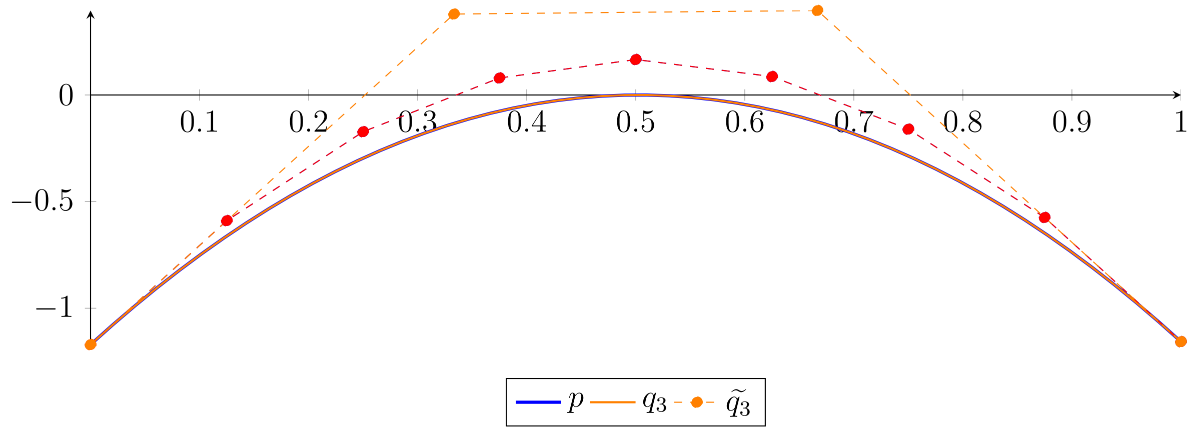
$$\begin{aligned}p &= -1.936623061789877\cdot 10^{-17}X^8 - 9.265403983052604\cdot 10^{-14}X^7 - 1.838110077974511\cdot 10^{-10}X^6\\ &\quad - 1.935168004397404\cdot 10^{-07}X^5 - 0.0001140127070697086X^4 - 0.0356257658581906X^3\\ &\quad - 4.602344943530705X^2 + 4.651752639476855X - 1.170857231885768\\ &= -1.170857231885768B_{0,8}(X) - 0.5893881519511612B_{1,8}(X) - 0.172288534285508B_{2,8}(X)\\ &\quad + 0.07980544672086649B_{3,8}(X) + 0.1662559879246795B_{4,8}(X) + 0.08642365397403269B_{5,8}(X)\\ &\quad - 0.1603326261538093B_{6,8}(X) - 0.574655562621217B_{7,8}(X) - 1.157189508205582B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661\\ &= -1.170855596980661B_{0,3} + 0.3797177193764957B_{1,3}\\ &\quad + 0.3962250709273155B_{2,3} - 1.157187871760247B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -3.713970905558003\cdot 10^{-303}X^8 + 1.449155002864004\cdot 10^{-302}X^7 - 2.225710229630734\\ &\quad \cdot 10^{-302}X^6 + 1.689258272787799\cdot 10^{-302}X^5 - 6.343129302139404\cdot 10^{-303}X^4\\ &\quad - 0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661\\ &= -1.170855596980661B_{0,8} - 0.5893906033467271B_{1,8} - 0.1722898202277581B_{2,8}\\ &\quad + 0.07980649649353122B_{3,8} + 0.1662580909344257B_{4,8} + 0.08642470721221019B_{5,8}\\ &\quad - 0.1603339105558303B_{6,8} - 0.574658018252411B_{7,8} - 1.157187871760247B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.45563119394813 \cdot 10^{-06}$.
Bounding polynomials M and m:

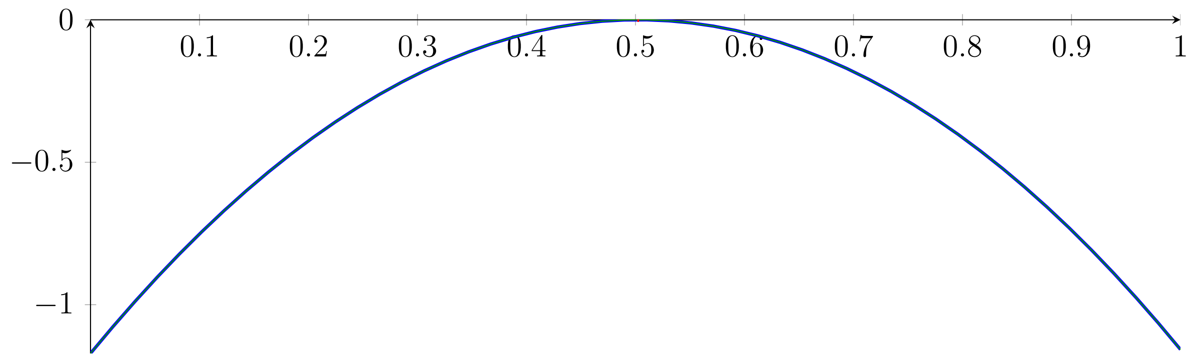
$$M = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170853141349467$$

$$m = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170858052611855$$

Root of M and m:

$N(M) = \{-129.3630802555855, 0.5016184355695172, 0.5032421204520033\}$ $N(m) = \{-129.3630802637075$

Intersection intervals:



$$[0.5016184355695172, 0.5032421204520033]$$

Longest intersection interval: 0.001623684882486142
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999934125800028, 0.5000066371751187]$,

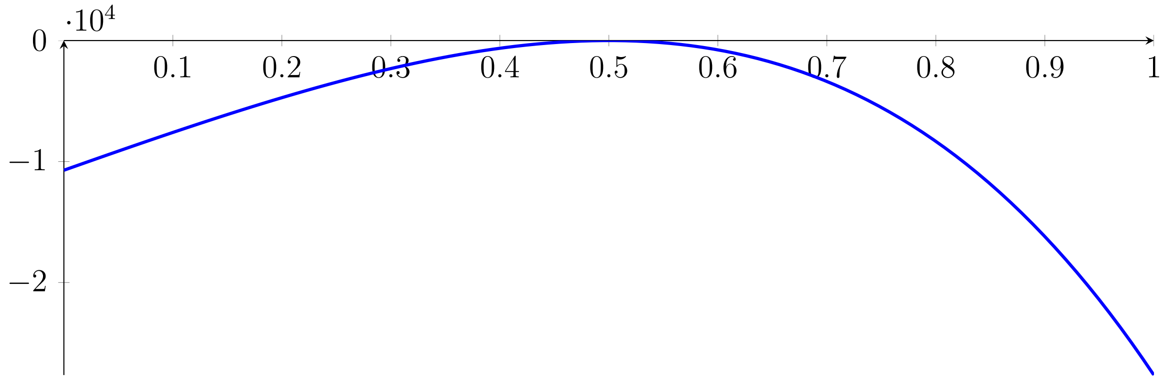
48.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4999934125800028, 0.50000663717511$

Found root in interval $[0.4999934125800028, 0.5000066371751187]$ at recursion depth 4!

48.5 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.4999934125800028, 0.5000066371751187]$$

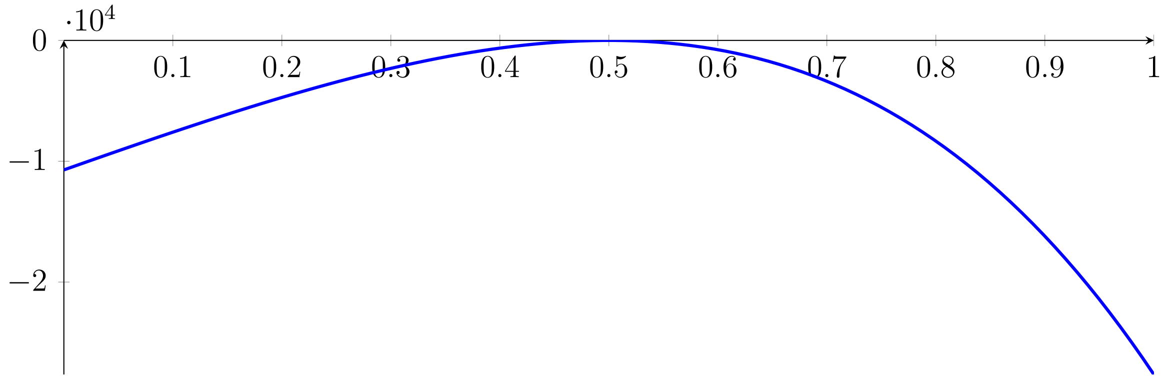
with precision $\varepsilon = 0.0001$.

49 Running `BezClip` on h_8 with epsilon 8

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

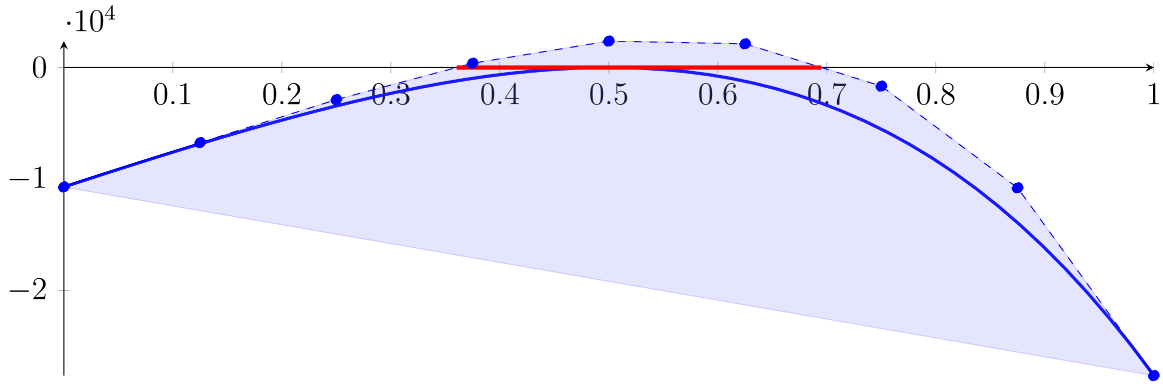
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



49.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

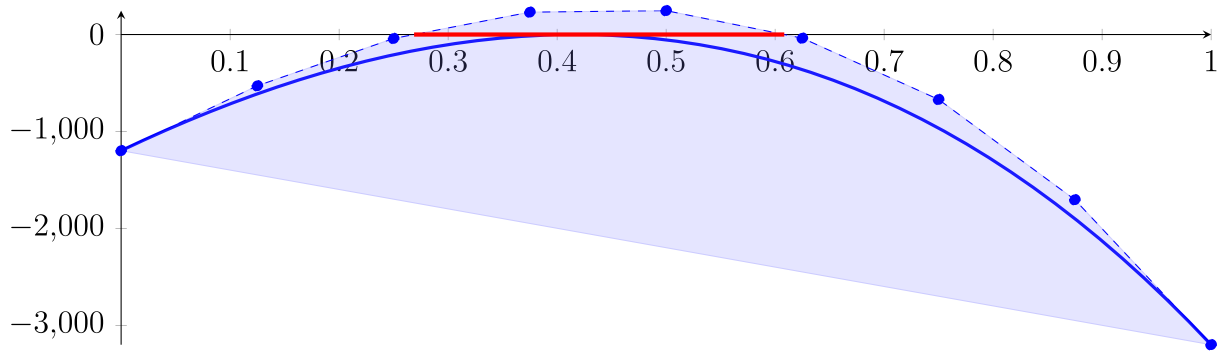
$$[0.3603640692588709, 0.6949437907012195]$$

Longest intersection interval: 0.3345797214423486
$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

49.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6 \\ &- 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3 \\ &- 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098 \\ = &-1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X) \\ &+ 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X) \\ &- 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

$$[0.2687909235445741, 0.6084084634355219]$$

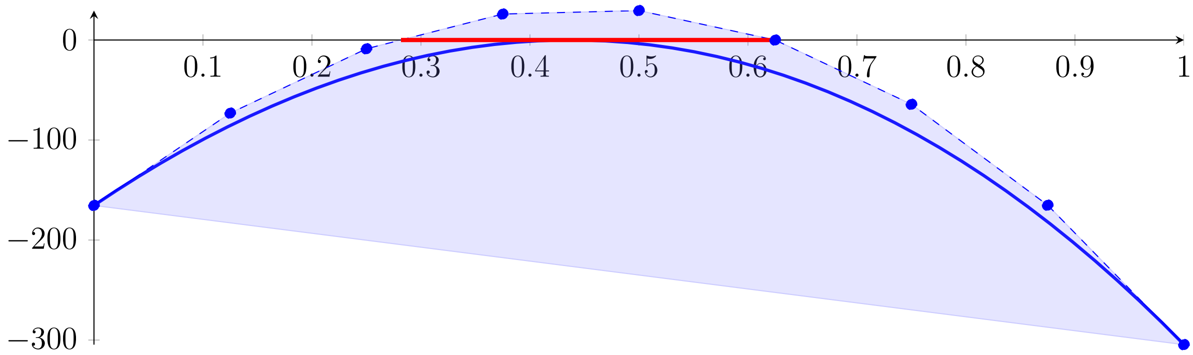
Longest intersection interval: 0.3396175398909478

$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

49.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-2.779187295559603 \cdot 10^{-08}X^8 - 9.441522673365855 \cdot 10^{-06}X^7 - 0.001328615938017263X^6 \\ &- 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3 \\ &- 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987 \\ = &-165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X) \\ &+ 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X) \\ &- 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

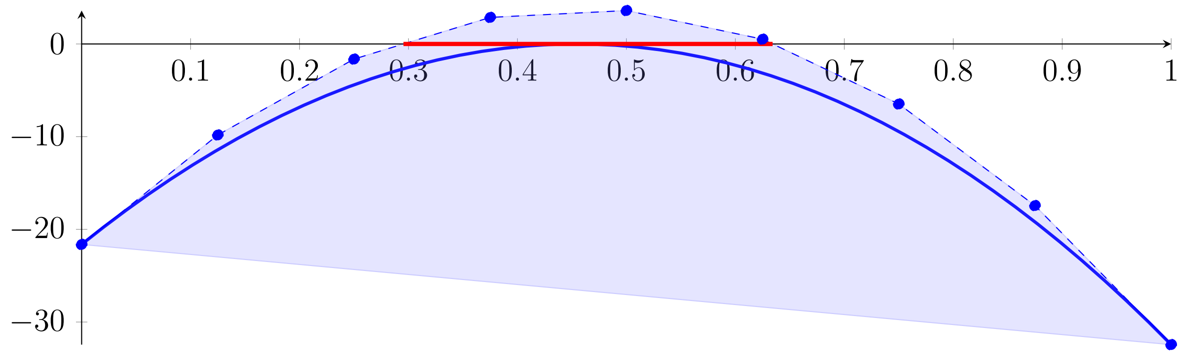
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\Longrightarrow$ Selective recursion: interval 1: $[0.4822990094256734, 0.5212832145711177]$,

49.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4822990094256734, 0.52128321457111$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -5.334693469973702\cdot 10^{-12}X^8 - 5.317476896558791\cdot 10^{-09}X^7 - 2.197128069269196\cdot 10^{-06}X^6 \\
& - 0.000481521724173462X^5 - 0.05899466848037266X^4 - 3.82353929224187X^3 \\
& - 101.3899724153744X^2 + 94.46052284159652X - 21.62667247491518 \\
= & -21.62667247491518B_{0,8}(X) - 9.819107119715613B_{1,8}(X) - 1.632612207922276B_{2,8}(X) \\
& + 2.86453477310337B_{3,8}(X) + 3.603213555021573B_{4,8}(X) + 0.5134524899120698B_{5,8}(X) \\
& - 6.475580126796985B_{6,8}(X) - 17.43558481253364B_{7,8}(X) - 32.43913973359036B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

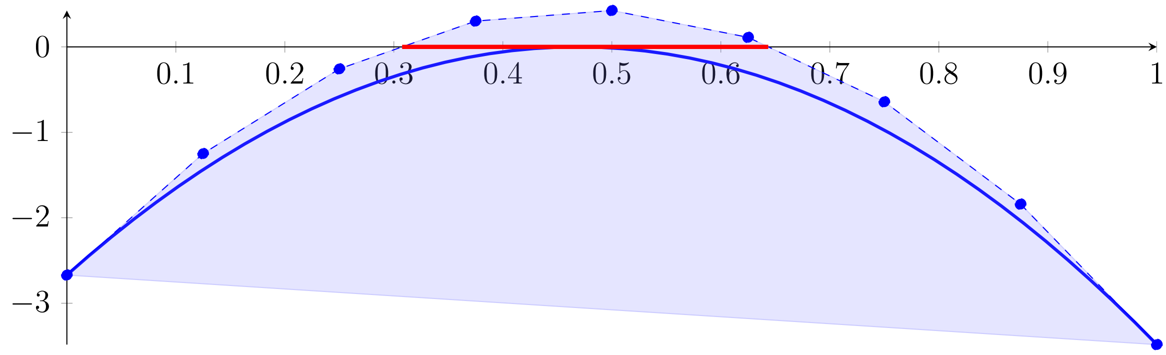
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4938141292321744, 0.5070221367077007]$,

49.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4938141292321744, 0.507022136707$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -9.261865042605128\cdot 10^{-16}X^8 - 2.731330938941274\cdot 10^{-12}X^7 - 3.339777336661422 \\
& \cdot 10^{-09}X^6 - 2.167033369334019\cdot 10^{-06}X^5 - 0.0007867416616259877X^4 \\
& - 0.1514273448103067X^3 - 12.0308548973337X^2 + 11.3691349552019X - 2.67015106585462 \\
= & -2.67015106585462B_{0,8}(X) - 1.249009196454382B_{1,8}(X) - 0.2575407162446335B_{2,8}(X) \\
& + 0.3015503150458701B_{3,8}(X) + 0.4255485985217787B_{4,8}(X) + 0.1117275574241232B_{5,8}(X) \\
& - 0.6426507016859872B_{6,8}(X) - 1.840335427863286B_{7,8}(X) - 3.484087264834231B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

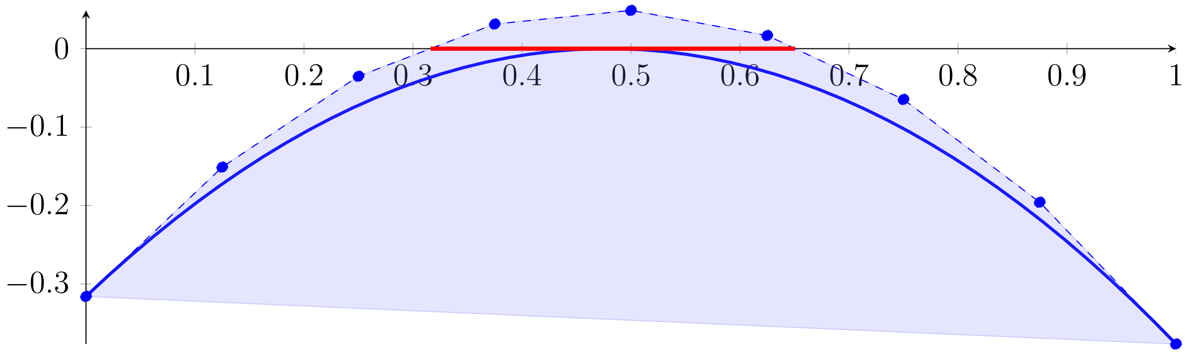
$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\Longrightarrow$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

49.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.502170909909546\cdot 10^{-19} X^8 - 1.319790000383349\cdot 10^{-15} X^7 - 4.808366303277033\cdot 10^{-12} X^6 \\ &- 9.297436410097437\cdot 10^{-09} X^5 - 1.006192361000331\cdot 10^{-05} X^4 - 0.005777437133356019 X^3 \\ &- 1.373512346650201 X^2 + 1.318590421802199 X - 0.3158295498425207 \\ = &-0.3158295498425207 B_{0,8}(X) - 0.1510057471172458 B_{1,8}(X) - 0.03523595677233526 B_{2,8}(X) \\ &+ 0.03137665267197247 B_{3,8}(X) + 0.04872876895367301 B_{4,8}(X) + 0.01671693590297049 B_{5,8}(X) \\ &- 0.06476244672391982 B_{6,8}(X) - 0.1958131234111408 B_{7,8}(X) - 0.376538983049735 B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3161210337394805, 0.6506459600024226\}$$

Intersection intervals with the x axis:

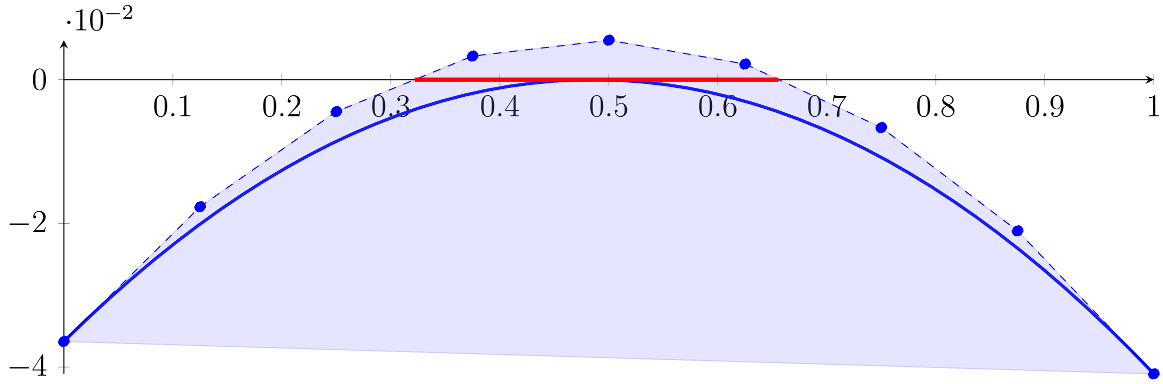
$$[0.3161210337394805, 0.6506459600024226]$$

Longest intersection interval: 0.3345249262629421
$\Longrightarrow$ Selective recursion: interval 1: $[0.499279281802493, 0.5007635705740208]$,

49.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.499279281802493, 0.5007635705$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.355847730899963\cdot 10^{-23}X^8 - 6.189124375986835\cdot 10^{-19}X^7 - 6.742674354644286\cdot 10^{-15}X^6\\ &- 3.898805488577673\cdot 10^{-11}X^5 - 1.261912061383275\cdot 10^{-07}X^4 - 0.000216758797755982X^3\\ &- 0.154319370565961X^2 + 0.1500226550818807X - 0.0364365303427455\\ = &-0.0364365303427455B_{0,8}(X) - 0.01768369845751041B_{1,8}(X) - 0.004442272663916792B_{2,8}(X)\\ &+ 0.003283876345218297B_{3,8}(X) + 0.005490876074346265B_{4,8}(X)\\ &+ 0.002174852224490793B_{5,8}(X) - 0.006668071307448625B_{6,8}(X)\\ &- 0.02104177242939338B_{7,8}(X) - 0.04095013085478269B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3218707447051634, 0.6557428337562166\}$$

Intersection intervals with the x axis:

$$[0.3218707447051634, 0.6557428337562166]$$

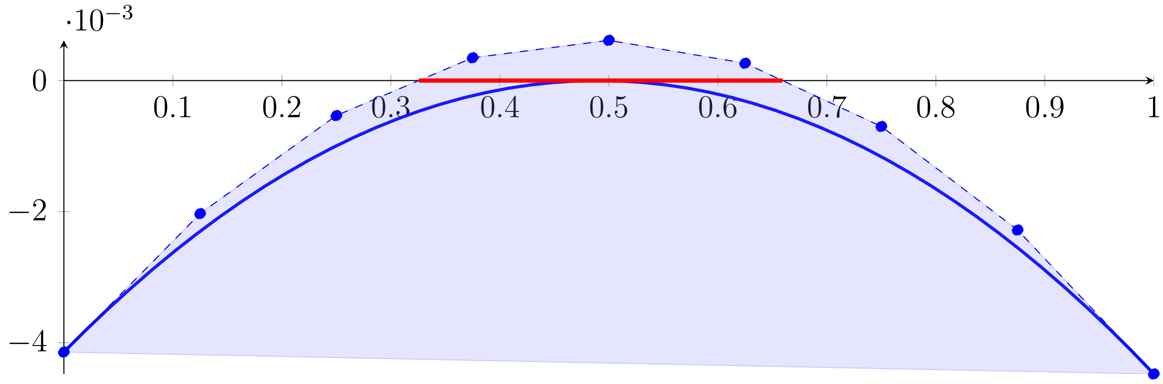
Longest intersection interval: 0.3338720890510532

$\Longrightarrow$ Selective recursion: interval 1: $[0.4997570309347421, 0.5002525935276471]$,

49.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4997570309347421, 0.5002525$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.637375463877353\cdot 10^{-27}X^8 - 2.862414855005092\cdot 10^{-22}X^7 - 9.341200314035287\cdot 10^{-18}X^6\\ &- 1.617995026179791\cdot 10^{-13}X^5 - 1.568792391901099\cdot 10^{-09}X^4 - 8.073141350089629\\ &\cdot 10^{-06}X^3 - 0.01722540857155108X^2 + 0.01689843085777645X - 0.004143462623424353\\ = &-0.004143462623424353B_{0,8}(X) - 0.002031158766202297B_{1,8}(X)\\ &- 0.0005340480722499233B_{2,8}(X) + 0.0003477252951943749B_{3,8}(X)\\ &+ 0.0006140171504808826B_{4,8}(X) + 0.0002646832855456765B_{5,8}(X)\\ &- 0.0007004205300922658B_{6,8}(X) - 0.002281438549333955B_{7,8}(X)\\ &- 0.004478515047503282B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3257065380923488, 0.6592817116222256\}$$

Intersection intervals with the x axis:

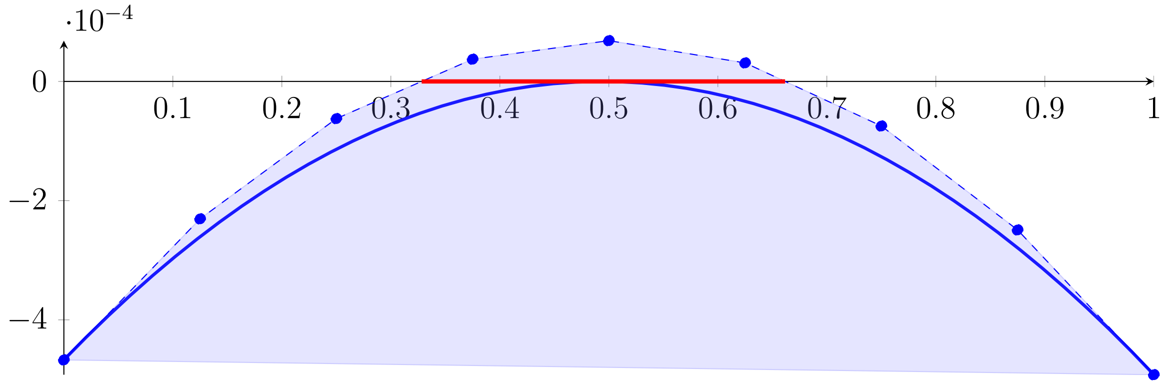
$$[0.3257065380923488, 0.6592817116222256]$$

Longest intersection interval: 0.3335751735298769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999184389112853, 0.5000837462892085]$,

49.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999184389112853, 0.50008$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-5.57619429667528 \cdot 10^{-31} X^8 - 1.315536799505273 \cdot 10^{-25} X^7 - 1.287049795843581 \cdot 10^{-20} X^6 \\ &- 6.683358895768893 \cdot 10^{-16} X^5 - 1.942733814710911 \cdot 10^{-11} X^4 - 2.997323807488328 \\ &\cdot 10^{-07} X^3 - 0.001917590365692151 X^2 + 0.001893040758741533 X - 0.0004671653183669499 \\ = &-0.0004671653183669499 B_{0,8}(X) - 0.0002305352235242582 B_{1,8}(X) - 6.239049888485767 \\ &\cdot 10^{-05} B_{2,8}(X) + 3.726350318730984 \cdot 10^{-05} B_{3,8}(X) + 6.842143005076897 \\ &\cdot 10^{-05} B_{4,8}(X) + 3.107792878649904 \cdot 10^{-05} B_{5,8}(X) - 7.47723538020779 \\ &\cdot 10^{-05} B_{6,8}(X) - 0.000249134771189109 B_{7,8}(X) - 0.0004920146771263226 B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3282588977707033, 0.661700337526842\}$$

Intersection intervals with the x axis:

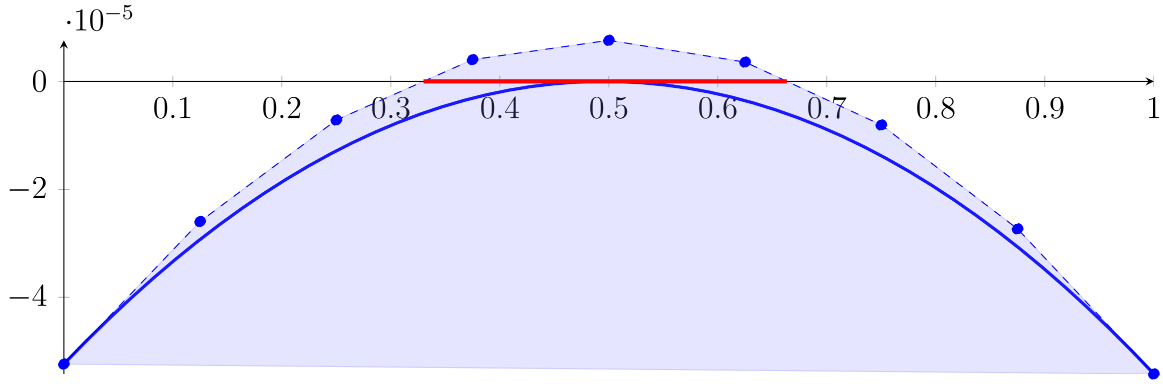
$$[0.3282588977707033, 0.661700337526842]$$

Longest intersection interval: 0.3334414397561387
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999727025289557, 0.5000278228590528]$,

49.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999727025289557, 0.500$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-8.521076768712742 \cdot 10^{-35} X^8 - 6.02899387826599 \cdot 10^{-29} X^7 - 1.76898025411792 \cdot 10^{-23} X^6 \\ &- 2.754920799910016 \cdot 10^{-18} X^5 - 2.401685361883718 \cdot 10^{-13} X^4 - 1.111294950492419 \cdot 10^{-08} X^3 \\ &- 0.0002132366404355359 X^2 + 0.0002114057621846039 X - 5.239629569054843 \cdot 10^{-05} \\ = &-5.239629569054843 \cdot 10^{-05} B_{0,8}(X) - 2.597057541747294 \cdot 10^{-05} B_{1,8}(X) - 7.160449445666593 \\ &\cdot 10^{-06} B_{2,8}(X) + 4.033883779343745 \cdot 10^{-06} B_{3,8}(X) + 7.612225808600218 \cdot 10^{-06} B_{4,8}(X) \\ &+ 3.574378189713946 \cdot 10^{-06} B_{5,8}(X) - 8.079857533135031 \cdot 10^{-06} B_{6,8}(X) \\ &- 2.73506798191978 \cdot 10^{-05} B_{7,8}(X) - 5.423828713115661 \cdot 10^{-05} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3299561852159799, 0.6633377584201652\}$$

Intersection intervals with the x axis:

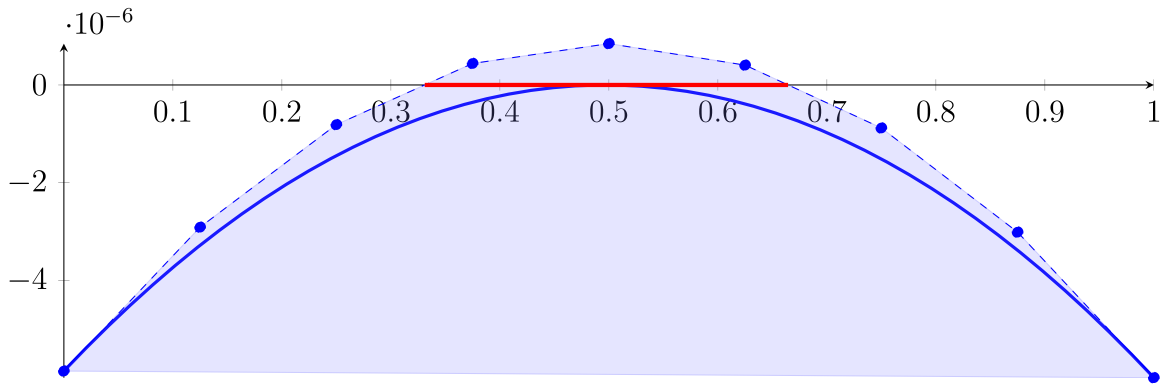
$$[0.3299561852159799, 0.6633377584201652]$$

Longest intersection interval: 0.3333815732041853
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999908898228024, 0.5000092659251657]$,

49.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999908898228024, 0.5$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.300251044438937\cdot 10^{-38}X^8 - 2.759545789497528\cdot 10^{-32}X^7 - 2.428711637904422\cdot 10^{-26}X^6\\ &- 1.134547275299063\cdot 10^{-20}X^5 - 2.966816573305973\cdot 10^{-15}X^4 - 4.117811892390909\cdot 10^{-10}X^3\\ &- 2.370104084998438\cdot 10^{-05}X^2 + 2.356495503794524\cdot 10^{-05}X - 5.857360304639568\cdot 10^{-06}\\ = &-5.857360304639568\cdot 10^{-06}B_{0,8}(X) - 2.911740924896412\cdot 10^{-06}B_{1,8}(X) - 8.125872897955564\\ &\cdot 10^{-07}B_{2,8}(X) + 4.400932474274781\cdot 10^{-07}B_{3,8}(X) + 8.462933334947859\cdot 10^{-07}B_{4,8}(X)\\ &+ 4.060056150860783\cdot 10^{-07}B_{5,8}(X) - 8.807772611613165\cdot 10^{-07}B_{6,8}(X)\\ &- 3.014062648652454\cdot 10^{-06}B_{7,8}(X) - 5.993857900834775\cdot 10^{-06}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.331084848216461, 0.6644399885346334\}$$

Intersection intervals with the x axis:

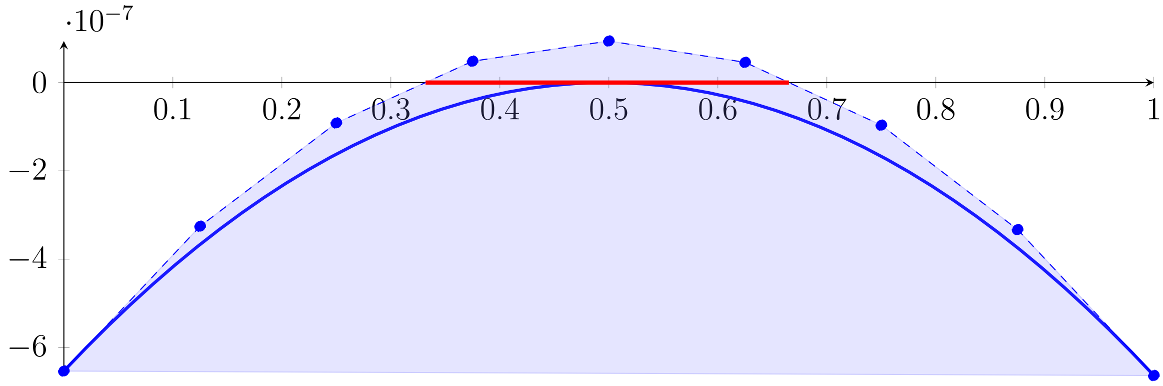
$$[0.331084848216461, 0.6644399885346334]$$

Longest intersection interval: 0.3333551403181724
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999969738718641, 0.500003099640046]$,

49.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999969738718641, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1.982825346126094 \cdot 10^{-42} X^8 - 1.262374580662423 \cdot 10^{-35} X^7 - 3.332882748505763 \cdot 10^{-29} X^6 \\ & - 4.670466114941116 \cdot 10^{-23} X^5 - 3.663718276431707 \cdot 10^{-17} X^4 - 1.52542941384219 \cdot 10^{-11} X^3 \\ & - 2.633839011095458 \cdot 10^{-06} X^2 + 2.623741169043712 \cdot 10^{-06} X - 6.534168705792709 \cdot 10^{-07} \\ = & -6.534168705792709 \cdot 10^{-07} B_{0,8}(X) - 3.254492244488069 \cdot 10^{-07} B_{1,8}(X) - 9.154725728603783 \\ & \cdot 10^{-08} B_{2,8}(X) + 4.828875851092667 \cdot 10^{-08} B_{3,8}(X) + 9.405855054345363 \cdot 10^{-08} B_{4,8}(X) \\ & + 4.576184641238665 \cdot 10^{-08} B_{5,8}(X) - 9.660162628195405 \cdot 10^{-08} B_{6,8}(X) \\ & - 3.330321399397716 \cdot 10^{-07} B_{7,8}(X) - 6.635299669617927 \cdot 10^{-07} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3318344765869905, 0.6651804668942704\}$$

Intersection intervals with the x axis:

$$[0.3318344765869905, 0.6651804668942704]$$

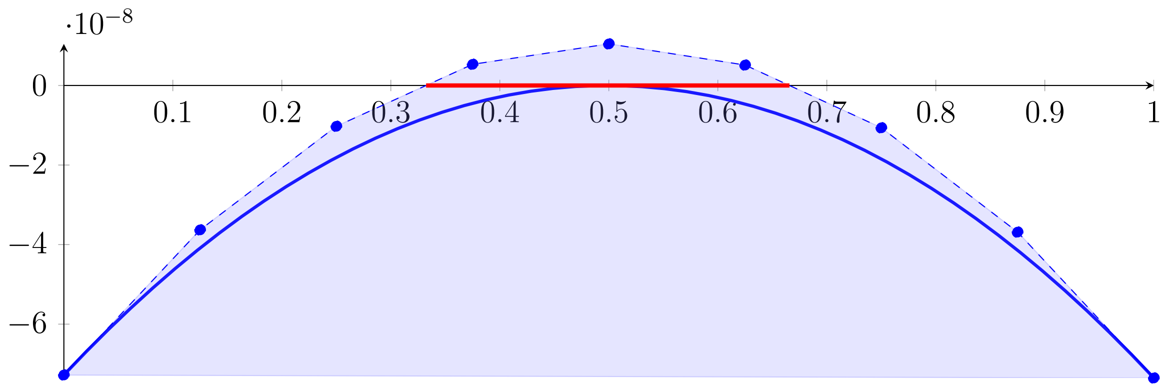
Longest intersection interval: 0.3333459903072799

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999990066129424, 0.5000010486132034]$,

49.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999990066129424$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -3.023057070307937 \cdot 10^{-46} X^8 - 5.773711385862107 \cdot 10^{-39} X^7 - 4.572901329678827 \cdot 10^{-32} X^6 \\ & - 1.922370176484083 \cdot 10^{-25} X^5 - 4.523805577109385 \cdot 10^{-19} X^4 - 5.650400184721212 \cdot 10^{-13} X^3 \\ & - 2.926726911514393 \cdot 10^{-07} X^2 + 2.919240677305221 \cdot 10^{-07} X - 7.27925149751094 \cdot 10^{-08} \\ = & -7.27925149751094 \cdot 10^{-08} B_{0,8}(X) - 3.630200650879414 \cdot 10^{-08} B_{1,8}(X) - 1.026409415503029 \\ & \cdot 10^{-08} B_{2,8}(X) + 5.321211996181834 \cdot 10^{-09} B_{3,8}(X) + 1.045390185483543 \cdot 10^{-08} B_{4,8}(X) \\ & + 5.133965330917245 \cdot 10^{-09} B_{5,8}(X) - 1.063860766559244 \cdot 10^{-08} B_{6,8}(X) \\ & - 3.68638272247198 \cdot 10^{-08} B_{7,8}(X) - 7.354170343649749 \cdot 10^{-08} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3323218842755297, 0.6656874431018115\}$$

Intersection intervals with the x axis:

$$[0.3323218842755297, 0.6656874431018115]$$

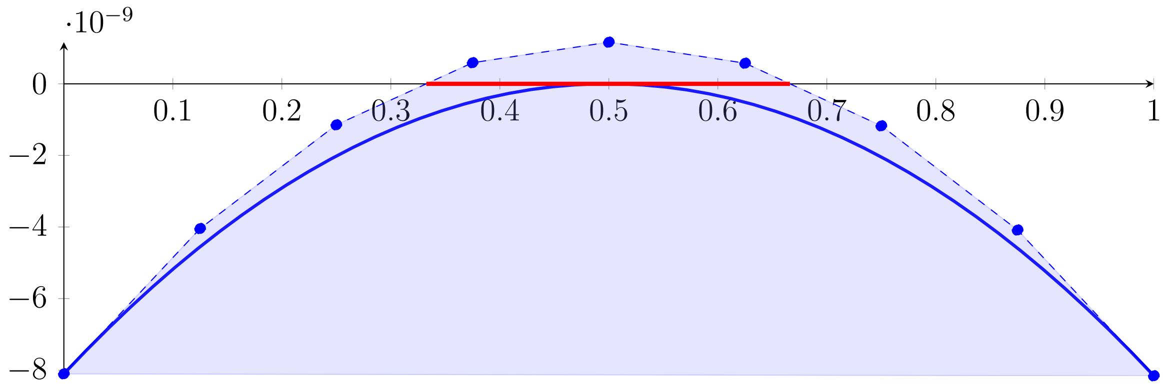
Longest intersection interval: 0.3333655588262817

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999996852143169, 0.500000365946875]$,

49.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999996852143169, 0.500000365946875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.61118111521237\cdot 10^{-50}X^8 - 2.641801828130962\cdot 10^{-42}X^7 - 6.276482670428966\cdot 10^{-35}X^6\\ &\quad - 7.91481690665562\cdot 10^{-28}X^5 - 5.587109146485419\cdot 10^{-21}X^4 - 2.093350052120201\cdot 10^{-14}X^3\\ &\quad - 3.252553849467158\cdot 10^{-08}X^2 + 3.247007216419568\cdot 10^{-08}X - 8.101917765629863\cdot 10^{-09}\\ &= -8.101917765629863\cdot 10^{-09}B_{0,8}(X) - 4.043158745105403\cdot 10^{-09}B_{1,8}(X)\\ &\quad - 1.146026099390642\cdot 10^{-09}B_{2,8}(X) + 5.8947979770191\cdot 10^{-10}B_{3,8}(X) + 1.163358572359665\\ &\quad \cdot 10^{-09}B_{4,8}(X) + 5.75609850769953\cdot 10^{-10}B_{5,8}(X) - 1.173766740879974\cdot 10^{-09}B_{6,8}(X)\\ &\quad - 4.084771576402945\cdot 10^{-09}B_{7,8}(X) - 8.157405029611868\cdot 10^{-09}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.332542653795541, 0.6661296410902487\}$$

Intersection intervals with the x axis:

$$[0.332542653795541, 0.6661296410902487]$$

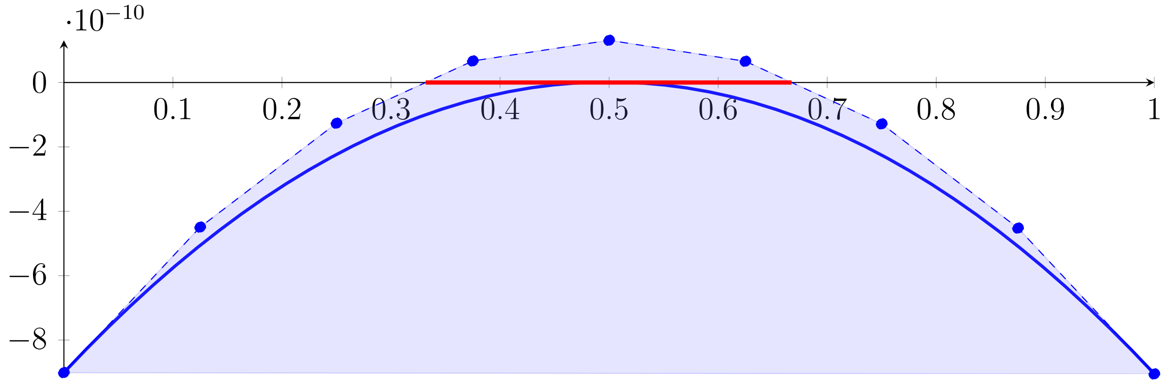
Longest intersection interval: 0.3335869872947078

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999999115869283, 0.5000001386704515]$,

49.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999115869283, 0.5000001386704515]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.071067608997674\cdot 10^{-54}X^8 - 1.214406168666063\cdot 10^{-45}X^7 - 8.649101296569956\cdot 10^{-38}X^6\\ &\quad - 3.269538436354099\cdot 10^{-30}X^5 - 6.918686662336931\cdot 10^{-23}X^4 - 7.770864123205297\cdot 10^{-16}X^3\\ &\quad - 3.61945329275134\cdot 10^{-09}X^2 + 3.615351534501487\cdot 10^{-09}X - 9.01058772952768\cdot 10^{-10}\\ &= -9.01058772952768\cdot 10^{-10}B_{0,8}(X) - 4.491398311400821\cdot 10^{-10}B_{1,8}(X) - 1.264870783542298\\ &\quad \cdot 10^{-10}B_{2,8}(X) + 6.689947152824597\cdot 10^{-11}B_{3,8}(X) + 1.31019804630801\cdot 10^{-10}B_{4,8}(X)\\ &\quad + 6.587390707689027\cdot 10^{-11}B_{5,8}(X) - 1.285382350100323\cdot 10^{-10}B_{6,8}(X)\\ &\quad - 4.522166355065136\cdot 10^{-10}B_{7,8}(X) - 9.051613082891018\cdot 10^{-10}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3317579340646351, 0.6673545479012814\}$$

Intersection intervals with the x axis:

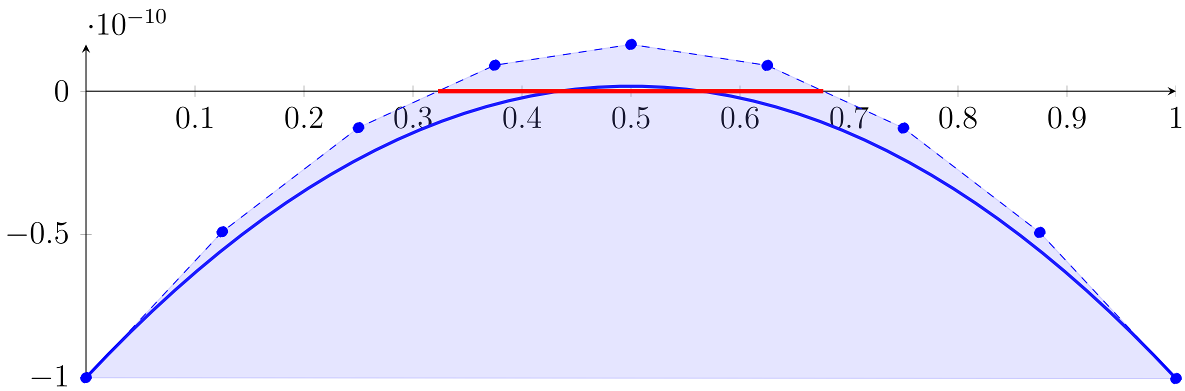
$$[0.3317579340646351, 0.6673545479012814]$$

Longest intersection interval: 0.3355966138366463
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999999869236888, 0.5000000631321502]$,

49.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999869236888, 0.5000000631321502]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.137694218100245\cdot 10^{-57}X^8 - 5.822197888022611\cdot 10^{-49}X^7 - 1.235595786383296\cdot 10^{-40}X^6 \\ &- 1.391791952088144\cdot 10^{-32}X^5 - 8.775946704095466\cdot 10^{-25}X^4 - 2.937122613307657\cdot 10^{-17}X^3 \\ &- 4.076413298856481\cdot 10^{-10}X^2 + 4.07342667585279\cdot 10^{-10}X - 1.000063159750977\cdot 10^{-10} \\ = &-1.000063159750977\cdot 10^{-10}B_{0,8}(X) - 4.908848252693777\cdot 10^{-11}B_{1,8}(X) - 1.272926800326533 \\ &\cdot 10^{-11}B_{2,8}(X) + 9.071327071433506\cdot 10^{-12}B_{3,8}(X) + 1.631330217267253\cdot 10^{-11}B_{4,8}(X) \\ &+ 8.996656775965545\cdot 10^{-12}B_{5,8}(X) - 1.287860964317367\cdot 10^{-11}B_{6,8}(X) \\ &- 4.931249760923136\cdot 10^{-11}B_{7,8}(X) - 1.003050076466937\cdot 10^{-10}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3229869297125206, 0.6764088411746962\}$$

Intersection intervals with the x axis:

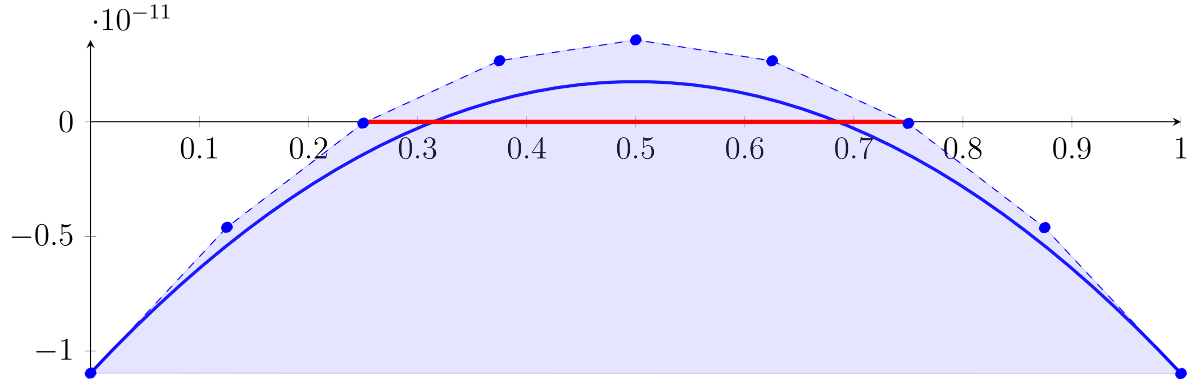
$$[0.3229869297125206, 0.6764088411746962]$$

Longest intersection interval: 0.3534219114621756
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000115380258, 0.5000000384717659]$,

49.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000115380258, 0.5000000384717659]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.769321148091343\cdot 10^{-61}X^8 - 4.009971301105125\cdot 10^{-52}X^7 - 2.40789337418918\cdot 10^{-43}X^6\\&\quad - 7.674351474736958\cdot 10^{-35}X^5 - 1.36920310040832\cdot 10^{-26}X^4 - 1.29658952293724\cdot 10^{-18}X^3\\&\quad - 5.091727851043352\cdot 10^{-11}X^2 + 5.08987688183243\cdot 10^{-11}X - 1.096532991460483\cdot 10^{-11}\\&= -1.096532991460483\cdot 10^{-11}B_{0,8}(X) - 4.602983812314296\cdot 10^{-12}B_{1,8}(X)\\&\quad - 5.91119425392419\cdot 10^{-14}B_{2,8}(X) + 2.666285671566945\cdot 10^{-12}B_{3,8}(X) + 3.57320900685088\\&\quad \cdot 10^{-12}B_{4,8}(X) + 2.661658040159178\cdot 10^{-12}B_{5,8}(X) - 6.83672516615453\cdot 10^{-14}B_{6,8}(X)\\&\quad - 4.616866891764675\cdot 10^{-12}B_{7,8}(X) - 1.09838409033036\cdot 10^{-11}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.252711161402344, 0.7468696603349071\}$$

Intersection intervals with the x axis:

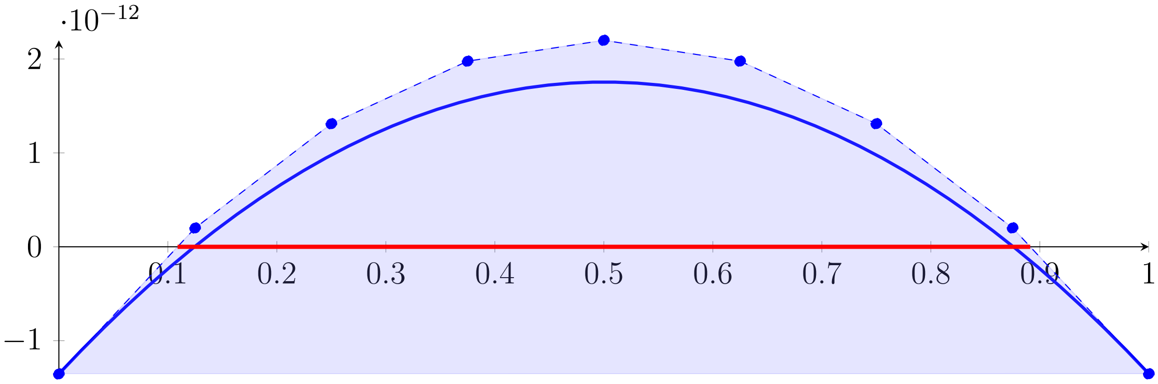
$$[0.252711161402344, 0.7468696603349071]$$

Longest intersection interval: 0.4941584989325631
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000183444825, 0.5000000316540191]$,

49.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000183444825, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -9.84698950588755\cdot 10^{-64}X^8 - 2.885394162330348\cdot 10^{-54}X^7 - 3.506185324343073\cdot 10^{-45}X^6\\&\quad - 2.261377324294573\cdot 10^{-36}X^5 - 8.164563166712554\cdot 10^{-28}X^4 - 1.564592773244276\cdot 10^{-19}X^3\\&\quad - 1.243362398803133\cdot 10^{-11}X^2 + 1.243502393415518\cdot 10^{-11}X - 1.354369602677131\cdot 10^{-12}\\&= -1.354369602677131\cdot 10^{-12}B_{0,8}(X) + 2.000083890922661\cdot 10^{-13}B_{1,8}(X)\\&\quad + 1.310328381289116\cdot 10^{-12}B_{2,8}(X) + 1.976590371119503\cdot 10^{-12}B_{3,8}(X)\\&\quad + 2.19879435578951\cdot 10^{-12}B_{4,8}(X) + 1.976940332505223\cdot 10^{-12}B_{5,8}(X) + 1.311028298472726\\&\quad \cdot 10^{-12}B_{6,8}(X) + 2.01058250898102\cdot 10^{-13}B_{7,8}(X) - 1.352969813012563\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.108915721421098, 0.8911723471704997\}$$

Intersection intervals with the x axis:

$$[0.108915721421098, 0.8911723471704997]$$

Longest intersection interval: 0.7822566257494017
$\Longrightarrow$ Bisection: first half $[0.5000000183444825, 0.5000000249992508]$ und second half $[0.5000000249992508, 0.50$

49.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.5000000183444825, 0.5000000249992508]$

Found root in interval $[0.5000000183444825, 0.5000000249992508]$ at recursion depth 19!

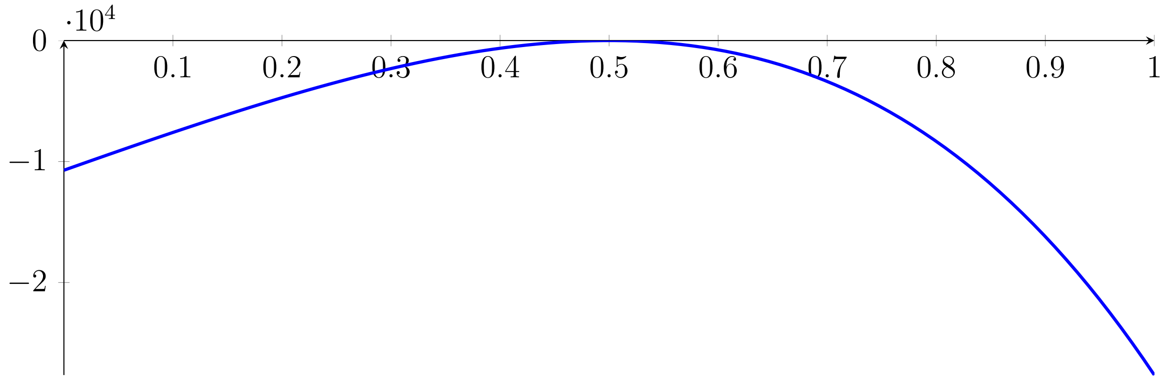
49.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.5000000249992508, 0.5000000316540191]$

Found root in interval $[0.5000000249992508, 0.5000000316540191]$ at recursion depth 19!

49.21 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.5000000183444825, 0.5000000249992508],\ [0.5000000249992508, 0.5000000316540191]$$

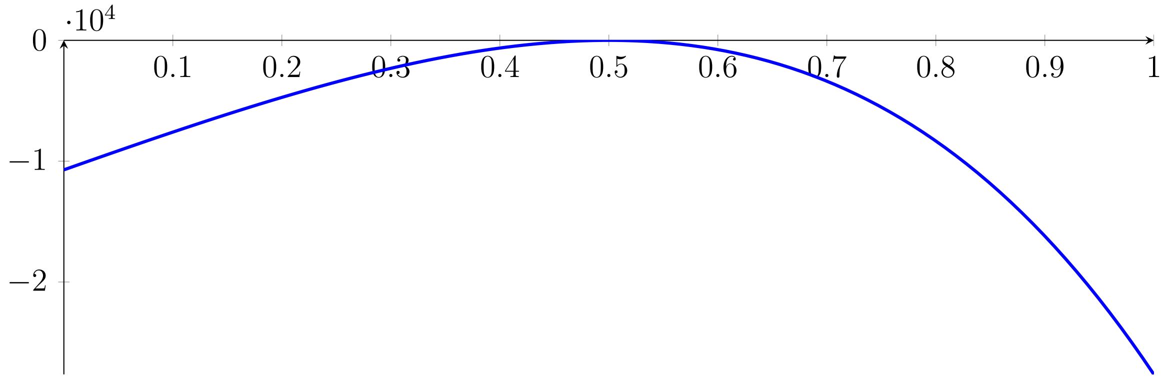
with precision $\varepsilon = 1 \cdot 10^{-08}$.

50 Running `QuadClip` on h_8 with epsilon 8

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

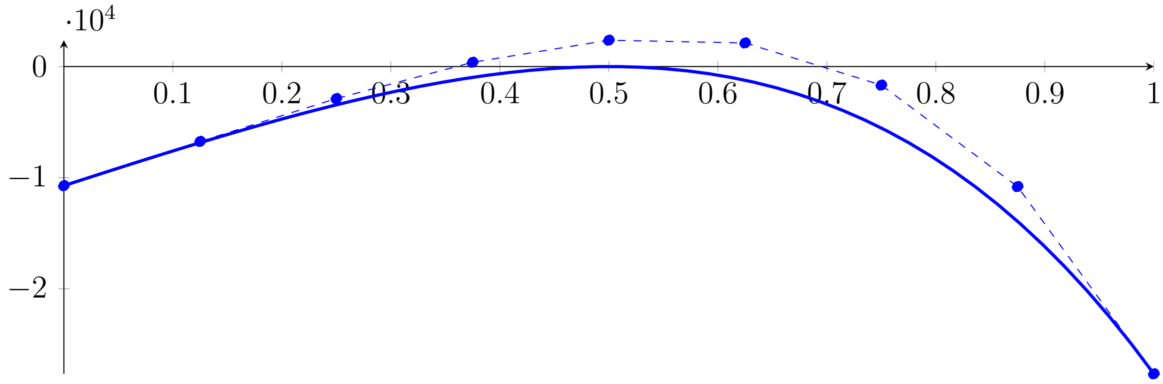
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



50.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

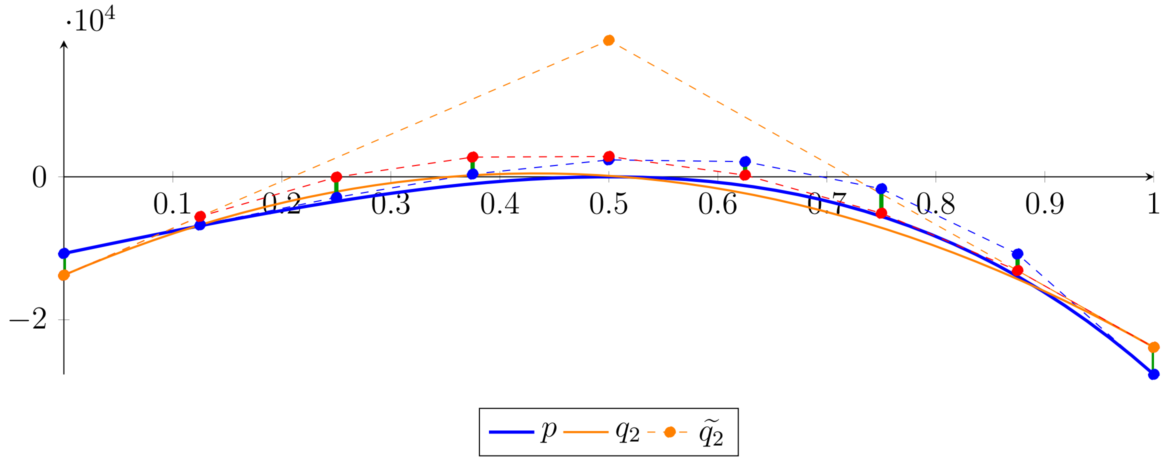
$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2} \end{aligned}$$

$$\begin{aligned} \tilde{q}_2 &= 8.415084934481114 \cdot 10^{-299}X^8 - 3.337656926892332 \cdot 10^{-298}X^7 + 5.61048922956057 \cdot 10^{-298}X^6 \\ &\quad - 5.189733160056009 \cdot 10^{-298}X^5 + 2.787623795639522 \cdot 10^{-298}X^4 - 8.054794139884735 \\ &\quad \cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8} \\ &\quad + 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8} \\ &\quad - 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.
Bounding polynomials M and m:

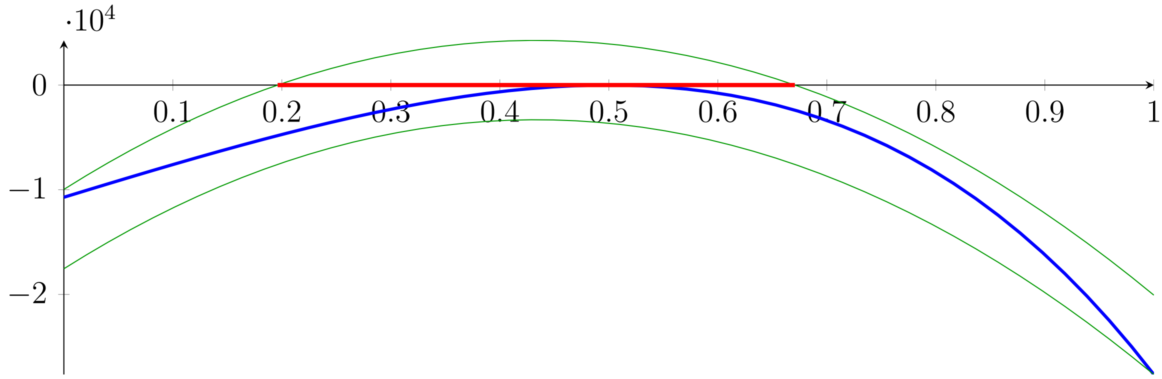
$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$

$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



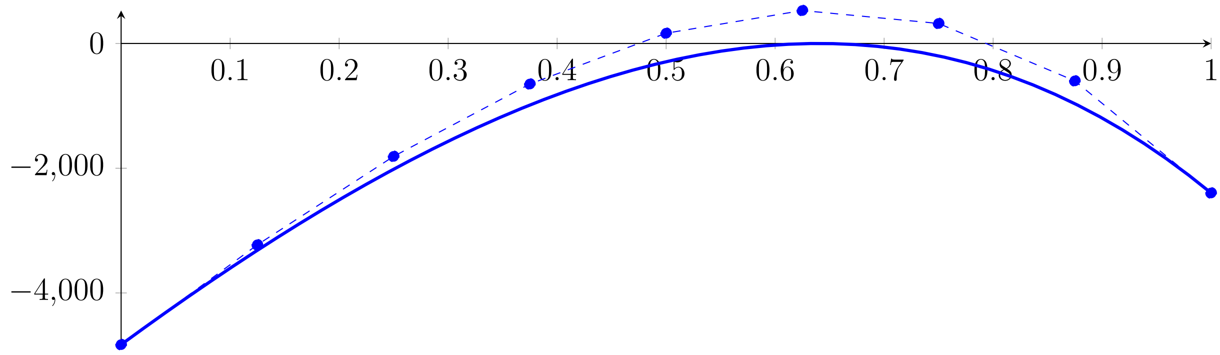
$$[0.196099166583006, 0.6706514347613278]$$

Longest intersection interval: 0.4745522681783218
$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

50.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

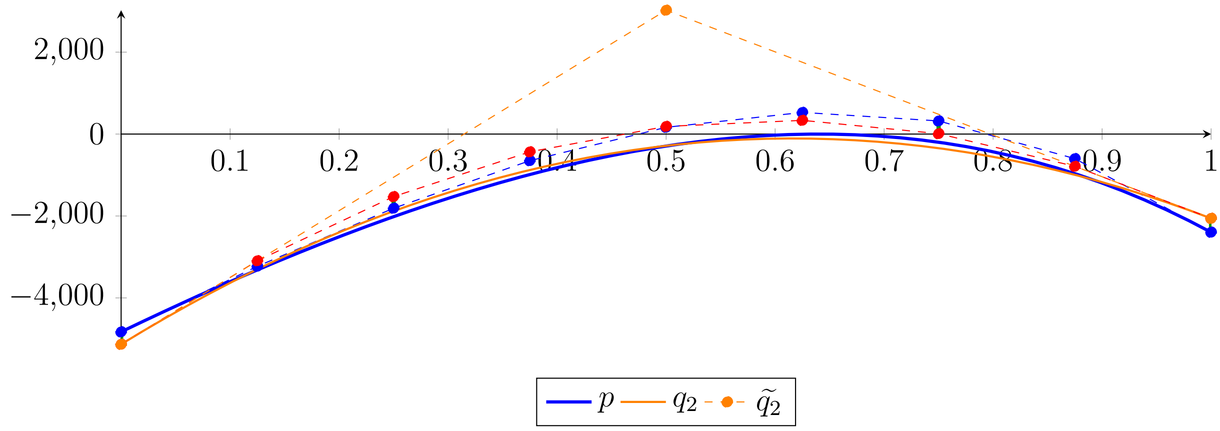
$$\begin{aligned} p = &-0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6 \\ &- 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3 \\ &- 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684 \\ = &-4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X) \\ &- 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X) \\ &+ 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.476670942620757\cdot 10^{-299}X^8 - 6.381627458836932\cdot 10^{-299}X^7 + 1.1779791924066\cdot 10^{-298}X^6 \\ &\quad - 1.190713137013755\cdot 10^{-298}X^5 + 6.850874958113058\cdot 10^{-299}X^4 - 2.05172173630317 \\ &\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\ &\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\ &\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

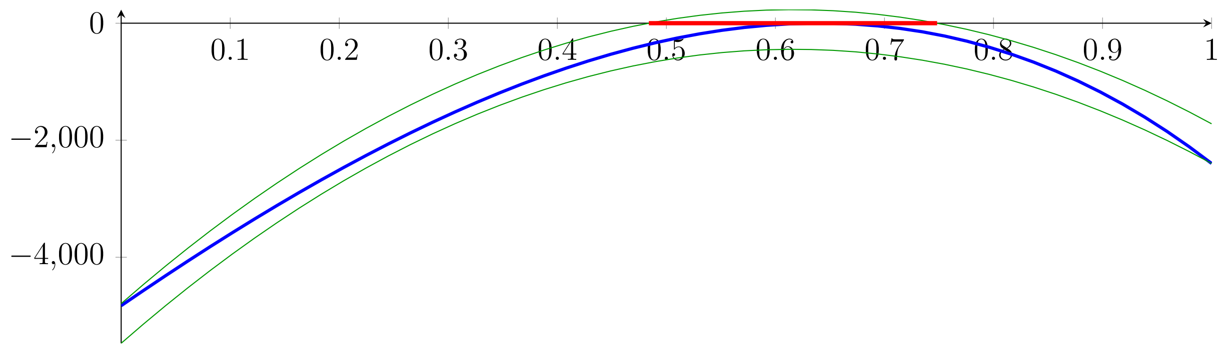
Bounding polynomials M and m:

$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$
$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.4839311609916354, 0.7482816274420809]$$

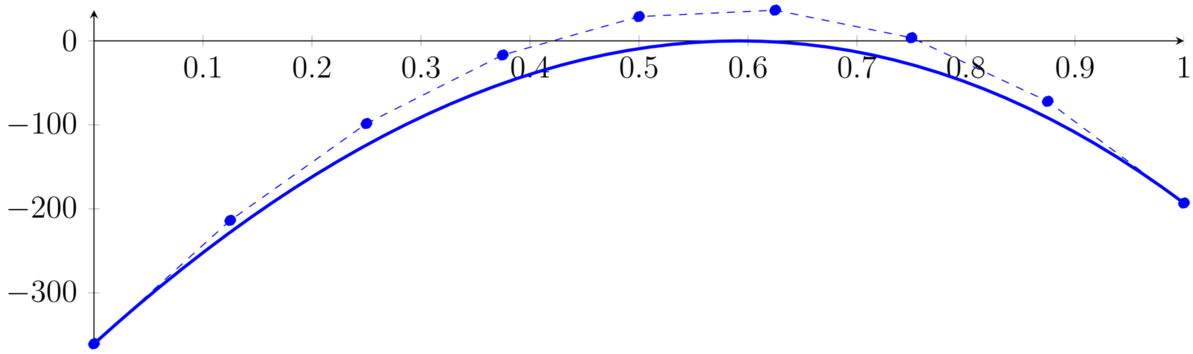
Longest intersection interval: 0.2643504664504455

$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

50.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.551197910121811$

Normalized monomial und Bézier representations and the Bézier polygon:

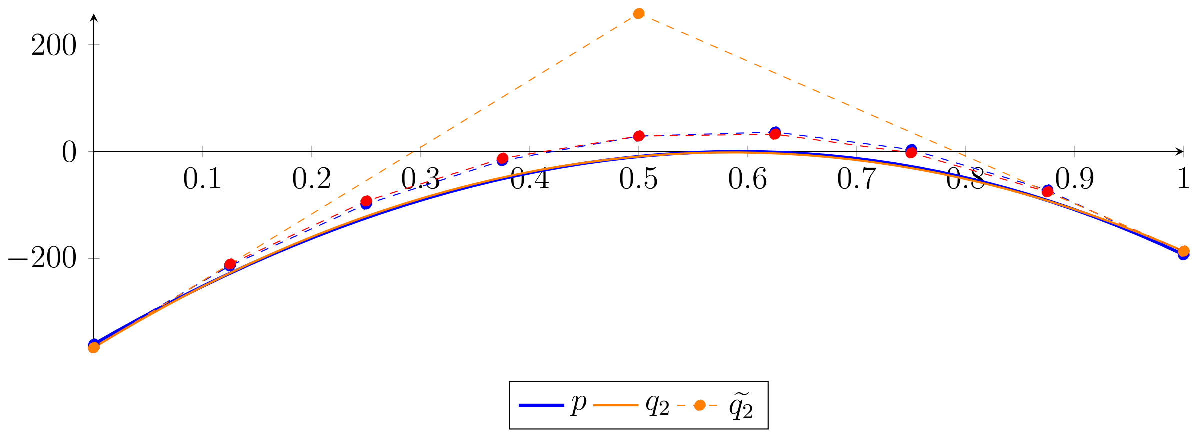
$$\begin{aligned} p = & -6.133566475094875 \cdot 10^{-08} X^8 - 1.877794231289375 \cdot 10^{-05} X^7 - 0.002379939031153127 X^6 \\ & - 0.1596298583101837 X^5 - 5.958684697205818 X^4 - 116.3336704708797 X^3 \\ & - 885.1606719876373 X^2 + 1175.121582406799 X - 360.5751654247078 \\ = & -360.5751654247078 B_{0,8}(X) - 213.6849676238579 B_{1,8}(X) - 98.40765096542357 B_{2,8}(X) \\ & - 16.82060242209915 B_{3,8}(X) + 28.91366696631814 B_{4,8}(X) + 36.54467155974404 B_{5,8}(X) \\ & + 3.731015586800578 B_{6,8}(X) - 71.9626297312559 B_{7,8}(X) - 193.0686388102505 B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -1070.165401921332 X^2 + 1250.543498884255 X - 366.9199822310984 \\ = & -366.9199822310984 B_{0,2} + 258.3517672110292 B_{1,2} - 186.5418852681748 B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & 1.223655337688364 \cdot 10^{-300} X^8 - 5.205587704887534 \cdot 10^{-300} X^7 + 9.409306890019087 \cdot 10^{-300} X^6 \\ & - 9.282190734559383 \cdot 10^{-300} X^5 + 5.21941805196838 \cdot 10^{-300} X^4 - 1.541981168110934 \\ & \cdot 10^{-300} X^3 - 1070.165401921332 X^2 + 1250.543498884255 X - 366.9199822310984 \\ = & -366.9199822310984 B_{0,8} - 210.6020448705665 B_{1,8} - 92.50430043579644 B_{2,8} \\ & - 12.62674892678823 B_{3,8} + 29.03060965645814 B_{4,8} + 32.46777531394266 B_{5,8} \\ & - 2.315251954334665 B_{6,8} - 75.31847214837383 B_{7,8} - 186.5418852681748 B_{8,8} \end{aligned}$$



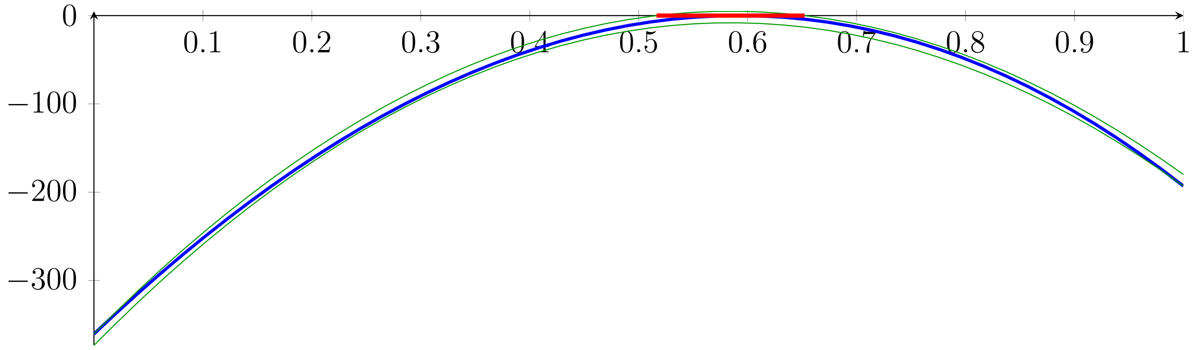
The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.
Bounding polynomials M and m:

$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$
$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



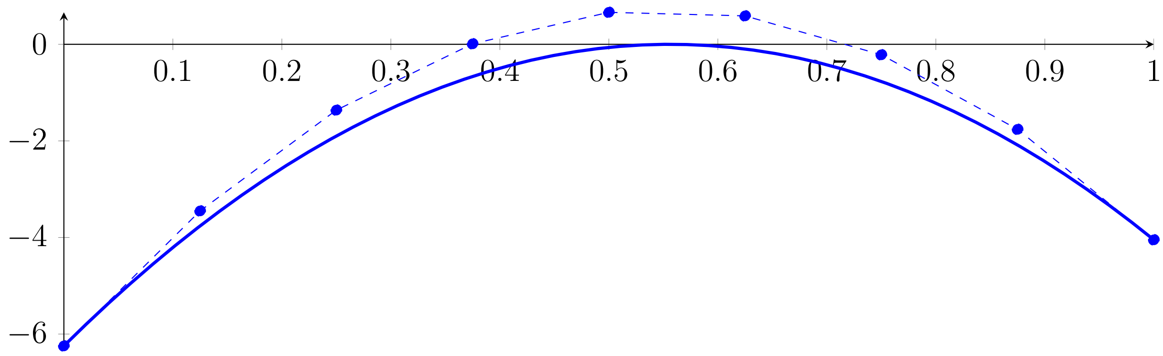
$$[0.5163481231478299, 0.652203482649816]$$

Longest intersection interval: 0.1358553595019861
$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

50.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.507567493156426$

Normalized monomial und Bézier representations and the Bézier polygon:

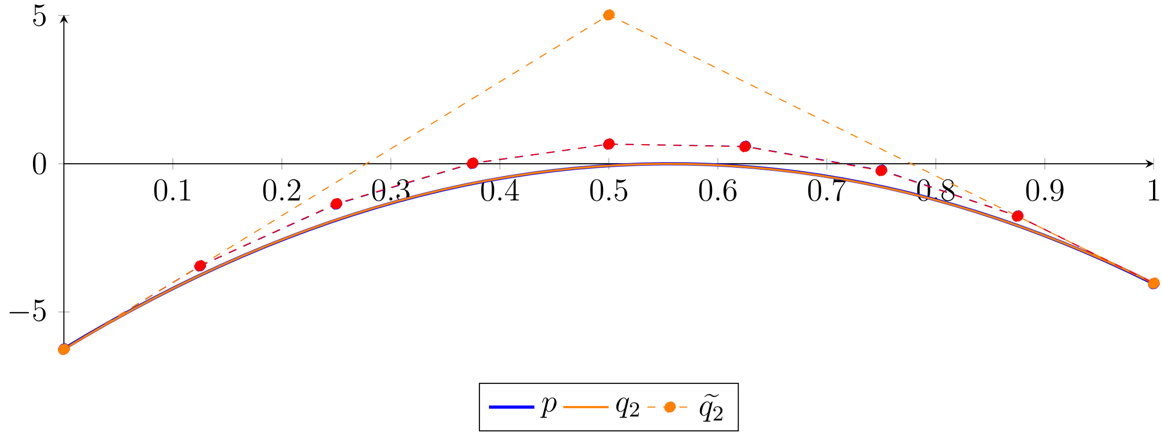
$$\begin{aligned} p = &-7.117496868854 28\cdot 10^{-15}X^8 - 1.625571369884896\cdot 10^{-11}X^7 - 1.539287417434116\cdot 10^{-08}X^6 \\ &- 7.733623372922873\cdot 10^{-06}X^5 - 0.002173482359120639X^4 - 0.3236423633498969X^3 \\ &- 19.84316397394663X^2 + 22.36613094108362X - 6.245496834711843 \\ = &-6.245496834711843B_{0,8}(X) - 3.449730467076391B_{1,8}(X) - 1.36264852708189B_{2,8}(X) \\ &+ 0.009969657354697074B_{3,8}(X) + 0.662313708568421B_{4,8}(X) + 0.5885420610459267B_{5,8}(X) \\ &- 0.217218177224708B_{6,8}(X) - 1.760871363955914B_{7,8}(X) - 4.048353462316386B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q_2} &= 2.334129178460356 \cdot 10^{-302} X^8 - 9.797943630323215 \cdot 10^{-302} X^7 + 1.744532667551292 \cdot 10^{-301} X^6 \\
&\quad - 1.695264192146682 \cdot 10^{-301} X^5 + 9.412284928871313 \cdot 10^{-302} X^4 - 2.760729238908791 \\
&\quad \cdot 10^{-302} X^3 - 20.33236732628744 X^2 + 22.56231184660062 X - 6.261866081804285 \\
&= -6.261866081804285 B_{0,8} - 3.441577100979207 B_{1,8} - 1.347444096092966 B_{2,8} \\
&\quad + 0.02053293285443696 B_{3,8} + 0.6623539858630032 B_{4,8} + 0.5780190629327322 B_{5,8} \\
&\quad - 0.232471835936376 B_{6,8} - 1.769118710744321 B_{7,8} - 4.031921561491104 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.

Bounding polynomials M and m:

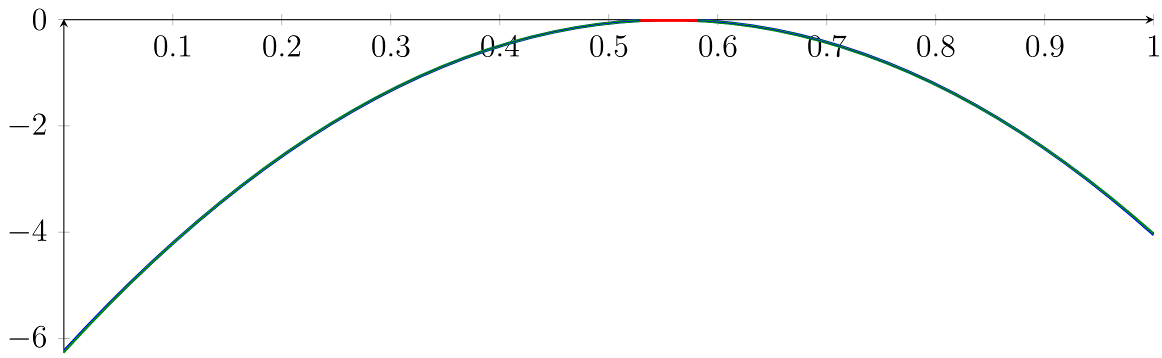
$$M = -20.33236732628744 X^2 + 22.56231184660062 X - 6.245434180979003$$

$$m = -20.33236732628744 X^2 + 22.56231184660062 X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

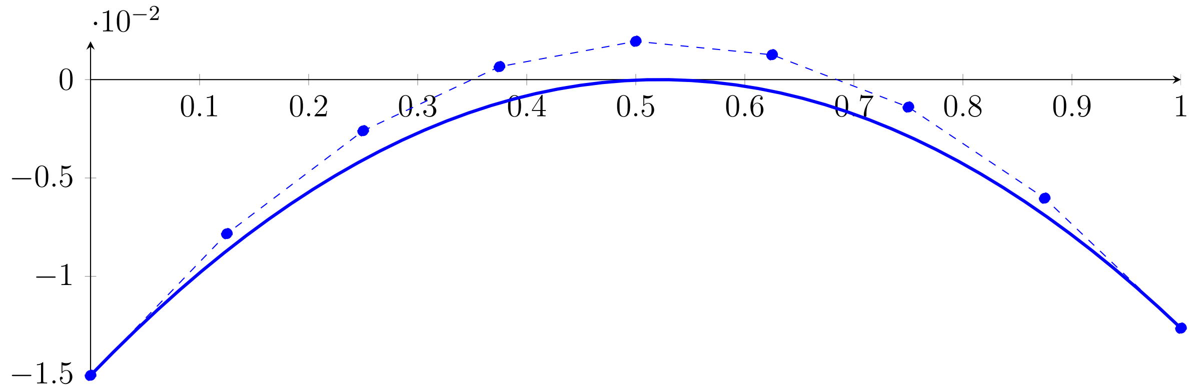
Longest intersection interval: 0.05205162765602134

$\Longrightarrow$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

50.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Normalized monomial und Bézier representations and the Bézier polygon:

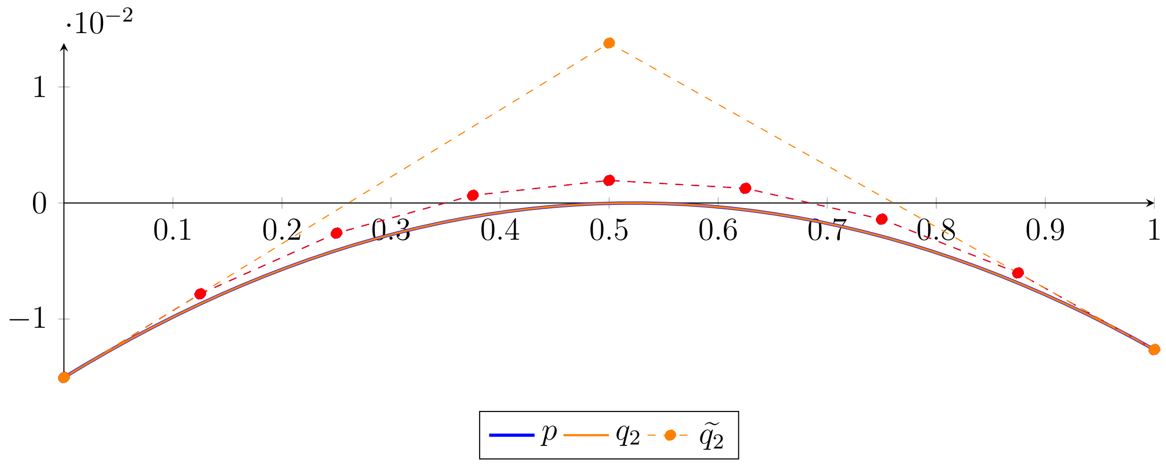
$$p = -3.835321724591002\cdot10^{-25}X^8 - 1.685970393664051\cdot10^{-20}X^7 - 3.073417757909172\cdot10^{-16}X^6 - 2.973678167232232\cdot10^{-12}X^5 - 1.610545215791654\cdot10^{-08}X^4 - 4.629380451575473\cdot10^{-05}X^3 - 0.05516351610802166X^2 + 0.05760784492782384X - 0.01503353559864481$$
$$= -0.01503353559864481B_{0,8}(X) - 0.007832554982666833B_{1,8}(X) - 0.002601699941975341B_{2,8}(X) + 0.0006582028483490249B_{3,8}(X) + 0.001946326483147738B_{4,8}(X) + 0.001261843827131283B_{5,8}(X) - 0.001396072485173959B_{6,8}(X) - 0.006028250049478829B_{7,8}(X) - 0.01263551669178453B_{8,8}(X)$$



Degree reduction and raising:

$$q_2 = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657$$
$$= -0.01503585166965657B_{0,2} + 0.01378196629969571B_{1,2} - 0.01263320016040455B_{2,2}$$

$$\widetilde{q_2} = 6.306459041947834\cdot10^{-305}X^8 - 2.608238530297949\cdot10^{-304}X^7 + 4.573872253581115\cdot10^{-304}X^6 - 4.384946099604334\cdot10^{-304}X^5 + 2.410615466544519\cdot10^{-304}X^4 - 7.036239809064398\cdot10^{-305}X^3 - 0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657$$
$$= -0.01503585166965657B_{0,8} - 0.007831397177318504B_{1,8} - 0.002599549271746596B_{2,8} + 0.0006596920470591496B_{3,8} + 0.001946326779098733B_{4,8} + 0.001260354924372155B_{5,8} - 0.001398223517120585B_{6,8} - 0.006029408545379487B_{7,8} - 0.01263320016040455B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 2.316531379978639\cdot10^{-06}$.
Bounding polynomials M and m:

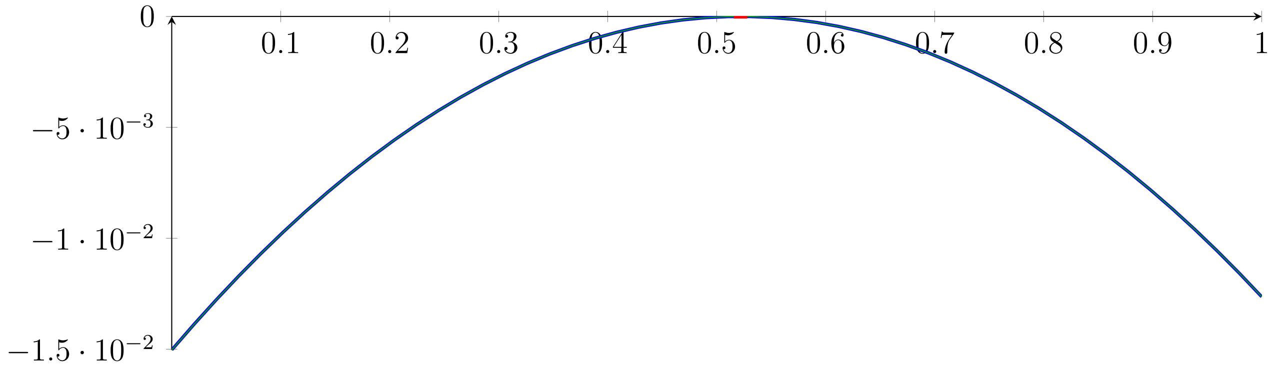
$$M = -0.05523298442945254X^2 + 0.05763563593870456X - 0.0150335351382766$$

$$m = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503816820103655$$

Root of M and m:

$$N(M) = \{0.5154882826278122, 0.5280120194846123\} \qquad N(m) = \{\}$$

Intersection intervals:



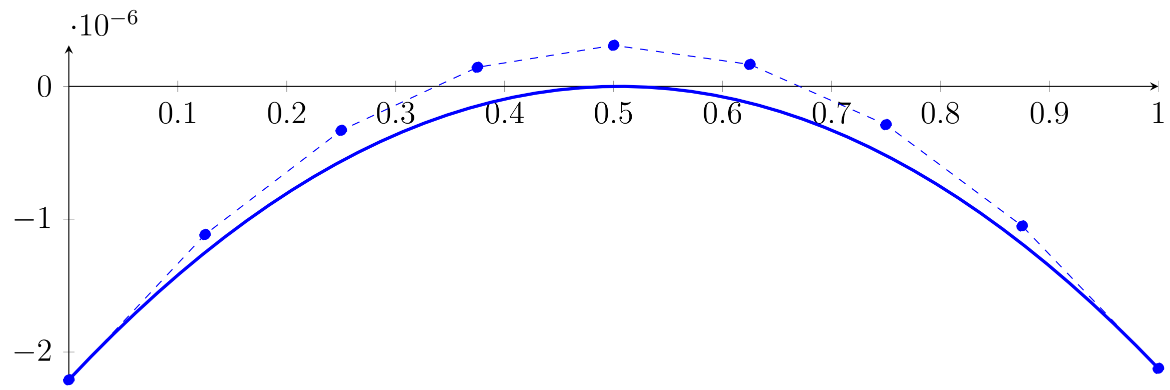
$$[0.5154882826278122, 0.5280120194846123]$$

Longest intersection interval: 0.01252373685680011
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999944147887801, 0.5000055246634291]$,

50.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4999944147887801, 0.500005524$

Normalized monomial und Bézier representations and the Bézier polygon:

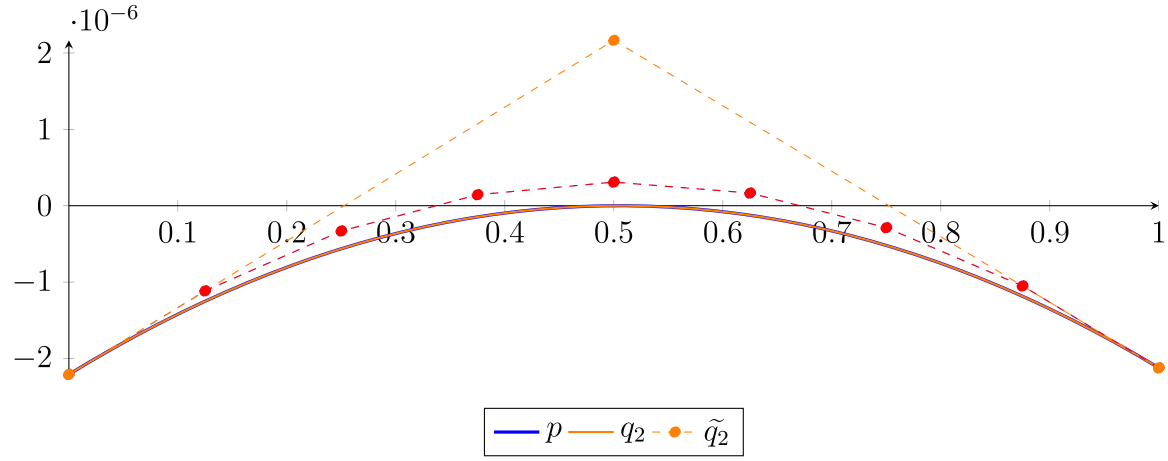
$$\begin{aligned} p = &-2.32099000990716\cdot 10^{-40}X^8 - 8.147572265547704\cdot 10^{-34}X^7 - 1.186072294581764\cdot 10^{-27}X^6 \\ &- 9.164366615560225\cdot 10^{-22}X^5 - 3.963832728671687\cdot 10^{-16}X^4 - 9.099890718270267\cdot 10^{-11}X^3 \\ &- 8.663298447262891\cdot 10^{-06}X^2 + 8.749571419899133\cdot 10^{-06}X - 2.209162411567082\cdot 10^{-06} \\ = &-2.209162411567082\cdot 10^{-06}B_{0,8}(X) - 1.11546598407969\cdot 10^{-06}B_{1,8}(X) - 3.311730725659734 \\ &\cdot 10^{-07}B_{2,8}(X) + 1.437146979935834\cdot 10^{-07}B_{3,8}(X) + 3.091957026128322 \\ &\cdot 10^{-07}B_{4,8}(X) + 1.652683162999621\cdot 10^{-07}B_{5,8}(X) - 2.880690859425\cdot 10^{-07}B_{6,8}(X) \\ &- 1.05081812911769\cdot 10^{-06}B_{7,8}(X) - 2.122980438234407\cdot 10^{-06}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209166961546417\cdot 10^{-06} \\ &= -2.209166961546417\cdot 10^{-06}B_{0,2} + 2.165646048256509\cdot 10^{-06}B_{1,2} - 2.122975888243747\cdot 10^{-06}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 9.84075548060145 \cdot 10^{-309} X^8 - 4.039091668354938 \cdot 10^{-308} X^7 + 7.030443670929191 \cdot 10^{-308} X^6 \\ &\quad - 6.698784216784483 \cdot 10^{-308} X^5 + 3.668633643168221 \cdot 10^{-308} X^4 - 1.069248215117909 \cdot 10^{-308} X^3 \\ &\quad - 8.663434946303181 \cdot 10^{-06} X^2 + 8.749626019605851 \cdot 10^{-06} X - 2.209166961546417 \cdot 10^{-06} \\ &= -2.209166961546417 \cdot 10^{-06} B_{0,8} - 1.115463709095685 \cdot 10^{-06} B_{1,8} - 3.311688475843534 \cdot 10^{-07} B_{2,8} \\ &\quad + 1.437176229875793 \cdot 10^{-07} B_{3,8} + 3.091957026201127 \cdot 10^{-07} B_{4,8} + 1.652653913132468 \cdot 10^{-07} B_{5,8} \\ &\quad - 2.880733109330184 \cdot 10^{-07} B_{6,8} - 1.050820404118683 \cdot 10^{-06} B_{7,8} - 2.122975888243747 \cdot 10^{-06} B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.549990660244254 \cdot 10^{-12}$.

Bounding polynomials M and m:

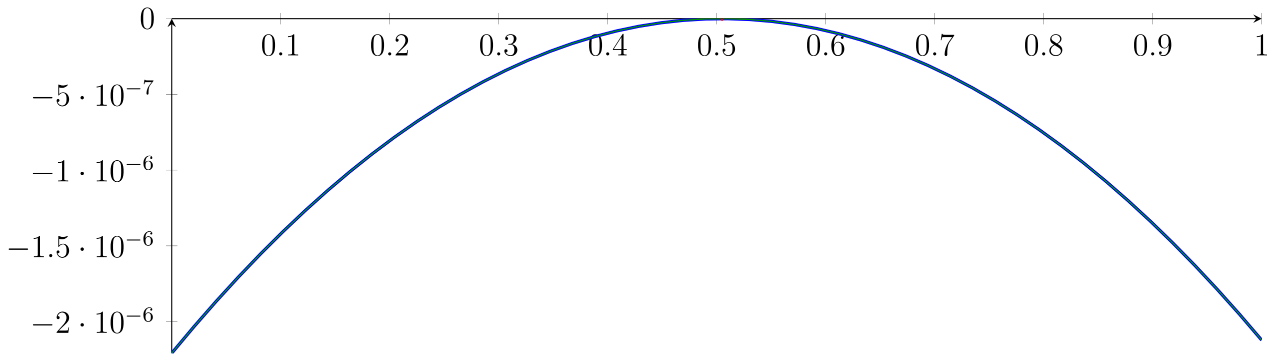
$$M = -8.663434946303181 \cdot 10^{-06} X^2 + 8.749626019605851 \cdot 10^{-06} X - 2.209162411555757 \cdot 10^{-06}$$

$$m = -8.663434946303181 \cdot 10^{-06} X^2 + 8.749626019605851 \cdot 10^{-06} X - 2.209171511537077 \cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{0.5041259457474217, 0.505822887923852\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5041259457474217, 0.505822887923852]$$

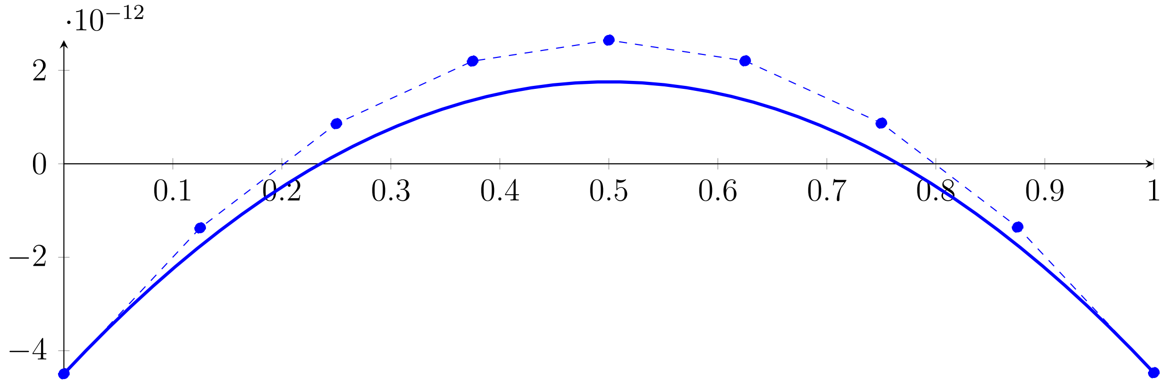
Longest intersection interval: 0.00169694217643035

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000155648447, 0.5000000344176595]$,

50.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.5000000155648447, 0.500000034$

Normalized monomial und Bézier representations and the Bézier polygon:

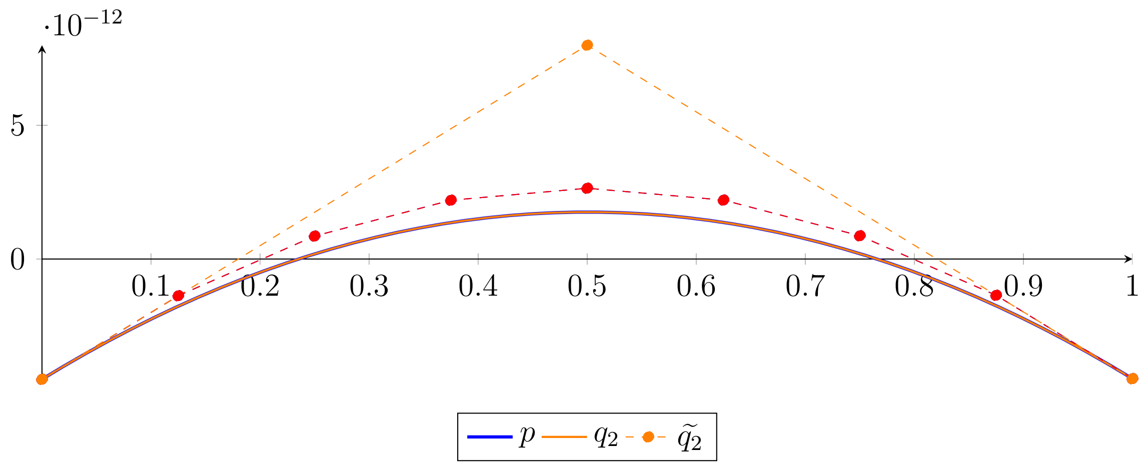
$$\begin{aligned}p &= -1.595914506899923\cdot 10^{-62}X^8 - 3.301399092257474\cdot 10^{-53}X^7 - 2.83213843356941\cdot 10^{-44}X^6\\ &\quad - 1.289553529043531\cdot 10^{-35}X^5 - 3.286896476505693\cdot 10^{-27}X^4 - 4.446733715730271\cdot 10^{-19}X^3\\ &\quad - 2.494734097474612\cdot 10^{-11}X^2 + 2.497049303064054\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}\\ &= -4.493680200151054\cdot 10^{-12}B_{0,8}(X) - 1.372368571320986\cdot 10^{-12}B_{1,8}(X) + 8.579665941252918\\ &\quad \cdot 10^{-13}B_{2,8}(X) + 2.197325288247184\cdot 10^{-12}B_{3,8}(X) + 2.645707503104094\cdot 10^{-12}B_{4,8}(X)\\ &\quad + 2.203113230755426\cdot 10^{-12}B_{5,8}(X) + 8.695424632605847\cdot 10^{-13}B_{6,8}(X)\\ &\quad - 1.355004807321027\cdot 10^{-12}B_{7,8}(X) - 4.470528588930005\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12}\\ &= -4.493680222384723\cdot 10^{-12}B_{0,2} + 7.991566426337562\cdot 10^{-12}B_{1,2} - 4.470528566696336\cdot 10^{-12}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 3.394595061927522\cdot 10^{-314}X^8 - 1.386657030412925\cdot 10^{-313}X^7 + 2.314937908564659\cdot 10^{-313}X^6\\ &\quad - 2.018101017184903\cdot 10^{-313}X^5 + 9.746671569929334\cdot 10^{-314}X^4 - 2.577278121543316\cdot 10^{-314}X^3\\ &\quad - 2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12}\\ &= -4.493680222384723\cdot 10^{-12}B_{0,8} - 1.372368560204152\cdot 10^{-12}B_{1,8} + 8.579666147708415\cdot 10^{-13}B_{2,8}\\ &\quad + 2.197325302540257\cdot 10^{-12}B_{3,8} + 2.645707503104094\cdot 10^{-12}B_{4,8} + 2.203113216462353\cdot 10^{-12}B_{5,8}\\ &\quad + 8.695424426150349\cdot 10^{-13}B_{6,8} - 1.355004818437861\cdot 10^{-12}B_{7,8} - 4.470528566696336\cdot 10^{-12}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.223366895429667\cdot 10^{-20}$.

Bounding polynomials M and m:

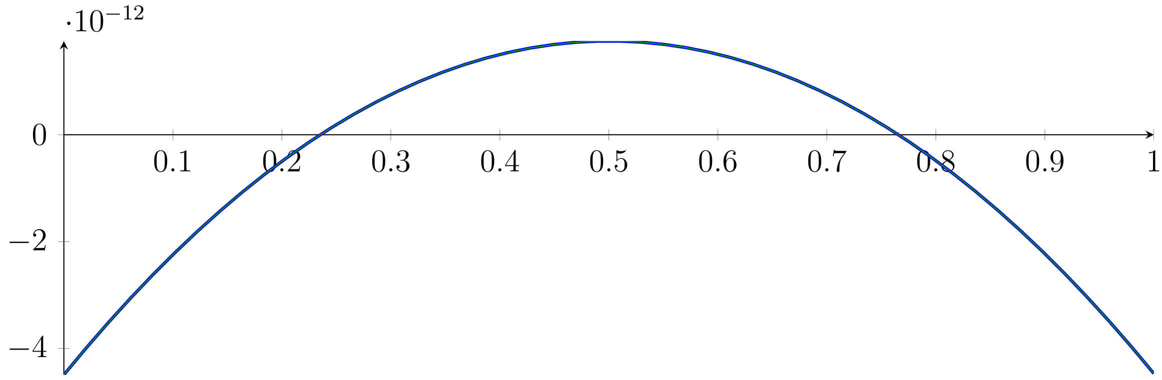
$$M = -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}$$

$$m = -2.494734164175618 \cdot 10^{-11} X^2 + 2.497049329744457 \cdot 10^{-11} X - 4.493680244618392 \cdot 10^{-12}$$

Root of M and m:

$N(M) = \{0.235251622185842, 0.765676398764079\}$ $N(m) = \{0.2352516255462581, 0.7656763954036629\}$

Intersection intervals:



$$[0.235251622185842, 0.2352516255462581], \ [0.7656763954036629, 0.765676398764079]$$

Longest intersection interval: $3.360416099786671 \cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.50000002]$, interval 2: $[0.50000003, 0.500000030000$

50.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000$

Found root in interval $[0.5000000199999999, 0.50000002]$ at recursion depth 8!

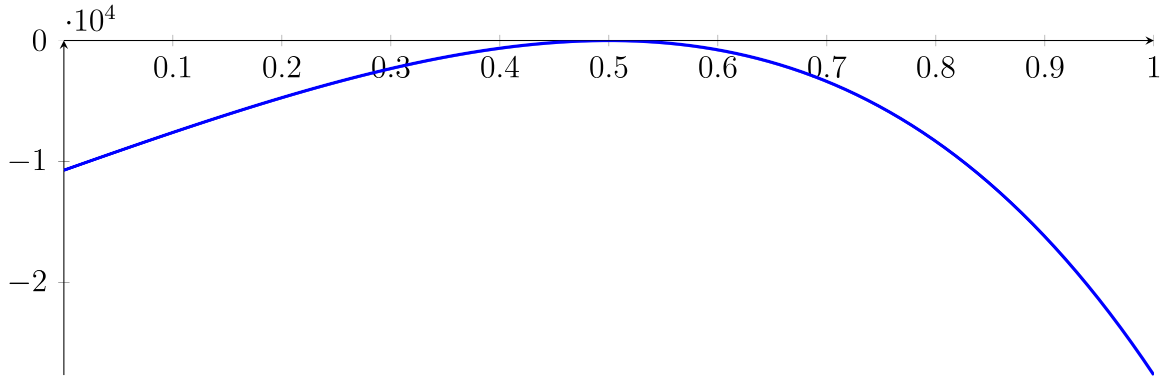
50.9 Recursion Branch 1 1 1 1 1 1 1 2 in Interval 2: $[0.50000003, 0.500000030000000$

Found root in interval $[0.50000003, 0.5000000300000001]$ at recursion depth 8!

50.10 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.5000000199999999, 0.50000002],\ [0.50000003, 0.5000000300000001]$$

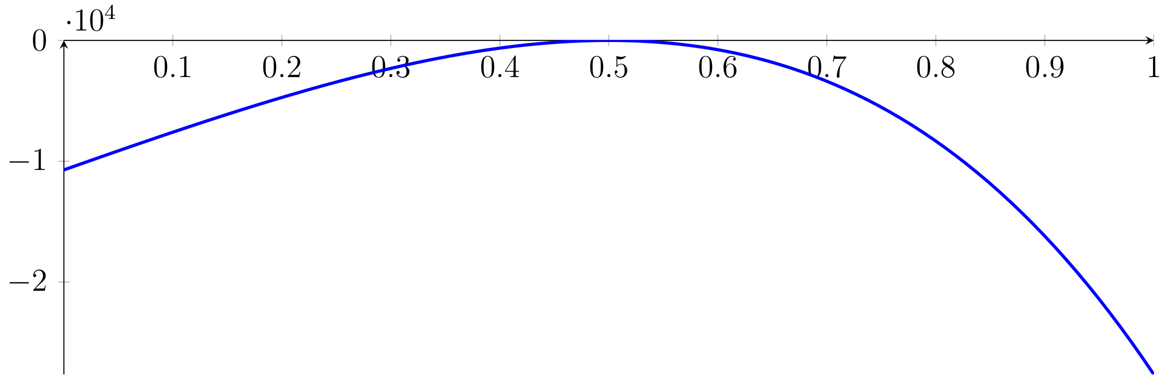
with precision $\varepsilon = 1{\cdot}10^{-08}$.

51 Running `CubeClip` on h_8 with epsilon 8

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

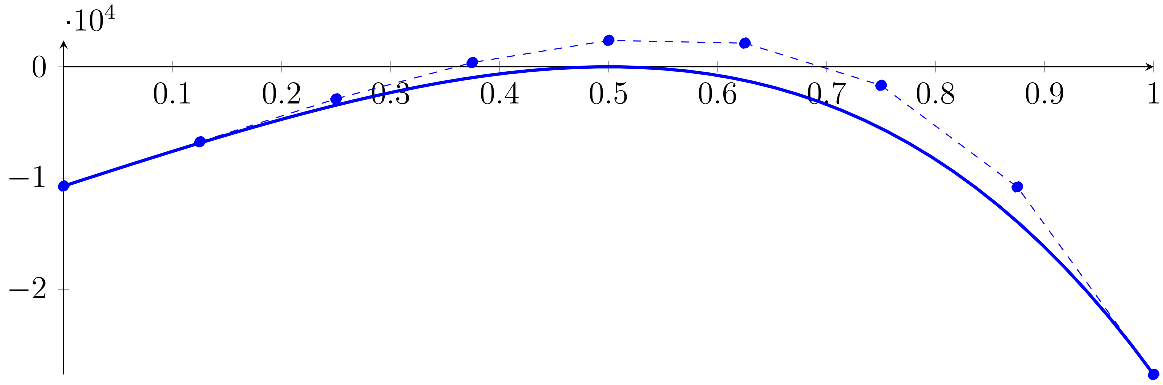
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



51.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

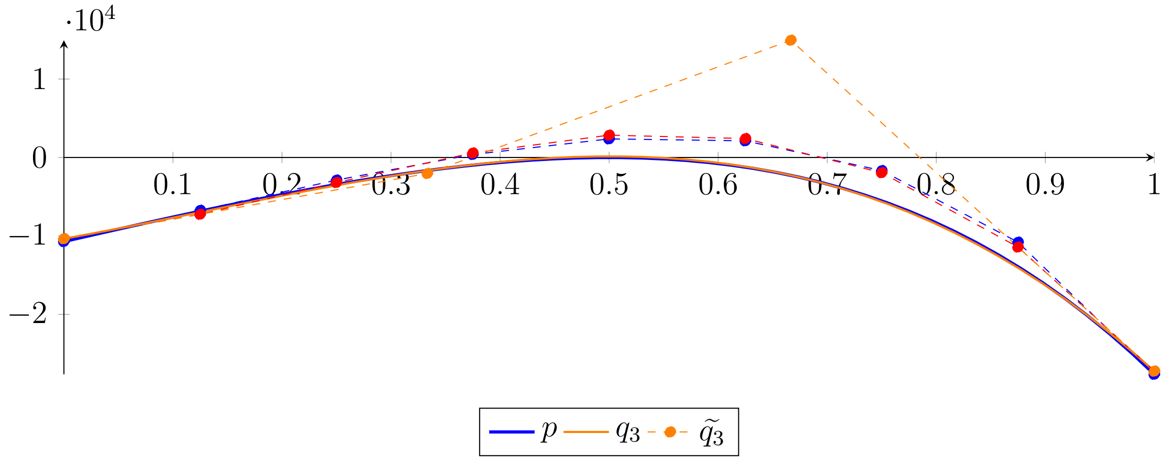
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ = &-10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &+ 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q_3} &= -5.703152099307112 \cdot 10^{-299} X^8 + 1.997854926822688 \cdot 10^{-298} X^7 - 2.620079279971631 \\
&\quad \cdot 10^{-298} X^6 + 1.512761996617561 \cdot 10^{-298} X^5 - 2.636105166170616 \cdot 10^{-299} X^4 \\
&\quad - 67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10364.86272786554 \\
&= -10364.86272786554 B_{0,8} - 7243.22568371353 B_{1,8} - 3192.719831416224 B_{2,8} \\
&\quad + 574.7343076805747 B_{3,8} + 2847.216212231063 B_{4,8} + 2412.80536088944 B_{5,8} \\
&\quad - 1940.418767690097 B_{6,8} - 11424.37669485335 B_{7,8} - 27250.98894194612 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

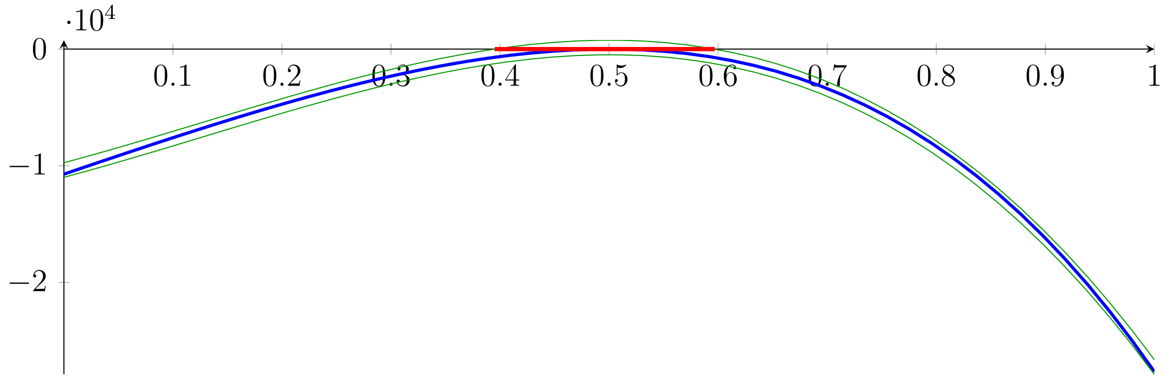
$$M = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 9740.484261812251$$

$$m = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $\quad N(m) = \{-0.623487965792376$

Intersection intervals:



$$[0.3950604114095972, 0.5968461976869074]$$

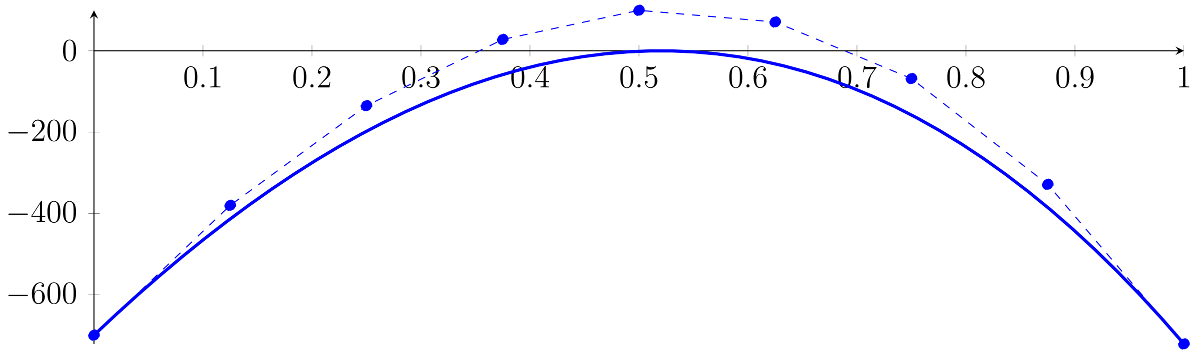
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

51.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

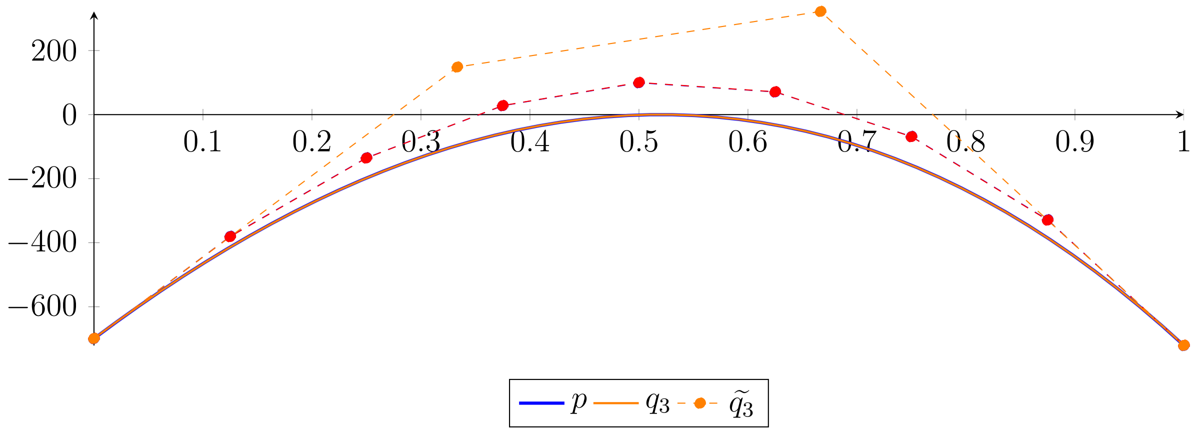
$$\begin{aligned} p &= -2.748682461629333\cdot10^{-06}X^8 - 0.0005198138726602274X^7 - 0.04066637698425721X^6 \\ &\quad - 1.681520506726176X^5 - 38.59639865464246X^4 - 460.2831920090805X^3 \\ &\quad - 2074.780222477586X^2 + 2553.796807278985X - 699.3475151603227 \\ &= -699.3475151603227B_{0,8}(X) - 380.1229142504495B_{1,8}(X) - 134.9976070004902B_{2,8}(X) \\ &\quad + 27.8090638751075B_{3,8}(X) + 99.52637853825791B_{4,8}(X) + 70.80221287533186B_{5,8}(X) \\ &\quad - 68.32844102535583B_{6,8}(X) - 328.4764717433994B_{7,8}(X) - 720.9332304689123B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323 \\ &= -698.7407882409323B_{0,3} + 148.5035278252892B_{1,3} \\ &\quad + 322.0745267793334B_{2,3} - 720.3121660793493B_{3,3} \end{aligned}$$

$$\begin{aligned} \tilde{q}_3 &= -2.24002635483637\cdot10^{-300}X^8 + 8.593407551274415\cdot10^{-300}X^7 - 1.289726574039331 \\ &\quad \cdot10^{-299}X^6 + 9.445830854385802\cdot10^{-300}X^5 - 3.303106122422159\cdot10^{-300}X^4 \\ &\quad - 542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323 \\ &= -698.7407882409323B_{0,8} - 381.0241697160992B_{1,8} - 135.4868351675709B_{2,8} \\ &\quad + 28.18756585642868B_{3,8} + 100.3153838076753B_{4,8} + 71.21296913794494B_{5,8} \\ &\quad - 68.80332770098655B_{6,8} - 329.4171562573433B_{7,8} - 720.3121660793493B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

Bounding polynomials M and m:

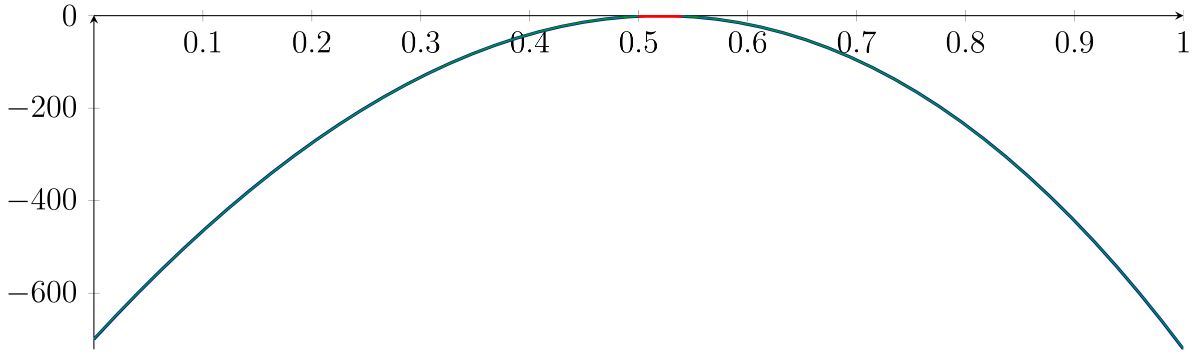
$$M = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 697.8001037269884$$

$$m = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 699.6814727548762$$

Root of M and m:

$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\}$ $\quad N(m) = \{-4.76690070753073\}$

Intersection intervals:



$[0.4997746332555131, 0.5401382492675976]$

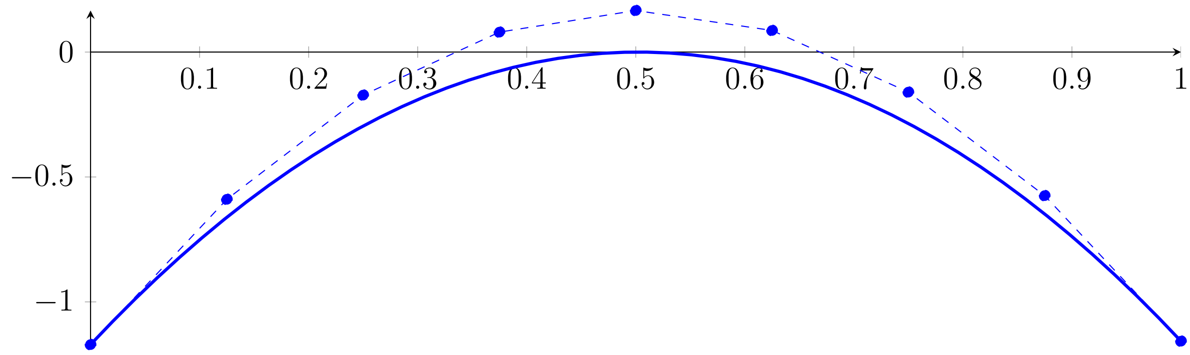
Longest intersection interval: 0.04036361601208447

$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

51.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Normalized monomial und Bézier representations and the Bézier polygon:

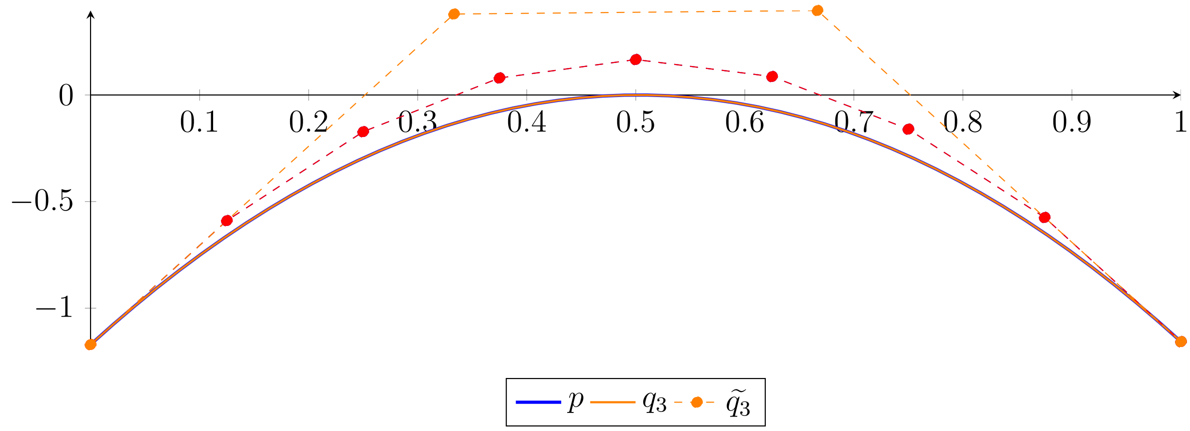
$$\begin{aligned}
p &= -1.936623061789877 \cdot 10^{-17} X^8 - 9.265403983052604 \cdot 10^{-14} X^7 - 1.838110077974511 \cdot 10^{-10} X^6 \\
&\quad - 1.935168004397404 \cdot 10^{-07} X^5 - 0.0001140127070697086 X^4 - 0.0356257658581906 X^3 \\
&\quad - 4.602344943530705 X^2 + 4.651752639476855 X - 1.170857231885768 \\
&= -1.170857231885768 B_{0,8}(X) - 0.5893881519511612 B_{1,8}(X) - 0.172288534285508 B_{2,8}(X) \\
&\quad + 0.07980544672086649 B_{3,8}(X) + 0.1662559879246795 B_{4,8}(X) + 0.08642365397403269 B_{5,8}(X) \\
&\quad - 0.1603326261538093 B_{6,8}(X) - 0.574655562621217 B_{7,8}(X) - 1.157189508205582 B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= -0.03585432943204529 X^3 - 4.60219789441901 X^2 + 4.65171994907147 X - 1.170855596980661 \\
&= -1.170855596980661 B_{0,3} + 0.3797177193764957 B_{1,3} \\
&\quad + 0.3962250709273155 B_{2,3} - 1.157187871760247 B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -3.713970905558003 \cdot 10^{-303} X^8 + 1.449155002864004 \cdot 10^{-302} X^7 - 2.225710229630734 \\
&\quad \cdot 10^{-302} X^6 + 1.689258272787799 \cdot 10^{-302} X^5 - 6.343129302139404 \cdot 10^{-303} X^4 \\
&\quad - 0.03585432943204529 X^3 - 4.60219789441901 X^2 + 4.65171994907147 X - 1.170855596980661 \\
&= -1.170855596980661 B_{0,8} - 0.5893906033467271 B_{1,8} - 0.1722898202277581 B_{2,8} \\
&\quad + 0.07980649649353122 B_{3,8} + 0.1662580909344257 B_{4,8} + 0.08642470721221019 B_{5,8} \\
&\quad - 0.1603339105558303 B_{6,8} - 0.574658018252411 B_{7,8} - 1.157187871760247 B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.45563119394813 \cdot 10^{-06}$.
Bounding polynomials M and m:

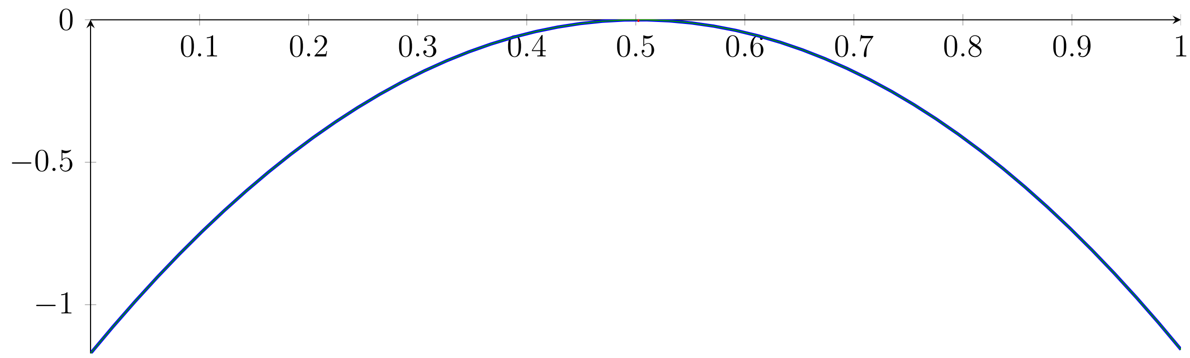
$$M = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170853141349467$$

$$m = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170858052611855$$

Root of M and m:

$$N(M) = \{-129.3630802555855, 0.5016184355695172, 0.5032421204520033\} \quad N(m) = \{-129.3630802637075$$

Intersection intervals:



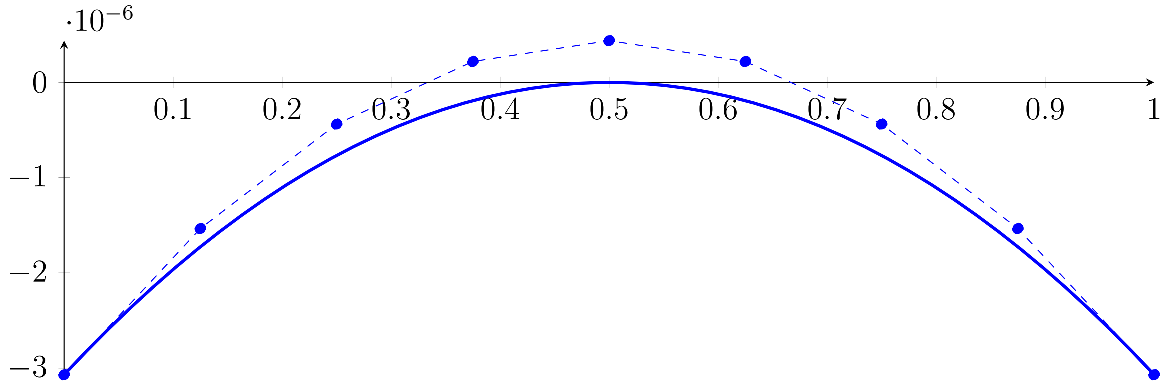
$$[0.5016184355695172, 0.5032421204520033]$$

Longest intersection interval: 0.001623684882486142
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999934125800028, 0.5000066371751187]$,

51.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4999934125800028, 0.50000663717511$

Normalized monomial und Bézier representations and the Bézier polygon:

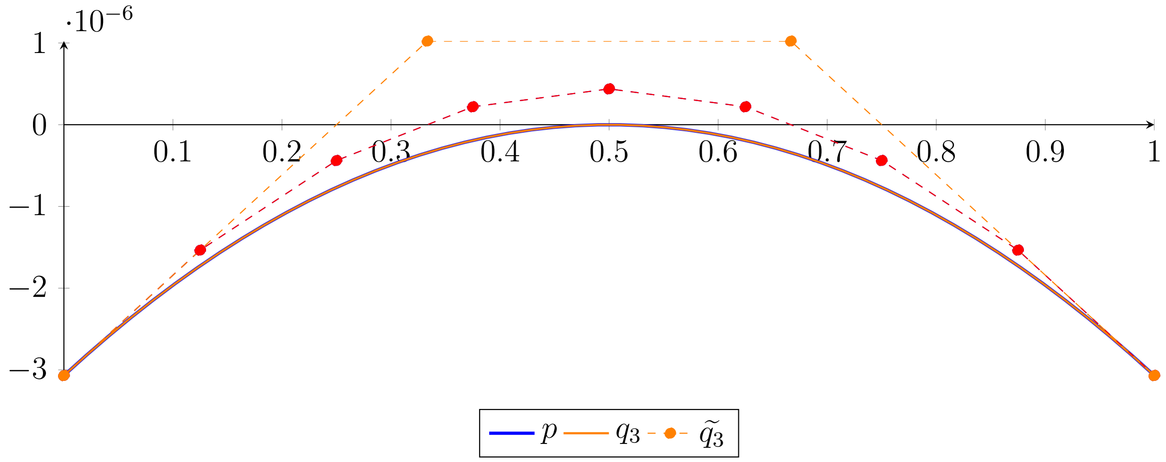
$$\begin{aligned} p = &-9.355329246270906 \cdot 10^{-40} X^8 - 2.758930189698976 \cdot 10^{-33} X^7 - 3.374040776758279 \cdot 10^{-27} X^6 \\ &- 2.190121853313308 \cdot 10^{-21} X^5 - 7.958070257617345 \cdot 10^{-16} X^4 - 1.534811947451426 \cdot 10^{-10} X^3 \\ &- 1.227519762250185 \cdot 10^{-05} X^2 + 1.227554003668957 \cdot 10^{-05} X - 3.068949673836091 \cdot 10^{-06} \\ = &-3.068949673836091 \cdot 10^{-06} B_{0,8}(X) - 1.534507169249895 \cdot 10^{-06} B_{1,8}(X) - 4.384645797530504 \\ &\cdot 10^{-07} B_{2,8}(X) + 2.191753539188221 \cdot 10^{-07} B_{3,8}(X) + 4.384098910187333 \cdot 10^{-07} B_{4,8}(X) \\ &+ 2.192362907883254 \cdot 10^{-07} B_{5,8}(X) - 4.383481875421282 \cdot 10^{-07} B_{6,8}(X) \\ &- 1.534346284753723 \cdot 10^{-06} B_{7,8}(X) - 3.068760741638923 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2 \\ &\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673824723\cdot 10^{-06} \\ &= -3.068949673824723\cdot 10^{-06}B_{0,3} + 1.022897004996009\cdot 10^{-06}B_{1,3} \\ &\quad + 1.023011143323854\cdot 10^{-06}B_{2,3} - 3.068760741627554\cdot 10^{-06}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -9.792734715851906\cdot 10^{-309}X^8 + 3.822699025575463\cdot 10^{-308}X^7 - 5.87427002023018\cdot 10^{-308}X^6 \\ &\quad + 4.462726908063245\cdot 10^{-308}X^5 - 1.679538448591103\cdot 10^{-308}X^4 - 1.534827863652778\cdot 10^{-10}X^3 \\ &\quad - 1.227519762147866\cdot 10^{-05}X^2 + 1.22755400364622\cdot 10^{-05}X - 3.068949673824723\cdot 10^{-06} \\ &= -3.068949673824723\cdot 10^{-06}B_{0,8} - 1.534507169266948\cdot 10^{-06}B_{1,8} - 4.38464579761983\cdot 10^{-07}B_{2,8} \\ &\quad + 2.191753539261305\cdot 10^{-07}B_{3,8} + 4.384098910333502\cdot 10^{-07}B_{4,8} + 2.192362907956339\cdot 10^{-07}B_{5,8} \\ &\quad - 4.383481875510608\cdot 10^{-07}B_{6,8} - 1.534346284770776\cdot 10^{-06}B_{7,8} - 3.068760741627554\cdot 10^{-06}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.705314892341177\cdot 10^{-17}$.
Bounding polynomials M and m:

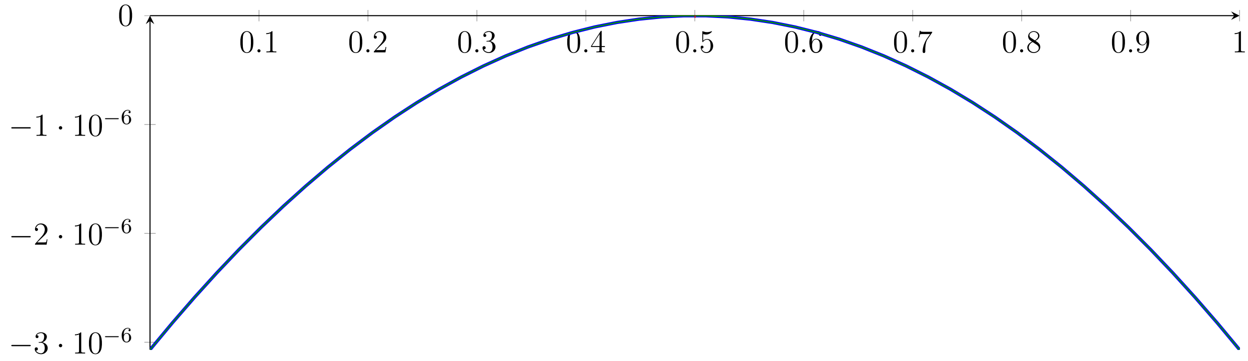
$$\begin{aligned} M &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2 \\ &\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673807669\cdot 10^{-06} \end{aligned}$$

$$\begin{aligned} m &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2 \\ &\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673841776\cdot 10^{-06} \end{aligned}$$

Root of M and m:

$N(M) = \{-79978.68293772447, 0.4996311727354866, 0.5003873441571286\}$ $\quad N(m) = \{-79978.68293772447,$

Intersection intervals:



$$[0.4996311727354866, 0.4996311764098305],\ [0.5003873404827848, 0.5003873441571286]$$

Longest intersection interval: $3.674343879435088 \cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999695, 0.5000000200000181]$, interval 2: $[0.500000029999981$

51.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.5000000199999695, 0.500000020000$

Found root in interval $[0.5000000199999695, 0.5000000200000181]$ at recursion depth 5!

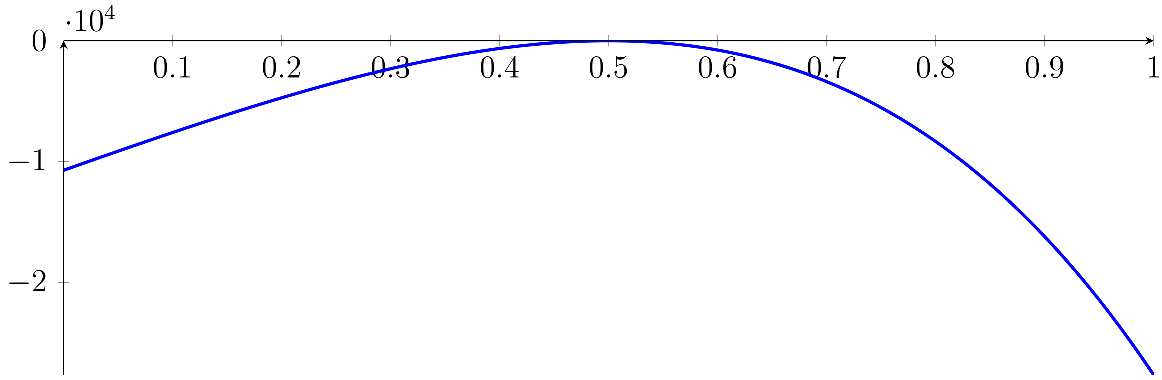
51.6 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.5000000299999818, 0.500000030000$

Found root in interval $[0.5000000299999818, 0.5000000300000304]$ at recursion depth 5!

51.7 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.5000000199999695, 0.5000000200000181],\ [0.5000000299999818, 0.5000000300000304]$$

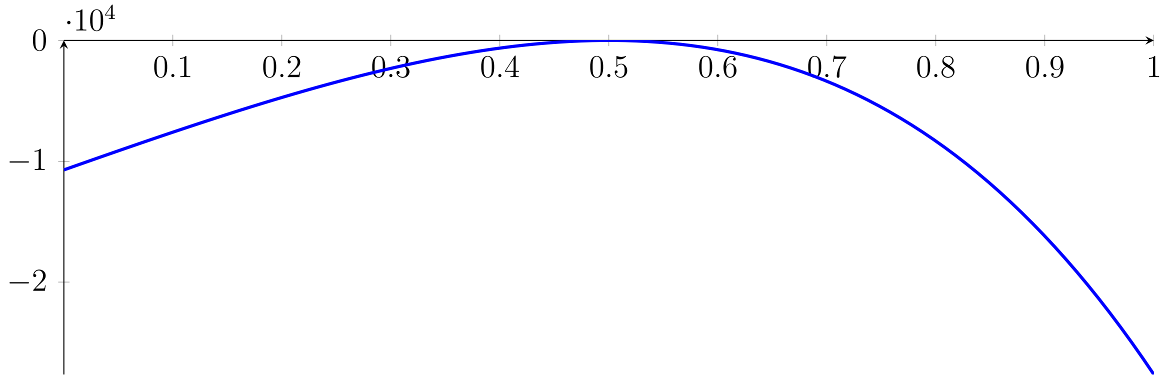
with precision $\varepsilon = 1{\cdot}10^{-08}$.

52 Running `BezClip` on h_8 with epsilon 16

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

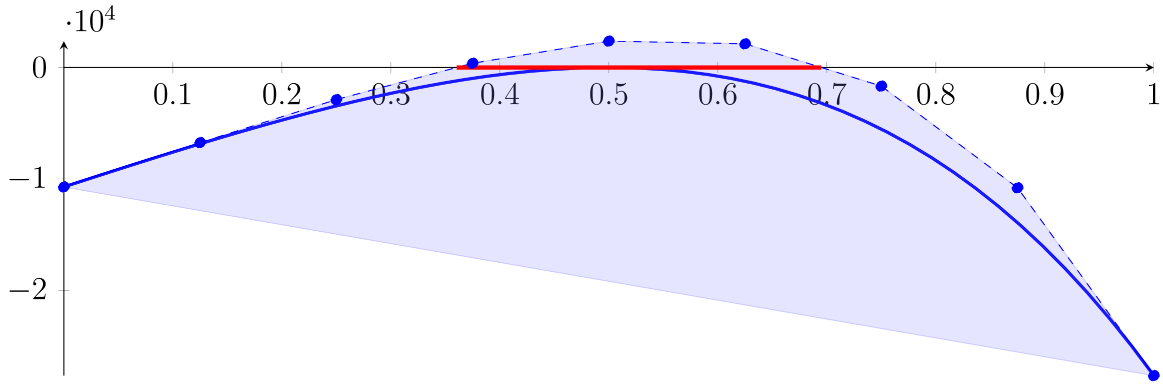
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



52.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

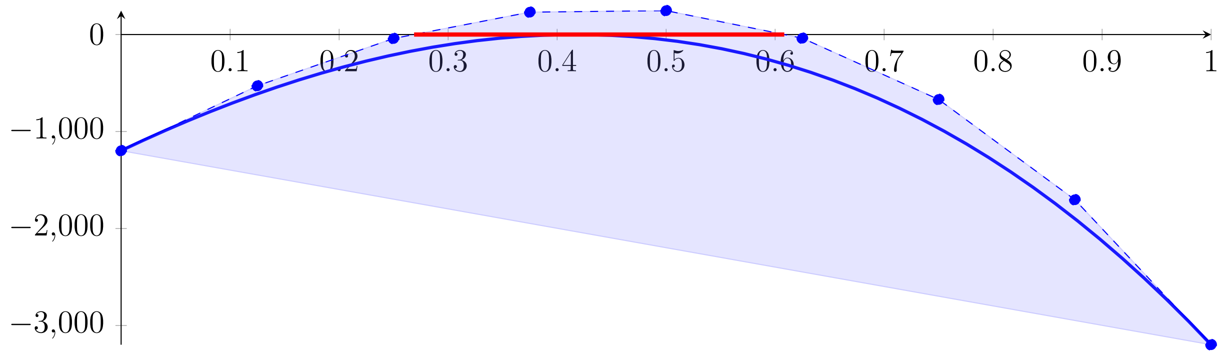
$$[0.3603640692588709, 0.6949437907012195]$$

Longest intersection interval: 0.3345797214423486
$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

52.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6\\&- 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3\\&- 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098\\= &-1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X)\\&+ 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X)\\&- 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

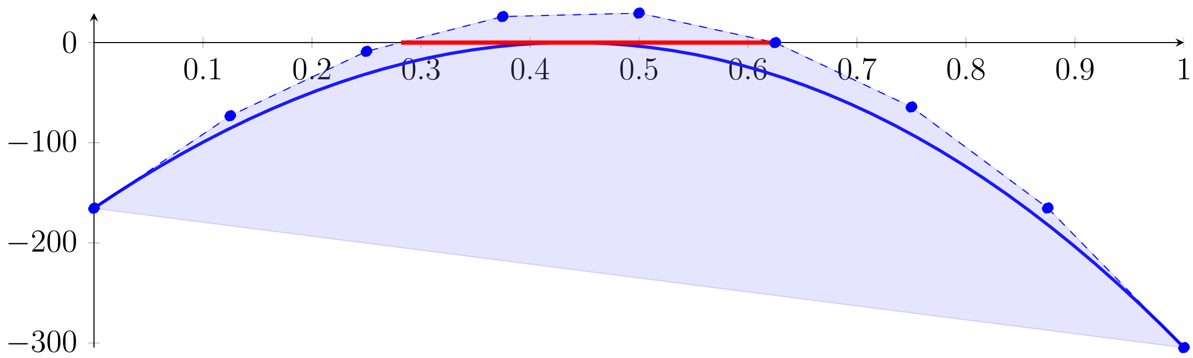
$$[0.2687909235445741, 0.6084084634355219]$$

Longest intersection interval: 0.3396175398909478
$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

52.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.779187295559603\cdot10^{-08}X^8 - 9.441522673365855\cdot10^{-06}X^7 - 0.001328615938017263X^6\\&- 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3\\&- 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987\\= &-165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X)\\&+ 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X)\\&- 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

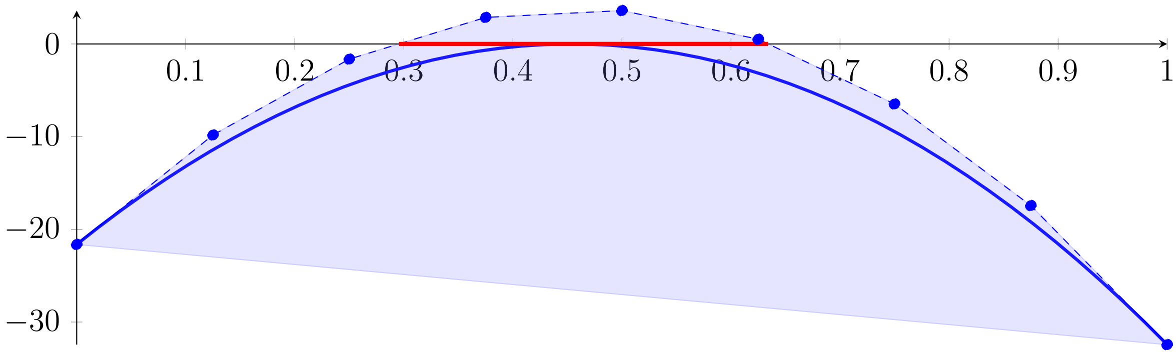
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\implies$ Selective recursion: interval 1: $[0.4822990094256734, 0.5212832145711177]$,

52.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4822990094256734, 0.52128321457111$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-5.334693469973702 \cdot 10^{-12} X^8 - 5.317476896558791 \cdot 10^{-09} X^7 - 2.197128069269196 \cdot 10^{-06} X^6 \\
&- 0.000481521724173462 X^5 - 0.05899466848037266 X^4 - 3.82353929224187 X^3 \\
&- 101.3899724153744 X^2 + 94.46052284159652 X - 21.62667247491518 \\
= &-21.62667247491518 B_{0,8}(X) - 9.819107119715613 B_{1,8}(X) - 1.632612207922276 B_{2,8}(X) \\
&+ 2.86453477310337 B_{3,8}(X) + 3.603213555021573 B_{4,8}(X) + 0.5134524899120698 B_{5,8}(X) \\
&- 6.475580126796985 B_{6,8}(X) - 17.43558481253364 B_{7,8}(X) - 32.43913973359036 B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

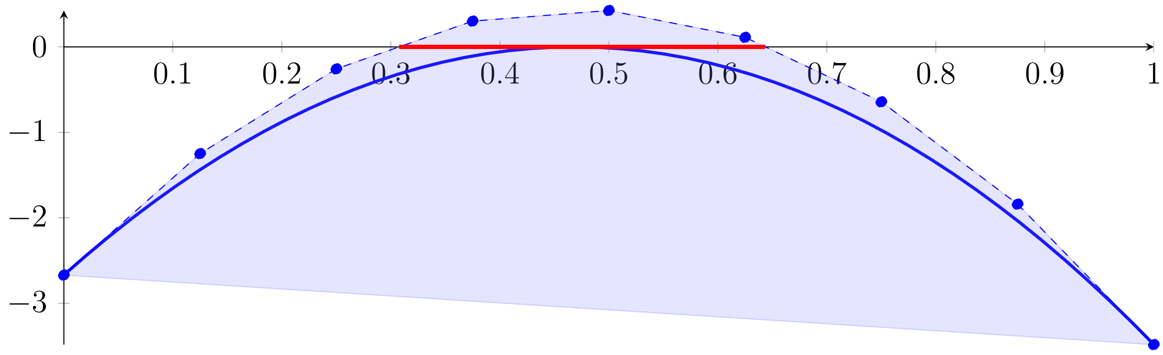
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\implies$ Selective recursion: interval 1: $[0.4938141292321744, 0.5070221367077007]$,

52.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4938141292321744, 0.507022136707$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-9.261865042605128 \cdot 10^{-16} X^8 - 2.731330938941274 \cdot 10^{-12} X^7 - 3.339777336661422 \\
&\cdot 10^{-09} X^6 - 2.167033369334019 \cdot 10^{-06} X^5 - 0.0007867416616259877 X^4 \\
&- 0.1514273448103067 X^3 - 12.0308548973337 X^2 + 11.3691349552019 X - 2.67015106585462 \\
= &-2.67015106585462 B_{0,8}(X) - 1.249009196454382 B_{1,8}(X) - 0.2575407162446335 B_{2,8}(X) \\
&+ 0.3015503150458701 B_{3,8}(X) + 0.4255485985217787 B_{4,8}(X) + 0.1117275574241232 B_{5,8}(X) \\
&- 0.6426507016859872 B_{6,8}(X) - 1.840335427863286 B_{7,8}(X) - 3.484087264834231 B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

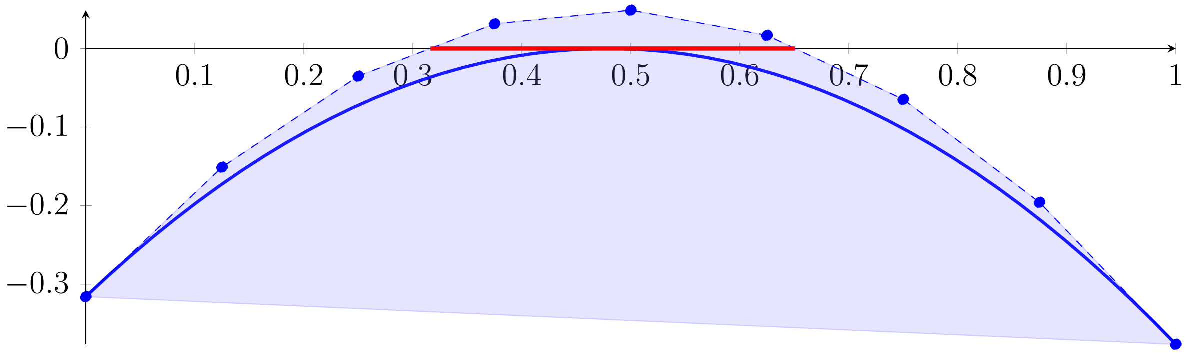
$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\Longrightarrow$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

52.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.502170909909546 \cdot 10^{-19} X^8 - 1.319790000383349 \cdot 10^{-15} X^7 - 4.808366303277033 \cdot 10^{-12} X^6 \\ &- 9.297436410097437 \cdot 10^{-09} X^5 - 1.006192361000331 \cdot 10^{-05} X^4 - 0.005777437133356019 X^3 \\ &- 1.373512346650201 X^2 + 1.318590421802199 X - 0.3158295498425207 \\ = &-0.3158295498425207 B_{0,8}(X) - 0.1510057471172458 B_{1,8}(X) - 0.03523595677233526 B_{2,8}(X) \\ &+ 0.03137665267197247 B_{3,8}(X) + 0.04872876895367301 B_{4,8}(X) + 0.01671693590297049 B_{5,8}(X) \\ &- 0.06476244672391982 B_{6,8}(X) - 0.1958131234111408 B_{7,8}(X) - 0.376538983049735 B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3161210337394805, 0.6506459600024226\}$$

Intersection intervals with the x axis:

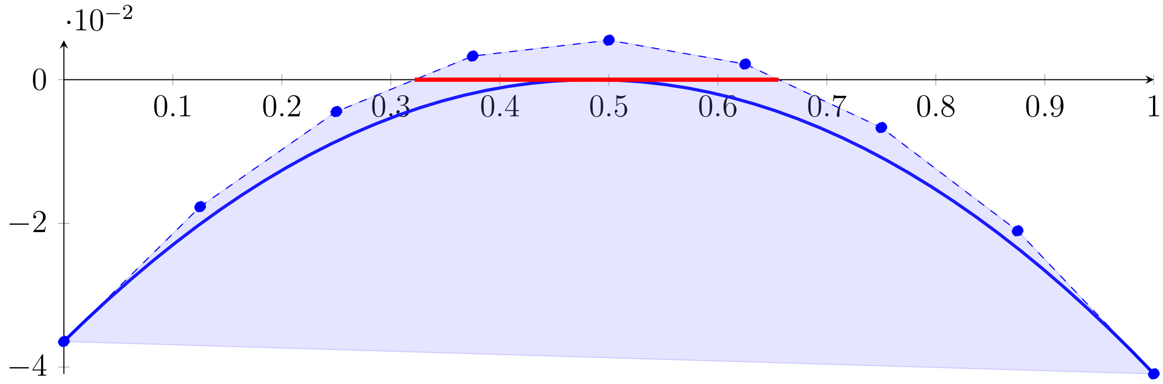
$$[0.3161210337394805, 0.6506459600024226]$$

Longest intersection interval: 0.3345249262629421
$\Longrightarrow$ Selective recursion: interval 1: $[0.499279281802493, 0.5007635705740208]$,

52.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.499279281802493, 0.5007635705$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.355847730899963\cdot10^{-23}X^8 - 6.189124375986835\cdot10^{-19}X^7 - 6.742674354644286\cdot10^{-15}X^6 - 3.898805488577673\cdot10^{-11}X^5 - 1.261912061383275\cdot10^{-07}X^4 - 0.000216758797755982X^3 - 0.154319370565961X^2 + 0.1500226550818807X - 0.0364365303427455$$
$$= -0.0364365303427455B_{0,8}(X) - 0.01768369845751041B_{1,8}(X) - 0.004442272663916792B_{2,8}(X) + 0.003283876345218297B_{3,8}(X) + 0.005490876074346265B_{4,8}(X) + 0.002174852224490793B_{5,8}(X) - 0.006668071307448625B_{6,8}(X) - 0.02104177242939338B_{7,8}(X) - 0.04095013085478269B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3218707447051634, 0.6557428337562166\}$$

Intersection intervals with the x axis:

$$[0.3218707447051634, 0.6557428337562166]$$

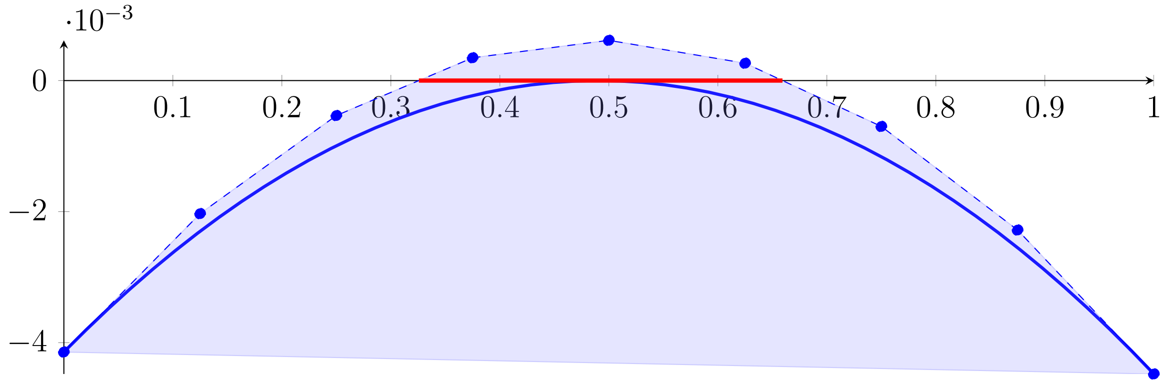
Longest intersection interval: 0.3338720890510532

$\Longrightarrow$ Selective recursion: interval 1: $[0.4997570309347421, 0.5002525935276471]$,

52.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4997570309347421, 0.5002525$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.637375463877353\cdot10^{-27}X^8 - 2.862414855005092\cdot10^{-22}X^7 - 9.341200314035287\cdot10^{-18}X^6 - 1.617995026179791\cdot10^{-13}X^5 - 1.568792391901099\cdot10^{-09}X^4 - 8.073141350089629\cdot10^{-06}X^3 - 0.01722540857155108X^2 + 0.01689843085777645X - 0.004143462623424353$$
$$= -0.004143462623424353B_{0,8}(X) - 0.002031158766202297B_{1,8}(X) - 0.0005340480722499233B_{2,8}(X) + 0.0003477252951943749B_{3,8}(X) + 0.0006140171504808826B_{4,8}(X) + 0.0002646832855456765B_{5,8}(X) - 0.0007004205300922658B_{6,8}(X) - 0.002281438549333955B_{7,8}(X) - 0.004478515047503282B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3257065380923488, 0.6592817116222256\}$$

Intersection intervals with the x axis:

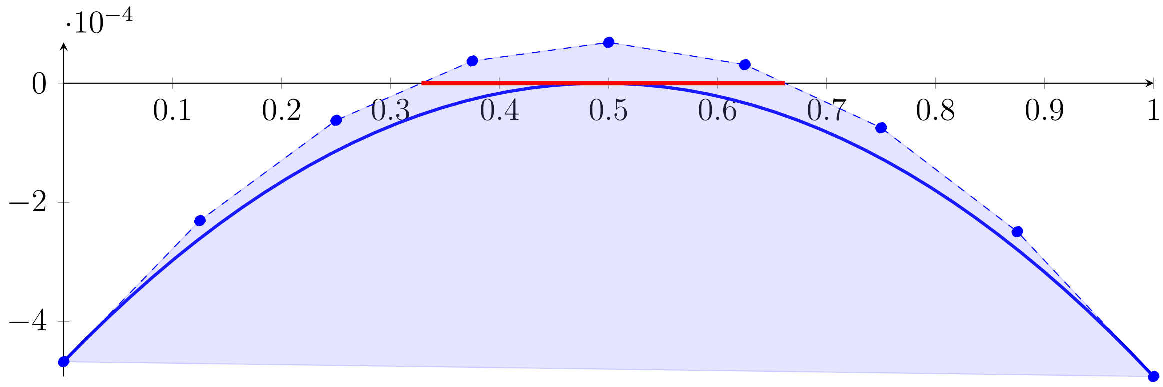
$$[0.3257065380923488, 0.6592817116222256]$$

Longest intersection interval: 0.3335751735298769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999184389112853, 0.5000837462892085]$,

52.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999184389112853, 0.50008$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -5.57619429667528\cdot 10^{-31}X^8 - 1.315536799505273\cdot 10^{-25}X^7 - 1.287049795843581\cdot 10^{-20}X^6 \\ & - 6.683358895768893\cdot 10^{-16}X^5 - 1.942733814710911\cdot 10^{-11}X^4 - 2.997323807488328 \\ & \cdot 10^{-07}X^3 - 0.001917590365692151X^2 + 0.001893040758741533X - 0.0004671653183669499 \\ = & -0.0004671653183669499B_{0,8}(X) - 0.0002305352235242582B_{1,8}(X) - 6.239049888485767 \\ & \cdot 10^{-05}B_{2,8}(X) + 3.726350318730984\cdot 10^{-05}B_{3,8}(X) + 6.842143005076897 \\ & \cdot 10^{-05}B_{4,8}(X) + 3.107792878649904\cdot 10^{-05}B_{5,8}(X) - 7.47723538020779 \\ & \cdot 10^{-05}B_{6,8}(X) - 0.000249134771189109B_{7,8}(X) - 0.0004920146771263226B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3282588977707033, 0.661700337526842\}$$

Intersection intervals with the x axis:

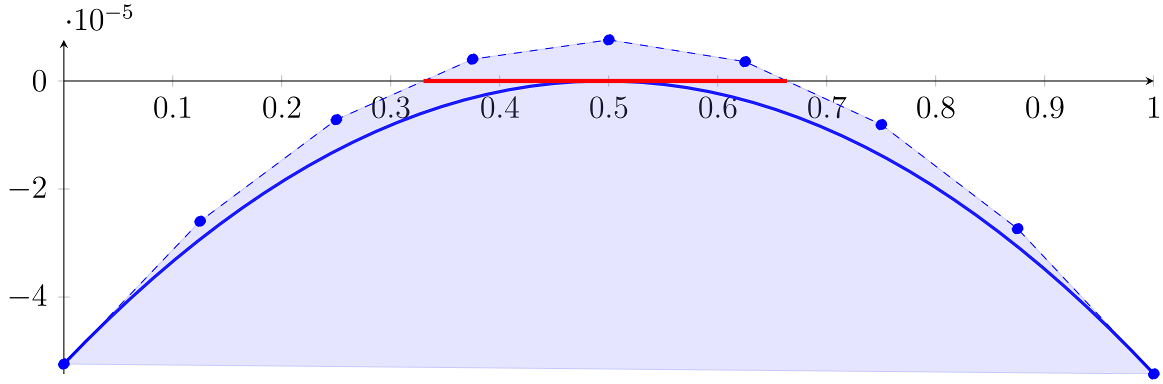
$$[0.3282588977707033, 0.661700337526842]$$

Longest intersection interval: 0.3334414397561387
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999727025289557, 0.5000278228590528]$,

52.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999727025289557, 0.500$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -8.521076768712742\cdot 10^{-35}X^8 - 6.02899387826599\cdot 10^{-29}X^7 - 1.76898025411792\cdot 10^{-23}X^6 \\ & - 2.754920799910016\cdot 10^{-18}X^5 - 2.401685361883718\cdot 10^{-13}X^4 - 1.111294950492419\cdot 10^{-08}X^3 \\ & - 0.0002132366404355359X^2 + 0.0002114057621846039X - 5.239629569054843\cdot 10^{-05} \\ = & -5.239629569054843\cdot 10^{-05}B_{0,8}(X) - 2.597057541747294\cdot 10^{-05}B_{1,8}(X) - 7.160449445666593 \\ & \cdot 10^{-06}B_{2,8}(X) + 4.033883779343745\cdot 10^{-06}B_{3,8}(X) + 7.612225808600218\cdot 10^{-06}B_{4,8}(X) \\ & + 3.574378189713946\cdot 10^{-06}B_{5,8}(X) - 8.079857533135031\cdot 10^{-06}B_{6,8}(X) \\ & - 2.73506798191978\cdot 10^{-05}B_{7,8}(X) - 5.423828713115661\cdot 10^{-05}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3299561852159799, 0.6633377584201652\}$$

Intersection intervals with the x axis:

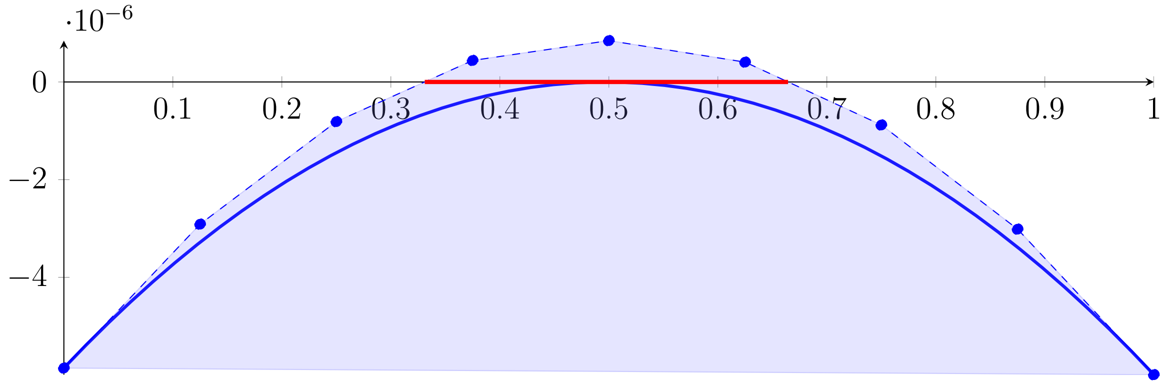
$$[0.3299561852159799, 0.6633377584201652]$$

Longest intersection interval: 0.3333815732041853
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999908898228024, 0.5000092659251657]$,

52.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999908898228024, 0.5$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.300251044438937 \cdot 10^{-38} X^8 - 2.759545789497528 \cdot 10^{-32} X^7 - 2.428711637904422 \cdot 10^{-26} X^6 \\
&- 1.134547275299063 \cdot 10^{-20} X^5 - 2.966816573305973 \cdot 10^{-15} X^4 - 4.117811892390909 \cdot 10^{-10} X^3 \\
&- 2.370104084998438 \cdot 10^{-05} X^2 + 2.356495503794524 \cdot 10^{-05} X - 5.857360304639568 \cdot 10^{-06} \\
= &-5.857360304639568 \cdot 10^{-06} B_{0,8}(X) - 2.911740924896412 \cdot 10^{-06} B_{1,8}(X) - 8.125872897955564 \\
&\cdot 10^{-07} B_{2,8}(X) + 4.400932474274781 \cdot 10^{-07} B_{3,8}(X) + 8.462933334947859 \cdot 10^{-07} B_{4,8}(X) \\
&+ 4.060056150860783 \cdot 10^{-07} B_{5,8}(X) - 8.807772611613165 \cdot 10^{-07} B_{6,8}(X) \\
&- 3.014062648652454 \cdot 10^{-06} B_{7,8}(X) - 5.993857900834775 \cdot 10^{-06} B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.331084848216461, 0.6644399885346334\}$$

Intersection intervals with the x axis:

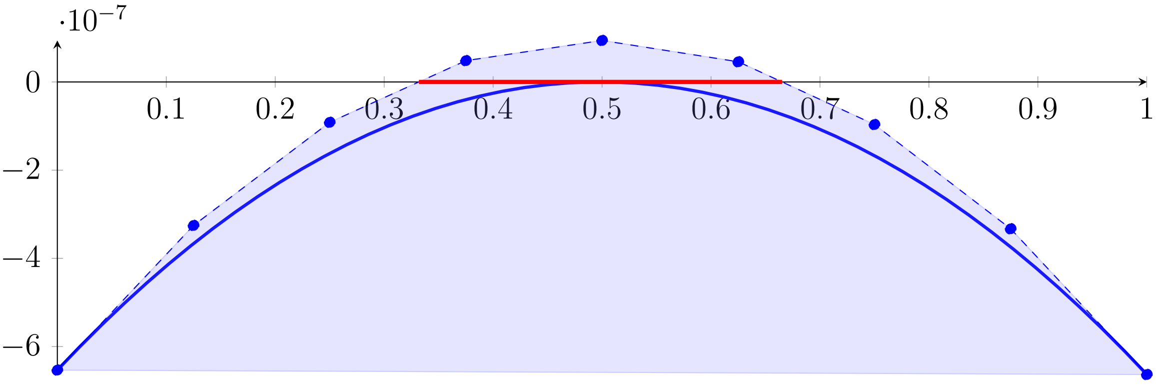
$$[0.331084848216461, 0.6644399885346334]$$

Longest intersection interval: 0.3333551403181724
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999969738718641, 0.500003099640046]$,

52.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999969738718641, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.982825346126094\cdot 10^{-42}X^8 - 1.262374580662423\cdot 10^{-35}X^7 - 3.332882748505763\cdot 10^{-29}X^6 \\ &- 4.670466114941116\cdot 10^{-23}X^5 - 3.663718276431707\cdot 10^{-17}X^4 - 1.52542941384219\cdot 10^{-11}X^3 \\ &- 2.633839011095458\cdot 10^{-06}X^2 + 2.623741169043712\cdot 10^{-06}X - 6.534168705792709\cdot 10^{-07} \\ = &-6.534168705792709\cdot 10^{-07}B_{0,8}(X) - 3.254492244488069\cdot 10^{-07}B_{1,8}(X) - 9.154725728603783 \\ &\cdot 10^{-08}B_{2,8}(X) + 4.828875851092667\cdot 10^{-08}B_{3,8}(X) + 9.405855054345363\cdot 10^{-08}B_{4,8}(X) \\ &+ 4.576184641238665\cdot 10^{-08}B_{5,8}(X) - 9.660162628195405\cdot 10^{-08}B_{6,8}(X) \\ &- 3.330321399397716\cdot 10^{-07}B_{7,8}(X) - 6.635299669617927\cdot 10^{-07}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3318344765869905, 0.6651804668942704\}$$

Intersection intervals with the x axis:

$$[0.3318344765869905, 0.6651804668942704]$$

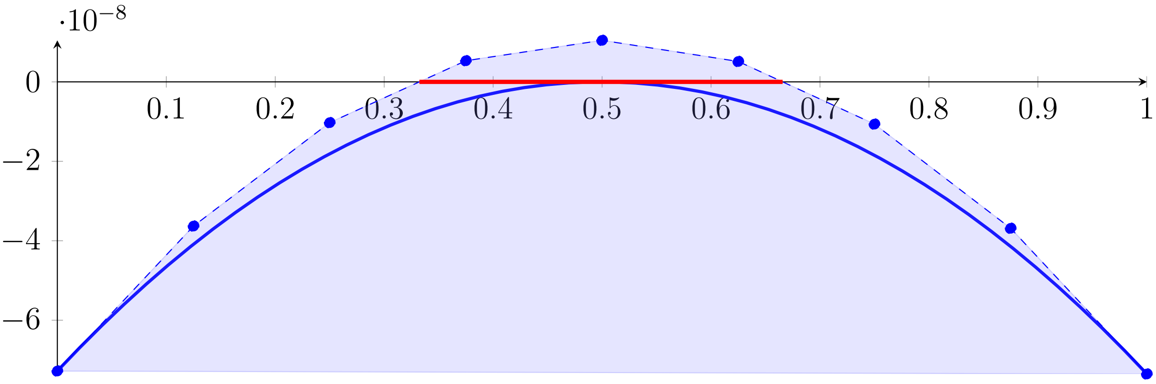
Longest intersection interval: 0.3333459903072799

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999990066129424, 0.5000010486132034]$,

52.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999990066129424$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-3.023057070307937\cdot 10^{-46}X^8 - 5.773711385862107\cdot 10^{-39}X^7 - 4.572901329678827\cdot 10^{-32}X^6 \\ &- 1.922370176484083\cdot 10^{-25}X^5 - 4.523805577109385\cdot 10^{-19}X^4 - 5.650400184721212\cdot 10^{-13}X^3 \\ &- 2.926726911514393\cdot 10^{-07}X^2 + 2.919240677305221\cdot 10^{-07}X - 7.27925149751094\cdot 10^{-08} \\ = &-7.27925149751094\cdot 10^{-08}B_{0,8}(X) - 3.630200650879414\cdot 10^{-08}B_{1,8}(X) - 1.026409415503029 \\ &\cdot 10^{-08}B_{2,8}(X) + 5.321211996181834\cdot 10^{-09}B_{3,8}(X) + 1.045390185483543\cdot 10^{-08}B_{4,8}(X) \\ &+ 5.133965330917245\cdot 10^{-09}B_{5,8}(X) - 1.063860766559244\cdot 10^{-08}B_{6,8}(X) \\ &- 3.68638272247198\cdot 10^{-08}B_{7,8}(X) - 7.354170343649749\cdot 10^{-08}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3323218842755297, 0.6656874431018115\}$$

Intersection intervals with the x axis:

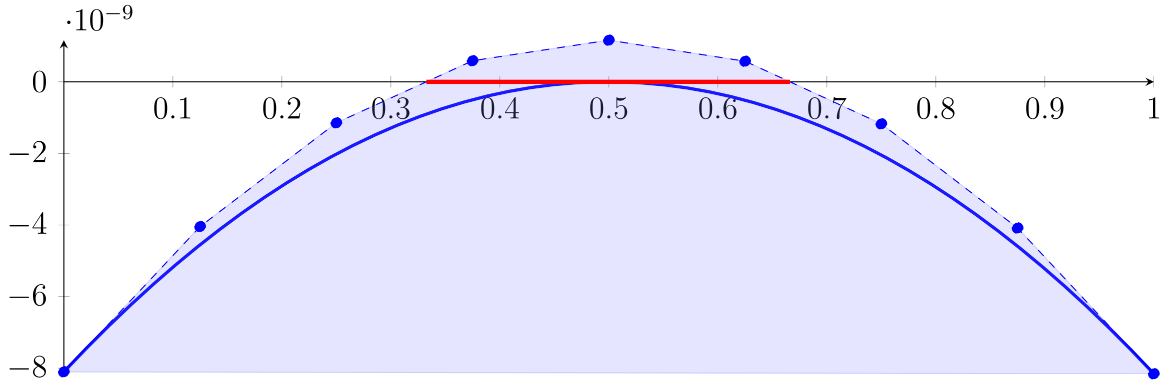
$$[0.3323218842755297, 0.6656874431018115]$$

Longest intersection interval: 0.3333655588262817
$\Longrightarrow$ Selective recursion: interval 1: [0.4999996852143169, 0.500000365946875],

52.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999996852143169, 0.500000365946875]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -4.61118111521237 \cdot 10^{-50} X^8 - 2.641801828130962 \cdot 10^{-42} X^7 - 6.276482670428966 \cdot 10^{-35} X^6 \\ & - 7.91481690665562 \cdot 10^{-28} X^5 - 5.587109146485419 \cdot 10^{-21} X^4 - 2.093350052120201 \cdot 10^{-14} X^3 \\ & - 3.252553849467158 \cdot 10^{-08} X^2 + 3.247007216419568 \cdot 10^{-08} X - 8.101917765629863 \cdot 10^{-09} \\ = & -8.101917765629863 \cdot 10^{-09} B_{0,8}(X) - 4.043158745105403 \cdot 10^{-09} B_{1,8}(X) \\ & - 1.146026099390642 \cdot 10^{-09} B_{2,8}(X) + 5.8947979770191 \cdot 10^{-10} B_{3,8}(X) + 1.163358572359665 \\ & \cdot 10^{-09} B_{4,8}(X) + 5.75609850769953 \cdot 10^{-10} B_{5,8}(X) - 1.173766740879974 \cdot 10^{-09} B_{6,8}(X) \\ & - 4.084771576402945 \cdot 10^{-09} B_{7,8}(X) - 8.157405029611868 \cdot 10^{-09} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.332542653795541, 0.6661296410902487\}$$

Intersection intervals with the x axis:

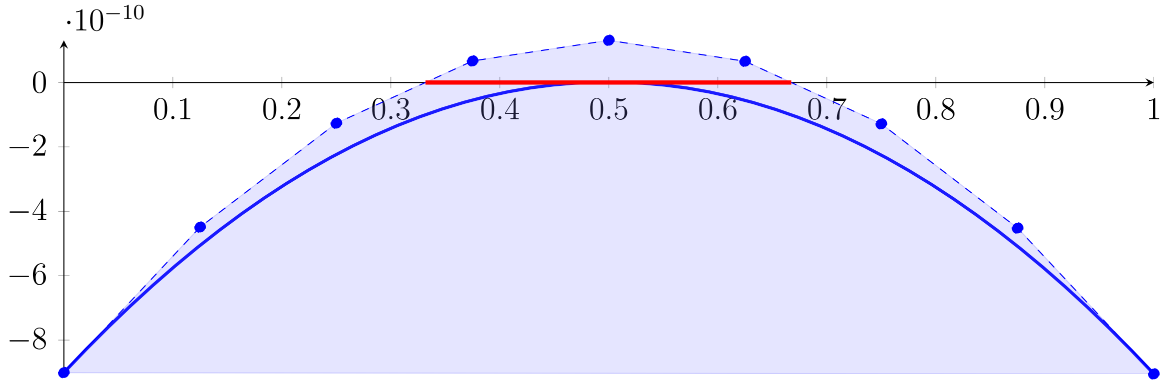
$$[0.332542653795541, 0.6661296410902487]$$

Longest intersection interval: 0.3335869872947078
$\Longrightarrow$ Selective recursion: interval 1: [0.4999999115869283, 0.5000001386704515],

52.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999999115869283, 0.5000001386704515]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -7.071067608997674 \cdot 10^{-54} X^8 - 1.214406168666063 \cdot 10^{-45} X^7 - 8.649101296569956 \cdot 10^{-38} X^6 \\ & - 3.269538436354099 \cdot 10^{-30} X^5 - 6.918686662336931 \cdot 10^{-23} X^4 - 7.770864123205297 \cdot 10^{-16} X^3 \\ & - 3.61945329275134 \cdot 10^{-09} X^2 + 3.615351534501487 \cdot 10^{-09} X - 9.01058772952768 \cdot 10^{-10} \\ = & -9.01058772952768 \cdot 10^{-10} B_{0,8}(X) - 4.491398311400821 \cdot 10^{-10} B_{1,8}(X) - 1.264870783542298 \\ & \cdot 10^{-10} B_{2,8}(X) + 6.689947152824597 \cdot 10^{-11} B_{3,8}(X) + 1.31019804630801 \cdot 10^{-10} B_{4,8}(X) \\ & + 6.587390707689027 \cdot 10^{-11} B_{5,8}(X) - 1.285382350100323 \cdot 10^{-10} B_{6,8}(X) \\ & - 4.522166355065136 \cdot 10^{-10} B_{7,8}(X) - 9.051613082891018 \cdot 10^{-10} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3317579340646351, 0.6673545479012814\}$$

Intersection intervals with the x axis:

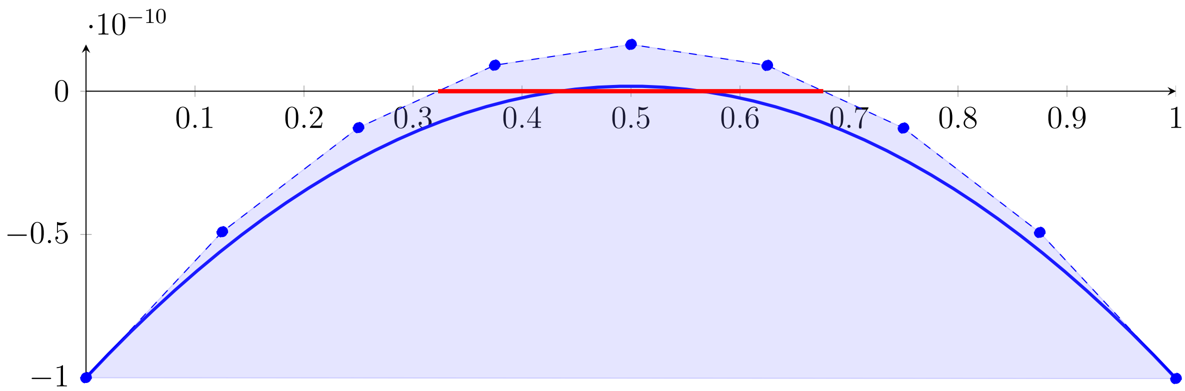
$$[0.3317579340646351, 0.6673545479012814]$$

Longest intersection interval: 0.3355966138366463
$\Longrightarrow$ Selective recursion: interval 1: [0.4999999869236888, 0.5000000631321502],

52.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999999869236888, 0.5000000631321502]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.137694218100245\cdot 10^{-57}X^8 - 5.822197888022611\cdot 10^{-49}X^7 - 1.235595786383296\cdot 10^{-40}X^6 \\ &- 1.391791952088144\cdot 10^{-32}X^5 - 8.775946704095466\cdot 10^{-25}X^4 - 2.937122613307657\cdot 10^{-17}X^3 \\ &- 4.076413298856481\cdot 10^{-10}X^2 + 4.07342667585279\cdot 10^{-10}X - 1.000063159750977\cdot 10^{-10} \\ = &-1.000063159750977\cdot 10^{-10}B_{0,8}(X) - 4.908848252693777\cdot 10^{-11}B_{1,8}(X) - 1.272926800326533 \\ &\cdot 10^{-11}B_{2,8}(X) + 9.071327071433506\cdot 10^{-12}B_{3,8}(X) + 1.631330217267253\cdot 10^{-11}B_{4,8}(X) \\ &+ 8.996656775965545\cdot 10^{-12}B_{5,8}(X) - 1.287860964317367\cdot 10^{-11}B_{6,8}(X) \\ &- 4.931249760923136\cdot 10^{-11}B_{7,8}(X) - 1.003050076466937\cdot 10^{-10}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3229869297125206, 0.6764088411746962\}$$

Intersection intervals with the x axis:

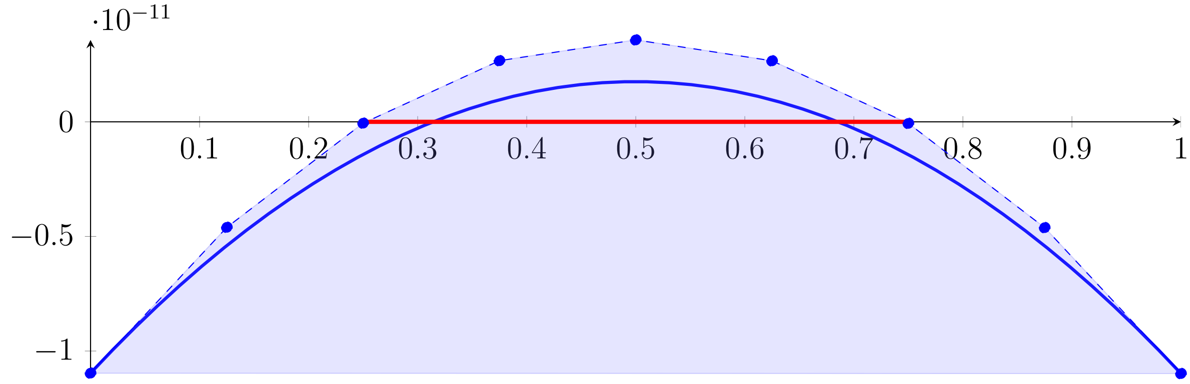
$$[0.3229869297125206, 0.6764088411746962]$$

Longest intersection interval: 0.3534219114621756
$\Longrightarrow$ Selective recursion: interval 1: [0.5000000115380258, 0.5000000384717659],

52.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000115380258, 0.5000000384717659]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.769321148091343\cdot 10^{-61}X^8 - 4.009971301105125\cdot 10^{-52}X^7 - 2.40789337418918\cdot 10^{-43}X^6\\&- 7.674351474736958\cdot 10^{-35}X^5 - 1.36920310040832\cdot 10^{-26}X^4 - 1.29658952293724\cdot 10^{-18}X^3\\&- 5.091727851043352\cdot 10^{-11}X^2 + 5.08987688183243\cdot 10^{-11}X - 1.096532991460483\cdot 10^{-11}\\= &-1.096532991460483\cdot 10^{-11}B_{0,8}(X) - 4.602983812314296\cdot 10^{-12}B_{1,8}(X)\\&- 5.91119425392419\cdot 10^{-14}B_{2,8}(X) + 2.666285671566945\cdot 10^{-12}B_{3,8}(X) + 3.57320900685088\\&\cdot 10^{-12}B_{4,8}(X) + 2.661658040159178\cdot 10^{-12}B_{5,8}(X) - 6.83672516615453\cdot 10^{-14}B_{6,8}(X)\\&- 4.616866891764675\cdot 10^{-12}B_{7,8}(X) - 1.09838409033036\cdot 10^{-11}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.252711161402344, 0.7468696603349071\}$$

Intersection intervals with the x axis:

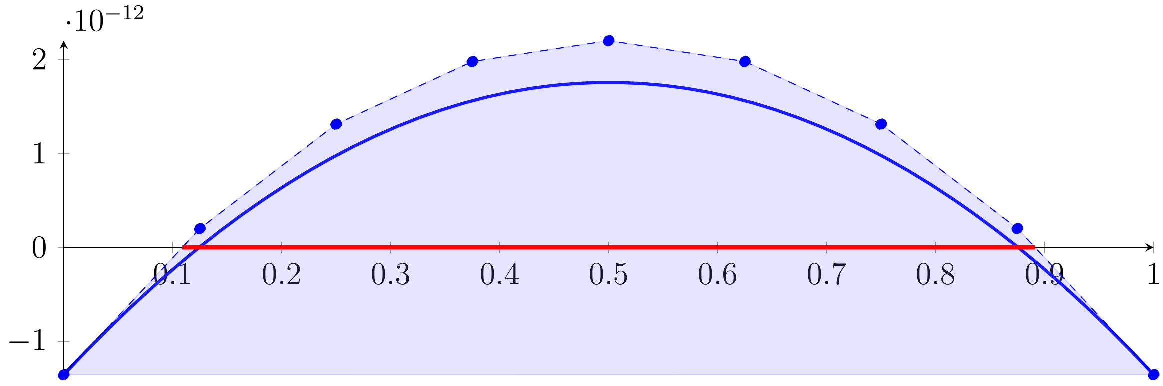
$$[0.252711161402344, 0.7468696603349071]$$

Longest intersection interval: 0.4941584989325631
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000183444825, 0.5000000316540191]$,

52.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000183444825, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-9.84698950588755\cdot 10^{-64}X^8 - 2.885394162330348\cdot 10^{-54}X^7 - 3.506185324343073\cdot 10^{-45}X^6\\&- 2.261377324294573\cdot 10^{-36}X^5 - 8.164563166712554\cdot 10^{-28}X^4 - 1.564592773244276\cdot 10^{-19}X^3\\&- 1.243362398803133\cdot 10^{-11}X^2 + 1.243502393415518\cdot 10^{-11}X - 1.354369602677131\cdot 10^{-12}\\= &-1.354369602677131\cdot 10^{-12}B_{0,8}(X) + 2.000083890922661\cdot 10^{-13}B_{1,8}(X)\\&+ 1.310328381289116\cdot 10^{-12}B_{2,8}(X) + 1.976590371119503\cdot 10^{-12}B_{3,8}(X)\\&+ 2.19879435578951\cdot 10^{-12}B_{4,8}(X) + 1.976940332505223\cdot 10^{-12}B_{5,8}(X) + 1.311028298472726\\&\cdot 10^{-12}B_{6,8}(X) + 2.01058250898102\cdot 10^{-13}B_{7,8}(X) - 1.352969813012563\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.108915721421098, 0.8911723471704997\}$$

Intersection intervals with the x axis:

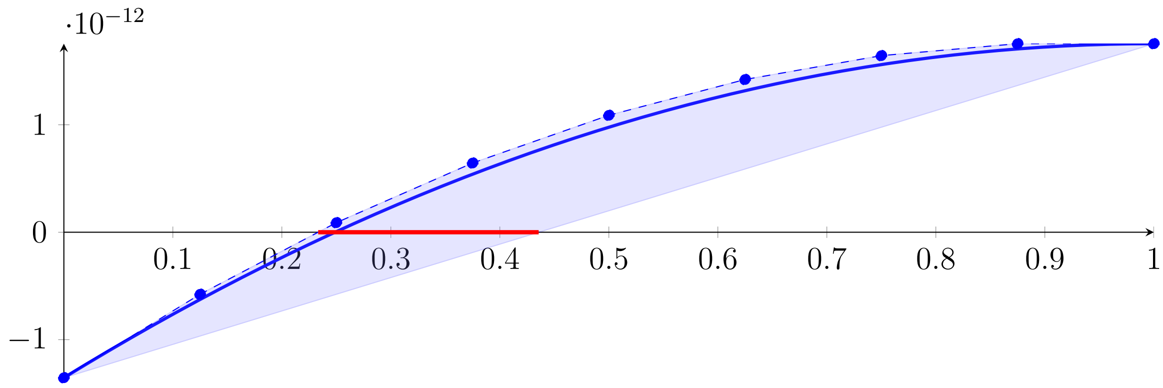
$$[0.108915721421098, 0.8911723471704997]$$

Longest intersection interval: 0.7822566257494017
$\Longrightarrow$ Bisection: first half [0.5000000183444825, 0.5000000249992508] und second half [0.5000000249992508, 0.50

52.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half [0.5000000183444825, 0.5000000249992508]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.846480275737324\cdot10^{-66}X^8 - 2.254214189320585\cdot10^{-56}X^7 - 5.478414569286052\cdot10^{-47}X^6\\ &\quad - 7.06680413842054\cdot10^{-38}X^5 - 5.102851979195346\cdot10^{-29}X^4 - 1.955740966555345\cdot10^{-20}X^3\\ &\quad - 3.108405997007834\cdot10^{-12}X^2 + 6.21751196707759\cdot10^{-12}X - 1.354369602677131\cdot10^{-12}\\ &= -1.354369602677131\cdot10^{-12}B_{0,8}(X) - 5.771806067924327\cdot10^{-13}B_{1,8}(X) + 8.899388919912914\\ &\quad \cdot10^{-14}B_{2,8}(X) + 6.441538849483146\cdot10^{-13}B_{3,8}(X) + 1.088299380105884\cdot10^{-12}B_{4,8}(X)\\ &\quad + 1.421430374322599\cdot10^{-12}B_{5,8}(X) + 1.643546867249218\cdot10^{-12}B_{6,8}(X)\\ &\quad + 1.754648858536504\cdot10^{-12}B_{7,8}(X) + 1.754736347835215\cdot10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2333013178726793, 0.4356138466281428\}$$

Intersection intervals with the x axis:

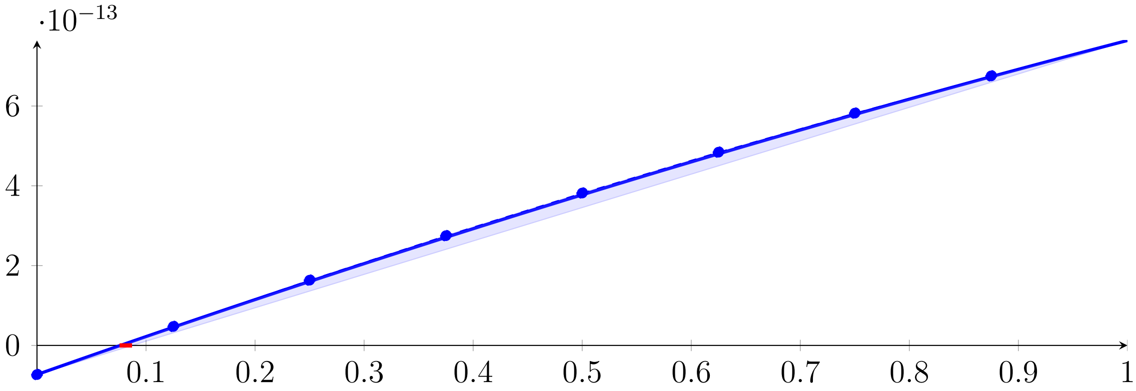
$$[0.2333013178726793, 0.4356138466281428]$$

Longest intersection interval: 0.2023125287554634
$\Longrightarrow$ Selective recursion: interval 1: [0.5000000198970487, 0.5000000212433917],

52.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.5000000198970487, 0.5000000212433917]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.079557397202381\cdot 10^{-71}X^8 - 3.12719259192325\cdot 10^{-61}X^7 - 3.756570682132575\cdot 10^{-51}X^6\\ &\quad - 2.395173331861966\cdot 10^{-41}X^5 - 8.548778908133761\cdot 10^{-32}X^4 - 1.619495215947232\cdot 10^{-22}X^3\\ &\quad - 1.272281748414343\cdot 10^{-13}X^2 + 9.644484121679\cdot 10^{-13}X - 7.300486662812878\cdot 10^{-14}\\ &= -7.300486662812878\cdot 10^{-14}B_{0,8}(X) + 4.755118489285872\cdot 10^{-14}B_{1,8}(X) + 1.635633730266521\\ &\quad\cdot 10^{-13}B_{2,8}(X) + 2.750316977703595\cdot 10^{-13}B_{3,8}(X) + 3.819561591210889\cdot 10^{-13}B_{4,8}(X)\\ &\quad + 4.843367570759483\cdot 10^{-13}B_{5,8}(X) + 5.821734916320458\cdot 10^{-13}B_{6,8}(X)\\ &\quad + 6.754663627864895\cdot 10^{-13}B_{7,8}(X) + 7.642153705363873\cdot 10^{-13}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.07569597886944255, 0.08719911844866588\}$$

Intersection intervals with the x axis:

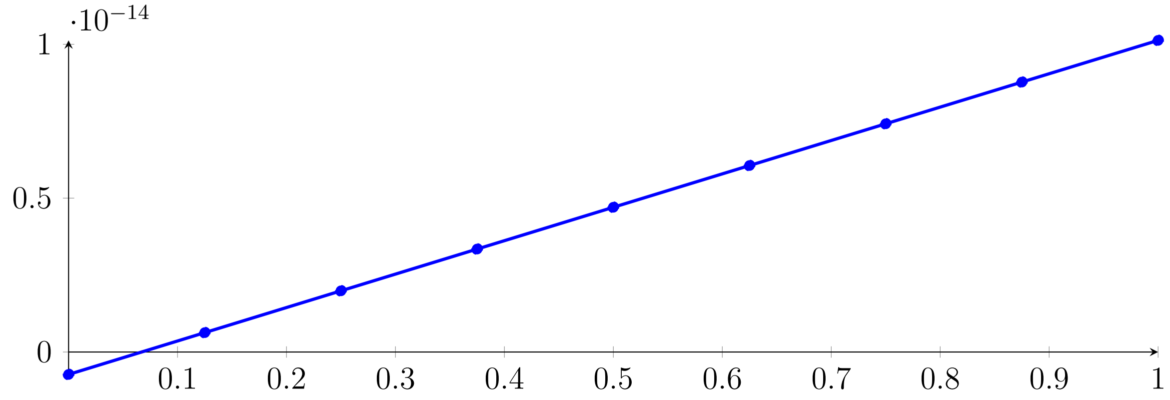
$$[0.07569597886944255, 0.08719911844866588]$$

Longest intersection interval: 0.01150313957922333
$\Longrightarrow$ Selective recursion: interval 1: [0.5000000199989615, 0.5000000200144486],

52.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.5000000199989615, 0.5000000200144486]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.309610258465664\cdot 10^{-87}X^8 - 8.334304343480435\cdot 10^{-75}X^7 - 8.703419206351739\cdot 10^{-63}X^6\\ &\quad - 4.824128808781935\cdot 10^{-51}X^5 - 1.496820228100742\cdot 10^{-39}X^4 - 2.465067626410487\cdot 10^{-28}X^3\\ &\quad - 1.683511456921895\cdot 10^{-17}X^2 + 1.087261902125672\cdot 10^{-14}X - 7.290023293677624\cdot 10^{-16}\\ &= -7.290023293677624\cdot 10^{-16}B_{0,8}(X) + 6.300750482893273\cdot 10^{-16}B_{1,8}(X) + 1.988551171854659\\ &\quad\cdot 10^{-15}B_{2,8}(X) + 3.346426041328229\cdot 10^{-15}B_{3,8}(X) + 4.703699656710032\cdot 10^{-15}B_{4,8}(X)\\ &\quad + 6.060372018000064\cdot 10^{-15}B_{5,8}(X) + 7.41644312519832\cdot 10^{-15}B_{6,8}(X)\\ &\quad + 8.771912978304797\cdot 10^{-15}B_{7,8}(X) + 1.012678157731949\cdot 10^{-14}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06704937678240291, 0.0671533567390459\}$$

Intersection intervals with the x axis:

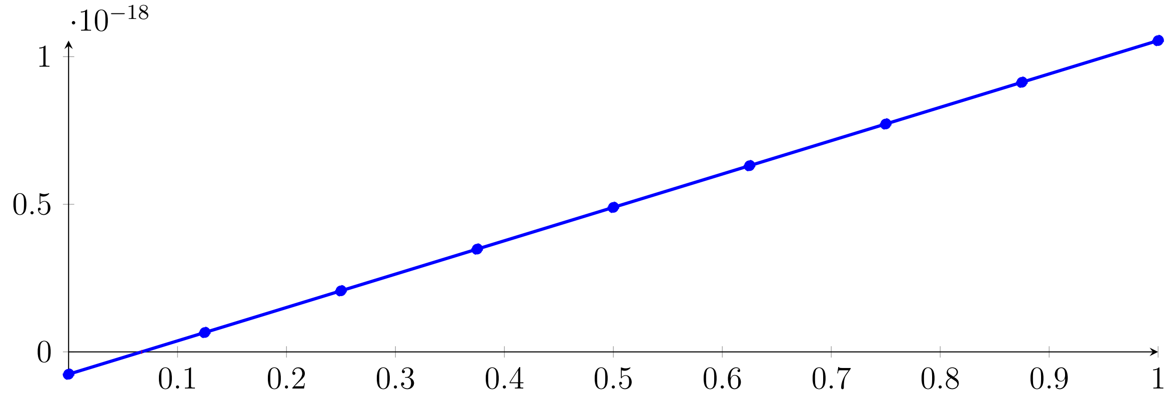
$$[0.06704937678240291, 0.0671533567390459]$$

Longest intersection interval: 0.0001039799566429901
$\implies$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000200000015]$,

52.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000200000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-4.522451416965012\cdot 10^{-119}X^8 - 1.095258871423056\cdot 10^{-102}X^7 - 1.099987358691302\cdot 10^{-86}X^6 \\ &- 5.863636713606292\cdot 10^{-71}X^5 - 1.749718451421176\cdot 10^{-55}X^4 - 2.771262941212064\cdot 10^{-40}X^3 \\ &- 1.820184200444671\cdot 10^{-25}X^2 + 1.130299712615757\cdot 10^{-18}X - 7.568425969413026\cdot 10^{-20} \\ = &-7.568425969413026\cdot 10^{-20}B_{0,8}(X) + 6.560320438283937\cdot 10^{-20}B_{1,8}(X) + 2.068906619591512 \\ &\cdot 10^{-19}B_{2,8}(X) + 3.481781130348051\cdot 10^{-19}B_{3,8}(X) + 4.894655576098011\cdot 10^{-19}B_{4,8}(X) \\ &+ 6.307529956841393\cdot 10^{-19}B_{5,8}(X) + 7.720404272578197\cdot 10^{-19}B_{6,8}(X) \\ &+ 9.133278523308422\cdot 10^{-19}B_{7,8}(X) + 1.054615270903207\cdot 10^{-18}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695946114945086, 0.06695947193230531\}$$

Intersection intervals with the x axis:

$$[0.06695946114945086, 0.06695947193230531]$$

Longest intersection interval: $1.078285445187343\cdot 10^{-08}$
$\implies$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

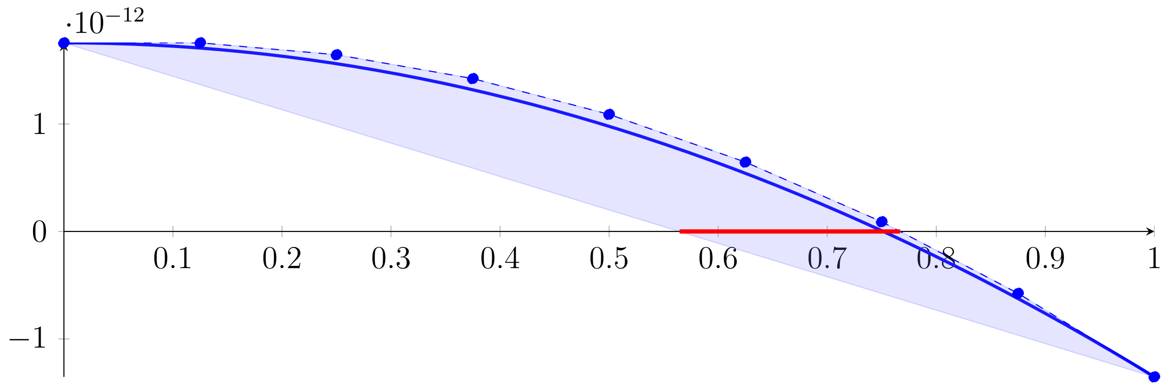
52.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 23!

52.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.5000000249992508, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.846480275737324\cdot 10^{-66}X^8 - 2.254214192397769\cdot 10^{-56}X^7 - 5.478414585065551\cdot 10^{-47}X^6\\ &- 7.066804171291027\cdot 10^{-38}X^5 - 5.102852014529367\cdot 10^{-29}X^4 - 1.955740986966753\cdot 10^{-20}X^3\\ &- 3.108406055680063\cdot 10^{-12}X^2 + 6.999143896938875\cdot 10^{-16}X + 1.754736347835215\cdot 10^{-12}\\ = &\ 1.754736347835215\cdot 10^{-12}B_{0,8}(X) + 1.754823837133927\cdot 10^{-12}B_{1,8}(X) + 1.643896824444065\\ &\cdot 10^{-12}B_{2,8}(X) + 1.42195530941639\cdot 10^{-12}B_{3,8}(X) + 1.088999291701662\cdot 10^{-12}B_{4,8}(X)\\ &+ 6.450287709506428\cdot 10^{-13}B_{5,8}(X) + 9.00437468140915\cdot 10^{-14}B_{6,8}(X)\\ &- 5.759557810572307\cdot 10^{-13}B_{7,8}(X) - 1.352969813012563\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.5646403672078589, 0.7669001146107961\}$$

Intersection intervals with the x axis:

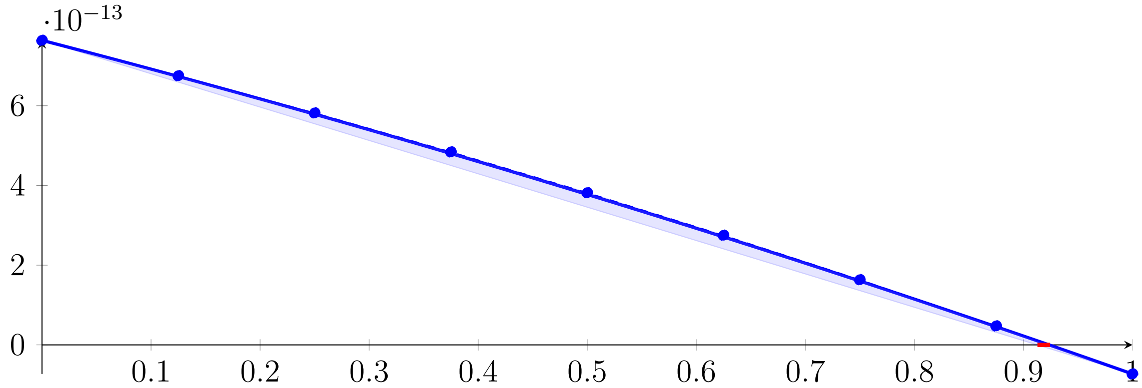
$$[0.5646403672078589, 0.7669001146107961]$$

Longest intersection interval: 0.2022597474029372
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000287568016, 0.5000000301027934]$,

52.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.5000000287568016, 0.5000000301027934]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.077306286121587\cdot 10^{-71}X^8 - 3.121486088546909\cdot 10^{-61}X^7 - 3.750694216009664\cdot 10^{-51}X^6\\ &- 2.392050590455968\cdot 10^{-41}X^5 - 8.5398613074672\cdot 10^{-32}X^4 - 1.618228037882981\cdot 10^{-22}X^3\\ &- 1.271618015330847\cdot 10^{-13}X^2 - 7.098433618236863\cdot 10^{-13}X + 7.641134288463311\cdot 10^{-13}\\ = &\ 7.641134288463311\cdot 10^{-13}B_{0,8}(X) + 6.753830086183703\cdot 10^{-13}B_{1,8}(X) + 5.821110954785137\\ &\cdot 10^{-13}B_{2,8}(X) + 4.842976894238714\cdot 10^{-13}B_{3,8}(X) + 3.819427904515539\cdot 10^{-13}B_{4,8}(X)\\ &+ 2.750463985586714\cdot 10^{-13}B_{5,8}(X) + 1.636085137423343\cdot 10^{-13}B_{6,8}(X)\\ &+ 4.762913599965286\cdot 10^{-14}B_{7,8}(X) - 7.289173467226265\cdot 10^{-14}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9129136379925769, 0.9243992614454615\}$$

Intersection intervals with the x axis:

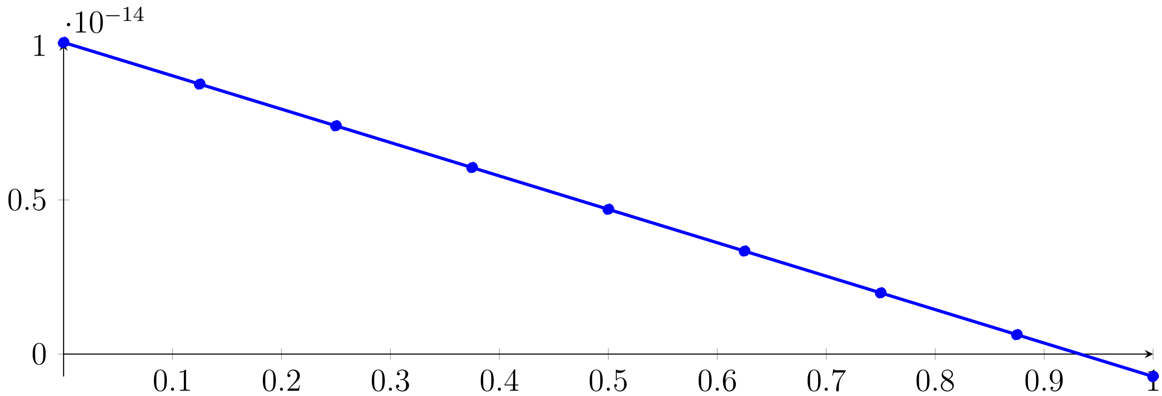
$$[0.9129136379925769, 0.9243992614454615]$$

Longest intersection interval: 0.01148562345288463
$\Longrightarrow$ Selective recursion: interval 1: [0.5000000299855758, 0.5000000300010354],

52.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: [0.5000000299855758, 0.5000000300010354]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.26268981617174\cdot 10^{-87}X^8 - 8.230826044001589\cdot 10^{-75}X^7 - 8.610712798456319\cdot 10^{-63}X^6\\ &- 4.781269588122392\cdot 10^{-51}X^5 - 1.48617213692691\cdot 10^{-39}X^4 - 2.451903903103066\cdot 10^{-28}X^3\\ &- 1.677512719815316\cdot 10^{-17}X^2 - 1.081967377285333\cdot 10^{-14}X + 1.010965922317952\cdot 10^{-14}\\ = &\ 1.010965922317952\cdot 10^{-14}B_{0,8}(X) + 8.757200001572855\cdot 10^{-15}B_{1,8}(X) + 7.40414166828054\\ &\cdot 10^{-15}B_{2,8}(X) + 6.050484223302573\cdot 10^{-15}B_{3,8}(X) + 4.696227666638948\cdot 10^{-15}B_{4,8}(X)\\ &+ 3.341371998289662\cdot 10^{-15}B_{5,8}(X) + 1.98591721825471\cdot 10^{-15}B_{6,8}(X)\\ &+ 6.298633265340878\cdot 10^{-16}B_{7,8}(X) - 7.267896768722089\cdot 10^{-16}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9329310105574586, 0.9330346747614\}$$

Intersection intervals with the x axis:

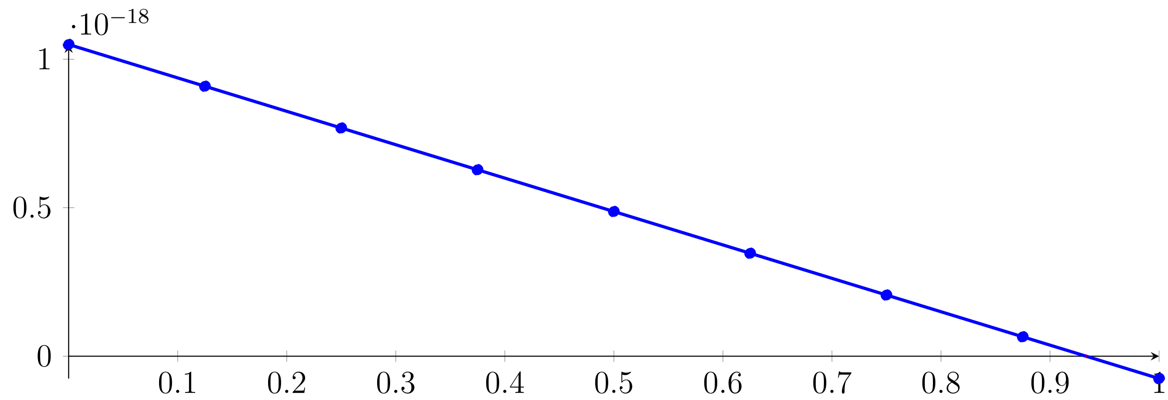
$$[0.9329310105574586, 0.9330346747614]$$

Longest intersection interval: 0.0001036642039414486
$\Longrightarrow$ Selective recursion: interval 1: [0.5000000299999985, 0.5000000300000001],

52.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.5000000299999985, 0.5000000300000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-4.351172724257996\cdot 10^{-119}X^8 - 1.058876090749164\cdot 10^{-102}X^7 - 1.068592215336126\cdot 10^{-86}X^6 \\ &- 5.723837778458117\cdot 10^{-71}X^5 - 1.716265145654202\cdot 10^{-55}X^4 - 2.731428873735883\cdot 10^{-40}X^3 \\ &- 1.802699988373219\cdot 10^{-25}X^2 - 1.124857565692813\cdot 10^{-18}X + 1.049632124051115\cdot 10^{-18} \\ = &\ 1.049632124051115\cdot 10^{-18}B_{0,8}(X) + 9.090249283395136\cdot 10^{-19}B_{1,8}(X) + 7.684177261896977 \\ &\cdot 10^{-19}B_{2,8}(X) + 6.278105176016676\cdot 10^{-19}B_{3,8}(X) + 4.872033025754233\cdot 10^{-19}B_{4,8}(X) \\ &+ 3.465960811109647\cdot 10^{-19}B_{5,8}(X) + 2.059888532082919\cdot 10^{-19}B_{6,8}(X) \\ &+ 6.538161886740477\cdot 10^{-20}B_{7,8}(X) - 7.522562191169655\cdot 10^{-20}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243242252754, 0.9331243349427871\}$$

Intersection intervals with the x axis:

$$[0.9331243242252754, 0.9331243349427871]$$

Longest intersection interval: $1.071751170792712\cdot 10^{-08}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

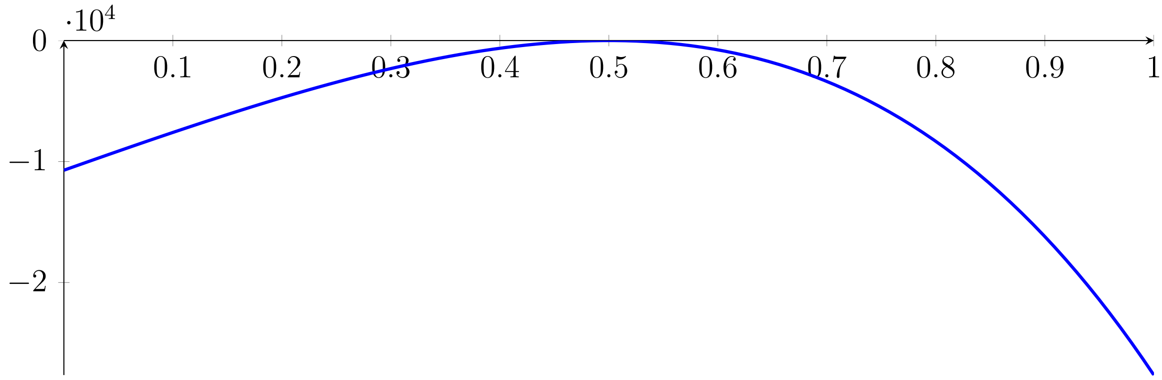
52.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 23!

52.29 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

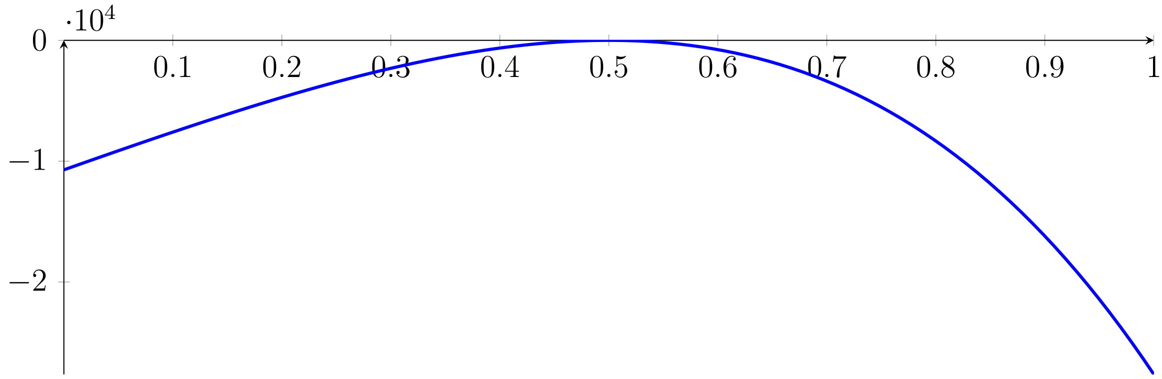
with precision $\varepsilon = 1 \cdot 10^{-16}$.

53 Running QuadClip on h_8 with epsilon 16

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called QuadClip with input polynomial on interval $[0, 1]$:

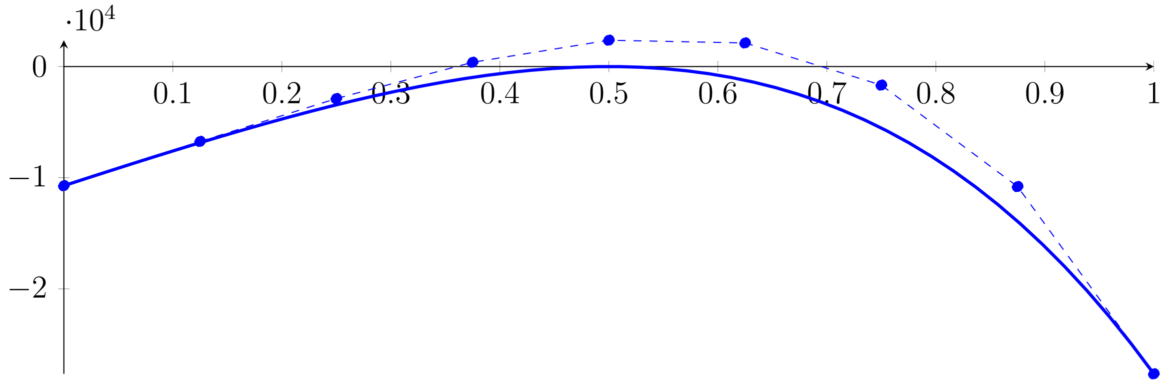
$$\begin{aligned}p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4\\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503\end{aligned}$$



53.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

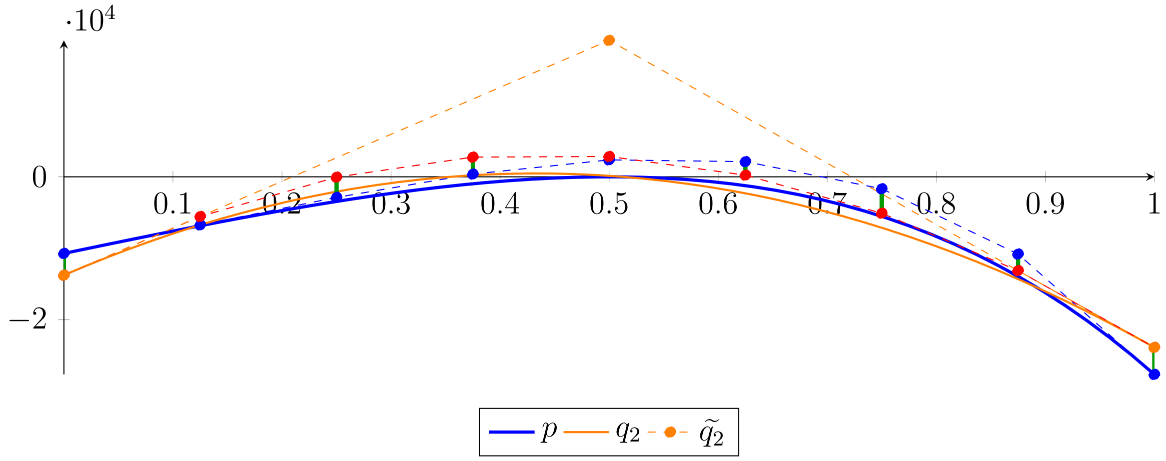
$$\begin{aligned}p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4\\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503\\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X)\\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X)\\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379\\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2}\end{aligned}$$

$$\begin{aligned}\tilde{q}_2 = &\ 8.415084934481114\cdot 10^{-299}X^8 - 3.337656926892332\cdot 10^{-298}X^7 + 5.61048922956057\cdot 10^{-298}X^6\\ &- 5.189733160056009\cdot 10^{-298}X^5 + 2.787623795639522\cdot 10^{-298}X^4 - 8.054794139884735\\ &\cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379\\ = &-13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8}\\ &+ 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8}\\ &- 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.

Bounding polynomials M and m:

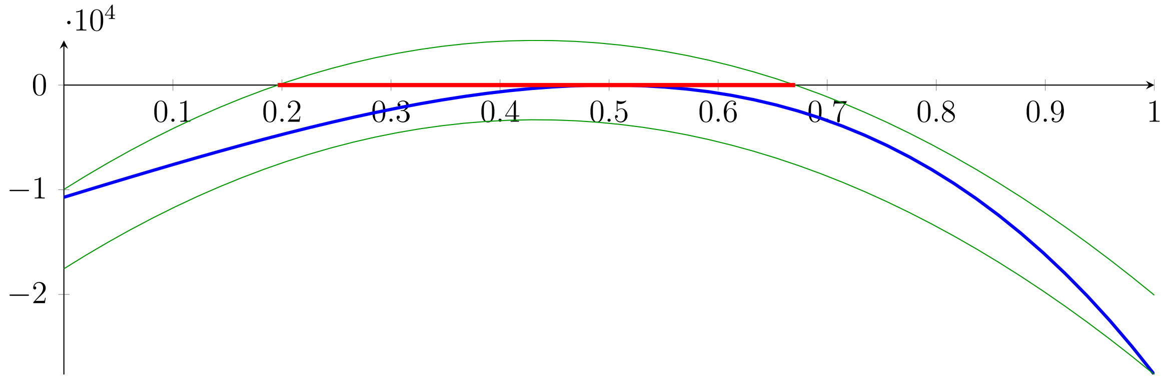
$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$

$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.196099166583006, 0.6706514347613278]$$

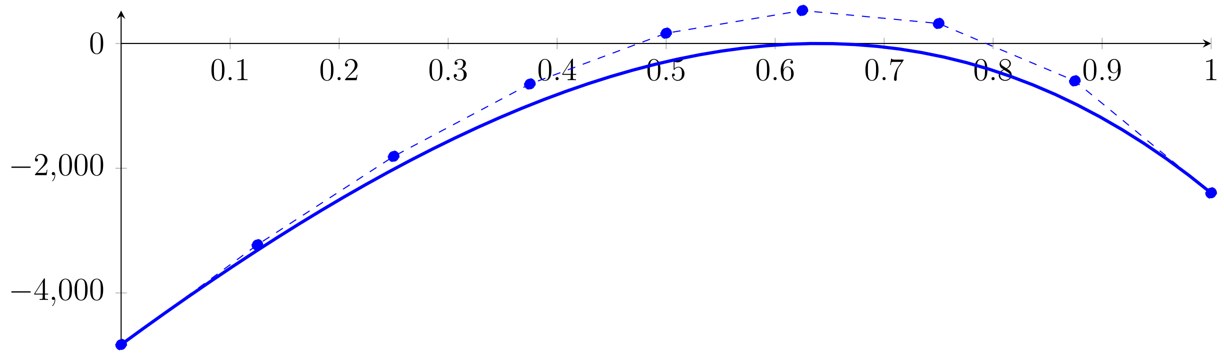
Longest intersection interval: 0.4745522681783218

$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

53.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

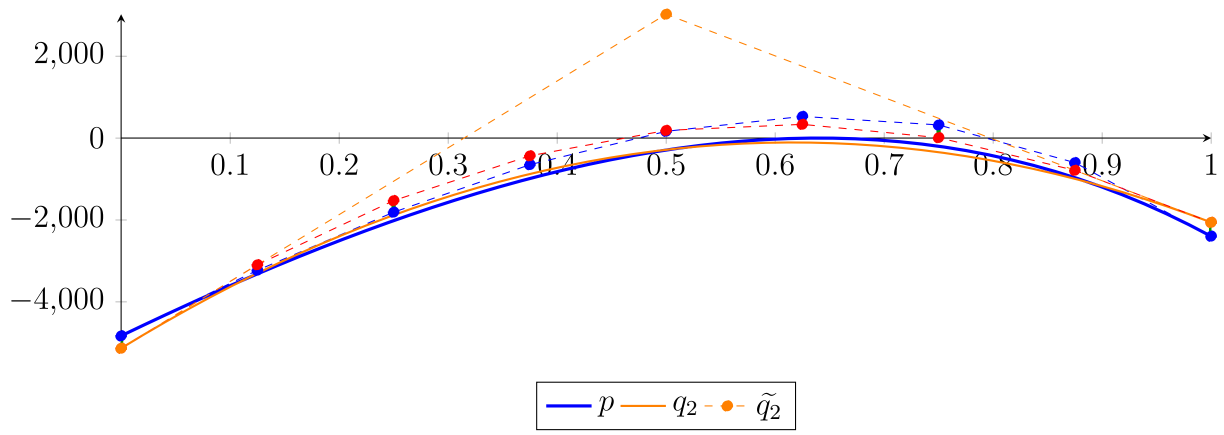
$$\begin{aligned} p = &-0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6 \\ &- 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3 \\ &- 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684 \\ = &-4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X) \\ &- 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X) \\ &+ 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.476670942620757\cdot 10^{-299}X^8 - 6.381627458836932\cdot 10^{-299}X^7 + 1.1779791924066\cdot 10^{-298}X^6 \\ &\quad - 1.190713137013755\cdot 10^{-298}X^5 + 6.850874958113058\cdot 10^{-299}X^4 - 2.05172173630317 \\ &\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\ &\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\ &\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

Bounding polynomials M and m:

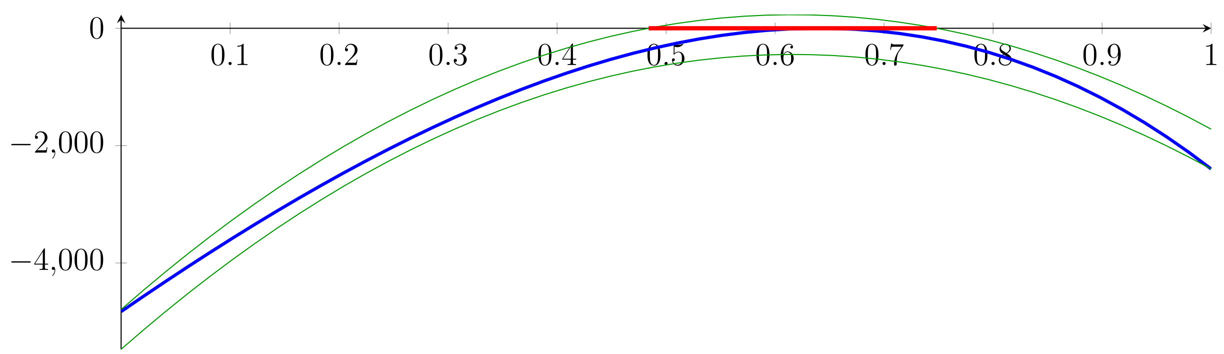
$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$

$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.4839311609916354, 0.7482816274420809]$$

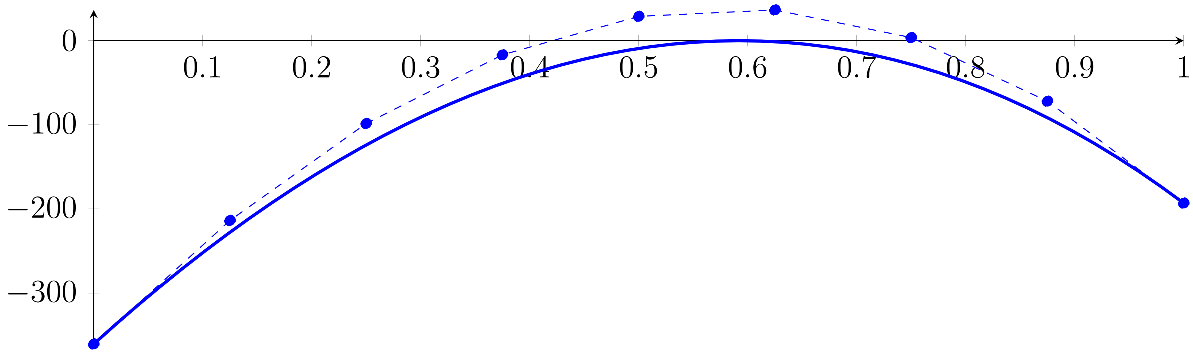
Longest intersection interval: 0.2643504664504455

$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

53.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.551197910121811$

Normalized monomial und Bézier representations and the Bézier polygon:

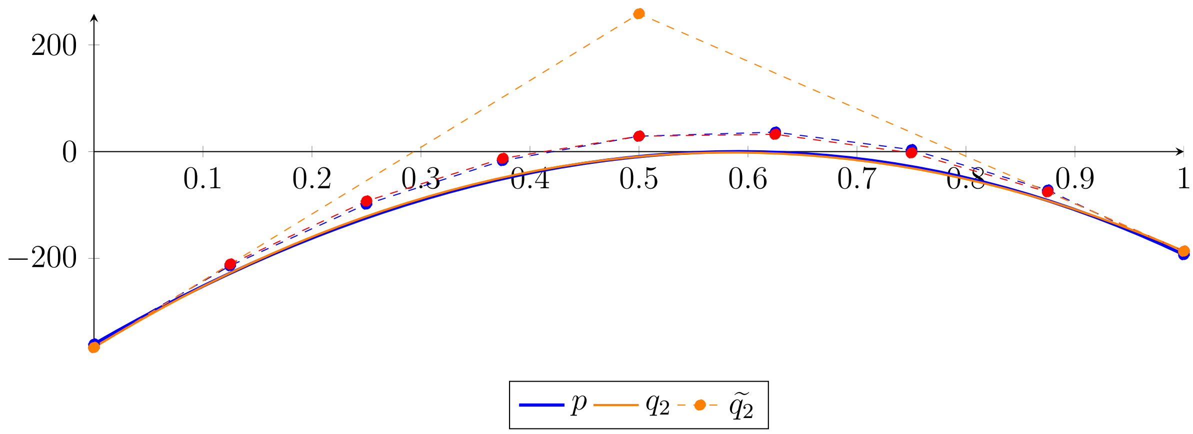
$$\begin{aligned}
p &= -6.133566475094875\cdot 10^{-08}X^8 - 1.877794231289375\cdot 10^{-05}X^7 - 0.002379939031153127X^6\\
&\quad - 0.1596298583101837X^5 - 5.958684697205818X^4 - 116.3336704708797X^3\\
&\quad - 885.1606719876373X^2 + 1175.121582406799X - 360.5751654247078\\
&= -360.5751654247078B_{0,8}(X) - 213.6849676238579B_{1,8}(X) - 98.40765096542357B_{2,8}(X)\\
&\quad - 16.82060242209915B_{3,8}(X) + 28.91366696631814B_{4,8}(X) + 36.54467155974404B_{5,8}(X)\\
&\quad + 3.731015586800578B_{6,8}(X) - 71.9626297312559B_{7,8}(X) - 193.0686388102505B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= -1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984\\
&= -366.9199822310984B_{0,2} + 258.3517672110292B_{1,2} - 186.5418852681748B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 &= 1.223655337688364\cdot 10^{-300}X^8 - 5.205587704887534\cdot 10^{-300}X^7 + 9.409306890019087\cdot 10^{-300}X^6\\
&\quad - 9.282190734559383\cdot 10^{-300}X^5 + 5.21941805196838\cdot 10^{-300}X^4 - 1.541981168110934\\
&\quad \cdot 10^{-300}X^3 - 1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984\\
&= -366.9199822310984B_{0,8} - 210.6020448705665B_{1,8} - 92.50430043579644B_{2,8}\\
&\quad - 12.62674892678823B_{3,8} + 29.03060965645814B_{4,8} + 32.46777531394266B_{5,8}\\
&\quad - 2.315251954334665B_{6,8} - 75.31847214837383B_{7,8} - 186.5418852681748B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.
Bounding polynomials M and m:

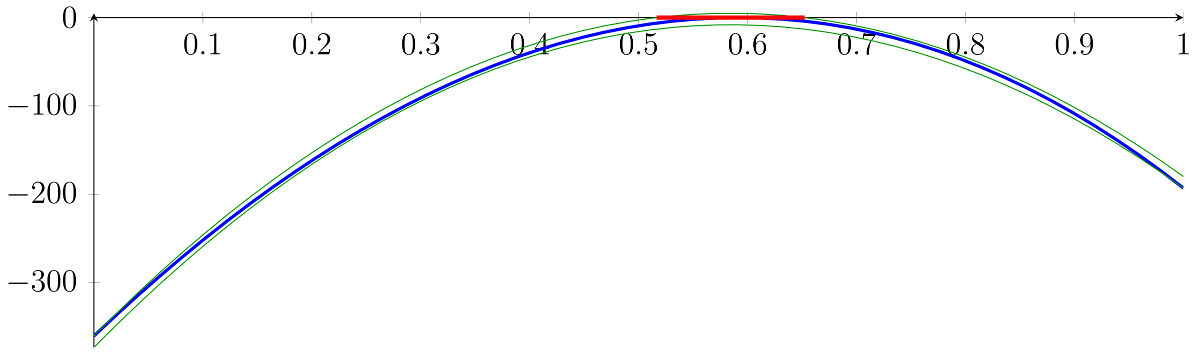
$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$

$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



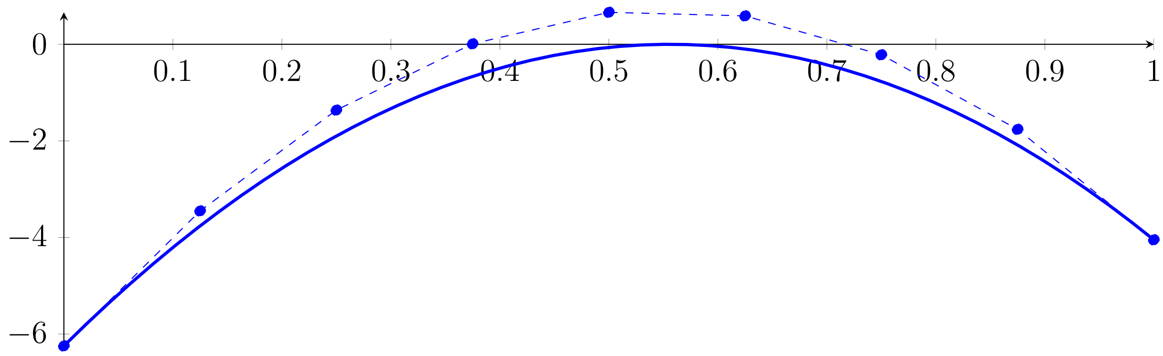
$$[0.5163481231478299, 0.652203482649816]$$

Longest intersection interval: 0.1358553595019861
$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

53.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.507567493156426$

Normalized monomial und Bézier representations and the Bézier polygon:

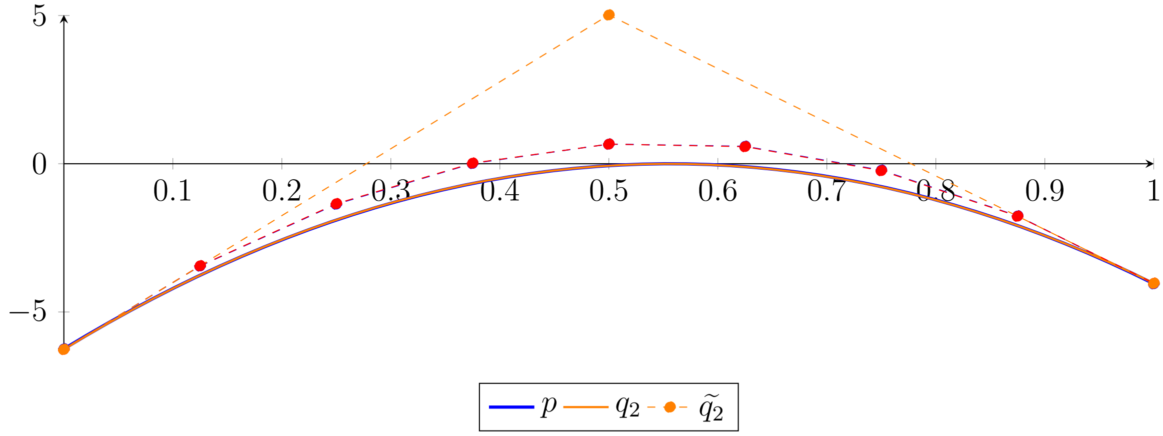
$$\begin{aligned} p = &-7.11749686885428 \cdot 10^{-15} X^8 - 1.625571369884896 \cdot 10^{-11} X^7 - 1.539287417434116 \cdot 10^{-08} X^6 \\ &- 7.733623372922873 \cdot 10^{-06} X^5 - 0.002173482359120639 X^4 - 0.3236423633498969 X^3 \\ &- 19.84316397394663 X^2 + 22.36613094108362 X - 6.245496834711843 \\ = &-6.245496834711843 B_{0,8}(X) - 3.449730467076391 B_{1,8}(X) - 1.36264852708189 B_{2,8}(X) \\ &+ 0.009969657354697074 B_{3,8}(X) + 0.662313708568421 B_{4,8}(X) + 0.5885420610459267 B_{5,8}(X) \\ &- 0.217218177224708 B_{6,8}(X) - 1.760871363955914 B_{7,8}(X) - 4.048353462316386 B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 2.334129178460356\cdot 10^{-302}X^8 - 9.797943630323215\cdot 10^{-302}X^7 + 1.744532667551292\cdot 10^{-301}X^6 \\ &\quad - 1.695264192146682\cdot 10^{-301}X^5 + 9.412284928871313\cdot 10^{-302}X^4 - 2.760729238908791 \\ &\quad \cdot 10^{-302}X^3 - 20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,8} - 3.441577100979207B_{1,8} - 1.347444096092966B_{2,8} \\ &\quad + 0.02053293285443696B_{3,8} + 0.6623539858630032B_{4,8} + 0.5780190629327322B_{5,8} \\ &\quad - 0.232471835936376B_{6,8} - 1.769118710744321B_{7,8} - 4.031921561491104B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.

Bounding polynomials M and m:

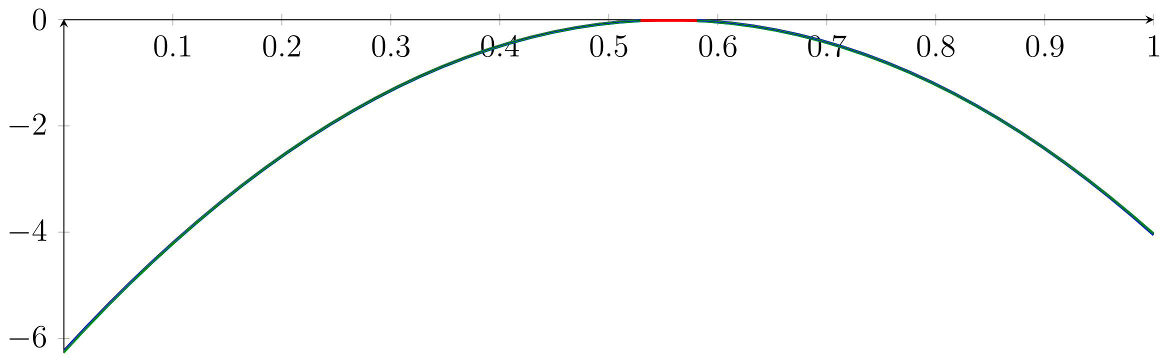
$$M = -20.33236732628744X^2 + 22.56231184660062X - 6.245434180979003$$

$$m = -20.33236732628744X^2 + 22.56231184660062X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

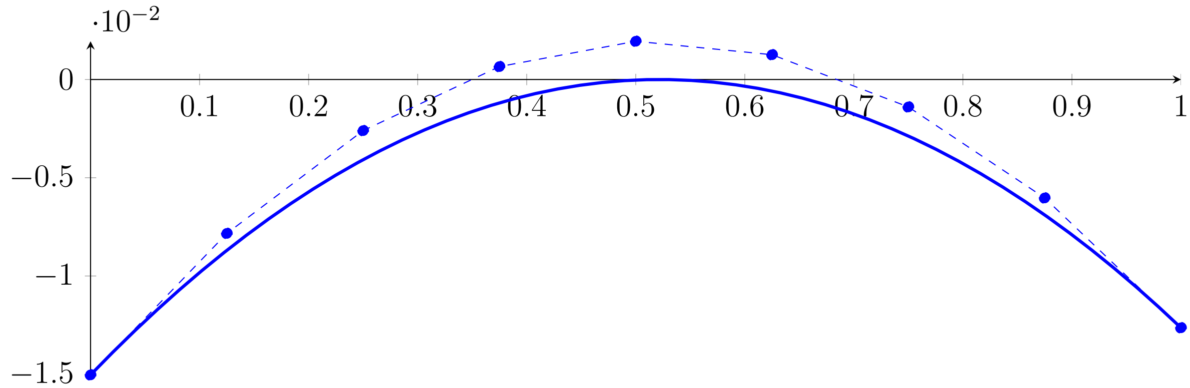
Longest intersection interval: 0.05205162765602134

$\Longrightarrow$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

53.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Normalized monomial und Bézier representations and the Bézier polygon:

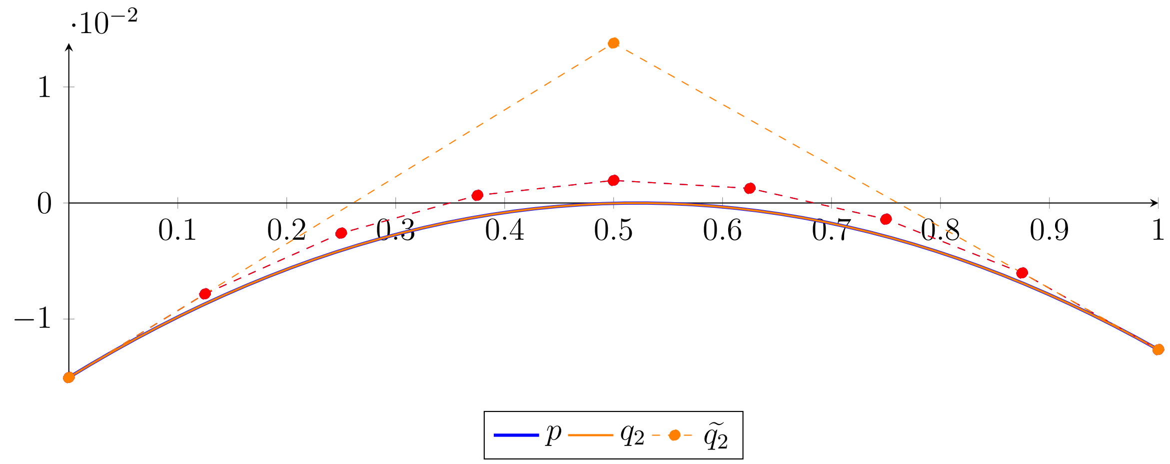
$$\begin{aligned}p &= -3.835321724591002\cdot 10^{-25}X^8 - 1.685970393664051\cdot 10^{-20}X^7 - 3.073417757909172\cdot 10^{-16}X^6\\ &\quad - 2.973678167232232\cdot 10^{-12}X^5 - 1.610545215791654\cdot 10^{-08}X^4 - 4.629380451575473\\ &\quad \cdot 10^{-05}X^3 - 0.05516351610802166X^2 + 0.05760784492782384X - 0.01503353559864481\\ &= -0.01503353559864481B_{0,8}(X) - 0.007832554982666833B_{1,8}(X) - 0.002601699941975341B_{2,8}(X)\\ &\quad + 0.0006582028483490249B_{3,8}(X) + 0.001946326483147738B_{4,8}(X)\\ &\quad + 0.001261843827131283B_{5,8}(X) - 0.001396072485173959B_{6,8}(X)\\ &\quad - 0.006028250049478829B_{7,8}(X) - 0.01263551669178453B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657\\ &= -0.01503585166965657B_{0,2} + 0.01378196629969571B_{1,2} - 0.01263320016040455B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 6.306459041947834\cdot 10^{-305}X^8 - 2.608238530297949\cdot 10^{-304}X^7 + 4.573872253581115\cdot 10^{-304}X^6\\ &\quad - 4.384946099604334\cdot 10^{-304}X^5 + 2.410615466544519\cdot 10^{-304}X^4 - 7.036239809064398\\ &\quad \cdot 10^{-305}X^3 - 0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657\\ &= -0.01503585166965657B_{0,8} - 0.007831397177318504B_{1,8} - 0.002599549271746596B_{2,8}\\ &\quad + 0.0006596920470591496B_{3,8} + 0.001946326779098733B_{4,8} + 0.001260354924372155B_{5,8}\\ &\quad - 0.001398223517120585B_{6,8} - 0.006029408545379487B_{7,8} - 0.01263320016040455B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.316531379978639\cdot 10^{-06}$.

Bounding polynomials M and m:

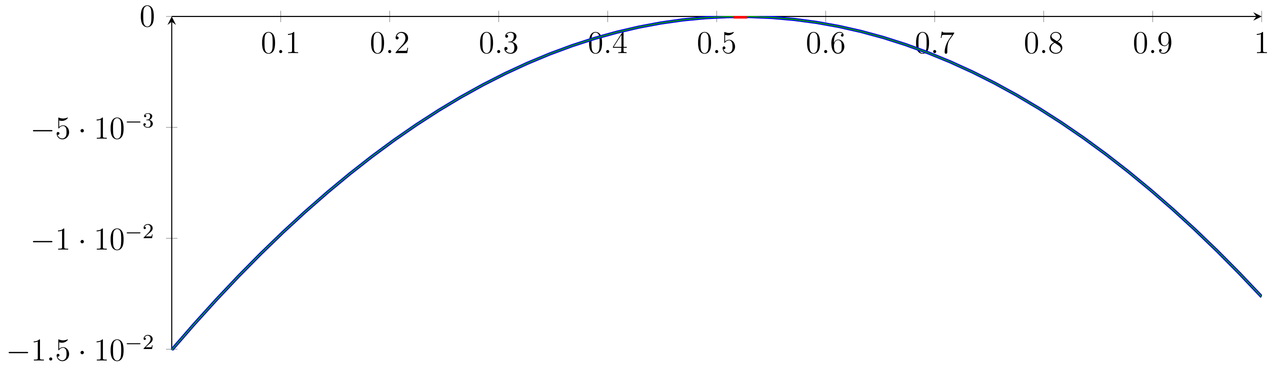
$$M = -0.05523298442945254X^2 + 0.05763563593870456X - 0.0150335351382766$$

$$m = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503816820103655$$

Root of M and m:

$$N(M) = \{0.5154882826278122, 0.5280120194846123\} \qquad N(m) = \{\}$$

Intersection intervals:



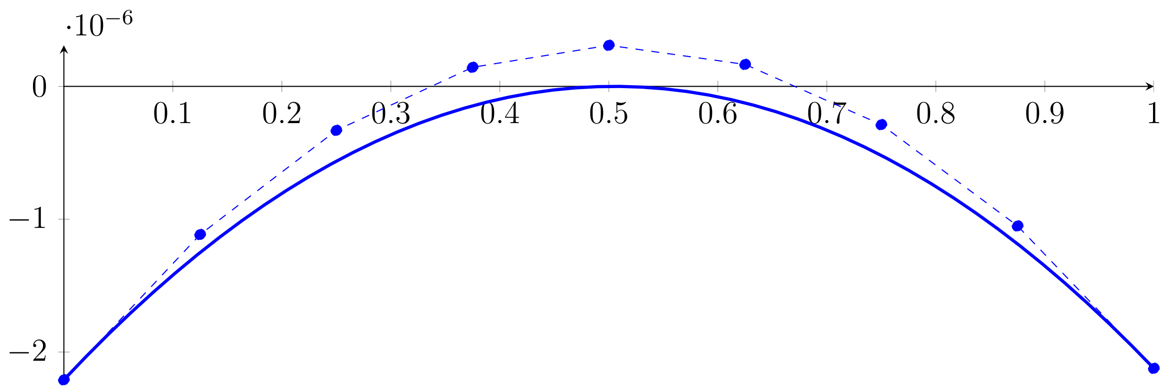
$$[0.5154882826278122, 0.5280120194846123]$$

Longest intersection interval: 0.01252373685680011
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999944147887801, 0.5000055246634291]$,

53.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4999944147887801, 0.500005524$

Normalized monomial und Bézier representations and the Bézier polygon:

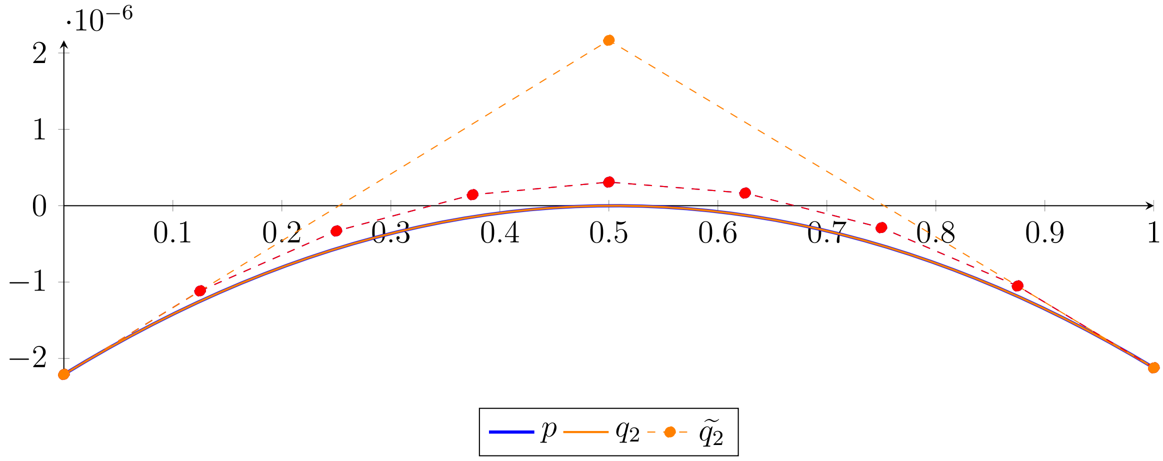
$$\begin{aligned} p = &-2.32099000990716 \cdot 10^{-40} X^8 - 8.147572265547704 \cdot 10^{-34} X^7 - 1.186072294581764 \cdot 10^{-27} X^6 \\ &- 9.164366615560225 \cdot 10^{-22} X^5 - 3.963832728671687 \cdot 10^{-16} X^4 - 9.099890718270267 \cdot 10^{-11} X^3 \\ &- 8.663298447262891 \cdot 10^{-06} X^2 + 8.749571419899133 \cdot 10^{-06} X - 2.209162411567082 \cdot 10^{-06} \\ = &-2.209162411567082 \cdot 10^{-06} B_{0,8}(X) - 1.11546598407969 \cdot 10^{-06} B_{1,8}(X) - 3.311730725659734 \\ &\cdot 10^{-07} B_{2,8}(X) + 1.437146979935834 \cdot 10^{-07} B_{3,8}(X) + 3.091957026128322 \\ &\cdot 10^{-07} B_{4,8}(X) + 1.652683162999621 \cdot 10^{-07} B_{5,8}(X) - 2.880690859425 \cdot 10^{-07} B_{6,8}(X) \\ &- 1.05081812911769 \cdot 10^{-06} B_{7,8}(X) - 2.122980438234407 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -8.663434946303181 \cdot 10^{-06} X^2 + 8.749626019605851 \cdot 10^{-06} X - 2.209166961546417 \cdot 10^{-06} \\ &= -2.209166961546417 \cdot 10^{-06} B_{0,2} + 2.165646048256509 \cdot 10^{-06} B_{1,2} - 2.122975888243747 \cdot 10^{-06} B_{2,2} \end{aligned}$$

$$\widetilde{q}_2 = 9.840755480601 45\cdot 10^{-309}X^8 - 4.039091668354938\cdot 10^{-308}X^7 + 7.030443670929191\cdot 10^{-308}X^6 - 6.698784216784483\cdot 10^{-308}X^5 + 3.668633643168221\cdot 10^{-308}X^4 - 1.069248215117909\cdot 10^{-308}X^3 - 8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209166961546417\cdot 10^{-06}$$
$$= -2.209166961546417\cdot 10^{-06}B_{0,8} - 1.115463709095685\cdot 10^{-06}B_{1,8} - 3.311688475843534\cdot 10^{-07}B_{2,8} + 1.437176229875793\cdot 10^{-07}B_{3,8} + 3.091957026201127\cdot 10^{-07}B_{4,8} + 1.652653913132468\cdot 10^{-07}B_{5,8} - 2.880733109330184\cdot 10^{-07}B_{6,8} - 1.050820404118683\cdot 10^{-06}B_{7,8} - 2.122975888243747\cdot 10^{-06}B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 4.549990660244254\cdot 10^{-12}$.

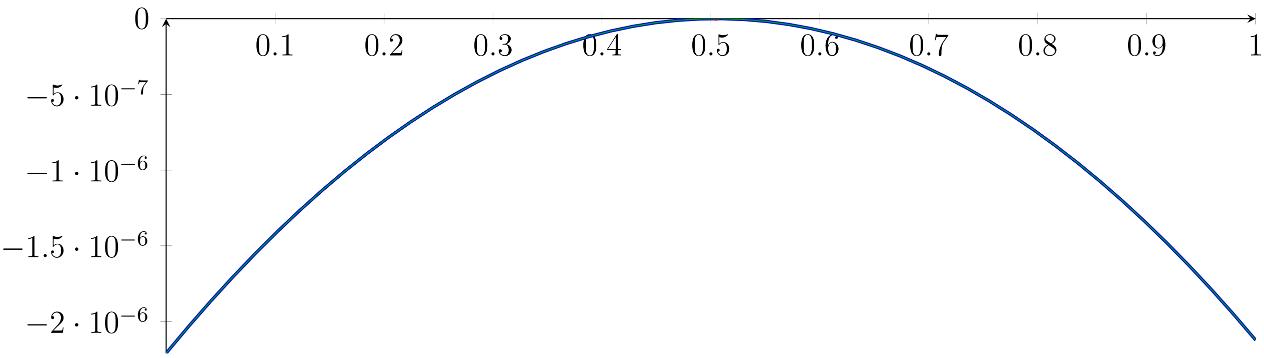
Bounding polynomials M and m:

$$M = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209162411555757\cdot 10^{-06}$$
$$m = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209171511537077\cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{0.5041259457474217, 0.505822887923852\} \qquad N(m) = \{\}$$

Intersection intervals:



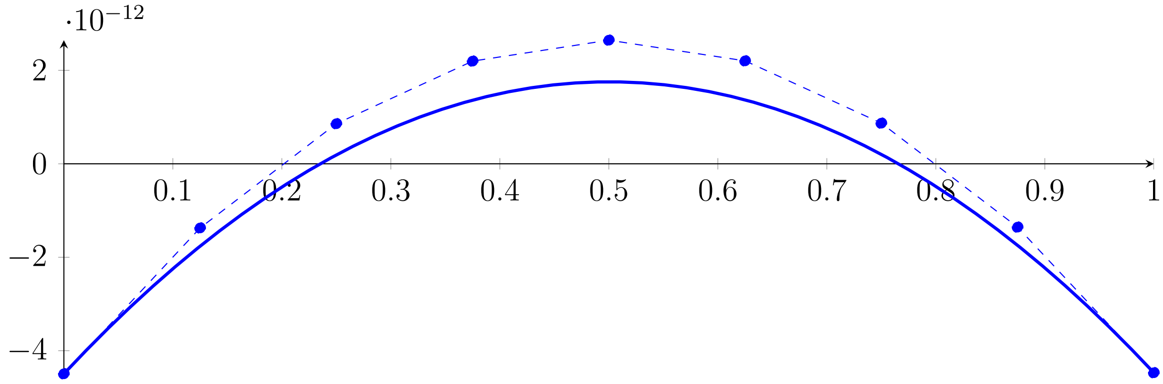
$$[0.5041259457474217, 0.505822887923852]$$

Longest intersection interval: 0.00169694217643035
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000155648447, 0.5000000344176595]$,

53.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.5000000155648447, 0.50000003$

Normalized monomial und Bézier representations and the Bézier polygon:

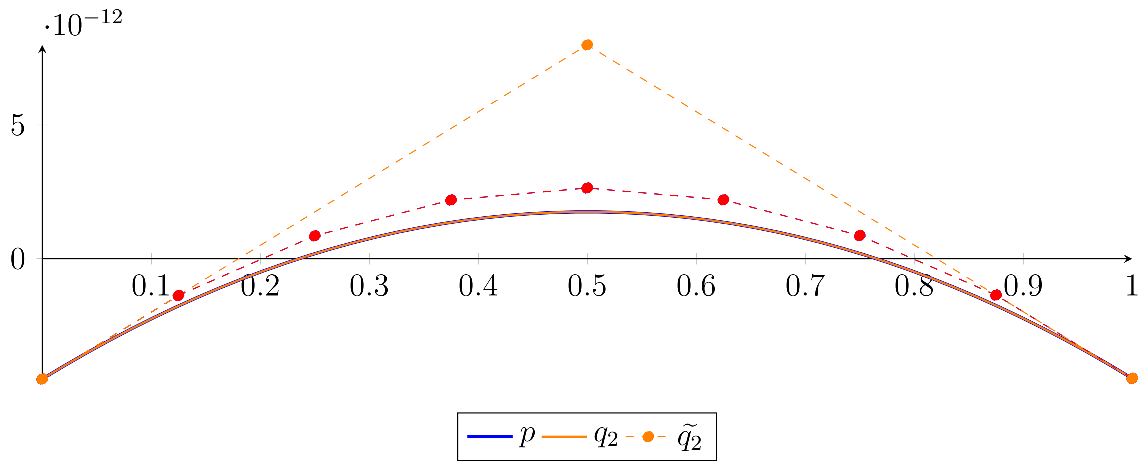
$$\begin{aligned}p &= -1.595914506899923\cdot 10^{-62}X^8 - 3.301399092257474\cdot 10^{-53}X^7 - 2.83213843356941\cdot 10^{-44}X^6\\ &\quad - 1.289553529043531\cdot 10^{-35}X^5 - 3.286896476505693\cdot 10^{-27}X^4 - 4.446733715730271\cdot 10^{-19}X^3\\ &\quad - 2.494734097474612\cdot 10^{-11}X^2 + 2.497049303064054\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}\\ &= -4.493680200151054\cdot 10^{-12}B_{0,8}(X) - 1.372368571320986\cdot 10^{-12}B_{1,8}(X) + 8.579665941252918\\ &\quad \cdot 10^{-13}B_{2,8}(X) + 2.197325288247184\cdot 10^{-12}B_{3,8}(X) + 2.645707503104094\cdot 10^{-12}B_{4,8}(X)\\ &\quad + 2.203113230755426\cdot 10^{-12}B_{5,8}(X) + 8.695424632605847\cdot 10^{-13}B_{6,8}(X)\\ &\quad - 1.355004807321027\cdot 10^{-12}B_{7,8}(X) - 4.470528588930005\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12}\\ &= -4.493680222384723\cdot 10^{-12}B_{0,2} + 7.991566426337562\cdot 10^{-12}B_{1,2} - 4.470528566696336\cdot 10^{-12}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 3.394595061927522\cdot 10^{-314}X^8 - 1.386657030412925\cdot 10^{-313}X^7 + 2.314937908564659\cdot 10^{-313}X^6\\ &\quad - 2.018101017184903\cdot 10^{-313}X^5 + 9.746671569929334\cdot 10^{-314}X^4 - 2.577278121543316\cdot 10^{-314}X^3\\ &\quad - 2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12}\\ &= -4.493680222384723\cdot 10^{-12}B_{0,8} - 1.372368560204152\cdot 10^{-12}B_{1,8} + 8.579666147708415\cdot 10^{-13}B_{2,8}\\ &\quad + 2.197325302540257\cdot 10^{-12}B_{3,8} + 2.645707503104094\cdot 10^{-12}B_{4,8} + 2.203113216462353\cdot 10^{-12}B_{5,8}\\ &\quad + 8.695424426150349\cdot 10^{-13}B_{6,8} - 1.355004818437861\cdot 10^{-12}B_{7,8} - 4.470528566696336\cdot 10^{-12}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.223366895429667\cdot 10^{-20}$.

Bounding polynomials M and m:

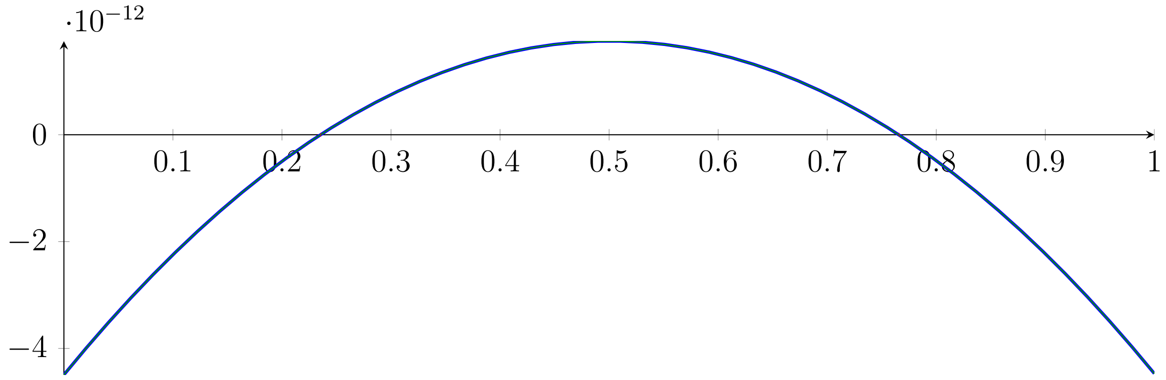
$$M = -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}$$

$$m = -2.494734164175618 \cdot 10^{-11} X^2 + 2.497049329744457 \cdot 10^{-11} X - 4.493680244618392 \cdot 10^{-12}$$

Root of M and m:

$N(M) = \{0.235251622185842, 0.765676398764079\}$ $\quad N(m) = \{0.2352516255462581, 0.7656763954036629\}$

Intersection intervals:



$$[0.235251622185842, 0.2352516255462581],\ [0.7656763954036629, 0.765676398764079]$$

Longest intersection interval: $3.360416099786671 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.50000002]$, interval 2: $[0.50000003, 0.500000030000$

53.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000$

Found root in interval $[0.5000000199999999, 0.50000002]$ at recursion depth 8!

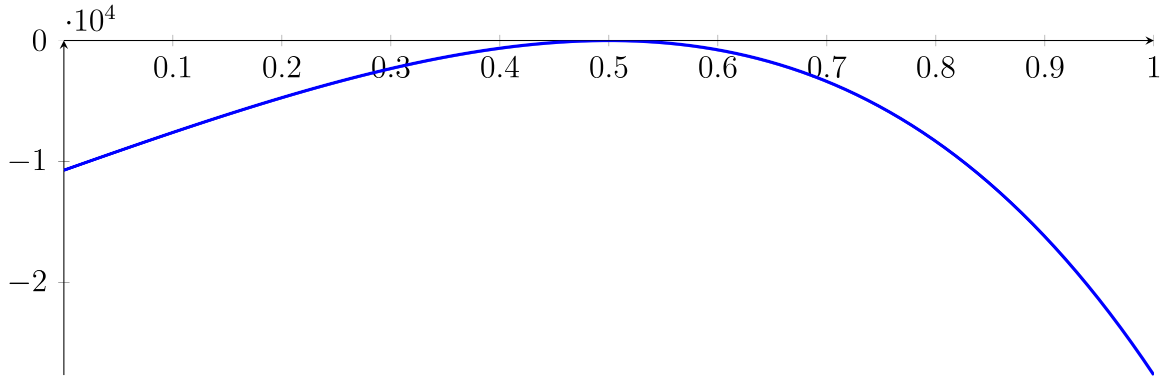
53.9 Recursion Branch 1 1 1 1 1 1 1 2 in Interval 2: $[0.50000003, 0.500000030000000$

Found root in interval $[0.50000003, 0.5000000300000001]$ at recursion depth 8!

53.10 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.5000000199999999, 0.50000002], [0.50000003, 0.5000000300000001]$$

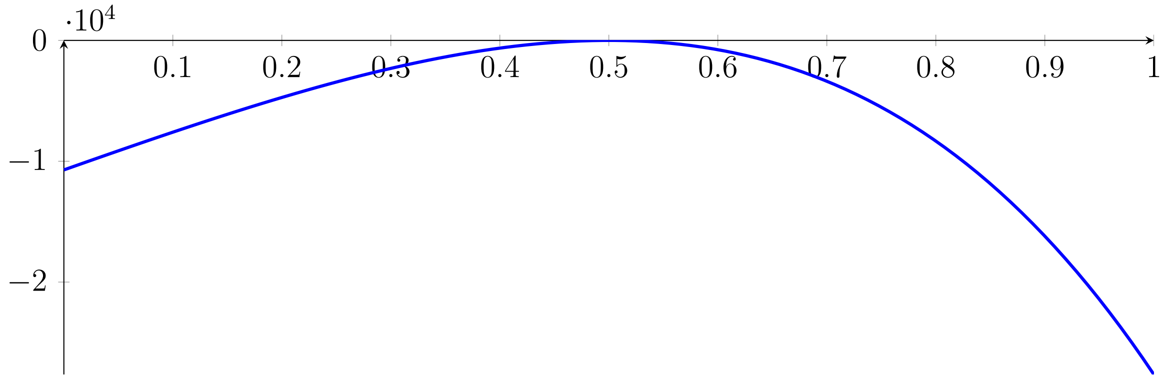
with precision $\varepsilon = 1 \cdot 10^{-16}$.

54 Running CubeClip on h_8 with epsilon 16

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called CubeClip with input polynomial on interval $[0, 1]$:

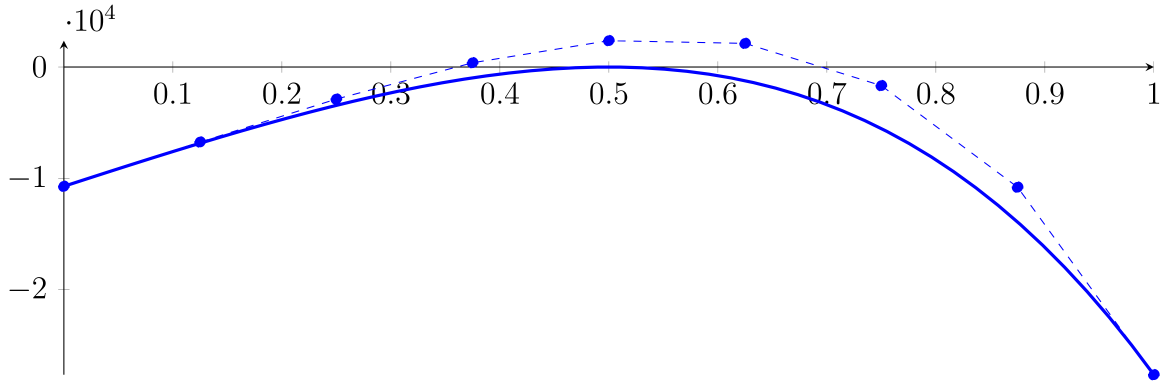
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



54.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

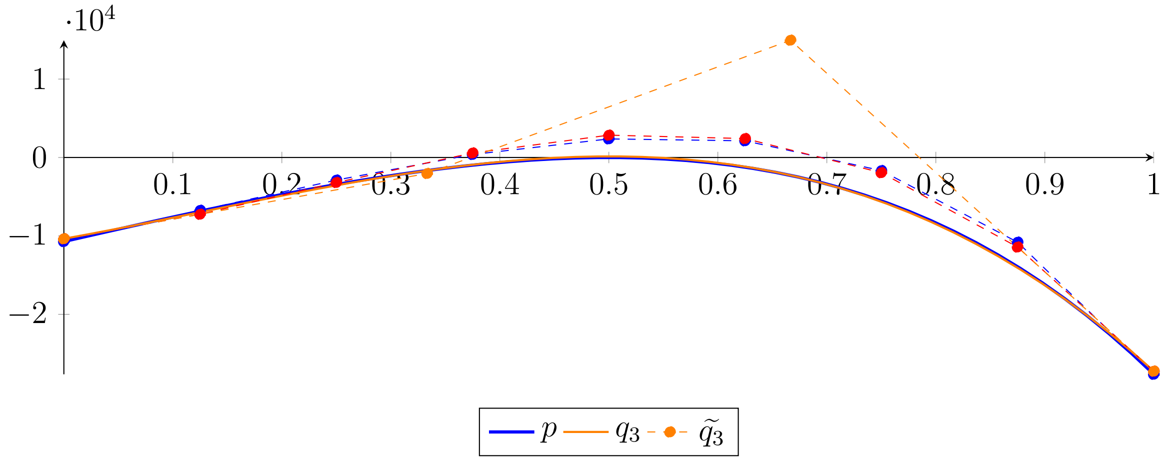
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ = &-10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &+ 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= -5.703152099307112\cdot 10^{-299}X^8 + 1.997854926822688\cdot 10^{-298}X^7 - 2.620079279971631\\ &\quad \cdot 10^{-298}X^6 + 1.512761996617561\cdot 10^{-298}X^5 - 2.636105166170616\cdot 10^{-299}X^4\\ &\quad - 67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554\\ &= -10364.86272786554B_{0,8} - 7243.22568371353B_{1,8} - 3192.719831416224B_{2,8}\\ &\quad + 574.7343076805747B_{3,8} + 2847.216212231063B_{4,8} + 2412.80536088944B_{5,8}\\ &\quad - 1940.418767690097B_{6,8} - 11424.37669485335B_{7,8} - 27250.98894194612B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

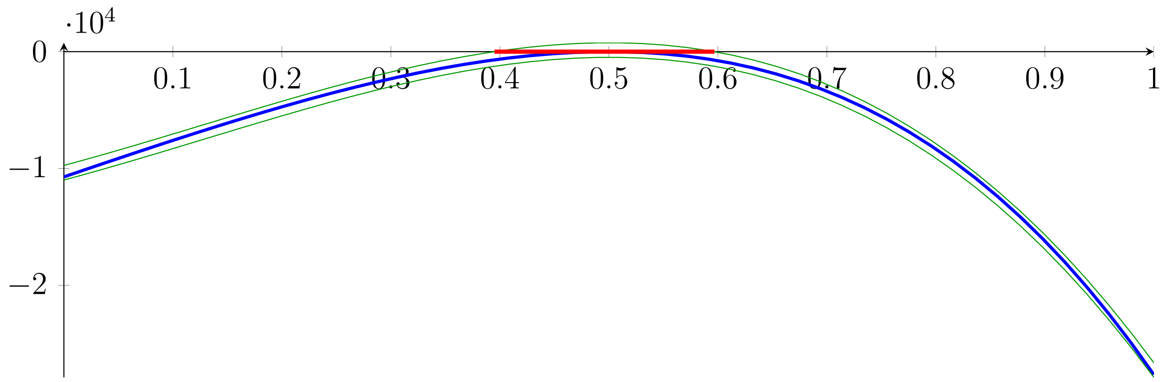
$$M = -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 9740.484261812251$$

$$m = -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $\quad N(m) = \{-0.62348796579237$

Intersection intervals:



$[0.3950604114095972, 0.5968461976869074]$

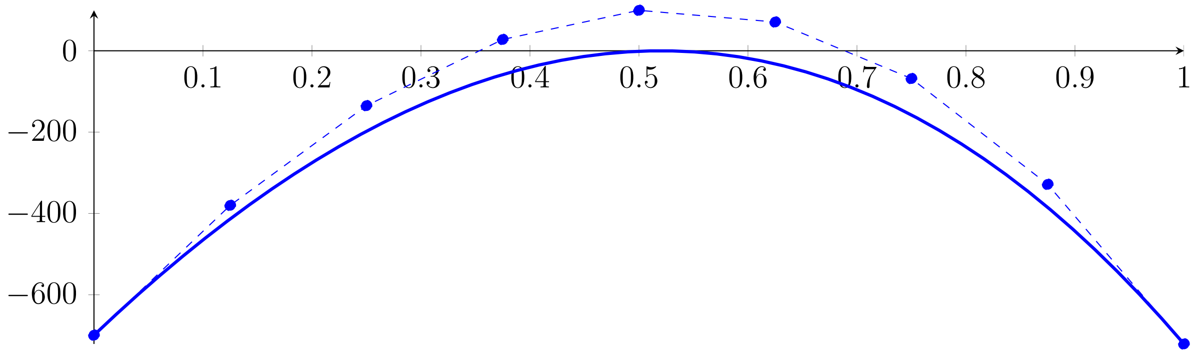
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

54.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

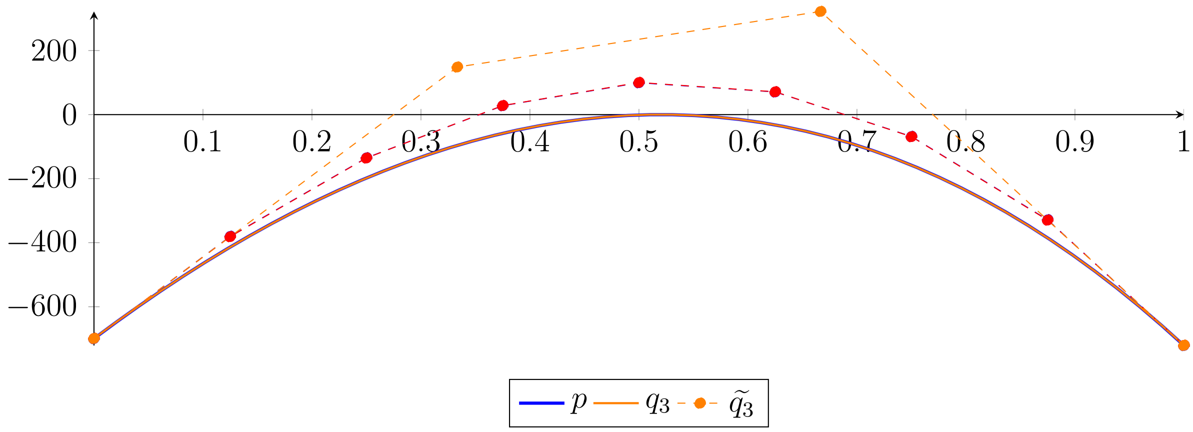
$$\begin{aligned}p = &-2.748682461629333\cdot 10^{-06}X^8 - 0.0005198138726602274X^7 - 0.04066637698425721X^6\\ &- 1.681520506726176X^5 - 38.59639865464246X^4 - 460.2831920090805X^3\\ &- 2074.780222477586X^2 + 2553.796807278985X - 699.3475151603227\\ = &-699.3475151603227B_{0,8}(X) - 380.1229142504495B_{1,8}(X) - 134.9976070004902B_{2,8}(X)\\ &+ 27.8090638751075B_{3,8}(X) + 99.52637853825791B_{4,8}(X) + 70.80221287533186B_{5,8}(X)\\ &- 68.32844102535583B_{6,8}(X) - 328.4764717433994B_{7,8}(X) - 720.9332304689123B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\ &= -698.7407882409323B_{0,3} + 148.5035278252892B_{1,3}\\ &\quad + 322.0745267793334B_{2,3} - 720.3121660793493B_{3,3}\end{aligned}$$

$$\begin{aligned}\tilde{q}_3 = &-2.24002635483637\cdot 10^{-300}X^8 + 8.593407551274415\cdot 10^{-300}X^7 - 1.289726574039331\\ &\cdot 10^{-299}X^6 + 9.445830854385802\cdot 10^{-300}X^5 - 3.303106122422159\cdot 10^{-300}X^4\\ &- 542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\ = &-698.7407882409323B_{0,8} - 381.0241697160992B_{1,8} - 135.4868351675709B_{2,8}\\ &+ 28.18756585642868B_{3,8} + 100.3153838076753B_{4,8} + 71.21296913794494B_{5,8}\\ &- 68.80332770098655B_{6,8} - 329.4171562573433B_{7,8} - 720.3121660793493B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

Bounding polynomials M and m:

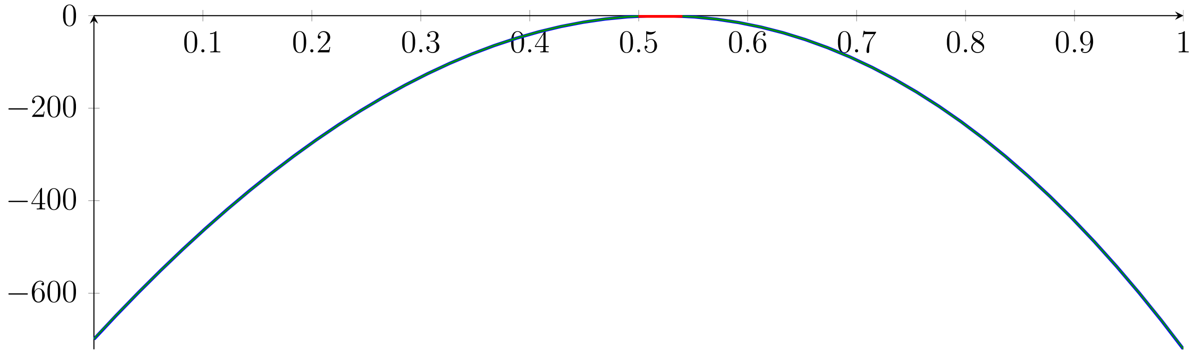
$$M = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 697.8001037269884$$

$$m = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 699.6814727548762$$

Root of M and m:

$$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\} \quad N(m) = \{-4.76690070753073\}$$

Intersection intervals:



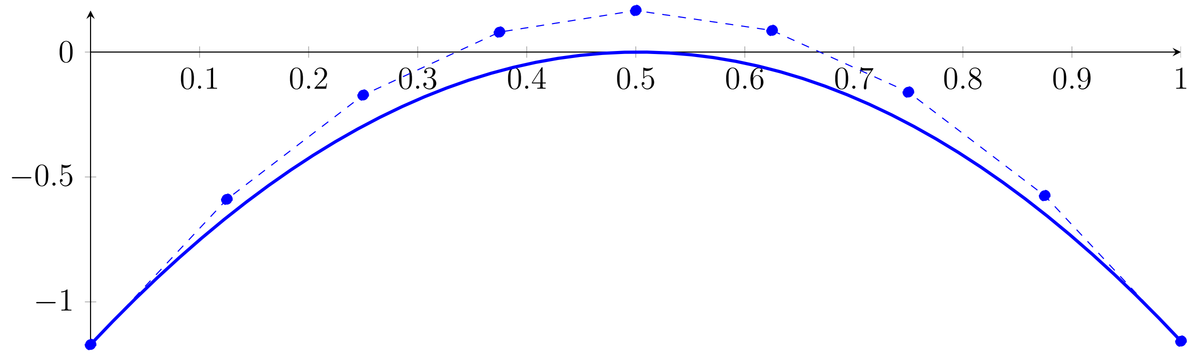
$$[0.4997746332555131, 0.5401382492675976]$$

Longest intersection interval: 0.04036361601208447
$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

54.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Normalized monomial und Bézier representations and the Bézier polygon:

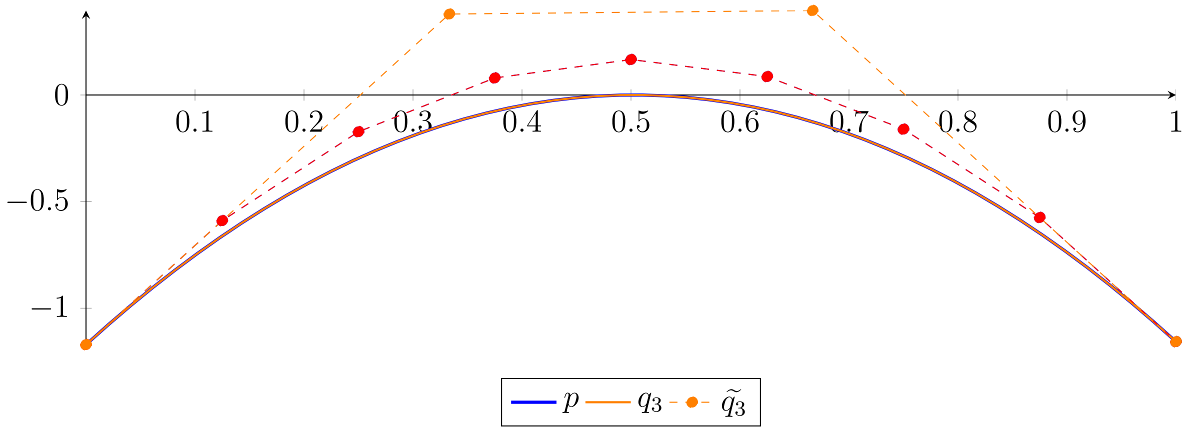
$$\begin{aligned}
p = &-1.936623061789877\cdot 10^{-17}X^8 - 9.265403983052604\cdot 10^{-14}X^7 - 1.838110077974511\cdot 10^{-10}X^6\\
&- 1.935168004397404\cdot 10^{-07}X^5 - 0.0001140127070697086X^4 - 0.0356257658581906X^3\\
&- 4.602344943530705X^2 + 4.651752639476855X - 1.170857231885768\\
= &-1.170857231885768B_{0,8}(X) - 0.5893881519511612B_{1,8}(X) - 0.172288534285508B_{2,8}(X)\\
&+ 0.07980544672086649B_{3,8}(X) + 0.1662559879246795B_{4,8}(X) + 0.08642365397403269B_{5,8}(X)\\
&- 0.1603326261538093B_{6,8}(X) - 0.574655562621217B_{7,8}(X) - 1.157189508205582B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &-0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661\\
= &-1.170855596980661B_{0,3} + 0.3797177193764957B_{1,3}\\
&+ 0.3962250709273155B_{2,3} - 1.157187871760247B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 = &-3.713970905558003\cdot 10^{-303}X^8 + 1.449155002864004\cdot 10^{-302}X^7 - 2.225710229630734\\
&\cdot 10^{-302}X^6 + 1.689258272787799\cdot 10^{-302}X^5 - 6.343129302139404\cdot 10^{-303}X^4\\
&- 0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661\\
= &-1.170855596980661B_{0,8} - 0.5893906033467271B_{1,8} - 0.1722898202277581B_{2,8}\\
&+ 0.07980649649353122B_{3,8} + 0.1662580909344257B_{4,8} + 0.08642470721221019B_{5,8}\\
&- 0.1603339105558303B_{6,8} - 0.574658018252411B_{7,8} - 1.157187871760247B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.45563119394813 \cdot 10^{-06}$.
Bounding polynomials M and m:

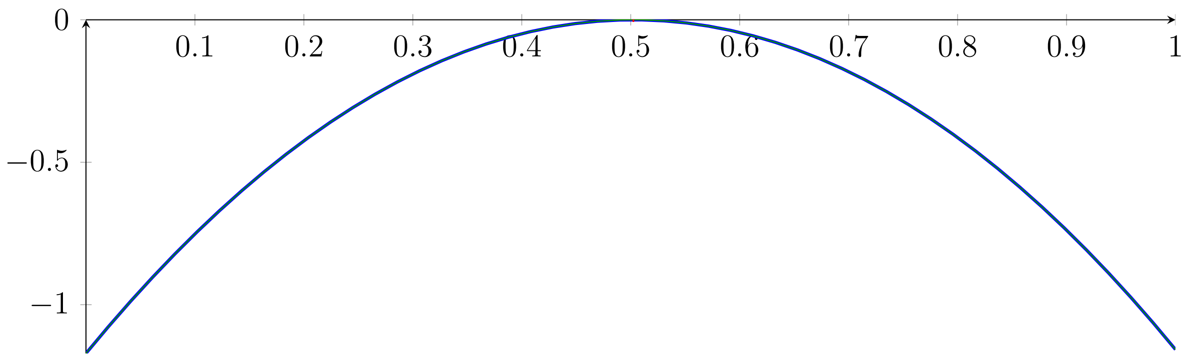
$$M = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170853141349467$$

$$m = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170858052611855$$

Root of M and m:

$$N(M) = \{-129.3630802555855, 0.5016184355695172, 0.5032421204520033\} \quad N(m) = \{-129.3630802637075\}$$

Intersection intervals:



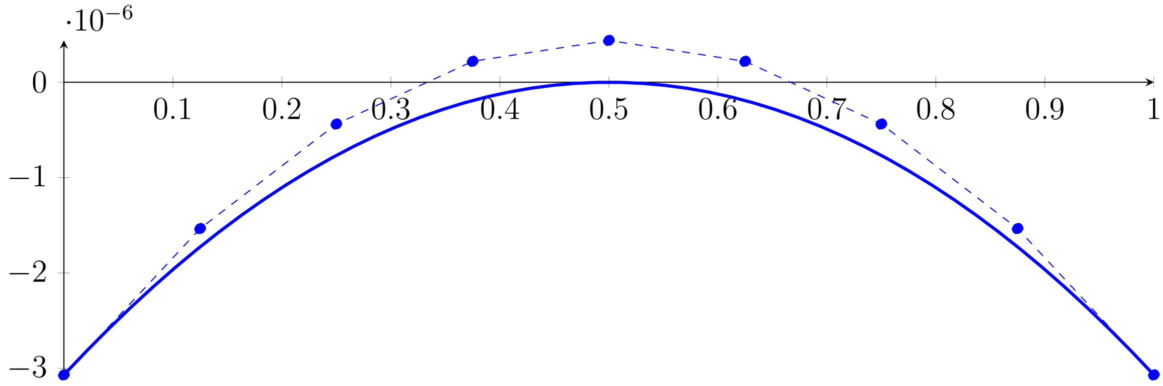
$$[0.5016184355695172, 0.5032421204520033]$$

Longest intersection interval: 0.001623684882486142
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999934125800028, 0.5000066371751187]$,

54.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4999934125800028, 0.50000663717511$

Normalized monomial und Bézier representations and the Bézier polygon:

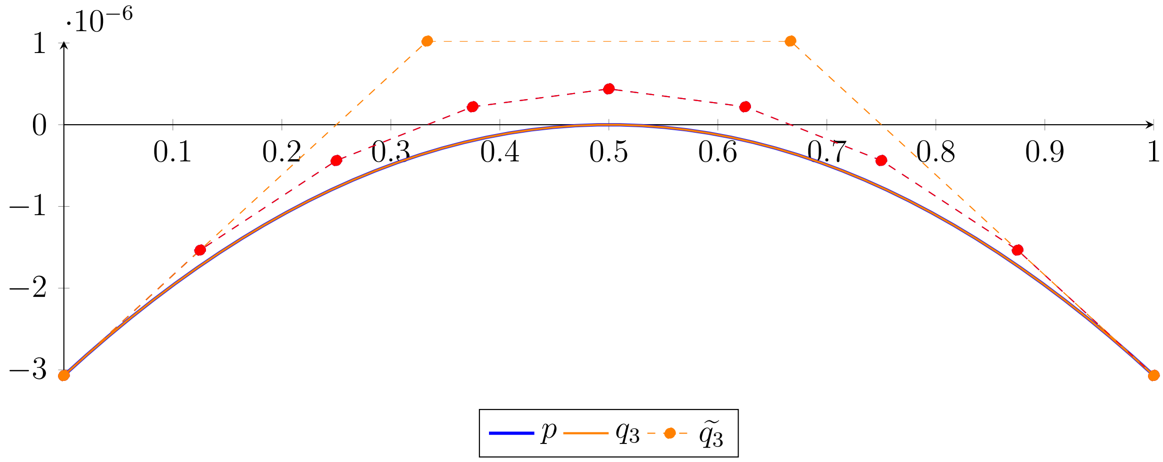
$$\begin{aligned}
p = &-9.355329246270906 \cdot 10^{-40} X^8 - 2.758930189698976 \cdot 10^{-33} X^7 - 3.374040776758279 \cdot 10^{-27} X^6 \\
&- 2.190121853313308 \cdot 10^{-21} X^5 - 7.958070257617345 \cdot 10^{-16} X^4 - 1.534811947451426 \cdot 10^{-10} X^3 \\
&- 1.227519762250185 \cdot 10^{-05} X^2 + 1.227554003668957 \cdot 10^{-05} X - 3.068949673836091 \cdot 10^{-06} \\
= &-3.068949673836091 \cdot 10^{-06} B_{0,8}(X) - 1.534507169249895 \cdot 10^{-06} B_{1,8}(X) - 4.384645797530504 \\
&\cdot 10^{-07} B_{2,8}(X) + 2.191753539188221 \cdot 10^{-07} B_{3,8}(X) + 4.384098910187333 \cdot 10^{-07} B_{4,8}(X) \\
&+ 2.192362907883254 \cdot 10^{-07} B_{5,8}(X) - 4.383481875421282 \cdot 10^{-07} B_{6,8}(X) \\
&- 1.534346284753723 \cdot 10^{-06} B_{7,8}(X) - 3.068760741638923 \cdot 10^{-06} B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2 \\
&\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673824723\cdot 10^{-06} \\
&= -3.068949673824723\cdot 10^{-06}B_{0,3} + 1.022897004996009\cdot 10^{-06}B_{1,3} \\
&\quad + 1.023011143323854\cdot 10^{-06}B_{2,3} - 3.068760741627554\cdot 10^{-06}B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -9.792734715851906\cdot 10^{-309}X^8 + 3.822699025575463\cdot 10^{-308}X^7 - 5.87427002023018\cdot 10^{-308}X^6 \\
&\quad + 4.462726908063245\cdot 10^{-308}X^5 - 1.679538448591103\cdot 10^{-308}X^4 - 1.534827863652778\cdot 10^{-10}X^3 \\
&\quad - 1.227519762147866\cdot 10^{-05}X^2 + 1.22755400364622\cdot 10^{-05}X - 3.068949673824723\cdot 10^{-06} \\
&= -3.068949673824723\cdot 10^{-06}B_{0,8} - 1.534507169266948\cdot 10^{-06}B_{1,8} - 4.38464579761983\cdot 10^{-07}B_{2,8} \\
&\quad + 2.191753539261305\cdot 10^{-07}B_{3,8} + 4.384098910333502\cdot 10^{-07}B_{4,8} + 2.192362907956339\cdot 10^{-07}B_{5,8} \\
&\quad - 4.383481875510608\cdot 10^{-07}B_{6,8} - 1.534346284770776\cdot 10^{-06}B_{7,8} - 3.068760741627554\cdot 10^{-06}B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.705314892341177\cdot 10^{-17}$.
Bounding polynomials M and m:

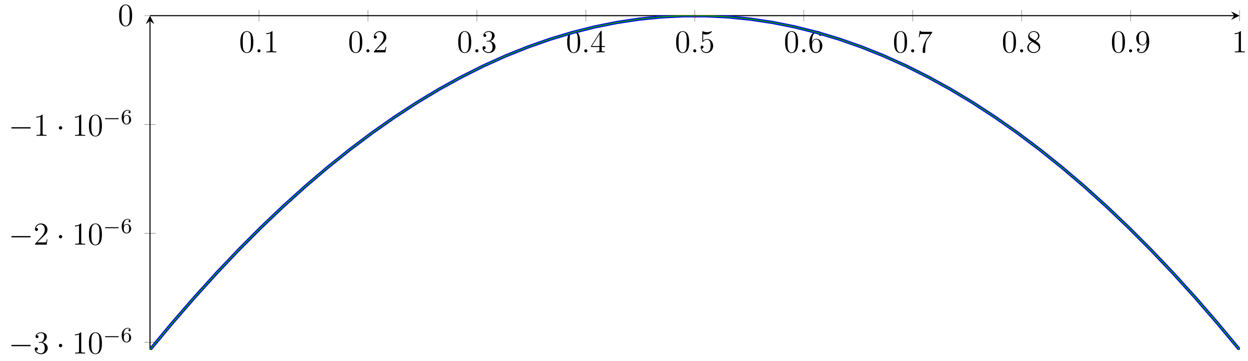
$$\begin{aligned}
M = &-1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2 \\
&+ 1.22755400364622\cdot 10^{-05}X - 3.068949673807669\cdot 10^{-06}
\end{aligned}$$

$$\begin{aligned}
m = &-1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2 \\
&+ 1.22755400364622\cdot 10^{-05}X - 3.068949673841776\cdot 10^{-06}
\end{aligned}$$

Root of M and m:

$N(M) = \{-79978.68293772447, 0.4996311727354866, 0.5003873441571286\}$ $\quad N(m) = \{-79978.68293772447,$

Intersection intervals:



$$[0.4996311727354866, 0.4996311764098305],\ [0.5003873404827848, 0.5003873441571286]$$

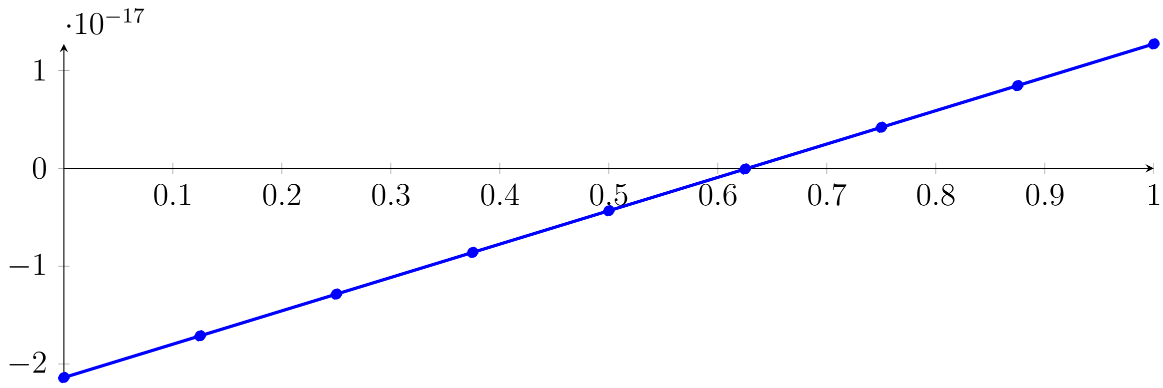
Longest intersection interval: $3.674343879435088 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999695, 0.5000000200000181]$, interval 2: $[0.500000029999981$

54.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.5000000199999695, 0.500000020000$

Normalized monomial und Bézier representations and the Bézier polygon:

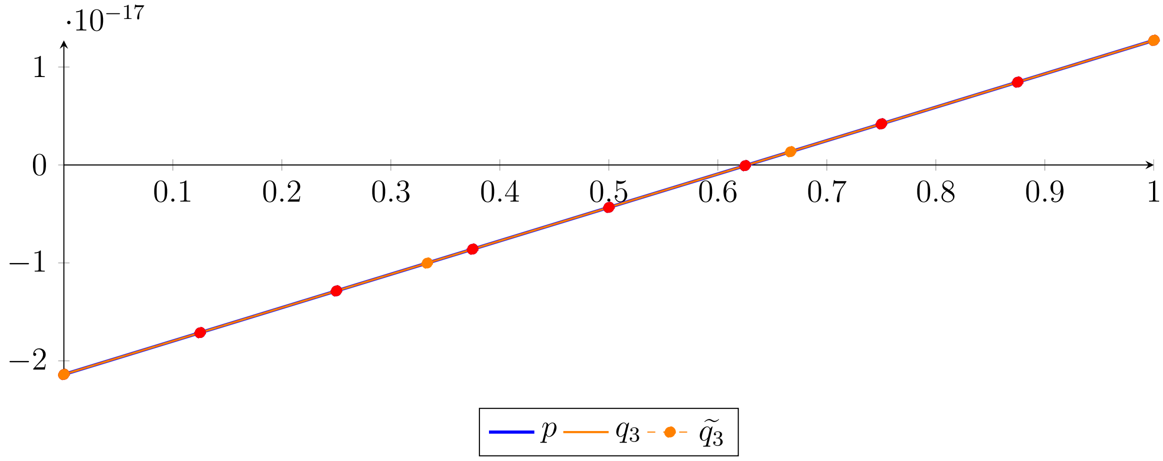
$$\begin{aligned}
p = &-3.10811779683933 \cdot 10^{-107} X^8 - 2.494594121391514 \cdot 10^{-92} X^7 - 8.302910645986526 \cdot 10^{-78} X^6 \\
&- 1.466794525552917 \cdot 10^{-63} X^5 - 1.450540809134678 \cdot 10^{-49} X^4 - 7.613758006193666 \cdot 10^{-36} X^3 \\
&- 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
= &-2.138639484655546 \cdot 10^{-17} B_{0,8}(X) - 1.71230868147238 \cdot 10^{-17} B_{1,8}(X) - 1.285978470175393 \\
&\cdot 10^{-17} B_{2,8}(X) - 8.596488507645843 \cdot 10^{-18} B_{3,8}(X) - 4.333198232399549 \\
&\cdot 10^{-18} B_{4,8}(X) - 6.991387601504461 \cdot 10^{-20} B_{5,8}(X) + 4.19336456150767 \cdot 10^{-18} B_{6,8}(X) \\
&+ 8.456637080168595 \cdot 10^{-18} B_{7,8}(X) + 1.271990367996773 \cdot 10^{-17} B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &-7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 \\
&+ 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
= &-2.138639484655546 \cdot 10^{-17} B_{0,3} - 1.001757342833769 \cdot 10^{-17} B_{1,3} \\
&+ 1.351192747170036 \cdot 10^{-18} B_{2,3} + 1.271990367996773 \cdot 10^{-17} B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 = &-3.883510371826523 \cdot 10^{-321} X^8 + 2.549502248952292 \cdot 10^{-320} X^7 - 4.188626787236358 \cdot 10^{-320} X^6 \\
&+ 1.055694768751284 \cdot 10^{-320} X^5 + 2.46739471443405 \cdot 10^{-320} X^4 - 7.613758006193956 \cdot 10^{-36} X^3 \\
&- 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
= &-2.138639484655546 \cdot 10^{-17} B_{0,8} - 1.71230868147238 \cdot 10^{-17} B_{1,8} - 1.285978470175393 \cdot 10^{-17} B_{2,8} \\
&- 8.596488507645843 \cdot 10^{-18} B_{3,8} - 4.333198232399549 \cdot 10^{-18} B_{4,8} - 6.991387601504461 \cdot 10^{-20} B_{5,8} \\
&+ 4.19336456150767 \cdot 10^{-18} B_{6,8} + 8.456637080168595 \cdot 10^{-18} B_{7,8} + 1.271990367996773 \cdot 10^{-17} B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301733860119 \cdot 10^{-51}$.
Bounding polynomials M and m:

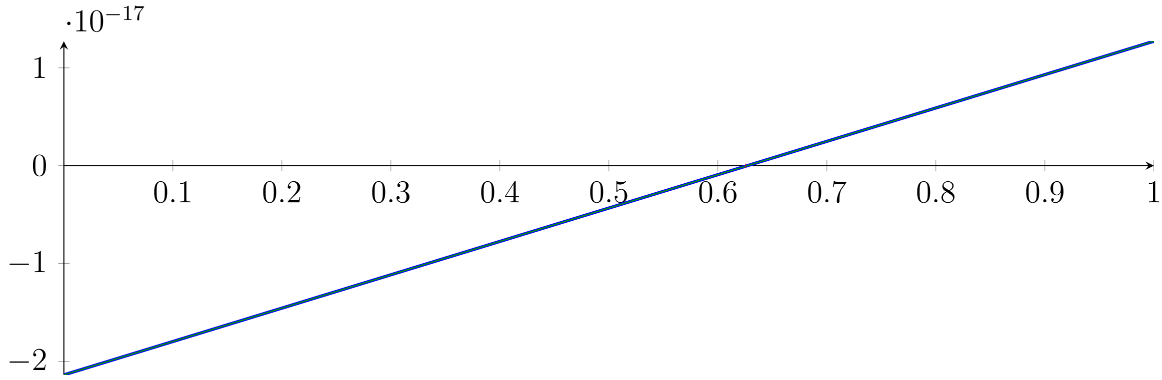
$$M = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

$$m = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929227157.35, 0.6249980054152174, 205797.0483973646\}$ $N(m) = \{-21766929227157.35, 0$

Intersection intervals:



$[0.6249980054152174, 0.6249980054152174]$

Longest intersection interval: $1.822716451808538 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000199999999]$,

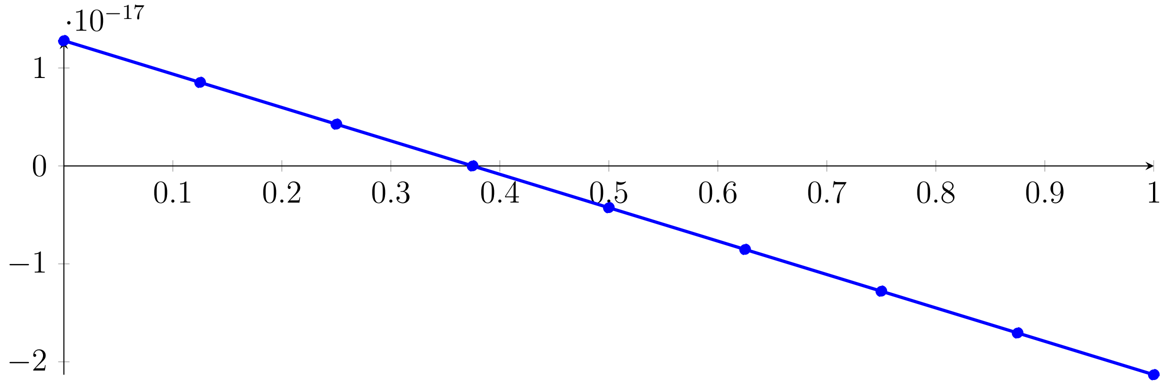
54.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.50000001999$

Found root in interval $[0.5000000199999999, 0.5000000199999999]$ at recursion depth 6!

54.7 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.5000000299999818, 0.500000030000$

Normalized monomial und Bézier representations and the Bézier polygon:

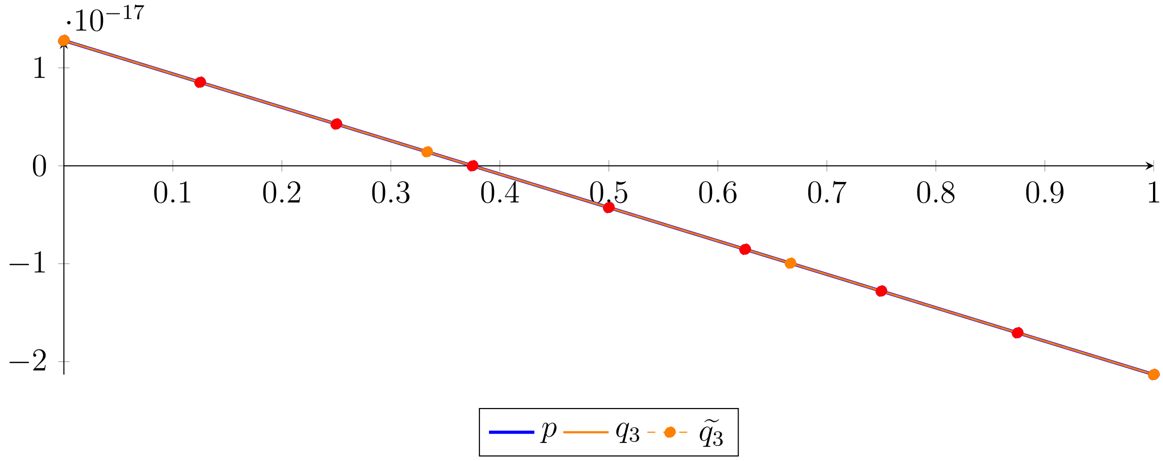
$$\begin{aligned}p = &-3.108117561752323 \cdot 10^{-107} X^8 - 2.494593961411665 \cdot 10^{-92} X^7 - 8.302910210921025 \cdot 10^{-78} X^6 \\ &- 1.466794466465724 \cdot 10^{-63} X^5 - 1.450540769370867 \cdot 10^{-49} X^4 - 7.613757909646128 \cdot 10^{-36} X^3 \\ &- 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17} \\ = &\ 1.27898932402398 \cdot 10^{-17} B_{0,8}(X) + 8.526626725463981 \cdot 10^{-18} B_{1,8}(X) + 4.263354291826315 \\ &\cdot 10^{-18} B_{2,8}(X) + 7.593932680254395 \cdot 10^{-23} B_{3,8}(X) - 4.263208332034555 \cdot 10^{-18} B_{4,8}(X) \\ &- 8.526498522257758 \cdot 10^{-18} B_{5,8}(X) - 1.278979463134281 \cdot 10^{-17} B_{6,8}(X) \\ &- 1.70530966592897 \cdot 10^{-17} B_{7,8}(X) - 2.131640460609844 \cdot 10^{-17} B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 = &-7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 \\ &- 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17} \\ = &\ 1.27898932402398 \cdot 10^{-17} B_{0,3} + 1.421182534170946 \cdot 10^{-18} B_{1,3} \\ &- 9.947583414608468 \cdot 10^{-18} B_{2,3} - 2.131640460609844 \cdot 10^{-17} B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 = &-6.944710234356022 \cdot 10^{-322} X^8 - 5.839855933843534 \cdot 10^{-321} X^7 + 4.132859127462027 \cdot 10^{-320} X^6 \\ &- 7.976936884929846 \cdot 10^{-320} X^5 + 6.21874252599804 \cdot 10^{-320} X^4 - 7.613757909646418 \cdot 10^{-36} X^3 \\ &- 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17} \\ = &\ 1.27898932402398 \cdot 10^{-17} B_{0,8} + 8.526626725463981 \cdot 10^{-18} B_{1,8} + 4.263354291826315 \cdot 10^{-18} B_{2,8} \\ &+ 7.593932680254395 \cdot 10^{-23} B_{3,8} - 4.263208332034555 \cdot 10^{-18} B_{4,8} - 8.526498522257758 \cdot 10^{-18} B_{5,8} \\ &- 1.278979463134281 \cdot 10^{-17} B_{6,8} - 1.70530966592897 \cdot 10^{-17} B_{7,8} - 2.131640460609844 \cdot 10^{-17} B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301648651952 \cdot 10^{-51}$.
Bounding polynomials M and m:

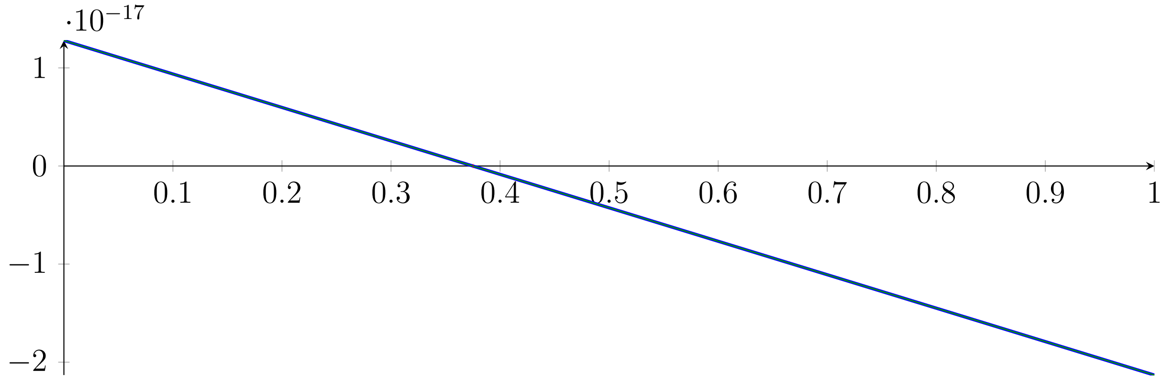
$$M = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

$$m = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929297379.88, -205796.0503430093, 0.375002063855762\}$ $N(m) = \{-21766929297379.88,$

Intersection intervals:



$[0.375002063855762, 0.375002063855762]$

Longest intersection interval: $1.822716401842262 \cdot 10^{-34}$
$\implies$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

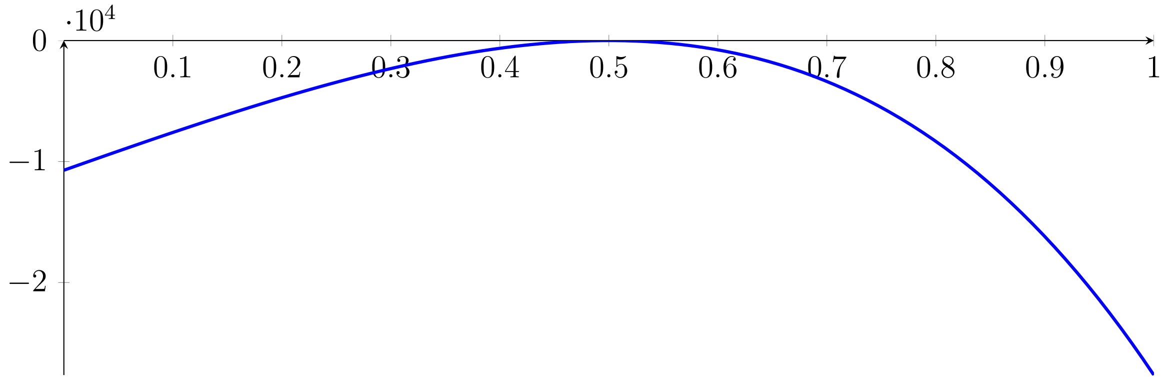
54.8 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 6!

54.9 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.5000000199999999, 0.5000000199999999],\ [0.50000003, 0.50000003]$$

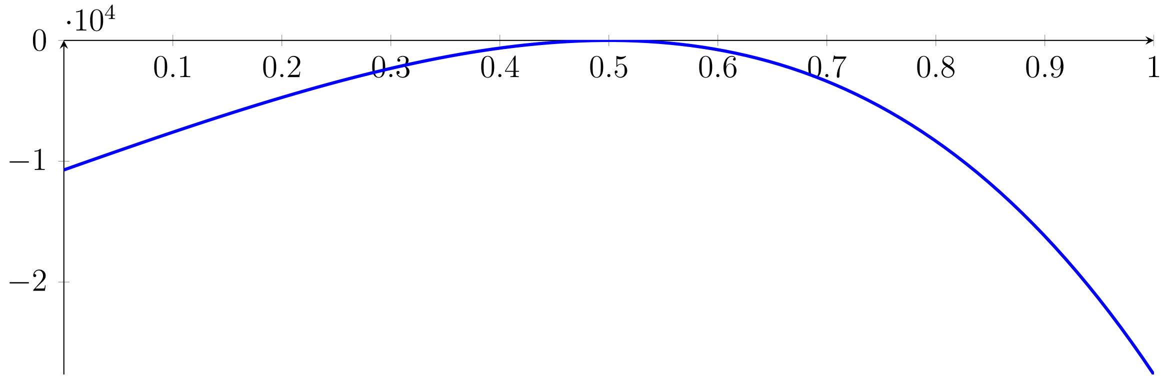
with precision $\varepsilon = 1 \cdot 10^{-16}$.

55 Running BezClip on h_8 with epsilon 32

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called BezClip with input polynomial on interval $[0, 1]$:

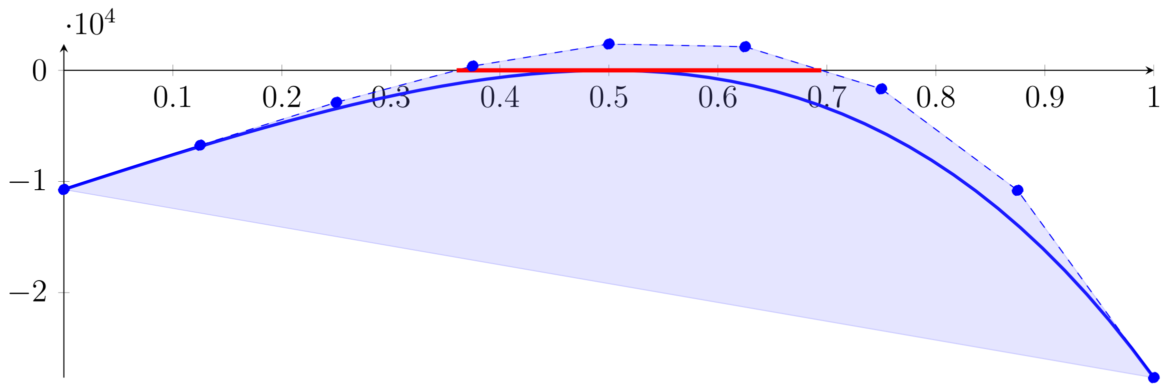
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



55.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

$$[0.3603640692588709, 0.6949437907012195]$$

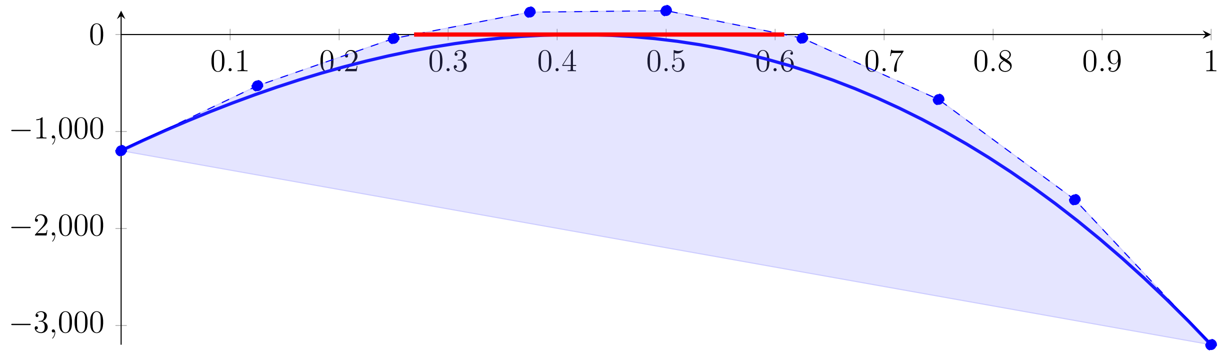
Longest intersection interval: 0.3345797214423486

$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

55.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6\\&\quad - 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3\\&\quad - 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098\\&= -1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X)\\&\quad + 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X)\\&\quad - 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

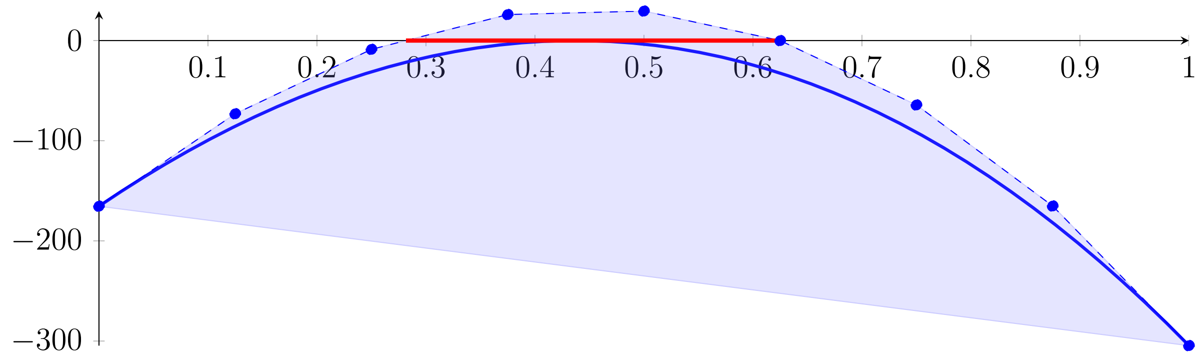
$$[0.2687909235445741, 0.6084084634355219]$$

Longest intersection interval: 0.3396175398909478
$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

55.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.779187295559603\cdot10^{-08}X^8 - 9.441522673365855\cdot10^{-06}X^7 - 0.001328615938017263X^6\\&\quad - 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3\\&\quad - 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987\\&= -165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X)\\&\quad + 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X)\\&\quad - 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

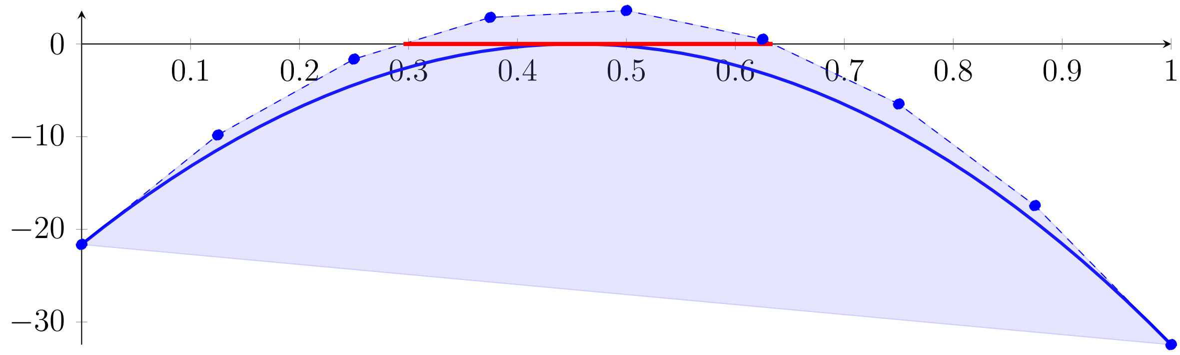
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\Longrightarrow$ Selective recursion: interval 1: $[0.4822990094256734, 0.5212832145711177]$,

55.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4822990094256734, 0.52128321457111$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -5.334693469973702\cdot 10^{-12}X^8 - 5.317476896558791\cdot 10^{-09}X^7 - 2.197128069269196\cdot 10^{-06}X^6\\ &\quad - 0.000481521724173462X^5 - 0.05899466848037266X^4 - 3.82353929224187X^3\\ &\quad - 101.3899724153744X^2 + 94.46052284159652X - 21.62667247491518\\ &= -21.62667247491518B_{0,8}(X) - 9.819107119715613B_{1,8}(X) - 1.632612207922276B_{2,8}(X)\\ &\quad + 2.86453477310337B_{3,8}(X) + 3.603213555021573B_{4,8}(X) + 0.5134524899120698B_{5,8}(X)\\ &\quad - 6.475580126796985B_{6,8}(X) - 17.43558481253364B_{7,8}(X) - 32.43913973359036B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

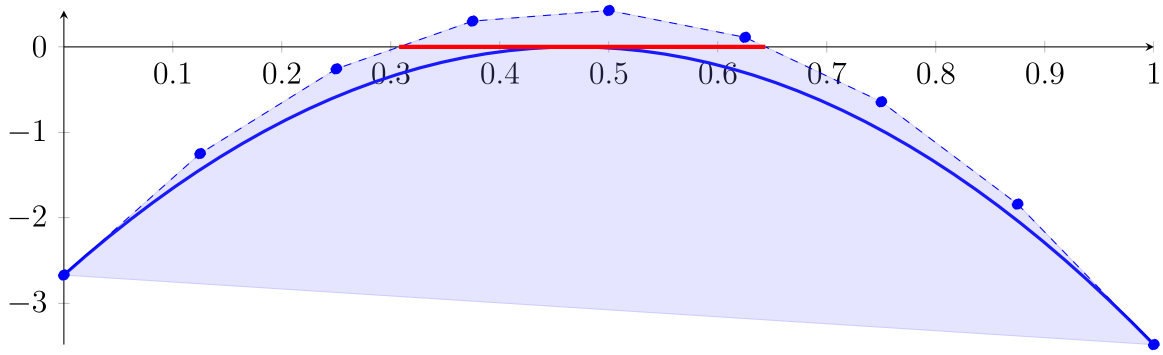
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4938141292321744, 0.5070221367077007]$,

55.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4938141292321744, 0.507022136707$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -9.261865042605128\cdot 10^{-16}X^8 - 2.731330938941274\cdot 10^{-12}X^7 - 3.339777336661422\\ &\quad \cdot 10^{-09}X^6 - 2.167033369334019\cdot 10^{-06}X^5 - 0.0007867416616259877X^4\\ &\quad - 0.1514273448103067X^3 - 12.0308548973337X^2 + 11.3691349552019X - 2.67015106585462\\ &= -2.67015106585462B_{0,8}(X) - 1.249009196454382B_{1,8}(X) - 0.2575407162446335B_{2,8}(X)\\ &\quad + 0.3015503150458701B_{3,8}(X) + 0.4255485985217787B_{4,8}(X) + 0.1117275574241232B_{5,8}(X)\\ &\quad - 0.6426507016859872B_{6,8}(X) - 1.840335427863286B_{7,8}(X) - 3.484087264834231B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

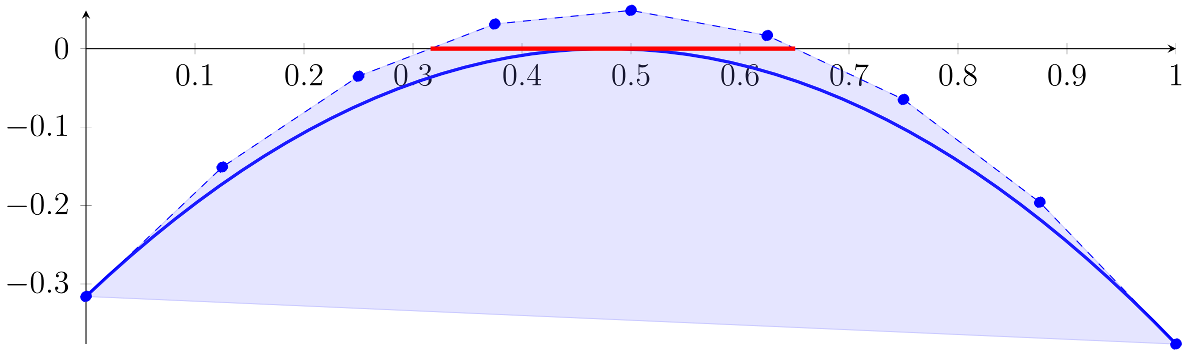
$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\Longrightarrow$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

55.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.502170909909546\cdot10^{-19}X^8 - 1.319790000383349\cdot10^{-15}X^7 - 4.808366303277033\cdot10^{-12}X^6\\ &\quad - 9.297436410097437\cdot10^{-09}X^5 - 1.006192361000331\cdot10^{-05}X^4 - 0.005777437133356019X^3\\ &\quad - 1.373512346650201X^2 + 1.318590421802199X - 0.3158295498425207\\ &= -0.3158295498425207B_{0,8}(X) - 0.1510057471172458B_{1,8}(X) - 0.03523595677233526B_{2,8}(X)\\ &\quad + 0.03137665267197247B_{3,8}(X) + 0.04872876895367301B_{4,8}(X) + 0.01671693590297049B_{5,8}(X)\\ &\quad - 0.06476244672391982B_{6,8}(X) - 0.1958131234111408B_{7,8}(X) - 0.376538983049735B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3161210337394805, 0.6506459600024226\}$$

Intersection intervals with the x axis:

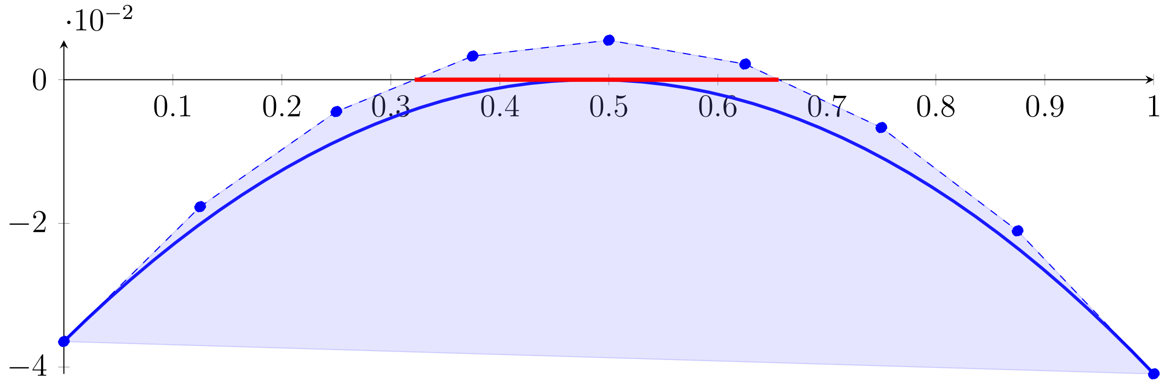
$$[0.3161210337394805, 0.6506459600024226]$$

Longest intersection interval: 0.3345249262629421
$\Longrightarrow$ Selective recursion: interval 1: $[0.499279281802493, 0.5007635705740208]$,

55.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.499279281802493, 0.5007635705$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.355847730899963\cdot 10^{-23}X^8 - 6.189124375986835\cdot 10^{-19}X^7 - 6.742674354644286\cdot 10^{-15}X^6\\ &- 3.898805488577673\cdot 10^{-11}X^5 - 1.261912061383275\cdot 10^{-07}X^4 - 0.000216758797755982X^3\\ &- 0.154319370565961X^2 + 0.1500226550818807X - 0.0364365303427455\\ = &-0.0364365303427455B_{0,8}(X) - 0.01768369845751041B_{1,8}(X) - 0.004442272663916792B_{2,8}(X)\\ &+ 0.003283876345218297B_{3,8}(X) + 0.005490876074346265B_{4,8}(X)\\ &+ 0.002174852224490793B_{5,8}(X) - 0.006668071307448625B_{6,8}(X)\\ &- 0.02104177242939338B_{7,8}(X) - 0.04095013085478269B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3218707447051634, 0.6557428337562166\}$$

Intersection intervals with the x axis:

$$[0.3218707447051634, 0.6557428337562166]$$

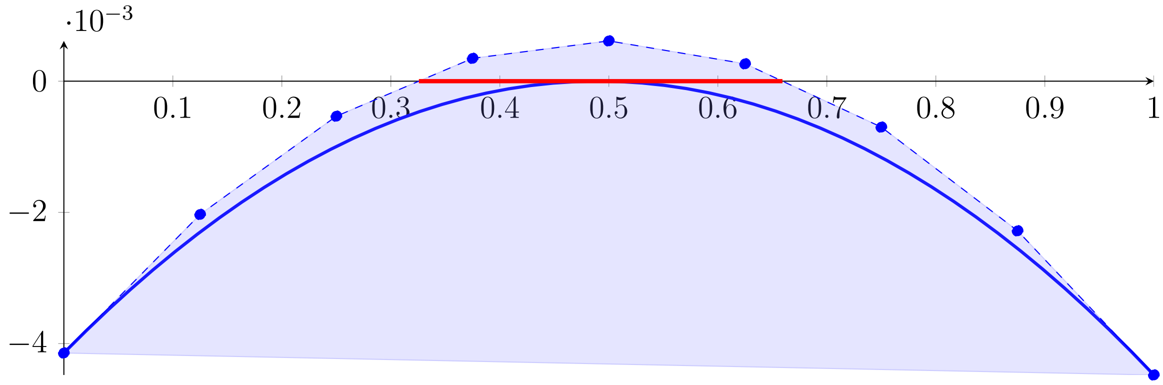
Longest intersection interval: 0.3338720890510532

$\Longrightarrow$ Selective recursion: interval 1: $[0.4997570309347421, 0.5002525935276471]$,

55.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4997570309347421, 0.5002525$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.637375463877353\cdot 10^{-27}X^8 - 2.862414855005092\cdot 10^{-22}X^7 - 9.341200314035287\cdot 10^{-18}X^6\\ &- 1.617995026179791\cdot 10^{-13}X^5 - 1.568792391901099\cdot 10^{-09}X^4 - 8.073141350089629\\ &\cdot 10^{-06}X^3 - 0.01722540857155108X^2 + 0.01689843085777645X - 0.004143462623424353\\ = &-0.004143462623424353B_{0,8}(X) - 0.002031158766202297B_{1,8}(X)\\ &- 0.0005340480722499233B_{2,8}(X) + 0.0003477252951943749B_{3,8}(X)\\ &+ 0.0006140171504808826B_{4,8}(X) + 0.0002646832855456765B_{5,8}(X)\\ &- 0.0007004205300922658B_{6,8}(X) - 0.002281438549333955B_{7,8}(X)\\ &- 0.004478515047503282B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3257065380923488, 0.6592817116222256\}$$

Intersection intervals with the x axis:

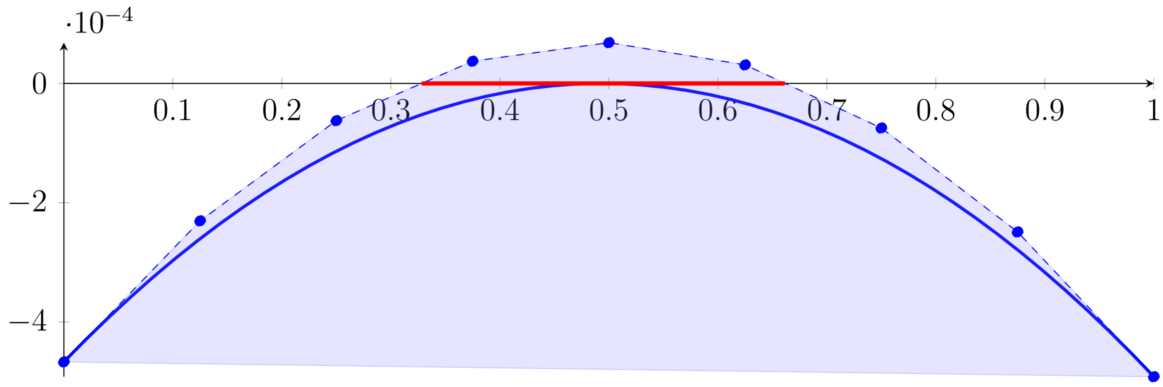
$$[0.3257065380923488, 0.6592817116222256]$$

Longest intersection interval: 0.3335751735298769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999184389112853, 0.5000837462892085]$,

55.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999184389112853, 0.50008$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -5.57619429667528\cdot 10^{-31}X^8 - 1.315536799505273\cdot 10^{-25}X^7 - 1.287049795843581\cdot 10^{-20}X^6\\ &\quad - 6.683358895768893\cdot 10^{-16}X^5 - 1.942733814710911\cdot 10^{-11}X^4 - 2.997323807488328\\ &\quad \cdot 10^{-07}X^3 - 0.001917590365692151X^2 + 0.001893040758741533X - 0.0004671653183669499\\ &= -0.0004671653183669499B_{0,8}(X) - 0.0002305352235242582B_{1,8}(X) - 6.239049888485767\\ &\quad \cdot 10^{-05}B_{2,8}(X) + 3.726350318730984\cdot 10^{-05}B_{3,8}(X) + 6.842143005076897\\ &\quad \cdot 10^{-05}B_{4,8}(X) + 3.107792878649904\cdot 10^{-05}B_{5,8}(X) - 7.47723538020779\\ &\quad \cdot 10^{-05}B_{6,8}(X) - 0.000249134771189109B_{7,8}(X) - 0.0004920146771263226B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3282588977707033, 0.661700337526842\}$$

Intersection intervals with the x axis:

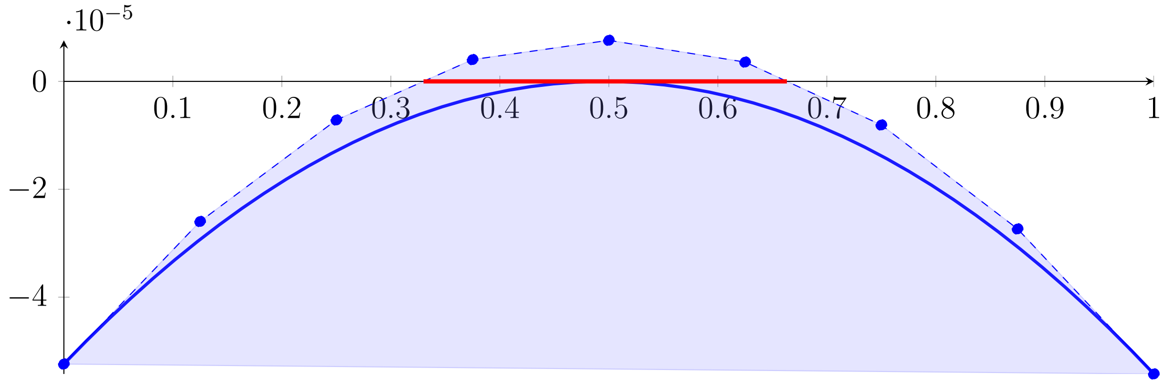
$$[0.3282588977707033, 0.661700337526842]$$

Longest intersection interval: 0.3334414397561387
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999727025289557, 0.5000278228590528]$,

55.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999727025289557, 0.500$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.521076768712742\cdot 10^{-35}X^8 - 6.02899387826599\cdot 10^{-29}X^7 - 1.76898025411792\cdot 10^{-23}X^6\\ &\quad - 2.754920799910016\cdot 10^{-18}X^5 - 2.401685361883718\cdot 10^{-13}X^4 - 1.111294950492419\cdot 10^{-08}X^3\\ &\quad - 0.0002132366404355359X^2 + 0.0002114057621846039X - 5.239629569054843\cdot 10^{-05}\\ &= -5.239629569054843\cdot 10^{-05}B_{0,8}(X) - 2.597057541747294\cdot 10^{-05}B_{1,8}(X) - 7.160449445666593\\ &\quad \cdot 10^{-06}B_{2,8}(X) + 4.033883779343745\cdot 10^{-06}B_{3,8}(X) + 7.612225808600218\cdot 10^{-06}B_{4,8}(X)\\ &\quad + 3.574378189713946\cdot 10^{-06}B_{5,8}(X) - 8.079857533135031\cdot 10^{-06}B_{6,8}(X)\\ &\quad - 2.73506798191978\cdot 10^{-05}B_{7,8}(X) - 5.423828713115661\cdot 10^{-05}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3299561852159799, 0.6633377584201652\}$$

Intersection intervals with the x axis:

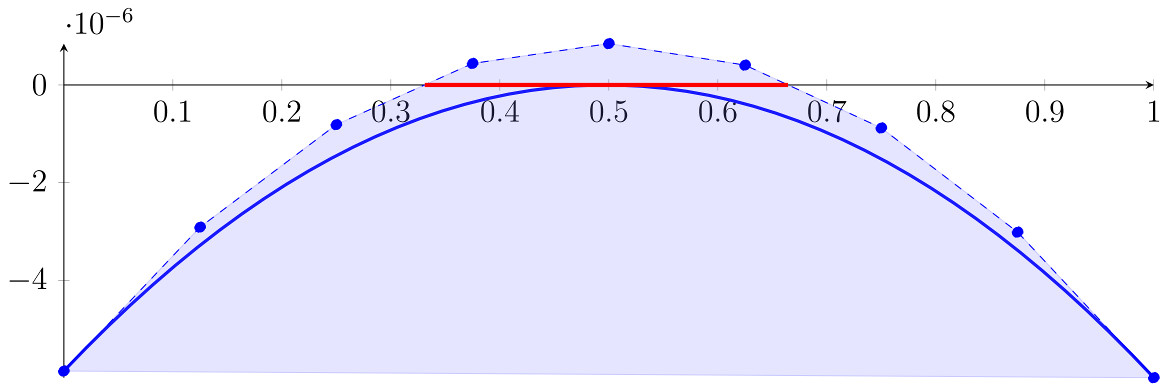
$$[0.3299561852159799, 0.6633377584201652]$$

Longest intersection interval: 0.3333815732041853
$\Longrightarrow$ Selective recursion: interval 1: [0.4999908898228024, 0.5000092659251657],

55.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999908898228024, 0.5

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.300251044438937\cdot 10^{-38}X^8 - 2.759545789497528\cdot 10^{-32}X^7 - 2.428711637904422\cdot 10^{-26}X^6\\
&- 1.134547275299063\cdot 10^{-20}X^5 - 2.966816573305973\cdot 10^{-15}X^4 - 4.117811892390909\cdot 10^{-10}X^3\\
&- 2.370104084998438\cdot 10^{-05}X^2 + 2.356495503794524\cdot 10^{-05}X - 5.857360304639568\cdot 10^{-06}\\
= &-5.857360304639568\cdot 10^{-06}B_{0,8}(X) - 2.911740924896412\cdot 10^{-06}B_{1,8}(X) - 8.125872897955564\\
&\cdot 10^{-07}B_{2,8}(X) + 4.400932474274781\cdot 10^{-07}B_{3,8}(X) + 8.462933334947859\cdot 10^{-07}B_{4,8}(X)\\
&+ 4.060056150860783\cdot 10^{-07}B_{5,8}(X) - 8.807772611613165\cdot 10^{-07}B_{6,8}(X)\\
&- 3.014062648652454\cdot 10^{-06}B_{7,8}(X) - 5.993857900834775\cdot 10^{-06}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.331084848216461, 0.6644399885346334\}$$

Intersection intervals with the x axis:

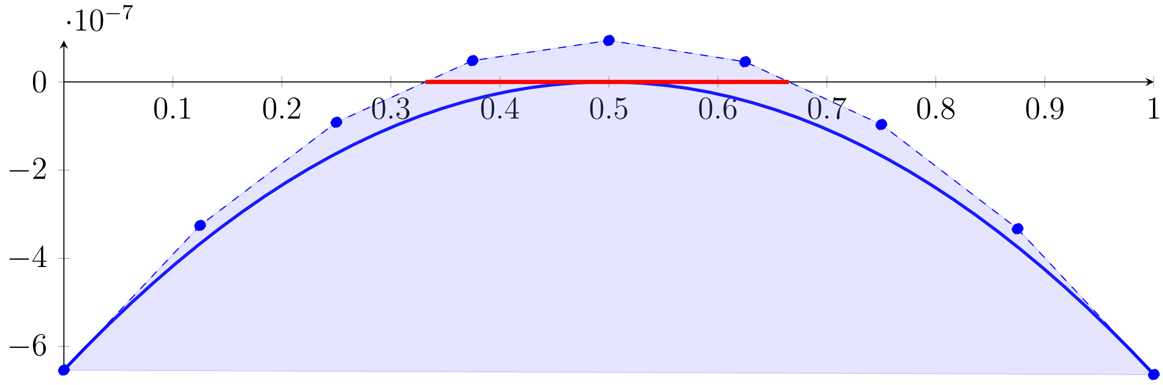
$$[0.331084848216461, 0.6644399885346334]$$

Longest intersection interval: 0.3333551403181724
$\Longrightarrow$ Selective recursion: interval 1: [0.4999969738718641, 0.500003099640046],

55.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999969738718641, 0

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1.982825346126094 \cdot 10^{-42} X^8 - 1.262374580662423 \cdot 10^{-35} X^7 - 3.332882748505763 \cdot 10^{-29} X^6 \\ & - 4.670466114941116 \cdot 10^{-23} X^5 - 3.663718276431707 \cdot 10^{-17} X^4 - 1.52542941384219 \cdot 10^{-11} X^3 \\ & - 2.633839011095458 \cdot 10^{-06} X^2 + 2.623741169043712 \cdot 10^{-06} X - 6.534168705792709 \cdot 10^{-07} \\ = & -6.534168705792709 \cdot 10^{-07} B_{0,8}(X) - 3.254492244488069 \cdot 10^{-07} B_{1,8}(X) - 9.154725728603783 \\ & \cdot 10^{-08} B_{2,8}(X) + 4.828875851092667 \cdot 10^{-08} B_{3,8}(X) + 9.405855054345363 \cdot 10^{-08} B_{4,8}(X) \\ & + 4.576184641238665 \cdot 10^{-08} B_{5,8}(X) - 9.660162628195405 \cdot 10^{-08} B_{6,8}(X) \\ & - 3.330321399397716 \cdot 10^{-07} B_{7,8}(X) - 6.635299669617927 \cdot 10^{-07} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3318344765869905, 0.6651804668942704\}$$

Intersection intervals with the x axis:

$$[0.3318344765869905, 0.6651804668942704]$$

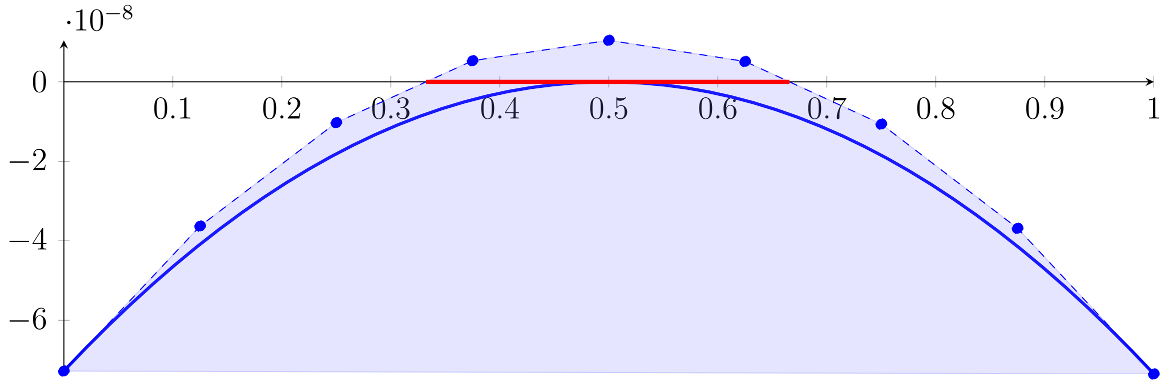
Longest intersection interval: 0.3333459903072799

$\Longrightarrow$ Selective recursion: interval 1: [0.4999990066129424, 0.5000010486132034],

55.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999990066129424

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -3.023057070307937 \cdot 10^{-46} X^8 - 5.773711385862107 \cdot 10^{-39} X^7 - 4.572901329678827 \cdot 10^{-32} X^6 \\ & - 1.922370176484083 \cdot 10^{-25} X^5 - 4.523805577109385 \cdot 10^{-19} X^4 - 5.650400184721212 \cdot 10^{-13} X^3 \\ & - 2.926726911514393 \cdot 10^{-07} X^2 + 2.919240677305221 \cdot 10^{-07} X - 7.27925149751094 \cdot 10^{-08} \\ = & -7.27925149751094 \cdot 10^{-08} B_{0,8}(X) - 3.630200650879414 \cdot 10^{-08} B_{1,8}(X) - 1.026409415503029 \\ & \cdot 10^{-08} B_{2,8}(X) + 5.321211996181834 \cdot 10^{-09} B_{3,8}(X) + 1.045390185483543 \cdot 10^{-08} B_{4,8}(X) \\ & + 5.133965330917245 \cdot 10^{-09} B_{5,8}(X) - 1.063860766559244 \cdot 10^{-08} B_{6,8}(X) \\ & - 3.68638272247198 \cdot 10^{-08} B_{7,8}(X) - 7.354170343649749 \cdot 10^{-08} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3323218842755297, 0.6656874431018115\}$$

Intersection intervals with the x axis:

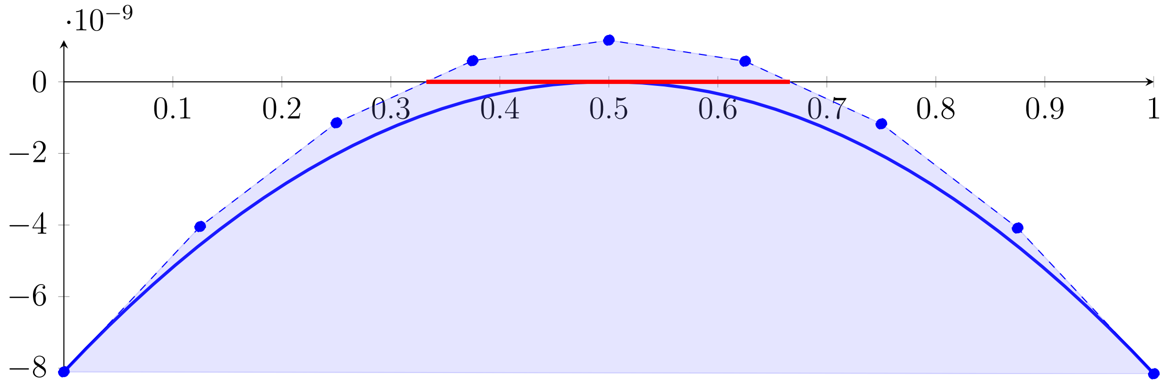
$$[0.3323218842755297, 0.6656874431018115]$$

Longest intersection interval: 0.3333655588262817
$\Longrightarrow$ Selective recursion: interval 1: [0.4999996852143169, 0.500000365946875],

55.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999996852143169, 0.500000365946875]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -4.61118111521237 \cdot 10^{-50} X^8 - 2.641801828130962 \cdot 10^{-42} X^7 - 6.276482670428966 \cdot 10^{-35} X^6 \\
& - 7.91481690665562 \cdot 10^{-28} X^5 - 5.587109146485419 \cdot 10^{-21} X^4 - 2.093350052120201 \cdot 10^{-14} X^3 \\
& - 3.252553849467158 \cdot 10^{-08} X^2 + 3.247007216419568 \cdot 10^{-08} X - 8.101917765629863 \cdot 10^{-09} \\
= & -8.101917765629863 \cdot 10^{-09} B_{0,8}(X) - 4.043158745105403 \cdot 10^{-09} B_{1,8}(X) \\
& - 1.146026099390642 \cdot 10^{-09} B_{2,8}(X) + 5.8947979770191 \cdot 10^{-10} B_{3,8}(X) + 1.163358572359665 \\
& \cdot 10^{-09} B_{4,8}(X) + 5.75609850769953 \cdot 10^{-10} B_{5,8}(X) - 1.173766740879974 \cdot 10^{-09} B_{6,8}(X) \\
& - 4.084771576402945 \cdot 10^{-09} B_{7,8}(X) - 8.157405029611868 \cdot 10^{-09} B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.332542653795541, 0.6661296410902487\}$$

Intersection intervals with the x axis:

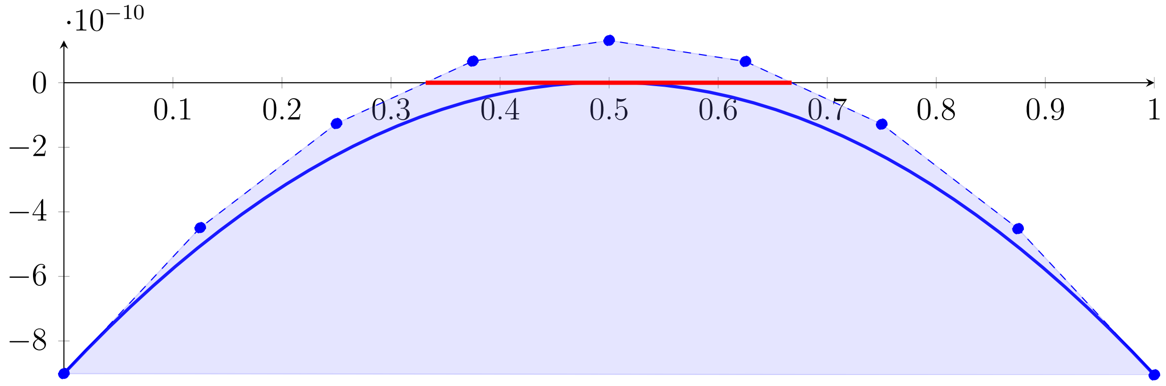
$$[0.332542653795541, 0.6661296410902487]$$

Longest intersection interval: 0.3335869872947078
$\Longrightarrow$ Selective recursion: interval 1: [0.4999999115869283, 0.5000001386704515],

55.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999999115869283, 0.5000001386704515]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -7.071067608997674 \cdot 10^{-54} X^8 - 1.214406168666063 \cdot 10^{-45} X^7 - 8.649101296569956 \cdot 10^{-38} X^6 \\
& - 3.269538436354099 \cdot 10^{-30} X^5 - 6.918686662336931 \cdot 10^{-23} X^4 - 7.770864123205297 \cdot 10^{-16} X^3 \\
& - 3.61945329275134 \cdot 10^{-09} X^2 + 3.615351534501487 \cdot 10^{-09} X - 9.01058772952768 \cdot 10^{-10} \\
= & -9.01058772952768 \cdot 10^{-10} B_{0,8}(X) - 4.491398311400821 \cdot 10^{-10} B_{1,8}(X) - 1.264870783542298 \\
& \cdot 10^{-10} B_{2,8}(X) + 6.689947152824597 \cdot 10^{-11} B_{3,8}(X) + 1.31019804630801 \cdot 10^{-10} B_{4,8}(X) \\
& + 6.587390707689027 \cdot 10^{-11} B_{5,8}(X) - 1.285382350100323 \cdot 10^{-10} B_{6,8}(X) \\
& - 4.522166355065136 \cdot 10^{-10} B_{7,8}(X) - 9.051613082891018 \cdot 10^{-10} B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3317579340646351, 0.6673545479012814\}$$

Intersection intervals with the x axis:

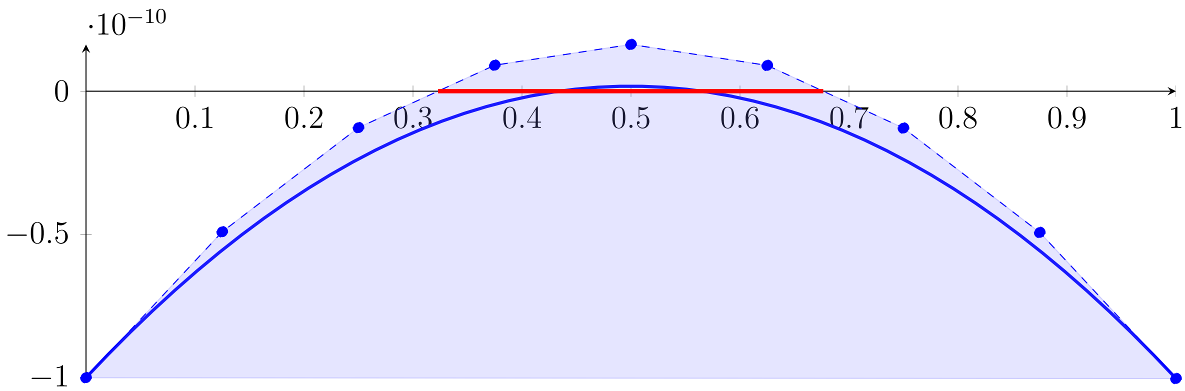
$$[0.3317579340646351, 0.6673545479012814]$$

Longest intersection interval: 0.3355966138366463
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999999869236888, 0.5000000631321502]$,

55.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999869236888, 0.5000000631321502]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1.137694218100245\cdot 10^{-57}X^8 - 5.822197888022611\cdot 10^{-49}X^7 - 1.235595786383296\cdot 10^{-40}X^6 \\ & - 1.391791952088144\cdot 10^{-32}X^5 - 8.775946704095466\cdot 10^{-25}X^4 - 2.937122613307657\cdot 10^{-17}X^3 \\ & - 4.076413298856481\cdot 10^{-10}X^2 + 4.07342667585279\cdot 10^{-10}X - 1.000063159750977\cdot 10^{-10} \\ = & -1.000063159750977\cdot 10^{-10}B_{0,8}(X) - 4.908848252693777\cdot 10^{-11}B_{1,8}(X) - 1.272926800326533 \\ & \cdot 10^{-11}B_{2,8}(X) + 9.071327071433506\cdot 10^{-12}B_{3,8}(X) + 1.631330217267253\cdot 10^{-11}B_{4,8}(X) \\ & + 8.996656775965545\cdot 10^{-12}B_{5,8}(X) - 1.287860964317367\cdot 10^{-11}B_{6,8}(X) \\ & - 4.931249760923136\cdot 10^{-11}B_{7,8}(X) - 1.003050076466937\cdot 10^{-10}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3229869297125206, 0.6764088411746962\}$$

Intersection intervals with the x axis:

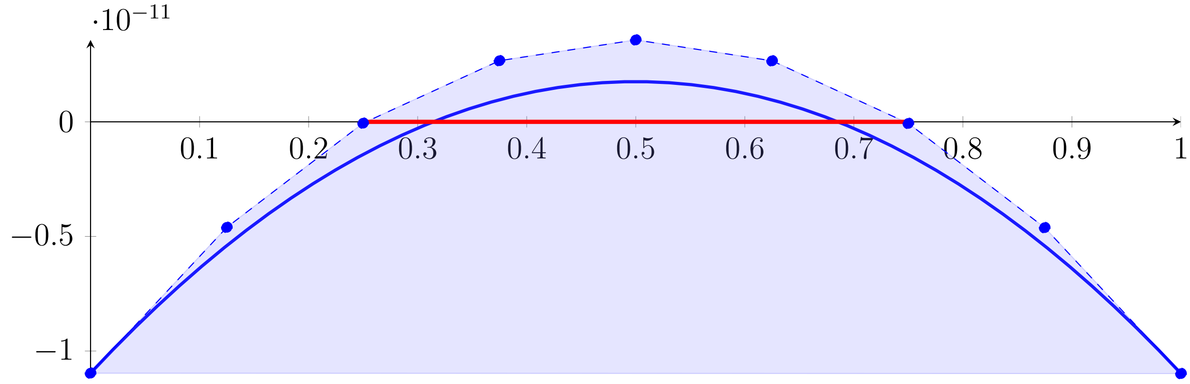
$$[0.3229869297125206, 0.6764088411746962]$$

Longest intersection interval: 0.3534219114621756
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000115380258, 0.5000000384717659]$,

55.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000115380258, 0.5000000384717659]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.769321148091343\cdot10^{-61}X^8 - 4.009971301105125\cdot10^{-52}X^7 - 2.40789337418918\cdot10^{-43}X^6\\ &- 7.674351474736958\cdot10^{-35}X^5 - 1.36920310040832\cdot10^{-26}X^4 - 1.29658952293724\cdot10^{-18}X^3\\ &- 5.091727851043352\cdot10^{-11}X^2 + 5.08987688183243\cdot10^{-11}X - 1.096532991460483\cdot10^{-11}\\ = &-1.096532991460483\cdot10^{-11}B_{0,8}(X) - 4.602983812314296\cdot10^{-12}B_{1,8}(X)\\ &- 5.91119425392419\cdot10^{-14}B_{2,8}(X) + 2.666285671566945\cdot10^{-12}B_{3,8}(X) + 3.57320900685088\\ &\cdot10^{-12}B_{4,8}(X) + 2.661658040159178\cdot10^{-12}B_{5,8}(X) - 6.83672516615453\cdot10^{-14}B_{6,8}(X)\\ &- 4.616866891764675\cdot10^{-12}B_{7,8}(X) - 1.09838409033036\cdot10^{-11}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.252711161402344, 0.7468696603349071\}$$

Intersection intervals with the x axis:

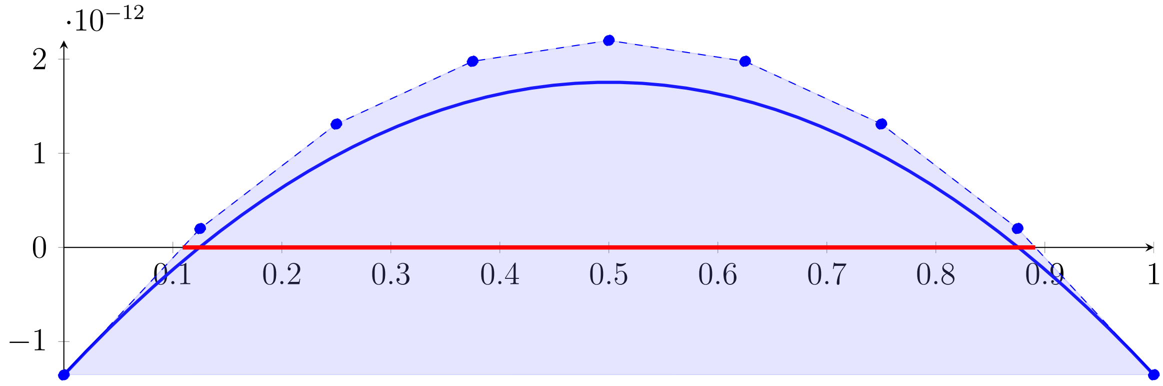
$$[0.252711161402344, 0.7468696603349071]$$

Longest intersection interval: 0.4941584989325631
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000183444825, 0.5000000316540191]$,

55.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000183444825, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-9.84698950588755\cdot10^{-64}X^8 - 2.885394162330348\cdot10^{-54}X^7 - 3.506185324343073\cdot10^{-45}X^6\\ &- 2.261377324294573\cdot10^{-36}X^5 - 8.164563166712554\cdot10^{-28}X^4 - 1.564592773244276\cdot10^{-19}X^3\\ &- 1.243362398803133\cdot10^{-11}X^2 + 1.243502393415518\cdot10^{-11}X - 1.354369602677131\cdot10^{-12}\\ = &-1.354369602677131\cdot10^{-12}B_{0,8}(X) + 2.000083890922661\cdot10^{-13}B_{1,8}(X)\\ &+ 1.310328381289116\cdot10^{-12}B_{2,8}(X) + 1.976590371119503\cdot10^{-12}B_{3,8}(X)\\ &+ 2.19879435578951\cdot10^{-12}B_{4,8}(X) + 1.976940332505223\cdot10^{-12}B_{5,8}(X) + 1.311028298472726\\ &\cdot10^{-12}B_{6,8}(X) + 2.01058250898102\cdot10^{-13}B_{7,8}(X) - 1.352969813012563\cdot10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.108915721421098, 0.8911723471704997\}$$

Intersection intervals with the x axis:

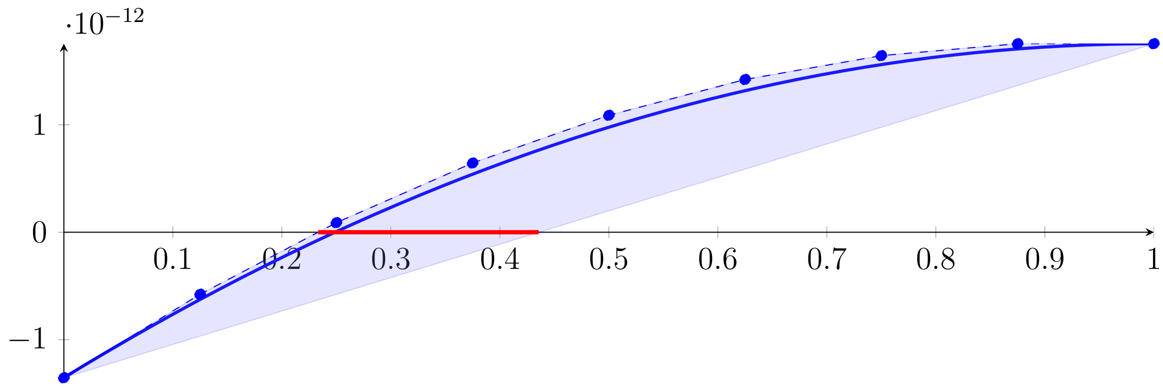
$$[0.108915721421098, 0.8911723471704997]$$

Longest intersection interval: 0.7822566257494017
$\Longrightarrow$ Bisection: first half $[0.5000000183444825, 0.5000000249992508]$ und second half $[0.5000000249992508, 0.50$

55.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.5000000183444825, 0.5000000249992508]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-3.846480275737324\cdot 10^{-66}X^8 - 2.254214189320585\cdot 10^{-56}X^7 - 5.478414569286052\cdot 10^{-47}X^6 \\ &- 7.06680413842054\cdot 10^{-38}X^5 - 5.102851979195346\cdot 10^{-29}X^4 - 1.955740966555345\cdot 10^{-20}X^3 \\ &- 3.108405997007834\cdot 10^{-12}X^2 + 6.21751196707759\cdot 10^{-12}X - 1.354369602677131\cdot 10^{-12} \\ = &-1.354369602677131\cdot 10^{-12}B_{0,8}(X) - 5.771806067924327\cdot 10^{-13}B_{1,8}(X) + 8.899388919912914 \\ &\cdot 10^{-14}B_{2,8}(X) + 6.441538849483146\cdot 10^{-13}B_{3,8}(X) + 1.088299380105884\cdot 10^{-12}B_{4,8}(X) \\ &+ 1.421430374322599\cdot 10^{-12}B_{5,8}(X) + 1.643546867249218\cdot 10^{-12}B_{6,8}(X) \\ &+ 1.754648858536504\cdot 10^{-12}B_{7,8}(X) + 1.754736347835215\cdot 10^{-12}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2333013178726793, 0.4356138466281428\}$$

Intersection intervals with the x axis:

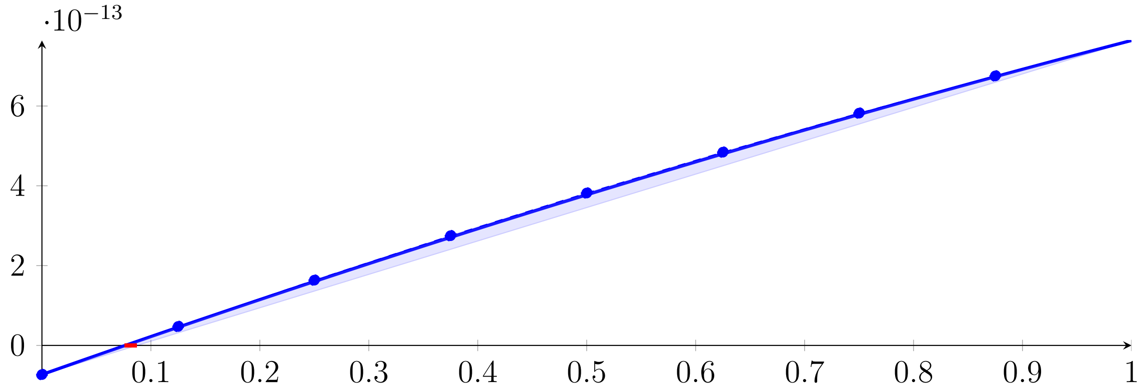
$$[0.2333013178726793, 0.4356138466281428]$$

Longest intersection interval: 0.2023125287554634
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000198970487, 0.5000000212433917]$,

55.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000198970487, 0.5000000212433917]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.079557397202381\cdot 10^{-71}X^8 - 3.12719259192325\cdot 10^{-61}X^7 - 3.756570682132575\cdot 10^{-51}X^6\\ &- 2.395173331861966\cdot 10^{-41}X^5 - 8.548778908133761\cdot 10^{-32}X^4 - 1.619495215947232\cdot 10^{-22}X^3\\ &- 1.272281748414343\cdot 10^{-13}X^2 + 9.644484121679\cdot 10^{-13}X - 7.300486662812878\cdot 10^{-14}\\ = &-7.300486662812878\cdot 10^{-14}B_{0,8}(X) + 4.755118489285872\cdot 10^{-14}B_{1,8}(X) + 1.635633730266521\\ &\cdot 10^{-13}B_{2,8}(X) + 2.750316977703595\cdot 10^{-13}B_{3,8}(X) + 3.819561591210889\cdot 10^{-13}B_{4,8}(X)\\ &+ 4.843367570759483\cdot 10^{-13}B_{5,8}(X) + 5.821734916320458\cdot 10^{-13}B_{6,8}(X)\\ &+ 6.754663627864895\cdot 10^{-13}B_{7,8}(X) + 7.642153705363873\cdot 10^{-13}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.07569597886944255, 0.08719911844866588\}$$

Intersection intervals with the x axis:

$$[0.07569597886944255, 0.08719911844866588]$$

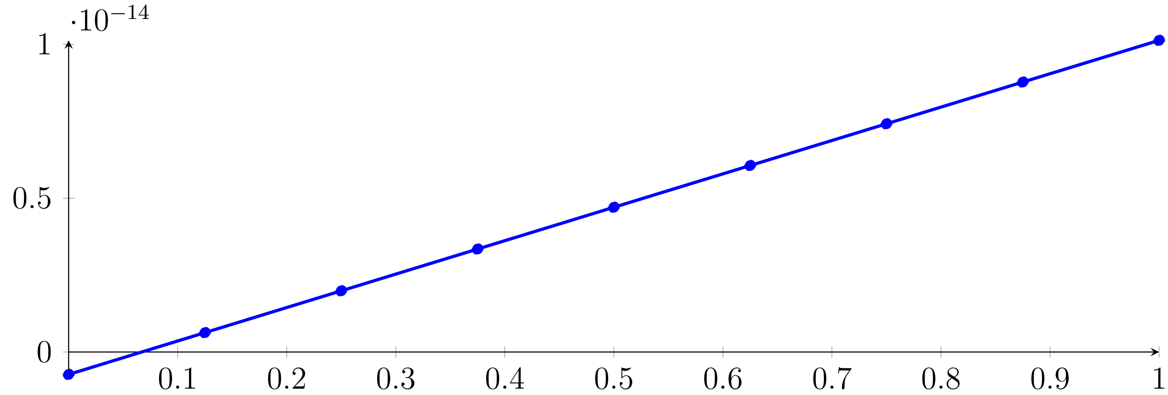
Longest intersection interval: 0.01150313957922333

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199989615, 0.5000000200144486]$,

55.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199989615, 0.5000000200144486]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.309610258465664\cdot 10^{-87}X^8 - 8.334304343480435\cdot 10^{-75}X^7 - 8.703419206351739\cdot 10^{-63}X^6\\ &- 4.824128808781935\cdot 10^{-51}X^5 - 1.496820228100742\cdot 10^{-39}X^4 - 2.465067626410487\cdot 10^{-28}X^3\\ &- 1.683511456921895\cdot 10^{-17}X^2 + 1.087261902125672\cdot 10^{-14}X - 7.290023293677624\cdot 10^{-16}\\ = &-7.290023293677624\cdot 10^{-16}B_{0,8}(X) + 6.300750482893273\cdot 10^{-16}B_{1,8}(X) + 1.988551171854659\\ &\cdot 10^{-15}B_{2,8}(X) + 3.346426041328229\cdot 10^{-15}B_{3,8}(X) + 4.703699656710032\cdot 10^{-15}B_{4,8}(X)\\ &+ 6.060372018000064\cdot 10^{-15}B_{5,8}(X) + 7.41644312519832\cdot 10^{-15}B_{6,8}(X)\\ &+ 8.771912978304797\cdot 10^{-15}B_{7,8}(X) + 1.012678157731949\cdot 10^{-14}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06704937678240291, 0.0671533567390459\}$$

Intersection intervals with the x axis:

$$[0.06704937678240291, 0.0671533567390459]$$

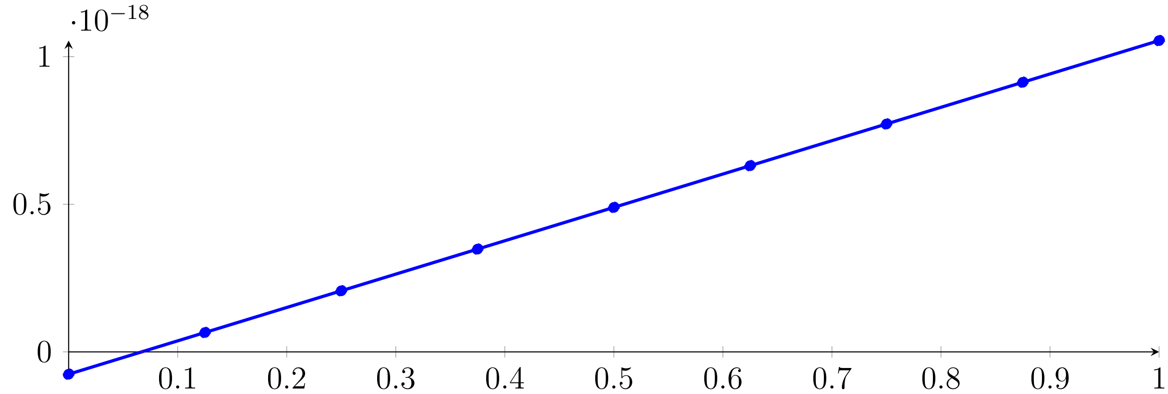
Longest intersection interval: 0.0001039799566429901

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000200000015]$,

55.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000200000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -4.522451416965012\cdot 10^{-119}X^8 - 1.095258871423056\cdot 10^{-102}X^7 - 1.099987358691302\cdot 10^{-86}X^6 \\
&\quad - 5.863636713606292\cdot 10^{-71}X^5 - 1.749718451421176\cdot 10^{-55}X^4 - 2.771262941212064\cdot 10^{-40}X^3 \\
&\quad - 1.820184200444671\cdot 10^{-25}X^2 + 1.130299712615757\cdot 10^{-18}X - 7.568425969413026\cdot 10^{-20} \\
&= -7.568425969413026\cdot 10^{-20}B_{0,8}(X) + 6.560320438283937\cdot 10^{-20}B_{1,8}(X) + 2.068906619591512 \\
&\quad \cdot 10^{-19}B_{2,8}(X) + 3.481781130348051\cdot 10^{-19}B_{3,8}(X) + 4.894655576098011\cdot 10^{-19}B_{4,8}(X) \\
&\quad + 6.307529956841393\cdot 10^{-19}B_{5,8}(X) + 7.720404272578197\cdot 10^{-19}B_{6,8}(X) \\
&\quad + 9.133278523308422\cdot 10^{-19}B_{7,8}(X) + 1.054615270903207\cdot 10^{-18}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695946114945086, 0.06695947193230531\}$$

Intersection intervals with the x axis:

$$[0.06695946114945086, 0.06695947193230531]$$

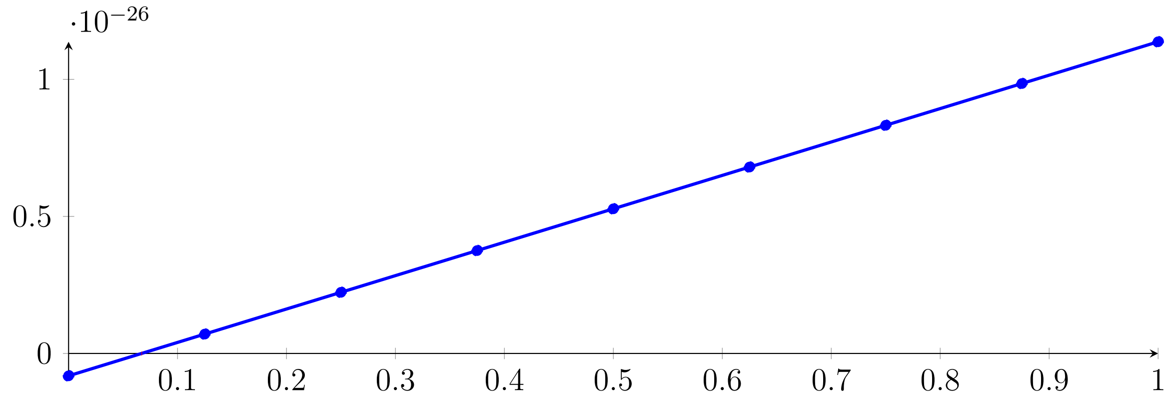
Longest intersection interval: $1.078285445187343\cdot 10^{-08}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

55.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -8.265018972796948\cdot 10^{-183}X^8 - 1.856320587559019\cdot 10^{-158}X^7 - 1.7289807291512\cdot 10^{-134}X^6 \\
& - 8.547434292245386\cdot 10^{-111}X^5 - 2.365392110963433\cdot 10^{-87}X^4 - 3.474393176904993\cdot 10^{-64}X^3 \\
& - 2.116327262136374\cdot 10^{-41}X^2 + 1.218785702529033\cdot 10^{-26}X - 8.160922251597258\cdot 10^{-28} \\
= & -8.160922251597258\cdot 10^{-28}B_{0,8}(X) + 7.073899030015655\cdot 10^{-28}B_{1,8}(X) + 2.230872031162856 \\
& \cdot 10^{-27}B_{2,8}(X) + 3.754354159324146\cdot 10^{-27}B_{3,8}(X) + 5.277836287485435\cdot 10^{-27}B_{4,8}(X) \\
& + 6.801318415646723\cdot 10^{-27}B_{5,8}(X) + 8.32480054380801\cdot 10^{-27}B_{6,8}(X) \\
& + 9.848282671969297\cdot 10^{-27}B_{7,8}(X) + 1.137176480013058\cdot 10^{-26}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695945181062587, 0.06695945181062598\}$$

Intersection intervals with the x axis:

$$[0.06695945181062587, 0.06695945181062598]$$

Longest intersection interval: $1.162699176979875\cdot 10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

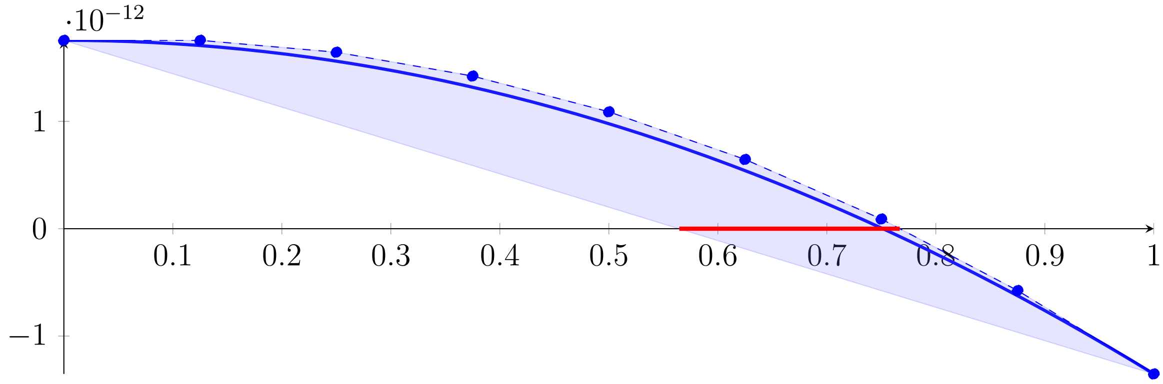
55.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 24!

55.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.5000000249992508, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.846480275737324\cdot 10^{-66}X^8 - 2.254214192397769\cdot 10^{-56}X^7 - 5.478414585065551\cdot 10^{-47}X^6 \\
& - 7.066804171291027\cdot 10^{-38}X^5 - 5.102852014529367\cdot 10^{-29}X^4 - 1.955740986966753\cdot 10^{-20}X^3 \\
& - 3.108406055680063\cdot 10^{-12}X^2 + 6.999143896938875\cdot 10^{-16}X + 1.754736347835215\cdot 10^{-12} \\
= & \ 1.754736347835215\cdot 10^{-12}B_{0,8}(X) + 1.754823837133927\cdot 10^{-12}B_{1,8}(X) + 1.643896824444065 \\
& \cdot 10^{-12}B_{2,8}(X) + 1.42195530941639\cdot 10^{-12}B_{3,8}(X) + 1.088999291701662\cdot 10^{-12}B_{4,8}(X) \\
& + 6.450287709506428\cdot 10^{-13}B_{5,8}(X) + 9.00437468140915\cdot 10^{-14}B_{6,8}(X) \\
& - 5.759557810572307\cdot 10^{-13}B_{7,8}(X) - 1.352969813012563\cdot 10^{-12}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.5646403672078589, 0.7669001146107961\}$$

Intersection intervals with the x axis:

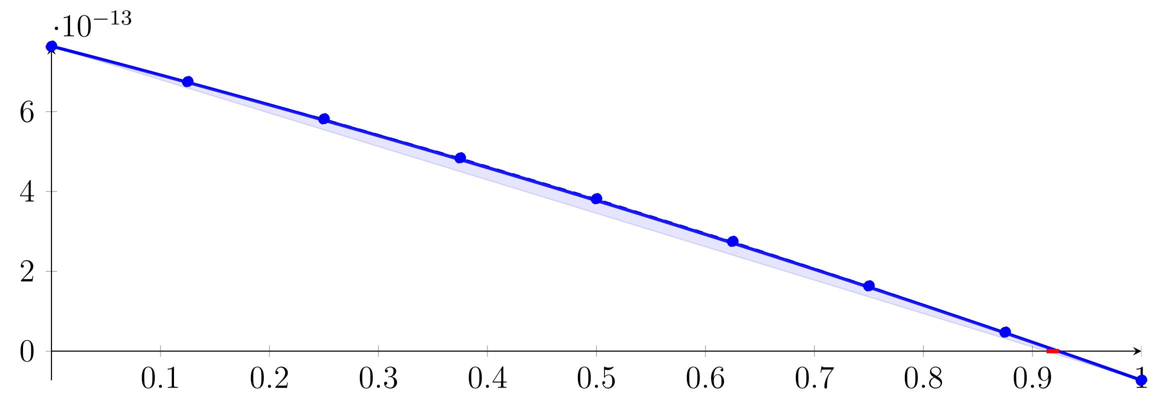
$$[0.5646403672078589, 0.7669001146107961]$$

Longest intersection interval: 0.2022597474029372
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000287568016, 0.5000000301027934]$,

55.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.5000000287568016, 0.5000000301027934]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -1.077306286121587 \cdot 10^{-71} X^8 - 3.121486088546909 \cdot 10^{-61} X^7 - 3.750694216009664 \cdot 10^{-51} X^6 \\
&\quad - 2.392050590455968 \cdot 10^{-41} X^5 - 8.5398613074672 \cdot 10^{-32} X^4 - 1.618228037882981 \cdot 10^{-22} X^3 \\
&\quad - 1.271618015330847 \cdot 10^{-13} X^2 - 7.098433618236863 \cdot 10^{-13} X + 7.641134288463311 \cdot 10^{-13} \\
&= 7.641134288463311 \cdot 10^{-13} B_{0,8}(X) + 6.753830086183703 \cdot 10^{-13} B_{1,8}(X) + 5.821110954785137 \\
&\quad \cdot 10^{-13} B_{2,8}(X) + 4.842976894238714 \cdot 10^{-13} B_{3,8}(X) + 3.819427904515539 \cdot 10^{-13} B_{4,8}(X) \\
&\quad + 2.750463985586714 \cdot 10^{-13} B_{5,8}(X) + 1.636085137423343 \cdot 10^{-13} B_{6,8}(X) \\
&\quad + 4.762913599965286 \cdot 10^{-14} B_{7,8}(X) - 7.289173467226265 \cdot 10^{-14} B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9129136379925769, 0.9243992614454615\}$$

Intersection intervals with the x axis:

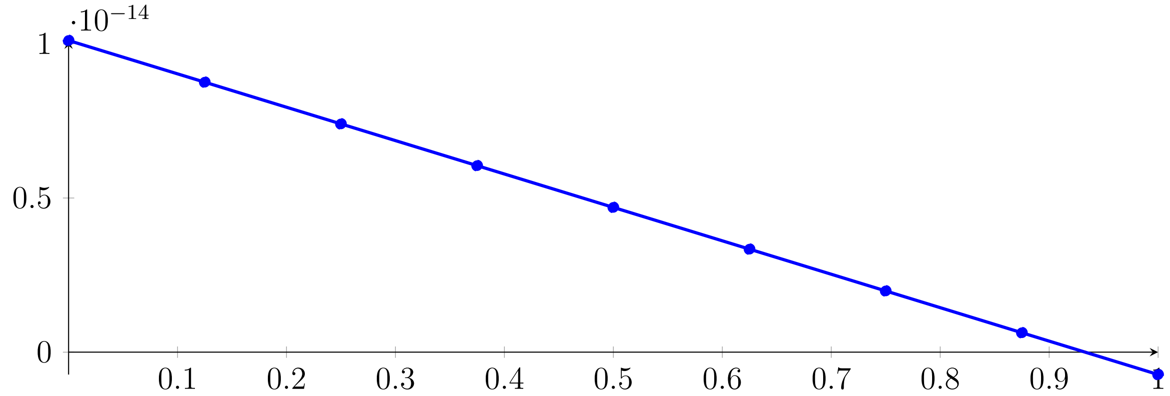
$$[0.9129136379925769, 0.9243992614454615]$$

Longest intersection interval: 0.01148562345288463
$\Longrightarrow$ Selective recursion: interval 1: $[0.500000299855758, 0.500000300010354]$,

55.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.5000000299855758, 0.5000000300010354]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.26268981617174\cdot10^{-87}X^8 - 8.230826044001589\cdot10^{-75}X^7 - 8.610712798456319\cdot10^{-63}X^6\\&\quad - 4.781269588122392\cdot10^{-51}X^5 - 1.48617213692691\cdot10^{-39}X^4 - 2.451903903103066\cdot10^{-28}X^3\\&\quad - 1.677512719815316\cdot10^{-17}X^2 - 1.081967377285333\cdot10^{-14}X + 1.010965922317952\cdot10^{-14}\\&= 1.010965922317952\cdot10^{-14}B_{0,8}(X) + 8.757200001572855\cdot10^{-15}B_{1,8}(X) + 7.40414166828054\\&\quad\cdot10^{-15}B_{2,8}(X) + 6.050484223302573\cdot10^{-15}B_{3,8}(X) + 4.696227666638948\cdot10^{-15}B_{4,8}(X)\\&\quad + 3.341371998289662\cdot10^{-15}B_{5,8}(X) + 1.98591721825471\cdot10^{-15}B_{6,8}(X)\\&\quad + 6.298633265340878\cdot10^{-16}B_{7,8}(X) - 7.267896768722089\cdot10^{-16}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9329310105574586, 0.9330346747614\}$$

Intersection intervals with the x axis:

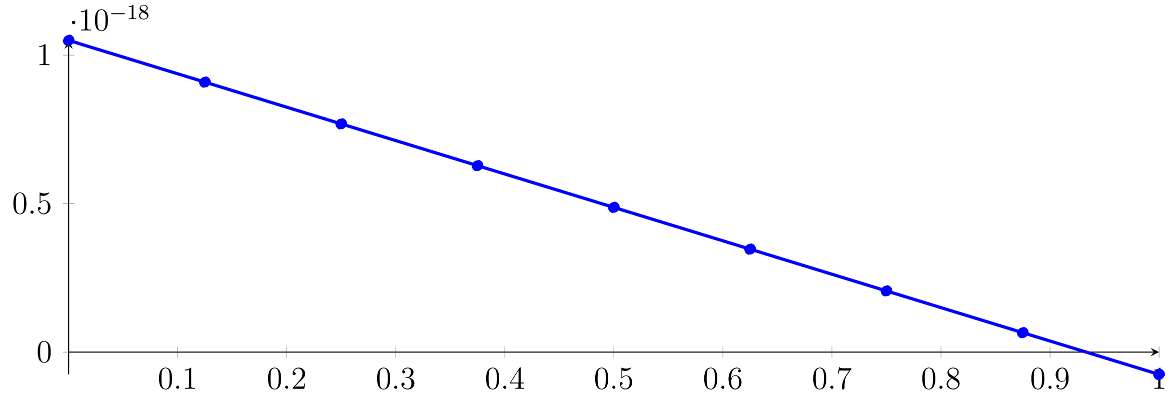
$$[0.9329310105574586, 0.9330346747614]$$

Longest intersection interval: 0.0001036642039414486
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000299999985, 0.5000000300000001]$,

55.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.5000000299999985, 0.5000000300000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.351172724257996\cdot10^{-119}X^8 - 1.058876090749164\cdot10^{-102}X^7 - 1.068592215336126\cdot10^{-86}X^6\\&\quad - 5.723837778458117\cdot10^{-71}X^5 - 1.716265145654202\cdot10^{-55}X^4 - 2.731428873735883\cdot10^{-40}X^3\\&\quad - 1.802699988373219\cdot10^{-25}X^2 - 1.124857565692813\cdot10^{-18}X + 1.049632124051115\cdot10^{-18}\\&= 1.049632124051115\cdot10^{-18}B_{0,8}(X) + 9.090249283395136\cdot10^{-19}B_{1,8}(X) + 7.684177261896977\\&\quad\cdot10^{-19}B_{2,8}(X) + 6.278105176016676\cdot10^{-19}B_{3,8}(X) + 4.872033025754233\cdot10^{-19}B_{4,8}(X)\\&\quad + 3.465960811109647\cdot10^{-19}B_{5,8}(X) + 2.059888532082919\cdot10^{-19}B_{6,8}(X)\\&\quad + 6.538161886740477\cdot10^{-20}B_{7,8}(X) - 7.522562191169655\cdot10^{-20}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243242252754, 0.9331243349427871\}$$

Intersection intervals with the x axis:

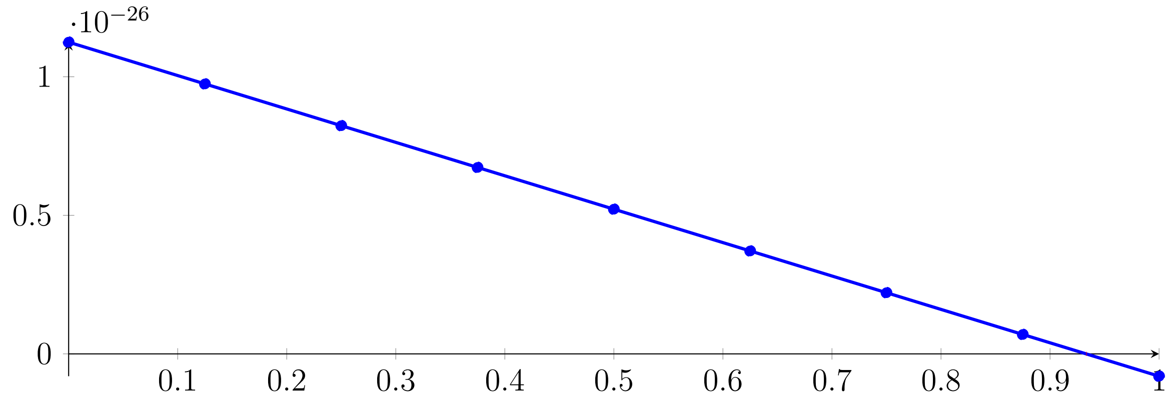
$$[0.9331243242252754, 0.9331243349427871]$$

Longest intersection interval: $1.071751170792712 \cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

55.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-7.574571270278352 \cdot 10^{-183} X^8 - 1.719898859058455 \cdot 10^{-158} X^7 - 1.619480808086848 \cdot 10^{-134} X^6 \\ &- 8.09388724153584 \cdot 10^{-111} X^5 - 2.264437035388769 \cdot 10^{-87} X^4 - 3.36257357580136 \cdot 10^{-64} X^3 \\ &- 2.070672372961545 \cdot 10^{-41} X^2 - 1.205567773574102 \cdot 10^{-26} X + 1.124944638137319 \cdot 10^{-26} \\ = &\ 1.124944638137319 \cdot 10^{-26} B_{0,8}(X) + 9.742486664405558 \cdot 10^{-27} B_{1,8}(X) + 8.235526947437929 \\ &\cdot 10^{-27} B_{2,8}(X) + 6.7285672304703 \cdot 10^{-27} B_{3,8}(X) + 5.22160751350267 \cdot 10^{-27} B_{4,8}(X) \\ &+ 3.714647796535039 \cdot 10^{-27} B_{5,8}(X) + 2.207688079567407 \cdot 10^{-27} B_{6,8}(X) \\ &+ 7.007283625997749 \cdot 10^{-28} B_{7,8}(X) - 8.062313543678582 \cdot 10^{-28} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243442268158, 0.933124344226816\}$$

Intersection intervals with the x axis:

$$[0.9331243442268158, 0.933124344226816]$$

Longest intersection interval: $1.148650253172236 \cdot 10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

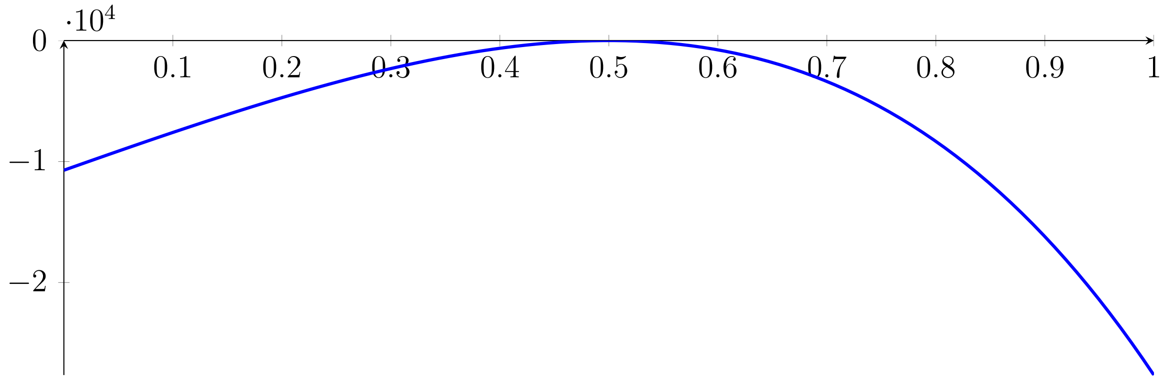
55.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 24!

55.31 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

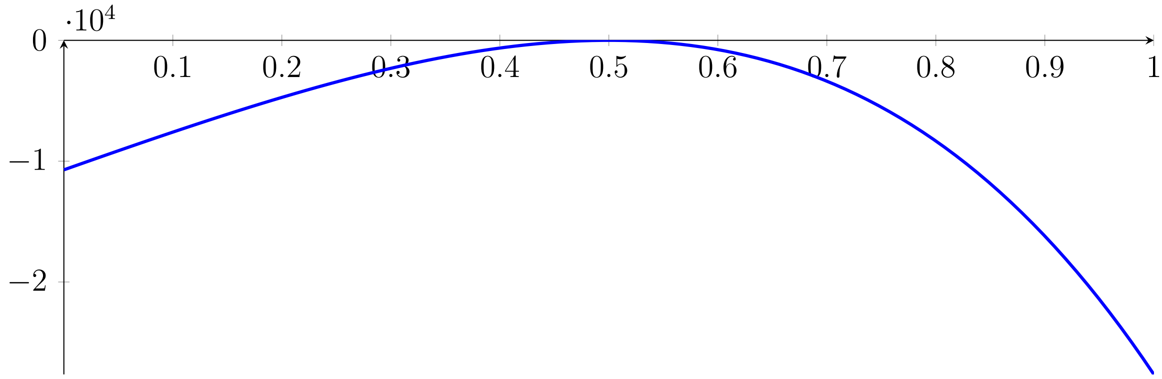
with precision $\varepsilon = 1 \cdot 10^{-32}$.

56 Running QuadClip on h_8 with epsilon 32

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called QuadClip with input polynomial on interval $[0, 1]$:

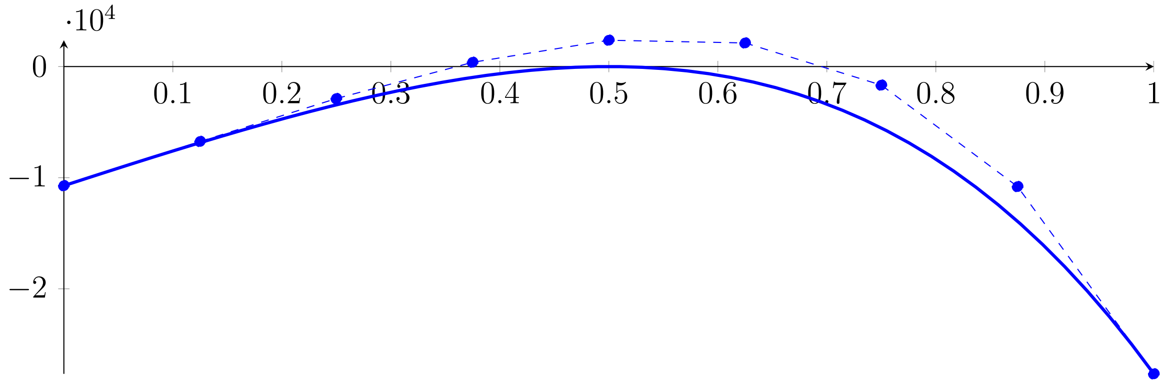
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



56.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

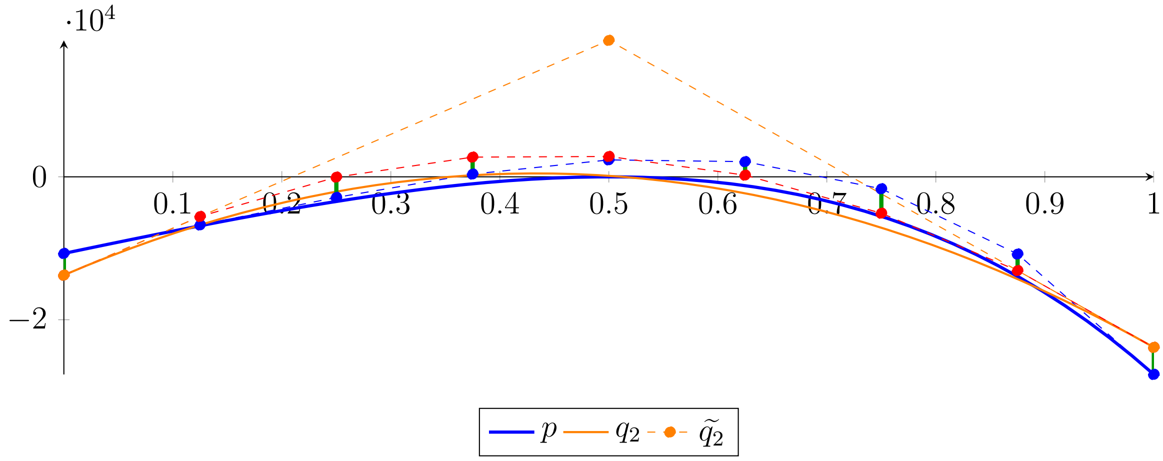
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q_2} = &\ 8.415084934481114\cdot 10^{-299}X^8 - 3.337656926892332\cdot 10^{-298}X^7 + 5.61048922956057\cdot 10^{-298}X^6 \\ &- 5.189733160056009\cdot 10^{-298}X^5 + 2.787623795639522\cdot 10^{-298}X^4 - 8.054794139884735 \\ &\cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ = &-13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8} \\ &+ 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8} \\ &- 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.
Bounding polynomials M and m:

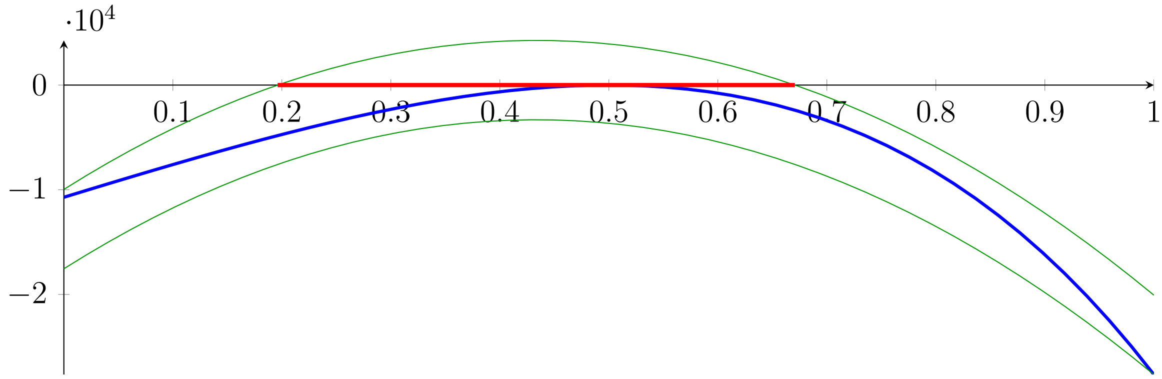
$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$

$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



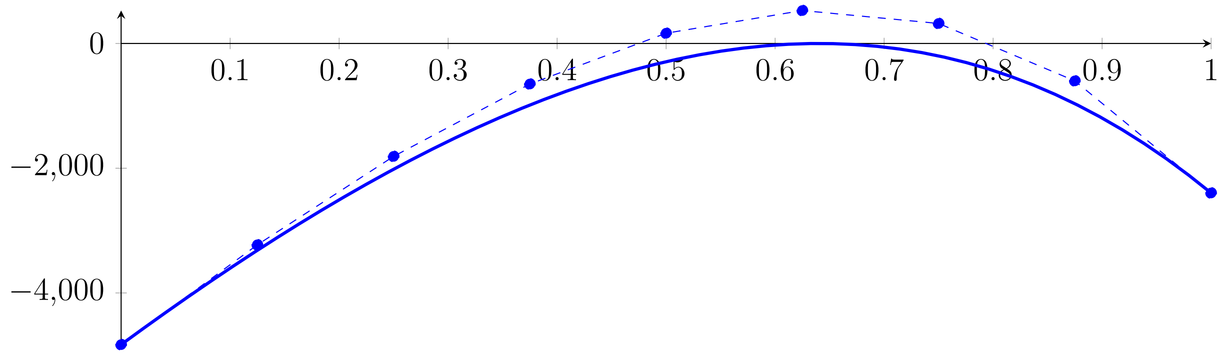
$$[0.196099166583006, 0.6706514347613278]$$

Longest intersection interval: 0.4745522681783218
$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

56.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

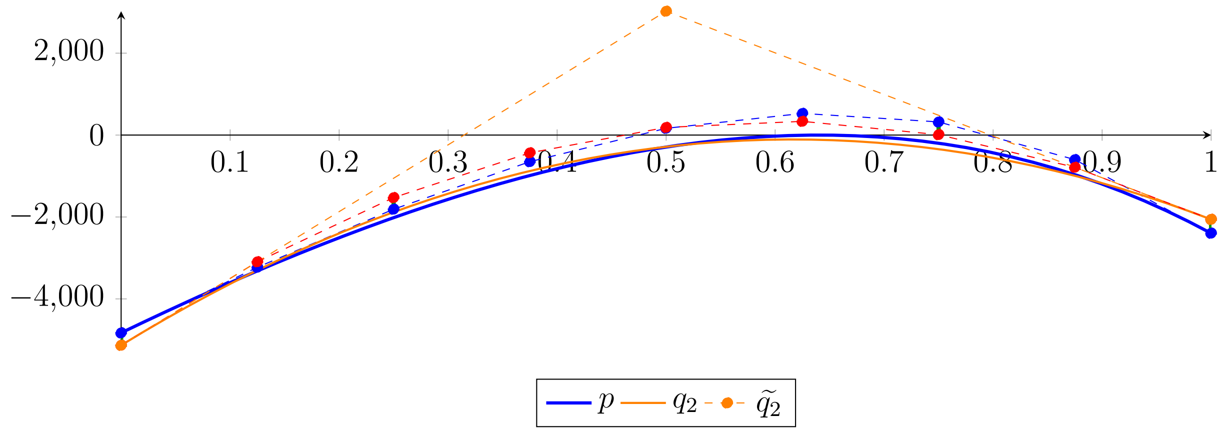
$$\begin{aligned}
p &= -0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6 \\
&\quad - 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3 \\
&\quad - 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684 \\
&= -4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X) \\
&\quad - 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X) \\
&\quad + 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.476670942620757\cdot 10^{-299}X^8 - 6.381627458836932\cdot 10^{-299}X^7 + 1.1779791924066\cdot 10^{-298}X^6 \\ &\quad - 1.190713137013755\cdot 10^{-298}X^5 + 6.850874958113058\cdot 10^{-299}X^4 - 2.05172173630317 \\ &\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\ &\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\ &\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

Bounding polynomials M and m:

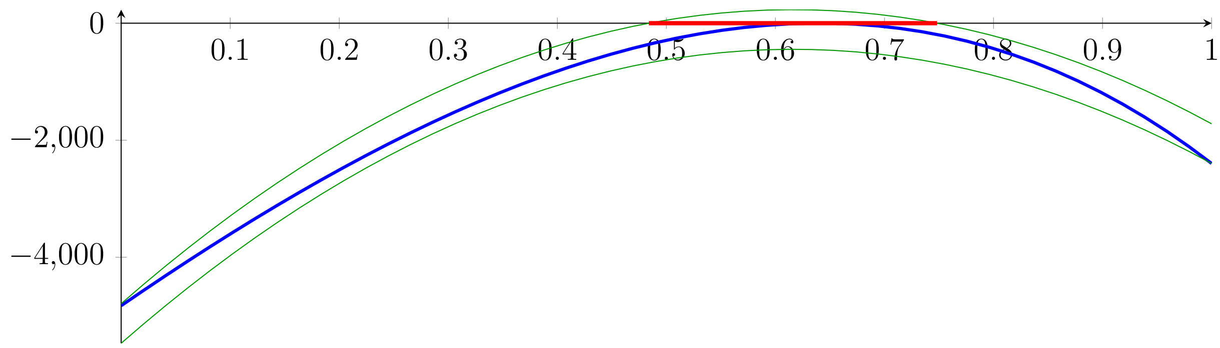
$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$

$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.4839311609916354, 0.7482816274420809]$$

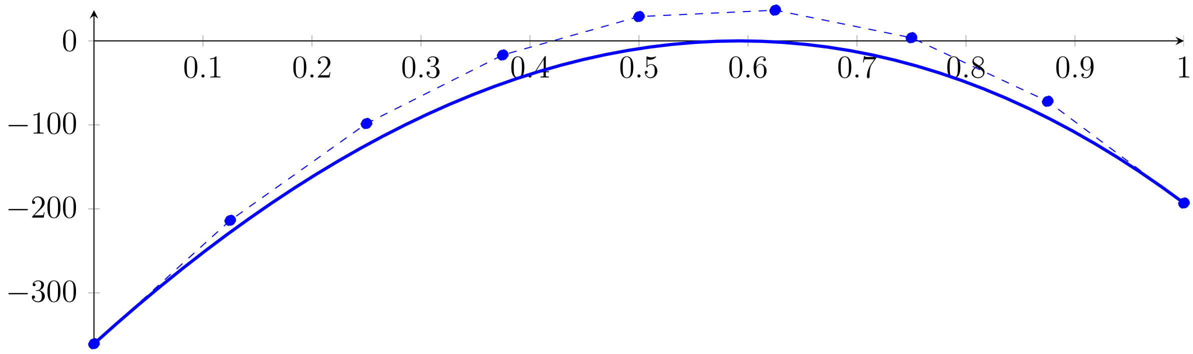
Longest intersection interval: 0.2643504664504455

$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

56.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.551197910121811$

Normalized monomial und Bézier representations and the Bézier polygon:

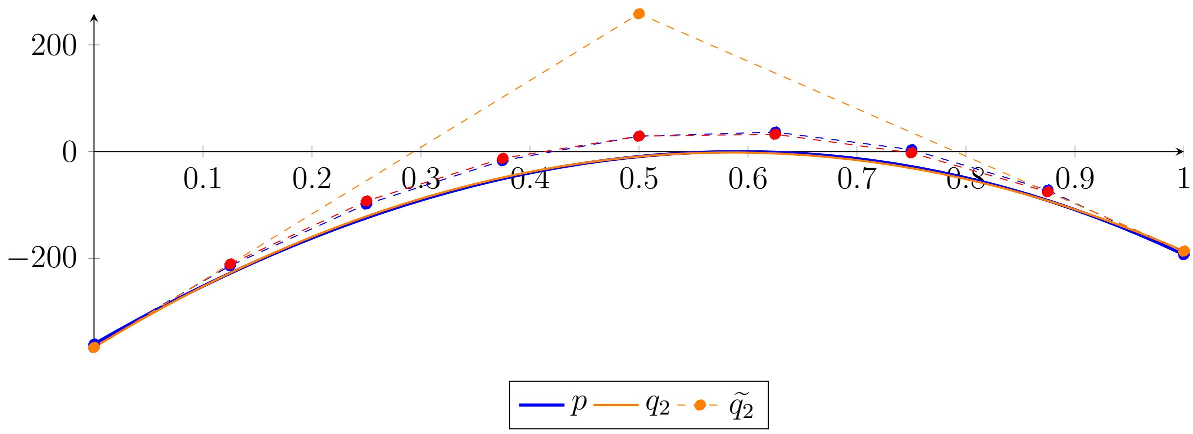
$$\begin{aligned} p = &-6.133566475094875\cdot 10^{-08}X^8 - 1.877794231289375\cdot 10^{-05}X^7 - 0.002379939031153127X^6 \\ &- 0.1596298583101837X^5 - 5.958684697205818X^4 - 116.3336704708797X^3 \\ &- 885.1606719876373X^2 + 1175.121582406799X - 360.5751654247078 \\ = &-360.5751654247078B_{0,8}(X) - 213.6849676238579B_{1,8}(X) - 98.40765096542357B_{2,8}(X) \\ &- 16.82060242209915B_{3,8}(X) + 28.91366696631814B_{4,8}(X) + 36.54467155974404B_{5,8}(X) \\ &+ 3.731015586800578B_{6,8}(X) - 71.9626297312559B_{7,8}(X) - 193.0686388102505B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = &-1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984 \\ = &-366.9199822310984B_{0,2} + 258.3517672110292B_{1,2} - 186.5418852681748B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &\ 1.223655337688364\cdot 10^{-300}X^8 - 5.205587704887534\cdot 10^{-300}X^7 + 9.409306890019087\cdot 10^{-300}X^6 \\ &- 9.282190734559383\cdot 10^{-300}X^5 + 5.21941805196838\cdot 10^{-300}X^4 - 1.541981168110934 \\ &\cdot 10^{-300}X^3 - 1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984 \\ = &-366.9199822310984B_{0,8} - 210.6020448705665B_{1,8} - 92.50430043579644B_{2,8} \\ &- 12.62674892678823B_{3,8} + 29.03060965645814B_{4,8} + 32.46777531394266B_{5,8} \\ &- 2.315251954334665B_{6,8} - 75.31847214837383B_{7,8} - 186.5418852681748B_{8,8} \end{aligned}$$



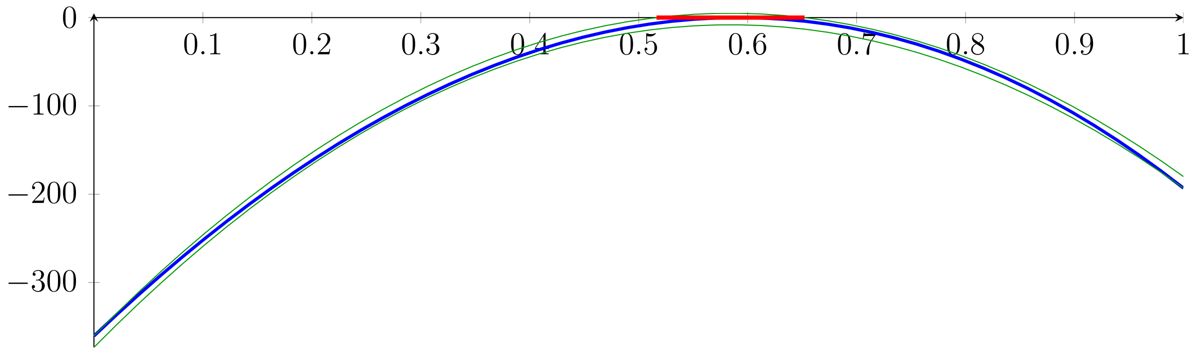
The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.
Bounding polynomials M and m:

$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$
$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



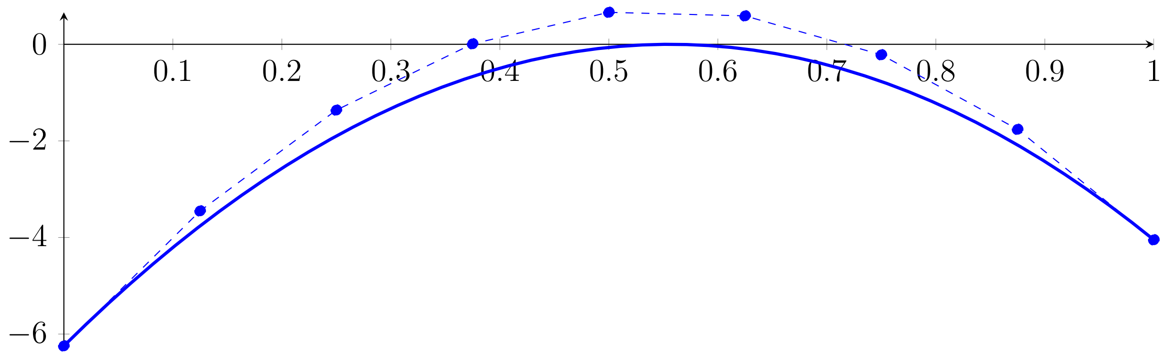
$$[0.5163481231478299, 0.652203482649816]$$

Longest intersection interval: 0.1358553595019861
$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

56.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.507567493156426$

Normalized monomial und Bézier representations and the Bézier polygon:

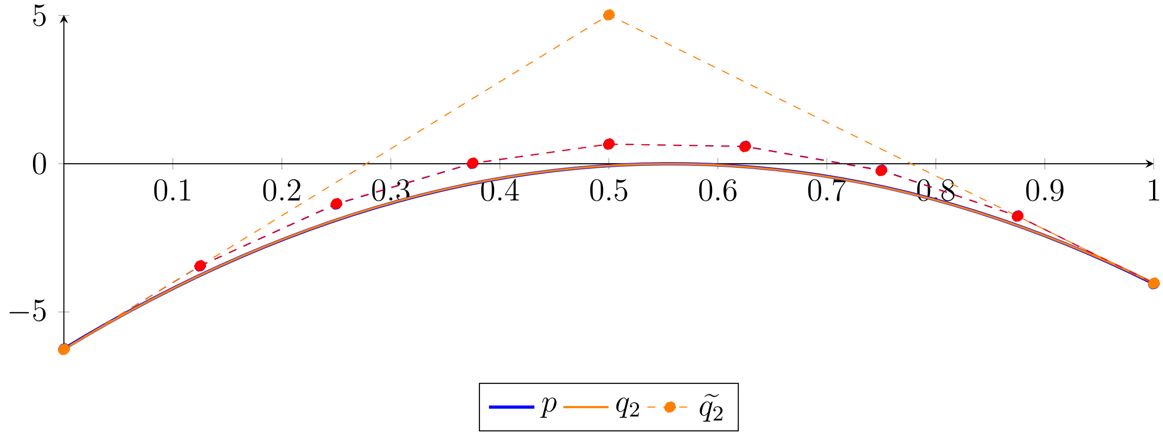
$$\begin{aligned} p = &-7.117496868854 28\cdot 10^{-15}X^8 - 1.625571369884896\cdot 10^{-11}X^7 - 1.539287417434116\cdot 10^{-08}X^6 \\ &- 7.733623372922873\cdot 10^{-06}X^5 - 0.002173482359120639X^4 - 0.3236423633498969X^3 \\ &- 19.84316397394663X^2 + 22.36613094108362X - 6.245496834711843 \\ = &-6.245496834711843B_{0,8}(X) - 3.449730467076391B_{1,8}(X) - 1.36264852708189B_{2,8}(X) \\ &+ 0.009969657354697074B_{3,8}(X) + 0.662313708568421B_{4,8}(X) + 0.5885420610459267B_{5,8}(X) \\ &- 0.217218177224708B_{6,8}(X) - 1.760871363955914B_{7,8}(X) - 4.048353462316386B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q_2} &= 2.334129178460356\cdot 10^{-302}X^8 - 9.797943630323215\cdot 10^{-302}X^7 + 1.744532667551292\cdot 10^{-301}X^6 \\ &\quad - 1.695264192146682\cdot 10^{-301}X^5 + 9.412284928871313\cdot 10^{-302}X^4 - 2.760729238908791 \\ &\quad \cdot 10^{-302}X^3 - 20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,8} - 3.441577100979207B_{1,8} - 1.347444096092966B_{2,8} \\ &\quad + 0.02053293285443696B_{3,8} + 0.6623539858630032B_{4,8} + 0.5780190629327322B_{5,8} \\ &\quad - 0.232471835936376B_{6,8} - 1.769118710744321B_{7,8} - 4.031921561491104B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.

Bounding polynomials M and m:

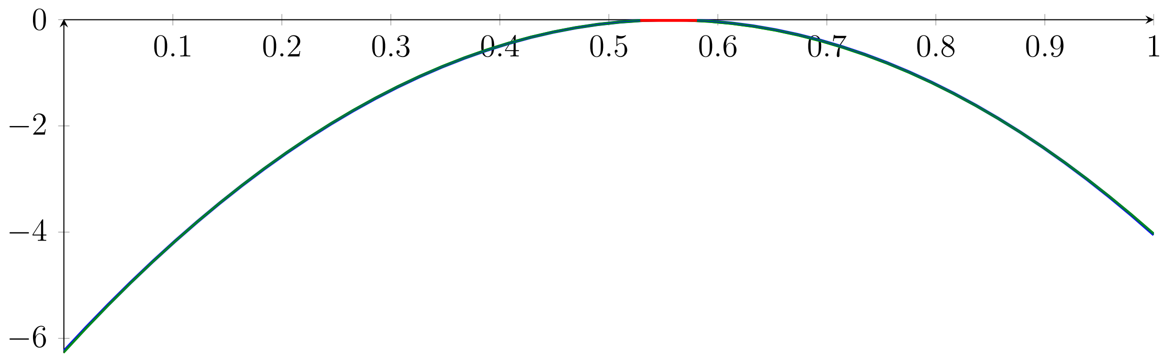
$$M = -20.33236732628744X^2 + 22.56231184660062X - 6.245434180979003$$

$$m = -20.33236732628744X^2 + 22.56231184660062X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

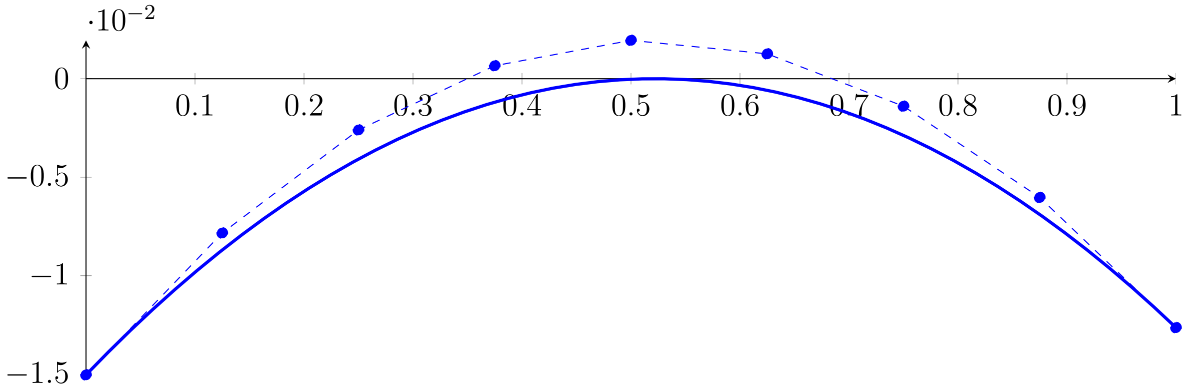
Longest intersection interval: 0.05205162765602134

$\implies$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

56.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Normalized monomial und Bézier representations and the Bézier polygon:

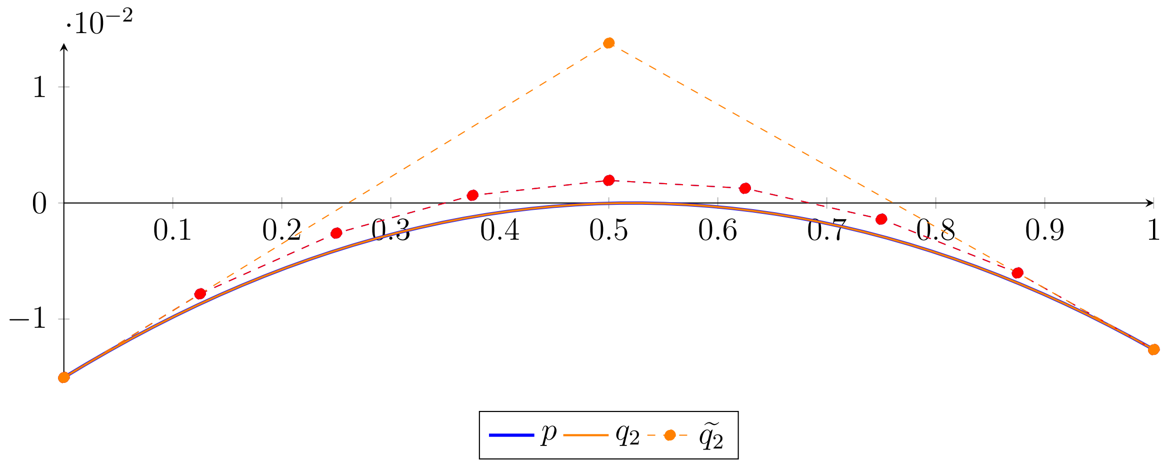
$$p = -3.835321724591002 \cdot 10^{-25} X^8 - 1.685970393664051 \cdot 10^{-20} X^7 - 3.073417757909172 \cdot 10^{-16} X^6 - 2.973678167232232 \cdot 10^{-12} X^5 - 1.610545215791654 \cdot 10^{-08} X^4 - 4.629380451575473 \cdot 10^{-05} X^3 - 0.05516351610802166 X^2 + 0.05760784492782384 X - 0.01503353559864481$$
$$= -0.01503353559864481 B_{0,8}(X) - 0.007832554982666833 B_{1,8}(X) - 0.002601699941975341 B_{2,8}(X) + 0.0006582028483490249 B_{3,8}(X) + 0.001946326483147738 B_{4,8}(X) + 0.001261843827131283 B_{5,8}(X) - 0.001396072485173959 B_{6,8}(X) - 0.006028250049478829 B_{7,8}(X) - 0.01263551669178453 B_{8,8}(X)$$



Degree reduction and raising:

$$q_2 = -0.05523298442945254 X^2 + 0.05763563593870456 X - 0.01503585166965657$$
$$= -0.01503585166965657 B_{0,2} + 0.01378196629969571 B_{1,2} - 0.01263320016040455 B_{2,2}$$

$$\widetilde{q}_2 = 6.306459041947834 \cdot 10^{-305} X^8 - 2.608238530297949 \cdot 10^{-304} X^7 + 4.573872253581115 \cdot 10^{-304} X^6 - 4.384946099604334 \cdot 10^{-304} X^5 + 2.410615466544519 \cdot 10^{-304} X^4 - 7.036239809064398 \cdot 10^{-305} X^3 - 0.05523298442945254 X^2 + 0.05763563593870456 X - 0.01503585166965657$$
$$= -0.01503585166965657 B_{0,8} - 0.007831397177318504 B_{1,8} - 0.002599549271746596 B_{2,8} + 0.0006596920470591496 B_{3,8} + 0.001946326779098733 B_{4,8} + 0.001260354924372155 B_{5,8} - 0.001398223517120585 B_{6,8} - 0.006029408545379487 B_{7,8} - 0.01263320016040455 B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 2.316531379978639 \cdot 10^{-06}$.
Bounding polynomials M and m:

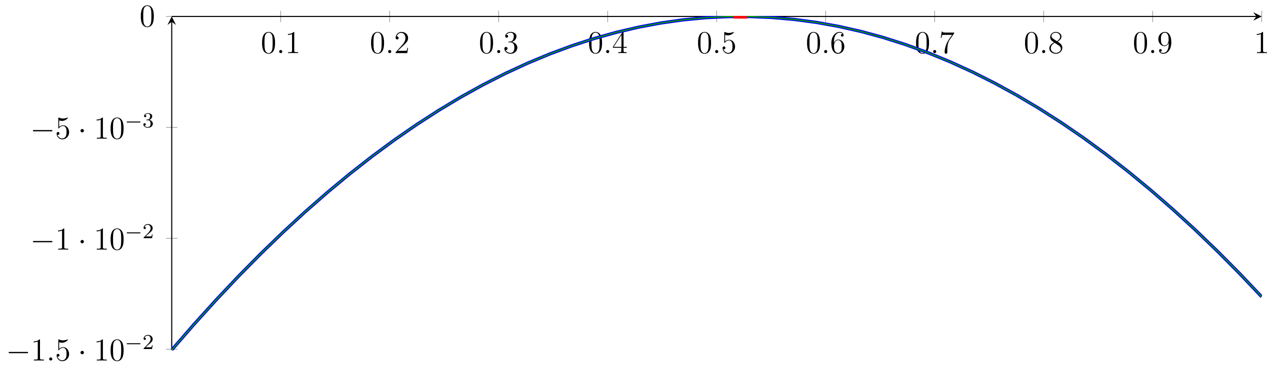
$$M = -0.05523298442945254 X^2 + 0.05763563593870456 X - 0.0150335351382766$$

$$m = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503816820103655$$

Root of M and m:

$$N(M) = \{0.5154882826278122, 0.5280120194846123\} \qquad N(m) = \{\}$$

Intersection intervals:



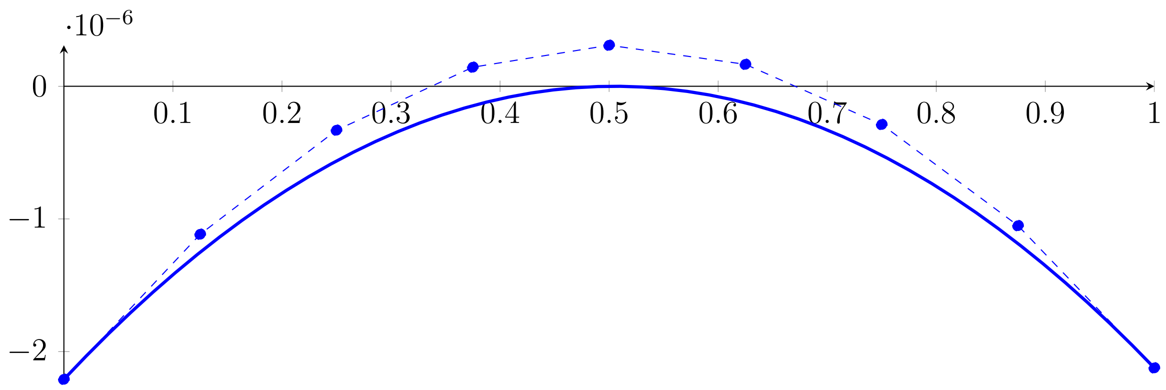
$$[0.5154882826278122, 0.5280120194846123]$$

Longest intersection interval: 0.01252373685680011
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999944147887801, 0.5000055246634291]$,

56.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4999944147887801, 0.50000552466$

Normalized monomial und Bézier representations and the Bézier polygon:

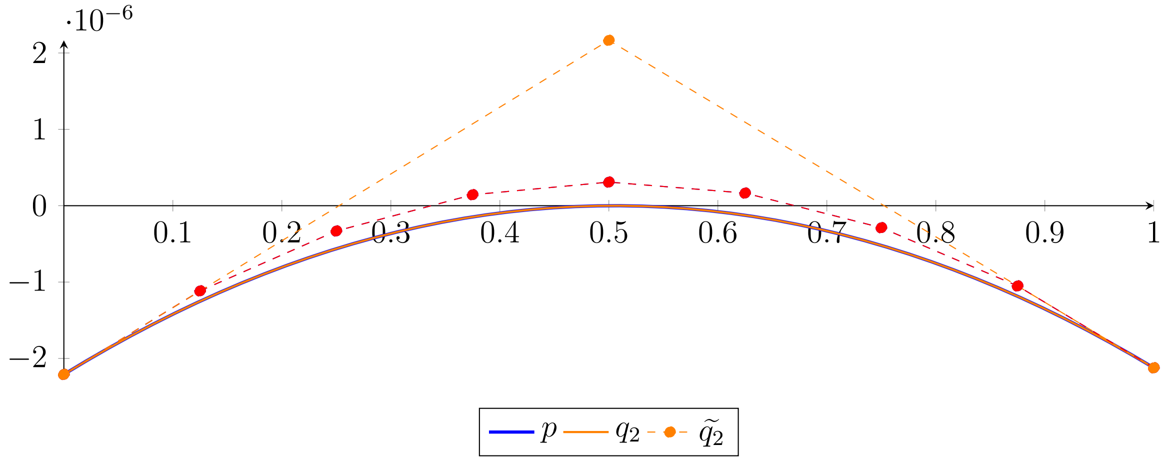
$$\begin{aligned} p = &-2.32099000990716 \cdot 10^{-40} X^8 - 8.147572265547704 \cdot 10^{-34} X^7 - 1.186072294581764 \cdot 10^{-27} X^6 \\ &- 9.164366615560225 \cdot 10^{-22} X^5 - 3.963832728671687 \cdot 10^{-16} X^4 - 9.099890718270267 \cdot 10^{-11} X^3 \\ &- 8.663298447262891 \cdot 10^{-06} X^2 + 8.749571419899133 \cdot 10^{-06} X - 2.209162411567082 \cdot 10^{-06} \\ = &-2.209162411567082 \cdot 10^{-06} B_{0,8}(X) - 1.11546598407969 \cdot 10^{-06} B_{1,8}(X) - 3.311730725659734 \\ &\cdot 10^{-07} B_{2,8}(X) + 1.437146979935834 \cdot 10^{-07} B_{3,8}(X) + 3.091957026128322 \\ &\cdot 10^{-07} B_{4,8}(X) + 1.652683162999621 \cdot 10^{-07} B_{5,8}(X) - 2.880690859425 \cdot 10^{-07} B_{6,8}(X) \\ &- 1.05081812911769 \cdot 10^{-06} B_{7,8}(X) - 2.122980438234407 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -8.663434946303181 \cdot 10^{-06} X^2 + 8.749626019605851 \cdot 10^{-06} X - 2.209166961546417 \cdot 10^{-06} \\ &= -2.209166961546417 \cdot 10^{-06} B_{0,2} + 2.165646048256509 \cdot 10^{-06} B_{1,2} - 2.122975888243747 \cdot 10^{-06} B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 9.84075548060145\cdot 10^{-309}X^8 - 4.039091668354938\cdot 10^{-308}X^7 + 7.030443670929191\cdot 10^{-308}X^6\\ &\quad - 6.698784216784483\cdot 10^{-308}X^5 + 3.668633643168221\cdot 10^{-308}X^4 - 1.069248215117909\cdot 10^{-308}X^3\\ &\quad - 8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209166961546417\cdot 10^{-06}\\ &= -2.209166961546417\cdot 10^{-06}B_{0,8} - 1.115463709095685\cdot 10^{-06}B_{1,8} - 3.311688475843534\cdot 10^{-07}B_{2,8}\\ &\quad + 1.437176229875793\cdot 10^{-07}B_{3,8} + 3.091957026201127\cdot 10^{-07}B_{4,8} + 1.652653913132468\cdot 10^{-07}B_{5,8}\\ &\quad - 2.880733109330184\cdot 10^{-07}B_{6,8} - 1.050820404118683\cdot 10^{-06}B_{7,8} - 2.122975888243747\cdot 10^{-06}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.549990660244254\cdot 10^{-12}$.

Bounding polynomials M and m:

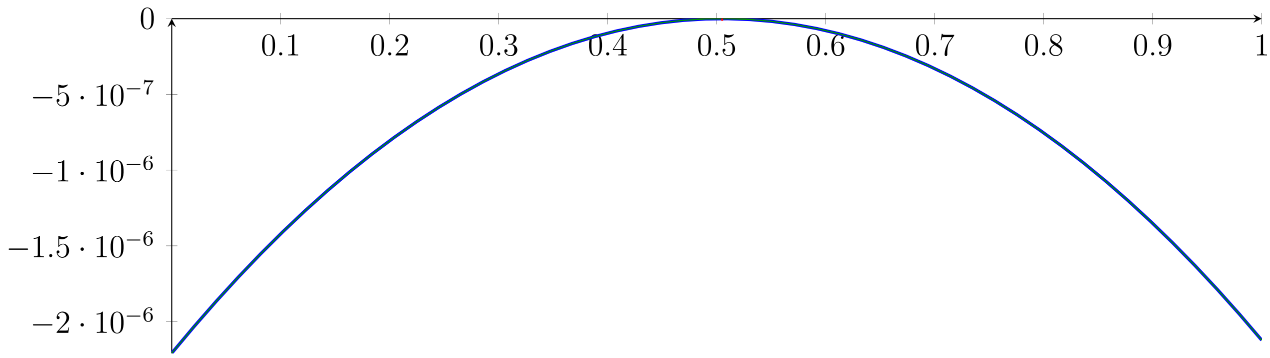
$$M = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209162411555757\cdot 10^{-06}$$

$$m = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209171511537077\cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{0.5041259457474217, 0.505822887923852\} \qquad N(m) = \{\}$$

Intersection intervals:



$[0.5041259457474217, 0.505822887923852]$

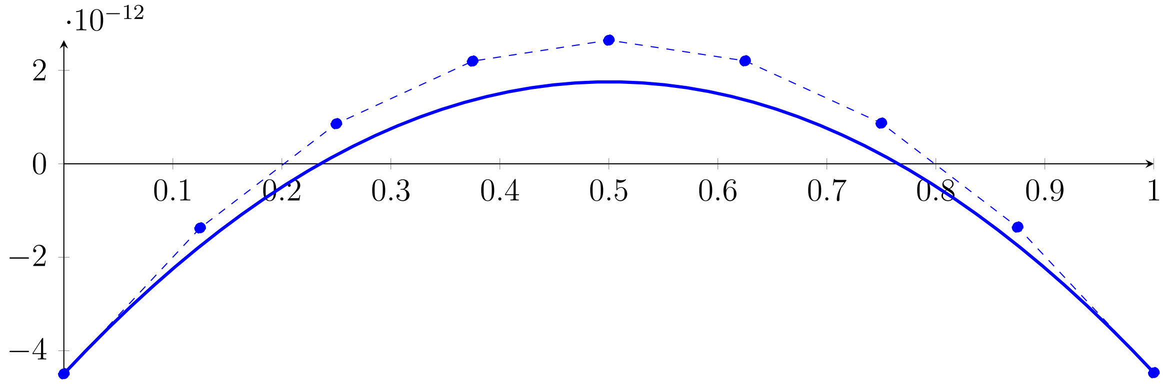
Longest intersection interval: 0.00169694217643035

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000155648447, 0.5000000344176595]$,

56.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.5000000155648447, 0.500000034$

Normalized monomial und Bézier representations and the Bézier polygon:

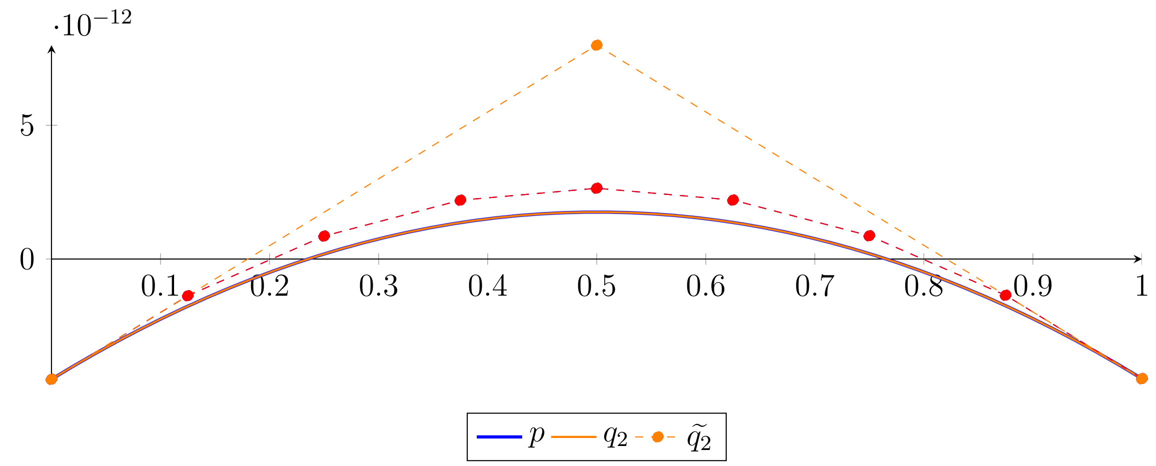
$$\begin{aligned}p &= -1.595914506899923\cdot 10^{-62}X^8 - 3.301399092257474\cdot 10^{-53}X^7 - 2.83213843356941\cdot 10^{-44}X^6\\&\quad - 1.289553529043531\cdot 10^{-35}X^5 - 3.286896476505693\cdot 10^{-27}X^4 - 4.446733715730271\cdot 10^{-19}X^3\\&\quad - 2.494734097474612\cdot 10^{-11}X^2 + 2.497049303064054\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}\\&= -4.493680200151054\cdot 10^{-12}B_{0,8}(X) - 1.372368571320986\cdot 10^{-12}B_{1,8}(X) + 8.579665941252918\\&\quad \cdot 10^{-13}B_{2,8}(X) + 2.197325288247184\cdot 10^{-12}B_{3,8}(X) + 2.645707503104094\cdot 10^{-12}B_{4,8}(X)\\&\quad + 2.203113230755426\cdot 10^{-12}B_{5,8}(X) + 8.695424632605847\cdot 10^{-13}B_{6,8}(X)\\&\quad - 1.355004807321027\cdot 10^{-12}B_{7,8}(X) - 4.470528588930005\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12}\\&= -4.493680222384723\cdot 10^{-12}B_{0,2} + 7.991566426337562\cdot 10^{-12}B_{1,2} - 4.470528566696336\cdot 10^{-12}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 3.394595061927522\cdot 10^{-314}X^8 - 1.386657030412925\cdot 10^{-313}X^7 + 2.314937908564659\cdot 10^{-313}X^6\\&\quad - 2.018101017184903\cdot 10^{-313}X^5 + 9.746671569929334\cdot 10^{-314}X^4 - 2.577278121543316\cdot 10^{-314}X^3\\&\quad - 2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12}\\&= -4.493680222384723\cdot 10^{-12}B_{0,8} - 1.372368560204152\cdot 10^{-12}B_{1,8} + 8.579666147708415\cdot 10^{-13}B_{2,8}\\&\quad + 2.197325302540257\cdot 10^{-12}B_{3,8} + 2.645707503104094\cdot 10^{-12}B_{4,8} + 2.203113216462353\cdot 10^{-12}B_{5,8}\\&\quad + 8.695424426150349\cdot 10^{-13}B_{6,8} - 1.355004818437861\cdot 10^{-12}B_{7,8} - 4.470528566696336\cdot 10^{-12}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.223366895429667\cdot 10^{-20}$.
Bounding polynomials M and m:

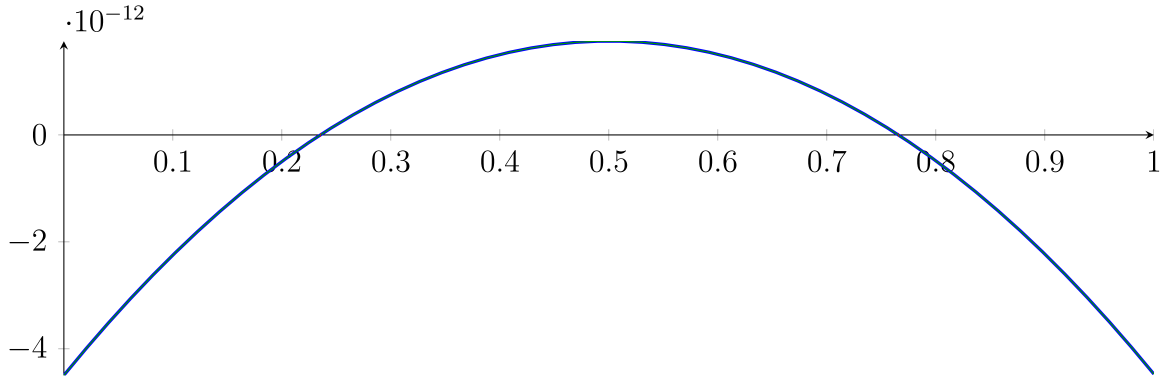
$$M = -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}$$

$$m = -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680244618392\cdot 10^{-12}$$

Root of M and m:

$$N(M) = \{0.235251622185842, 0.765676398764079\} \quad N(m) = \{0.2352516255462581, 0.7656763954036629\}$$

Intersection intervals:



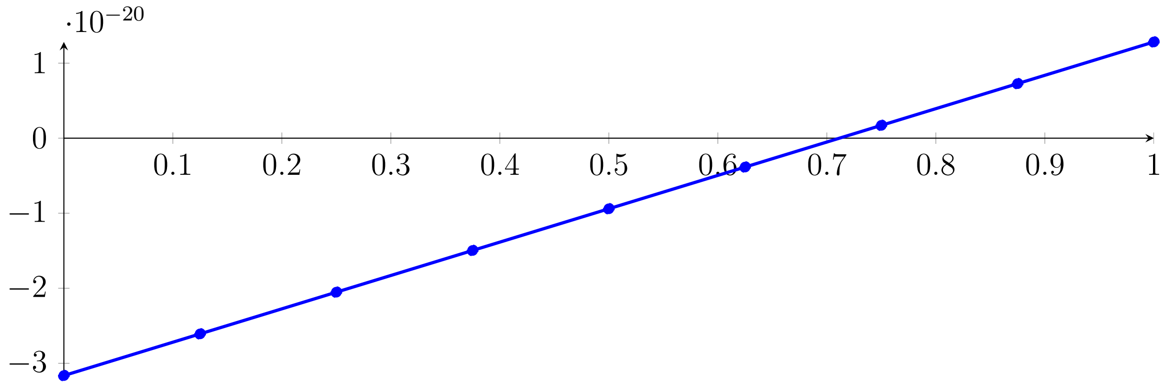
$$[0.235251622185842, 0.2352516255462581],\ [0.7656763954036629, 0.765676398764079]$$

Longest intersection interval: $3.360416099786671\cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.50000002]$, interval 2: $[0.50000003, 0.50000003000000$

56.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000$

Normalized monomial und Bézier representations and the Bézier polygon:

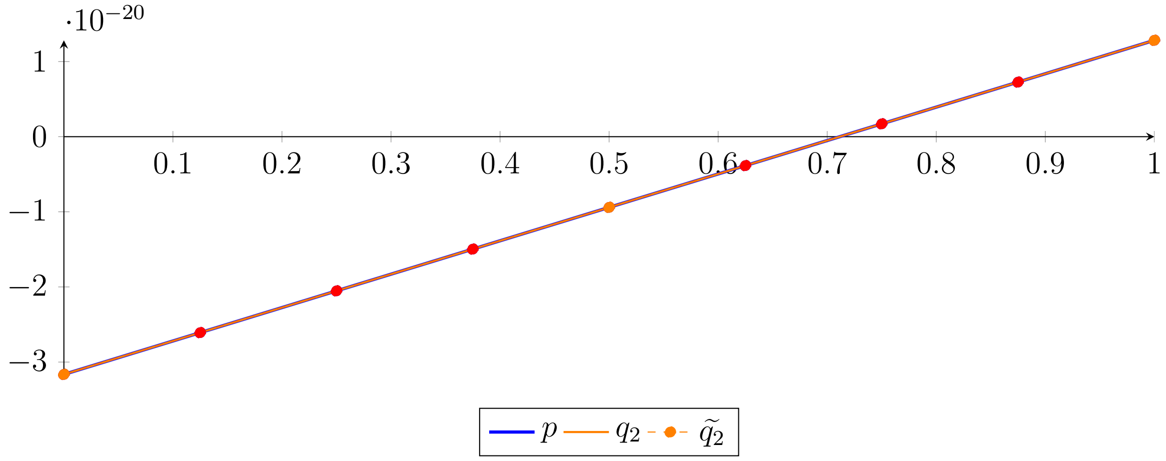
$$\begin{aligned} p = &-2.595099873436077\cdot 10^{-130}X^8 - 1.59753148120066\cdot 10^{-112}X^7 - 4.078240366489356\cdot 10^{-95}X^6 \\ &- 5.52592084177806\cdot 10^{-78}X^5 - 4.191391755953253\cdot 10^{-61}X^4 - 1.687408749214622\cdot 10^{-44}X^3 \\ &- 2.817152660512407\cdot 10^{-28}X^2 + 4.446733810024026\cdot 10^{-20}X - 3.164099743671992\cdot 10^{-20} \\ = &-3.164099743671992\cdot 10^{-20}B_{0,8}(X) - 2.608258017418989\cdot 10^{-20}B_{1,8}(X) - 2.052416292172112 \\ &\cdot 10^{-20}B_{2,8}(X) - 1.49657456793136\cdot 10^{-20}B_{3,8}(X) - 9.407328446967348\cdot 10^{-21}B_{4,8}(X) \\ &- 3.848911224682354\cdot 10^{-21}B_{5,8}(X) + 1.709505987541381\cdot 10^{-21}B_{6,8}(X) \\ &+ 7.267923189703856\cdot 10^{-21}B_{7,8}(X) + 1.282634038180507\cdot 10^{-20}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -2.817152660512407\cdot 10^{-28}X^2 + 4.446733810024026\cdot 10^{-20}X - 3.164099743671992\cdot 10^{-20} \\ &= -3.164099743671992\cdot 10^{-20}B_{0,2} - 9.407328386599791\cdot 10^{-21}B_{1,2} + 1.282634038180507\cdot 10^{-20}B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -2.04607227392422\cdot10^{-323}X^8 + 3.432164034855183\cdot10^{-323}X^7 + 6.927895597874022\cdot10^{-323}X^6\\ &\quad - 2.227227667041084\cdot10^{-322}X^5 + 2.093778075420854\cdot10^{-322}X^4 - 7.69541017416500 2\cdot10^{-323}X^3\\ &\quad - 2.817152660512407\cdot10^{-28}X^2 + 4.446733810024026\cdot10^{-20}X - 3.164099743671992\cdot10^{-20}\\ &= -3.164099743671992\cdot10^{-20}B_{0,8} - 2.608258017418989\cdot10^{-20}B_{1,8} - 2.052416292172112\cdot10^{-20}B_{2,8}\\ &\quad - 1.49657456793136\cdot10^{-20}B_{3,8} - 9.407328446967348\cdot10^{-21}B_{4,8} - 3.848911224682354\cdot10^{-21}B_{5,8}\\ &\quad + 1.709505987541381\cdot10^{-21}B_{6,8} + 7.267923189703856\cdot10^{-21}B_{7,8} + 1.282634038180507\cdot10^{-20}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.437043746073111\cdot10^{-46}$.
Bounding polynomials M and m:

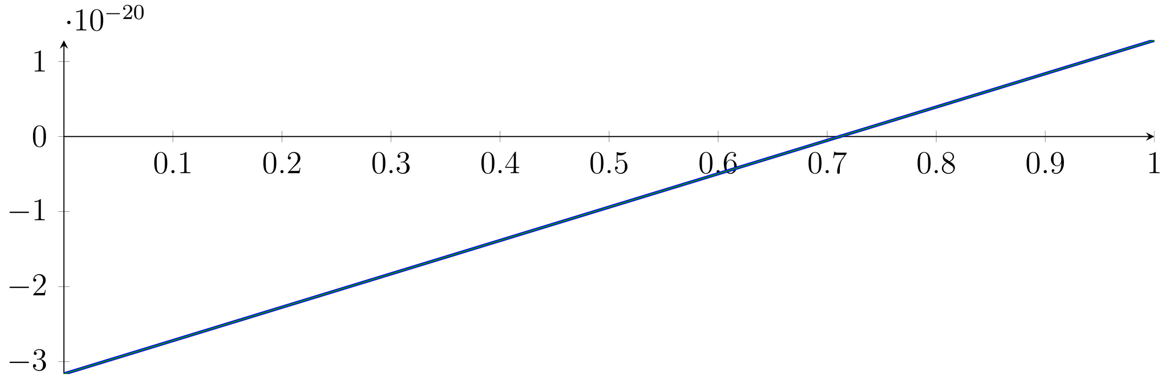
$$M = -2.817152660512407\cdot10^{-28}X^2 + 4.446733810024026\cdot10^{-20}X - 3.164099743671992\cdot10^{-20}$$

$$m = -2.817152660512407\cdot10^{-28}X^2 + 4.446733810024026\cdot10^{-20}X - 3.164099743671992\cdot10^{-20}$$

Root of M and m:

$N(M) = \{0.7115559179195557, 157844970.6438557\}$ $N(m) = \{0.7115559179195557, 157844970.6438557\}$

Intersection intervals:



$$[0.7115559179195557, 0.7115559179195557]$$

Longest intersection interval: $3.794715034716651\cdot10^{-26}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

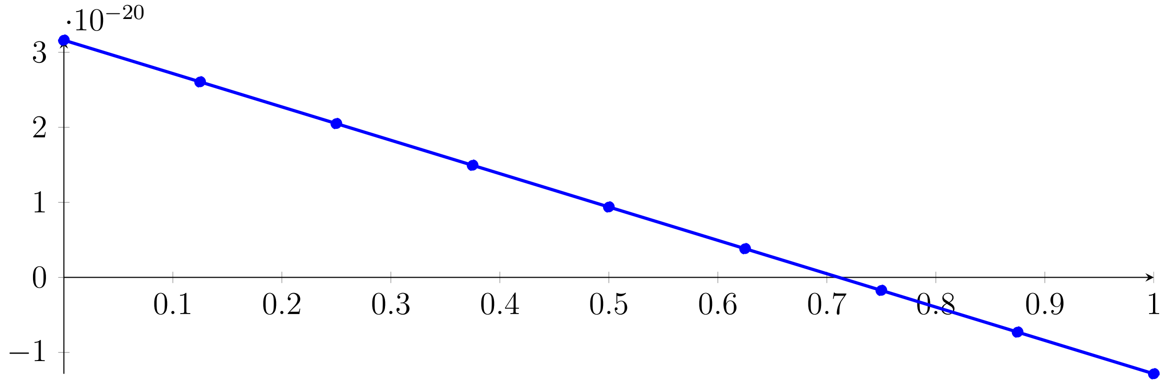
56.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 9!

56.10 Recursion Branch 1 1 1 1 1 1 1 2 in Interval 2: $[0.50000003, 0.5000000300000$

Normalized monomial und Bézier representations and the Bézier polygon:

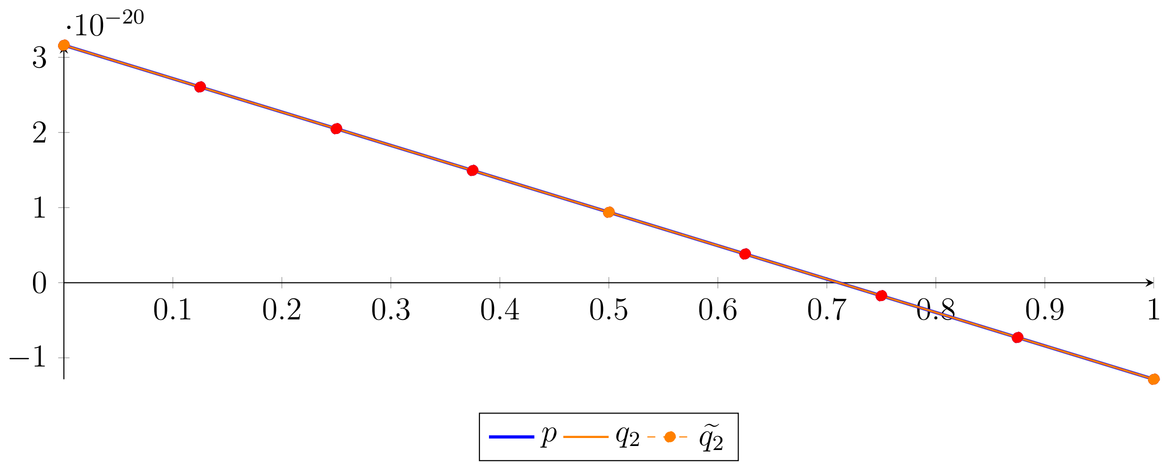
$$\begin{aligned}p &= -2.595099873436077\cdot 10^{-130}X^8 - 1.597531484477648\cdot 10^{-112}X^7 - 4.078240384140717\cdot 10^{-95}X^6\\ &\quad - 5.525920880401843\cdot 10^{-78}X^5 - 4.191391799565194\cdot 10^{-61}X^4 - 1.687408775678226\cdot 10^{-44}X^3\\ &\quad - 2.817152740417101\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}\\ &= 3.161581953700541\cdot 10^{-20}B_{0,8}(X) + 2.605740232211127\cdot 10^{-20}B_{1,8}(X) + 2.049898509715588\\ &\quad \cdot 10^{-20}B_{2,8}(X) + 1.494056786213922\cdot 10^{-20}B_{3,8}(X) + 9.382150617061309\cdot 10^{-21}B_{4,8}(X)\\ &\quad + 3.823733361922135\cdot 10^{-21}B_{5,8}(X) - 1.734683903278299\cdot 10^{-21}B_{6,8}(X)\\ &\quad - 7.293101178539992\cdot 10^{-21}B_{7,8}(X) - 1.285151846386295\cdot 10^{-20}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}\\ &= 3.161581953700541\cdot 10^{-20}B_{0,2} + 9.382150677428867\cdot 10^{-21}B_{1,2} - 1.285151846386295\cdot 10^{-20}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 2.039046071807459\cdot 10^{-323}X^8 - 3.404903576857106\cdot 10^{-323}X^7 - 6.962513967101668\cdot 10^{-323}X^6\\ &\quad + 2.228747497885225\cdot 10^{-322}X^5 - 2.092722637334645\cdot 10^{-322}X^4 + 7.6903440713512\cdot 10^{-323}X^3\\ &\quad - 2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}\\ &= 3.161581953700541\cdot 10^{-20}B_{0,8} + 2.605740232211127\cdot 10^{-20}B_{1,8} + 2.049898509715588\cdot 10^{-20}B_{2,8}\\ &\quad + 1.494056786213922\cdot 10^{-20}B_{3,8} + 9.382150617061309\cdot 10^{-21}B_{4,8} + 3.823733361922135\cdot 10^{-21}B_{5,8}\\ &\quad - 1.734683903278299\cdot 10^{-21}B_{6,8} - 7.293101178539992\cdot 10^{-21}B_{7,8} - 1.285151846386295\cdot 10^{-20}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.437043878391132\cdot 10^{-46}$.
Bounding polynomials M and m:

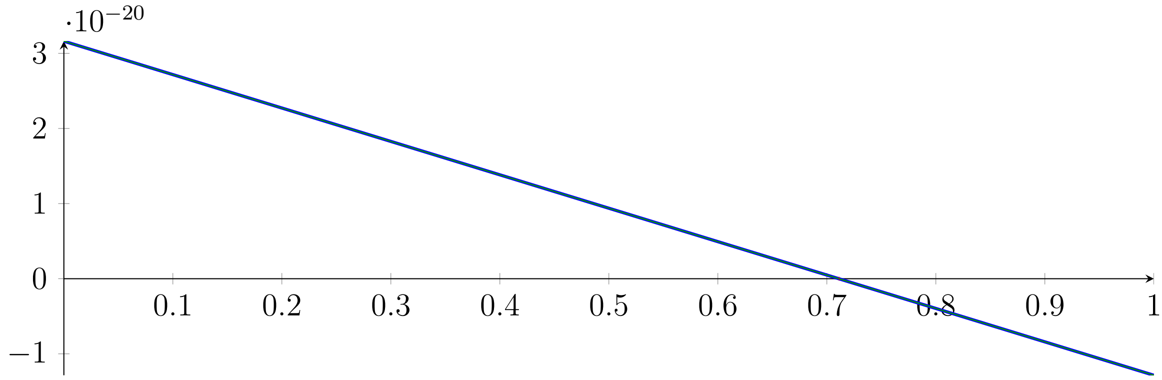
$$M = -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}$$

$$m = -2.817152740417102 \cdot 10^{-28} X^2 - 4.446733771915308 \cdot 10^{-20} X + 3.161581953700541 \cdot 10^{-20}$$

Root of M and m:

$N(M) = \{-157844966.2366053, 0.7109897065184296\}$ $N(m) = \{-157844966.2366053, 0.7109897065184296\}$

Intersection intervals:



$$[0.7109897065184296, 0.7109897065184296]$$

Longest intersection interval: $3.794715058351799 \cdot 10^{-26}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

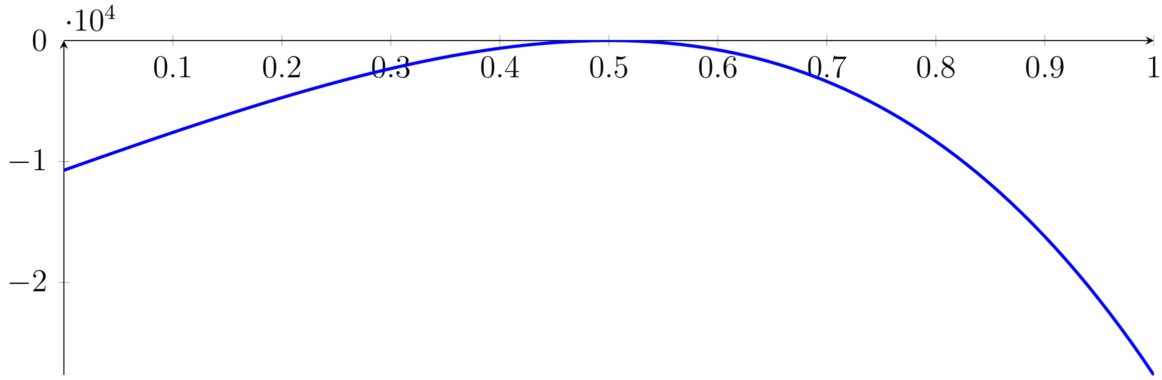
56.11 Recursion Branch 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 9!

56.12 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

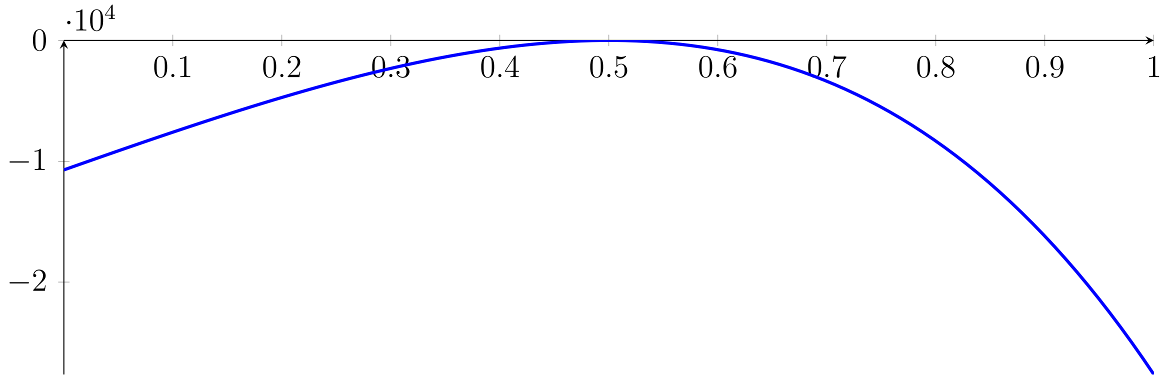
with precision $\varepsilon = 1 \cdot 10^{-32}$.

57 Running CubeClip on h_8 with epsilon 32

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called CubeClip with input polynomial on interval $[0,1]$:

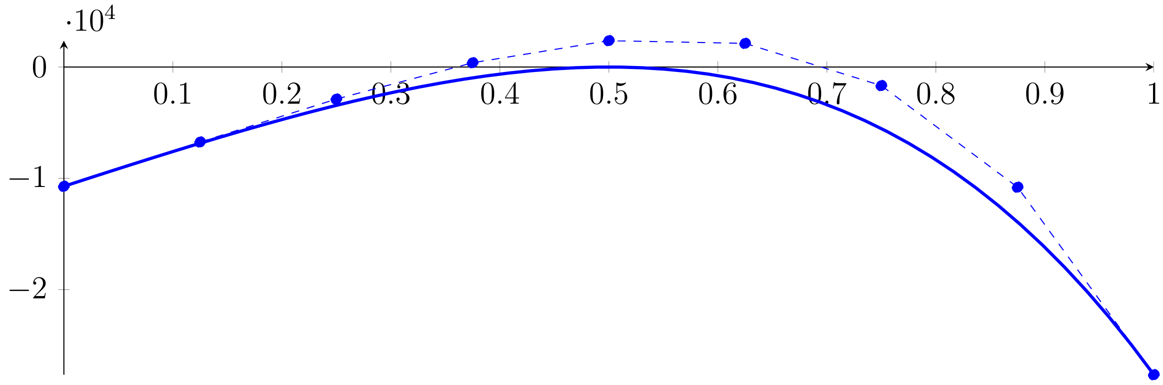
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



57.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

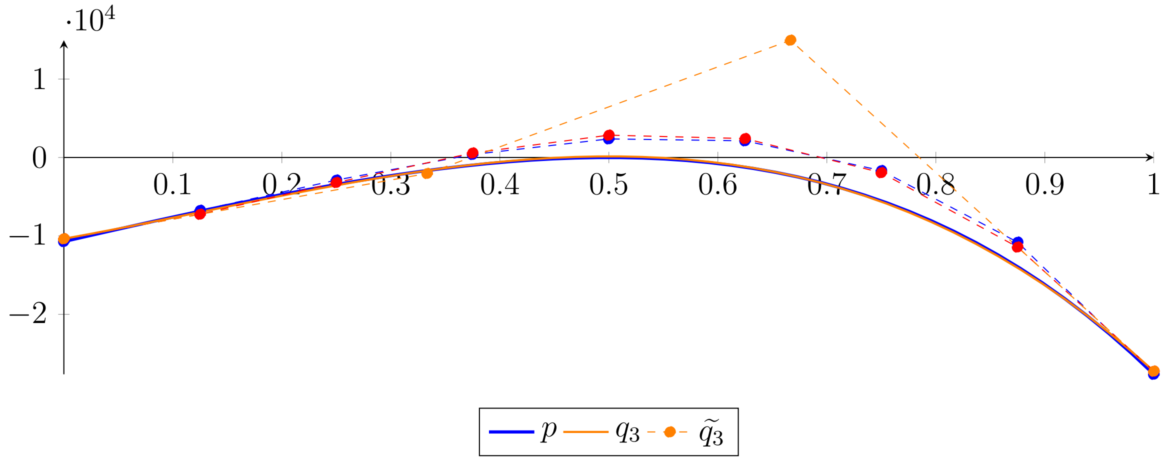
$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ &= -10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &\quad + 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -5.703152099307112\cdot10^{-299}X^8 + 1.997854926822688\cdot10^{-298}X^7 - 2.620079279971631\\ &\quad\cdot10^{-298}X^6 + 1.512761996617561\cdot10^{-298}X^5 - 2.636105166170616\cdot10^{-299}X^4\\ &\quad - 67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554\\ &= -10364.86272786554B_{0,8} - 7243.22568371353B_{1,8} - 3192.719831416224B_{2,8}\\ &\quad + 574.7343076805747B_{3,8} + 2847.216212231063B_{4,8} + 2412.80536088944B_{5,8}\\ &\quad - 1940.418767690097B_{6,8} - 11424.37669485335B_{7,8} - 27250.98894194612B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

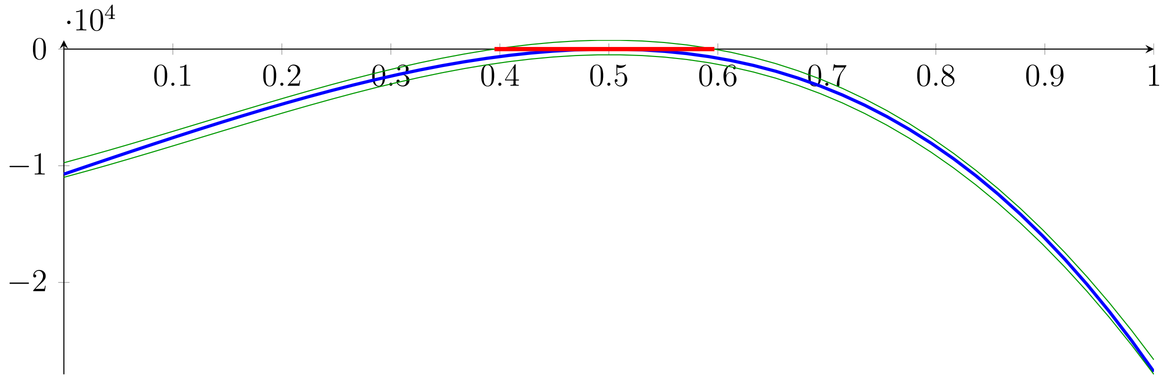
$$M = -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 9740.484261812251$$

$$m = -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $N(m) = \{-0.623487965792376$

Intersection intervals:



$[0.3950604114095972, 0.5968461976869074]$

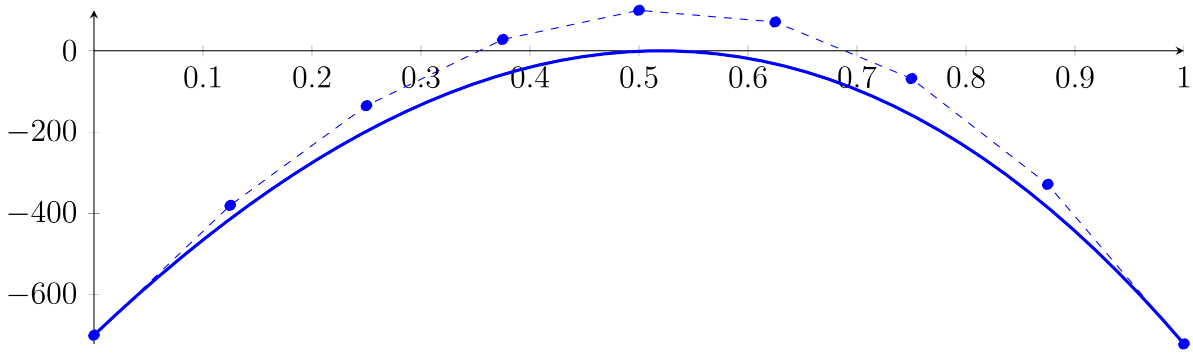
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

57.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

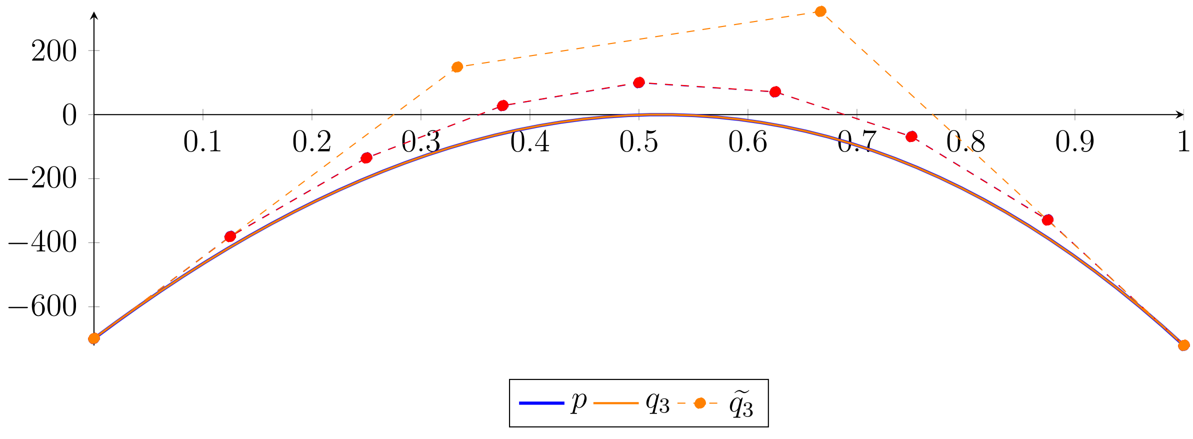
$$\begin{aligned} p = &-2.748682461629333 \cdot 10^{-06} X^8 - 0.0005198138726602274 X^7 - 0.04066637698425721 X^6 \\ &- 1.681520506726176 X^5 - 38.59639865464246 X^4 - 460.2831920090805 X^3 \\ &- 2074.780222477586 X^2 + 2553.796807278985 X - 699.3475151603227 \\ = &-699.3475151603227 B_{0,8}(X) - 380.1229142504495 B_{1,8}(X) - 134.9976070004902 B_{2,8}(X) \\ &+ 27.8090638751075 B_{3,8}(X) + 99.52637853825791 B_{4,8}(X) + 70.80221287533186 B_{5,8}(X) \\ &- 68.32844102535583 B_{6,8}(X) - 328.4764717433994 B_{7,8}(X) - 720.9332304689123 B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 698.7407882409323 \\ &= -698.7407882409323 B_{0,3} + 148.5035278252892 B_{1,3} \\ &\quad + 322.0745267793334 B_{2,3} - 720.3121660793493 B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = &-2.24002635483637 \cdot 10^{-300} X^8 + 8.593407551274415 \cdot 10^{-300} X^7 - 1.289726574039331 \\ &\cdot 10^{-299} X^6 + 9.445830854385802 \cdot 10^{-300} X^5 - 3.303106122422159 \cdot 10^{-300} X^4 \\ &- 542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 698.7407882409323 \\ = &-698.7407882409323 B_{0,8} - 381.0241697160992 B_{1,8} - 135.4868351675709 B_{2,8} \\ &+ 28.18756585642868 B_{3,8} + 100.3153838076753 B_{4,8} + 71.21296913794494 B_{5,8} \\ &- 68.80332770098655 B_{6,8} - 329.4171562573433 B_{7,8} - 720.3121660793493 B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

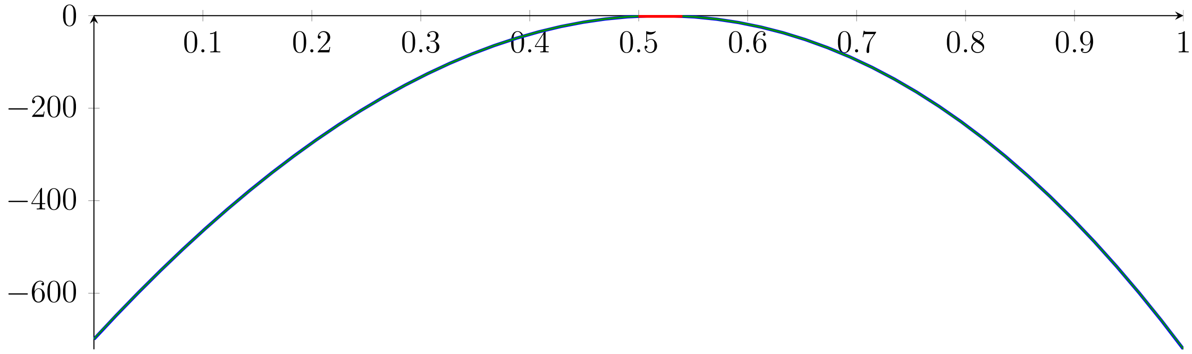
Bounding polynomials M and m:

$$M = -542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 697.8001037269884$$
$$m = -542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 699.6814727548762$$

Root of M and m:

$$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\} \quad N(m) = \{-4.76690070753073\}$$

Intersection intervals:



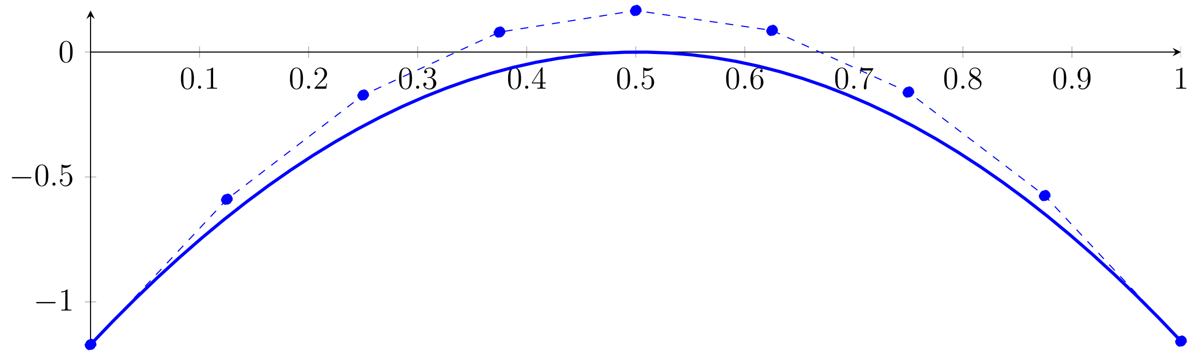
$$[0.4997746332555131, 0.5401382492675976]$$

Longest intersection interval: 0.04036361601208447
$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

57.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Normalized monomial und Bézier representations and the Bézier polygon:

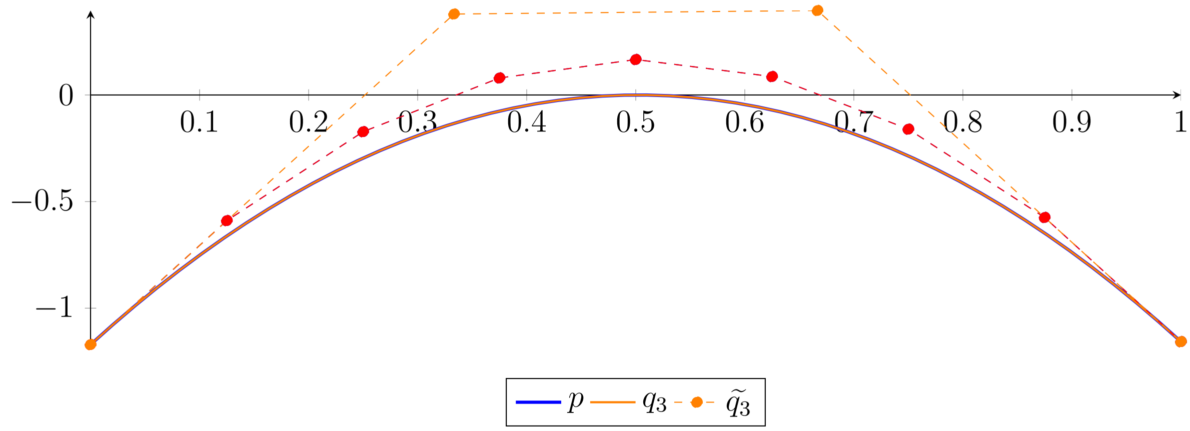
$$\begin{aligned} p = &-1.936623061789877\cdot 10^{-17}X^8 - 9.265403983052604\cdot 10^{-14}X^7 - 1.838110077974511\cdot 10^{-10}X^6 \\ &- 1.935168004397404\cdot 10^{-07}X^5 - 0.0001140127070697086X^4 - 0.0356257658581906X^3 \\ &- 4.602344943530705X^2 + 4.651752639476855X - 1.170857231885768 \\ = &-1.170857231885768B_{0,8}(X) - 0.5893881519511612B_{1,8}(X) - 0.172288534285508B_{2,8}(X) \\ &+ 0.07980544672086649B_{3,8}(X) + 0.1662559879246795B_{4,8}(X) + 0.08642365397403269B_{5,8}(X) \\ &- 0.1603326261538093B_{6,8}(X) - 0.574655562621217B_{7,8}(X) - 1.157189508205582B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661 \\ = &-1.170855596980661B_{0,3} + 0.3797177193764957B_{1,3} \\ &+ 0.3962250709273155B_{2,3} - 1.157187871760247B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = &-3.713970905558003\cdot 10^{-303}X^8 + 1.449155002864004\cdot 10^{-302}X^7 - 2.225710229630734 \\ &\cdot 10^{-302}X^6 + 1.689258272787799\cdot 10^{-302}X^5 - 6.343129302139404\cdot 10^{-303}X^4 \\ &- 0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661 \\ = &-1.170855596980661B_{0,8} - 0.5893906033467271B_{1,8} - 0.1722898202277581B_{2,8} \\ &+ 0.07980649649353122B_{3,8} + 0.1662580909344257B_{4,8} + 0.08642470721221019B_{5,8} \\ &- 0.1603339105558303B_{6,8} - 0.574658018252411B_{7,8} - 1.157187871760247B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.45563119394813 \cdot 10^{-06}$.
Bounding polynomials M and m:

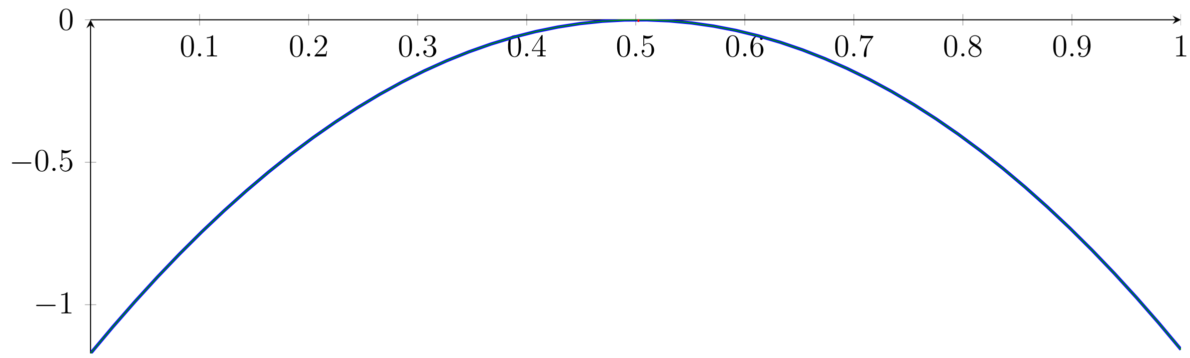
$$M = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170853141349467$$

$$m = -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170858052611855$$

Root of M and m:

$$N(M) = \{-129.3630802555855, 0.5016184355695172, 0.5032421204520033\} \quad N(m) = \{-129.3630802637075\}$$

Intersection intervals:



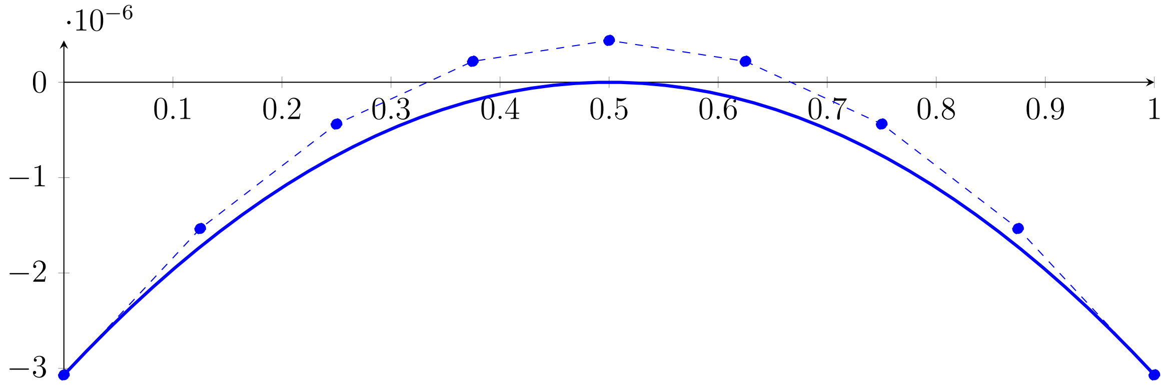
$$[0.5016184355695172, 0.5032421204520033]$$

Longest intersection interval: 0.001623684882486142
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999934125800028, 0.5000066371751187]$,

57.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4999934125800028, 0.50000663717511$

Normalized monomial und Bézier representations and the Bézier polygon:

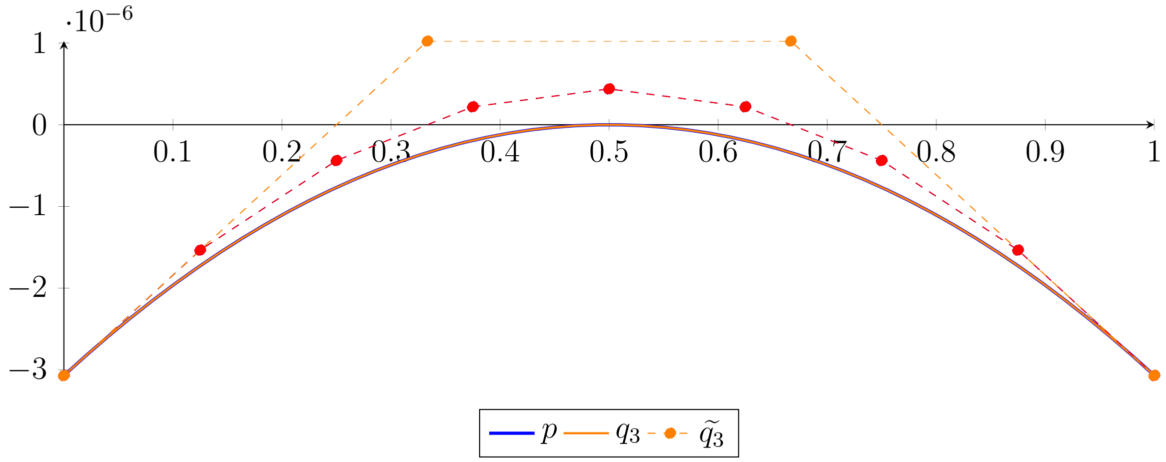
$$\begin{aligned} p = & -9.355329246270906 \cdot 10^{-40} X^8 - 2.758930189698976 \cdot 10^{-33} X^7 - 3.374040776758279 \cdot 10^{-27} X^6 \\ & - 2.190121853313308 \cdot 10^{-21} X^5 - 7.958070257617345 \cdot 10^{-16} X^4 - 1.534811947451426 \cdot 10^{-10} X^3 \\ & - 1.227519762250185 \cdot 10^{-05} X^2 + 1.227554003668957 \cdot 10^{-05} X - 3.068949673836091 \cdot 10^{-06} \\ = & -3.068949673836091 \cdot 10^{-06} B_{0,8}(X) - 1.534507169249895 \cdot 10^{-06} B_{1,8}(X) - 4.384645797530504 \\ & \cdot 10^{-07} B_{2,8}(X) + 2.191753539188221 \cdot 10^{-07} B_{3,8}(X) + 4.384098910187333 \cdot 10^{-07} B_{4,8}(X) \\ & + 2.192362907883254 \cdot 10^{-07} B_{5,8}(X) - 4.383481875421282 \cdot 10^{-07} B_{6,8}(X) \\ & - 1.534346284753723 \cdot 10^{-06} B_{7,8}(X) - 3.068760741638923 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -1.534827863652778 \cdot 10^{-10} X^3 - 1.227519762147866 \cdot 10^{-05} X^2 \\ &\quad + 1.22755400364622 \cdot 10^{-05} X - 3.068949673824723 \cdot 10^{-06} \\ &= -3.068949673824723 \cdot 10^{-06} B_{0,3} + 1.022897004996009 \cdot 10^{-06} B_{1,3} \\ &\quad + 1.023011143323854 \cdot 10^{-06} B_{2,3} - 3.068760741627554 \cdot 10^{-06} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -9.792734715851906 \cdot 10^{-309} X^8 + 3.822699025575463 \cdot 10^{-308} X^7 - 5.87427002023018 \cdot 10^{-308} X^6 \\ &\quad + 4.462726908063245 \cdot 10^{-308} X^5 - 1.679538448591103 \cdot 10^{-308} X^4 - 1.534827863652778 \cdot 10^{-10} X^3 \\ &\quad - 1.227519762147866 \cdot 10^{-05} X^2 + 1.22755400364622 \cdot 10^{-05} X - 3.068949673824723 \cdot 10^{-06} \\ &= -3.068949673824723 \cdot 10^{-06} B_{0,8} - 1.534507169266948 \cdot 10^{-06} B_{1,8} - 4.38464579761983 \cdot 10^{-07} B_{2,8} \\ &\quad + 2.191753539261305 \cdot 10^{-07} B_{3,8} + 4.384098910333502 \cdot 10^{-07} B_{4,8} + 2.192362907956339 \cdot 10^{-07} B_{5,8} \\ &\quad - 4.383481875510608 \cdot 10^{-07} B_{6,8} - 1.534346284770776 \cdot 10^{-06} B_{7,8} - 3.068760741627554 \cdot 10^{-06} B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.705314892341177 \cdot 10^{-17}$.
Bounding polynomials M and m:

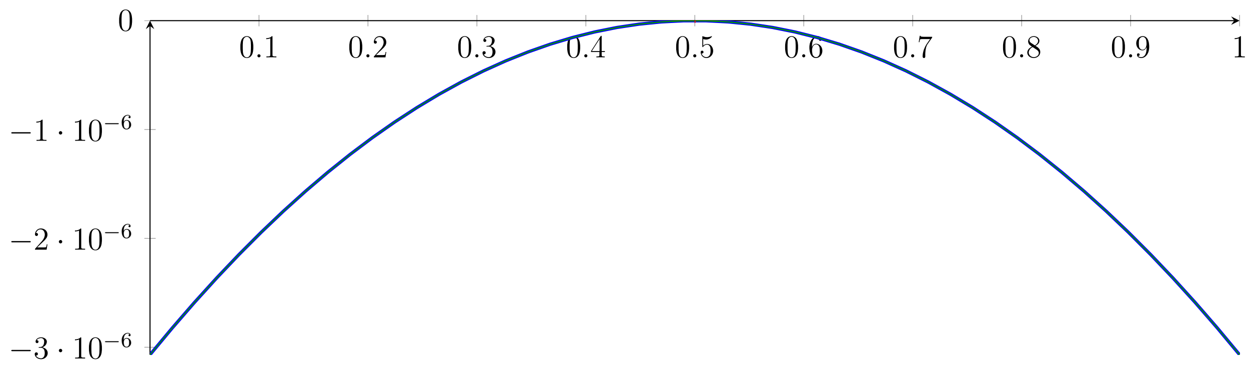
$$\begin{aligned} M &= -1.534827863652778 \cdot 10^{-10} X^3 - 1.227519762147866 \cdot 10^{-05} X^2 \\ &\quad + 1.22755400364622 \cdot 10^{-05} X - 3.068949673807669 \cdot 10^{-06} \end{aligned}$$

$$\begin{aligned} m &= -1.534827863652778 \cdot 10^{-10} X^3 - 1.227519762147866 \cdot 10^{-05} X^2 \\ &\quad + 1.22755400364622 \cdot 10^{-05} X - 3.068949673841776 \cdot 10^{-06} \end{aligned}$$

Root of M and m:

$N(M) = \{-79978.68293772447, 0.4996311727354866, 0.5003873441571286\}$ $\quad N(m) = \{-79978.68293772447,$

Intersection intervals:



$$[0.4996311727354866, 0.4996311764098305],\ [0.5003873404827848, 0.5003873441571286]$$

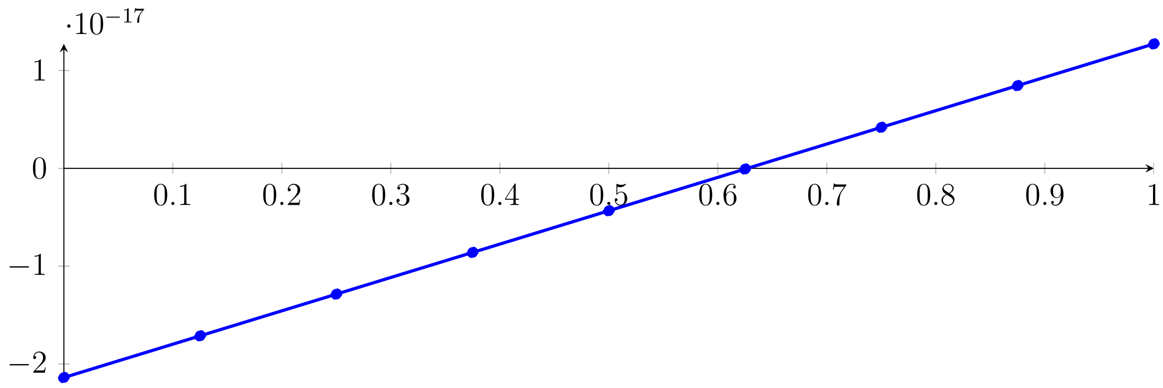
Longest intersection interval: $3.674343879435088 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999695, 0.5000000200000181]$, interval 2: $[0.500000029999981$

57.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.5000000199999695, 0.500000020000$

Normalized monomial und Bézier representations and the Bézier polygon:

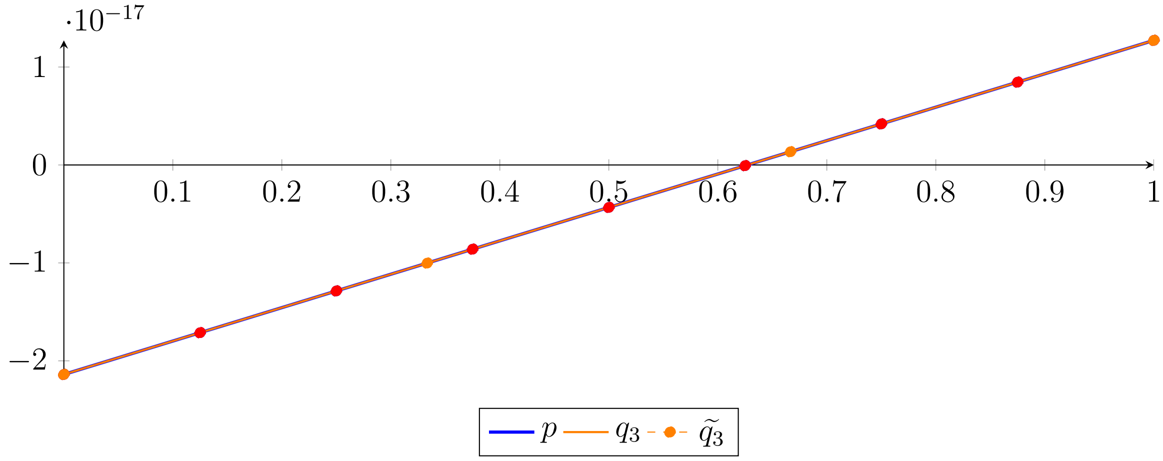
$$\begin{aligned}
p = & -3.10811779683933 \cdot 10^{-107} X^8 - 2.494594121391514 \cdot 10^{-92} X^7 - 8.302910645986526 \cdot 10^{-78} X^6 \\
& - 1.466794525552917 \cdot 10^{-63} X^5 - 1.450540809134678 \cdot 10^{-49} X^4 - 7.613758006193666 \cdot 10^{-36} X^3 \\
& - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
= & -2.138639484655546 \cdot 10^{-17} B_{0,8}(X) - 1.71230868147238 \cdot 10^{-17} B_{1,8}(X) - 1.285978470175393 \\
& \cdot 10^{-17} B_{2,8}(X) - 8.596488507645843 \cdot 10^{-18} B_{3,8}(X) - 4.333198232399549 \\
& \cdot 10^{-18} B_{4,8}(X) - 6.991387601504461 \cdot 10^{-20} B_{5,8}(X) + 4.19336456150767 \cdot 10^{-18} B_{6,8}(X) \\
& + 8.456637080168595 \cdot 10^{-18} B_{7,8}(X) + 1.271990367996773 \cdot 10^{-17} B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = & -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 \\
& + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
= & -2.138639484655546 \cdot 10^{-17} B_{0,3} - 1.001757342833769 \cdot 10^{-17} B_{1,3} \\
& + 1.351192747170036 \cdot 10^{-18} B_{2,3} + 1.271990367996773 \cdot 10^{-17} B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 = & -3.883510371826523 \cdot 10^{-321} X^8 + 2.549502248952292 \cdot 10^{-320} X^7 - 4.188626787236358 \cdot 10^{-320} X^6 \\
& + 1.055694768751284 \cdot 10^{-320} X^5 + 2.46739471443405 \cdot 10^{-320} X^4 - 7.613758006193956 \cdot 10^{-36} X^3 \\
& - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
= & -2.138639484655546 \cdot 10^{-17} B_{0,8} - 1.71230868147238 \cdot 10^{-17} B_{1,8} - 1.285978470175393 \cdot 10^{-17} B_{2,8} \\
& - 8.596488507645843 \cdot 10^{-18} B_{3,8} - 4.333198232399549 \cdot 10^{-18} B_{4,8} - 6.991387601504461 \cdot 10^{-20} B_{5,8} \\
& + 4.19336456150767 \cdot 10^{-18} B_{6,8} + 8.456637080168595 \cdot 10^{-18} B_{7,8} + 1.271990367996773 \cdot 10^{-17} B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301733860119 \cdot 10^{-51}$.
Bounding polynomials M and m:

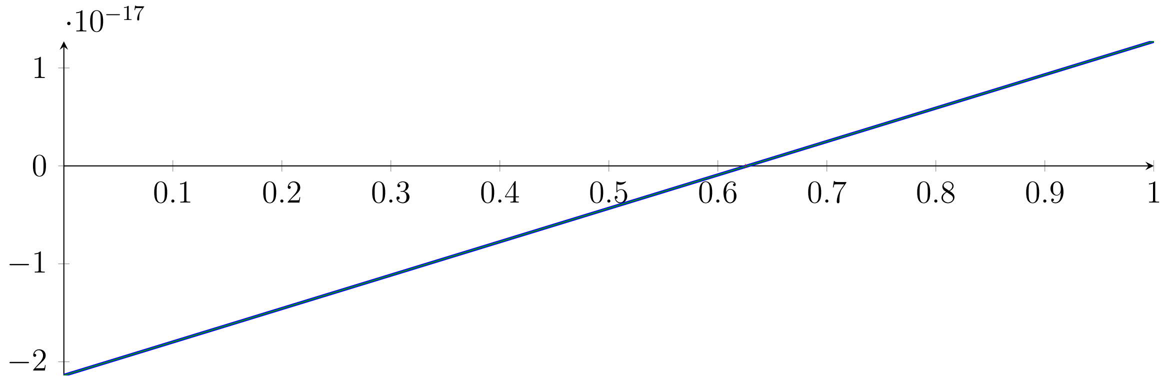
$$M = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

$$m = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929227157.35, 0.6249980054152174, 205797.0483973646\}$ $N(m) = \{-21766929227157.35, 0$

Intersection intervals:



$$[0.6249980054152174, 0.6249980054152174]$$

Longest intersection interval: $1.822716451808538 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000199999999]$,

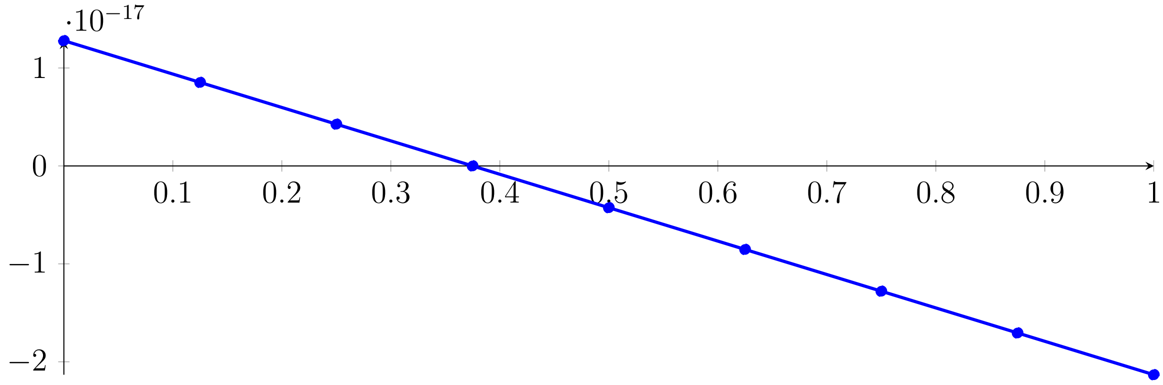
57.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.50000001999$

Reached interval $[0.5000000199999999, 0.5000000199999999]$ **without sign change** at depth 6!
p(0) = -6.998745276936609e-20 - p(1) -6.998745276936609e-20

57.7 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.5000000299999818, 0.500000030000$

Normalized monomial und Bézier representations and the Bézier polygon:

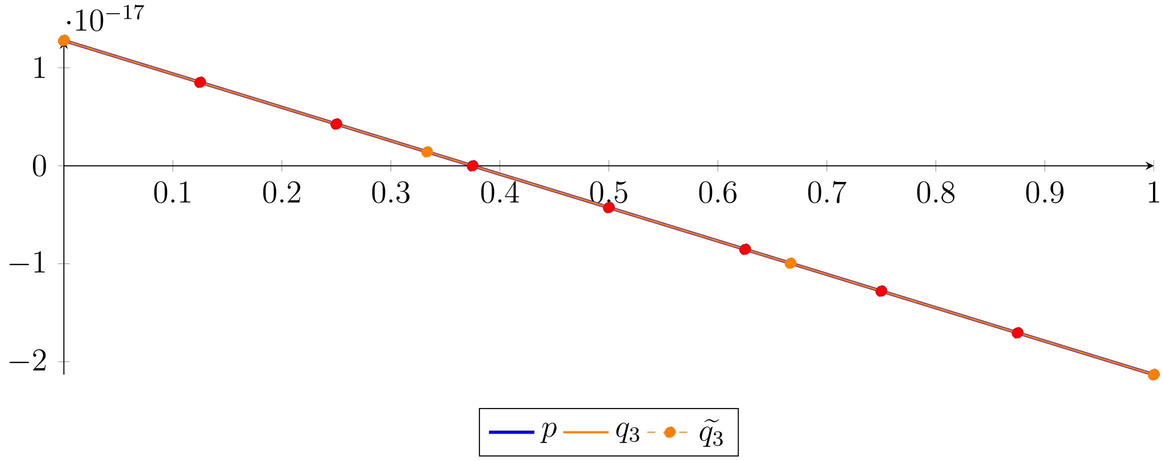
$$
\begin{aligned}
p &= -3.108117561752323\cdot 10^{-107}X^8 - 2.494593961411665\cdot 10^{-92}X^7 - 8.302910210921025\cdot 10^{-78}X^6 \\
&\quad - 1.466794466465724\cdot 10^{-63}X^5 - 1.450540769370867\cdot 10^{-49}X^4 - 7.613757909646128\cdot 10^{-36}X^3 \\
&\quad - 1.657281316735189\cdot 10^{-22}X^2 - 3.410613211820657\cdot 10^{-17}X + 1.27898932402398\cdot 10^{-17} \\
&= 1.27898932402398\cdot 10^{-17}B_{0,8}(X) + 8.526626725463981\cdot 10^{-18}B_{1,8}(X) + 4.263354291826315 \\
&\quad \cdot 10^{-18}B_{2,8}(X) + 7.593932680254395\cdot 10^{-23}B_{3,8}(X) - 4.263208332034555\cdot 10^{-18}B_{4,8}(X) \\
&\quad - 8.526498522257758\cdot 10^{-18}B_{5,8}(X) - 1.278979463134281\cdot 10^{-17}B_{6,8}(X) \\
&\quad - 1.70530966592897\cdot 10^{-17}B_{7,8}(X) - 2.131640460609844\cdot 10^{-17}B_{8,8}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= -7.613757909646418\cdot 10^{-36}X^3 - 1.657281316735189\cdot 10^{-22}X^2 \\
&\quad - 3.410613211820657\cdot 10^{-17}X + 1.27898932402398\cdot 10^{-17} \\
&= 1.27898932402398\cdot 10^{-17}B_{0,3} + 1.421182534170946\cdot 10^{-18}B_{1,3} \\
&\quad - 9.947583414608468\cdot 10^{-18}B_{2,3} - 2.131640460609844\cdot 10^{-17}B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 &= -6.944710234356022\cdot 10^{-322}X^8 - 5.839855933843534\cdot 10^{-321}X^7 + 4.132859127462027\cdot 10^{-320}X^6 \\
&\quad - 7.976936884929846\cdot 10^{-320}X^5 + 6.21874252599804\cdot 10^{-320}X^4 - 7.613757909646418\cdot 10^{-36}X^3 \\
&\quad - 1.657281316735189\cdot 10^{-22}X^2 - 3.410613211820657\cdot 10^{-17}X + 1.27898932402398\cdot 10^{-17} \\
&= 1.27898932402398\cdot 10^{-17}B_{0,8} + 8.526626725463981\cdot 10^{-18}B_{1,8} + 4.263354291826315\cdot 10^{-18}B_{2,8} \\
&\quad + 7.593932680254395\cdot 10^{-23}B_{3,8} - 4.263208332034555\cdot 10^{-18}B_{4,8} - 8.526498522257758\cdot 10^{-18}B_{5,8} \\
&\quad - 1.278979463134281\cdot 10^{-17}B_{6,8} - 1.70530966592897\cdot 10^{-17}B_{7,8} - 2.131640460609844\cdot 10^{-17}B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301648651952 \cdot 10^{-51}$.
Bounding polynomials M and m:

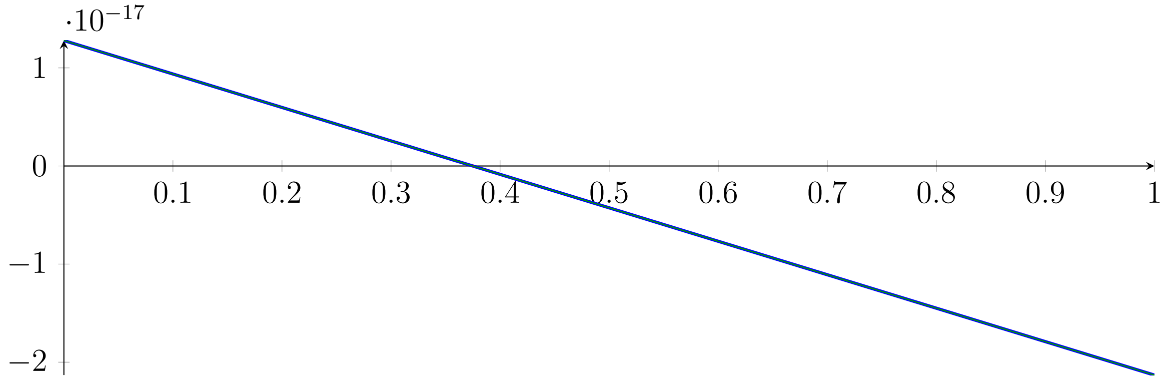
$$M = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 \\ - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

$$m = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 \\ - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929297379.88, -205796.0503430093, 0.375002063855762\}$ $N(m) = \{-21766929297379.88,$

Intersection intervals:



$$[0.375002063855762, 0.375002063855762]$$

Longest intersection interval: $1.822716401842262 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

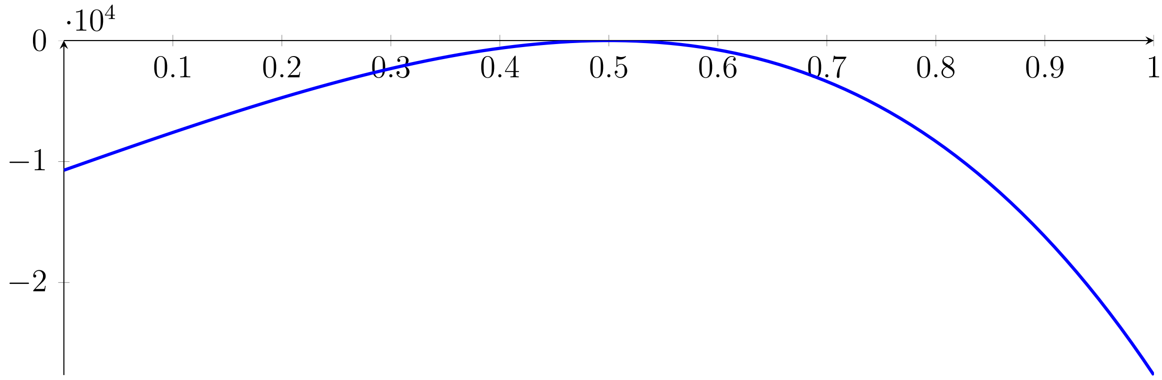
57.8 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 6!

57.9 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000003, 0.50000003]$$

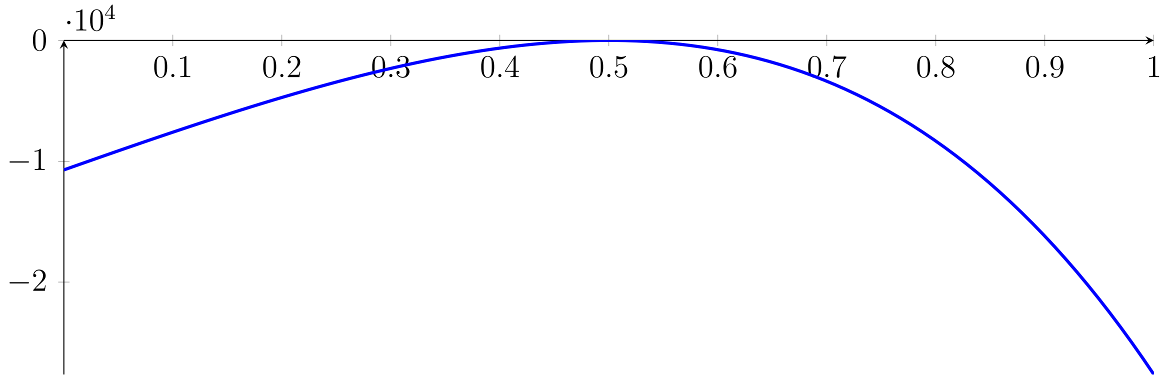
with precision $\varepsilon = 1 \cdot 10^{-32}$.

58 Running BezClip on h_8 with epsilon 64

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called BezClip with input polynomial on interval $[0, 1]$:

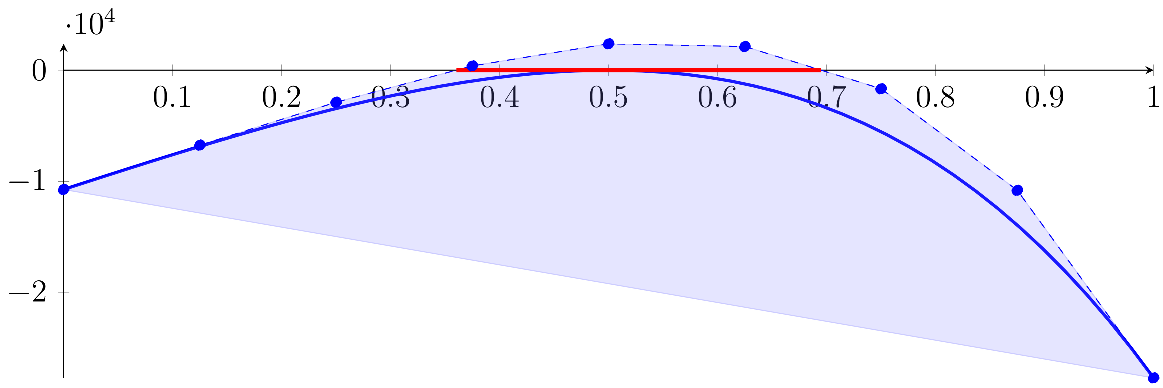
$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



58.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

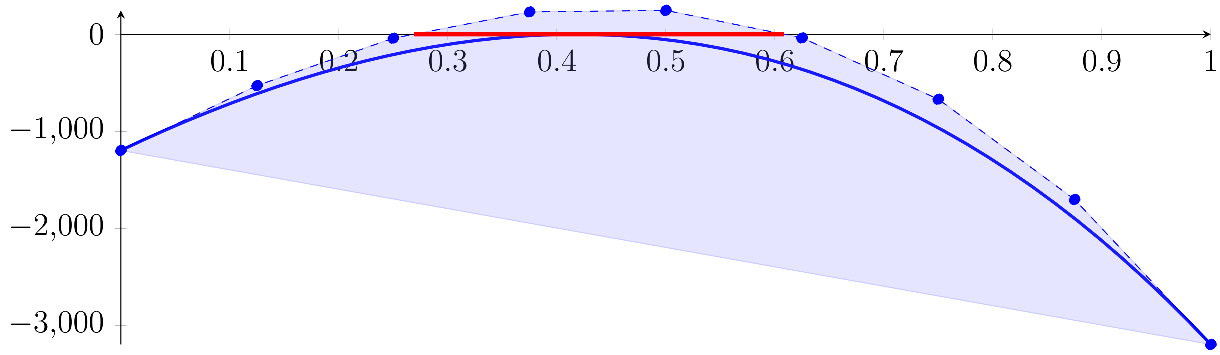
$$[0.3603640692588709, 0.6949437907012195]$$

Longest intersection interval: 0.3345797214423486
$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

58.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6 \\
& - 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3 \\
& - 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098 \\
= & -1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X) \\
& + 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X) \\
& - 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

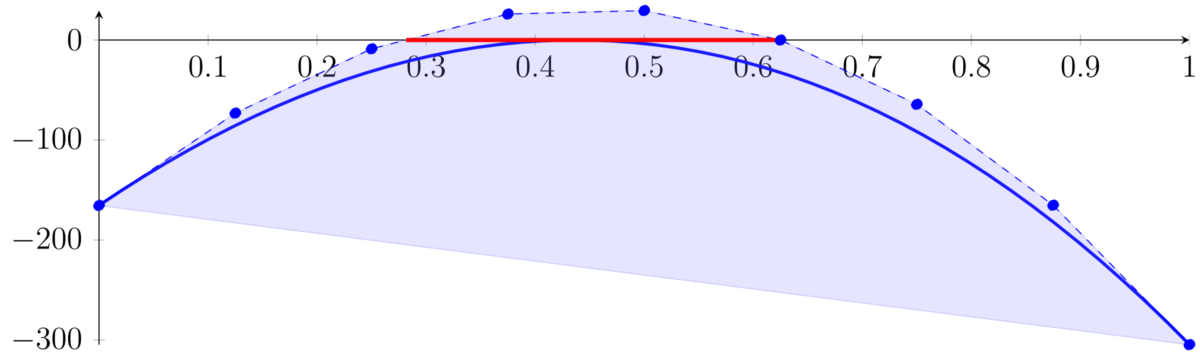
$$[0.2687909235445741, 0.6084084634355219]$$

Longest intersection interval: 0.3396175398909478
$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

58.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -2.779187295559603 \cdot 10^{-08}X^8 - 9.441522673365855 \cdot 10^{-06}X^7 - 0.001328615938017263X^6 \\
& - 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3 \\
& - 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987 \\
= & -165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X) \\
& + 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X) \\
& - 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

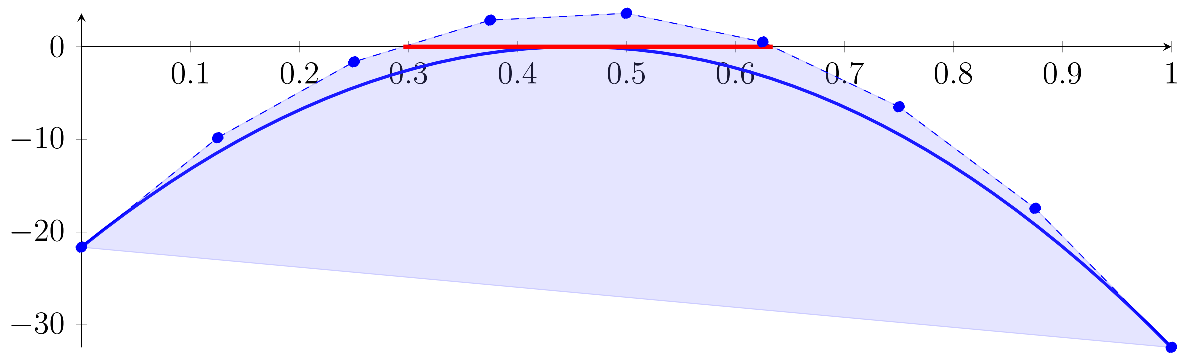
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\Longrightarrow$ Selective recursion: interval 1: $[0.4822990094256734, 0.5212832145711177]$,

58.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4822990094256734, 0.52128321457111$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -5.334693469973702 \cdot 10^{-12} X^8 - 5.317476896558791 \cdot 10^{-09} X^7 - 2.197128069269196 \cdot 10^{-06} X^6 \\ & - 0.000481521724173462 X^5 - 0.05899466848037266 X^4 - 3.82353929224187 X^3 \\ & - 101.3899724153744 X^2 + 94.46052284159652 X - 21.62667247491518 \\ = & -21.62667247491518 B_{0,8}(X) - 9.819107119715613 B_{1,8}(X) - 1.632612207922276 B_{2,8}(X) \\ & + 2.86453477310337 B_{3,8}(X) + 3.603213555021573 B_{4,8}(X) + 0.5134524899120698 B_{5,8}(X) \\ & - 6.475580126796985 B_{6,8}(X) - 17.43558481253364 B_{7,8}(X) - 32.43913973359036 B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

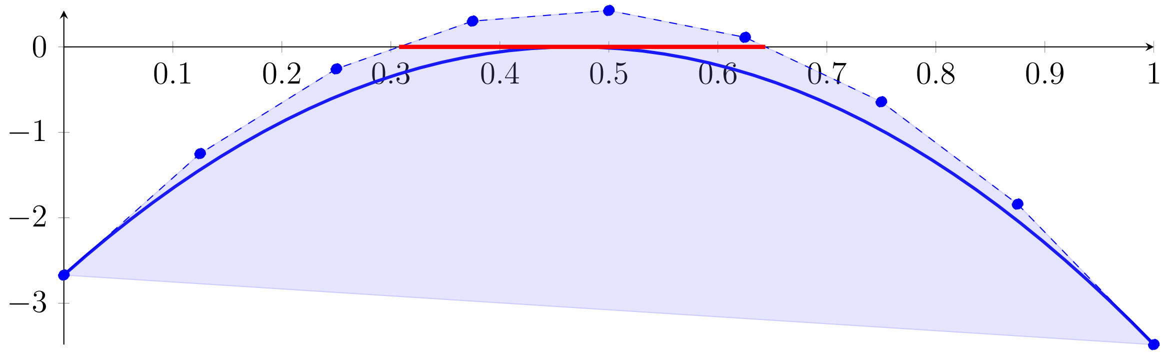
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4938141292321744, 0.5070221367077007]$,

58.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4938141292321744, 0.507022136707$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -9.261865042605128 \cdot 10^{-16} X^8 - 2.731330938941274 \cdot 10^{-12} X^7 - 3.339777336661422 \\ & \cdot 10^{-09} X^6 - 2.167033369334019 \cdot 10^{-06} X^5 - 0.0007867416616259877 X^4 \\ & - 0.1514273448103067 X^3 - 12.0308548973337 X^2 + 11.3691349552019 X - 2.67015106585462 \\ = & -2.67015106585462 B_{0,8}(X) - 1.249009196454382 B_{1,8}(X) - 0.2575407162446335 B_{2,8}(X) \\ & + 0.3015503150458701 B_{3,8}(X) + 0.4255485985217787 B_{4,8}(X) + 0.1117275574241232 B_{5,8}(X) \\ & - 0.6426507016859872 B_{6,8}(X) - 1.840335427863286 B_{7,8}(X) - 3.484087264834231 B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

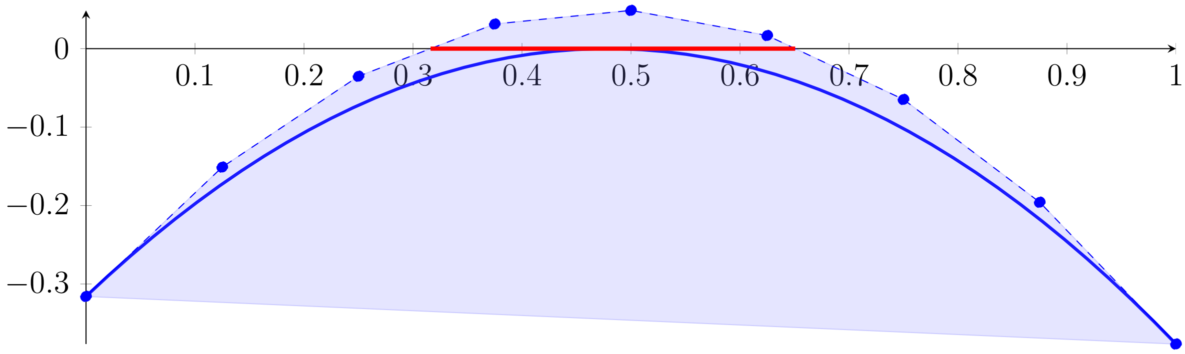
$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\Longrightarrow$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

58.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.502170909909546 \cdot 10^{-19} X^8 - 1.319790000383349 \cdot 10^{-15} X^7 - 4.808366303277033 \cdot 10^{-12} X^6 \\
&- 9.297436410097437 \cdot 10^{-09} X^5 - 1.006192361000331 \cdot 10^{-05} X^4 - 0.005777437133356019 X^3 \\
&- 1.373512346650201 X^2 + 1.318590421802199 X - 0.3158295498425207 \\
= &-0.3158295498425207 B_{0,8}(X) - 0.1510057471172458 B_{1,8}(X) - 0.03523595677233526 B_{2,8}(X) \\
&+ 0.03137665267197247 B_{3,8}(X) + 0.04872876895367301 B_{4,8}(X) + 0.01671693590297049 B_{5,8}(X) \\
&- 0.06476244672391982 B_{6,8}(X) - 0.1958131234111408 B_{7,8}(X) - 0.376538983049735 B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3161210337394805, 0.6506459600024226\}$$

Intersection intervals with the x axis:

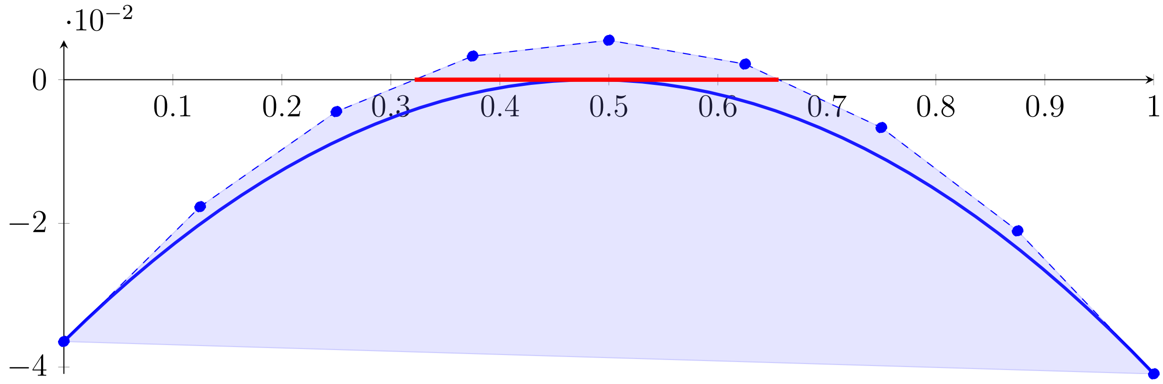
$$[0.3161210337394805, 0.6506459600024226]$$

Longest intersection interval: 0.3345249262629421
$\Longrightarrow$ Selective recursion: interval 1: $[0.499279281802493, 0.5007635705740208]$,

58.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.499279281802493, 0.5007635705$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.355847730899963 \cdot 10^{-23} X^8 - 6.189124375986835 \cdot 10^{-19} X^7 - 6.742674354644286 \cdot 10^{-15} X^6 - 3.898805488577673 \cdot 10^{-11} X^5 - 1.261912061383275 \cdot 10^{-07} X^4 - 0.000216758797755982 X^3 - 0.154319370565961 X^2 + 0.1500226550818807 X - 0.0364365303427455$$
$$= -0.0364365303427455 B_{0,8}(X) - 0.01768369845751041 B_{1,8}(X) - 0.004442272663916792 B_{2,8}(X) + 0.003283876345218297 B_{3,8}(X) + 0.005490876074346265 B_{4,8}(X) + 0.002174852224490793 B_{5,8}(X) - 0.006668071307448625 B_{6,8}(X) - 0.02104177242939338 B_{7,8}(X) - 0.04095013085478269 B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3218707447051634, 0.6557428337562166\}$$

Intersection intervals with the x axis:

$$[0.3218707447051634, 0.6557428337562166]$$

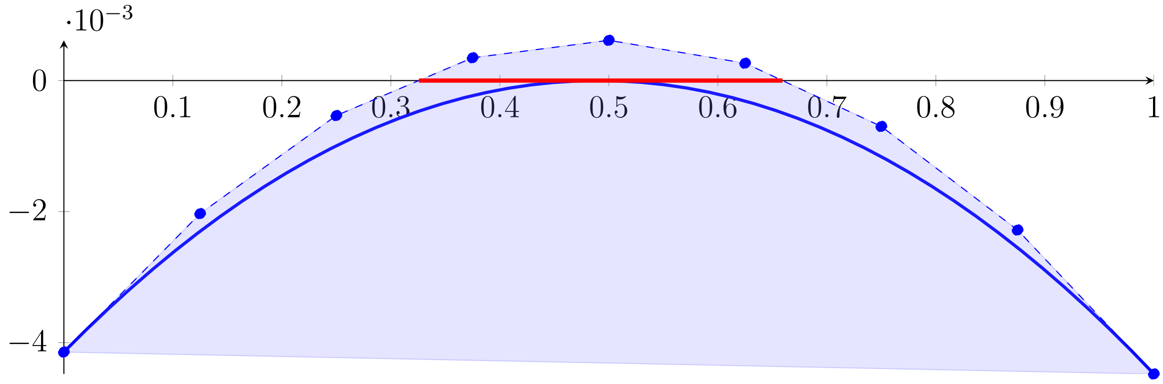
Longest intersection interval: 0.3338720890510532

$\implies$ Selective recursion: interval 1: $[0.4997570309347421, 0.5002525935276471]$,

58.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4997570309347421, 0.5002525$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.637375463877353 \cdot 10^{-27} X^8 - 2.862414855005092 \cdot 10^{-22} X^7 - 9.341200314035287 \cdot 10^{-18} X^6 - 1.617995026179791 \cdot 10^{-13} X^5 - 1.568792391901099 \cdot 10^{-09} X^4 - 8.073141350089629 \cdot 10^{-06} X^3 - 0.01722540857155108 X^2 + 0.01689843085777645 X - 0.004143462623424353$$
$$= -0.004143462623424353 B_{0,8}(X) - 0.002031158766202297 B_{1,8}(X) - 0.0005340480722499233 B_{2,8}(X) + 0.0003477252951943749 B_{3,8}(X) + 0.0006140171504808826 B_{4,8}(X) + 0.0002646832855456765 B_{5,8}(X) - 0.0007004205300922658 B_{6,8}(X) - 0.002281438549333955 B_{7,8}(X) - 0.004478515047503282 B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3257065380923488, 0.6592817116222256\}$$

Intersection intervals with the x axis:

$$[0.3257065380923488, 0.6592817116222256]$$

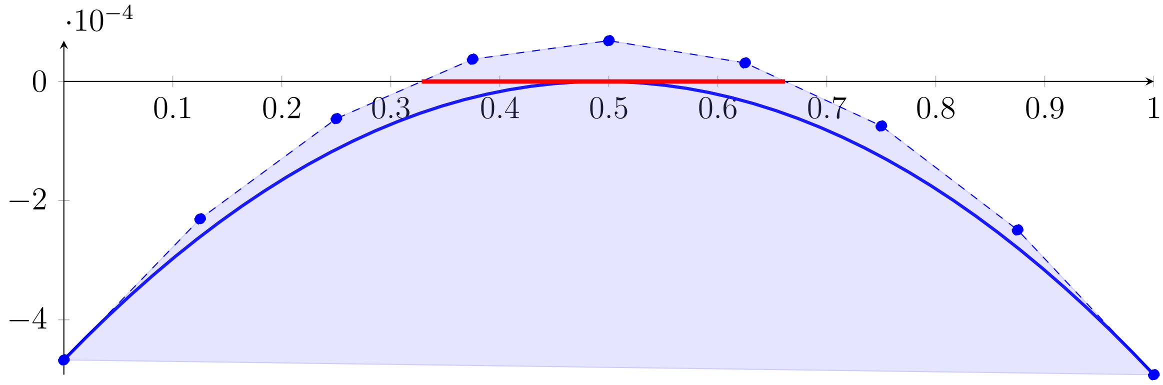
Longest intersection interval: 0.3335751735298769

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999184389112853, 0.5000837462892085]$,

58.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999184389112853, 0.50008$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -5.57619429667528\cdot 10^{-31}X^8 - 1.315536799505273\cdot 10^{-25}X^7 - 1.287049795843581\cdot 10^{-20}X^6\\ &\quad - 6.683358895768893\cdot 10^{-16}X^5 - 1.942733814710911\cdot 10^{-11}X^4 - 2.997323807488328\\ &\quad \cdot 10^{-07}X^3 - 0.001917590365692151X^2 + 0.001893040758741533X - 0.0004671653183669499\\ &= -0.0004671653183669499B_{0,8}(X) - 0.0002305352235242582B_{1,8}(X) - 6.239049888485767\\ &\quad \cdot 10^{-05}B_{2,8}(X) + 3.726350318730984\cdot 10^{-05}B_{3,8}(X) + 6.842143005076897\\ &\quad \cdot 10^{-05}B_{4,8}(X) + 3.107792878649904\cdot 10^{-05}B_{5,8}(X) - 7.47723538020779\\ &\quad \cdot 10^{-05}B_{6,8}(X) - 0.000249134771189109B_{7,8}(X) - 0.0004920146771263226B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3282588977707033, 0.661700337526842\}$$

Intersection intervals with the x axis:

$$[0.3282588977707033, 0.661700337526842]$$

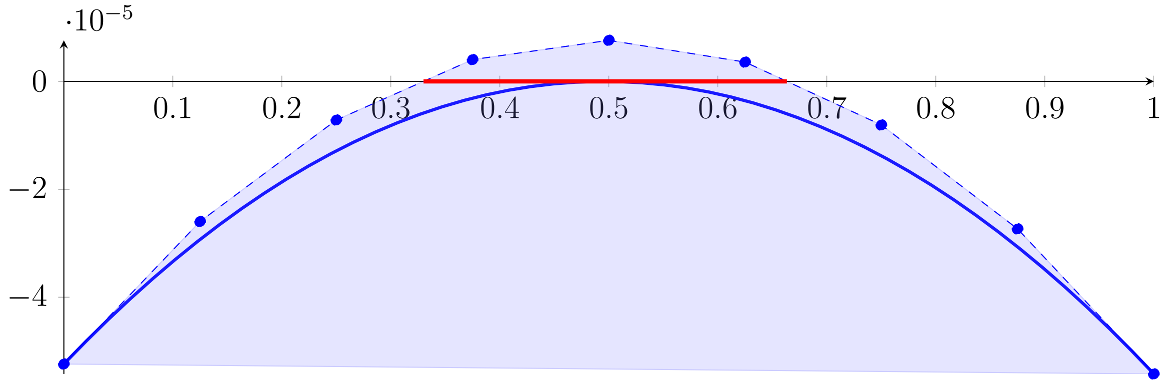
Longest intersection interval: 0.3334414397561387

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999727025289557, 0.5000278228590528]$,

58.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999727025289557, 0.500$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.521076768712742\cdot 10^{-35}X^8 - 6.02899387826599\cdot 10^{-29}X^7 - 1.76898025411792\cdot 10^{-23}X^6\\ &\quad - 2.754920799910016\cdot 10^{-18}X^5 - 2.401685361883718\cdot 10^{-13}X^4 - 1.111294950492419\cdot 10^{-08}X^3\\ &\quad - 0.0002132366404355359X^2 + 0.0002114057621846039X - 5.239629569054843\cdot 10^{-05}\\ &= -5.239629569054843\cdot 10^{-05}B_{0,8}(X) - 2.597057541747294\cdot 10^{-05}B_{1,8}(X) - 7.160449445666593\\ &\quad \cdot 10^{-06}B_{2,8}(X) + 4.033883779343745\cdot 10^{-06}B_{3,8}(X) + 7.612225808600218\cdot 10^{-06}B_{4,8}(X)\\ &\quad + 3.574378189713946\cdot 10^{-06}B_{5,8}(X) - 8.079857533135031\cdot 10^{-06}B_{6,8}(X)\\ &\quad - 2.73506798191978\cdot 10^{-05}B_{7,8}(X) - 5.423828713115661\cdot 10^{-05}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3299561852159799, 0.6633377584201652\}$$

Intersection intervals with the x axis:

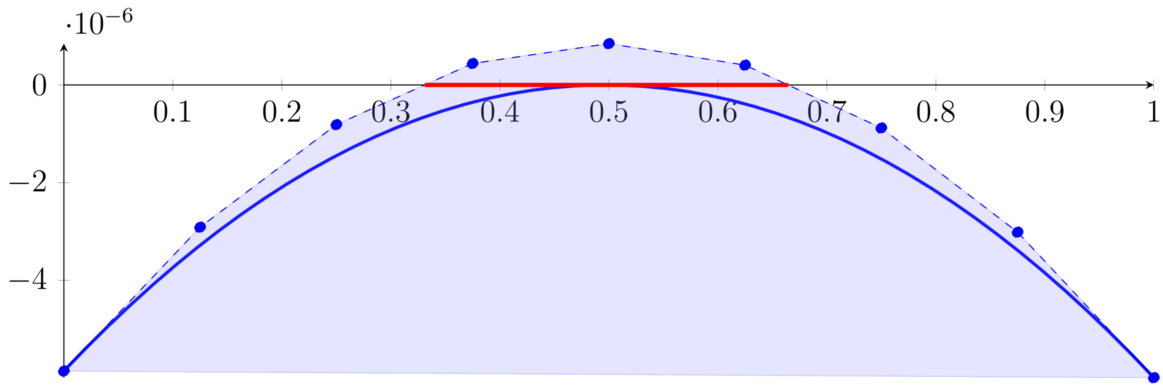
$$[0.3299561852159799, 0.6633377584201652]$$

Longest intersection interval: 0.3333815732041853
$\Longrightarrow$ Selective recursion: interval 1: [0.4999908898228024, 0.5000092659251657],

58.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999908898228024, 0.5

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.300251044438937\cdot 10^{-38}X^8 - 2.759545789497528\cdot 10^{-32}X^7 - 2.428711637904422\cdot 10^{-26}X^6\\&- 1.134547275299063\cdot 10^{-20}X^5 - 2.966816573305973\cdot 10^{-15}X^4 - 4.117811892390909\cdot 10^{-10}X^3\\&- 2.370104084998438\cdot 10^{-05}X^2 + 2.356495503794524\cdot 10^{-05}X - 5.857360304639568\cdot 10^{-06}\\= &-5.857360304639568\cdot 10^{-06}B_{0,8}(X) - 2.911740924896412\cdot 10^{-06}B_{1,8}(X) - 8.125872897955564\\&\cdot 10^{-07}B_{2,8}(X) + 4.400932474274781\cdot 10^{-07}B_{3,8}(X) + 8.462933334947859\cdot 10^{-07}B_{4,8}(X)\\&+ 4.060056150860783\cdot 10^{-07}B_{5,8}(X) - 8.807772611613165\cdot 10^{-07}B_{6,8}(X)\\&- 3.014062648652454\cdot 10^{-06}B_{7,8}(X) - 5.993857900834775\cdot 10^{-06}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.331084848216461, 0.6644399885346334\}$$

Intersection intervals with the x axis:

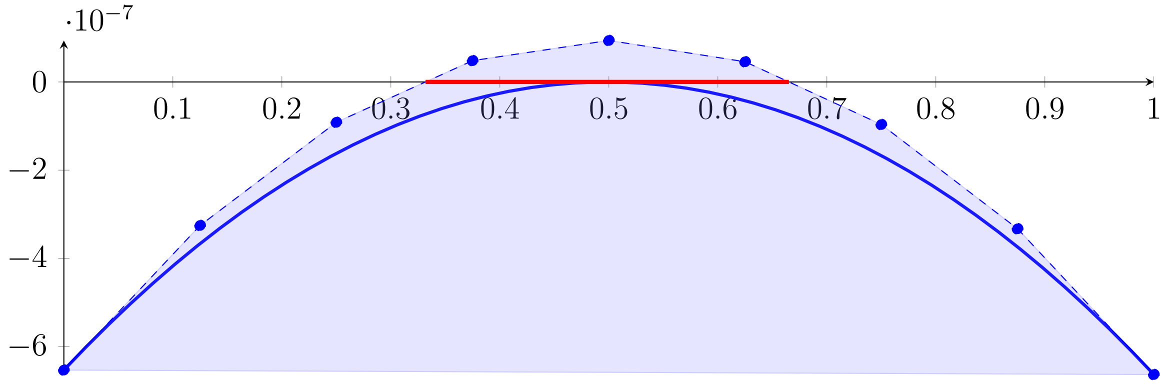
$$[0.331084848216461, 0.6644399885346334]$$

Longest intersection interval: 0.3333551403181724
$\Longrightarrow$ Selective recursion: interval 1: [0.4999969738718641, 0.500003099640046],

58.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999969738718641, 0

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.982825346126094\cdot10^{-42}X^8 - 1.262374580662423\cdot10^{-35}X^7 - 3.332882748505763\cdot10^{-29}X^6\\ &- 4.670466114941116\cdot10^{-23}X^5 - 3.663718276431707\cdot10^{-17}X^4 - 1.52542941384219\cdot10^{-11}X^3\\ &- 2.633839011095458\cdot10^{-06}X^2 + 2.623741169043712\cdot10^{-06}X - 6.534168705792709\cdot10^{-07}\\ = &-6.534168705792709\cdot10^{-07}B_{0,8}(X) - 3.254492244488069\cdot10^{-07}B_{1,8}(X) - 9.154725728603783\\ &\cdot10^{-08}B_{2,8}(X) + 4.828875851092667\cdot10^{-08}B_{3,8}(X) + 9.405855054345363\cdot10^{-08}B_{4,8}(X)\\ &+ 4.576184641238665\cdot10^{-08}B_{5,8}(X) - 9.660162628195405\cdot10^{-08}B_{6,8}(X)\\ &- 3.330321399397716\cdot10^{-07}B_{7,8}(X) - 6.635299669617927\cdot10^{-07}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3318344765869905, 0.6651804668942704\}$$

Intersection intervals with the x axis:

$$[0.3318344765869905, 0.6651804668942704]$$

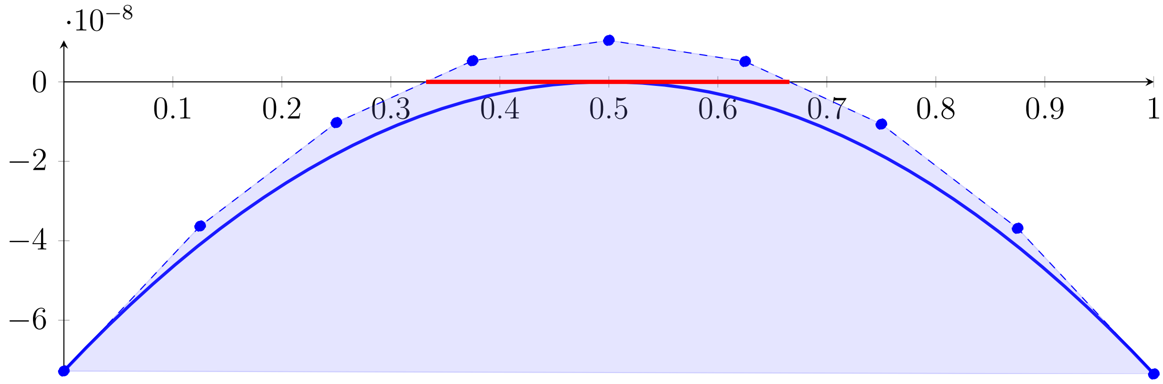
Longest intersection interval: 0.3333459903072799

$\Longrightarrow$ Selective recursion: interval 1: [0.4999990066129424, 0.5000010486132034],

58.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999990066129424

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.023057070307937\cdot10^{-46}X^8 - 5.773711385862107\cdot10^{-39}X^7 - 4.572901329678827\cdot10^{-32}X^6\\ &- 1.922370176484083\cdot10^{-25}X^5 - 4.523805577109385\cdot10^{-19}X^4 - 5.650400184721212\cdot10^{-13}X^3\\ &- 2.926726911514393\cdot10^{-07}X^2 + 2.919240677305221\cdot10^{-07}X - 7.27925149751094\cdot10^{-08}\\ = &-7.27925149751094\cdot10^{-08}B_{0,8}(X) - 3.630200650879414\cdot10^{-08}B_{1,8}(X) - 1.026409415503029\\ &\cdot10^{-08}B_{2,8}(X) + 5.321211996181834\cdot10^{-09}B_{3,8}(X) + 1.045390185483543\cdot10^{-08}B_{4,8}(X)\\ &+ 5.133965330917245\cdot10^{-09}B_{5,8}(X) - 1.063860766559244\cdot10^{-08}B_{6,8}(X)\\ &- 3.68638272247198\cdot10^{-08}B_{7,8}(X) - 7.354170343649749\cdot10^{-08}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3323218842755297, 0.6656874431018115\}$$

Intersection intervals with the x axis:

$$[0.3323218842755297, 0.6656874431018115]$$

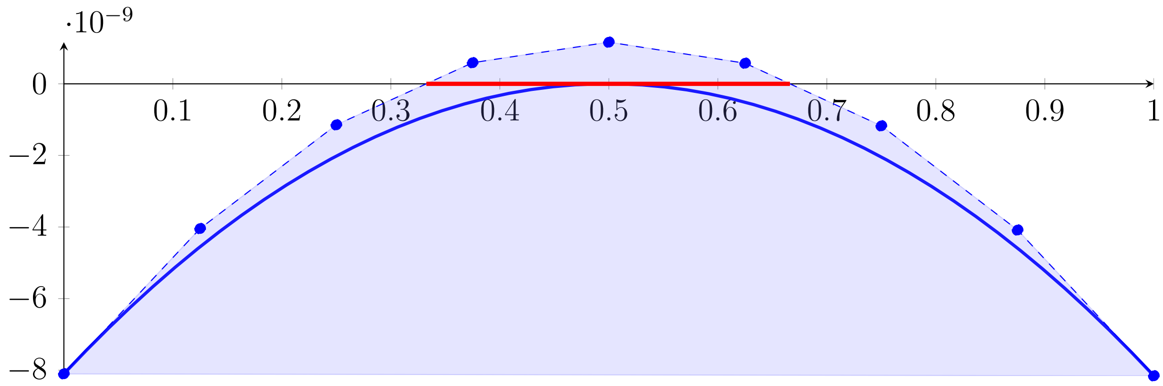
Longest intersection interval: 0.3333655588262817

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999996852143169, 0.500000365946875]$,

58.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999996852143169, 0.500000365946875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.61118111521237\cdot10^{-50}X^8 - 2.641801828130962\cdot10^{-42}X^7 - 6.276482670428966\cdot10^{-35}X^6\\&\quad - 7.91481690665562\cdot10^{-28}X^5 - 5.587109146485419\cdot10^{-21}X^4 - 2.093350052120201\cdot10^{-14}X^3\\&\quad - 3.252553849467158\cdot10^{-08}X^2 + 3.247007216419568\cdot10^{-08}X - 8.101917765629863\cdot10^{-09}\\&= -8.101917765629863\cdot10^{-09}B_{0,8}(X) - 4.043158745105403\cdot10^{-09}B_{1,8}(X)\\&\quad - 1.146026099390642\cdot10^{-09}B_{2,8}(X) + 5.8947979770191\cdot10^{-10}B_{3,8}(X) + 1.163358572359665\\&\quad \cdot10^{-09}B_{4,8}(X) + 5.75609850769953\cdot10^{-10}B_{5,8}(X) - 1.173766740879974\cdot10^{-09}B_{6,8}(X)\\&\quad - 4.084771576402945\cdot10^{-09}B_{7,8}(X) - 8.157405029611868\cdot10^{-09}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.332542653795541, 0.6661296410902487\}$$

Intersection intervals with the x axis:

$$[0.332542653795541, 0.6661296410902487]$$

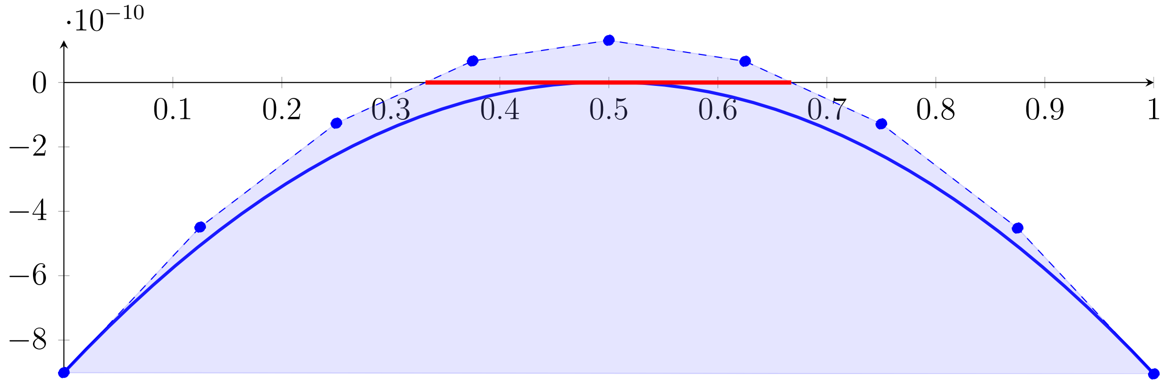
Longest intersection interval: 0.3335869872947078

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999999115869283, 0.5000001386704515]$,

58.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999115869283, 0.5000001386704515]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -7.071067608997674\cdot10^{-54}X^8 - 1.214406168666063\cdot10^{-45}X^7 - 8.649101296569956\cdot10^{-38}X^6\\&\quad - 3.269538436354099\cdot10^{-30}X^5 - 6.918686662336931\cdot10^{-23}X^4 - 7.770864123205297\cdot10^{-16}X^3\\&\quad - 3.61945329275134\cdot10^{-09}X^2 + 3.615351534501487\cdot10^{-09}X - 9.01058772952768\cdot10^{-10}\\&= -9.01058772952768\cdot10^{-10}B_{0,8}(X) - 4.491398311400821\cdot10^{-10}B_{1,8}(X) - 1.264870783542298\\&\quad \cdot10^{-10}B_{2,8}(X) + 6.689947152824597\cdot10^{-11}B_{3,8}(X) + 1.31019804630801\cdot10^{-10}B_{4,8}(X)\\&\quad + 6.587390707689027\cdot10^{-11}B_{5,8}(X) - 1.285382350100323\cdot10^{-10}B_{6,8}(X)\\&\quad - 4.522166355065136\cdot10^{-10}B_{7,8}(X) - 9.051613082891018\cdot10^{-10}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3317579340646351, 0.6673545479012814\}$$

Intersection intervals with the x axis:

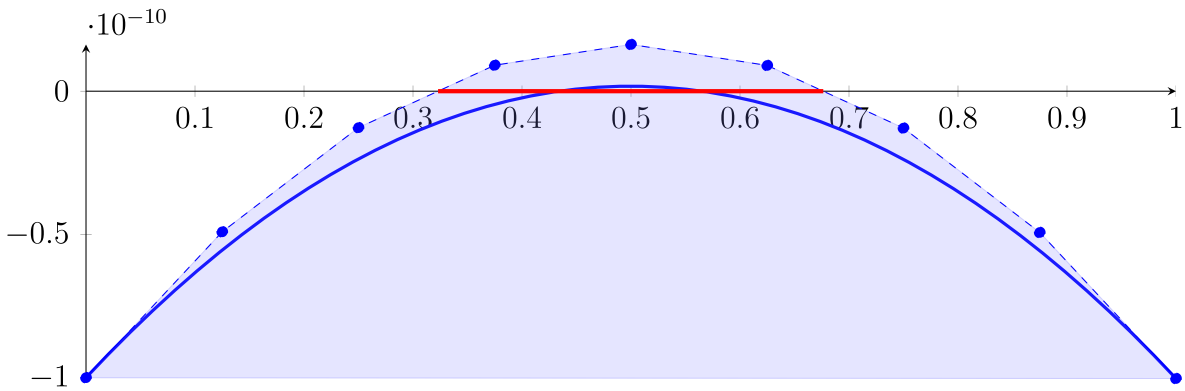
$$[0.3317579340646351, 0.6673545479012814]$$

Longest intersection interval: 0.3355966138366463
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999999869236888, 0.5000000631321502]$,

58.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999869236888, 0.5000000631321502]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1.137694218100245 \cdot 10^{-57} X^8 - 5.822197888022611 \cdot 10^{-49} X^7 - 1.235595786383296 \cdot 10^{-40} X^6 \\ & - 1.391791952088144 \cdot 10^{-32} X^5 - 8.775946704095466 \cdot 10^{-25} X^4 - 2.937122613307657 \cdot 10^{-17} X^3 \\ & - 4.076413298856481 \cdot 10^{-10} X^2 + 4.07342667585279 \cdot 10^{-10} X - 1.000063159750977 \cdot 10^{-10} \\ = & -1.000063159750977 \cdot 10^{-10} B_{0,8}(X) - 4.908848252693777 \cdot 10^{-11} B_{1,8}(X) - 1.272926800326533 \\ & \cdot 10^{-11} B_{2,8}(X) + 9.071327071433506 \cdot 10^{-12} B_{3,8}(X) + 1.631330217267253 \cdot 10^{-11} B_{4,8}(X) \\ & + 8.996656775965545 \cdot 10^{-12} B_{5,8}(X) - 1.287860964317367 \cdot 10^{-11} B_{6,8}(X) \\ & - 4.931249760923136 \cdot 10^{-11} B_{7,8}(X) - 1.003050076466937 \cdot 10^{-10} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3229869297125206, 0.6764088411746962\}$$

Intersection intervals with the x axis:

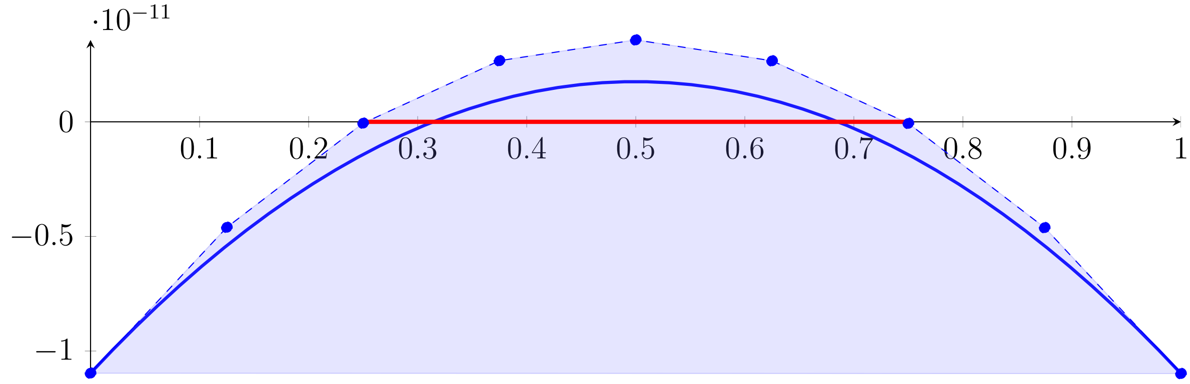
$$[0.3229869297125206, 0.6764088411746962]$$

Longest intersection interval: 0.3534219114621756
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000115380258, 0.5000000384717659]$,

58.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000115380258, 0.5000000384717659]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-2.769321148091343\cdot10^{-61}X^8 - 4.009971301105125\cdot10^{-52}X^7 - 2.40789337418918\cdot10^{-43}X^6 \\ &- 7.674351474736958\cdot10^{-35}X^5 - 1.36920310040832\cdot10^{-26}X^4 - 1.29658952293724\cdot10^{-18}X^3 \\ &- 5.091727851043352\cdot10^{-11}X^2 + 5.08987688183243\cdot10^{-11}X - 1.096532991460483\cdot10^{-11} \\ = &-1.096532991460483\cdot10^{-11}B_{0,8}(X) - 4.602983812314296\cdot10^{-12}B_{1,8}(X) \\ &- 5.91119425392419\cdot10^{-14}B_{2,8}(X) + 2.666285671566945\cdot10^{-12}B_{3,8}(X) + 3.57320900685088 \\ &\cdot10^{-12}B_{4,8}(X) + 2.661658040159178\cdot10^{-12}B_{5,8}(X) - 6.83672516615453\cdot10^{-14}B_{6,8}(X) \\ &- 4.616866891764675\cdot10^{-12}B_{7,8}(X) - 1.09838409033036\cdot10^{-11}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.252711161402344, 0.7468696603349071\}$$

Intersection intervals with the x axis:

$$[0.252711161402344, 0.7468696603349071]$$

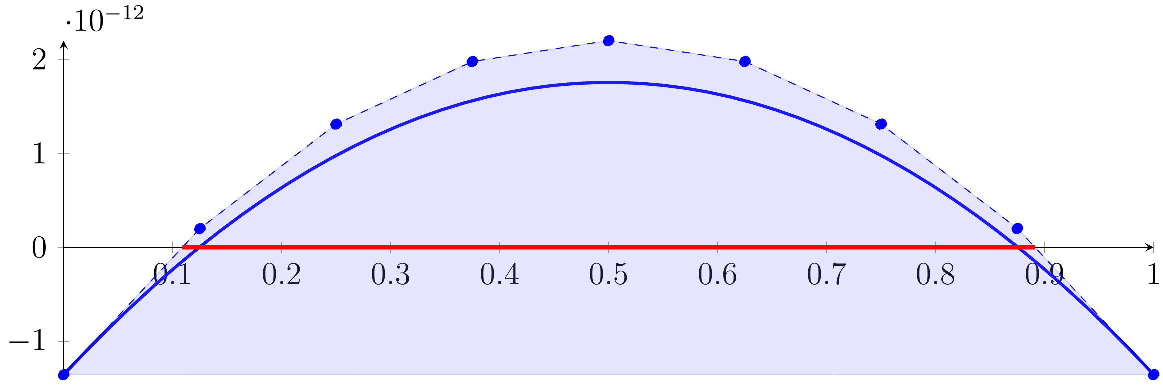
Longest intersection interval: 0.4941584989325631

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000183444825, 0.5000000316540191]$,

58.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000183444825, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-9.84698950588755\cdot10^{-64}X^8 - 2.885394162330348\cdot10^{-54}X^7 - 3.506185324343073\cdot10^{-45}X^6 \\ &- 2.261377324294573\cdot10^{-36}X^5 - 8.164563166712554\cdot10^{-28}X^4 - 1.564592773244276\cdot10^{-19}X^3 \\ &- 1.243362398803133\cdot10^{-11}X^2 + 1.243502393415518\cdot10^{-11}X - 1.354369602677131\cdot10^{-12} \\ = &-1.354369602677131\cdot10^{-12}B_{0,8}(X) + 2.000083890922661\cdot10^{-13}B_{1,8}(X) \\ &+ 1.310328381289116\cdot10^{-12}B_{2,8}(X) + 1.976590371119503\cdot10^{-12}B_{3,8}(X) \\ &+ 2.19879435578951\cdot10^{-12}B_{4,8}(X) + 1.976940332505223\cdot10^{-12}B_{5,8}(X) + 1.311028298472726 \\ &\cdot10^{-12}B_{6,8}(X) + 2.01058250898102\cdot10^{-13}B_{7,8}(X) - 1.352969813012563\cdot10^{-12}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.108915721421098, 0.8911723471704997\}$$

Intersection intervals with the x axis:

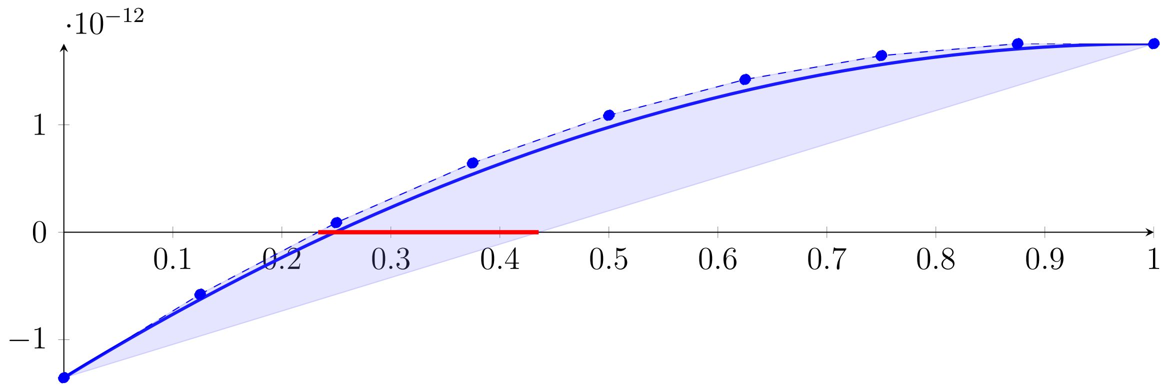
$$[0.108915721421098, 0.8911723471704997]$$

Longest intersection interval: 0.7822566257494017
$\Longrightarrow$ Bisection: first half $[0.5000000183444825, 0.5000000249992508]$ und second half $[0.5000000249992508, 0.50$

58.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.5000000183444825, 0.5000000249992508]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-3.846480275737324\cdot 10^{-66}X^8 - 2.254214189320585\cdot 10^{-56}X^7 - 5.478414569286052\cdot 10^{-47}X^6 \\
&- 7.06680413842054\cdot 10^{-38}X^5 - 5.102851979195346\cdot 10^{-29}X^4 - 1.955740966555345\cdot 10^{-20}X^3 \\
&- 3.108405997007834\cdot 10^{-12}X^2 + 6.21751196707759\cdot 10^{-12}X - 1.354369602677131\cdot 10^{-12} \\
= &-1.354369602677131\cdot 10^{-12}B_{0,8}(X) - 5.771806067924327\cdot 10^{-13}B_{1,8}(X) + 8.899388919912914 \\
&\cdot 10^{-14}B_{2,8}(X) + 6.441538849483146\cdot 10^{-13}B_{3,8}(X) + 1.088299380105884\cdot 10^{-12}B_{4,8}(X) \\
&+ 1.421430374322599\cdot 10^{-12}B_{5,8}(X) + 1.643546867249218\cdot 10^{-12}B_{6,8}(X) \\
&+ 1.754648858536504\cdot 10^{-12}B_{7,8}(X) + 1.754736347835215\cdot 10^{-12}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2333013178726793, 0.4356138466281428\}$$

Intersection intervals with the x axis:

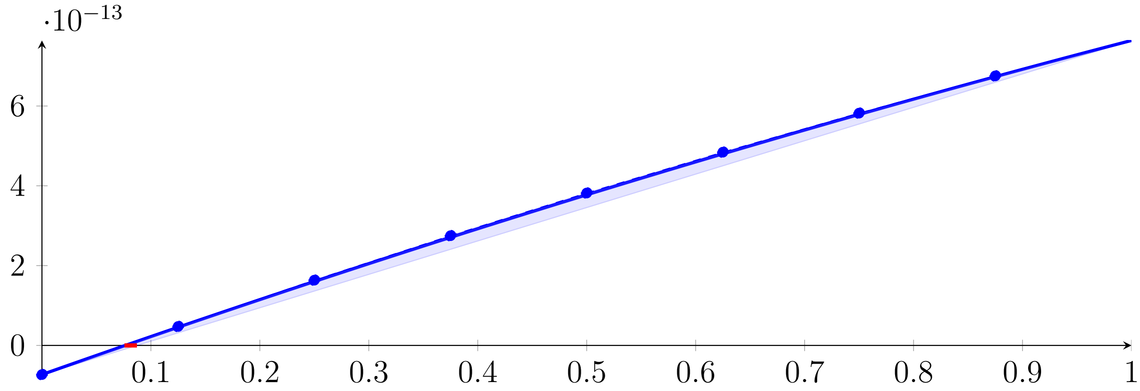
$$[0.2333013178726793, 0.4356138466281428]$$

Longest intersection interval: 0.2023125287554634
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000198970487, 0.5000000212433917]$,

58.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000198970487, 0.5000000212433917]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = &-1.079557397202381\cdot 10^{-71}X^8 - 3.12719259192325\cdot 10^{-61}X^7 - 3.756570682132575\cdot 10^{-51}X^6 \\
&- 2.395173331861966\cdot 10^{-41}X^5 - 8.548778908133761\cdot 10^{-32}X^4 - 1.619495215947232\cdot 10^{-22}X^3 \\
&- 1.272281748414343\cdot 10^{-13}X^2 + 9.644484121679\cdot 10^{-13}X - 7.300486662812878\cdot 10^{-14} \\
= &-7.300486662812878\cdot 10^{-14}B_{0,8}(X) + 4.755118489285872\cdot 10^{-14}B_{1,8}(X) + 1.635633730266521 \\
&\cdot 10^{-13}B_{2,8}(X) + 2.750316977703595\cdot 10^{-13}B_{3,8}(X) + 3.819561591210889\cdot 10^{-13}B_{4,8}(X) \\
&+ 4.843367570759483\cdot 10^{-13}B_{5,8}(X) + 5.821734916320458\cdot 10^{-13}B_{6,8}(X) \\
&+ 6.754663627864895\cdot 10^{-13}B_{7,8}(X) + 7.642153705363873\cdot 10^{-13}B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.07569597886944255, 0.08719911844866588\}$$

Intersection intervals with the x axis:

$$[0.07569597886944255, 0.08719911844866588]$$

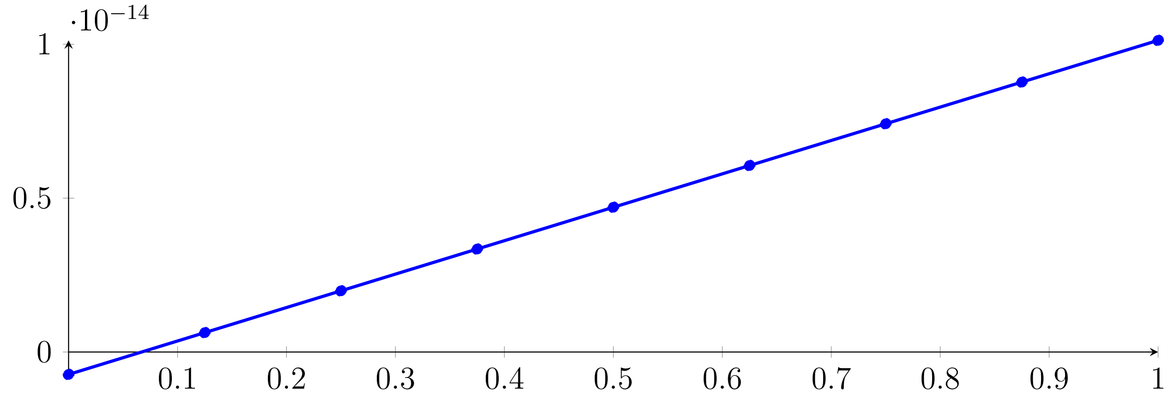
Longest intersection interval: 0.01150313957922333

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199989615, 0.5000000200144486]$,

58.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199989615, 0.5000000200144486]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = &-3.309610258465664\cdot 10^{-87}X^8 - 8.334304343480435\cdot 10^{-75}X^7 - 8.703419206351739\cdot 10^{-63}X^6 \\
&- 4.824128808781935\cdot 10^{-51}X^5 - 1.496820228100742\cdot 10^{-39}X^4 - 2.465067626410487\cdot 10^{-28}X^3 \\
&- 1.683511456921895\cdot 10^{-17}X^2 + 1.087261902125672\cdot 10^{-14}X - 7.290023293677624\cdot 10^{-16} \\
= &-7.290023293677624\cdot 10^{-16}B_{0,8}(X) + 6.300750482893273\cdot 10^{-16}B_{1,8}(X) + 1.988551171854659 \\
&\cdot 10^{-15}B_{2,8}(X) + 3.346426041328229\cdot 10^{-15}B_{3,8}(X) + 4.703699656710032\cdot 10^{-15}B_{4,8}(X) \\
&+ 6.060372018000064\cdot 10^{-15}B_{5,8}(X) + 7.41644312519832\cdot 10^{-15}B_{6,8}(X) \\
&+ 8.771912978304797\cdot 10^{-15}B_{7,8}(X) + 1.012678157731949\cdot 10^{-14}B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.06704937678240291, 0.0671533567390459\}$$

Intersection intervals with the x axis:

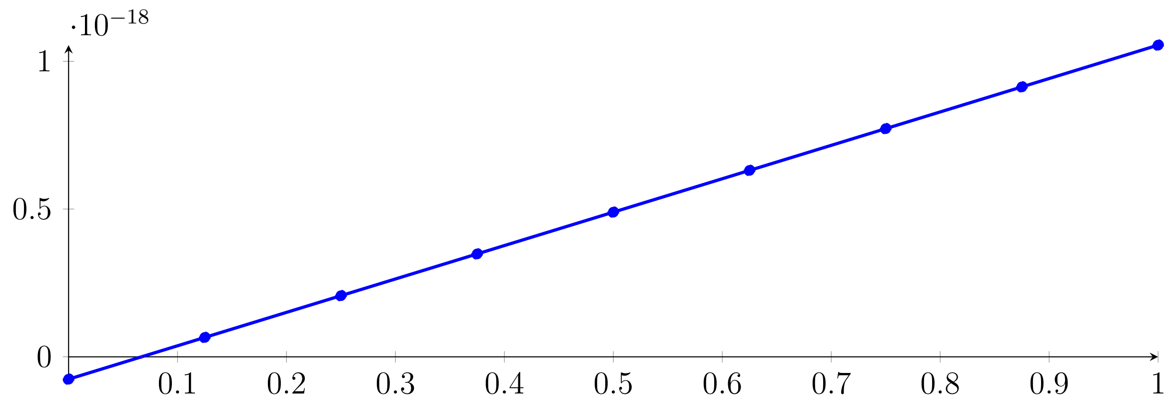
$$[0.06704937678240291, 0.0671533567390459]$$

Longest intersection interval: 0.0001039799566429901
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000200000015]$,

58.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000200000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.522451416965012\cdot 10^{-119}X^8 - 1.095258871423056\cdot 10^{-102}X^7 - 1.099987358691302\cdot 10^{-86}X^6\\ &\quad - 5.863636713606292\cdot 10^{-71}X^5 - 1.749718451421176\cdot 10^{-55}X^4 - 2.771262941212064\cdot 10^{-40}X^3\\ &\quad - 1.820184200444671\cdot 10^{-25}X^2 + 1.130299712615757\cdot 10^{-18}X - 7.568425969413026\cdot 10^{-20}\\ &= -7.568425969413026\cdot 10^{-20}B_{0,8}(X) + 6.560320438283937\cdot 10^{-20}B_{1,8}(X) + 2.068906619591512\\ &\quad \cdot 10^{-19}B_{2,8}(X) + 3.481781130348051\cdot 10^{-19}B_{3,8}(X) + 4.894655576098011\cdot 10^{-19}B_{4,8}(X)\\ &\quad + 6.307529956841393\cdot 10^{-19}B_{5,8}(X) + 7.720404272578197\cdot 10^{-19}B_{6,8}(X)\\ &\quad + 9.133278523308422\cdot 10^{-19}B_{7,8}(X) + 1.054615270903207\cdot 10^{-18}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695946114945086, 0.06695947193230531\}$$

Intersection intervals with the x axis:

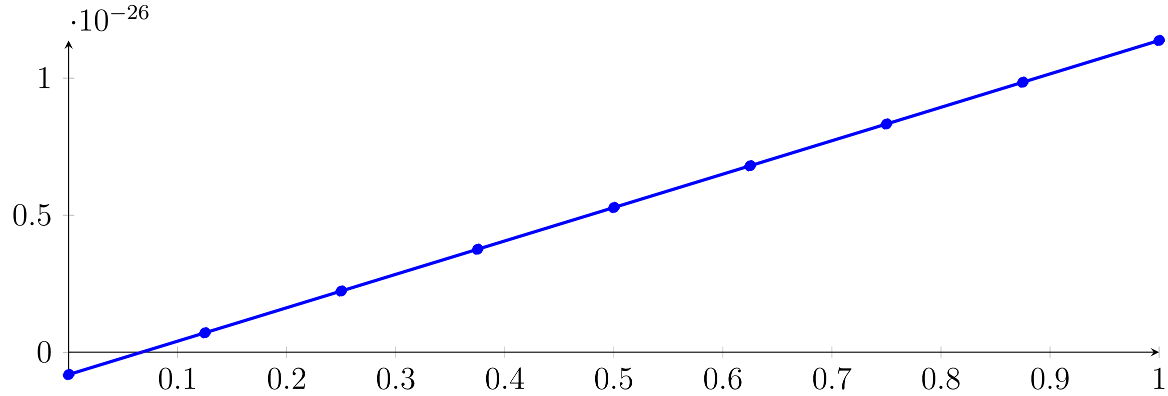
$$[0.06695946114945086, 0.06695947193230531]$$

Longest intersection interval: $1.078285445187343\cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

58.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -8.265018972796948\cdot10^{-183}X^8 - 1.856320587559019\cdot10^{-158}X^7 - 1.7289807291512\cdot10^{-134}X^6\\&\quad - 8.547434292245386\cdot10^{-111}X^5 - 2.365392110963433\cdot10^{-87}X^4 - 3.474393176904993\cdot10^{-64}X^3\\&\quad - 2.116327262136374\cdot10^{-41}X^2 + 1.218785702529033\cdot10^{-26}X - 8.160922251597258\cdot10^{-28}\\&= -8.160922251597258\cdot10^{-28}B_{0,8}(X) + 7.073899030015655\cdot10^{-28}B_{1,8}(X) + 2.230872031162856\\&\quad\cdot10^{-27}B_{2,8}(X) + 3.754354159324146\cdot10^{-27}B_{3,8}(X) + 5.277836287485435\cdot10^{-27}B_{4,8}(X)\\&\quad + 6.801318415646723\cdot10^{-27}B_{5,8}(X) + 8.32480054380801\cdot10^{-27}B_{6,8}(X)\\&\quad + 9.848282671969297\cdot10^{-27}B_{7,8}(X) + 1.137176480013058\cdot10^{-26}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695945181062587, 0.06695945181062598\}$$

Intersection intervals with the x axis:

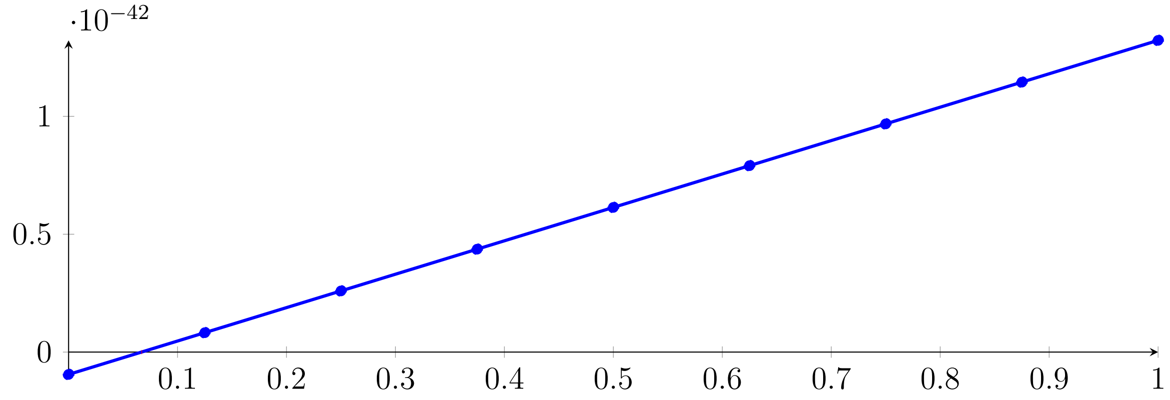
$$[0.06695945181062587, 0.06695945181062598]$$

Longest intersection interval: $1.162699176979875\cdot10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

58.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.76046836984994\cdot10^{-310}X^8 - 5.332422307789531\cdot10^{-270}X^7 - 4.271637033651821\cdot10^{-230}X^6\\&\quad - 1.816237323415445\cdot10^{-190}X^5 - 4.322874268764078\cdot10^{-151}X^4 - 5.461111688235365\cdot10^{-112}X^3\\&\quad - 2.860998015592859\cdot10^{-73}X^2 + 1.417081133245345\cdot10^{-42}X - 9.488697585328866\cdot10^{-44}\\&= -9.488697585328866\cdot10^{-44}B_{0,8}(X) + 8.224816580237952\cdot10^{-44}B_{1,8}(X) + 2.593833074580477\\&\quad\cdot10^{-43}B_{2,8}(X) + 4.365184491137159\cdot10^{-43}B_{3,8}(X) + 6.136535907693841\cdot10^{-43}B_{4,8}(X)\\&\quad + 7.907887324250522\cdot10^{-43}B_{5,8}(X) + 9.679238740807204\cdot10^{-43}B_{6,8}(X)\\&\quad + 1.145059015736389\cdot10^{-42}B_{7,8}(X) + 1.322194157392057\cdot10^{-42}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695945181062577, 0.06695945181062577\}$$

Intersection intervals with the x axis:

$$[0.06695945181062577, 0.06695945181062577]$$

Longest intersection interval: $1.351869376149675 \cdot 10^{-32}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

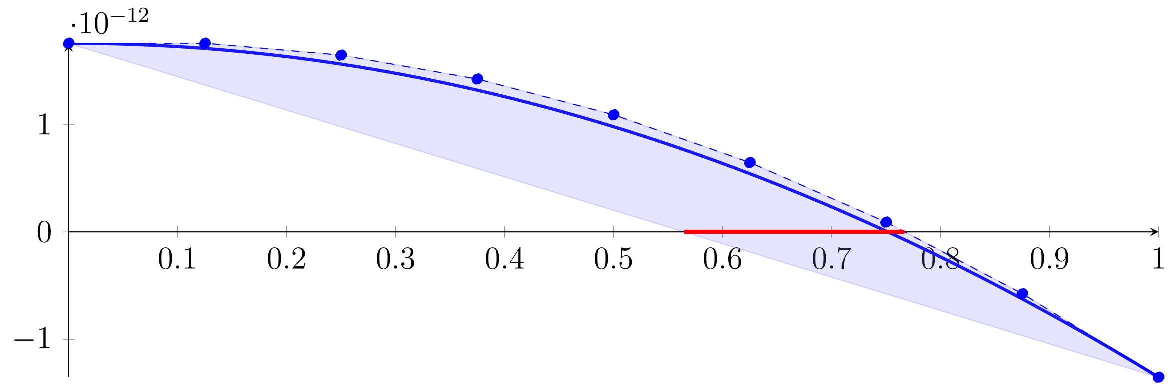
58.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 25!

58.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.5000000249992508, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-3.846480275737324 \cdot 10^{-66} X^8 - 2.254214192397769 \cdot 10^{-56} X^7 - 5.478414585065551 \cdot 10^{-47} X^6 \\
&- 7.066804171291027 \cdot 10^{-38} X^5 - 5.102852014529367 \cdot 10^{-29} X^4 - 1.955740986966753 \cdot 10^{-20} X^3 \\
&- 3.108406055680063 \cdot 10^{-12} X^2 + 6.999143896938875 \cdot 10^{-16} X + 1.754736347835215 \cdot 10^{-12} \\
= &\ 1.754736347835215 \cdot 10^{-12} B_{0,8}(X) + 1.754823837133927 \cdot 10^{-12} B_{1,8}(X) + 1.643896824444065 \\
&\cdot 10^{-12} B_{2,8}(X) + 1.42195530941639 \cdot 10^{-12} B_{3,8}(X) + 1.088999291701662 \cdot 10^{-12} B_{4,8}(X) \\
&+ 6.450287709506428 \cdot 10^{-13} B_{5,8}(X) + 9.00437468140915 \cdot 10^{-14} B_{6,8}(X) \\
&- 5.759557810572307 \cdot 10^{-13} B_{7,8}(X) - 1.352969813012563 \cdot 10^{-12} B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.5646403672078589, 0.7669001146107961\}$$

Intersection intervals with the x axis:

$$[0.5646403672078589, 0.7669001146107961]$$

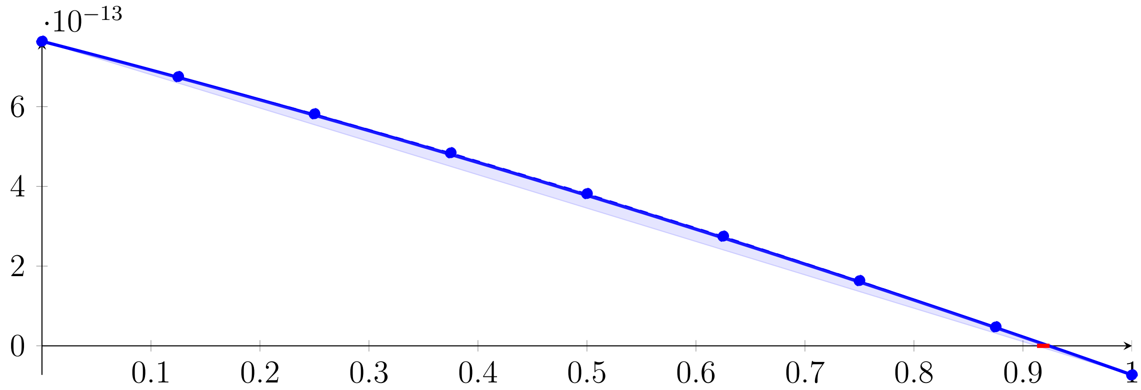
Longest intersection interval: 0.2022597474029372

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000287568016, 0.5000000301027934]$,

58.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.5000000287568016, 0.5000000301027934]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1.077306286121587\cdot 10^{-71} X^8 - 3.121486088546909\cdot 10^{-61} X^7 - 3.750694216009664\cdot 10^{-51} X^6 \\ &- 2.392050590455968\cdot 10^{-41} X^5 - 8.5398613074672\cdot 10^{-32} X^4 - 1.618228037882981\cdot 10^{-22} X^3 \\ &- 1.271618015330847\cdot 10^{-13} X^2 - 7.098433618236863\cdot 10^{-13} X + 7.641134288463311\cdot 10^{-13} \\ = &\ 7.641134288463311\cdot 10^{-13} B_{0,8}(X) + 6.753830086183703\cdot 10^{-13} B_{1,8}(X) + 5.821110954785137 \\ &\cdot 10^{-13} B_{2,8}(X) + 4.842976894238714\cdot 10^{-13} B_{3,8}(X) + 3.819427904515539\cdot 10^{-13} B_{4,8}(X) \\ &+ 2.750463985586714\cdot 10^{-13} B_{5,8}(X) + 1.636085137423343\cdot 10^{-13} B_{6,8}(X) \\ &+ 4.762913599965286\cdot 10^{-14} B_{7,8}(X) - 7.289173467226265\cdot 10^{-14} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9129136379925769, 0.9243992614454615\}$$

Intersection intervals with the x axis:

$$[0.9129136379925769, 0.9243992614454615]$$

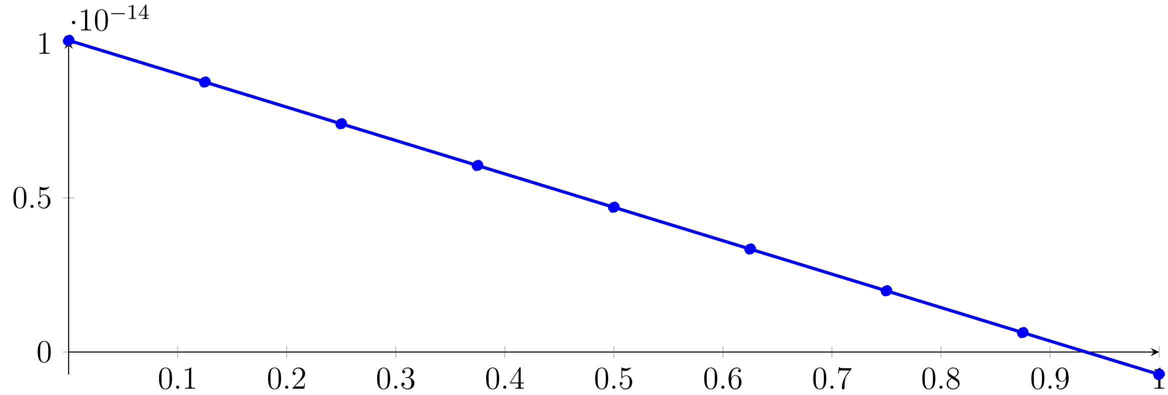
Longest intersection interval: 0.01148562345288463

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000299855758, 0.5000000300010354]$,

58.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.5000000299855758, 0.5000000300010354]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-3.26268981617174\cdot 10^{-87} X^8 - 8.230826044001589\cdot 10^{-75} X^7 - 8.610712798456319\cdot 10^{-63} X^6 \\ &- 4.781269588122392\cdot 10^{-51} X^5 - 1.48617213692691\cdot 10^{-39} X^4 - 2.451903903103066\cdot 10^{-28} X^3 \\ &- 1.677512719815316\cdot 10^{-17} X^2 - 1.081967377285333\cdot 10^{-14} X + 1.010965922317952\cdot 10^{-14} \\ = &\ 1.010965922317952\cdot 10^{-14} B_{0,8}(X) + 8.757200001572855\cdot 10^{-15} B_{1,8}(X) + 7.40414166828054 \\ &\cdot 10^{-15} B_{2,8}(X) + 6.050484223302573\cdot 10^{-15} B_{3,8}(X) + 4.696227666638948\cdot 10^{-15} B_{4,8}(X) \\ &+ 3.341371998289662\cdot 10^{-15} B_{5,8}(X) + 1.98591721825471\cdot 10^{-15} B_{6,8}(X) \\ &+ 6.298633265340878\cdot 10^{-16} B_{7,8}(X) - 7.267896768722089\cdot 10^{-16} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9329310105574586, 0.9330346747614\}$$

Intersection intervals with the x axis:

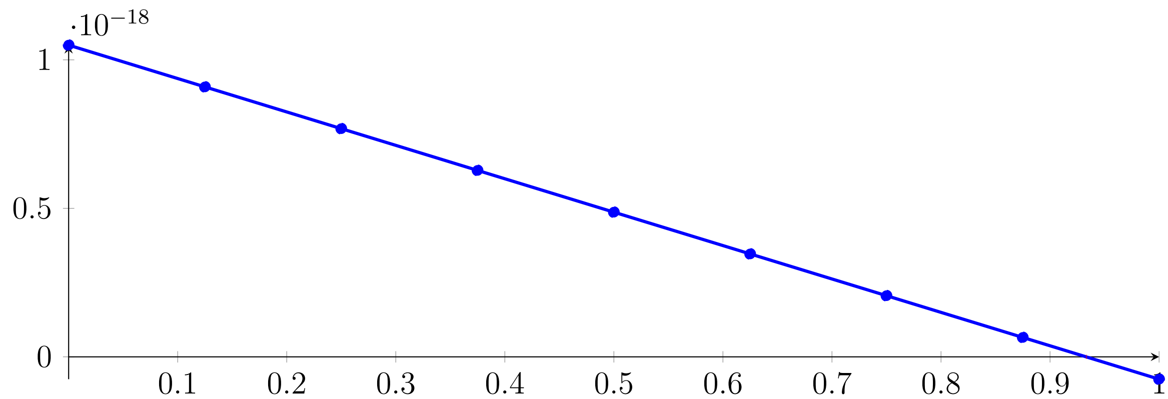
$$[0.9329310105574586, 0.9330346747614]$$

Longest intersection interval: 0.0001036642039414486
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000299999985, 0.5000000300000001]$,

58.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.5000000299999985, 0.5000000300000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-4.351172724257996\cdot 10^{-119}X^8 - 1.058876090749164\cdot 10^{-102}X^7 - 1.068592215336126\cdot 10^{-86}X^6\\ &- 5.723837778458117\cdot 10^{-71}X^5 - 1.716265145654202\cdot 10^{-55}X^4 - 2.731428873735883\cdot 10^{-40}X^3\\ &- 1.802699988373219\cdot 10^{-25}X^2 - 1.124857565692813\cdot 10^{-18}X + 1.049632124051115\cdot 10^{-18}\\ = &\ 1.049632124051115\cdot 10^{-18}B_{0,8}(X) + 9.090249283395136\cdot 10^{-19}B_{1,8}(X) + 7.684177261896977\\ &\cdot 10^{-19}B_{2,8}(X) + 6.278105176016676\cdot 10^{-19}B_{3,8}(X) + 4.872033025754233\cdot 10^{-19}B_{4,8}(X)\\ &+ 3.465960811109647\cdot 10^{-19}B_{5,8}(X) + 2.059888532082919\cdot 10^{-19}B_{6,8}(X)\\ &+ 6.538161886740477\cdot 10^{-20}B_{7,8}(X) - 7.522562191169655\cdot 10^{-20}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243242252754, 0.9331243349427871\}$$

Intersection intervals with the x axis:

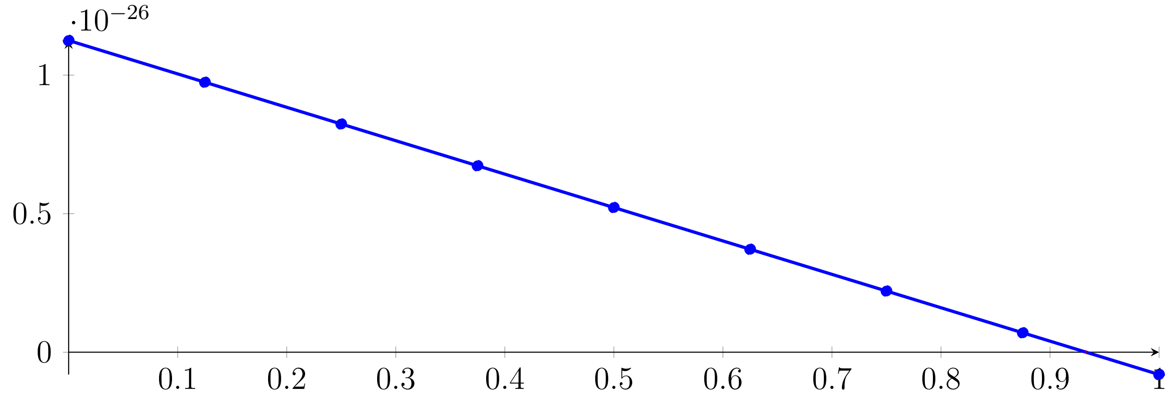
$$[0.9331243242252754, 0.9331243349427871]$$

Longest intersection interval: $1.071751170792712\cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

58.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-7.574571270278352\cdot10^{-183}X^8 - 1.719898859058455\cdot10^{-158}X^7 - 1.619480808086848\cdot10^{-134}X^6\\ &- 8.09388724153584\cdot10^{-111}X^5 - 2.264437035388769\cdot10^{-87}X^4 - 3.36257357580136\cdot10^{-64}X^3\\ &- 2.070672372961545\cdot10^{-41}X^2 - 1.205567773574102\cdot10^{-26}X + 1.124944638137319\cdot10^{-26}\\ = &\ 1.124944638137319\cdot10^{-26}B_{0,8}(X) + 9.742486664405558\cdot10^{-27}B_{1,8}(X) + 8.235526947437929\\ &\cdot10^{-27}B_{2,8}(X) + 6.7285672304703\cdot10^{-27}B_{3,8}(X) + 5.22160751350267\cdot10^{-27}B_{4,8}(X)\\ &+ 3.714647796535039\cdot10^{-27}B_{5,8}(X) + 2.207688079567407\cdot10^{-27}B_{6,8}(X)\\ &+ 7.007283625997749\cdot10^{-28}B_{7,8}(X) - 8.062313543678582\cdot10^{-28}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243442268158, 0.933124344226816\}$$

Intersection intervals with the x axis:

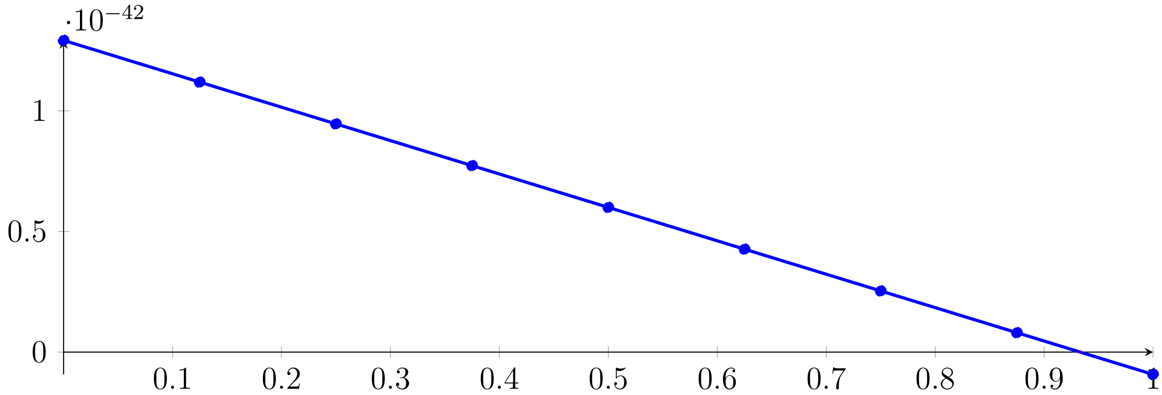
$$[0.9331243442268158, 0.933124344226816]$$

Longest intersection interval: $1.148650253172236\cdot10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

58.31 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.295411522634873\cdot10^{-310}X^8 - 4.537510017831649\cdot10^{-270}X^7 - 3.719655178468669\cdot10^{-230}X^6\\ &- 1.618438544591323\cdot10^{-190}X^5 - 3.941953525953829\cdot10^{-151}X^4 - 5.096068226680637\cdot10^{-112}X^3\\ &- 2.732039753653228\cdot10^{-73}X^2 - 1.384775728332186\cdot10^{-42}X + 1.292167943401183\cdot10^{-42}\\ = &\ 1.292167943401183\cdot10^{-42}B_{0,8}(X) + 1.11907097735966\cdot10^{-42}B_{1,8}(X) + 9.459740113181363\\ &\cdot10^{-43}B_{2,8}(X) + 7.72877045276613\cdot10^{-43}B_{3,8}(X) + 5.997800792350898\cdot10^{-43}B_{4,8}(X)\\ &+ 4.266831131935665\cdot10^{-43}B_{5,8}(X) + 2.535861471520432\cdot10^{-43}B_{6,8}(X)\\ &+ 8.048918111051992\cdot10^{-44}B_{7,8}(X) - 9.260778493100336\cdot10^{-44}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243442268161, 0.9331243442268161\}$$

Intersection intervals with the x axis:

$$[0.9331243442268161, 0.9331243442268161]$$

Longest intersection interval: $1.319397404112639 \cdot 10^{-32}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

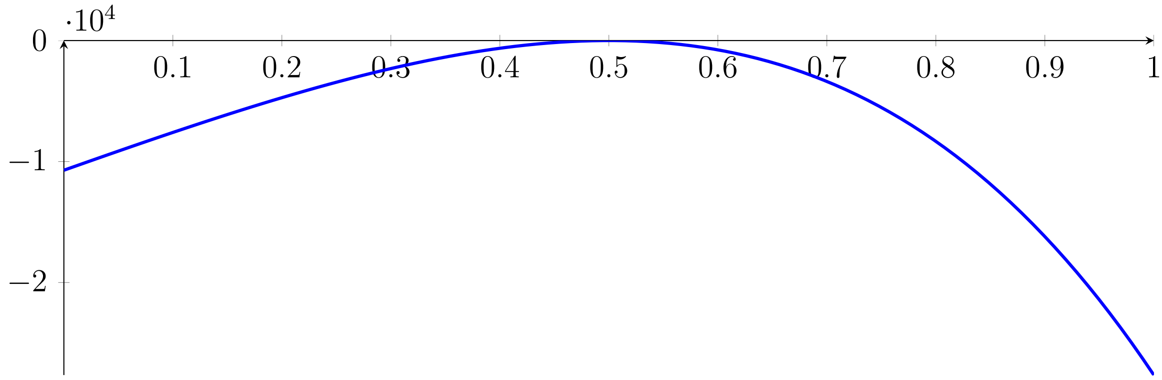
58.32 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 25!

58.33 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

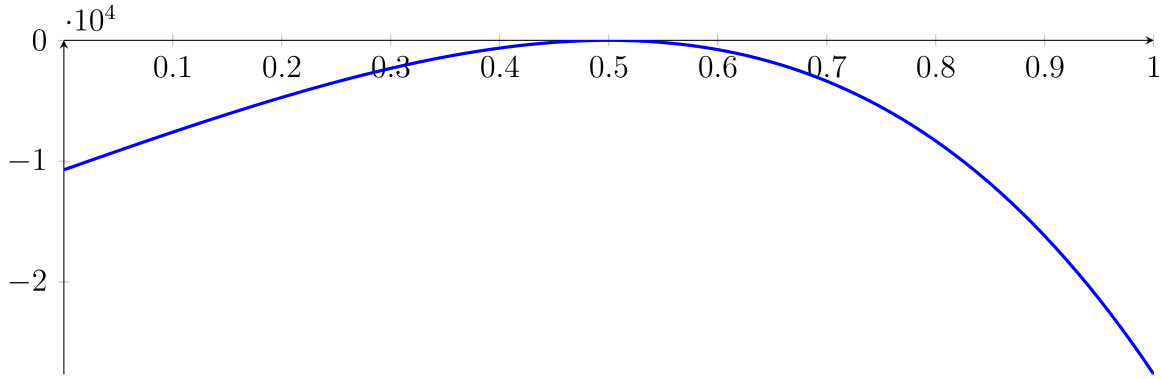
with precision $\varepsilon = 1 \cdot 10^{-64}$.

59 Running QuadClip on h_8 with epsilon 64

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called QuadClip with input polynomial on interval $[0, 1]$:

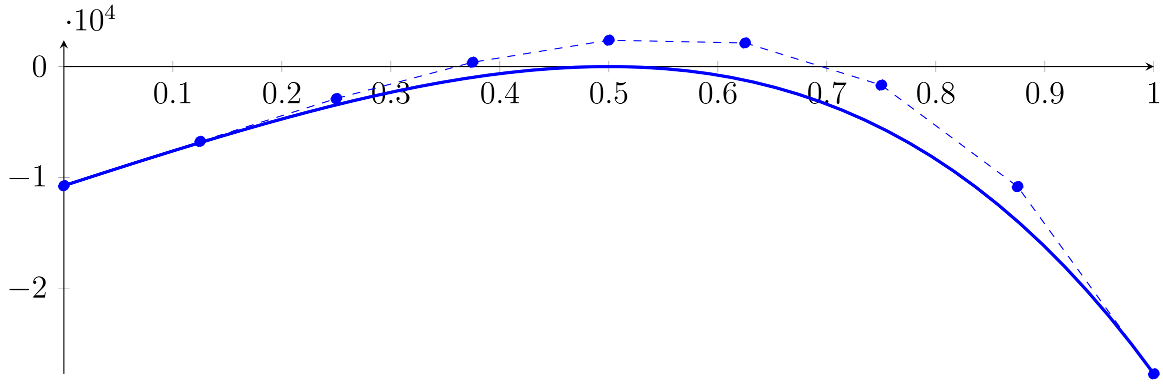
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



59.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

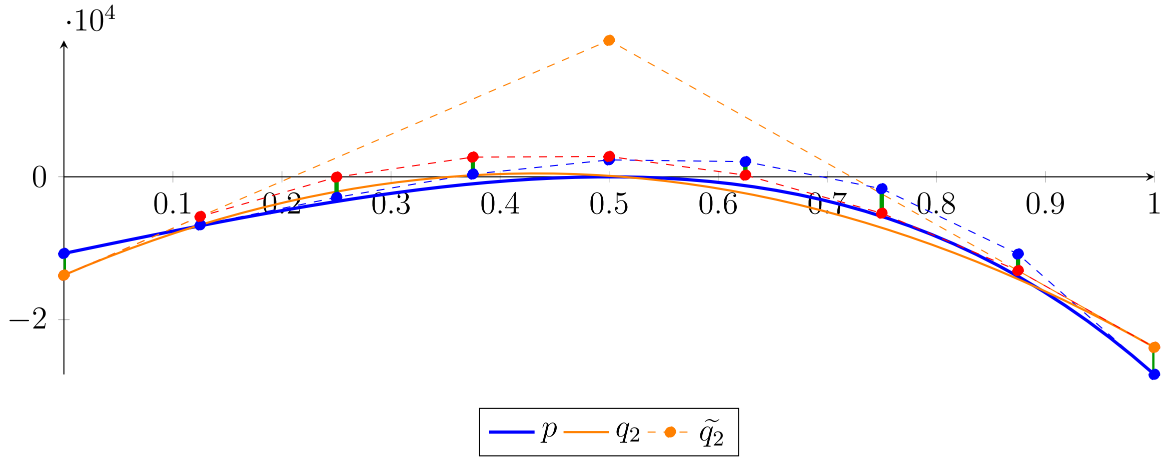
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &\ 8.415084934481114 \cdot 10^{-299}X^8 - 3.337656926892332 \cdot 10^{-298}X^7 + 5.61048922956057 \cdot 10^{-298}X^6 \\ &- 5.189733160056009 \cdot 10^{-298}X^5 + 2.787623795639522 \cdot 10^{-298}X^4 - 8.054794139884735 \\ &\cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ = &-13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8} \\ &+ 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8} \\ &- 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.
Bounding polynomials M and m:

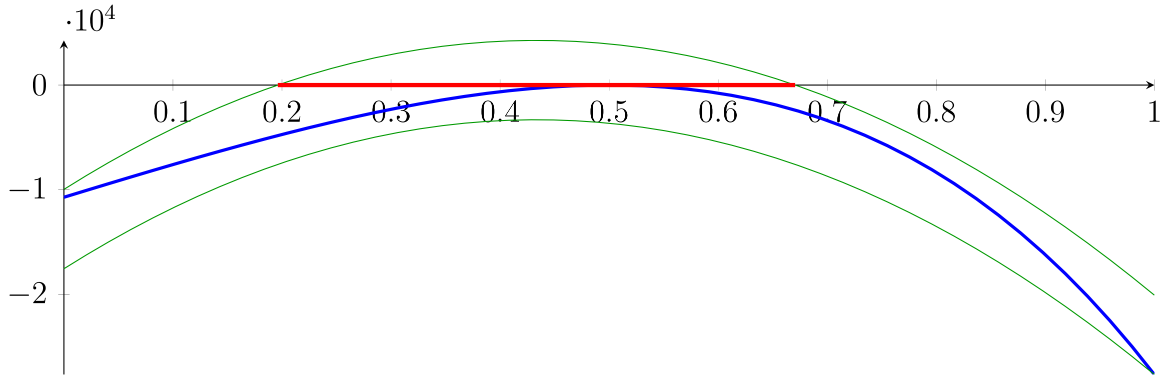
$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$

$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



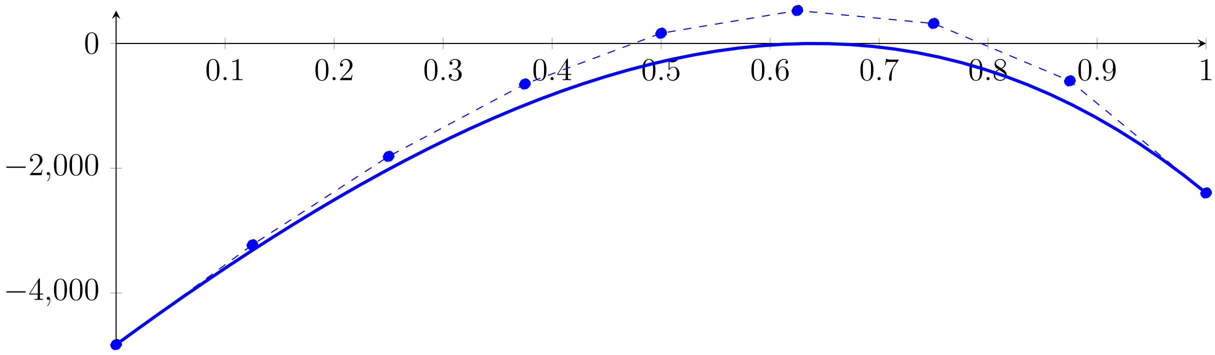
$$[0.196099166583006, 0.6706514347613278]$$

Longest intersection interval: 0.4745522681783218
$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

59.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

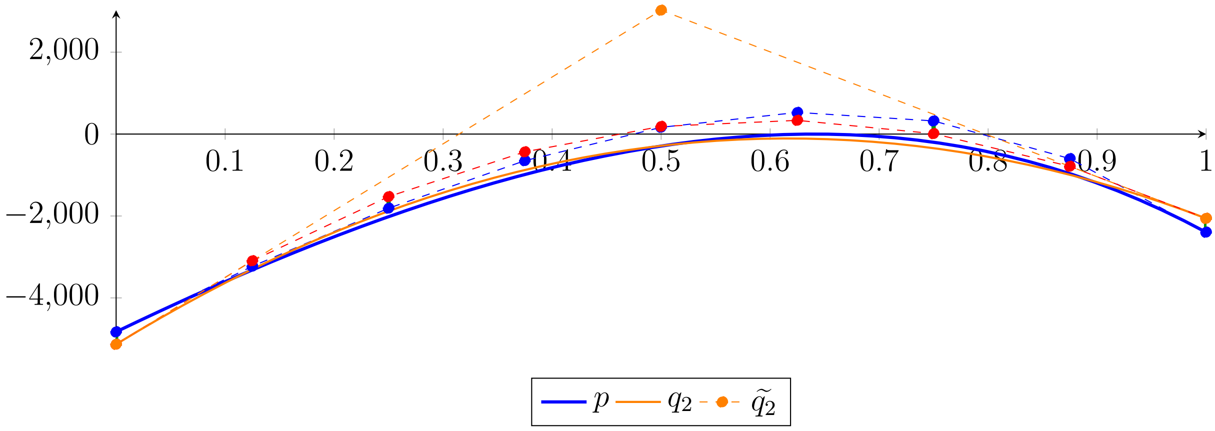
$$\begin{aligned} p = &-0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6 \\ &- 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3 \\ &- 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684 \\ = &-4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X) \\ &- 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X) \\ &+ 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.476670942620757 \cdot 10^{-299}X^8 - 6.381627458836932 \cdot 10^{-299}X^7 + 1.1779791924066 \cdot 10^{-298}X^6 \\ &\quad - 1.190713137013755 \cdot 10^{-298}X^5 + 6.850874958113058 \cdot 10^{-299}X^4 - 2.05172173630317 \\ &\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\ &\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\ &\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

Bounding polynomials M and m:

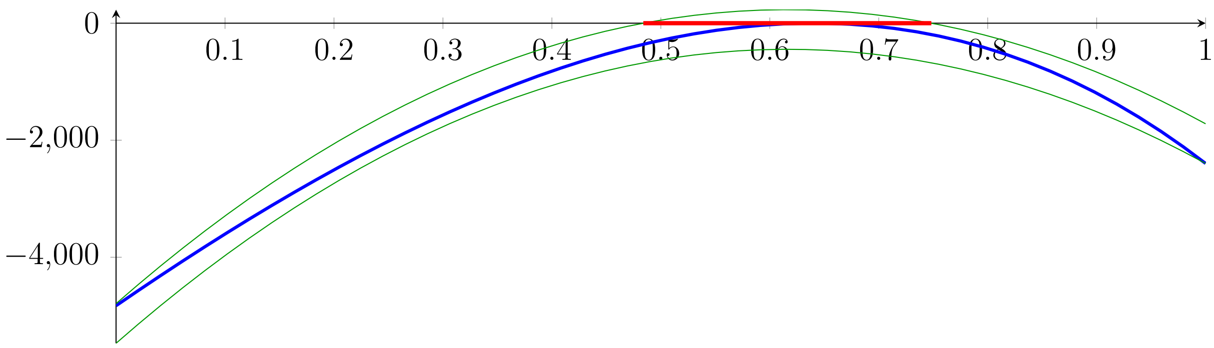
$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$

$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.4839311609916354, 0.7482816274420809]$$

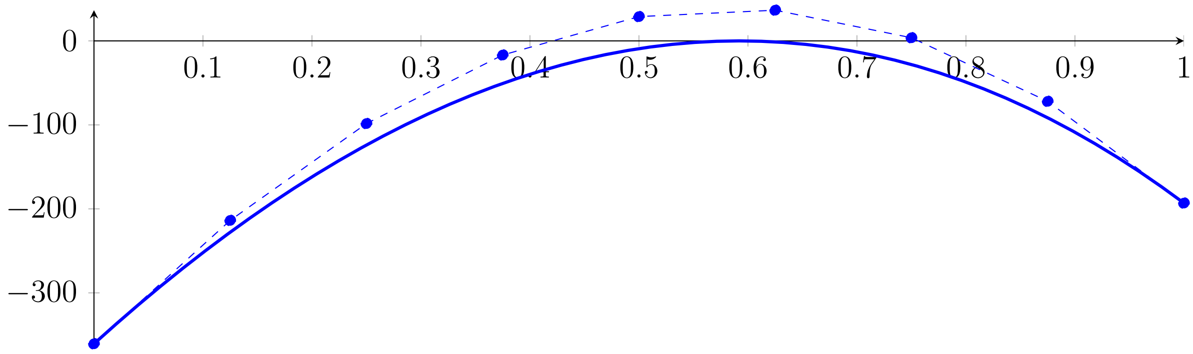
Longest intersection interval: 0.2643504664504455

$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

59.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.551197910121811$

Normalized monomial und Bézier representations and the Bézier polygon:

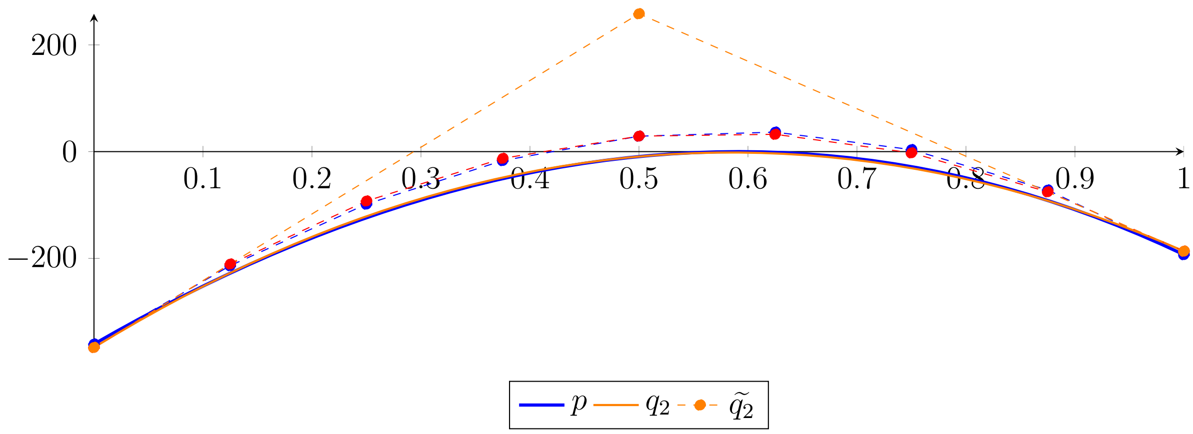
$$\begin{aligned} p = & -6.133566475094875 \cdot 10^{-08} X^8 - 1.877794231289375 \cdot 10^{-05} X^7 - 0.002379939031153127 X^6 \\ & - 0.1596298583101837 X^5 - 5.958684697205818 X^4 - 116.3336704708797 X^3 \\ & - 885.1606719876373 X^2 + 1175.121582406799 X - 360.5751654247078 \\ = & -360.5751654247078 B_{0,8}(X) - 213.6849676238579 B_{1,8}(X) - 98.40765096542357 B_{2,8}(X) \\ & - 16.82060242209915 B_{3,8}(X) + 28.91366696631814 B_{4,8}(X) + 36.54467155974404 B_{5,8}(X) \\ & + 3.731015586800578 B_{6,8}(X) - 71.9626297312559 B_{7,8}(X) - 193.0686388102505 B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -1070.165401921332 X^2 + 1250.543498884255 X - 366.9199822310984 \\ &= -366.9199822310984 B_{0,2} + 258.3517672110292 B_{1,2} - 186.5418852681748 B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & \, 1.223655337688364 \cdot 10^{-300} X^8 - 5.205587704887534 \cdot 10^{-300} X^7 + 9.409306890019087 \cdot 10^{-300} X^6 \\ & - 9.282190734559383 \cdot 10^{-300} X^5 + 5.21941805196838 \cdot 10^{-300} X^4 - 1.541981168110934 \\ & \cdot 10^{-300} X^3 - 1070.165401921332 X^2 + 1250.543498884255 X - 366.9199822310984 \\ = & -366.9199822310984 B_{0,8} - 210.6020448705665 B_{1,8} - 92.50430043579644 B_{2,8} \\ & - 12.62674892678823 B_{3,8} + 29.03060965645814 B_{4,8} + 32.46777531394266 B_{5,8} \\ & - 2.315251954334665 B_{6,8} - 75.31847214837383 B_{7,8} - 186.5418852681748 B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.

Bounding polynomials M and m:

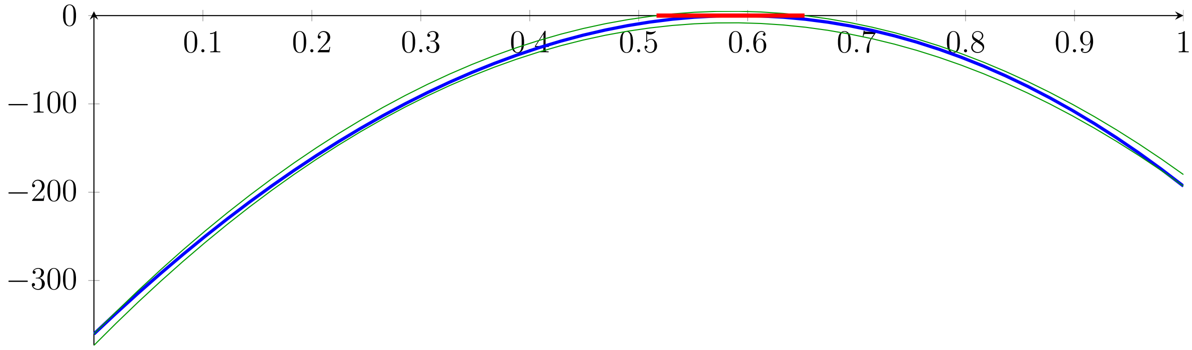
$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$

$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5163481231478299, 0.652203482649816]$$

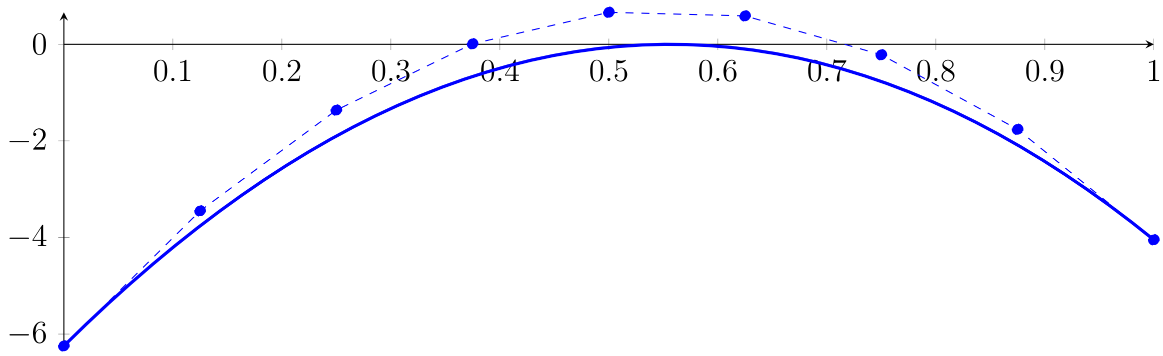
Longest intersection interval: 0.1358553595019861

$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

59.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.507567493156426$

Normalized monomial und Bézier representations and the Bézier polygon:

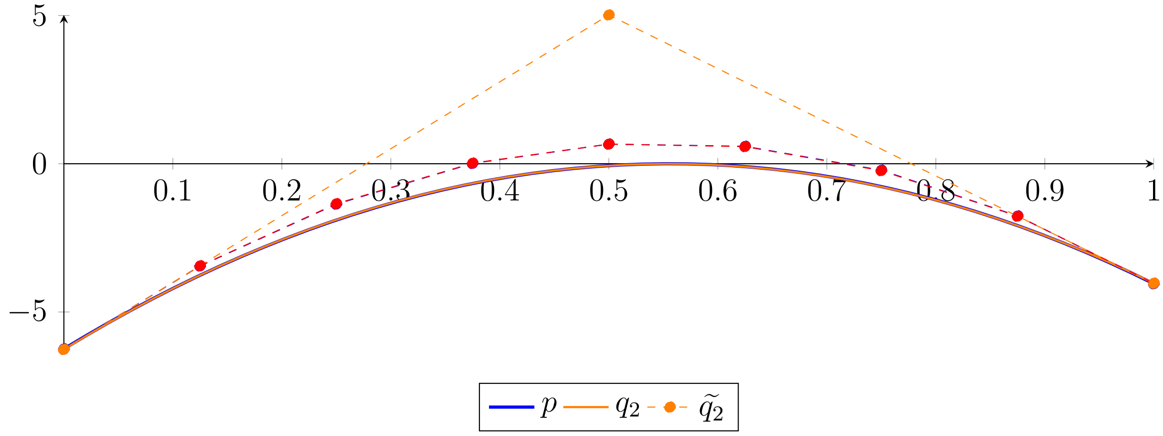
$$\begin{aligned} p = &-7.117496868854 28\cdot 10^{-15}X^8 - 1.625571369884896\cdot 10^{-11}X^7 - 1.539287417434116\cdot 10^{-08}X^6 \\ &- 7.733623372922873\cdot 10^{-06}X^5 - 0.002173482359120639X^4 - 0.3236423633498969X^3 \\ &- 19.84316397394663X^2 + 22.36613094108362X - 6.245496834711843 \\ = &-6.245496834711843B_{0,8}(X) - 3.449730467076391B_{1,8}(X) - 1.36264852708189B_{2,8}(X) \\ &+ 0.009969657354697074B_{3,8}(X) + 0.662313708568421B_{4,8}(X) + 0.5885420610459267B_{5,8}(X) \\ &- 0.217218177224708B_{6,8}(X) - 1.760871363955914B_{7,8}(X) - 4.048353462316386B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 2.334129178460356\cdot 10^{-302}X^8 - 9.797943630323215\cdot 10^{-302}X^7 + 1.744532667551292\cdot 10^{-301}X^6\\ &\quad - 1.695264192146682\cdot 10^{-301}X^5 + 9.412284928871313\cdot 10^{-302}X^4 - 2.760729238908791\\ &\quad \cdot 10^{-302}X^3 - 20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285\\ &= -6.261866081804285B_{0,8} - 3.441577100979207B_{1,8} - 1.347444096092966B_{2,8}\\ &\quad + 0.02053293285443696B_{3,8} + 0.6623539858630032B_{4,8} + 0.5780190629327322B_{5,8}\\ &\quad - 0.232471835936376B_{6,8} - 1.769118710744321B_{7,8} - 4.031921561491104B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.

Bounding polynomials M and m:

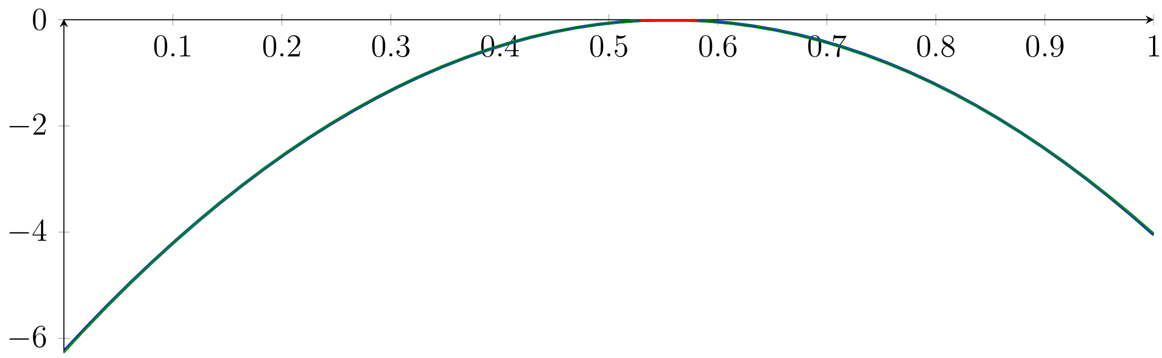
$$M = -20.33236732628744X^2 + 22.56231184660062X - 6.245434180979003$$

$$m = -20.33236732628744X^2 + 22.56231184660062X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

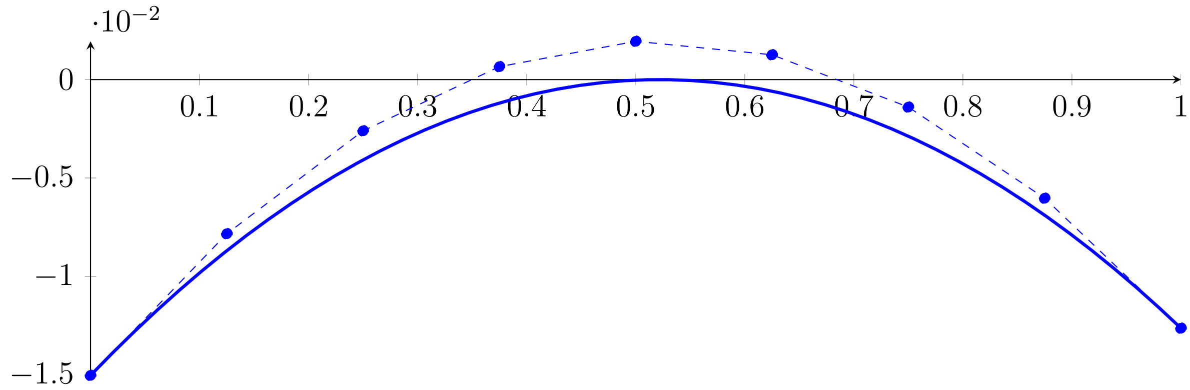
Longest intersection interval: 0.05205162765602134

$\implies$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

59.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Normalized monomial und Bézier representations and the Bézier polygon:

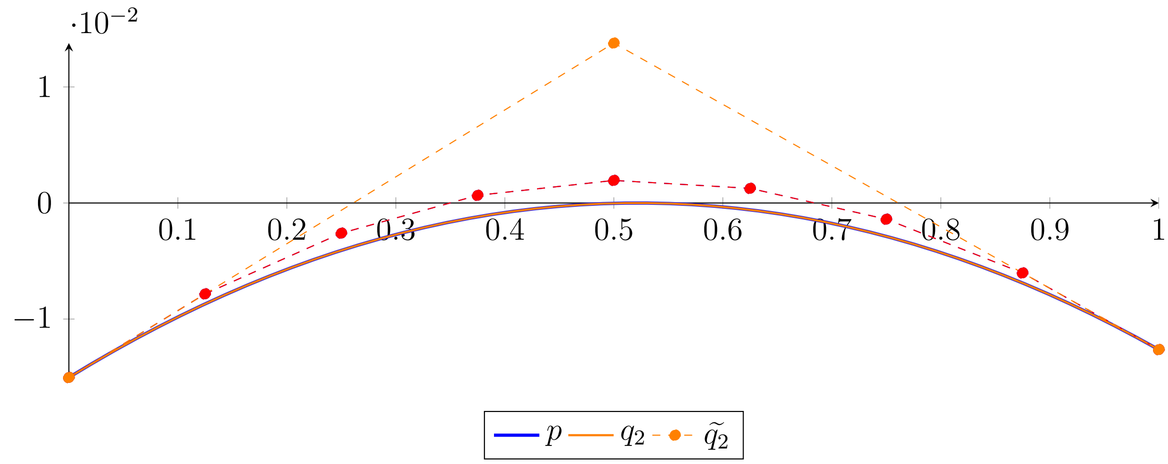
$$
\begin{aligned}
p = & -3.835321724591002 \cdot 10^{-25} X^8 - 1.685970393664051 \cdot 10^{-20} X^7 - 3.073417757909172 \cdot 10^{-16} X^6 \\
& - 2.973678167232232 \cdot 10^{-12} X^5 - 1.610545215791654 \cdot 10^{-08} X^4 - 4.629380451575473 \\
& \cdot 10^{-05} X^3 - 0.05516351610802166 X^2 + 0.05760784492782384 X - 0.01503353559864481 \\
= & -0.01503353559864481 B_{0,8}(X) - 0.007832554982666833 B_{1,8}(X) - 0.002601699941975341 B_{2,8}(X) \\
& + 0.0006582028483490249 B_{3,8}(X) + 0.001946326483147738 B_{4,8}(X) \\
& + 0.001261843827131283 B_{5,8}(X) - 0.001396072485173959 B_{6,8}(X) \\
& - 0.006028250049478829 B_{7,8}(X) - 0.01263551669178453 B_{8,8}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= -0.05523298442945254 X^2 + 0.05763563593870456 X - 0.01503585166965657 \\
&= -0.01503585166965657 B_{0,2} + 0.01378196629969571 B_{1,2} - 0.01263320016040455 B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = & \ 6.306459041947834 \cdot 10^{-305} X^8 - 2.608238530297949 \cdot 10^{-304} X^7 + 4.573872253581115 \cdot 10^{-304} X^6 \\
& - 4.384946099604334 \cdot 10^{-304} X^5 + 2.410615466544519 \cdot 10^{-304} X^4 - 7.036239809064398 \\
& \cdot 10^{-305} X^3 - 0.05523298442945254 X^2 + 0.05763563593870456 X - 0.01503585166965657 \\
= & -0.01503585166965657 B_{0,8} - 0.007831397177318504 B_{1,8} - 0.002599549271746596 B_{2,8} \\
& + 0.0006596920470591496 B_{3,8} + 0.001946326779098733 B_{4,8} + 0.001260354924372155 B_{5,8} \\
& - 0.001398223517120585 B_{6,8} - 0.006029408545379487 B_{7,8} - 0.01263320016040455 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 2.316531379978639 \cdot 10^{-06}$.

Bounding polynomials M and m:

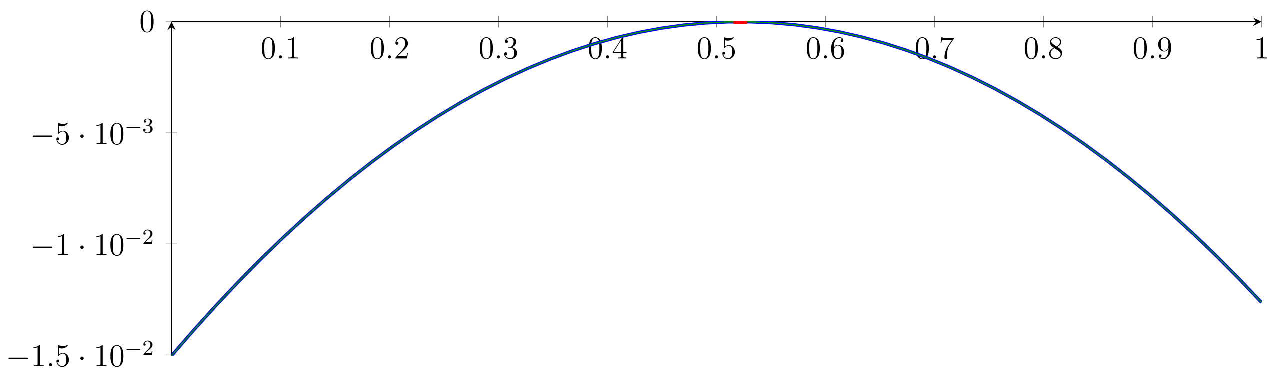
$$
M = -0.05523298442945254 X^2 + 0.05763563593870456 X - 0.0150335351382766
$$

$$m = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503816820103655$$

Root of M and m:

$$N(M) = \{0.5154882826278122, 0.5280120194846123\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5154882826278122, 0.5280120194846123]$$

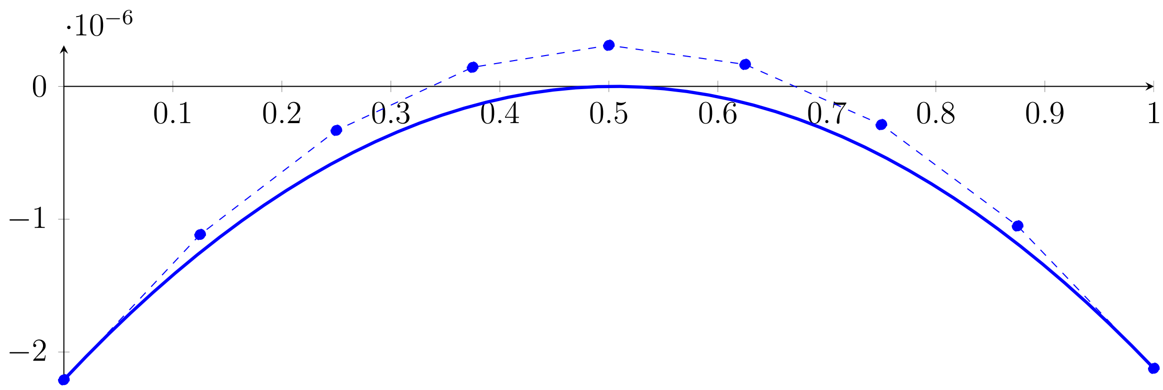
Longest intersection interval: 0.01252373685680011

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999944147887801, 0.5000055246634291]$,

59.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4999944147887801, 0.500005524\ldots$

Normalized monomial und Bézier representations and the Bézier polygon:

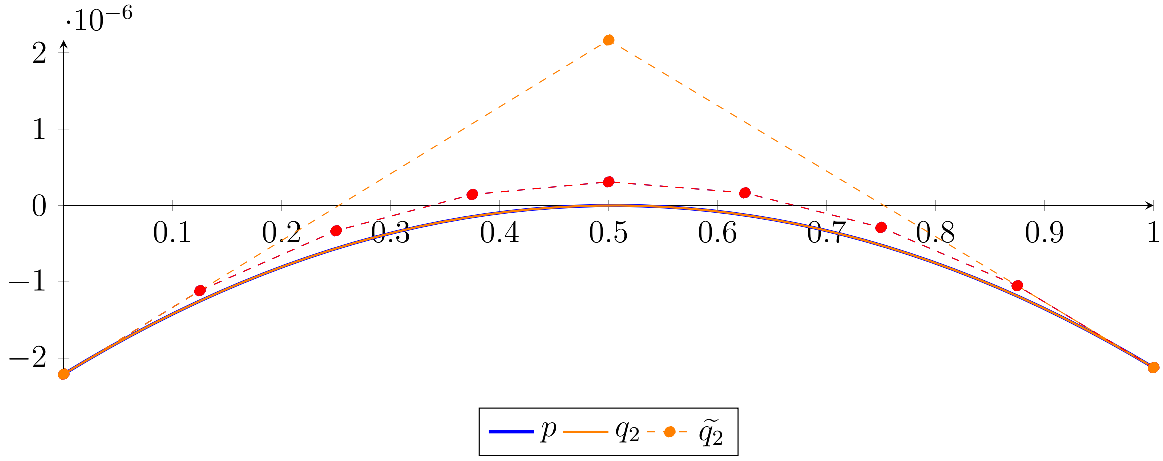
$$\begin{aligned} p = & -2.32099000990716 \cdot 10^{-40} X^8 - 8.147572265547704 \cdot 10^{-34} X^7 - 1.186072294581764 \cdot 10^{-27} X^6 \\ & - 9.164366615560225 \cdot 10^{-22} X^5 - 3.963832728671687 \cdot 10^{-16} X^4 - 9.099890718270267 \cdot 10^{-11} X^3 \\ & - 8.663298447262891 \cdot 10^{-06} X^2 + 8.749571419899133 \cdot 10^{-06} X - 2.209162411567082 \cdot 10^{-06} \\ = & -2.209162411567082 \cdot 10^{-06} B_{0,8}(X) - 1.11546598407969 \cdot 10^{-06} B_{1,8}(X) - 3.311730725659734 \\ & \cdot 10^{-07} B_{2,8}(X) + 1.437146979935834 \cdot 10^{-07} B_{3,8}(X) + 3.091957026128322 \\ & \cdot 10^{-07} B_{4,8}(X) + 1.652683162999621 \cdot 10^{-07} B_{5,8}(X) - 2.880690859425 \cdot 10^{-07} B_{6,8}(X) \\ & - 1.05081812911769 \cdot 10^{-06} B_{7,8}(X) - 2.122980438234407 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -8.663434946303181 \cdot 10^{-06} X^2 + 8.749626019605851 \cdot 10^{-06} X - 2.209166961546417 \cdot 10^{-06} \\ = & -2.209166961546417 \cdot 10^{-06} B_{0,2} + 2.165646048256509 \cdot 10^{-06} B_{1,2} - 2.122975888243747 \cdot 10^{-06} B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 9.84075548060145\cdot 10^{-309}X^8 - 4.039091668354938\cdot 10^{-308}X^7 + 7.030443670929191\cdot 10^{-308}X^6 \\ &\quad - 6.698784216784483\cdot 10^{-308}X^5 + 3.668633643168221\cdot 10^{-308}X^4 - 1.069248215117909\cdot 10^{-308}X^3 \\ &\quad - 8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209166961546417\cdot 10^{-06} \\ &= -2.209166961546417\cdot 10^{-06}B_{0,8} - 1.115463709095685\cdot 10^{-06}B_{1,8} - 3.311688475843534\cdot 10^{-07}B_{2,8} \\ &\quad + 1.437176229875793\cdot 10^{-07}B_{3,8} + 3.091957026201127\cdot 10^{-07}B_{4,8} + 1.652653913132468\cdot 10^{-07}B_{5,8} \\ &\quad - 2.880733109330184\cdot 10^{-07}B_{6,8} - 1.050820404118683\cdot 10^{-06}B_{7,8} - 2.122975888243747\cdot 10^{-06}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.549990660244254\cdot 10^{-12}$.

Bounding polynomials M and m:

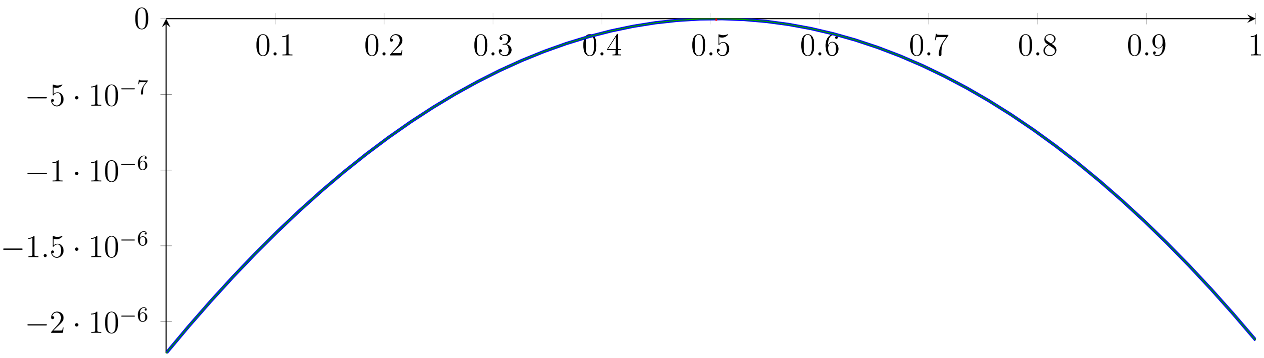
$$M = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209162411555757\cdot 10^{-06}$$

$$m = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209171511537077\cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{0.5041259457474217, 0.505822887923852\} \qquad N(m) = \{\}$$

Intersection intervals:



$[0.5041259457474217, 0.505822887923852]$

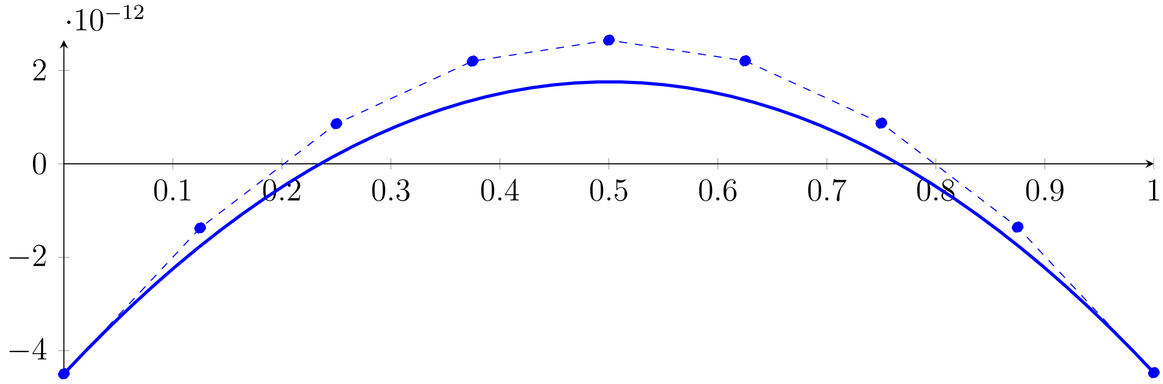
Longest intersection interval: 0.00169694217643035

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000155648447, 0.5000000344176595]$,

59.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.5000000155648447, 0.500000034$

Normalized monomial und Bézier representations and the Bézier polygon:

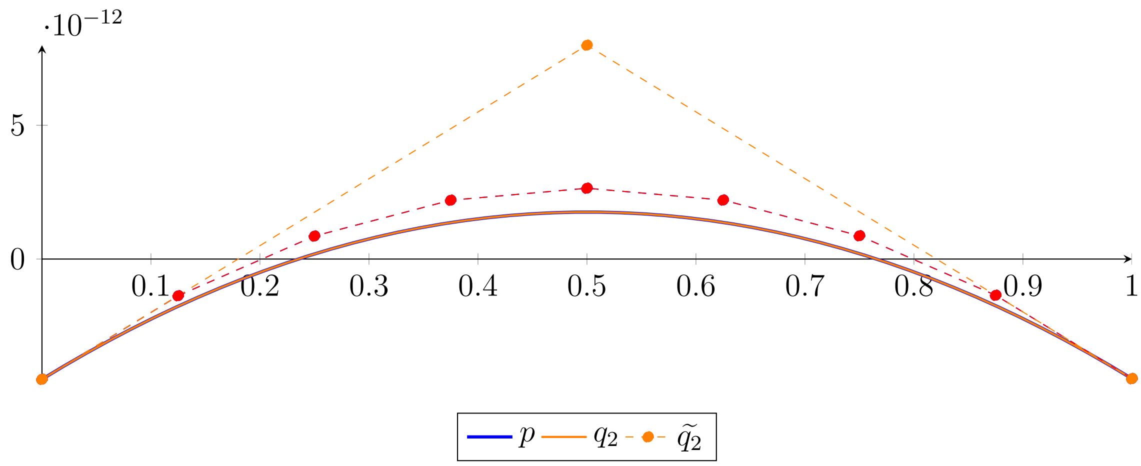
$$\begin{aligned} p = &-1.595914506899923\cdot 10^{-62}X^8 - 3.301399092257474\cdot 10^{-53}X^7 - 2.83213843356941\cdot 10^{-44}X^6 \\ &- 1.289553529043531\cdot 10^{-35}X^5 - 3.286896476505693\cdot 10^{-27}X^4 - 4.446733715730271\cdot 10^{-19}X^3 \\ &- 2.494734097474612\cdot 10^{-11}X^2 + 2.497049303064054\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12} \\ = &-4.493680200151054\cdot 10^{-12}B_{0,8}(X) - 1.372368571320986\cdot 10^{-12}B_{1,8}(X) + 8.579665941252918 \\ &\cdot 10^{-13}B_{2,8}(X) + 2.197325288247184\cdot 10^{-12}B_{3,8}(X) + 2.645707503104094\cdot 10^{-12}B_{4,8}(X) \\ &+ 2.203113230755426\cdot 10^{-12}B_{5,8}(X) + 8.695424632605847\cdot 10^{-13}B_{6,8}(X) \\ &- 1.355004807321027\cdot 10^{-12}B_{7,8}(X) - 4.470528588930005\cdot 10^{-12}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12} \\ &= -4.493680222384723\cdot 10^{-12}B_{0,2} + 7.991566426337562\cdot 10^{-12}B_{1,2} - 4.470528566696336\cdot 10^{-12}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = &\ 3.394595061927522\cdot 10^{-314}X^8 - 1.386657030412925\cdot 10^{-313}X^7 + 2.314937908564659\cdot 10^{-313}X^6 \\ &- 2.018101017184903\cdot 10^{-313}X^5 + 9.746671569929334\cdot 10^{-314}X^4 - 2.577278121543316\cdot 10^{-314}X^3 \\ &- 2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680222384723\cdot 10^{-12} \\ = &-4.493680222384723\cdot 10^{-12}B_{0,8} - 1.372368560204152\cdot 10^{-12}B_{1,8} + 8.579666147708415\cdot 10^{-13}B_{2,8} \\ &+ 2.197325302540257\cdot 10^{-12}B_{3,8} + 2.645707503104094\cdot 10^{-12}B_{4,8} + 2.203113216462353\cdot 10^{-12}B_{5,8} \\ &+ 8.695424426150349\cdot 10^{-13}B_{6,8} - 1.355004818437861\cdot 10^{-12}B_{7,8} - 4.470528566696336\cdot 10^{-12}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.223366895429667\cdot 10^{-20}$.

Bounding polynomials M and m:

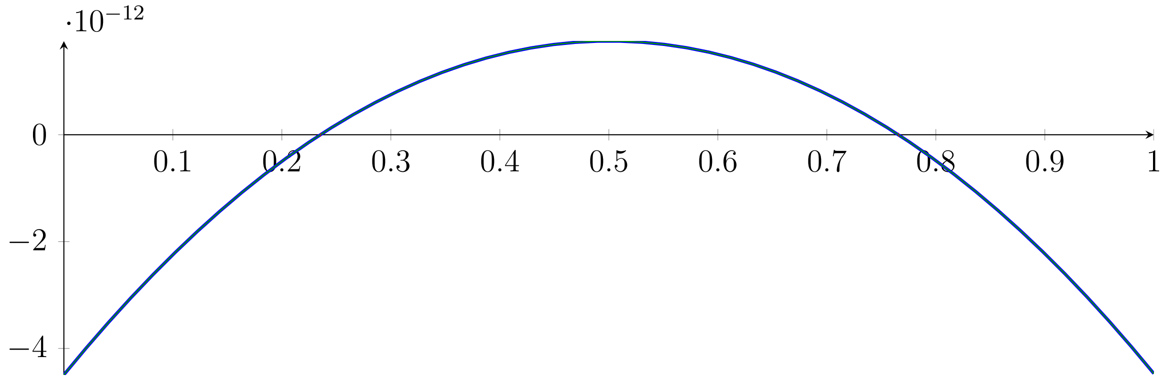
$$M = -2.494734164175618\cdot 10^{-11}X^2 + 2.497049329744457\cdot 10^{-11}X - 4.493680200151054\cdot 10^{-12}$$

$$m = -2.494734164175618 \cdot 10^{-11} X^2 + 2.497049329744457 \cdot 10^{-11} X - 4.493680244618392 \cdot 10^{-12}$$

Root of M and m:

$$N(M) = \{0.235251622185842, 0.765676398764079\} \quad N(m) = \{0.2352516255462581, 0.7656763954036629\}$$

Intersection intervals:



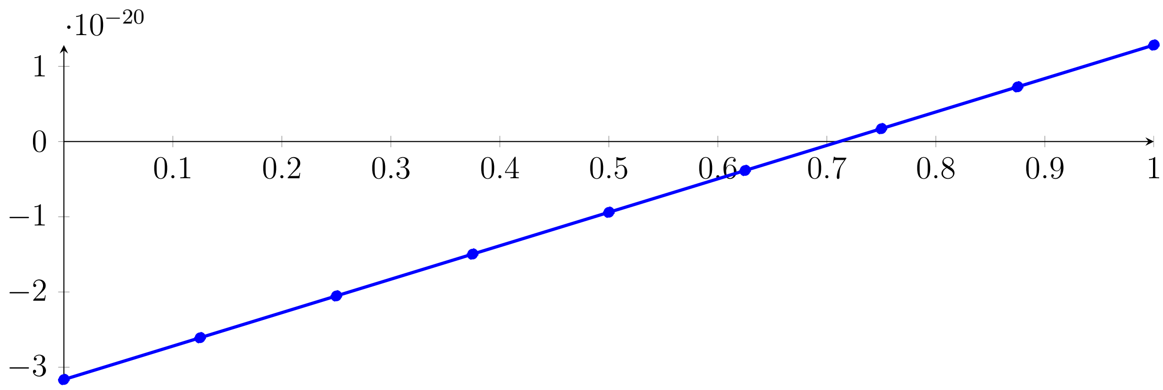
$$[0.235251622185842, 0.2352516255462581], \; [0.7656763954036629, 0.765676398764079]$$

Longest intersection interval: $3.360416099786671 \cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.50000002]$, interval 2: $[0.50000003, 0.500000030000$

59.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000$

Normalized monomial und Bézier representations and the Bézier polygon:

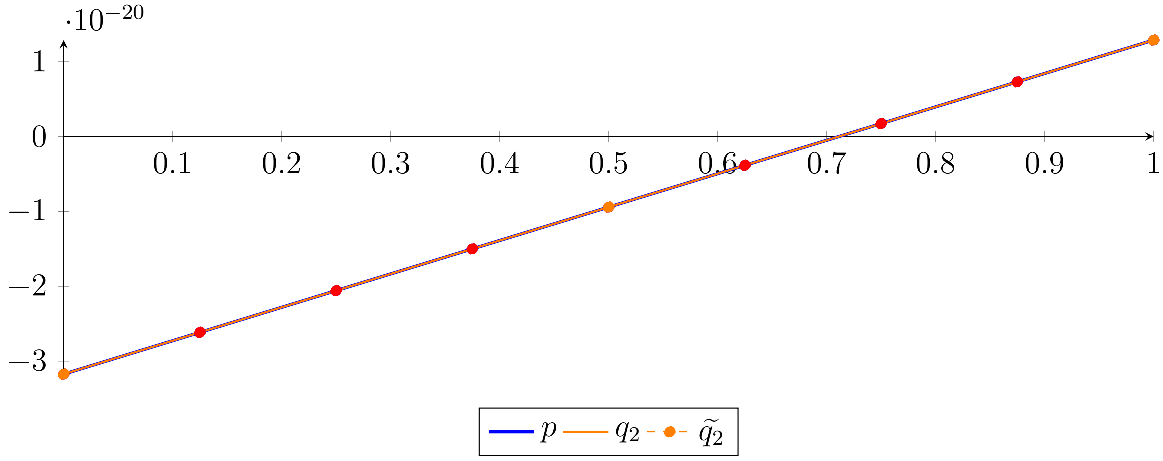
$$\begin{aligned} p = &-2.595099873436077 \cdot 10^{-130} X^8 - 1.59753148120066 \cdot 10^{-112} X^7 - 4.078240366489356 \cdot 10^{-95} X^6 \\ &- 5.52592084177806 \cdot 10^{-78} X^5 - 4.191391755953253 \cdot 10^{-61} X^4 - 1.687408749214622 \cdot 10^{-44} X^3 \\ &- 2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20} \\ = &-3.164099743671992 \cdot 10^{-20} B_{0,8}(X) - 2.608258017418989 \cdot 10^{-20} B_{1,8}(X) - 2.052416292172112 \\ &\cdot 10^{-20} B_{2,8}(X) - 1.49657456793136 \cdot 10^{-20} B_{3,8}(X) - 9.407328446967348 \cdot 10^{-21} B_{4,8}(X) \\ &- 3.848911224682354 \cdot 10^{-21} B_{5,8}(X) + 1.709505987541381 \cdot 10^{-21} B_{6,8}(X) \\ &+ 7.267923189703856 \cdot 10^{-21} B_{7,8}(X) + 1.282634038180507 \cdot 10^{-20} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20} \\ &= -3.164099743671992 \cdot 10^{-20} B_{0,2} - 9.407328386599791 \cdot 10^{-21} B_{1,2} + 1.282634038180507 \cdot 10^{-20} B_{2,2} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 &= -2.04607227392422\cdot 10^{-323}X^8 + 3.432164034855183\cdot 10^{-323}X^7 + 6.927895597874022\cdot 10^{-323}X^6 \\
&\quad - 2.227227667041084\cdot 10^{-322}X^5 + 2.093778075420854\cdot 10^{-322}X^4 - 7.695410174165002\cdot 10^{-323}X^3 \\
&\quad - 2.817152660512407\cdot 10^{-28}X^2 + 4.446733810024026\cdot 10^{-20}X - 3.164099743671992\cdot 10^{-20} \\
&= -3.164099743671992\cdot 10^{-20}B_{0,8} - 2.608258017418989\cdot 10^{-20}B_{1,8} - 2.052416292172112\cdot 10^{-20}B_{2,8} \\
&\quad - 1.49657456793136\cdot 10^{-20}B_{3,8} - 9.407328446967348\cdot 10^{-21}B_{4,8} - 3.848911224682354\cdot 10^{-21}B_{5,8} \\
&\quad + 1.709505987541381\cdot 10^{-21}B_{6,8} + 7.267923189703856\cdot 10^{-21}B_{7,8} + 1.282634038180507\cdot 10^{-20}B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 8.437043746073111\cdot 10^{-46}$.

Bounding polynomials M and m:

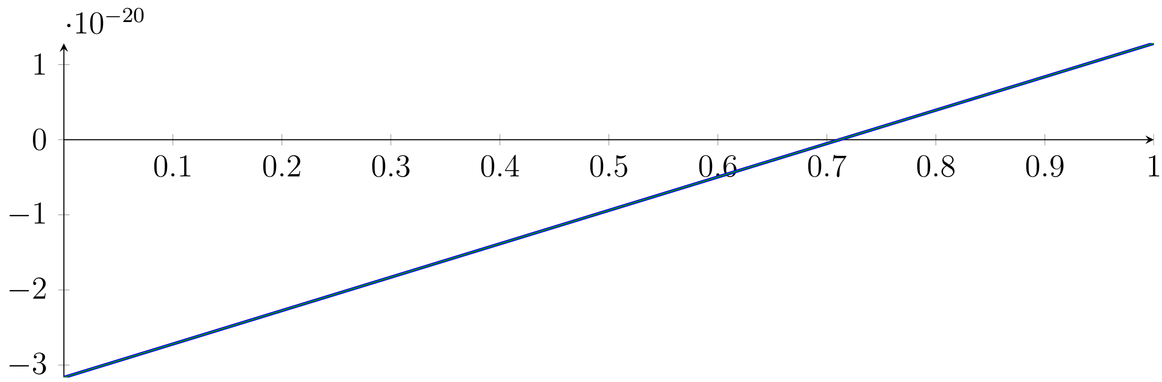
$$M = -2.817152660512407\cdot 10^{-28}X^2 + 4.446733810024026\cdot 10^{-20}X - 3.164099743671992\cdot 10^{-20}$$

$$m = -2.817152660512407\cdot 10^{-28}X^2 + 4.446733810024026\cdot 10^{-20}X - 3.164099743671992\cdot 10^{-20}$$

Root of M and m:

$N(M) = \{0.7115559179195557, 157844970.6438557\}$ $\quad N(m) = \{0.7115559179195557, 157844970.6438557\}$

Intersection intervals:



$[0.7115559179195557, 0.7115559179195557]$

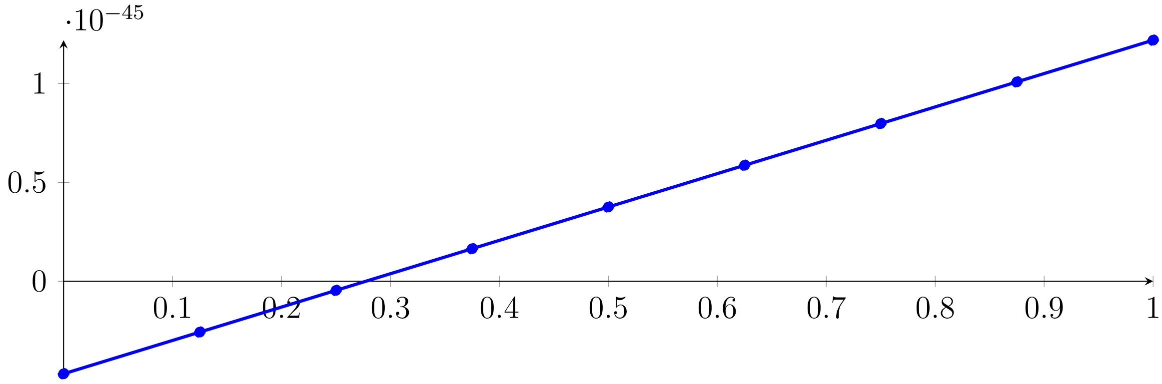
Longest intersection interval: $3.794715034716651\cdot 10^{-26}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

59.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

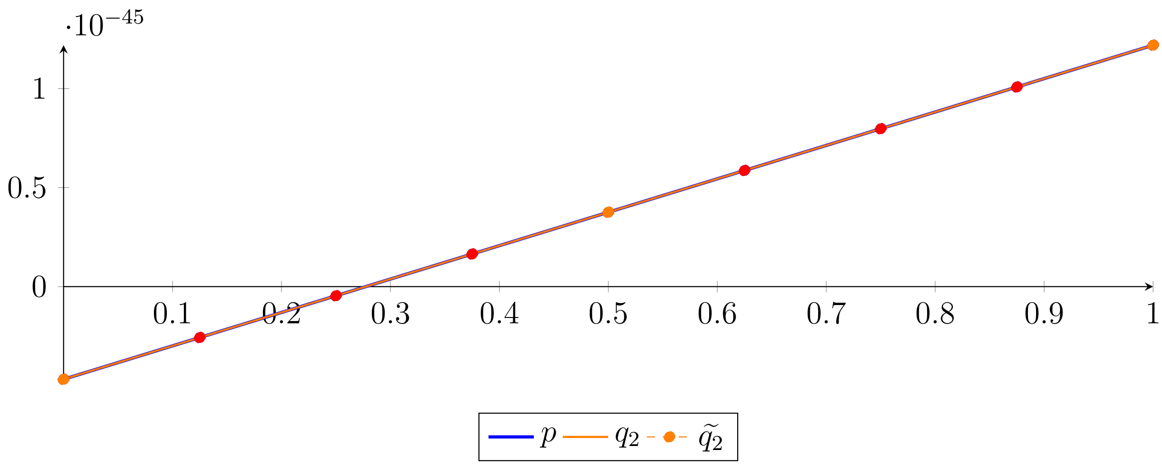
$$\begin{aligned}p &= -1.115802731757248\cdot10^{-333}X^8 - 1.81010430406932\cdot10^{-290}X^7 - 1.217721087493713\cdot10^{-247}X^6\\ &\quad - 4.348109767863853\cdot10^{-205}X^5 - 8.691103598298647\cdot10^{-163}X^4 - 9.22057066454039\cdot10^{-121}X^3\\ &\quad - 4.056661009282407\cdot10^{-79}X^2 + 1.687408749214622\cdot10^{-45}X - 4.680026339204605\cdot10^{-46}\\ &= -4.680026339204605\cdot10^{-46}B_{0,8}(X) - 2.570765402686327\cdot10^{-46}B_{1,8}(X) - 4.615044661680489\\ &\quad\cdot10^{-47}B_{2,8}(X) + 1.647756470350229\cdot10^{-46}B_{3,8}(X) + 3.757017406868507\cdot10^{-46}B_{4,8}(X)\\ &\quad + 5.866278343386785\cdot10^{-46}B_{5,8}(X) + 7.975539279905062\cdot10^{-46}B_{6,8}(X)\\ &\quad + 1.008480021642334\cdot10^{-45}B_{7,8}(X) + 1.219406115294162\cdot10^{-45}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -4.056661009282407\cdot10^{-79}X^2 + 1.687408749214622\cdot10^{-45}X - 4.680026339204605\cdot10^{-46}\\ &= -4.680026339204605\cdot10^{-46}B_{0,2} + 3.757017406868507\cdot10^{-46}B_{1,2} + 1.219406115294162\cdot10^{-45}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 1.583039910757457\cdot10^{-348}X^8 - 8.209830365232051\cdot10^{-348}X^7 + 1.523964084824618\cdot10^{-347}X^6\\ &\quad - 1.276185009399131\cdot10^{-347}X^5 + 4.899925340940039\cdot10^{-348}X^4 - 9.411348806295522\cdot10^{-349}X^3\\ &\quad - 4.056661009282407\cdot10^{-79}X^2 + 1.687408749214622\cdot10^{-45}X - 4.680026339204605\cdot10^{-46}\\ &= -4.680026339204605\cdot10^{-46}B_{0,8} - 2.570765402686327\cdot10^{-46}B_{1,8} - 4.615044661680489\cdot10^{-47}B_{2,8}\\ &\quad + 1.647756470350229\cdot10^{-46}B_{3,8} + 3.757017406868507\cdot10^{-46}B_{4,8} + 5.866278343386785\cdot10^{-46}B_{5,8}\\ &\quad + 7.975539279905062\cdot10^{-46}B_{6,8} + 1.008480021642334\cdot10^{-45}B_{7,8} + 1.219406115294162\cdot10^{-45}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.610285332270195\cdot10^{-122}$.

Bounding polynomials M and m:

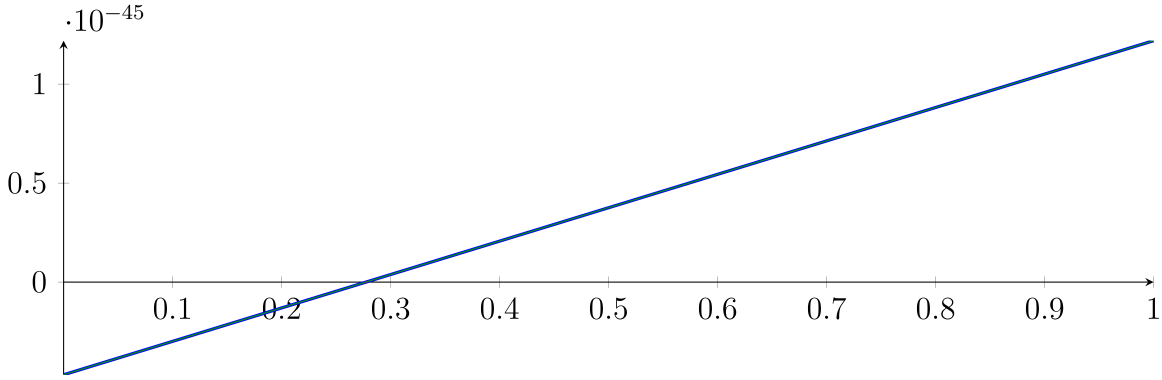
$$M = -4.056661009282407\cdot10^{-79}X^2 + 1.687408749214622\cdot10^{-45}X - 4.680026339204605\cdot10^{-46}$$

$$m = -4.056661009282407 \cdot 10^{-79} X^2 + 1.687408749214622 \cdot 10^{-45} X - 4.680026339204605 \cdot 10^{-46}$$

Root of M and m:

$N(M) = \{0.2773498917427594, 4.159600088233924 \cdot 10^{33}\}$ $N(m) = \{0.2773498917427594, 4.15960008823392$

Intersection intervals:



$$[0.2773498917427594, 0.2773498917427594]$$

Longest intersection interval: $5.464337356809344 \cdot 10^{-77}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

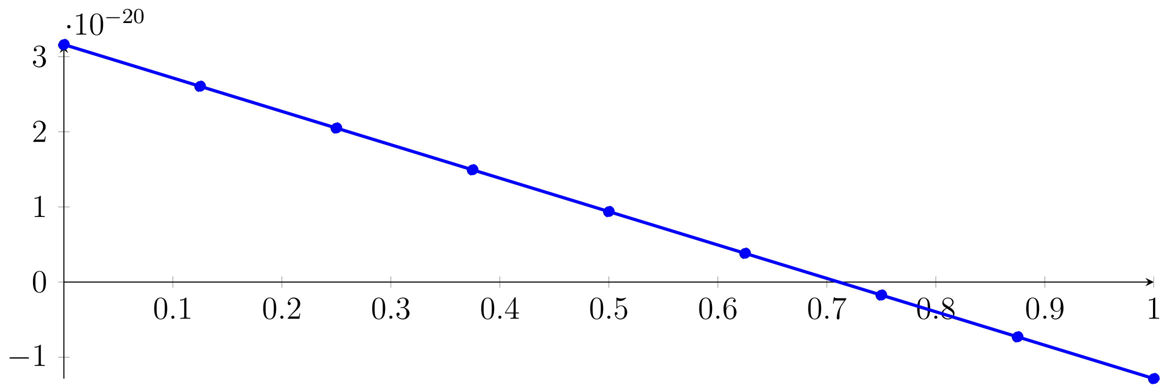
59.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 10!

59.11 Recursion Branch 1 1 1 1 1 1 1 2 in Interval 2: $[0.50000003, 0.5000000300000$

Normalized monomial und Bézier representations and the Bézier polygon:

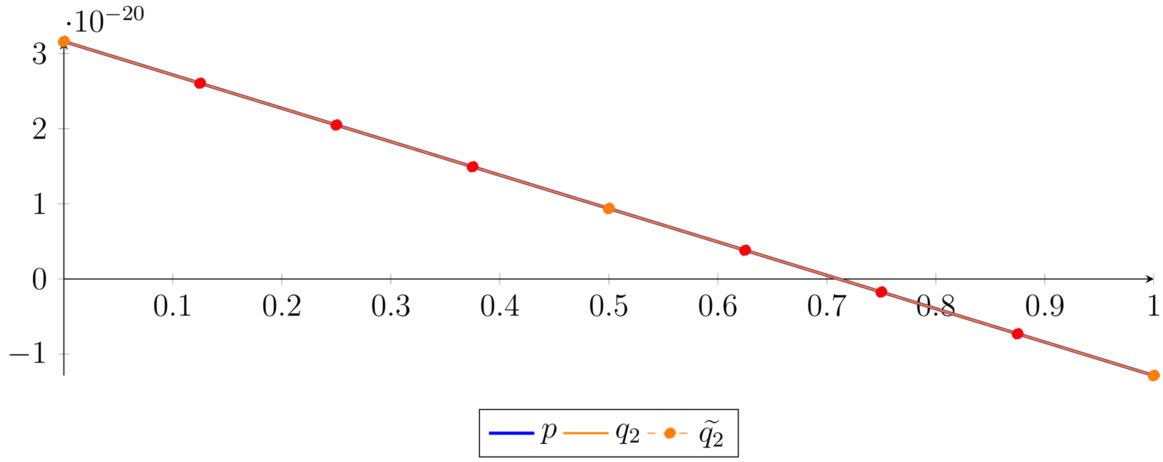
$$\begin{aligned} p = & -2.595099873436077 \cdot 10^{-130} X^8 - 1.597531484477648 \cdot 10^{-112} X^7 - 4.078240384140717 \cdot 10^{-95} X^6 \\ & - 5.525920880401843 \cdot 10^{-78} X^5 - 4.191391799565194 \cdot 10^{-61} X^4 - 1.687408775678226 \cdot 10^{-44} X^3 \\ & - 2.817152740417101 \cdot 10^{-28} X^2 - 4.446733771915308 \cdot 10^{-20} X + 3.161581953700541 \cdot 10^{-20} \\ = & \, 3.161581953700541 \cdot 10^{-20} B_{0,8}(X) + 2.605740232211127 \cdot 10^{-20} B_{1,8}(X) + 2.049898509715588 \\ & \cdot 10^{-20} B_{2,8}(X) + 1.494056786213922 \cdot 10^{-20} B_{3,8}(X) + 9.382150617061309 \cdot 10^{-21} B_{4,8}(X) \\ & + 3.823733361922135 \cdot 10^{-21} B_{5,8}(X) - 1.734683903278299 \cdot 10^{-21} B_{6,8}(X) \\ & - 7.293101178539992 \cdot 10^{-21} B_{7,8}(X) - 1.285151846386295 \cdot 10^{-20} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20} \\ &= 3.161581953700541\cdot 10^{-20}B_{0,2} + 9.382150677428867\cdot 10^{-21}B_{1,2} - 1.285151846386295\cdot 10^{-20}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 2.039046071807459\cdot 10^{-323}X^8 - 3.404903576857106\cdot 10^{-323}X^7 - 6.962513967101668\cdot 10^{-323}X^6 \\ &\quad + 2.228747497885225\cdot 10^{-322}X^5 - 2.092722637334645\cdot 10^{-322}X^4 + 7.6903440713512\cdot 10^{-323}X^3 \\ &\quad - 2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20} \\ &= 3.161581953700541\cdot 10^{-20}B_{0,8} + 2.605740232211127\cdot 10^{-20}B_{1,8} + 2.049898509715588\cdot 10^{-20}B_{2,8} \\ &\quad + 1.494056786213922\cdot 10^{-20}B_{3,8} + 9.382150617061309\cdot 10^{-21}B_{4,8} + 3.823733361922135\cdot 10^{-21}B_{5,8} \\ &\quad - 1.734683903278299\cdot 10^{-21}B_{6,8} - 7.293101178539992\cdot 10^{-21}B_{7,8} - 1.285151846386295\cdot 10^{-20}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.437043878391132\cdot 10^{-46}$.

Bounding polynomials M and m:

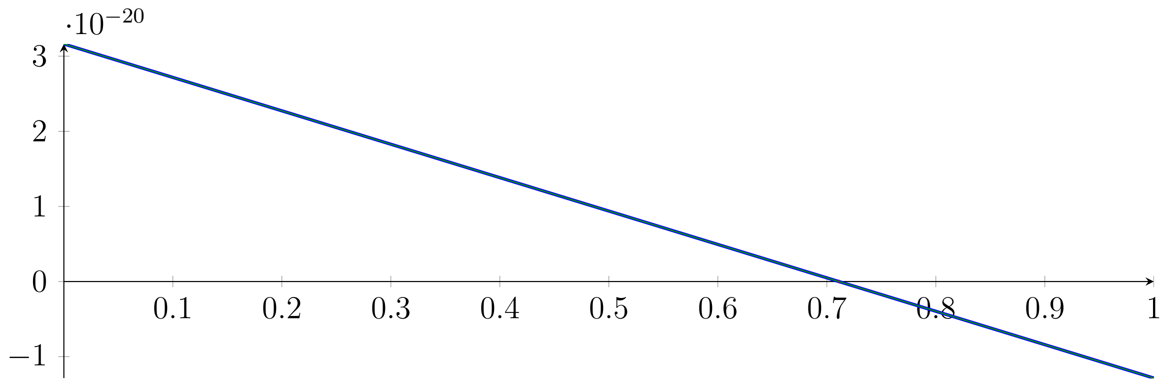
$$M = -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}$$

$$m = -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}$$

Root of M and m:

$N(M) = \{-157844966.2366053, 0.7109897065184296\}$ $\quad N(m) = \{-157844966.2366053, 0.7109897065184296\}$

Intersection intervals:



$$[0.7109897065184296, 0.7109897065184296]$$

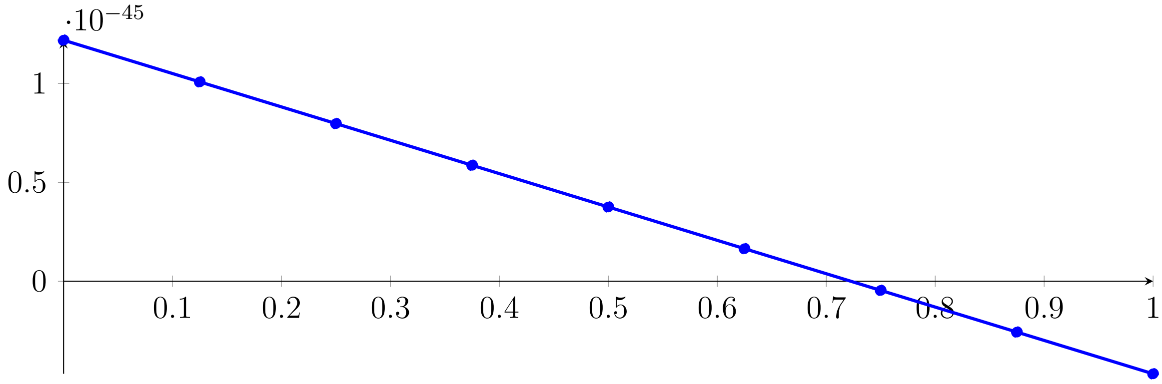
Longest intersection interval: $3.794715058351799\cdot 10^{-26}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

59.12 Recursion Branch 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

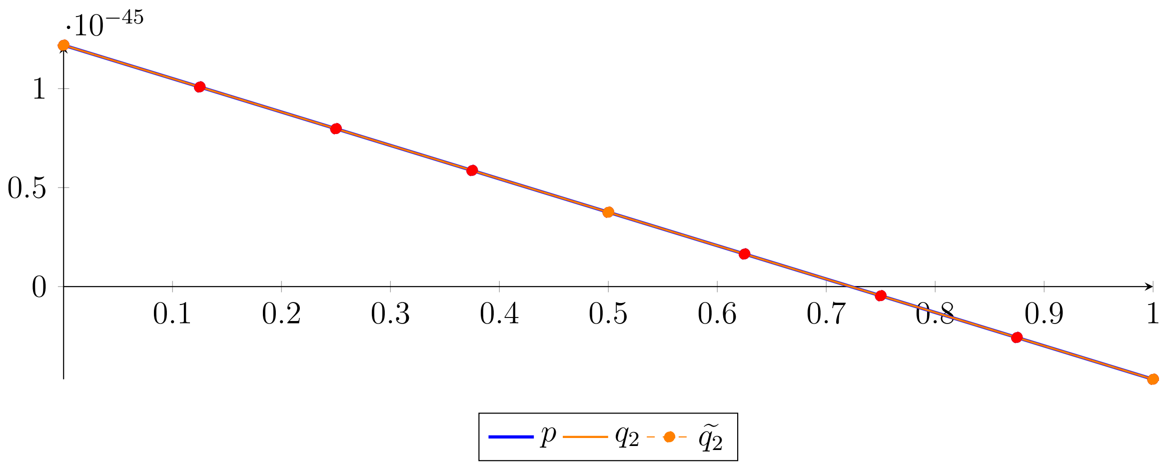
$$p = -1.115802787354916\cdot 10^{-333}X^8 - 1.810104386701217\cdot 10^{-290}X^7 - 1.217721138271235\cdot 10^{-247}X^6 - 4.348109933664918\cdot 10^{-205}X^5 - 8.691103905258648\cdot 10^{-163}X^4 - 9.220570981435719\cdot 10^{-121}X^3 - 4.056661174877394\cdot 10^{-79}X^2 - 1.687408775678226\cdot 10^{-45}X + 1.219252393140717\cdot 10^{-45}$$
$$= 1.219252393140717\cdot 10^{-45}B_{0,8}(X) + 1.008326296180938\cdot 10^{-45}B_{1,8}(X) + 7.974001992211601\cdot 10^{-46}B_{2,8}(X) + 5.864741022613818\cdot 10^{-46}B_{3,8}(X) + 3.755480053016035\cdot 10^{-46}B_{4,8}(X) + 1.646219083418252\cdot 10^{-46}B_{5,8}(X) - 4.630418861795306\cdot 10^{-47}B_{6,8}(X) - 2.572302855777314\cdot 10^{-46}B_{7,8}(X) - 4.681563825375097\cdot 10^{-46}B_{8,8}(X)$$



Degree reduction and raising:

$$q_2 = -4.056661174877394\cdot 10^{-79}X^2 - 1.687408775678226\cdot 10^{-45}X + 1.219252393140717\cdot 10^{-45}$$
$$= 1.219252393140717\cdot 10^{-45}B_{0,2} + 3.755480053016035\cdot 10^{-46}B_{1,2} - 4.681563825375097\cdot 10^{-46}B_{2,2}$$

$$\widetilde{q}_2 = 8.361910453367177\cdot 10^{-349}X^8 - 1.541597891866099\cdot 10^{-348}X^7 - 2.313332235573428\cdot 10^{-348}X^6 + 8.342313869995998\cdot 10^{-348}X^5 - 8.027038050174711\cdot 10^{-348}X^4 + 2.973130645625176\cdot 10^{-348}X^3 - 4.056661174877394\cdot 10^{-79}X^2 - 1.687408775678226\cdot 10^{-45}X + 1.219252393140717\cdot 10^{-45}$$
$$= 1.219252393140717\cdot 10^{-45}B_{0,8} + 1.008326296180938\cdot 10^{-45}B_{1,8} + 7.974001992211601\cdot 10^{-46}B_{2,8} + 5.864741022613818\cdot 10^{-46}B_{3,8} + 3.755480053016035\cdot 10^{-46}B_{4,8} + 1.646219083418252\cdot 10^{-46}B_{5,8} - 4.630418861795306\cdot 10^{-47}B_{6,8} - 2.572302855777314\cdot 10^{-46}B_{7,8} - 4.681563825375097\cdot 10^{-46}B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 4.61028549071786\cdot 10^{-122}$.

Bounding polynomials M and m:

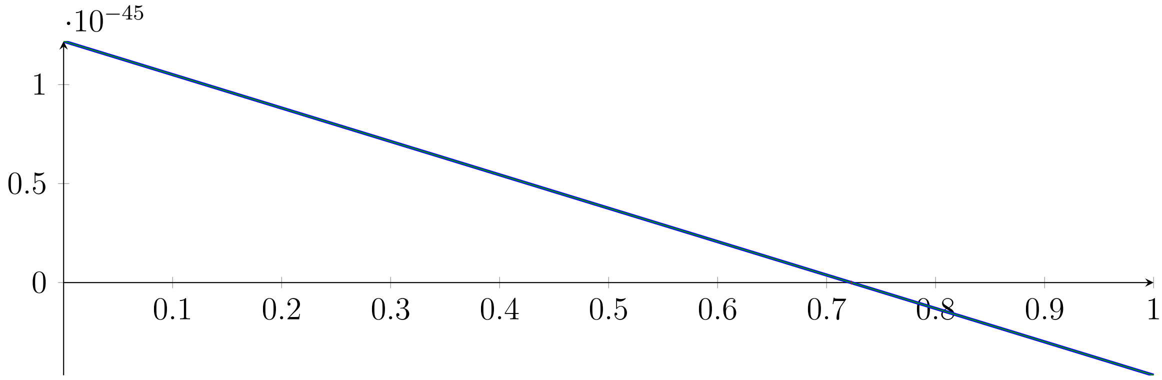
$$M = -4.056661174877394\cdot 10^{-79}X^2 - 1.687408775678226\cdot 10^{-45}X + 1.219252393140717\cdot 10^{-45}$$

$$m = -4.056661174877394 \cdot 10^{-79} X^2 - 1.687408775678226 \cdot 10^{-45} X + 1.219252393140717 \cdot 10^{-45}$$

Root of M and m:

$N(M) = \{-4.159599983671857 \cdot 10^{33}, 0.7225589973897452\}$ $N(m) = \{-4.159599983671857 \cdot 10^{33}, 0.72255899$

Intersection intervals:



$$[0.7225589973897452, 0.7225589973897452]$$

Longest intersection interval: $5.464337458912208 \cdot 10^{-77}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

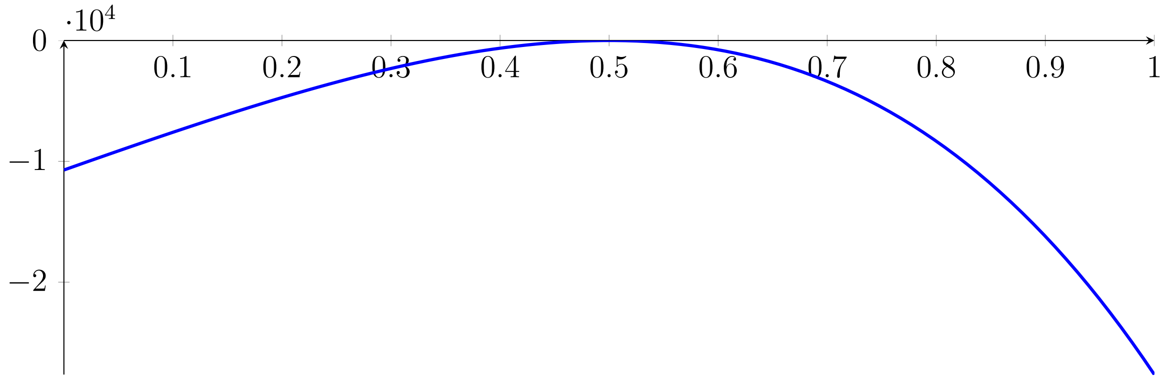
59.13 Recursion Branch 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 10!

59.14 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

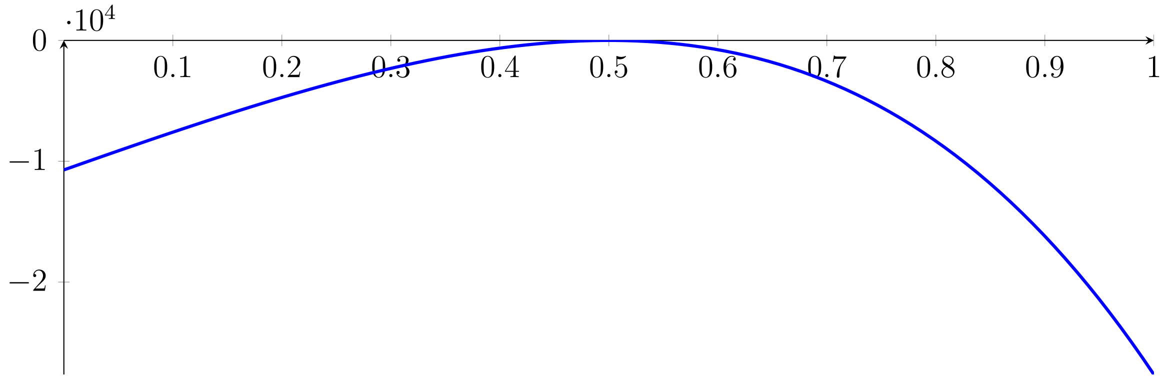
with precision $\varepsilon = 1 \cdot 10^{-64}$.

60 Running CubeClip on h_8 with epsilon 64

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called CubeClip with input polynomial on interval $[0, 1]$:

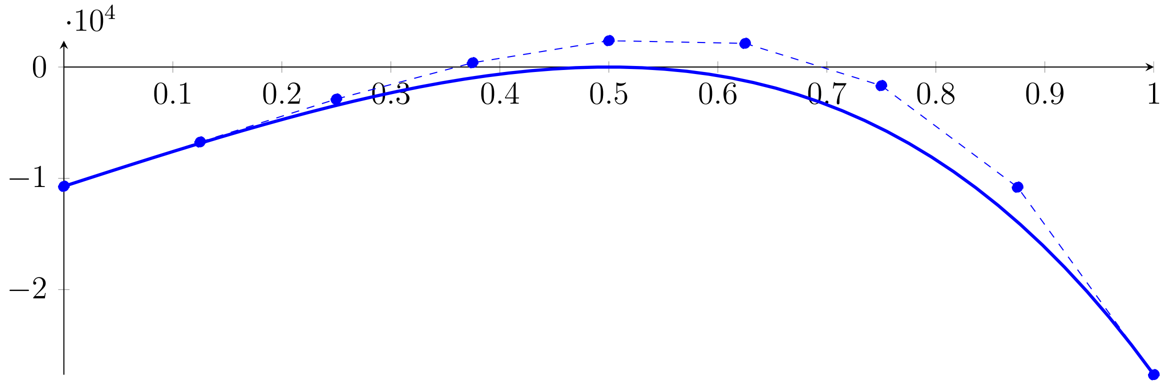
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



60.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

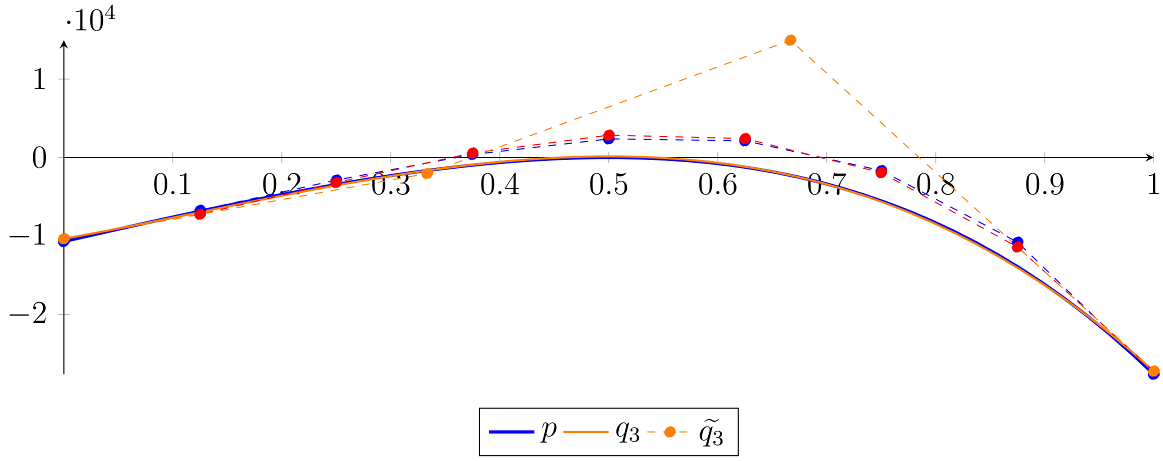
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ = &-10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &+ 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -5.703152099307112\cdot 10^{-299}X^8 + 1.997854926822688\cdot 10^{-298}X^7 - 2.620079279971631 \\
&\quad \cdot 10^{-298}X^6 + 1.512761996617561\cdot 10^{-298}X^5 - 2.636105166170616\cdot 10^{-299}X^4 \\
&\quad - 67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\
&= -10364.86272786554B_{0,8} - 7243.22568371353B_{1,8} - 3192.719831416224B_{2,8} \\
&\quad + 574.7343076805747B_{3,8} + 2847.216212231063B_{4,8} + 2412.80536088944B_{5,8} \\
&\quad - 1940.418767690097B_{6,8} - 11424.37669485335B_{7,8} - 27250.98894194612B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

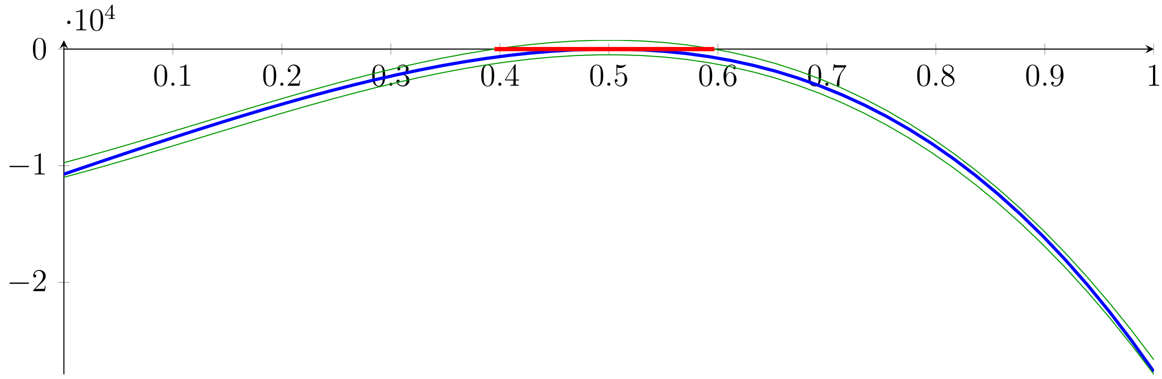
$$M = -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 9740.484261812251$$

$$m = -67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $\quad N(m) = \{-0.623487965792376$

Intersection intervals:



$[0.3950604114095972, 0.5968461976869074]$

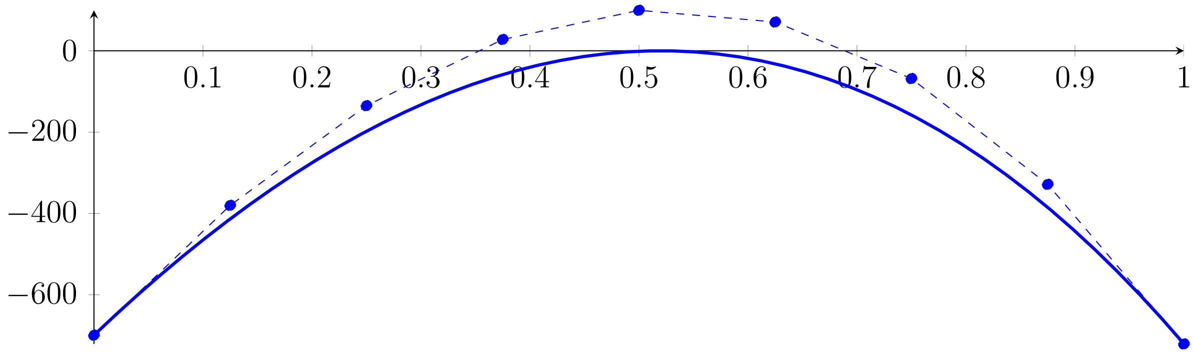
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

60.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

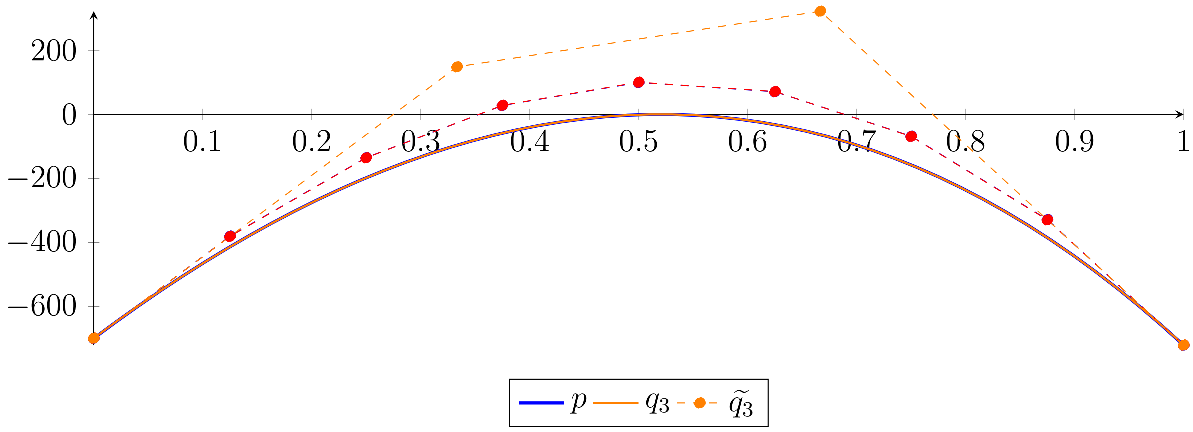
$$p = -2.748682461629333 \cdot 10^{-06} X^8 - 0.0005198138726602274 X^7 - 0.04066637698425721 X^6 - 1.681520506726176 X^5 - 38.59639865464246 X^4 - 460.2831920090805 X^3 - 2074.780222477586 X^2 + 2553.796807278985 X - 699.3475151603227$$
$$= -699.3475151603227 B_{0,8}(X) - 380.1229142504495 B_{1,8}(X) - 134.9976070004902 B_{2,8}(X) + 27.8090638751075 B_{3,8}(X) + 99.52637853825791 B_{4,8}(X) + 70.80221287533186 B_{5,8}(X) - 68.32844102535583 B_{6,8}(X) - 328.4764717433994 B_{7,8}(X) - 720.9332304689123 B_{8,8}(X)$$



Degree reduction and raising:

$$q_3 = -542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 698.7407882409323$$
$$= -698.7407882409323 B_{0,3} + 148.5035278252892 B_{1,3} + 322.0745267793334 B_{2,3} - 720.3121660793493 B_{3,3}$$

$$\tilde{q}_3 = -2.24002635483637 \cdot 10^{-300} X^8 + 8.593407551274415 \cdot 10^{-300} X^7 - 1.289726574039331 \cdot 10^{-299} X^6 + 9.445830854385802 \cdot 10^{-300} X^5 - 3.303106122422159 \cdot 10^{-300} X^4 - 542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 698.7407882409323$$
$$= -698.7407882409323 B_{0,8} - 381.0241697160992 B_{1,8} - 135.4868351675709 B_{2,8} + 28.18756585642868 B_{3,8} + 100.3153838076753 B_{4,8} + 71.21296913794494 B_{5,8} - 68.80332770098655 B_{6,8} - 329.4171562573433 B_{7,8} - 720.3121660793493 B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

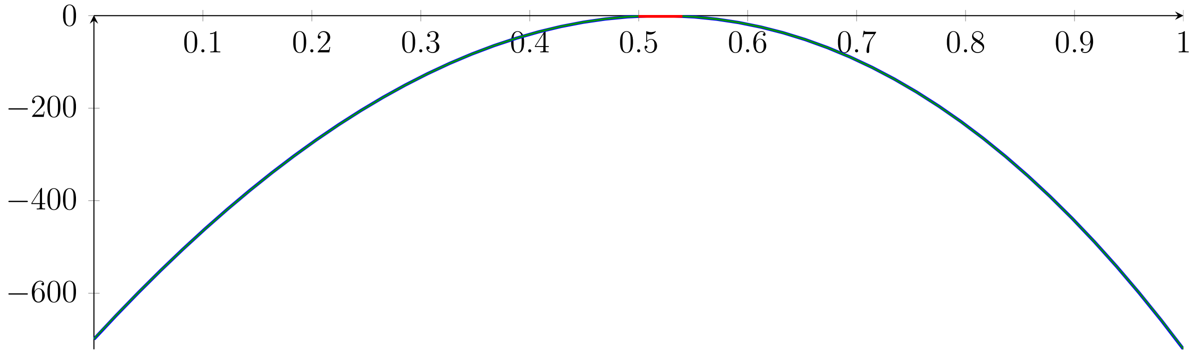
Bounding polynomials M and m:

$$M = -542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 697.8001037269884$$
$$m = -542.2843747005496 X^3 - 2021.019951336532 X^2 + 2541.732948198665 X - 699.6814727548762$$

Root of M and m:

$$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\} \quad N(m) = \{-4.76690070753073\}$$

Intersection intervals:



$[0.4997746332555131, 0.5401382492675976]$

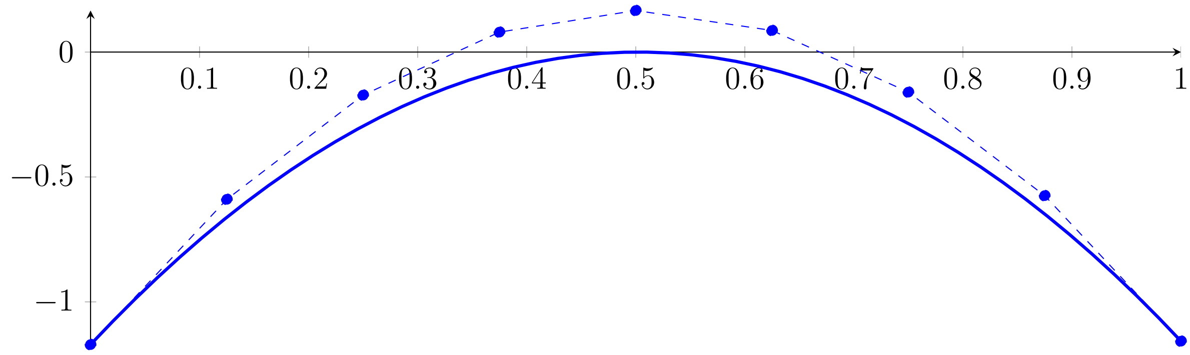
Longest intersection interval: 0.04036361601208447

$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

60.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Normalized monomial und Bézier representations and the Bézier polygon:

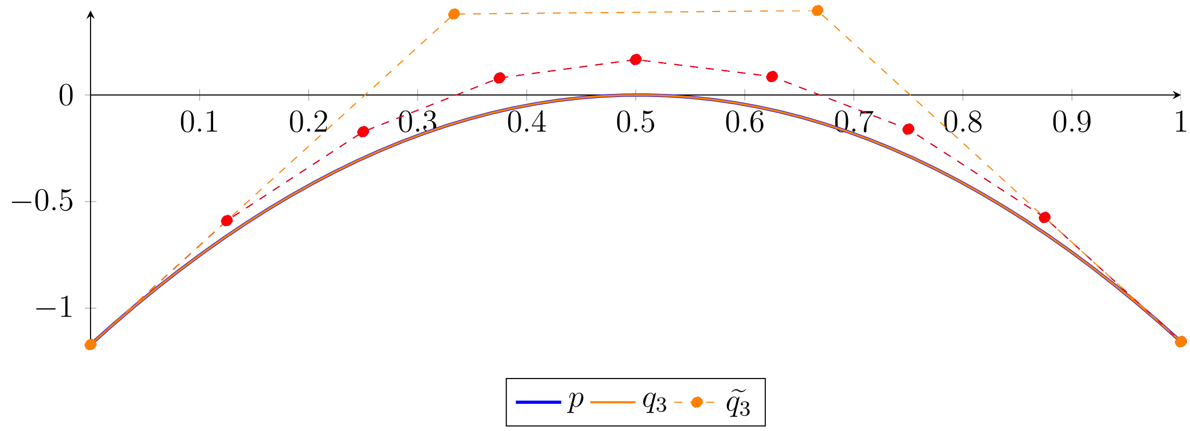
$$
\begin{aligned}
p &= -1.936623061789877 \cdot 10^{-17} X^8 - 9.265403983052604 \cdot 10^{-14} X^7 - 1.838110077974511 \cdot 10^{-10} X^6 \\
&\quad - 1.935168004397404 \cdot 10^{-07} X^5 - 0.0001140127070697086 X^4 - 0.0356257658581906 X^3 \\
&\quad - 4.602344943530705 X^2 + 4.651752639476855 X - 1.170857231885768 \\
&= -1.170857231885768 B_{0,8}(X) - 0.5893881519511612 B_{1,8}(X) - 0.172288534285508 B_{2,8}(X) \\
&\quad + 0.07980544672086649 B_{3,8}(X) + 0.1662559879246795 B_{4,8}(X) + 0.08642365397403269 B_{5,8}(X) \\
&\quad - 0.1603326261538093 B_{6,8}(X) - 0.574655562621217 B_{7,8}(X) - 1.157189508205582 B_{8,8}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= -0.03585432943204529 X^3 - 4.60219789441901 X^2 + 4.65171994907147 X - 1.170855596980661 \\
&= -1.170855596980661 B_{0,3} + 0.3797177193764957 B_{1,3} \\
&\quad + 0.3962250709273155 B_{2,3} - 1.157187871760247 B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 &= -3.713970905558003 \cdot 10^{-303} X^8 + 1.449155002864004 \cdot 10^{-302} X^7 - 2.225710229630734 \\
&\quad \cdot 10^{-302} X^6 + 1.689258272787799 \cdot 10^{-302} X^5 - 6.343129302139404 \cdot 10^{-303} X^4 \\
&\quad - 0.03585432943204529 X^3 - 4.60219789441901 X^2 + 4.65171994907147 X - 1.170855596980661 \\
&= -1.170855596980661 B_{0,8} - 0.5893906033467271 B_{1,8} - 0.1722898202277581 B_{2,8} \\
&\quad + 0.07980649649353122 B_{3,8} + 0.1662580909344257 B_{4,8} + 0.08642470721221019 B_{5,8} \\
&\quad - 0.1603339105558303 B_{6,8} - 0.574658018252411 B_{7,8} - 1.157187871760247 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 2.45563119394813 \cdot 10^{-06}$.

Bounding polynomials M and m:

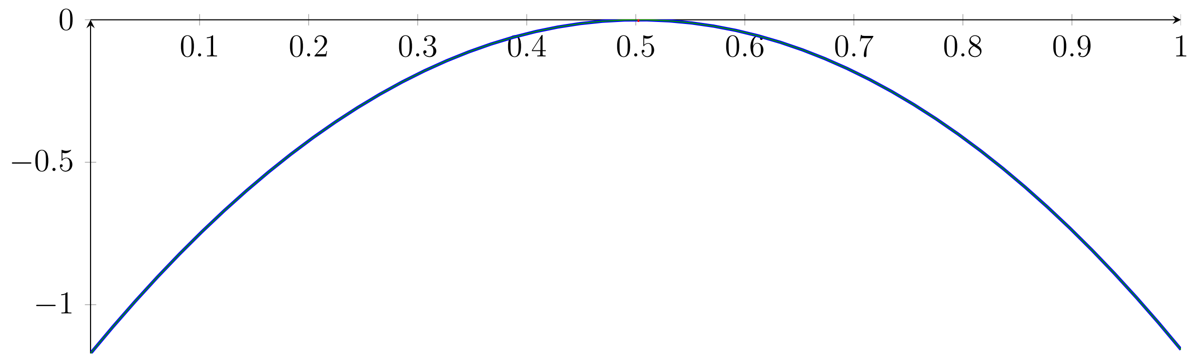
$$M = -0.0358543294320452 9X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170853141349467$$

$$m = -0.0358543294320452 9X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170858052611855$$

Root of M and m:

$$N(M) = \{-129.3630802555855, 0.5016184355695172, 0.5032421204520033\} \quad N(m) = \{-129.3630802637075\}$$

Intersection intervals:



$$[0.5016184355695172, 0.5032421204520033]$$

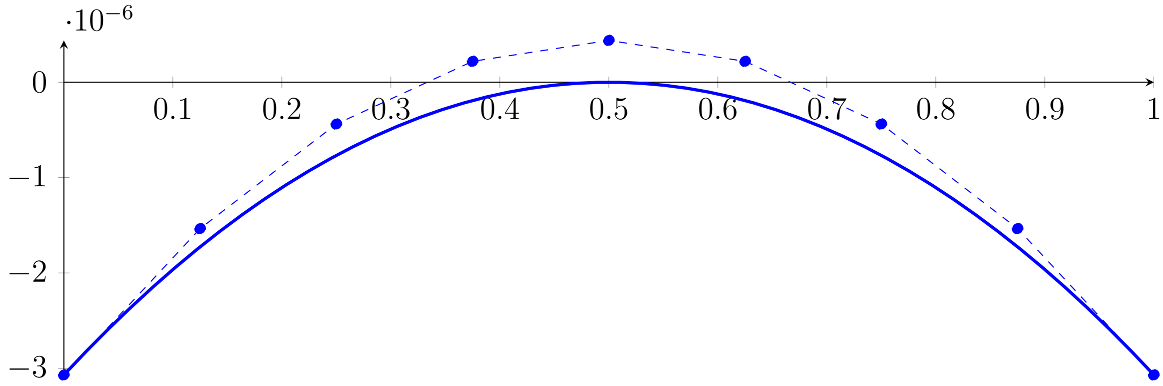
Longest intersection interval: 0.001623684882486142

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999934125800028, 0.5000066371751187]$,

60.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4999934125800028, 0.50000663717511$

Normalized monomial und Bézier representations and the Bézier polygon:

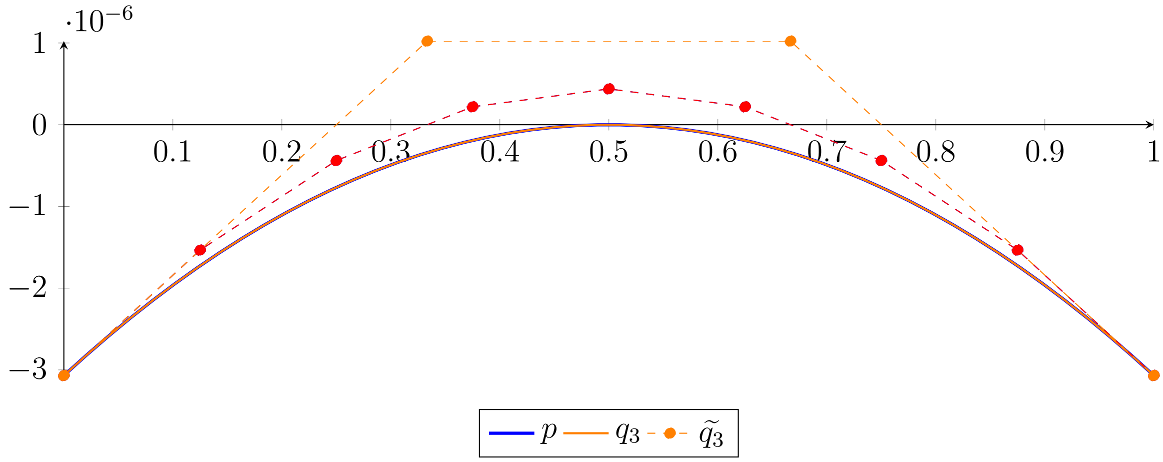
$$\begin{aligned} p = &-9.355329246270906 \cdot 10^{-40} X^8 - 2.758930189698976 \cdot 10^{-33} X^7 - 3.374040776758279 \cdot 10^{-27} X^6 \\ &- 2.190121853313308 \cdot 10^{-21} X^5 - 7.958070257617345 \cdot 10^{-16} X^4 - 1.534811947451426 \cdot 10^{-10} X^3 \\ &- 1.227519762250185 \cdot 10^{-05} X^2 + 1.227554003668957 \cdot 10^{-05} X - 3.068949673836091 \cdot 10^{-06} \\ = &-3.068949673836091 \cdot 10^{-06} B_{0,8}(X) - 1.534507169249895 \cdot 10^{-06} B_{1,8}(X) - 4.384645797530504 \\ &\cdot 10^{-07} B_{2,8}(X) + 2.191753539188221 \cdot 10^{-07} B_{3,8}(X) + 4.384098910187333 \cdot 10^{-07} B_{4,8}(X) \\ &+ 2.192362907883254 \cdot 10^{-07} B_{5,8}(X) - 4.383481875421282 \cdot 10^{-07} B_{6,8}(X) \\ &- 1.534346284753723 \cdot 10^{-06} B_{7,8}(X) - 3.068760741638923 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2\\&\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673824723\cdot 10^{-06}\\&= -3.068949673824723\cdot 10^{-06}B_{0,3} + 1.022897004996009\cdot 10^{-06}B_{1,3}\\&\quad + 1.023011143323854\cdot 10^{-06}B_{2,3} - 3.068760741627554\cdot 10^{-06}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -9.792734715851906\cdot 10^{-309}X^8 + 3.822699025575463\cdot 10^{-308}X^7 - 5.87427002023018\cdot 10^{-308}X^6\\&\quad + 4.462726908063245\cdot 10^{-308}X^5 - 1.679538448591103\cdot 10^{-308}X^4 - 1.534827863652778\cdot 10^{-10}X^3\\&\quad - 1.227519762147866\cdot 10^{-05}X^2 + 1.22755400364622\cdot 10^{-05}X - 3.068949673824723\cdot 10^{-06}\\&= -3.068949673824723\cdot 10^{-06}B_{0,8} - 1.534507169266948\cdot 10^{-06}B_{1,8} - 4.38464579761983\cdot 10^{-07}B_{2,8}\\&\quad + 2.191753539261305\cdot 10^{-07}B_{3,8} + 4.384098910333502\cdot 10^{-07}B_{4,8} + 2.192362907956339\cdot 10^{-07}B_{5,8}\\&\quad - 4.383481875510608\cdot 10^{-07}B_{6,8} - 1.534346284770776\cdot 10^{-06}B_{7,8} - 3.068760741627554\cdot 10^{-06}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.705314892341177\cdot 10^{-17}$.
Bounding polynomials M and m:

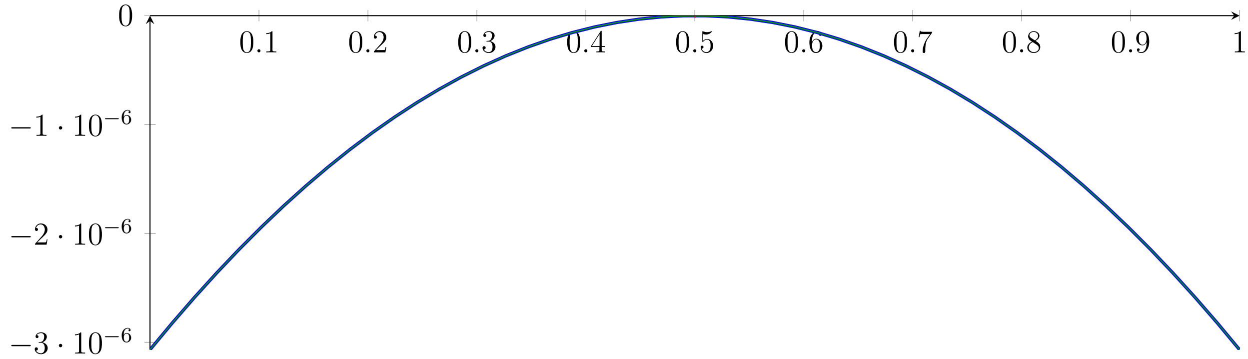
$$\begin{aligned}M &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2\\&\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673807669\cdot 10^{-06}\end{aligned}$$

$$\begin{aligned}m &= -1.534827863652778\cdot 10^{-10}X^3 - 1.227519762147866\cdot 10^{-05}X^2\\&\quad + 1.22755400364622\cdot 10^{-05}X - 3.068949673841776\cdot 10^{-06}\end{aligned}$$

Root of M and m:

$N(M) = \{-79978.68293772447, 0.4996311727354866, 0.5003873441571286\}$ $N(m) = \{-79978.68293772447,$

Intersection intervals:



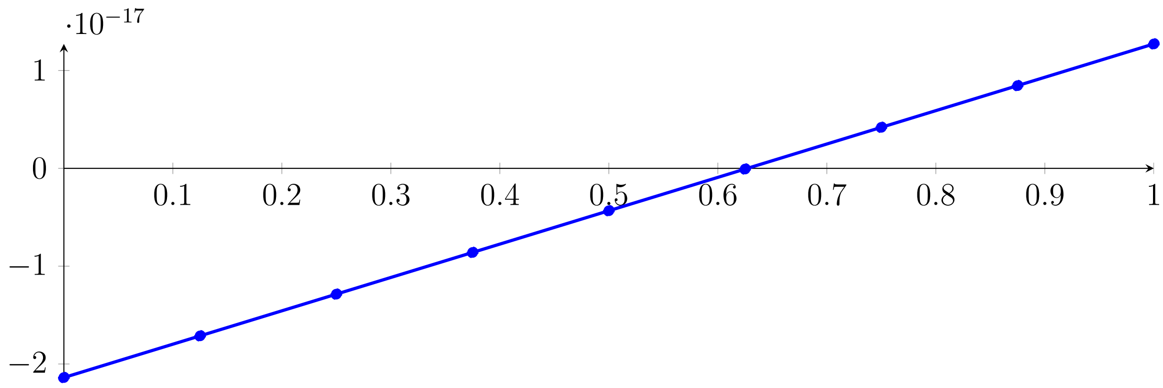
$$[0.4996311727354866, 0.4996311764098305],\ [0.5003873404827848, 0.5003873441571286]$$

Longest intersection interval: $3.674343879435088{\cdot}10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999695, 0.5000000200000181]$, interval 2: $[0.50000002999998$

60.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.5000000199999695, 0.50000002000$

Normalized monomial und Bézier representations and the Bézier polygon:

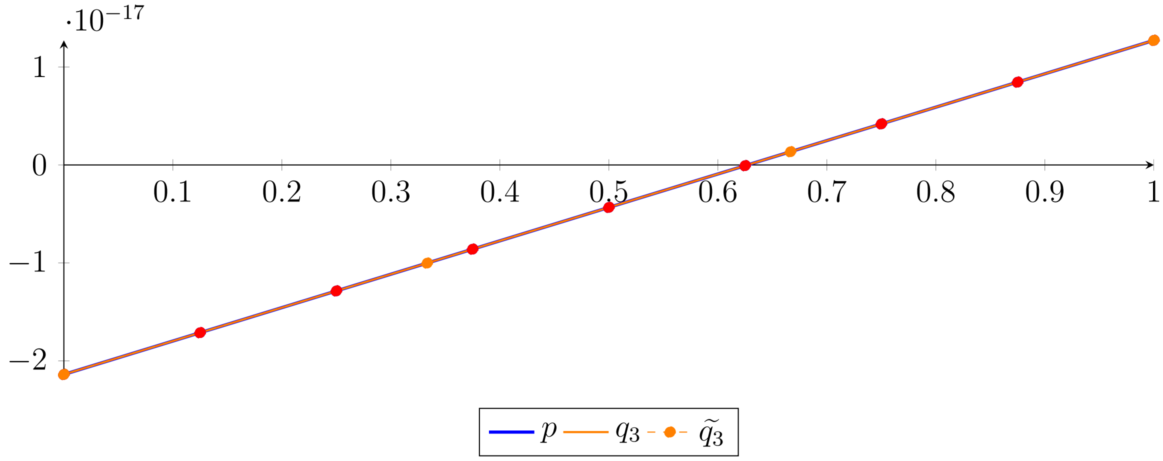
$$\begin{aligned} p = &-3.10811779683933{\cdot}10^{-107}X^8 - 2.494594121391514{\cdot}10^{-92}X^7 - 8.302910645986526{\cdot}10^{-78}X^6 \\ &- 1.466794525552917{\cdot}10^{-63}X^5 - 1.450540809134678{\cdot}10^{-49}X^4 - 7.613758006193666{\cdot}10^{-36}X^3 \\ &- 1.657281301066328{\cdot}10^{-22}X^2 + 3.41064642546533{\cdot}10^{-17}X - 2.138639484655546{\cdot}10^{-17} \\ = &-2.138639484655546{\cdot}10^{-17}B_{0,8}(X) - 1.71230868147238{\cdot}10^{-17}B_{1,8}(X) - 1.285978470175393 \\ &{\cdot}10^{-17}B_{2,8}(X) - 8.596488507645843{\cdot}10^{-18}B_{3,8}(X) - 4.333198232399549 \\ &{\cdot}10^{-18}B_{4,8}(X) - 6.991387601504461{\cdot}10^{-20}B_{5,8}(X) + 4.19336456150767{\cdot}10^{-18}B_{6,8}(X) \\ &+ 8.456637080168595{\cdot}10^{-18}B_{7,8}(X) + 1.271990367996773{\cdot}10^{-17}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-7.613758006193956{\cdot}10^{-36}X^3 - 1.657281301066328{\cdot}10^{-22}X^2 \\ &+ 3.41064642546533{\cdot}10^{-17}X - 2.138639484655546{\cdot}10^{-17} \\ = &-2.138639484655546{\cdot}10^{-17}B_{0,3} - 1.001757342833769{\cdot}10^{-17}B_{1,3} \\ &+ 1.351192747170036{\cdot}10^{-18}B_{2,3} + 1.271990367996773{\cdot}10^{-17}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 = &-3.883510371826523{\cdot}10^{-321}X^8 + 2.549502248952292{\cdot}10^{-320}X^7 - 4.188626787236358{\cdot}10^{-320}X^6 \\ &+ 1.055694768751284{\cdot}10^{-320}X^5 + 2.46739471443405{\cdot}10^{-320}X^4 - 7.613758006193956{\cdot}10^{-36}X^3 \\ &- 1.657281301066328{\cdot}10^{-22}X^2 + 3.41064642546533{\cdot}10^{-17}X - 2.138639484655546{\cdot}10^{-17} \\ = &-2.138639484655546{\cdot}10^{-17}B_{0,8} - 1.71230868147238{\cdot}10^{-17}B_{1,8} - 1.285978470175393{\cdot}10^{-17}B_{2,8} \\ &- 8.596488507645843{\cdot}10^{-18}B_{3,8} - 4.333198232399549{\cdot}10^{-18}B_{4,8} - 6.991387601504461{\cdot}10^{-20}B_{5,8} \\ &+ 4.19336456150767{\cdot}10^{-18}B_{6,8} + 8.456637080168595{\cdot}10^{-18}B_{7,8} + 1.271990367996773{\cdot}10^{-17}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301733860119 \cdot 10^{-51}$.
Bounding polynomials M and m:

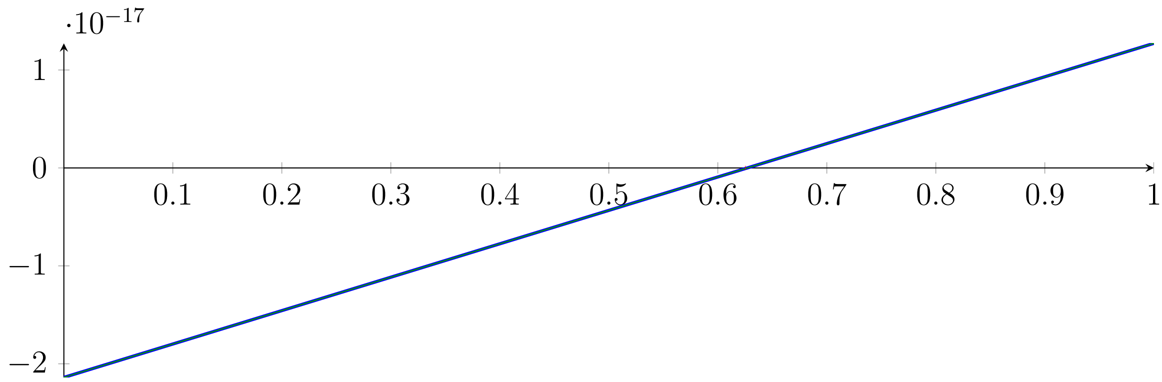
$$M = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

$$m = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929227157.35, 0.6249980054152174, 205797.0483973646\}$ $N(m) = \{-21766929227157.35, 0$

Intersection intervals:



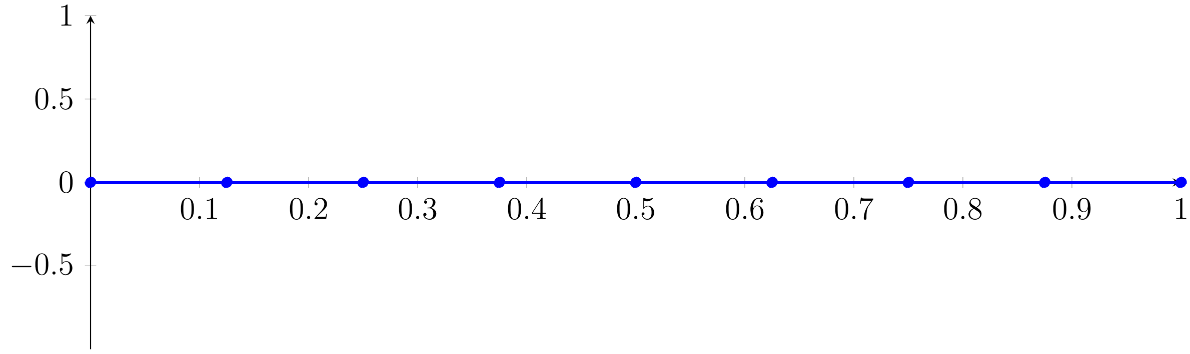
$$[0.6249980054152174, 0.6249980054152174]$$

Longest intersection interval: $1.822716451808538 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000199999999]$,

60.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.50000001999$

Normalized monomial und Bézier representations and the Bézier polygon:

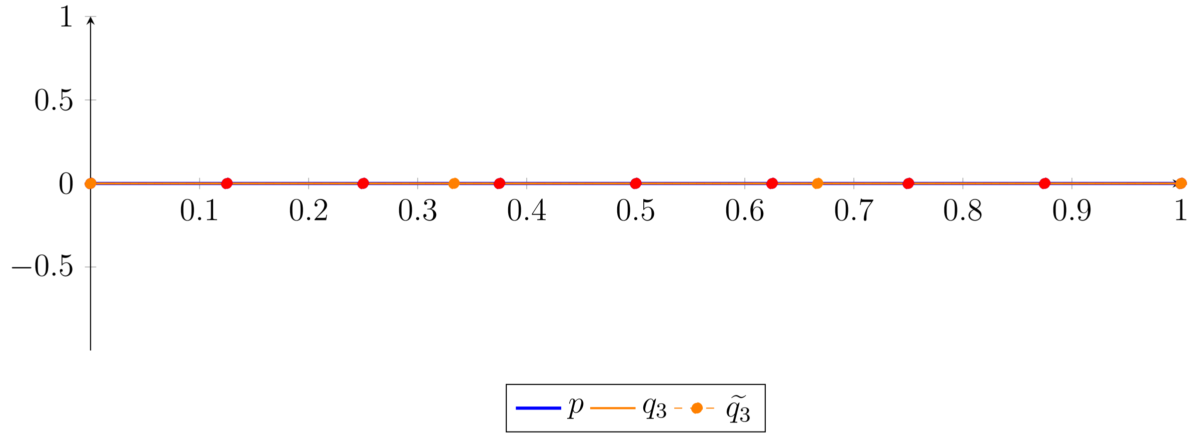
$$\begin{aligned} p = & -1.929943929067369 \cdot 10^{-326} X^8 + 3.859887858134739 \cdot 10^{-326} X^7 - 3.044703772619375 \cdot 10^{-280} X^6 \\ & - 2.950970364376253 \cdot 10^{-232} X^5 - 1.601055569746729 \cdot 10^{-184} X^4 - 4.610588999829434 \cdot 10^{-137} X^3 \\ & - 5.505977817140947 \cdot 10^{-90} X^2 + 6.216603591694483 \cdot 10^{-51} X - 6.998745276936609 \cdot 10^{-20} \\ = & -6.998745276936609 \cdot 10^{-20} B_{0,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{1,8}(X) - 6.998745276936609 \\ & \cdot 10^{-20} B_{2,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{3,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{4,8}(X) \\ & - 6.998745276936609 \cdot 10^{-20} B_{5,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{6,8}(X) \\ & - 6.998745276936609 \cdot 10^{-20} B_{7,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -4.610588999829434 \cdot 10^{-137} X^3 - 5.505977817140947 \cdot 10^{-90} X^2 \\ &\quad + 6.216603591694483 \cdot 10^{-51} X - 6.998745276936609 \cdot 10^{-20} \\ &= -6.998745276936609 \cdot 10^{-20} B_{0,3} - 6.998745276936609 \cdot 10^{-20} B_{1,3} \\ &\quad - 6.998745276936609 \cdot 10^{-20} B_{2,3} - 6.998745276936609 \cdot 10^{-20} B_{3,3} \end{aligned}$$

$$\begin{aligned} \tilde{q}_3 &= -3.734501813493143 \cdot 10^{-323} X^8 + 1.603059676081584 \cdot 10^{-322} X^7 - 4.947893748146468 \cdot 10^{-324} X^6 \\ &\quad - 5.631817628009717 \cdot 10^{-322} X^5 + 7.073546053770825 \cdot 10^{-322} X^4 - 4.610588999829434 \cdot 10^{-137} X^3 \\ &\quad - 5.505977817140947 \cdot 10^{-90} X^2 + 6.216603591694483 \cdot 10^{-51} X - 6.998745276936609 \cdot 10^{-20} \\ &= -6.998745276936609 \cdot 10^{-20} B_{0,8} - 6.998745276936609 \cdot 10^{-20} B_{1,8} - 6.998745276936609 \cdot 10^{-20} B_{2,8} \\ &\quad - 6.998745276936609 \cdot 10^{-20} B_{3,8} - 6.998745276936609 \cdot 10^{-20} B_{4,8} - 6.998745276936609 \cdot 10^{-20} B_{5,8} \\ &\quad - 6.998745276936609 \cdot 10^{-20} B_{6,8} - 6.998745276936609 \cdot 10^{-20} B_{7,8} - 6.998745276936609 \cdot 10^{-20} B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.43083336374299 \cdot 10^{-186}$.
Bounding polynomials M and m:

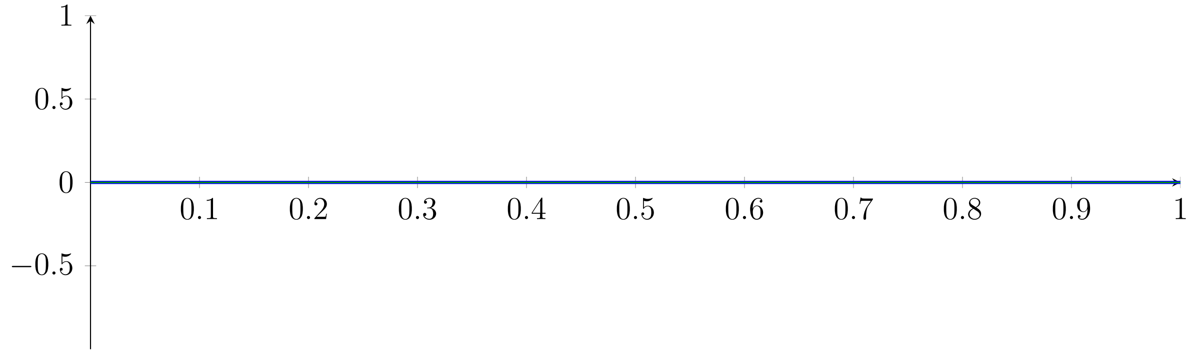
$$\begin{aligned} M &= -4.610588999829434 \cdot 10^{-137} X^3 - 5.505977817140947 \cdot 10^{-90} X^2 \\ &\quad + 6.216603591694483 \cdot 10^{-51} X - 6.998745276936609 \cdot 10^{-20} \end{aligned}$$

$$\begin{aligned} m &= -4.610588999829434 \cdot 10^{-137} X^3 - 5.505977817140947 \cdot 10^{-90} X^2 \\ &\quad + 6.216603591694483 \cdot 10^{-51} X - 6.998745276936609 \cdot 10^{-20} \end{aligned}$$

Root of M and m:

$N(M) = \{-1.19420270802736 \cdot 10^{47}, 1.07241027244706 \cdot 10^{17}, 1.129064387360764 \cdot 10^{39}\} \quad N(m) = \{-1.1942027$

Intersection intervals:

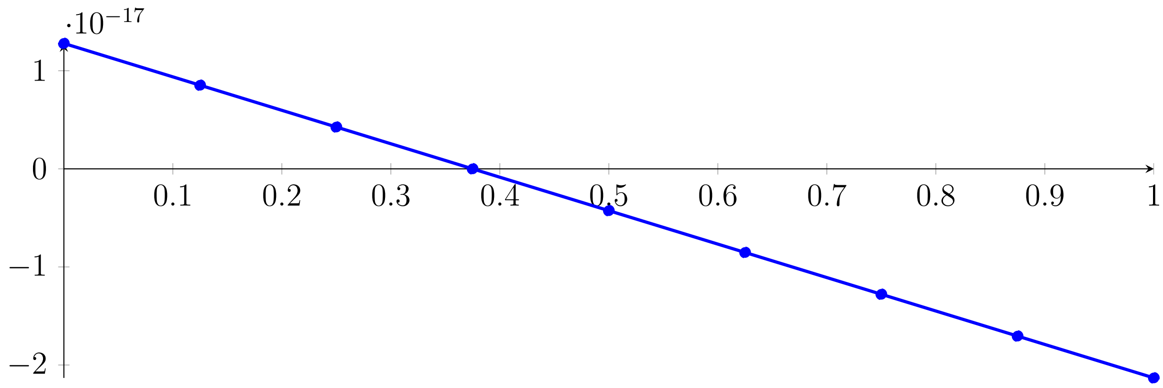


No intersection intervals with the x axis.

60.7 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.5000000299999818, 0.500000030000$

Normalized monomial und Bézier representations and the Bézier polygon:

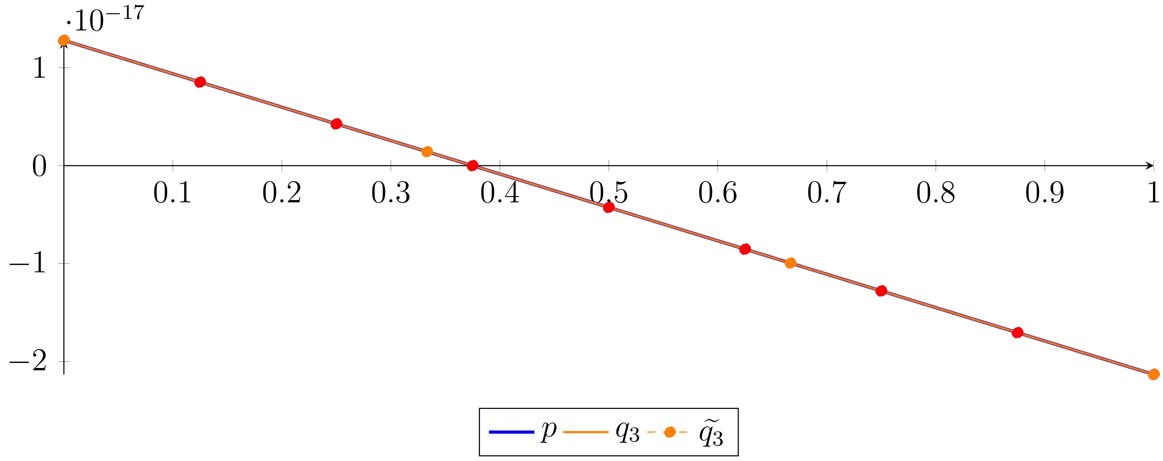
$$\begin{aligned}p &= -3.108117561752323\cdot 10^{-107}X^8 - 2.494593961411665\cdot 10^{-92}X^7 - 8.302910210921025\cdot 10^{-78}X^6\\&\quad - 1.466794466465724\cdot 10^{-63}X^5 - 1.450540769370867\cdot 10^{-49}X^4 - 7.613757909646128\cdot 10^{-36}X^3\\&\quad - 1.657281316735189\cdot 10^{-22}X^2 - 3.410613211820657\cdot 10^{-17}X + 1.27898932402398\cdot 10^{-17}\\&= 1.27898932402398\cdot 10^{-17}B_{0,8}(X) + 8.526626725463981\cdot 10^{-18}B_{1,8}(X) + 4.263354291826315\\&\quad \cdot 10^{-18}B_{2,8}(X) + 7.593932680254395\cdot 10^{-23}B_{3,8}(X) - 4.263208332034555\cdot 10^{-18}B_{4,8}(X)\\&\quad - 8.526498522257758\cdot 10^{-18}B_{5,8}(X) - 1.278979463134281\cdot 10^{-17}B_{6,8}(X)\\&\quad - 1.70530966592897\cdot 10^{-17}B_{7,8}(X) - 2.131640460609844\cdot 10^{-17}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -7.613757909646418\cdot 10^{-36}X^3 - 1.657281316735189\cdot 10^{-22}X^2\\&\quad - 3.410613211820657\cdot 10^{-17}X + 1.27898932402398\cdot 10^{-17}\\&= 1.27898932402398\cdot 10^{-17}B_{0,3} + 1.421182534170946\cdot 10^{-18}B_{1,3}\\&\quad - 9.947583414608468\cdot 10^{-18}B_{2,3} - 2.131640460609844\cdot 10^{-17}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -6.944710234356022\cdot 10^{-322}X^8 - 5.839855933843534\cdot 10^{-321}X^7 + 4.132859127462027\cdot 10^{-320}X^6\\&\quad - 7.976936884929846\cdot 10^{-320}X^5 + 6.21874252599804\cdot 10^{-320}X^4 - 7.613757909646418\cdot 10^{-36}X^3\\&\quad - 1.657281316735189\cdot 10^{-22}X^2 - 3.410613211820657\cdot 10^{-17}X + 1.27898932402398\cdot 10^{-17}\\&= 1.27898932402398\cdot 10^{-17}B_{0,8} + 8.526626725463981\cdot 10^{-18}B_{1,8} + 4.263354291826315\cdot 10^{-18}B_{2,8}\\&\quad + 7.593932680254395\cdot 10^{-23}B_{3,8} - 4.263208332034555\cdot 10^{-18}B_{4,8} - 8.526498522257758\cdot 10^{-18}B_{5,8}\\&\quad - 1.278979463134281\cdot 10^{-17}B_{6,8} - 1.70530966592897\cdot 10^{-17}B_{7,8} - 2.131640460609844\cdot 10^{-17}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301648651952 \cdot 10^{-51}$.
Bounding polynomials M and m:

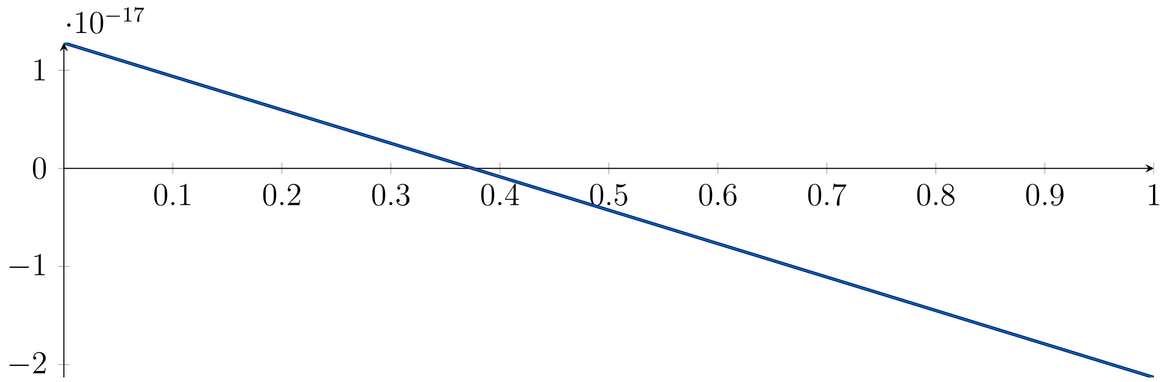
$$M = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

$$m = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929297379.88, -205796.0503430093, 0.375002063855762\}$ $N(m) = \{-21766929297379.88,$

Intersection intervals:



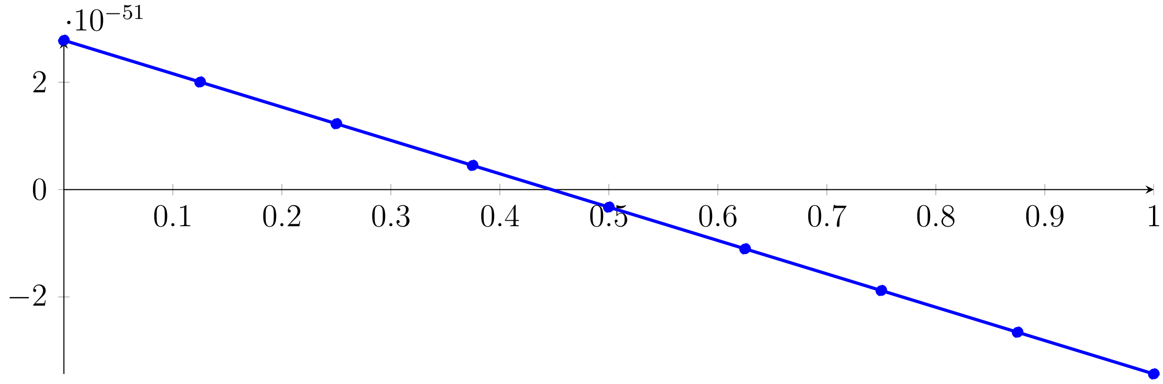
$[0.375002063855762, 0.375002063855762]$

Longest intersection interval: $1.822716401842262 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

60.8 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

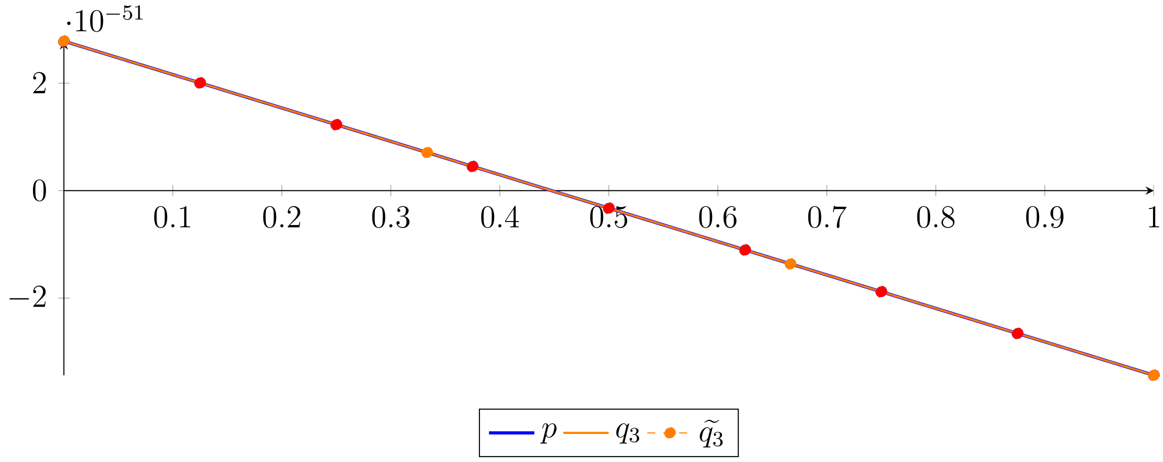
$$\begin{aligned} p &= 4.757679109093379 \cdot 10^{-358} X^8 - 1.667376123134876 \cdot 10^{-328} X^7 - 3.044703112291227 \cdot 10^{-280} X^6 \\ &\quad - 2.95096984102574 \cdot 10^{-232} X^5 - 1.601055350297363 \cdot 10^{-184} X^4 - 4.610588562192646 \cdot 10^{-137} X^3 \\ &\quad - 5.505977567325696 \cdot 10^{-90} X^2 - 6.216603297303905 \cdot 10^{-51} X + 2.781471077280957 \cdot 10^{-51} \\ &= 2.781471077280957 \cdot 10^{-51} B_{0,8}(X) + 2.004395665117968 \cdot 10^{-51} B_{1,8}(X) + 1.22732025295498 \\ &\quad \cdot 10^{-51} B_{2,8}(X) + 4.502448407919922 \cdot 10^{-52} B_{3,8}(X) - 3.268305713709959 \cdot 10^{-52} B_{4,8}(X) \\ &\quad - 1.103905983533984 \cdot 10^{-51} B_{5,8}(X) - 1.880981395696972 \cdot 10^{-51} B_{6,8}(X) \\ &\quad - 2.65805680785996 \cdot 10^{-51} B_{7,8}(X) - 3.435132220022948 \cdot 10^{-51} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= -4.610588562192646\cdot 10^{-137}X^3 - 5.505977567325696\cdot 10^{-90}X^2 \\
&\quad - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51} \\
&= 2.781471077280957\cdot 10^{-51}B_{0,3} + 7.092699781796549\cdot 10^{-52}B_{1,3} \\
&\quad - 1.362931120921647\cdot 10^{-51}B_{2,3} - 3.435132220022948\cdot 10^{-51}B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 &= 1.121622849968764\cdot 10^{-355}X^8 - 2.091475736357449\cdot 10^{-354}X^7 + 7.553291353596648\cdot 10^{-354}X^6 \\
&\quad - 1.090697935759657\cdot 10^{-353}X^5 + 6.781476860123975\cdot 10^{-354}X^4 - 4.610588562192646\cdot 10^{-137}X^3 \\
&\quad - 5.505977567325696\cdot 10^{-90}X^2 - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51} \\
&= 2.781471077280957\cdot 10^{-51}B_{0,8} + 2.004395665117968\cdot 10^{-51}B_{1,8} + 1.22732025295498\cdot 10^{-51}B_{2,8} \\
&\quad + 4.502448407919922\cdot 10^{-52}B_{3,8} - 3.268305713709959\cdot 10^{-52}B_{4,8} - 1.103905983533984\cdot 10^{-51}B_{5,8} \\
&\quad - 1.880981395696972\cdot 10^{-51}B_{6,8} - 2.65805680785996\cdot 10^{-51}B_{7,8} - 3.435132220022948\cdot 10^{-51}B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.430832893494348\cdot 10^{-186}$.
Bounding polynomials M and m:

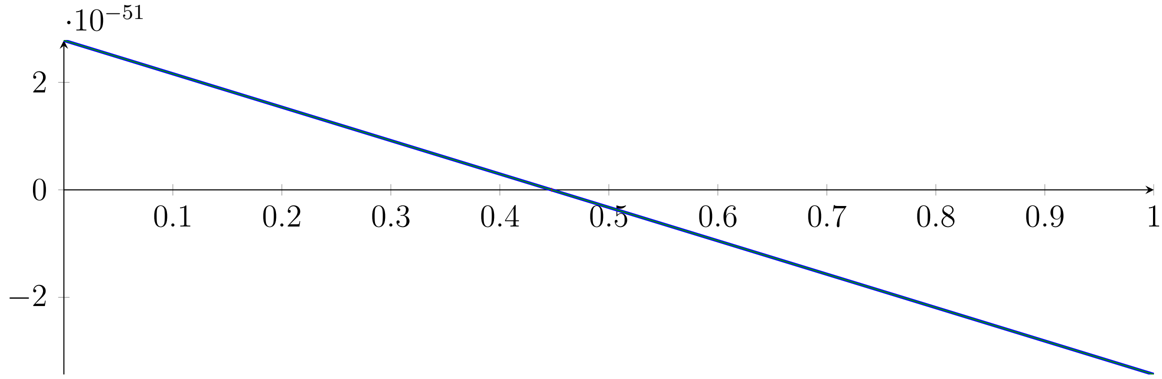
$$
\begin{aligned}
M &= -4.610588562192646\cdot 10^{-137}X^3 - 5.505977567325696\cdot 10^{-90}X^2 \\
&\quad - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51}
\end{aligned}
$$

$$
\begin{aligned}
m &= -4.610588562192646\cdot 10^{-137}X^3 - 5.505977567325696\cdot 10^{-90}X^2 \\
&\quad - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51}
\end{aligned}
$$

Root of M and m:

$N(M) = \{-1.194202744616778\cdot 10^{47}, -1.129064428986703\cdot 10^{39}, 0.4474261818326513\}$ $\quad N(m) = \{-1.194202$

Intersection intervals:



$$[0.4474261818326513, 0.4474261818326513]$$

Longest intersection interval: $1.103764460885666 \cdot 10^{-135}$
$\implies$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

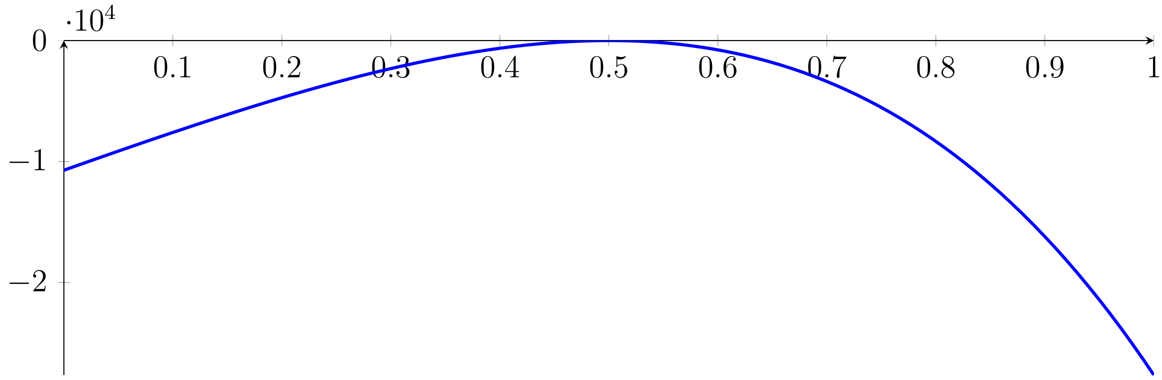
60.9 Recursion Branch 1 1 1 1 2 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 7!

60.10 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000003, 0.50000003]$$

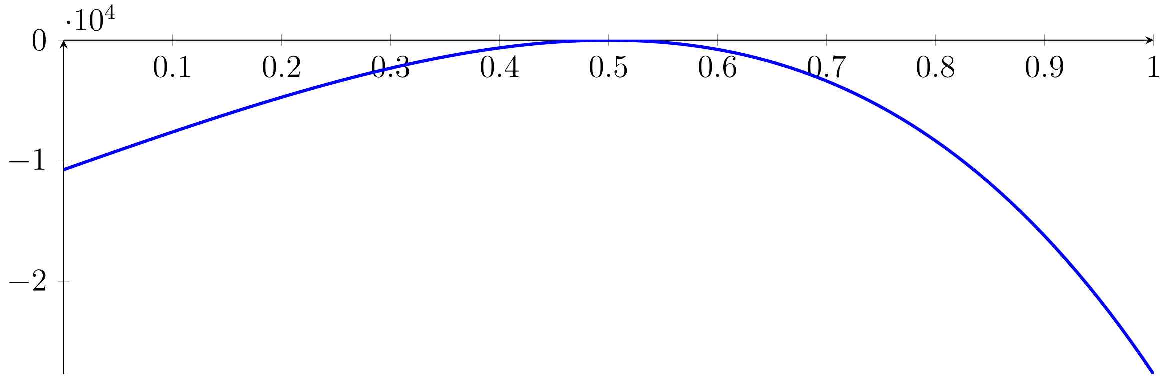
with precision $\varepsilon = 1 \cdot 10^{-64}$.

61 Running BezClip on h_8 with epsilon 128

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

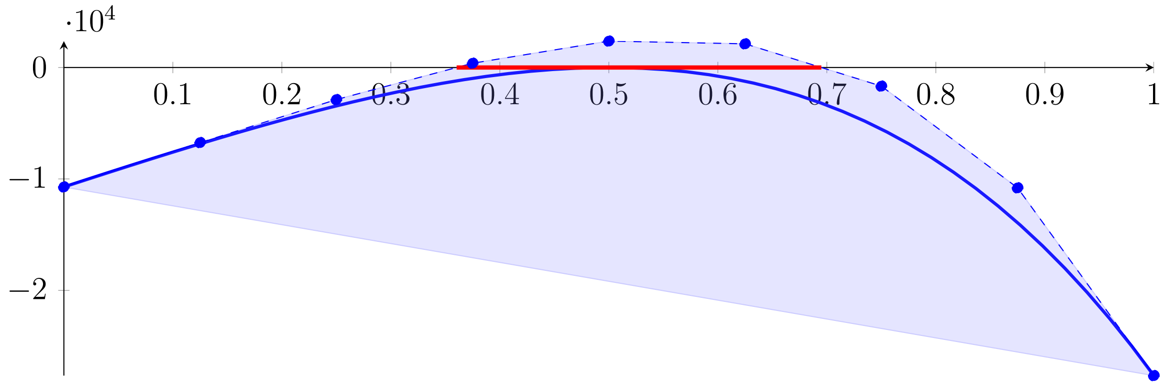
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



61.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3603640692588709, 0.6949437907012195\}$$

Intersection intervals with the x axis:

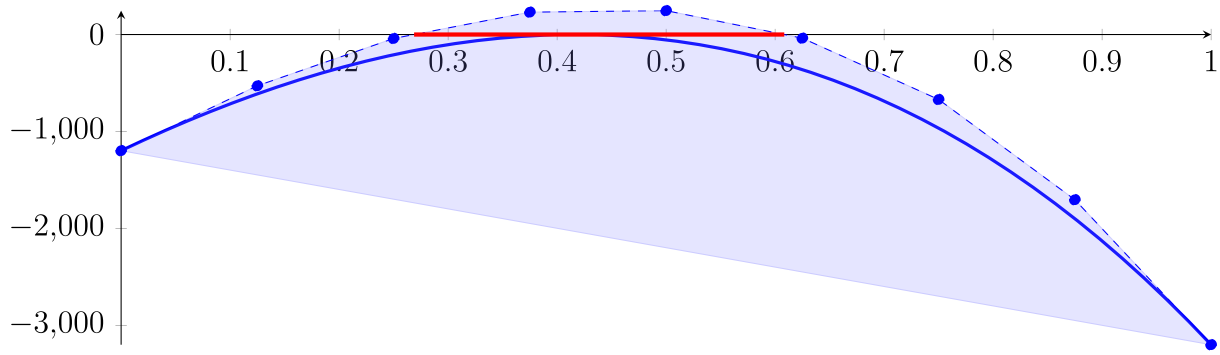
$$[0.3603640692588709, 0.6949437907012195]$$

Longest intersection interval: 0.3345797214423486
$\Longrightarrow$ Selective recursion: interval 1: $[0.3603640692588709, 0.6949437907012195]$,

61.2 Recursion Branch 1 1 in Interval 1: $[0.3603640692588709, 0.6949437907012195]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-0.0001570351674648188X^8 - 0.01778036482153305X^7 - 0.8321100708277401X^6\\&- 20.55224953791548X^5 - 280.9398596960248X^4 - 1979.461827852286X^3\\&- 5069.964094993441X^2 + 5351.083864999425X - 1197.499321131098\\= &-1197.499321131098B_{0,8}(X) - 528.61383800617B_{1,8}(X) - 40.79850113100757B_{2,8}(X)\\&+ 230.5991568541697B_{3,8}(X) + 246.2181767420565B_{4,8}(X) - 37.68283169777314B_{5,8}(X)\\&- 669.6224123917145B_{6,8}(X) - 1703.3249263974B_{7,8}(X) - 3198.183535682156B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2687909235445741, 0.6084084634355219\}$$

Intersection intervals with the x axis:

$$[0.2687909235445741, 0.6084084634355219]$$

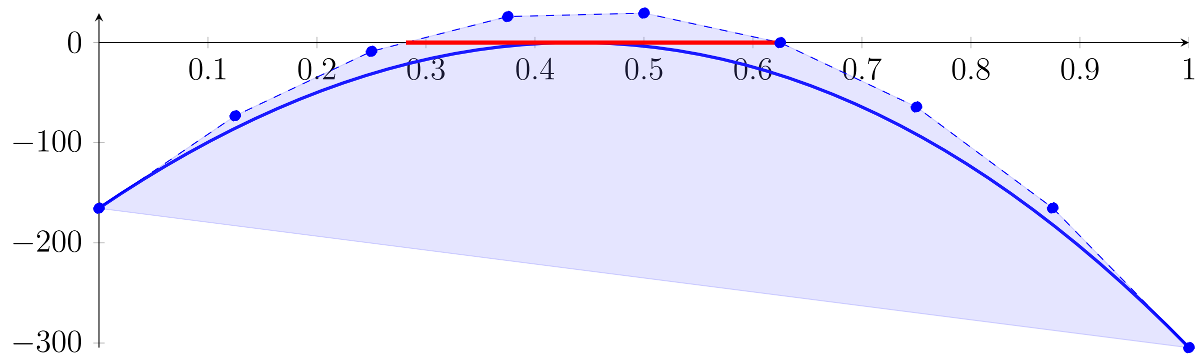
Longest intersection interval: 0.3396175398909478

$\Longrightarrow$ Selective recursion: interval 1: $[0.4502960615846461, 0.5639252034782951]$,

61.3 Recursion Branch 1 1 1 in Interval 1: $[0.4502960615846461, 0.563925203478295$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.779187295559603\cdot 10^{-08}X^8 - 9.441522673365855\cdot 10^{-06}X^7 - 0.001328615938017263X^6\\&- 0.09904177753952623X^5 - 4.117049822961124X^4 - 89.96494981674486X^3\\&- 783.3886356669557X^2 + 738.3817879690506X - 165.41044010987\\= &-165.41044010987B_{0,8}(X) - 73.11271661373872B_{1,8}(X) - 8.793158677141534B_{2,8}(X)\\&+ 25.94171673890822B_{3,8}(X) + 29.42657767592637B_{4,8}(X) - 0.06449142521248714B_{5,8}(X)\\&- 64.31980577801453B_{6,8}(X) - 165.1919449348658B_{7,8}(X) - 304.5996673102733B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2816438398433068, 0.6247266484940266\}$$

Intersection intervals with the x axis:

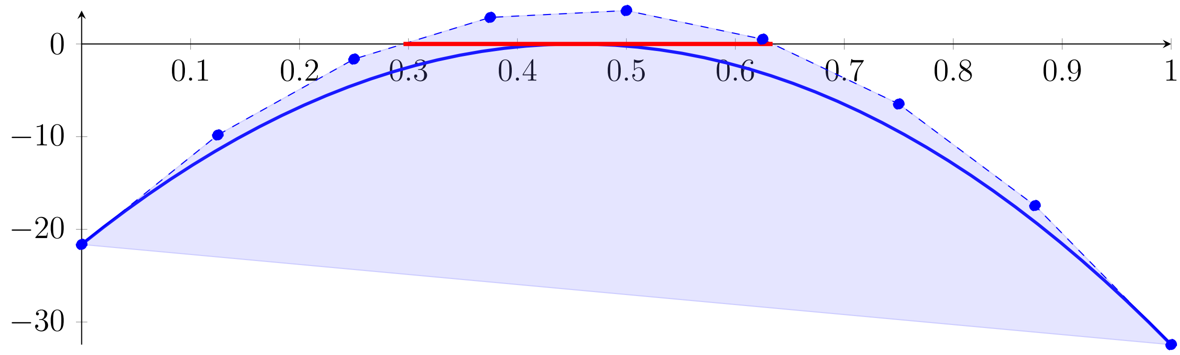
$$[0.2816438398433068, 0.6247266484940266]$$

Longest intersection interval: 0.3430828086507199
$\Longrightarrow$ Selective recursion: interval 1: $[0.4822990094256734, 0.5212832145711177]$,

61.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4822990094256734, 0.52128321457111$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -5.334693469973702\cdot 10^{-12}X^8 - 5.317476896558791\cdot 10^{-09}X^7 - 2.197128069269196\cdot 10^{-06}X^6 \\
&\quad - 0.000481521724173462X^5 - 0.05899466848037266X^4 - 3.82353929224187X^3 \\
&\quad - 101.3899724153744X^2 + 94.46052284159652X - 21.62667247491518 \\
&= -21.62667247491518B_{0,8}(X) - 9.819107119715613B_{1,8}(X) - 1.632612207922276B_{2,8}(X) \\
&\quad + 2.86453477310337B_{3,8}(X) + 3.603213555021573B_{4,8}(X) + 0.5134524899120698B_{5,8}(X) \\
&\quad - 6.475580126796985B_{6,8}(X) - 17.43558481253364B_{7,8}(X) - 32.43913973359036B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.295379109655816, 0.634183182388585\}$$

Intersection intervals with the x axis:

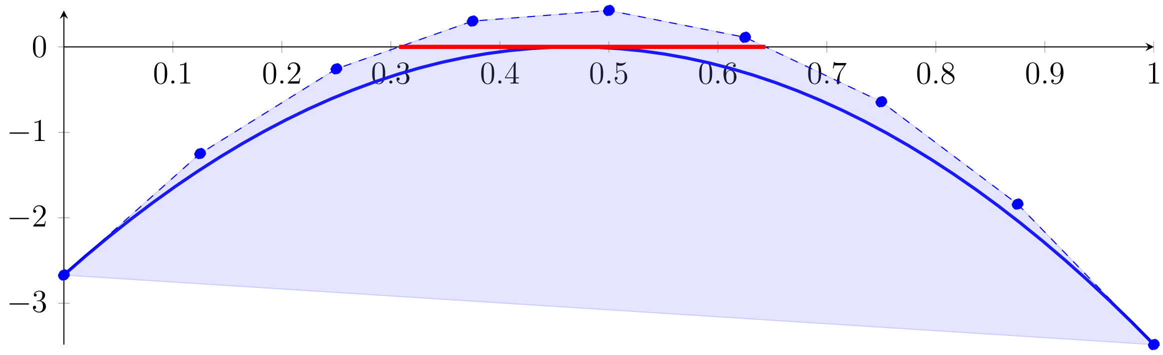
$$[0.295379109655816, 0.634183182388585]$$

Longest intersection interval: 0.338804072732769
$\Longrightarrow$ Selective recursion: interval 1: $[0.4938141292321744, 0.5070221367077007]$,

61.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4938141292321744, 0.507022136707$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -9.261865042605128\cdot 10^{-16}X^8 - 2.731330938941274\cdot 10^{-12}X^7 - 3.339777336661422 \\
&\quad \cdot 10^{-09}X^6 - 2.167033369334019\cdot 10^{-06}X^5 - 0.0007867416616259877X^4 \\
&\quad - 0.1514273448103067X^3 - 12.0308548973337X^2 + 11.3691349552019X - 2.67015106585462 \\
&= -2.67015106585462B_{0,8}(X) - 1.249009196454382B_{1,8}(X) - 0.2575407162446335B_{2,8}(X) \\
&\quad + 0.3015503150458701B_{3,8}(X) + 0.4255485985217787B_{4,8}(X) + 0.1117275574241232B_{5,8}(X) \\
&\quad - 0.6426507016859872B_{6,8}(X) - 1.840335427863286B_{7,8}(X) - 3.484087264834231B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3075802288515909, 0.6435131855396921\}$$

Intersection intervals with the x axis:

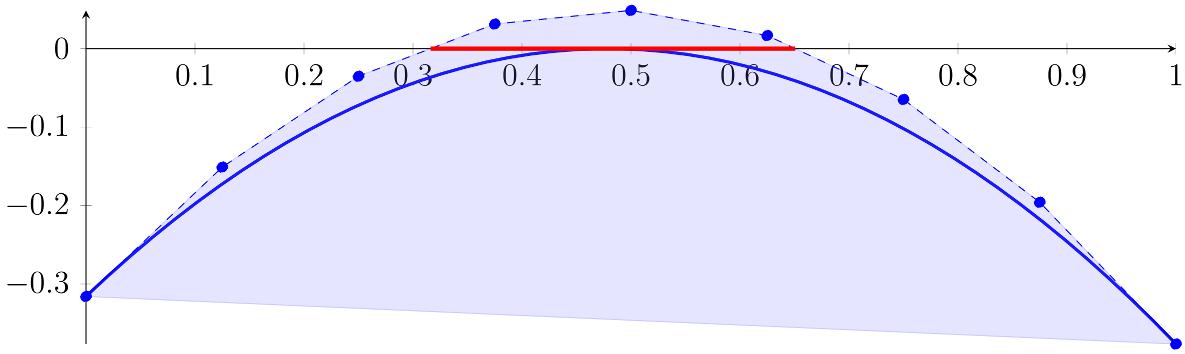
$$[0.3075802288515909, 0.6435131855396921]$$

Longest intersection interval: 0.3359329566881012
$\Longrightarrow$ Selective recursion: interval 1: $[0.4978766511941703, 0.5023136561973824]$,

61.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4978766511941703, 0.50231365619$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.502170909909546\cdot 10^{-19}X^8 - 1.319790000383349\cdot 10^{-15}X^7 - 4.808366303277033\cdot 10^{-12}X^6\\
&- 9.297436410097437\cdot 10^{-09}X^5 - 1.006192361000331\cdot 10^{-05}X^4 - 0.005777437133356019X^3\\
&- 1.373512346650201X^2 + 1.318590421802199X - 0.3158295498425207\\
= &-0.3158295498425207B_{0,8}(X) - 0.1510057471172458B_{1,8}(X) - 0.03523595677233526B_{2,8}(X)\\
&+ 0.03137665267197247B_{3,8}(X) + 0.04872876895367301B_{4,8}(X) + 0.01671693590297049B_{5,8}(X)\\
&- 0.06476244672391982B_{6,8}(X) - 0.1958131234111408B_{7,8}(X) - 0.376538983049735B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3161210337394805, 0.6506459600024226\}$$

Intersection intervals with the x axis:

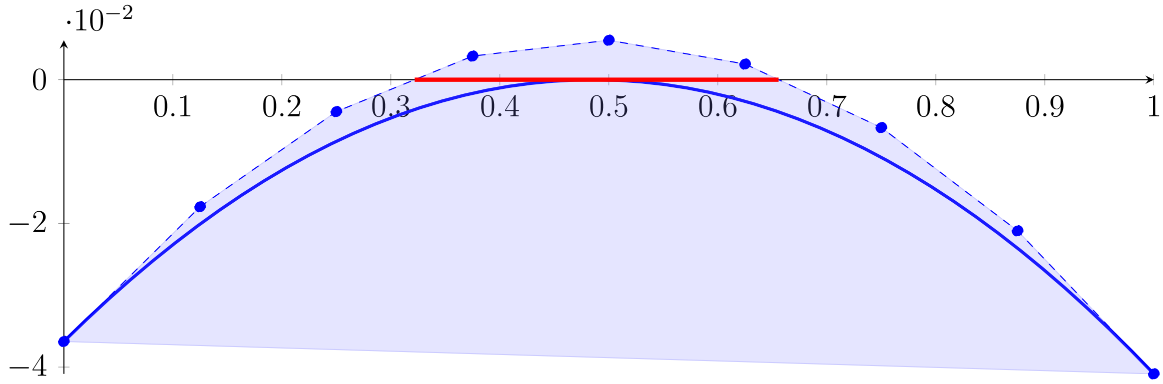
$$[0.3161210337394805, 0.6506459600024226]$$

Longest intersection interval: 0.3345249262629421
$\Longrightarrow$ Selective recursion: interval 1: $[0.499279281802493, 0.5007635705740208]$,

61.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.499279281802493, 0.5007635705$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -2.355847730899963 \cdot 10^{-23} X^8 - 6.189124375986835 \cdot 10^{-19} X^7 - 6.742674354644286 \cdot 10^{-15} X^6 \\
& - 3.898805488577673 \cdot 10^{-11} X^5 - 1.261912061383275 \cdot 10^{-07} X^4 - 0.000216758797755982 X^3 \\
& - 0.154319370565961 X^2 + 0.1500226550818807 X - 0.0364365303427455 \\
= & -0.0364365303427455 B_{0,8}(X) - 0.01768369845751041 B_{1,8}(X) - 0.004442272663916792 B_{2,8}(X) \\
& + 0.003283876345218297 B_{3,8}(X) + 0.005490876074346265 B_{4,8}(X) \\
& + 0.002174852224490793 B_{5,8}(X) - 0.006668071307448625 B_{6,8}(X) \\
& - 0.02104177242939338 B_{7,8}(X) - 0.04095013085478269 B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3218707447051634, 0.6557428337562166\}$$

Intersection intervals with the x axis:

$$[0.3218707447051634, 0.6557428337562166]$$

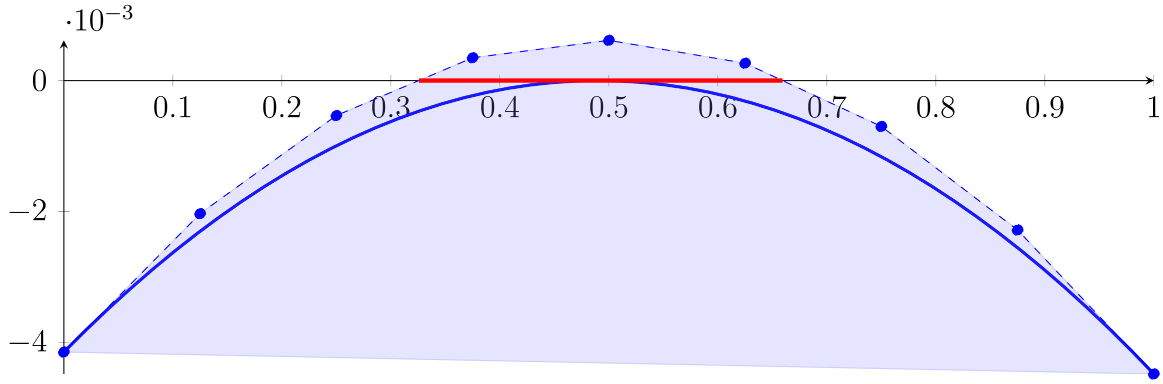
Longest intersection interval: 0.3338720890510532

$\Longrightarrow$ Selective recursion: interval 1: $[0.4997570309347421, 0.5002525935276471]$,

61.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.4997570309347421, 0.5002525$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.637375463877353 \cdot 10^{-27} X^8 - 2.862414855005092 \cdot 10^{-22} X^7 - 9.341200314035287 \cdot 10^{-18} X^6 \\
& - 1.617995026179791 \cdot 10^{-13} X^5 - 1.568792391901099 \cdot 10^{-09} X^4 - 8.073141350089629 \\
& \cdot 10^{-06} X^3 - 0.01722540857155108 X^2 + 0.01689843085777645 X - 0.004143462623424353 \\
= & -0.004143462623424353 B_{0,8}(X) - 0.002031158766202297 B_{1,8}(X) \\
& - 0.0005340480722499233 B_{2,8}(X) + 0.0003477252951943749 B_{3,8}(X) \\
& + 0.0006140171504808826 B_{4,8}(X) + 0.0002646832855456765 B_{5,8}(X) \\
& - 0.0007004205300922658 B_{6,8}(X) - 0.002281438549333955 B_{7,8}(X) \\
& - 0.004478515047503282 B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3257065380923488, 0.6592817116222256\}$$

Intersection intervals with the x axis:

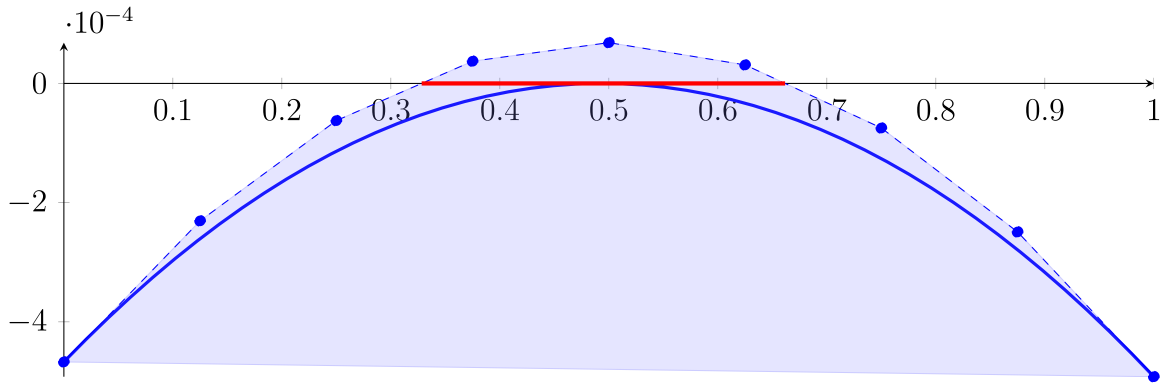
$$[0.3257065380923488, 0.6592817116222256]$$

Longest intersection interval: 0.3335751735298769
$\implies$ Selective recursion: interval 1: [0.4999184389112853, 0.5000837462892085],

61.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999184389112853, 0.50008

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-5.57619429667528\cdot 10^{-31}X^8 - 1.315536799505273\cdot 10^{-25}X^7 - 1.287049795843581\cdot 10^{-20}X^6\\&- 6.683358895768893\cdot 10^{-16}X^5 - 1.942733814710911\cdot 10^{-11}X^4 - 2.997323807488328\\&\cdot 10^{-07}X^3 - 0.001917590365692151X^2 + 0.001893040758741533X - 0.0004671653183669499\\= &-0.0004671653183669499B_{0,8}(X) - 0.0002305352235242582B_{1,8}(X) - 6.239049888485767\\&\cdot 10^{-05}B_{2,8}(X) + 3.726350318730984\cdot 10^{-05}B_{3,8}(X) + 6.842143005076897\\&\cdot 10^{-05}B_{4,8}(X) + 3.107792878649904\cdot 10^{-05}B_{5,8}(X) - 7.47723538020779\\&\cdot 10^{-05}B_{6,8}(X) - 0.0002491347711189109B_{7,8}(X) - 0.0004920146771263226B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3282588977707033, 0.661700337526842\}$$

Intersection intervals with the x axis:

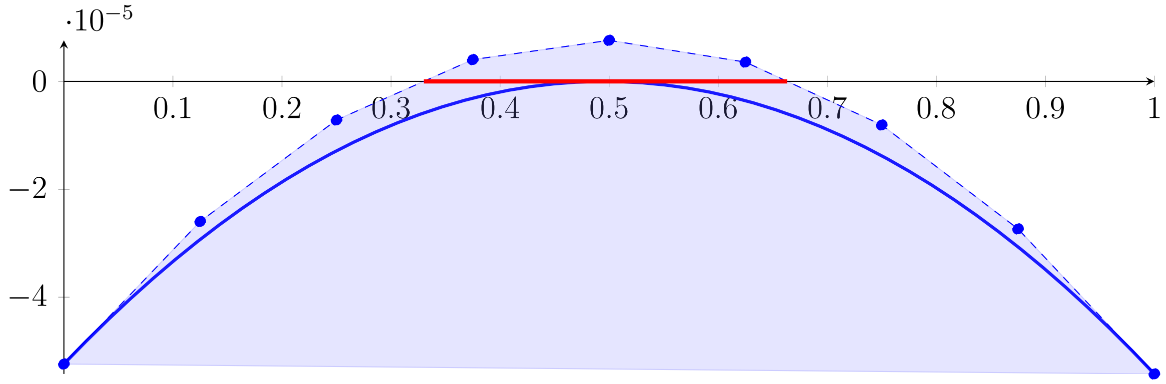
$$[0.3282588977707033, 0.661700337526842]$$

Longest intersection interval: 0.3334414397561387
$\implies$ Selective recursion: interval 1: [0.4999727025289557, 0.5000278228590528],

61.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.4999727025289557, 0.500

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-8.521076768712742\cdot 10^{-35}X^8 - 6.02899387826599\cdot 10^{-29}X^7 - 1.76898025411792\cdot 10^{-23}X^6\\&- 2.754920799910016\cdot 10^{-18}X^5 - 2.401685361883718\cdot 10^{-13}X^4 - 1.111294950492419\cdot 10^{-08}X^3\\&- 0.0002132366404355359X^2 + 0.0002114057621846039X - 5.239629569054843\cdot 10^{-05}\\= &-5.239629569054843\cdot 10^{-05}B_{0,8}(X) - 2.597057541747294\cdot 10^{-05}B_{1,8}(X) - 7.160449445666593\\&\cdot 10^{-06}B_{2,8}(X) + 4.033883779343745\cdot 10^{-06}B_{3,8}(X) + 7.612225808600218\cdot 10^{-06}B_{4,8}(X)\\&+ 3.574378189713946\cdot 10^{-06}B_{5,8}(X) - 8.079857533135031\cdot 10^{-06}B_{6,8}(X)\\&- 2.73506798191978\cdot 10^{-05}B_{7,8}(X) - 5.423828713115661\cdot 10^{-05}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3299561852159799, 0.6633377584201652\}$$

Intersection intervals with the x axis:

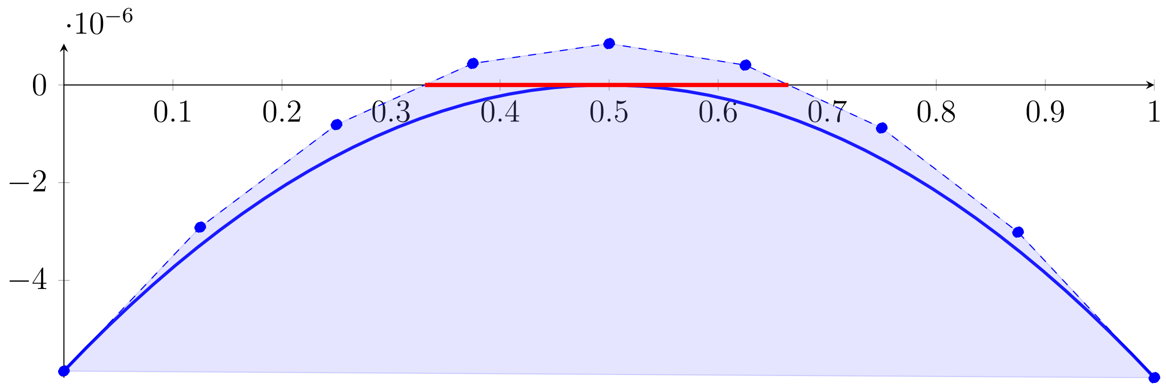
$$[0.3299561852159799, 0.6633377584201652]$$

Longest intersection interval: 0.3333815732041853
$\implies$ Selective recursion: interval 1: $[0.4999908898228024, 0.5000092659251657]$,

61.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999908898228024, 0.5$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.300251044438937\cdot10^{-38}X^8 - 2.759545789497528\cdot10^{-32}X^7 - 2.428711637904422\cdot10^{-26}X^6 \\ &\quad - 1.134547275299063\cdot10^{-20}X^5 - 2.966816573305973\cdot10^{-15}X^4 - 4.117811892390909\cdot10^{-10}X^3 \\ &\quad - 2.370104084998438\cdot10^{-05}X^2 + 2.356495503794524\cdot10^{-05}X - 5.857360304639568\cdot10^{-06} \\ &= -5.857360304639568\cdot10^{-06}B_{0,8}(X) - 2.911740924896412\cdot10^{-06}B_{1,8}(X) - 8.125872897955564 \\ &\quad \cdot10^{-07}B_{2,8}(X) + 4.400932474274781\cdot10^{-07}B_{3,8}(X) + 8.462933334947859\cdot10^{-07}B_{4,8}(X) \\ &\quad + 4.060056150860783\cdot10^{-07}B_{5,8}(X) - 8.807772611613165\cdot10^{-07}B_{6,8}(X) \\ &\quad - 3.014062648652454\cdot10^{-06}B_{7,8}(X) - 5.993857900834775\cdot10^{-06}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.331084848216461, 0.6644399885346334\}$$

Intersection intervals with the x axis:

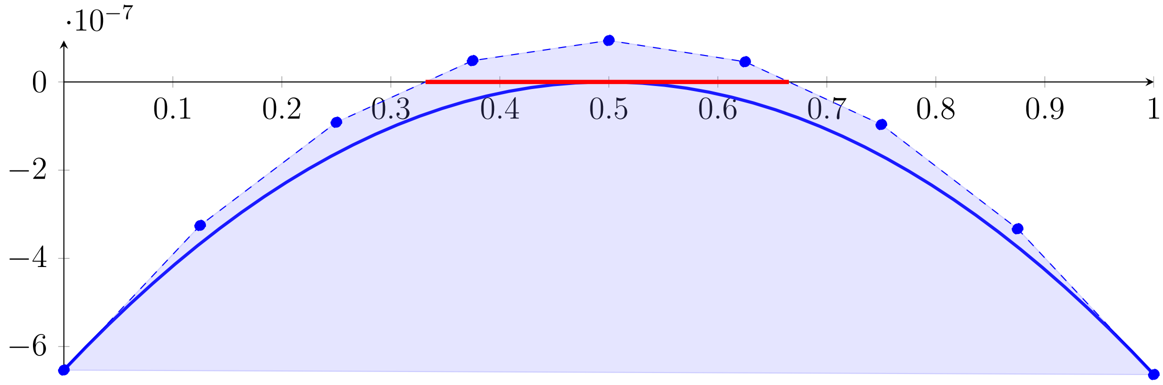
$$[0.331084848216461, 0.6644399885346334]$$

Longest intersection interval: 0.3333551403181724
$\implies$ Selective recursion: interval 1: $[0.4999969738718641, 0.500003099640046]$,

61.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999969738718641, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.982825346126094 \cdot 10^{-42} X^8 - 1.262374580662423 \cdot 10^{-35} X^7 - 3.332882748505763 \cdot 10^{-29} X^6 - 4.670466114941116 \cdot 10^{-23} X^5 - 3.663718276431707 \cdot 10^{-17} X^4 - 1.52542941384219 \cdot 10^{-11} X^3 - 2.633839011095458 \cdot 10^{-06} X^2 + 2.623741169043712 \cdot 10^{-06} X - 6.534168705792709 \cdot 10^{-07}$$
$$= -6.534168705792709 \cdot 10^{-07} B_{0,8}(X) - 3.254492244488069 \cdot 10^{-07} B_{1,8}(X) - 9.154725728603783 \cdot 10^{-08} B_{2,8}(X) + 4.828875851092667 \cdot 10^{-08} B_{3,8}(X) + 9.405855054345363 \cdot 10^{-08} B_{4,8}(X) + 4.576184641238665 \cdot 10^{-08} B_{5,8}(X) - 9.660162628195405 \cdot 10^{-08} B_{6,8}(X) - 3.330321399397716 \cdot 10^{-07} B_{7,8}(X) - 6.635299669617927 \cdot 10^{-07} B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3318344765869905, 0.6651804668942704\}$$

Intersection intervals with the x axis:

$$[0.3318344765869905, 0.6651804668942704]$$

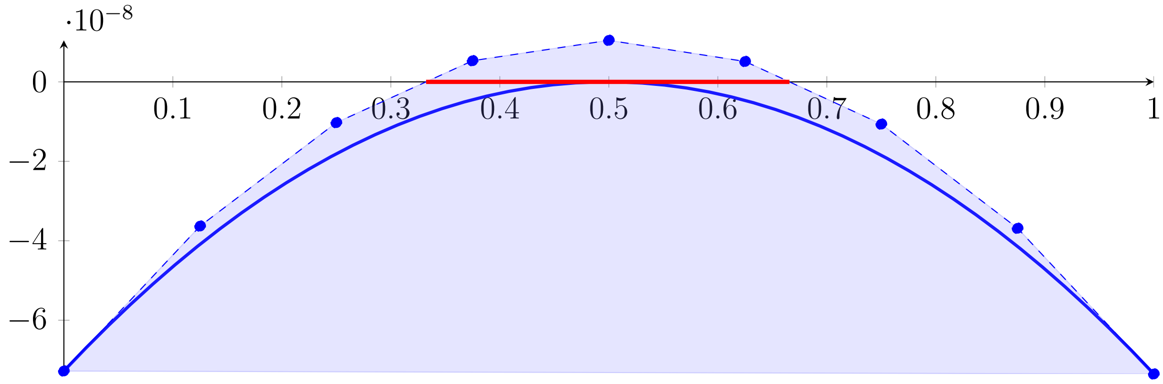
Longest intersection interval: 0.3333459903072799

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999990066129424, 0.5000010486132034]$,

61.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999990066129424$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -3.023057070307937 \cdot 10^{-46} X^8 - 5.773711385862107 \cdot 10^{-39} X^7 - 4.572901329678827 \cdot 10^{-32} X^6 - 1.922370176484083 \cdot 10^{-25} X^5 - 4.523805577109385 \cdot 10^{-19} X^4 - 5.650400184721212 \cdot 10^{-13} X^3 - 2.926726911514393 \cdot 10^{-07} X^2 + 2.919240677305221 \cdot 10^{-07} X - 7.27925149751094 \cdot 10^{-08}$$
$$= -7.27925149751094 \cdot 10^{-08} B_{0,8}(X) - 3.630200650879414 \cdot 10^{-08} B_{1,8}(X) - 1.026409415503029 \cdot 10^{-08} B_{2,8}(X) + 5.321211996181834 \cdot 10^{-09} B_{3,8}(X) + 1.045390185483543 \cdot 10^{-08} B_{4,8}(X) + 5.133965330917245 \cdot 10^{-09} B_{5,8}(X) - 1.063860766559244 \cdot 10^{-08} B_{6,8}(X) - 3.68638272247198 \cdot 10^{-08} B_{7,8}(X) - 7.354170343649749 \cdot 10^{-08} B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3323218842755297, 0.6656874431018115\}$$

Intersection intervals with the x axis:

$$[0.3323218842755297, 0.6656874431018115]$$

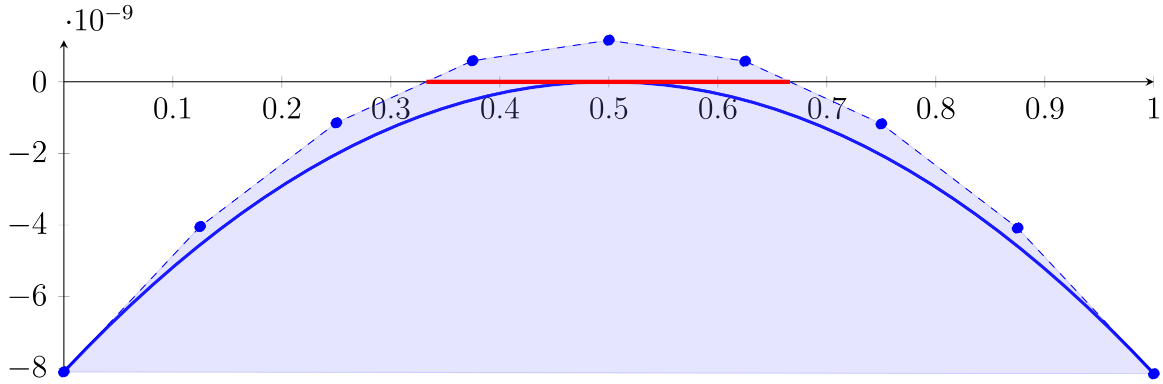
Longest intersection interval: 0.3333655588262817

$\implies$ Selective recursion: interval 1: $[0.4999996852143169, 0.500000365946875]$,

61.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999996852143169, 0.500000365946875]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -4.61118111521237\cdot 10^{-50}X^8 - 2.641801828130962\cdot 10^{-42}X^7 - 6.276482670428966\cdot 10^{-35}X^6\\
&\quad - 7.91481690665562\cdot 10^{-28}X^5 - 5.587109146485419\cdot 10^{-21}X^4 - 2.093350052120201\cdot 10^{-14}X^3\\
&\quad - 3.252553849467158\cdot 10^{-08}X^2 + 3.247007216419568\cdot 10^{-08}X - 8.101917765629863\cdot 10^{-09}\\
&= -8.101917765629863\cdot 10^{-09}B_{0,8}(X) - 4.043158745105403\cdot 10^{-09}B_{1,8}(X)\\
&\quad - 1.146026099390642\cdot 10^{-09}B_{2,8}(X) + 5.8947979770191\cdot 10^{-10}B_{3,8}(X) + 1.163358572359665\\
&\quad \cdot 10^{-09}B_{4,8}(X) + 5.75609850769953\cdot 10^{-10}B_{5,8}(X) - 1.173766740879974\cdot 10^{-09}B_{6,8}(X)\\
&\quad - 4.084771576402945\cdot 10^{-09}B_{7,8}(X) - 8.157405029611868\cdot 10^{-09}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.332542653795541, 0.6661296410902487\}$$

Intersection intervals with the x axis:

$$[0.332542653795541, 0.6661296410902487]$$

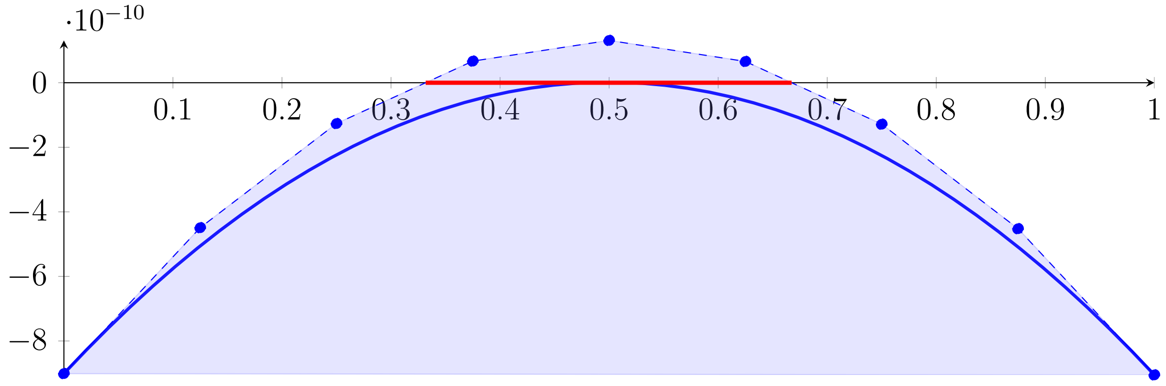
Longest intersection interval: 0.3335869872947078

$\implies$ Selective recursion: interval 1: $[0.4999999115869283, 0.5000001386704515]$,

61.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999115869283, 0.5000001386704515]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -7.071067608997674\cdot 10^{-54}X^8 - 1.214406168666063\cdot 10^{-45}X^7 - 8.649101296569956\cdot 10^{-38}X^6\\
&\quad - 3.269538436354099\cdot 10^{-30}X^5 - 6.918686662336931\cdot 10^{-23}X^4 - 7.770864123205297\cdot 10^{-16}X^3\\
&\quad - 3.61945329275134\cdot 10^{-09}X^2 + 3.615351534501487\cdot 10^{-09}X - 9.01058772952768\cdot 10^{-10}\\
&= -9.01058772952768\cdot 10^{-10}B_{0,8}(X) - 4.491398311400821\cdot 10^{-10}B_{1,8}(X) - 1.264870783542298\\
&\quad \cdot 10^{-10}B_{2,8}(X) + 6.689947152824597\cdot 10^{-11}B_{3,8}(X) + 1.31019804630801\cdot 10^{-10}B_{4,8}(X)\\
&\quad + 6.587390707689027\cdot 10^{-11}B_{5,8}(X) - 1.285382350100323\cdot 10^{-10}B_{6,8}(X)\\
&\quad - 4.522166355065136\cdot 10^{-10}B_{7,8}(X) - 9.051613082891018\cdot 10^{-10}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3317579340646351, 0.6673545479012814\}$$

Intersection intervals with the x axis:

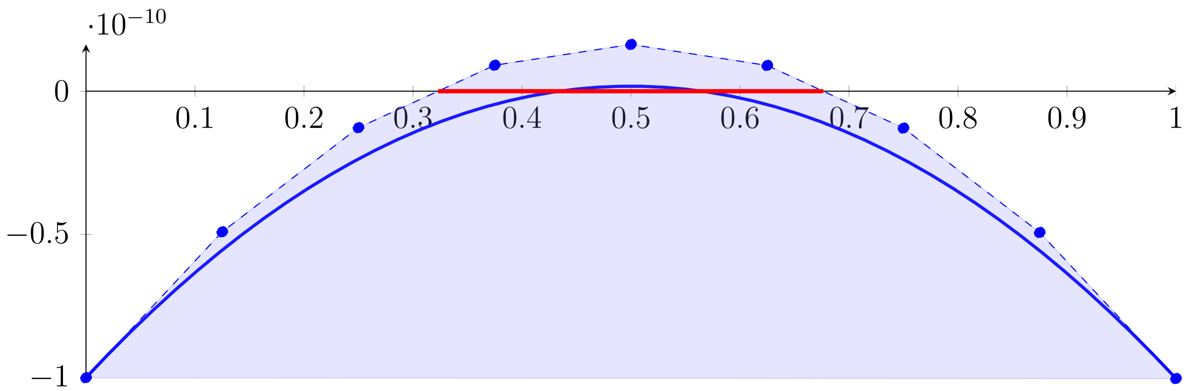
$$[0.3317579340646351, 0.6673545479012814]$$

Longest intersection interval: 0.3355966138366463
$\Longrightarrow$ Selective recursion: interval 1: $[0.4999999869236888, 0.5000000631321502]$,

61.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.4999999869236888, 0.5000000631321502]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1.137694218100245 \cdot 10^{-57} X^8 - 5.822197888022611 \cdot 10^{-49} X^7 - 1.235595786383296 \cdot 10^{-40} X^6 \\ & - 1.391791952088144 \cdot 10^{-32} X^5 - 8.775946704095466 \cdot 10^{-25} X^4 - 2.937122613307657 \cdot 10^{-17} X^3 \\ & - 4.076413298856481 \cdot 10^{-10} X^2 + 4.07342667585279 \cdot 10^{-10} X - 1.000063159750977 \cdot 10^{-10} \\ = & -1.000063159750977 \cdot 10^{-10} B_{0,8}(X) - 4.908848252693777 \cdot 10^{-11} B_{1,8}(X) - 1.272926800326533 \\ & \cdot 10^{-11} B_{2,8}(X) + 9.071327071433506 \cdot 10^{-12} B_{3,8}(X) + 1.631330217267253 \cdot 10^{-11} B_{4,8}(X) \\ & + 8.996656775965545 \cdot 10^{-12} B_{5,8}(X) - 1.287860964317367 \cdot 10^{-11} B_{6,8}(X) \\ & - 4.931249760923136 \cdot 10^{-11} B_{7,8}(X) - 1.003050076466937 \cdot 10^{-10} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3229869297125206, 0.6764088411746962\}$$

Intersection intervals with the x axis:

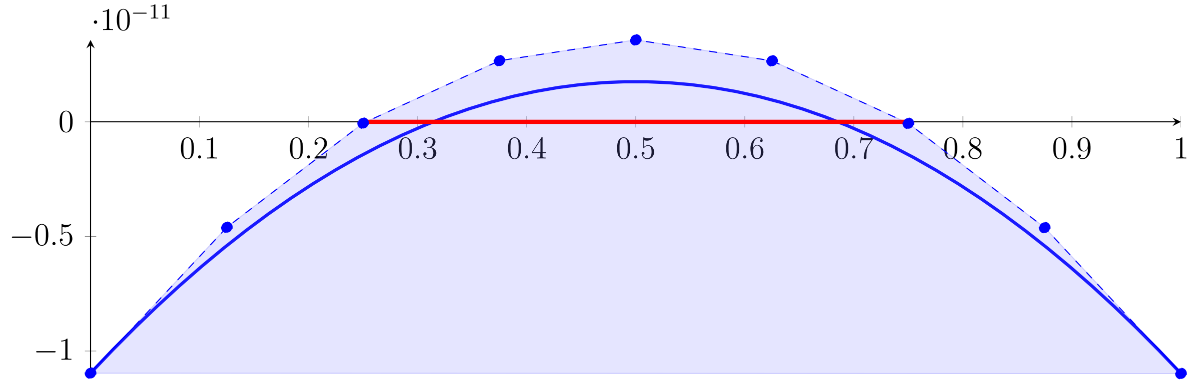
$$[0.3229869297125206, 0.6764088411746962]$$

Longest intersection interval: 0.3534219114621756
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000115380258, 0.5000000384717659]$,

61.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000115380258, 0.5000000384717659]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -2.769321148091343\cdot 10^{-61}X^8 - 4.009971301105125\cdot 10^{-52}X^7 - 2.40789337418918\cdot 10^{-43}X^6 \\ & - 7.674351474736958\cdot 10^{-35}X^5 - 1.36920310040832\cdot 10^{-26}X^4 - 1.29658952293724\cdot 10^{-18}X^3 \\ & - 5.091727851043352\cdot 10^{-11}X^2 + 5.08987688183243\cdot 10^{-11}X - 1.096532991460483\cdot 10^{-11} \\ = & -1.096532991460483\cdot 10^{-11}B_{0,8}(X) - 4.602983812314296\cdot 10^{-12}B_{1,8}(X) \\ & - 5.91119425392419\cdot 10^{-14}B_{2,8}(X) + 2.666285671566945\cdot 10^{-12}B_{3,8}(X) + 3.57320900685088 \\ & \cdot 10^{-12}B_{4,8}(X) + 2.661658040159178\cdot 10^{-12}B_{5,8}(X) - 6.83672516615453\cdot 10^{-14}B_{6,8}(X) \\ & - 4.616866891764675\cdot 10^{-12}B_{7,8}(X) - 1.09838409033036\cdot 10^{-11}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.252711161402344, 0.7468696603349071\}$$

Intersection intervals with the x axis:

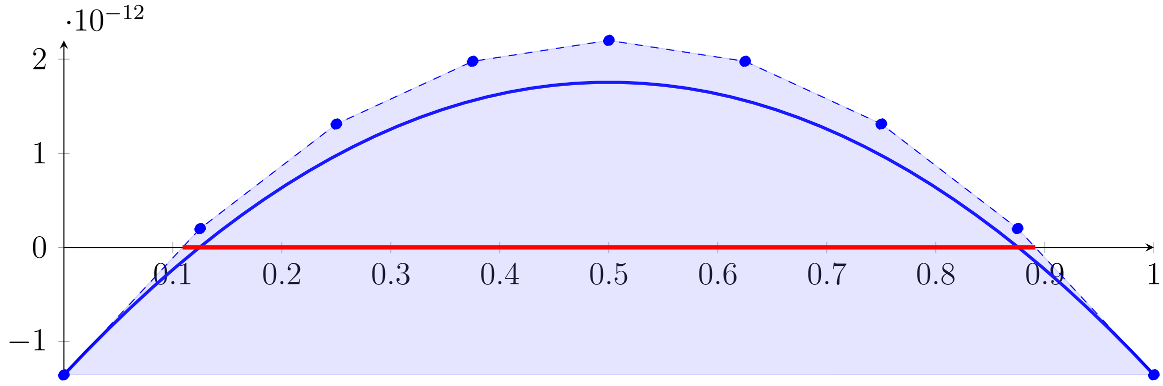
$$[0.252711161402344, 0.7468696603349071]$$

Longest intersection interval: 0.4941584989325631
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000183444825, 0.5000000316540191]$,

61.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000183444825, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -9.84698950588755\cdot 10^{-64}X^8 - 2.885394162330348\cdot 10^{-54}X^7 - 3.506185324343073\cdot 10^{-45}X^6 \\ & - 2.261377324294573\cdot 10^{-36}X^5 - 8.164563166712554\cdot 10^{-28}X^4 - 1.564592773244276\cdot 10^{-19}X^3 \\ & - 1.243362398803133\cdot 10^{-11}X^2 + 1.243502393415518\cdot 10^{-11}X - 1.354369602677131\cdot 10^{-12} \\ = & -1.354369602677131\cdot 10^{-12}B_{0,8}(X) + 2.000083890922661\cdot 10^{-13}B_{1,8}(X) \\ & + 1.310328381289116\cdot 10^{-12}B_{2,8}(X) + 1.976590371119503\cdot 10^{-12}B_{3,8}(X) \\ & + 2.19879435578951\cdot 10^{-12}B_{4,8}(X) + 1.976940332505223\cdot 10^{-12}B_{5,8}(X) + 1.311028298472726 \\ & \cdot 10^{-12}B_{6,8}(X) + 2.01058250898102\cdot 10^{-13}B_{7,8}(X) - 1.352969813012563\cdot 10^{-12}B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.108915721421098, 0.8911723471704997\}$$

Intersection intervals with the x axis:

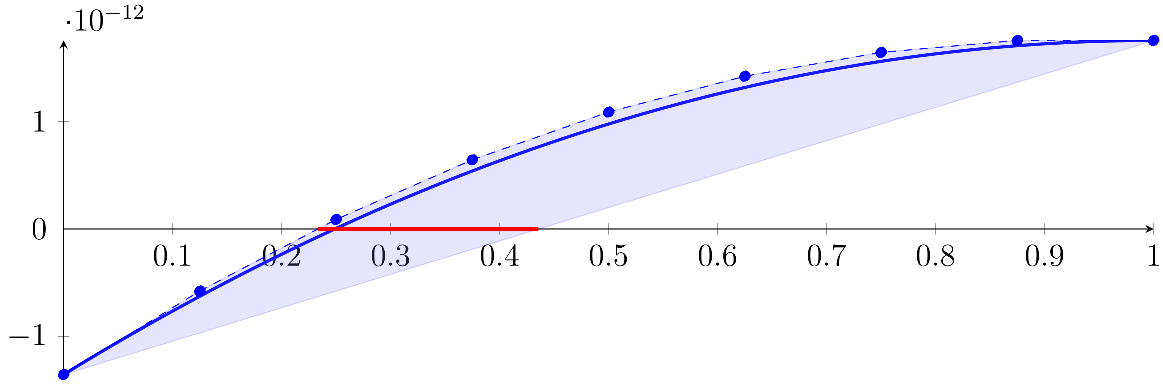
$$[0.108915721421098, 0.8911723471704997]$$

Longest intersection interval: 0.7822566257494017
$\Longrightarrow$ Bisection: first half $[0.5000000183444825, 0.5000000249992508]$ und second half $[0.5000000249992508, 0.50$

61.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.5000000183444825, 0.5000000249992508]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.846480275737324\cdot 10^{-66}X^8 - 2.254214189320585\cdot 10^{-56}X^7 - 5.478414569286052\cdot 10^{-47}X^6\\
&\quad - 7.06680413842054\cdot 10^{-38}X^5 - 5.102851979195346\cdot 10^{-29}X^4 - 1.955740966555345\cdot 10^{-20}X^3\\
&\quad - 3.108405997007834\cdot 10^{-12}X^2 + 6.21751196707759\cdot 10^{-12}X - 1.354369602677131\cdot 10^{-12}\\
&= -1.354369602677131\cdot 10^{-12}B_{0,8}(X) - 5.771806067924327\cdot 10^{-13}B_{1,8}(X) + 8.899388919912914\\
&\quad \cdot 10^{-14}B_{2,8}(X) + 6.441538849483146\cdot 10^{-13}B_{3,8}(X) + 1.088299380105884\cdot 10^{-12}B_{4,8}(X)\\
&\quad + 1.421430374322599\cdot 10^{-12}B_{5,8}(X) + 1.643546867249218\cdot 10^{-12}B_{6,8}(X)\\
&\quad + 1.754648858536504\cdot 10^{-12}B_{7,8}(X) + 1.754736347835215\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.2333013178726793, 0.4356138466281428\}$$

Intersection intervals with the x axis:

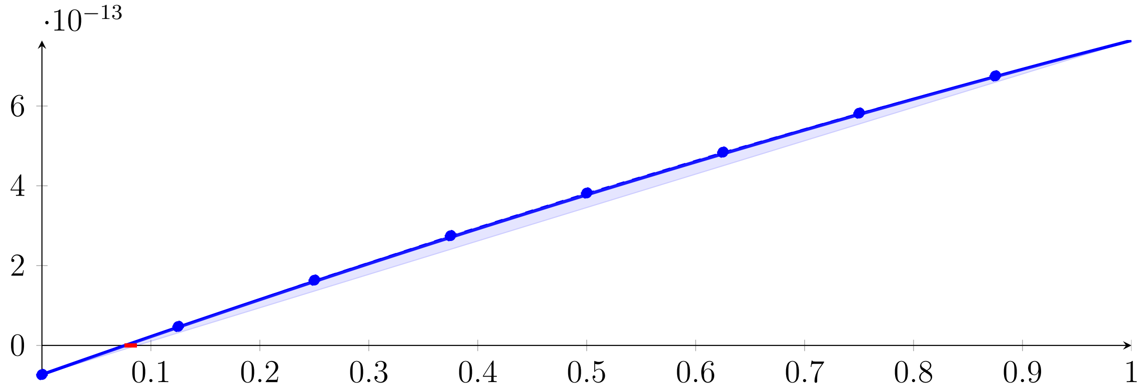
$$[0.2333013178726793, 0.4356138466281428]$$

Longest intersection interval: 0.2023125287554634
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000198970487, 0.5000000212433917]$,

61.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000198970487, 0.5000000212433917]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -1.079557397202381\cdot 10^{-71}X^8 - 3.12719259192325\cdot 10^{-61}X^7 - 3.756570682132575\cdot 10^{-51}X^6\\ &\quad - 2.395173331861966\cdot 10^{-41}X^5 - 8.548778908133761\cdot 10^{-32}X^4 - 1.619495215947232\cdot 10^{-22}X^3\\ &\quad - 1.272281748414343\cdot 10^{-13}X^2 + 9.644484121679\cdot 10^{-13}X - 7.300486662812878\cdot 10^{-14}\\ &= -7.300486662812878\cdot 10^{-14}B_{0,8}(X) + 4.755118489285872\cdot 10^{-14}B_{1,8}(X) + 1.635633730266521\\ &\quad \cdot 10^{-13}B_{2,8}(X) + 2.750316977703595\cdot 10^{-13}B_{3,8}(X) + 3.819561591210889\cdot 10^{-13}B_{4,8}(X)\\ &\quad + 4.843367570759483\cdot 10^{-13}B_{5,8}(X) + 5.821734916320458\cdot 10^{-13}B_{6,8}(X)\\ &\quad + 6.754663627864895\cdot 10^{-13}B_{7,8}(X) + 7.642153705363873\cdot 10^{-13}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.07569597886944255, 0.08719911844866588\}$$

Intersection intervals with the x axis:

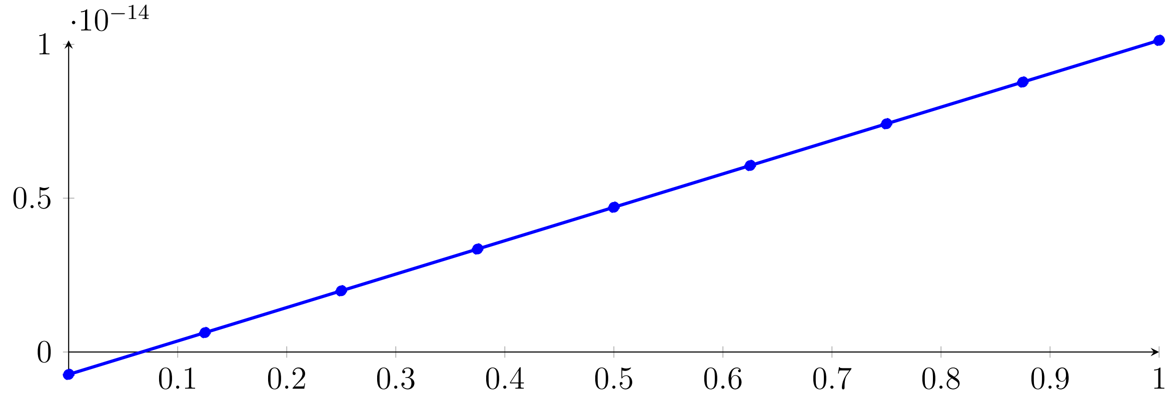
$$[0.07569597886944255, 0.08719911844866588]$$

Longest intersection interval: 0.01150313957922333
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199989615, 0.5000000200144486]$,

61.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199989615, 0.5000000200144486]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -3.309610258465664\cdot 10^{-87}X^8 - 8.334304343480435\cdot 10^{-75}X^7 - 8.703419206351739\cdot 10^{-63}X^6\\ &\quad - 4.824128808781935\cdot 10^{-51}X^5 - 1.496820228100742\cdot 10^{-39}X^4 - 2.465067626410487\cdot 10^{-28}X^3\\ &\quad - 1.683511456921895\cdot 10^{-17}X^2 + 1.087261902125672\cdot 10^{-14}X - 7.290023293677624\cdot 10^{-16}\\ &= -7.290023293677624\cdot 10^{-16}B_{0,8}(X) + 6.300750482893273\cdot 10^{-16}B_{1,8}(X) + 1.988551171854659\\ &\quad \cdot 10^{-15}B_{2,8}(X) + 3.346426041328229\cdot 10^{-15}B_{3,8}(X) + 4.703699656710032\cdot 10^{-15}B_{4,8}(X)\\ &\quad + 6.060372018000064\cdot 10^{-15}B_{5,8}(X) + 7.41644312519832\cdot 10^{-15}B_{6,8}(X)\\ &\quad + 8.771912978304797\cdot 10^{-15}B_{7,8}(X) + 1.012678157731949\cdot 10^{-14}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06704937678240291, 0.0671533567390459\}$$

Intersection intervals with the x axis:

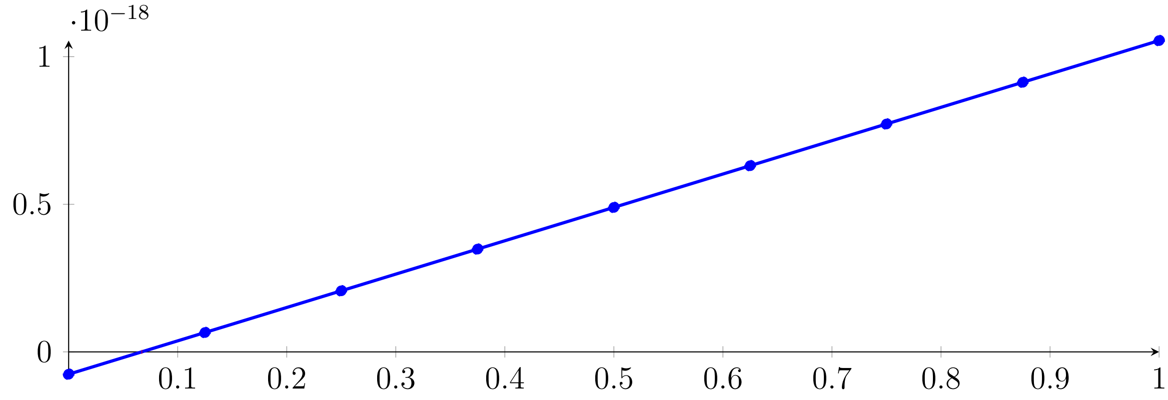
$$[0.06704937678240291, 0.0671533567390459]$$

Longest intersection interval: 0.0001039799566429901
$\implies$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000200000015]$,

61.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000200000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -4.522451416965012\cdot10^{-119}X^8 - 1.095258871423056\cdot10^{-102}X^7 - 1.099987358691302\cdot10^{-86}X^6\\&\quad - 5.863636713606292\cdot10^{-71}X^5 - 1.749718451421176\cdot10^{-55}X^4 - 2.771262941212064\cdot10^{-40}X^3\\&\quad - 1.820184200444671\cdot10^{-25}X^2 + 1.130299712615757\cdot10^{-18}X - 7.568425969413026\cdot10^{-20}\\&= -7.568425969413026\cdot10^{-20}B_{0,8}(X) + 6.560320438283937\cdot10^{-20}B_{1,8}(X) + 2.068906619591512\\&\quad\cdot10^{-19}B_{2,8}(X) + 3.481781130348051\cdot10^{-19}B_{3,8}(X) + 4.894655576098011\cdot10^{-19}B_{4,8}(X)\\&\quad + 6.307529956841393\cdot10^{-19}B_{5,8}(X) + 7.720404272578197\cdot10^{-19}B_{6,8}(X)\\&\quad + 9.133278523308422\cdot10^{-19}B_{7,8}(X) + 1.054615270903207\cdot10^{-18}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695946114945086, 0.06695947193230531\}$$

Intersection intervals with the x axis:

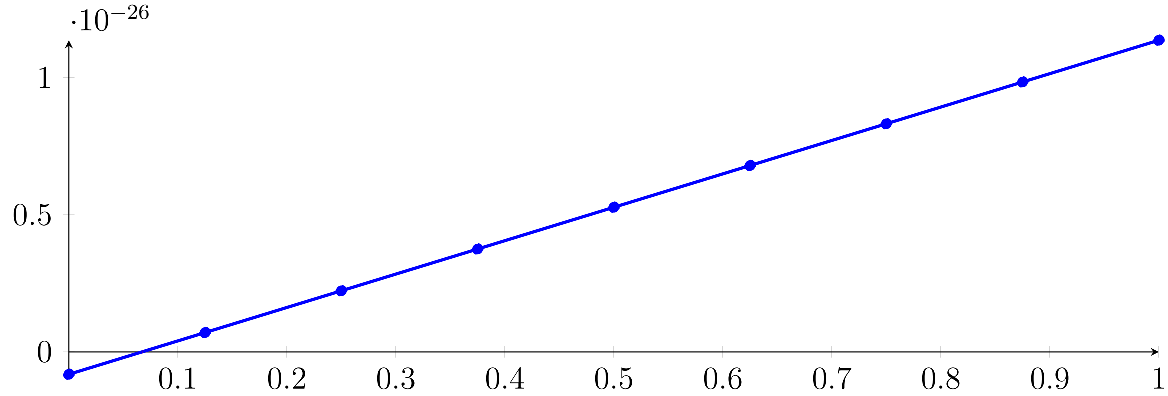
$$[0.06695946114945086, 0.06695947193230531]$$

Longest intersection interval: $1.078285445187343\cdot10^{-08}$
$\implies$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

61.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-8.265018972796948\cdot10^{-183}X^8 - 1.856320587559019\cdot10^{-158}X^7 - 1.7289807291512\cdot10^{-134}X^6\\ &- 8.547434292245386\cdot10^{-111}X^5 - 2.365392110963433\cdot10^{-87}X^4 - 3.474393176904993\cdot10^{-64}X^3\\ &- 2.116327262136374\cdot10^{-41}X^2 + 1.218785702529033\cdot10^{-26}X - 8.160922251597258\cdot10^{-28}\\ = &-8.160922251597258\cdot10^{-28}B_{0,8}(X) + 7.073899030015655\cdot10^{-28}B_{1,8}(X) + 2.230872031162856\\ &\cdot10^{-27}B_{2,8}(X) + 3.754354159324146\cdot10^{-27}B_{3,8}(X) + 5.277836287485435\cdot10^{-27}B_{4,8}(X)\\ &+ 6.801318415646723\cdot10^{-27}B_{5,8}(X) + 8.32480054380801\cdot10^{-27}B_{6,8}(X)\\ &+ 9.848282671969297\cdot10^{-27}B_{7,8}(X) + 1.137176480013058\cdot10^{-26}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695945181062587, 0.06695945181062598\}$$

Intersection intervals with the x axis:

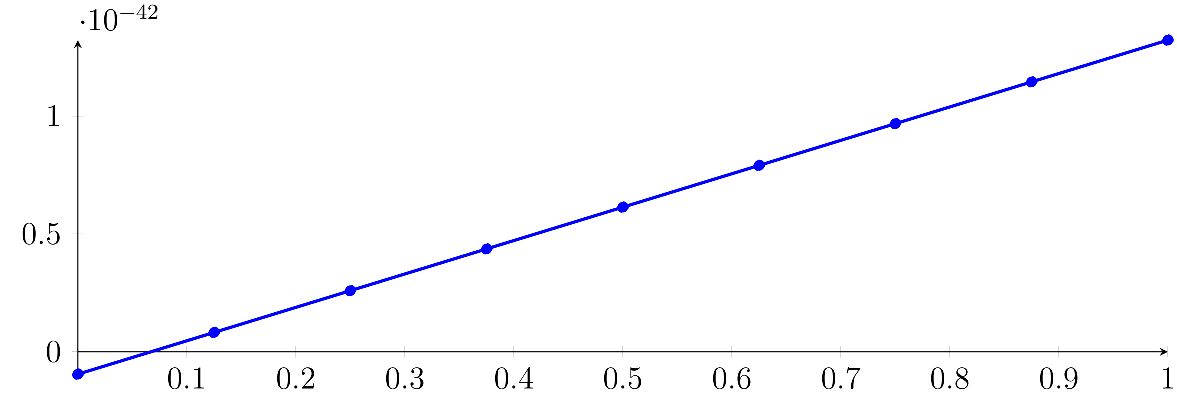
$$[0.06695945181062587, 0.06695945181062598]$$

Longest intersection interval: $1.162699176979875\cdot10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

61.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.76046836984994\cdot10^{-310}X^8 - 5.332422307789531\cdot10^{-270}X^7 - 4.271637033651821\cdot10^{-230}X^6\\ &- 1.816237323415445\cdot10^{-190}X^5 - 4.322874268764078\cdot10^{-151}X^4 - 5.461111688235365\cdot10^{-112}X^3\\ &- 2.860998015592859\cdot10^{-73}X^2 + 1.417081133245345\cdot10^{-42}X - 9.488697585328866\cdot10^{-44}\\ = &-9.488697585328866\cdot10^{-44}B_{0,8}(X) + 8.224816580237952\cdot10^{-44}B_{1,8}(X) + 2.593833074580477\\ &\cdot10^{-43}B_{2,8}(X) + 4.365184491137159\cdot10^{-43}B_{3,8}(X) + 6.136535907693841\cdot10^{-43}B_{4,8}(X)\\ &+ 7.907887324250522\cdot10^{-43}B_{5,8}(X) + 9.679238740807204\cdot10^{-43}B_{6,8}(X)\\ &+ 1.145059015736389\cdot10^{-42}B_{7,8}(X) + 1.322194157392057\cdot10^{-42}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695945181062577, 0.06695945181062577\}$$

Intersection intervals with the x axis:

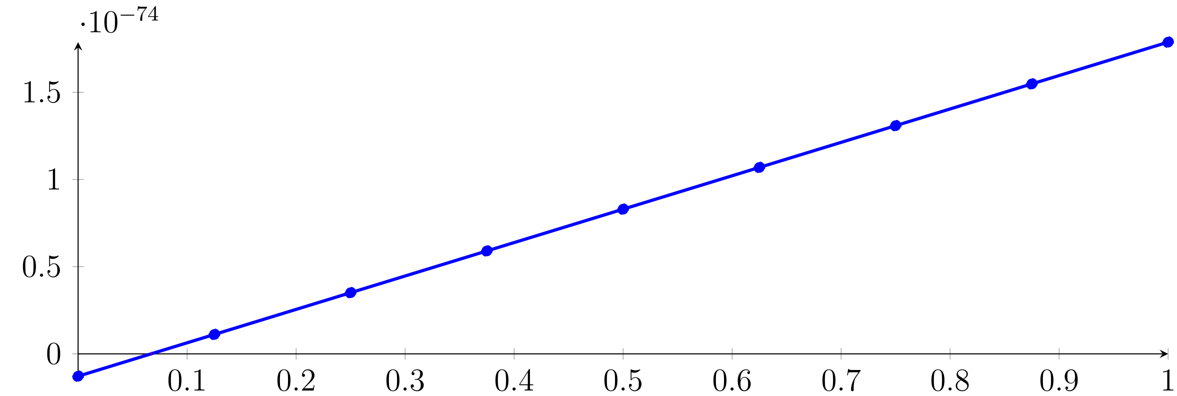
$$[0.06695945181062577, 0.06695945181062577]$$

Longest intersection interval: $1.351869376149675 \cdot 10^{-32}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

61.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-2.459662449869127 \cdot 10^{-381} X^8 + 1.731602364707866 \cdot 10^{-380} X^7 + 2.754821943853423 \cdot 10^{-381} X^6 \\
&- 8.200611664641817 \cdot 10^{-350} X^5 - 1.443814917429404 \cdot 10^{-278} X^4 - 1.349227700405142 \cdot 10^{-207} X^3 \\
&- 5.228619241295246 \cdot 10^{-137} X^2 + 1.91570858755386 \cdot 10^{-74} X - 1.282747968515146 \cdot 10^{-75} \\
= &-1.282747968515146 \cdot 10^{-75} B_{0,8}(X) + 1.111887765927179 \cdot 10^{-75} B_{1,8}(X) + 3.506523500369503 \\
&\cdot 10^{-75} B_{2,8}(X) + 5.901159234811828 \cdot 10^{-75} B_{3,8}(X) + 8.295794969254153 \cdot 10^{-75} B_{4,8}(X) \\
&+ 1.069043070369648 \cdot 10^{-74} B_{5,8}(X) + 1.30850664381388 \cdot 10^{-74} B_{6,8}(X) \\
&+ 1.547970217258113 \cdot 10^{-74} B_{7,8}(X) + 1.787433790702345 \cdot 10^{-74} B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.06695945181062577, 0.06695945181062577\}$$

Intersection intervals with the x axis:

$$[0.06695945181062577, 0.06695945181062577]$$

Longest intersection interval: $1.827550810171312 \cdot 10^{-64}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

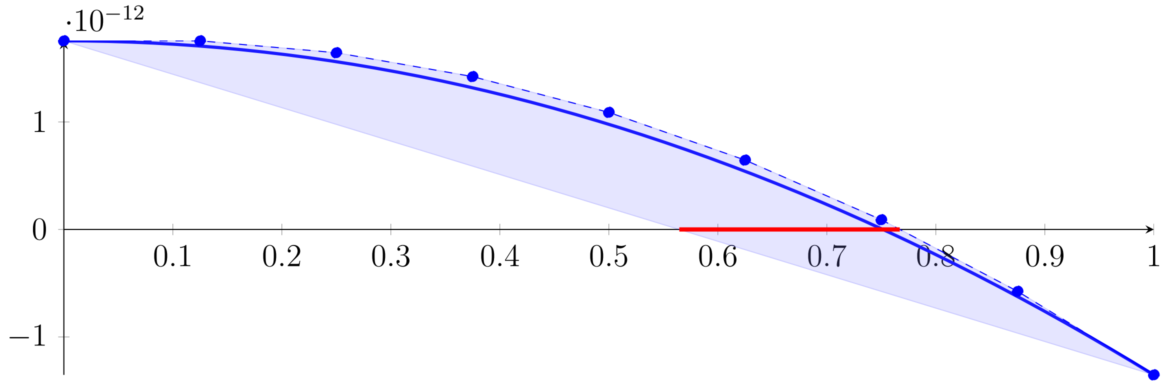
61.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 26!

61.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.5000000249992508, 0.5000000316540191]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.846480275737324\cdot 10^{-66}X^8 - 2.254214192397769\cdot 10^{-56}X^7 - 5.478414585065551\cdot 10^{-47}X^6\\ &- 7.066804171291027\cdot 10^{-38}X^5 - 5.102852014529367\cdot 10^{-29}X^4 - 1.955740986966753\cdot 10^{-20}X^3\\ &- 3.108406055680063\cdot 10^{-12}X^2 + 6.999143896938875\cdot 10^{-16}X + 1.754736347835215\cdot 10^{-12}\\ = &\ 1.754736347835215\cdot 10^{-12}B_{0,8}(X) + 1.754823837133927\cdot 10^{-12}B_{1,8}(X) + 1.643896824444065\\ &\cdot 10^{-12}B_{2,8}(X) + 1.42195530941639\cdot 10^{-12}B_{3,8}(X) + 1.088999291701662\cdot 10^{-12}B_{4,8}(X)\\ &+ 6.450287709506428\cdot 10^{-13}B_{5,8}(X) + 9.00437468140915\cdot 10^{-14}B_{6,8}(X)\\ &- 5.759557810572307\cdot 10^{-13}B_{7,8}(X) - 1.352969813012563\cdot 10^{-12}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.5646403672078589, 0.7669001146107961\}$$

Intersection intervals with the x axis:

$$[0.5646403672078589, 0.7669001146107961]$$

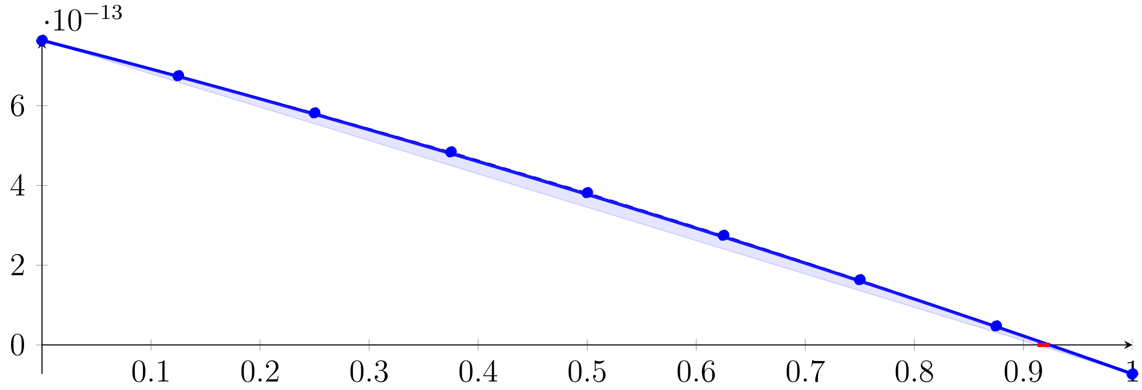
Longest intersection interval: 0.2022597474029372

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000287568016, 0.5000000301027934]$,

61.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.5000000287568016, 0.5000000301027934]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.077306286121587\cdot 10^{-71}X^8 - 3.121486088546909\cdot 10^{-61}X^7 - 3.750694216009664\cdot 10^{-51}X^6\\ &- 2.392050590455968\cdot 10^{-41}X^5 - 8.5398613074672\cdot 10^{-32}X^4 - 1.618228037882981\cdot 10^{-22}X^3\\ &- 1.271618015330847\cdot 10^{-13}X^2 - 7.098433618236863\cdot 10^{-13}X + 7.641134288463311\cdot 10^{-13}\\ = &\ 7.641134288463311\cdot 10^{-13}B_{0,8}(X) + 6.753830086183703\cdot 10^{-13}B_{1,8}(X) + 5.821110954785137\\ &\cdot 10^{-13}B_{2,8}(X) + 4.842976894238714\cdot 10^{-13}B_{3,8}(X) + 3.819427904515539\cdot 10^{-13}B_{4,8}(X)\\ &+ 2.750463985586714\cdot 10^{-13}B_{5,8}(X) + 1.636085137423343\cdot 10^{-13}B_{6,8}(X)\\ &+ 4.762913599965286\cdot 10^{-14}B_{7,8}(X) - 7.289173467226265\cdot 10^{-14}B_{8,8}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9129136379925769, 0.9243992614454615\}$$

Intersection intervals with the x axis:

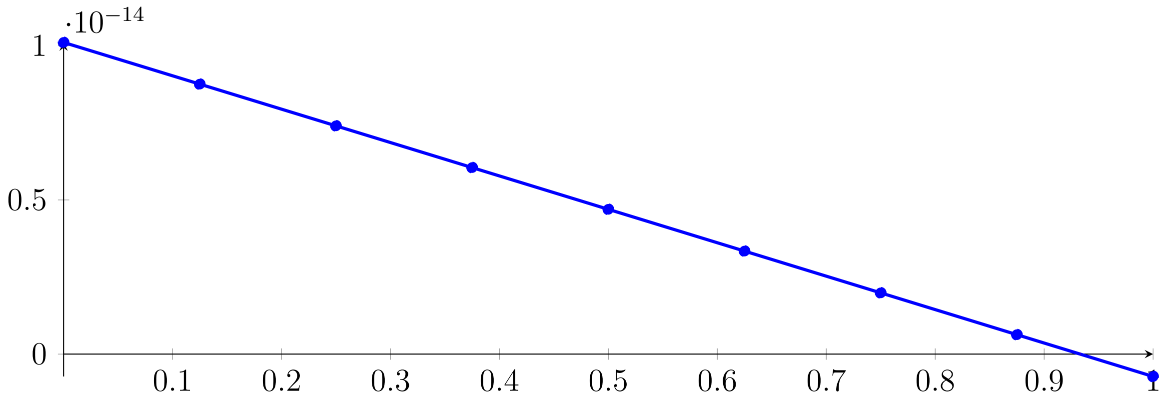
$$[0.9129136379925769, 0.9243992614454615]$$

Longest intersection interval: 0.01148562345288463
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000299855758, 0.5000000300010354]$,

61.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.5000000299855758, 0.5000000300010354]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.26268981617174\cdot 10^{-87}X^8 - 8.230826044001589\cdot 10^{-75}X^7 - 8.610712798456319\cdot 10^{-63}X^6 \\
& - 4.781269588122392\cdot 10^{-51}X^5 - 1.48617213692691\cdot 10^{-39}X^4 - 2.451903903103066\cdot 10^{-28}X^3 \\
& - 1.677512719815316\cdot 10^{-17}X^2 - 1.081967377285333\cdot 10^{-14}X + 1.010965922317952\cdot 10^{-14} \\
= & \ 1.010965922317952\cdot 10^{-14}B_{0,8}(X) + 8.757200001572855\cdot 10^{-15}B_{1,8}(X) + 7.40414166828054 \\
& \cdot 10^{-15}B_{2,8}(X) + 6.050484223302573\cdot 10^{-15}B_{3,8}(X) + 4.696227666638948\cdot 10^{-15}B_{4,8}(X) \\
& + 3.341371998289662\cdot 10^{-15}B_{5,8}(X) + 1.98591721825471\cdot 10^{-15}B_{6,8}(X) \\
& + 6.298633265340878\cdot 10^{-16}B_{7,8}(X) - 7.267896768722089\cdot 10^{-16}B_{8,8}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9329310105574586, 0.9330346747614\}$$

Intersection intervals with the x axis:

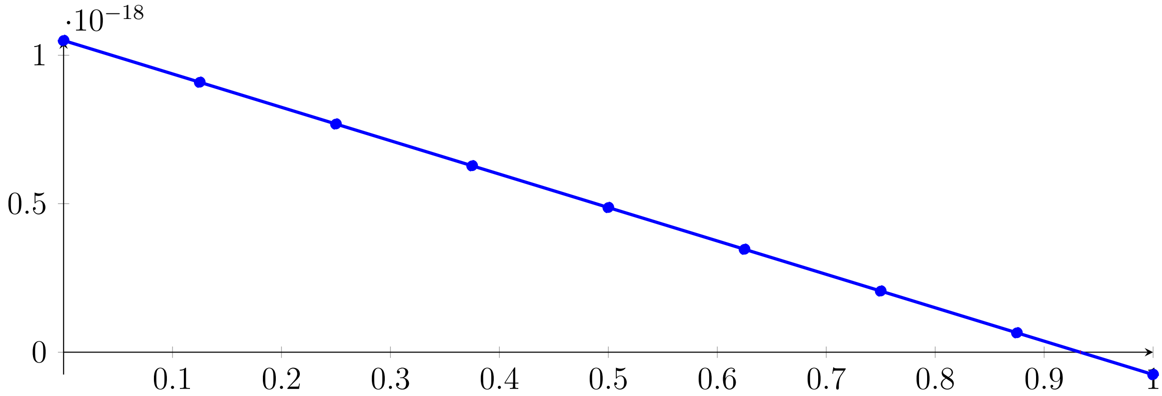
$$[0.9329310105574586, 0.9330346747614]$$

Longest intersection interval: 0.0001036642039414486
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000299999985, 0.5000000300000001]$,

61.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.5000000299999985, 0.5000000300000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -4.351172724257996\cdot 10^{-119} X^8 - 1.058876090749164\cdot 10^{-102} X^7 - 1.068592215336126\cdot 10^{-86} X^6 - 5.723837778458117\cdot 10^{-71} X^5 - 1.716265145654202\cdot 10^{-55} X^4 - 2.731428873735883\cdot 10^{-40} X^3 - 1.802699988373219\cdot 10^{-25} X^2 - 1.124857565692813\cdot 10^{-18} X + 1.049632124051115\cdot 10^{-18}$$
$$= 1.049632124051115\cdot 10^{-18} B_{0,8}(X) + 9.090249283395136\cdot 10^{-19} B_{1,8}(X) + 7.684177261896977\cdot 10^{-19} B_{2,8}(X) + 6.278105176016676\cdot 10^{-19} B_{3,8}(X) + 4.872033025754233\cdot 10^{-19} B_{4,8}(X) + 3.465960811109647\cdot 10^{-19} B_{5,8}(X) + 2.059888532082919\cdot 10^{-19} B_{6,8}(X) + 6.538161886740477\cdot 10^{-20} B_{7,8}(X) - 7.522562191169655\cdot 10^{-20} B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9331243242252754, 0.9331243349427871\}$$

Intersection intervals with the x axis:

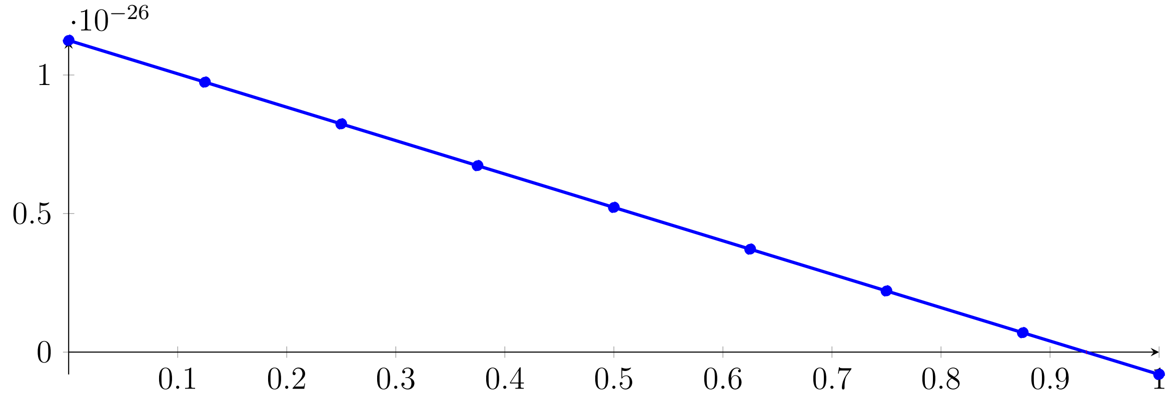
$$[0.9331243242252754, 0.9331243349427871]$$

Longest intersection interval: $1.071751170792712\cdot 10^{-08}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

61.31 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -7.574571270278352\cdot 10^{-183} X^8 - 1.719898859058455\cdot 10^{-158} X^7 - 1.619480808086848\cdot 10^{-134} X^6 - 8.09388724153584\cdot 10^{-111} X^5 - 2.264437035388769\cdot 10^{-87} X^4 - 3.36257357580136\cdot 10^{-64} X^3 - 2.070672372961545\cdot 10^{-41} X^2 - 1.205567773574102\cdot 10^{-26} X + 1.124944638137319\cdot 10^{-26}$$
$$= 1.124944638137319\cdot 10^{-26} B_{0,8}(X) + 9.742486664405558\cdot 10^{-27} B_{1,8}(X) + 8.235526947437929\cdot 10^{-27} B_{2,8}(X) + 6.7285672304703\cdot 10^{-27} B_{3,8}(X) + 5.22160751350267\cdot 10^{-27} B_{4,8}(X) + 3.714647796535039\cdot 10^{-27} B_{5,8}(X) + 2.207688079567407\cdot 10^{-27} B_{6,8}(X) + 7.007283625997749\cdot 10^{-28} B_{7,8}(X) - 8.062313543678582\cdot 10^{-28} B_{8,8}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.9331243442268158, 0.933124344226816\}$$

Intersection intervals with the x axis:

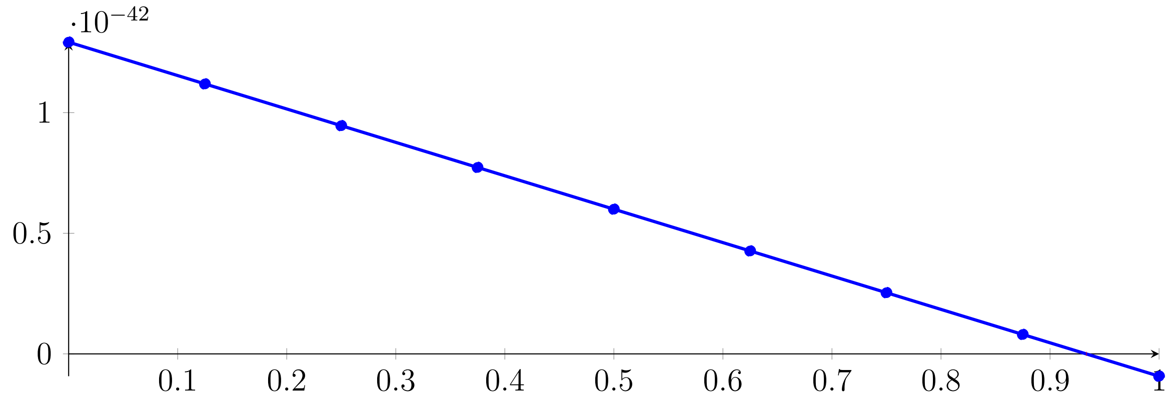
$$[0.9331243442268158, 0.933124344226816]$$

Longest intersection interval: $1.148650253172236 \cdot 10^{-16}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

61.32 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -2.295411522634873 \cdot 10^{-310} X^8 - 4.537510017831649 \cdot 10^{-270} X^7 - 3.719655178468669 \cdot 10^{-230} X^6 \\ & - 1.618438544591323 \cdot 10^{-190} X^5 - 3.941953525953829 \cdot 10^{-151} X^4 - 5.096068226680637 \cdot 10^{-112} X^3 \\ & - 2.732039753653228 \cdot 10^{-73} X^2 - 1.384775728332186 \cdot 10^{-42} X + 1.292167943401183 \cdot 10^{-42} \\ = & \, 1.292167943401183 \cdot 10^{-42} B_{0,8}(X) + 1.11907097735966 \cdot 10^{-42} B_{1,8}(X) + 9.459740113181363 \\ & \cdot 10^{-43} B_{2,8}(X) + 7.72877045276613 \cdot 10^{-43} B_{3,8}(X) + 5.997800792350898 \cdot 10^{-43} B_{4,8}(X) \\ & + 4.266831131935665 \cdot 10^{-43} B_{5,8}(X) + 2.535861471520432 \cdot 10^{-43} B_{6,8}(X) \\ & + 8.048918111051992 \cdot 10^{-44} B_{7,8}(X) - 9.260778493100336 \cdot 10^{-44} B_{8,8}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9331243442268161, 0.9331243442268161\}$$

Intersection intervals with the x axis:

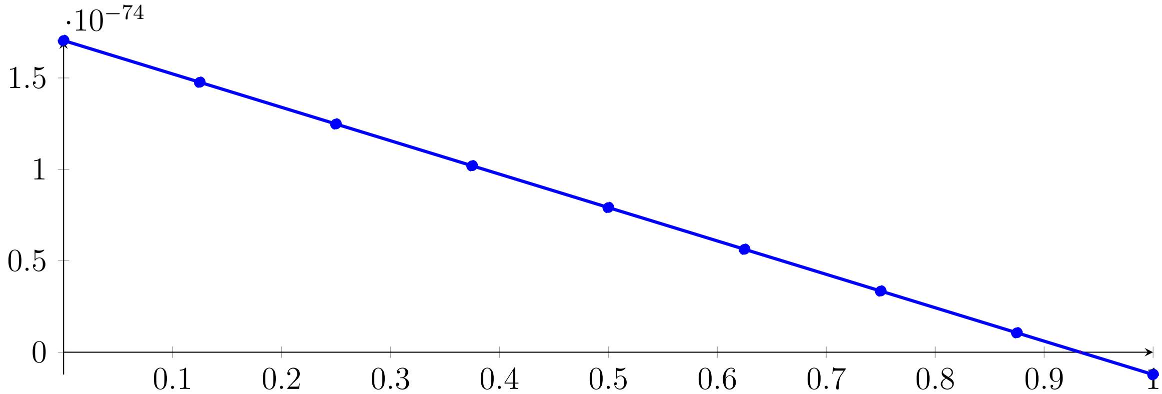
$$[0.9331243442268161, 0.9331243442268161]$$

Longest intersection interval: $1.319397404112639 \cdot 10^{-32}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

61.33 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -7.870919839581208\cdot 10^{-382}X^8 - 1.259347174332993\cdot 10^{-380}X^7 - 2.203857555082738\cdot 10^{-380}X^6 \\
& - 6.471043800004731\cdot 10^{-350}X^5 - 1.194576593485926\cdot 10^{-278}X^4 - 1.170474912363739\cdot 10^{-207}X^3 \\
& - 4.755960784800685\cdot 10^{-137}X^2 - 1.827069501239675\cdot 10^{-74}X + 1.704883030201088\cdot 10^{-74} \\
= & \ 1.704883030201088\cdot 10^{-74}B_{0,8}(X) + 1.476499342546128\cdot 10^{-74}B_{1,8}(X) + 1.248115654891169 \\
& \cdot 10^{-74}B_{2,8}(X) + 1.019731967236209\cdot 10^{-74}B_{3,8}(X) + 7.913482795812501\cdot 10^{-75}B_{4,8}(X) \\
& + 5.629645919262907\cdot 10^{-75}B_{5,8}(X) + 3.345809042713314\cdot 10^{-75}B_{6,8}(X) \\
& + 1.06197216616372\cdot 10^{-75}B_{7,8}(X) - 1.221864710385874\cdot 10^{-75}B_{8,8}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.9331243442268161, 0.9331243442268161\}$$

Intersection intervals with the x axis:

$$[0.9331243442268161, 0.9331243442268161]$$

Longest intersection interval: $1.740809509979169\cdot 10^{-64}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

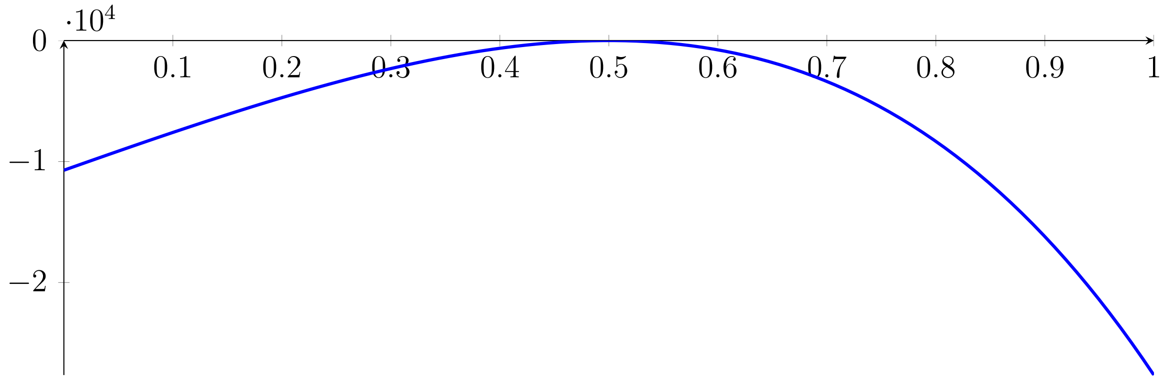
61.34 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 26!

61.35 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

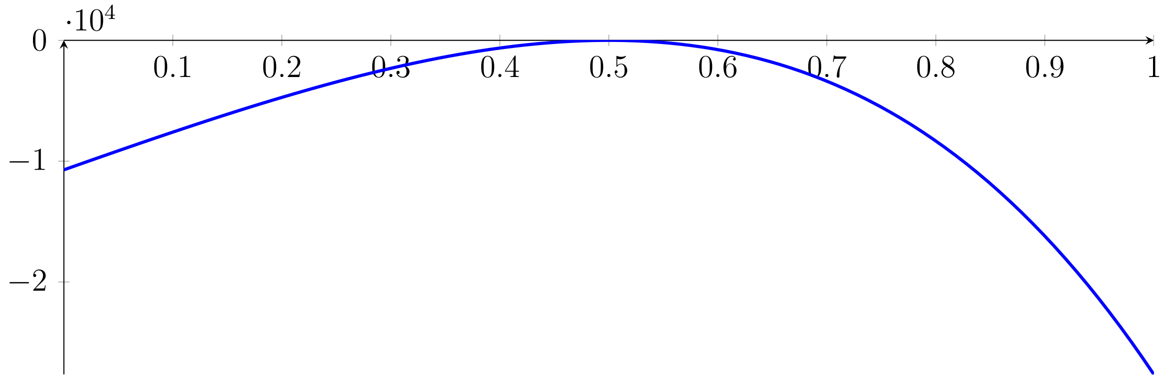
with precision $\varepsilon = 1 \cdot 10^{-128}$.

62 Running `QuadClip` on h_8 with epsilon 128

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

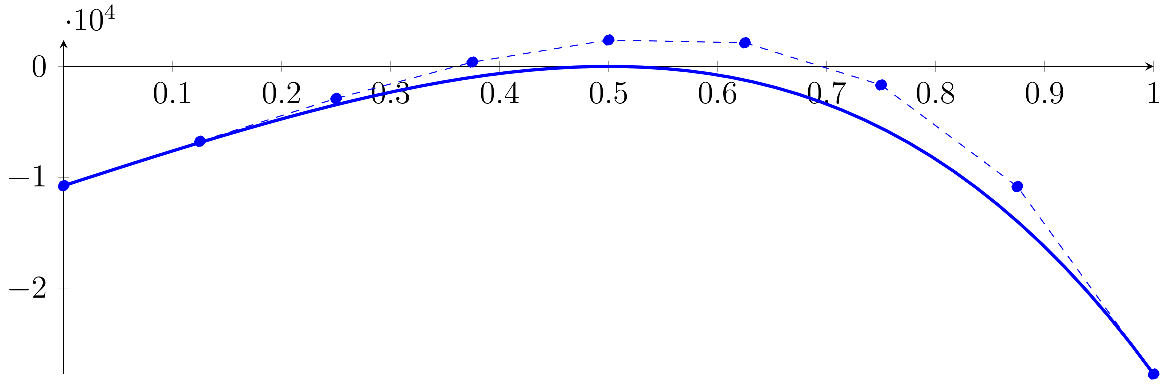
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



62.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

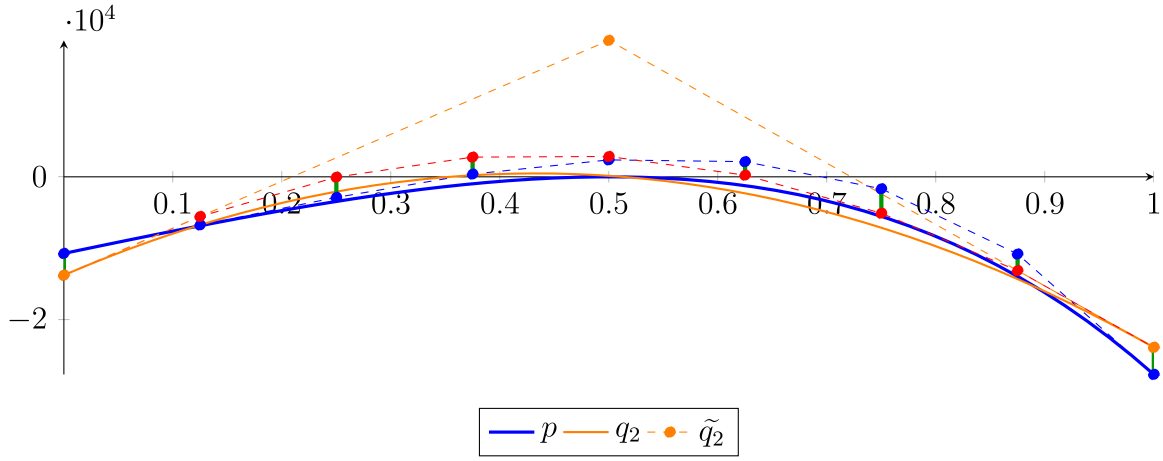
$$\begin{aligned} p &= -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &\quad - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ &= -10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &\quad + 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &\quad - 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,2} + 19088.57274758373B_{1,2} - 23857.61148217787B_{2,2} \end{aligned}$$

$$\begin{aligned} \tilde{q}_2 &= 8.415084934481114 \cdot 10^{-299}X^8 - 3.337656926892332 \cdot 10^{-298}X^7 + 5.61048922956057 \cdot 10^{-298}X^6 \\ &\quad - 5.189733160056009 \cdot 10^{-298}X^5 + 2.787623795639522 \cdot 10^{-298}X^4 - 8.054794139884735 \\ &\quad \cdot 10^{-299}X^3 - 75792.99716497913X^2 + 65693.62587043504X - 13758.24018763379 \\ &= -13758.24018763379B_{0,8} - 5546.536953829407B_{1,8} - 41.72647591713882B_{2,8} \\ &\quad + 2756.191246103018B_{3,8} + 2847.216212231063B_{4,8} + 231.3484224669965B_{5,8} \\ &\quad - 5091.412123189182B_{6,8} - 13121.06542473747B_{7,8} - 23857.61148217787B_{8,8} \end{aligned}$$



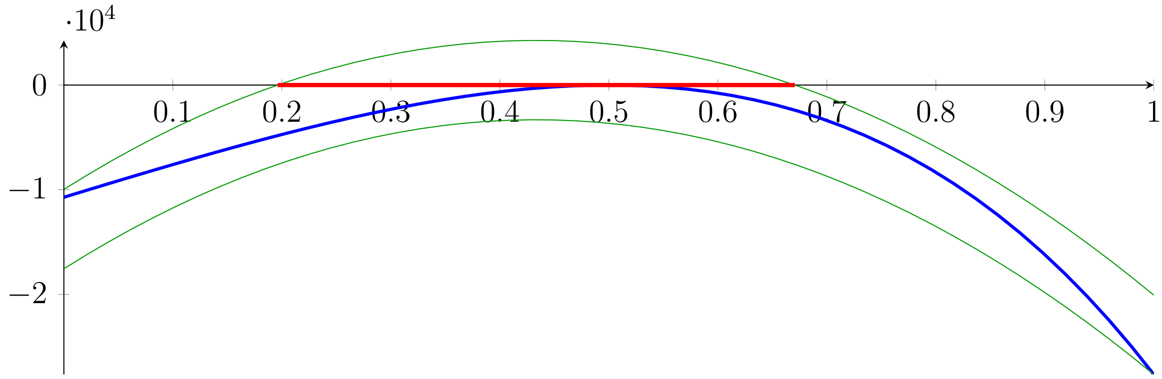
The maximum difference of the Bézier coefficients is $\delta = 3790.385753022192$.
Bounding polynomials M and m:

$$M = -75792.99716497913X^2 + 65693.62587043504X - 9967.854434611596$$
$$m = -75792.99716497913X^2 + 65693.62587043504X - 17548.62594065598$$

Root of M and m:

$$N(M) = \{0.196099166583006, 0.6706514347613278\} \qquad N(m) = \{\}$$

Intersection intervals:



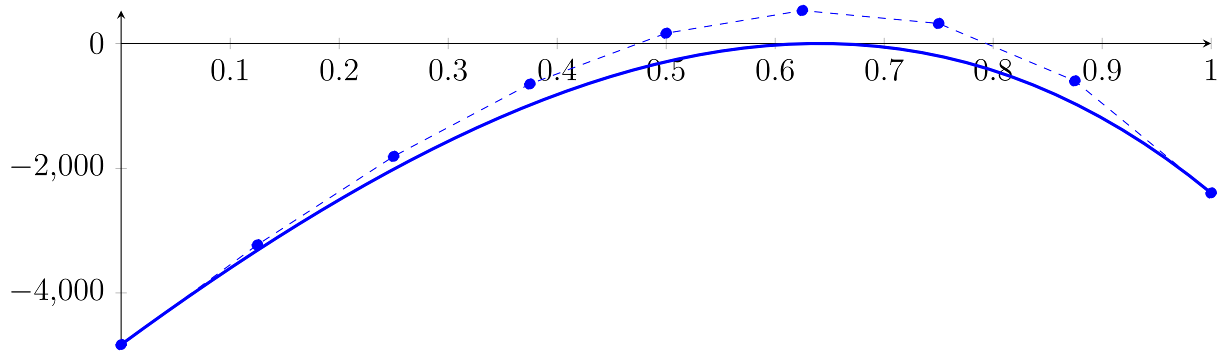
$$[0.196099166583006, 0.6706514347613278]$$

Longest intersection interval: 0.4745522681783218
$\Longrightarrow$ Selective recursion: interval 1: $[0.196099166583006, 0.6706514347613278]$,

62.2 Recursion Branch 1 1 in Interval 1: $[0.196099166583006, 0.6706514347613278]$

Normalized monomial und Bézier representations and the Bézier polygon:

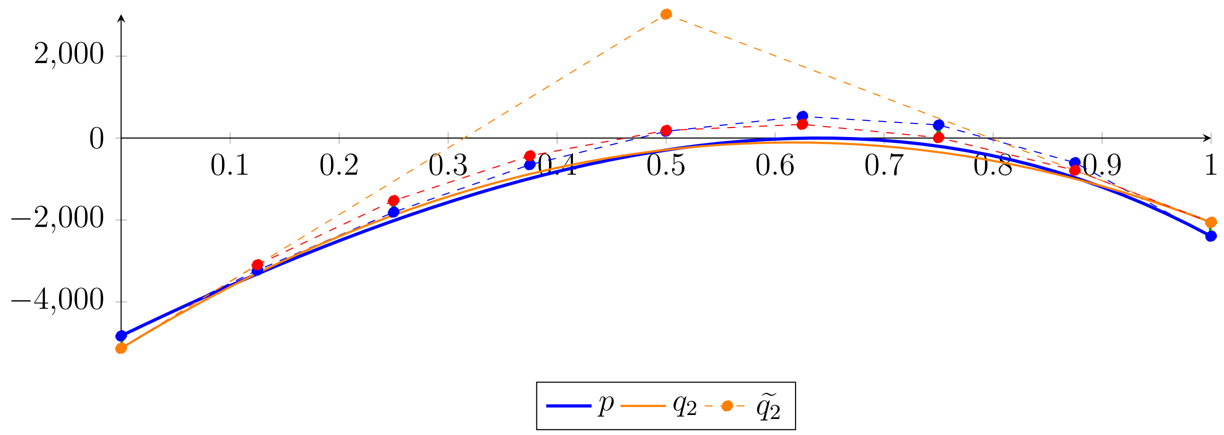
$$\begin{aligned} p = &-0.002572008668665384X^8 - 0.1981978796280906X^7 - 6.285790283786479X^6 \\ &- 104.4131864237547X^5 - 944.6763621987006X^4 - 4209.666927096399X^3 \\ &- 5104.073606289322X^2 + 12800.83304545001X - 4828.225717962684 \\ = &-4828.225717962684B_{0,8}(X) - 3228.121587281433B_{1,8}(X) - 1810.305799681943B_{2,8}(X) \\ &- 649.9509788623646B_{3,8}(X) + 164.2748748763137B_{4,8}(X) + 528.3438634441263B_{5,8}(X) \\ &+ 320.779177265857B_{6,8}(X) - 599.6831856603241B_{7,8}(X) - 2396.709314692933B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,2} + 3023.171418262273B_{1,2} - 2058.062446280739B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= 1.476670942620757\cdot 10^{-299}X^8 - 6.381627458836932\cdot 10^{-299}X^7 + 1.1779791924066\cdot 10^{-298}X^6 \\ &\quad - 1.190713137013755\cdot 10^{-298}X^5 + 6.850874958113058\cdot 10^{-299}X^4 - 2.05172173630317 \\ &\quad \cdot 10^{-299}X^3 - 13236.04714430528X^2 + 16309.62655952454X - 5131.641861499995 \\ &= -5131.641861499995B_{0,8} - 3092.938541559428B_{1,8} - 1526.951191058335B_{2,8} \\ &\quad - 433.6798099967171B_{3,8} + 186.8756016254271B_{4,8} + 334.7150438080969B_{5,8} \\ &\quad + 9.838516551292577B_{6,8} - 787.753980144986B_{7,8} - 2058.062446280739B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 338.646868412194$.

Bounding polynomials M and m:

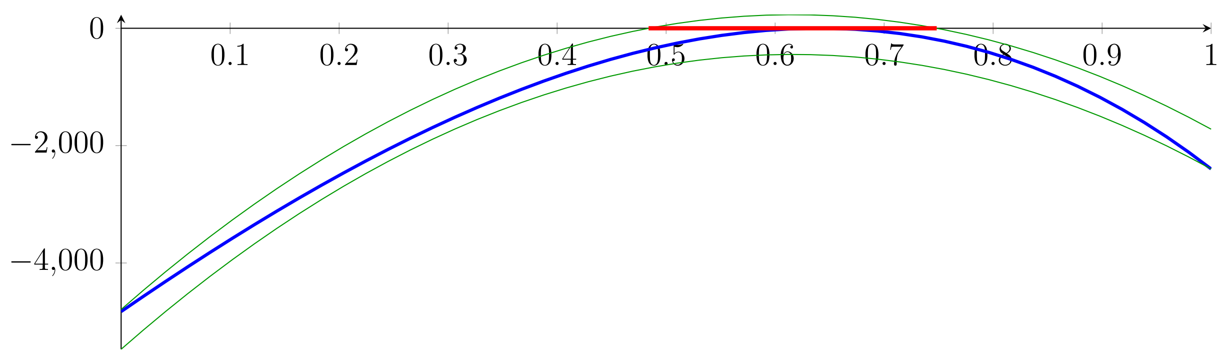
$$M = -13236.04714430528X^2 + 16309.62655952454X - 4792.994993087801$$

$$m = -13236.04714430528X^2 + 16309.62655952454X - 5470.288729912189$$

Root of M and m:

$$N(M) = \{0.4839311609916354, 0.7482816274420809\} \qquad N(m) = \{\}$$

Intersection intervals:



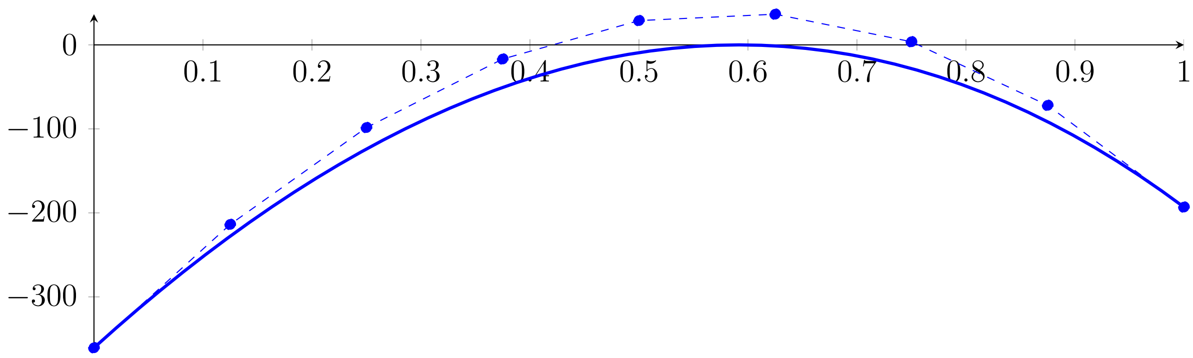
$$[0.4839311609916354, 0.7482816274420809]$$

Longest intersection interval: 0.2643504664504455
$\Longrightarrow$ Selective recursion: interval 1: $[0.4257497966737551, 0.5511979101218114]$,

62.3 Recursion Branch 1 1 1 in Interval 1: $[0.4257497966737551, 0.551197910121811$

Normalized monomial und Bézier representations and the Bézier polygon:

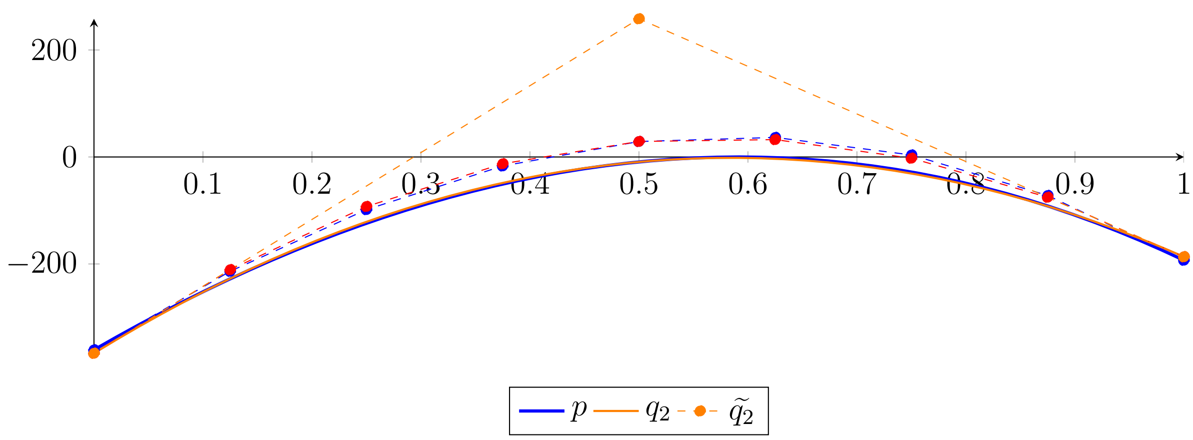
$$\begin{aligned} p = & -6.133566475094875\cdot 10^{-08}X^8 - 1.877794231289375\cdot 10^{-05}X^7 - 0.002379939031153127X^6 \\ & - 0.1596298583101837X^5 - 5.958684697205818X^4 - 116.3336704708797X^3 \\ & - 885.1606719876373X^2 + 1175.121582406799X - 360.5751654247078 \\ = & -360.5751654247078B_{0,8}(X) - 213.6849676238579B_{1,8}(X) - 98.40765096542357B_{2,8}(X) \\ & - 16.82060242209915B_{3,8}(X) + 28.91366696631814B_{4,8}(X) + 36.54467155974404B_{5,8}(X) \\ & + 3.731015586800578B_{6,8}(X) - 71.9626297312559B_{7,8}(X) - 193.0686388102505B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984 \\ = & -366.9199822310984B_{0,2} + 258.3517672110292B_{1,2} - 186.5418852681748B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & 1.223655337688364\cdot 10^{-300}X^8 - 5.205587704887534\cdot 10^{-300}X^7 + 9.409306890019087\cdot 10^{-300}X^6 \\ & - 9.282190734559383\cdot 10^{-300}X^5 + 5.21941805196838\cdot 10^{-300}X^4 - 1.541981168110934 \\ & \cdot 10^{-300}X^3 - 1070.165401921332X^2 + 1250.543498884255X - 366.9199822310984 \\ = & -366.9199822310984B_{0,8} - 210.6020448705665B_{1,8} - 92.50430043579644B_{2,8} \\ & - 12.62674892678823B_{3,8} + 29.03060965645814B_{4,8} + 32.46777531394266B_{5,8} \\ & - 2.315251954334665B_{6,8} - 75.31847214837383B_{7,8} - 186.5418852681748B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 6.52675354207568$.
Bounding polynomials M and m:

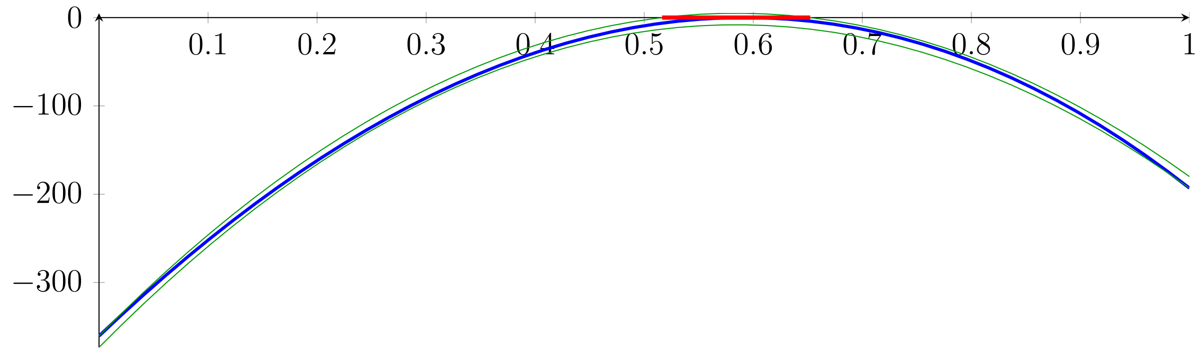
$$M = -1070.165401921332X^2 + 1250.543498884255X - 360.3932286890227$$

$$m = -1070.165401921332X^2 + 1250.543498884255X - 373.4467357731741$$

Root of M and m:

$$N(M) = \{0.5163481231478299, 0.652203482649816\} \qquad N(m) = \{\}$$

Intersection intervals:



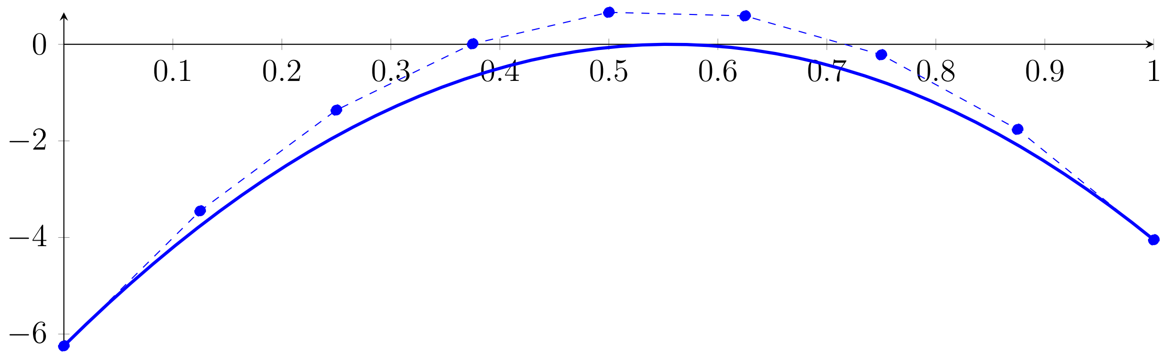
$$[0.5163481231478299, 0.652203482649816]$$

Longest intersection interval: 0.1358553595019861
$\Longrightarrow$ Selective recursion: interval 1: $[0.490524694605095, 0.5075674931564267]$,

62.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.490524694605095, 0.507567493156426$

Normalized monomial und Bézier representations and the Bézier polygon:

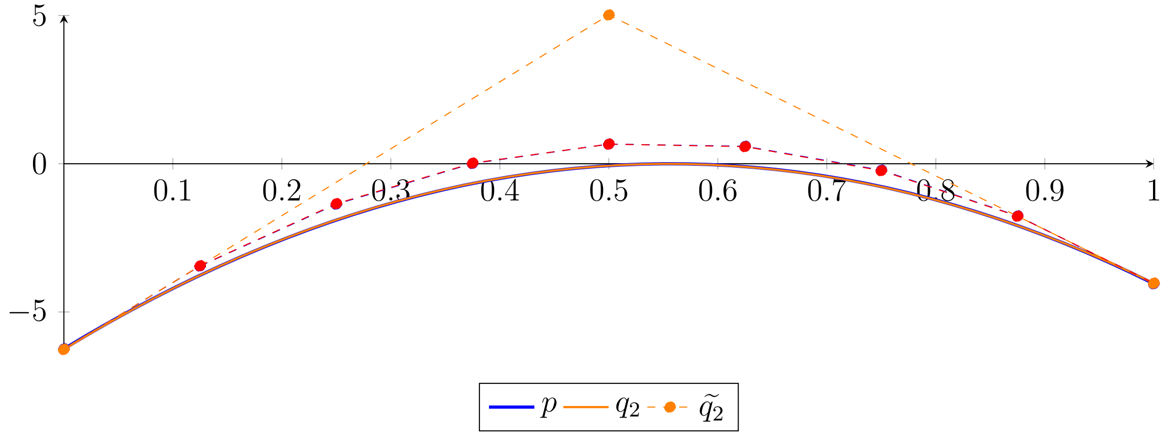
$$\begin{aligned}
p = &-7.117496868854 28\cdot 10^{-15}X^8 - 1.625571369884896\cdot 10^{-11}X^7 - 1.539287417434116\cdot 10^{-08}X^6 \\
&- 7.733623372922873\cdot 10^{-06}X^5 - 0.002173482359120639X^4 - 0.3236423633498969X^3 \\
&- 19.84316397394663X^2 + 22.36613094108362X - 6.245496834711843 \\
= &-6.245496834711843B_{0,8}(X) - 3.449730467076391B_{1,8}(X) - 1.36264852708189B_{2,8}(X) \\
&+ 0.009969657354697074B_{3,8}(X) + 0.662313708568421B_{4,8}(X) + 0.5885420610459267B_{5,8}(X) \\
&- 0.217218177224708B_{6,8}(X) - 1.760871363955914B_{7,8}(X) - 4.048353462316386B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= -20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\
&= -6.261866081804285B_{0,2} + 5.019289841496026B_{1,2} - 4.031921561491104B_{2,2}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 2.334129178460356\cdot 10^{-302}X^8 - 9.797943630323215\cdot 10^{-302}X^7 + 1.744532667551292\cdot 10^{-301}X^6 \\ &\quad - 1.695264192146682\cdot 10^{-301}X^5 + 9.412284928871313\cdot 10^{-302}X^4 - 2.760729238908791 \\ &\quad \cdot 10^{-302}X^3 - 20.33236732628744X^2 + 22.56231184660062X - 6.261866081804285 \\ &= -6.261866081804285B_{0,8} - 3.441577100979207B_{1,8} - 1.347444096092966B_{2,8} \\ &\quad + 0.02053293285443696B_{3,8} + 0.6623539858630032B_{4,8} + 0.5780190629327322B_{5,8} \\ &\quad - 0.232471835936376B_{6,8} - 1.769118710744321B_{7,8} - 4.031921561491104B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.01643190082528179$.

Bounding polynomials M and m:

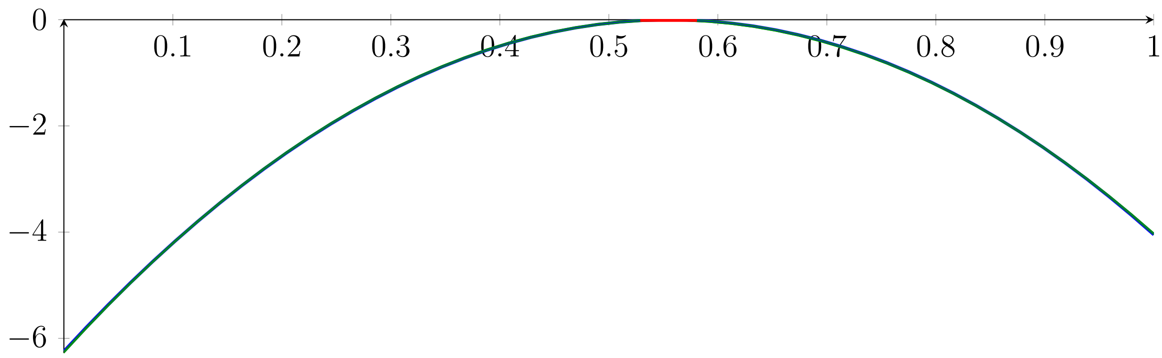
$$M = -20.33236732628744X^2 + 22.56231184660062X - 6.245434180979003$$

$$m = -20.33236732628744X^2 + 22.56231184660062X - 6.278297982629567$$

Root of M and m:

$$N(M) = \{0.5288114927317158, 0.5808631203877371\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5288114927317158, 0.5808631203877371]$$

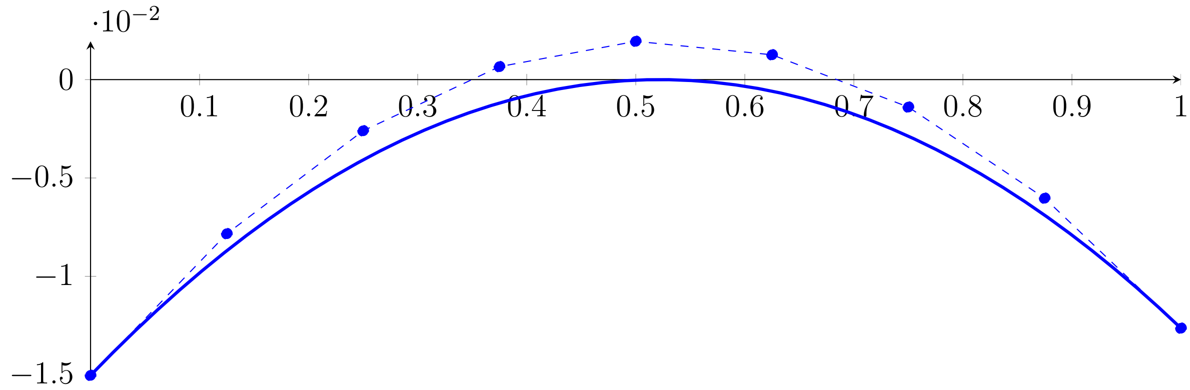
Longest intersection interval: 0.05205162765602134

$\Longrightarrow$ Selective recursion: interval 1: $[0.4995371223473506, 0.5004242277517611]$,

62.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.4995371223473506, 0.500424227751$

Normalized monomial und Bézier representations and the Bézier polygon:

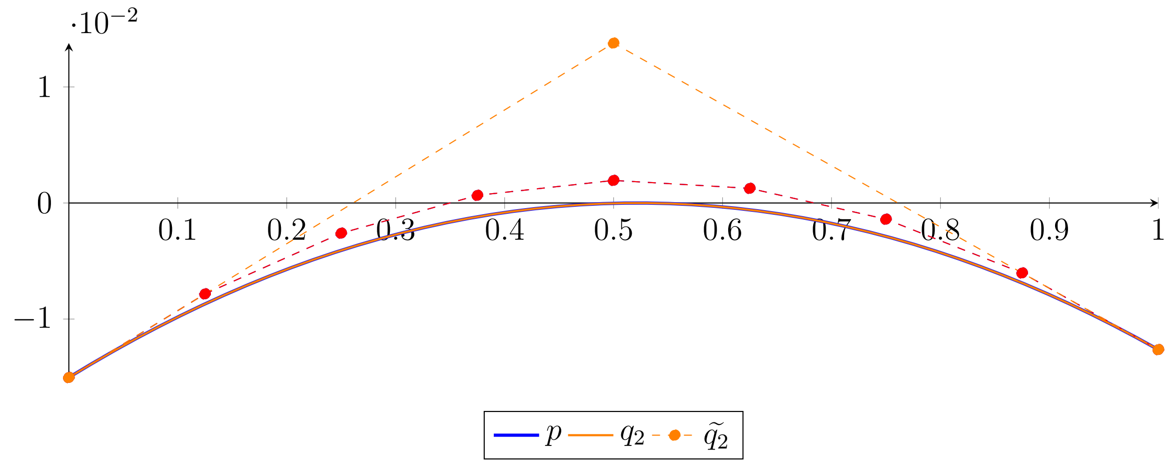
$$\begin{aligned}p = &-3.835321724591002\cdot 10^{-25}X^8 - 1.685970393664051\cdot 10^{-20}X^7 - 3.073417757909172\cdot 10^{-16}X^6\\&- 2.973678167232232\cdot 10^{-12}X^5 - 1.610545215791654\cdot 10^{-08}X^4 - 4.629380451575473\\&\cdot 10^{-05}X^3 - 0.05516351610802166X^2 + 0.05760784492782384X - 0.01503353559864481\\= &-0.01503353559864481B_{0,8}(X) - 0.007832554982666833B_{1,8}(X) - 0.002601699941975341B_{2,8}(X)\\&+ 0.0006582028483490249B_{3,8}(X) + 0.001946326483147738B_{4,8}(X)\\&+ 0.001261843827131283B_{5,8}(X) - 0.001396072485173959B_{6,8}(X)\\&- 0.006028250049478829B_{7,8}(X) - 0.01263551669178453B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657\\&= -0.01503585166965657B_{0,2} + 0.01378196629969571B_{1,2} - 0.01263320016040455B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q_2} = &\ 6.306459041947834\cdot 10^{-305}X^8 - 2.608238530297949\cdot 10^{-304}X^7 + 4.573872253581115\cdot 10^{-304}X^6\\&- 4.384946099604334\cdot 10^{-304}X^5 + 2.410615466544519\cdot 10^{-304}X^4 - 7.036239809064398\\&\cdot 10^{-305}X^3 - 0.05523298442945254X^2 + 0.05763563593870456X - 0.01503585166965657\\= &-0.01503585166965657B_{0,8} - 0.007831397177318504B_{1,8} - 0.002599549271746596B_{2,8}\\&+ 0.0006596920470591496B_{3,8} + 0.001946326779098733B_{4,8} + 0.001260354924372155B_{5,8}\\&- 0.001398223517120585B_{6,8} - 0.006029408545379487B_{7,8} - 0.01263320016040455B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.316531379978639\cdot 10^{-06}$.

Bounding polynomials M and m:

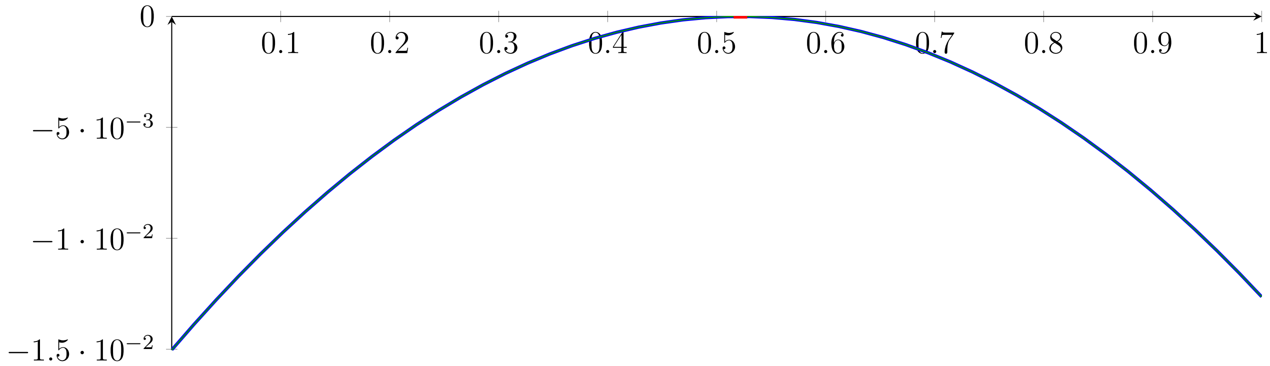
$$M = -0.05523298442945254X^2 + 0.05763563593870456X - 0.0150335351382766$$

$$m = -0.05523298442945254X^2 + 0.05763563593870456X - 0.01503816820103655$$

Root of M and m:

$$N(M) = \{0.5154882826278122, 0.5280120194846123\} \qquad N(m) = \{\}$$

Intersection intervals:



$$[0.5154882826278122, 0.5280120194846123]$$

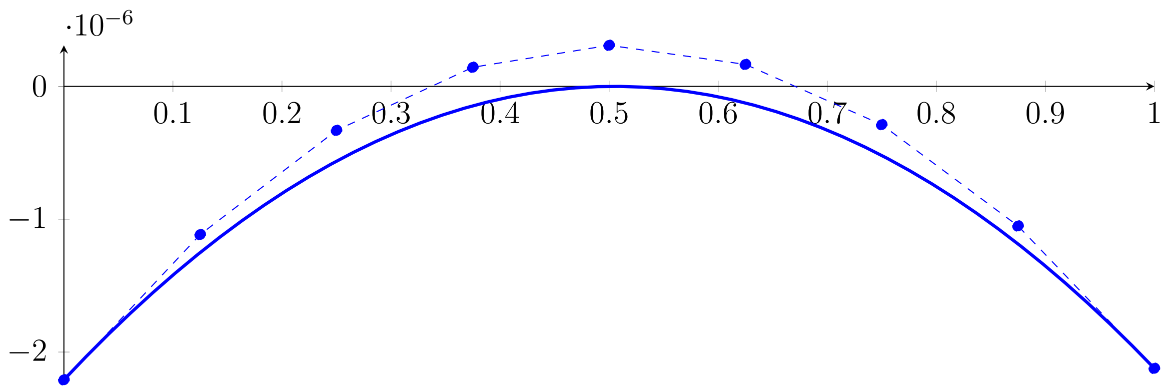
Longest intersection interval: 0.01252373685680011

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999944147887801, 0.5000055246634291]$,

62.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.4999944147887801, 0.5000055246$

Normalized monomial und Bézier representations and the Bézier polygon:

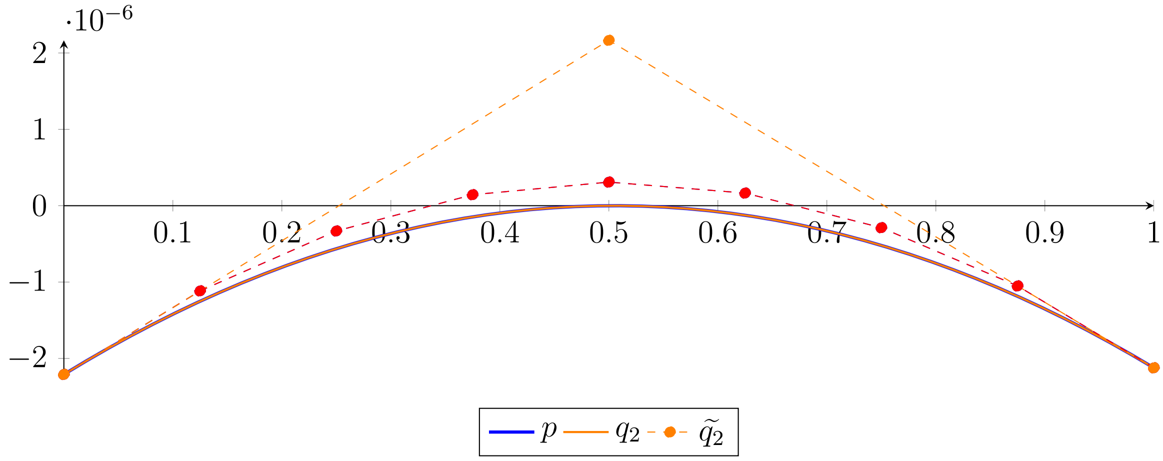
$$\begin{aligned} p = &-2.32099000990716\cdot10^{-40}X^8 - 8.147572265547704\cdot10^{-34}X^7 - 1.186072294581764\cdot10^{-27}X^6 \\ &- 9.164366615560225\cdot10^{-22}X^5 - 3.963832728671687\cdot10^{-16}X^4 - 9.099890718270267\cdot10^{-11}X^3 \\ &- 8.663298447262891\cdot10^{-06}X^2 + 8.749571419899133\cdot10^{-06}X - 2.209162411567082\cdot10^{-06} \\ = &-2.209162411567082\cdot10^{-06}B_{0,8}(X) - 1.11546598407969\cdot10^{-06}B_{1,8}(X) - 3.311730725659734 \\ &\cdot10^{-07}B_{2,8}(X) + 1.437146979935834\cdot10^{-07}B_{3,8}(X) + 3.091957026128322 \\ &\cdot10^{-07}B_{4,8}(X) + 1.652683162999621\cdot10^{-07}B_{5,8}(X) - 2.880690859425\cdot10^{-07}B_{6,8}(X) \\ &- 1.05081812911769\cdot10^{-06}B_{7,8}(X) - 2.122980438234407\cdot10^{-06}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= -8.663434946303181\cdot10^{-06}X^2 + 8.749626019605851\cdot10^{-06}X - 2.209166961546417\cdot10^{-06} \\ &= -2.209166961546417\cdot10^{-06}B_{0,2} + 2.165646048256509\cdot10^{-06}B_{1,2} - 2.122975888243747\cdot10^{-06}B_{2,2} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 &= 9.84075548060145\cdot 10^{-309}X^8 - 4.039091668354938\cdot 10^{-308}X^7 + 7.030443670929191\cdot 10^{-308}X^6\\
&\quad - 6.698784216784483\cdot 10^{-308}X^5 + 3.668633643168221\cdot 10^{-308}X^4 - 1.069248215117909\cdot 10^{-308}X^3\\
&\quad - 8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209166961546417\cdot 10^{-06}\\
&= -2.209166961546417\cdot 10^{-06}B_{0,8} - 1.115463709095685\cdot 10^{-06}B_{1,8} - 3.311688475843534\cdot 10^{-07}B_{2,8}\\
&\quad + 1.437176229875793\cdot 10^{-07}B_{3,8} + 3.091957026201127\cdot 10^{-07}B_{4,8} + 1.652653913132468\cdot 10^{-07}B_{5,8}\\
&\quad - 2.880733109330184\cdot 10^{-07}B_{6,8} - 1.050820404118683\cdot 10^{-06}B_{7,8} - 2.122975888243747\cdot 10^{-06}B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 4.549990660244254\cdot 10^{-12}$.

Bounding polynomials M and m:

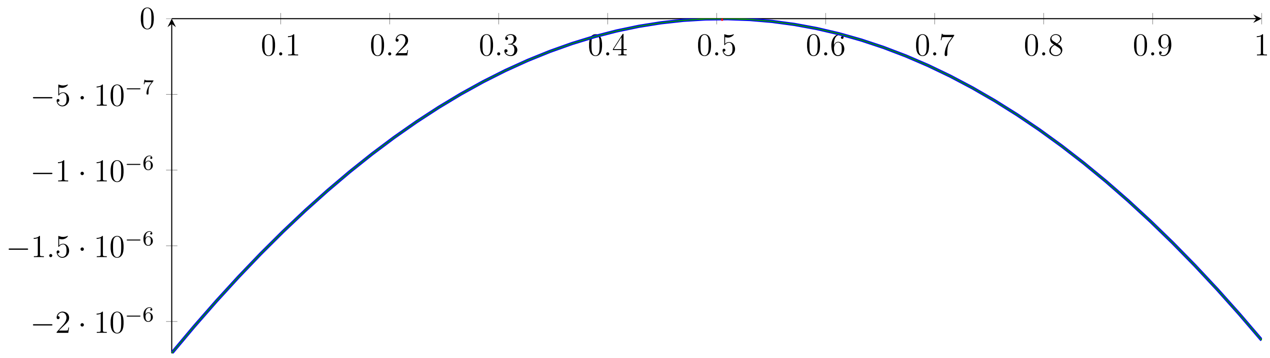
$$M = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209162411555757\cdot 10^{-06}$$

$$m = -8.663434946303181\cdot 10^{-06}X^2 + 8.749626019605851\cdot 10^{-06}X - 2.209171511537077\cdot 10^{-06}$$

Root of M and m:

$$N(M) = \{0.5041259457474217, 0.505822887923852\} \qquad N(m) = \{\}$$

Intersection intervals:



$[0.5041259457474217, 0.505822887923852]$

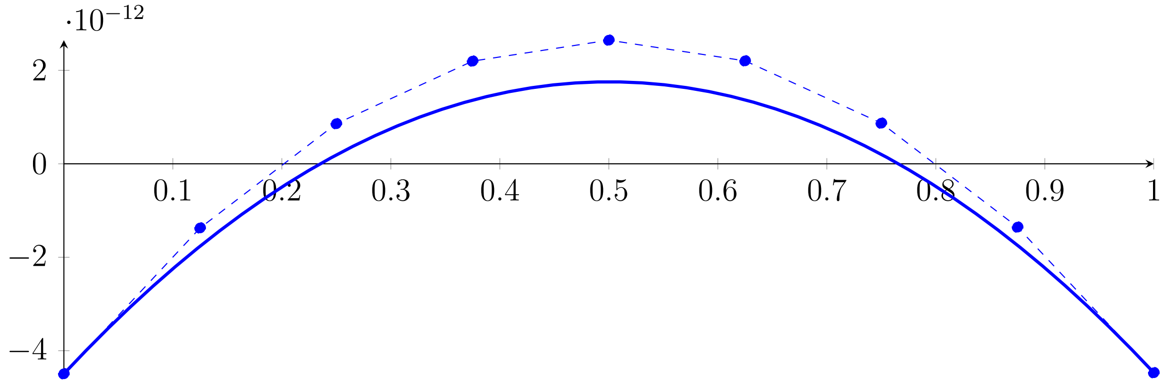
Longest intersection interval: 0.00169694217643035

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000155648447, 0.5000000344176595]$,

62.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.5000000155648447, 0.50000003$

Normalized monomial und Bézier representations and the Bézier polygon:

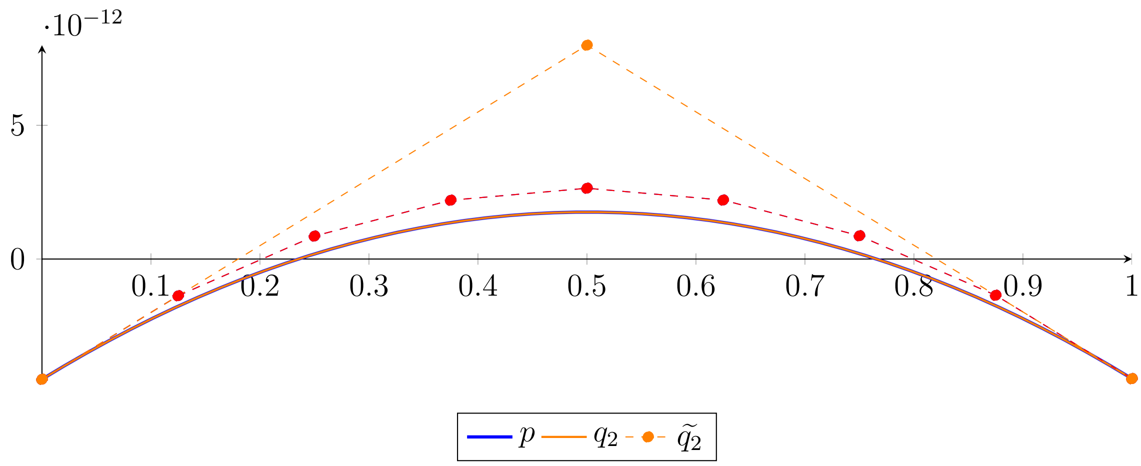
$$\begin{aligned}p = &-1.595914506899923\cdot10^{-62}X^8 - 3.301399092257474\cdot10^{-53}X^7 - 2.83213843356941\cdot10^{-44}X^6\\ &- 1.289553529043531\cdot10^{-35}X^5 - 3.286896476505693\cdot10^{-27}X^4 - 4.446733715730271\cdot10^{-19}X^3\\ &- 2.494734097474612\cdot10^{-11}X^2 + 2.497049303064054\cdot10^{-11}X - 4.493680200151054\cdot10^{-12}\\ = &-4.493680200151054\cdot10^{-12}B_{0,8}(X) - 1.372368571320986\cdot10^{-12}B_{1,8}(X) + 8.579665941252918\\ &\cdot10^{-13}B_{2,8}(X) + 2.197325288247184\cdot10^{-12}B_{3,8}(X) + 2.645707503104094\cdot10^{-12}B_{4,8}(X)\\ &+ 2.203113230755426\cdot10^{-12}B_{5,8}(X) + 8.695424632605847\cdot10^{-13}B_{6,8}(X)\\ &- 1.355004807321027\cdot10^{-12}B_{7,8}(X) - 4.470528588930005\cdot10^{-12}B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= -2.494734164175618\cdot10^{-11}X^2 + 2.497049329744457\cdot10^{-11}X - 4.493680222384723\cdot10^{-12}\\ &= -4.493680222384723\cdot10^{-12}B_{0,2} + 7.991566426337562\cdot10^{-12}B_{1,2} - 4.470528566696336\cdot10^{-12}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &\ 3.394595061927522\cdot10^{-314}X^8 - 1.386657030412925\cdot10^{-313}X^7 + 2.314937908564659\cdot10^{-313}X^6\\ &- 2.018101017184903\cdot10^{-313}X^5 + 9.746671569929334\cdot10^{-314}X^4 - 2.577278121543316\cdot10^{-314}X^3\\ &- 2.494734164175618\cdot10^{-11}X^2 + 2.497049329744457\cdot10^{-11}X - 4.493680222384723\cdot10^{-12}\\ = &-4.493680222384723\cdot10^{-12}B_{0,8} - 1.372368560204152\cdot10^{-12}B_{1,8} + 8.579666147708415\cdot10^{-13}B_{2,8}\\ &+ 2.197325302540257\cdot10^{-12}B_{3,8} + 2.645707503104094\cdot10^{-12}B_{4,8} + 2.203113216462353\cdot10^{-12}B_{5,8}\\ &+ 8.695424426150349\cdot10^{-13}B_{6,8} - 1.355004818437861\cdot10^{-12}B_{7,8} - 4.470528566696336\cdot10^{-12}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.223366895429667\cdot10^{-20}$.
Bounding polynomials M and m:

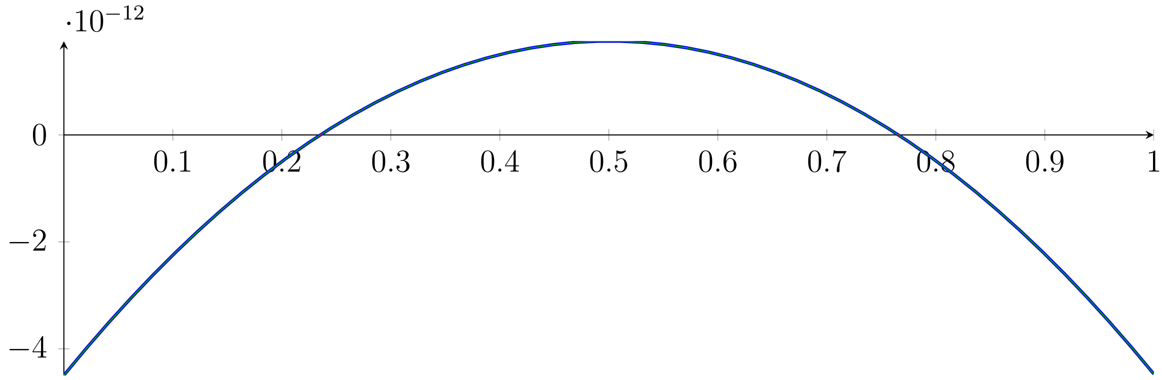
$$M = -2.494734164175618\cdot10^{-11}X^2 + 2.497049329744457\cdot10^{-11}X - 4.493680200151054\cdot10^{-12}$$

$$m = -2.494734164175618 \cdot 10^{-11} X^2 + 2.497049329744457 \cdot 10^{-11} X - 4.493680244618392 \cdot 10^{-12}$$

Root of M and m:

$N(M) = \{0.235251622185842, 0.765676398764079\}$ $N(m) = \{0.2352516255462581, 0.7656763954036629\}$

Intersection intervals:



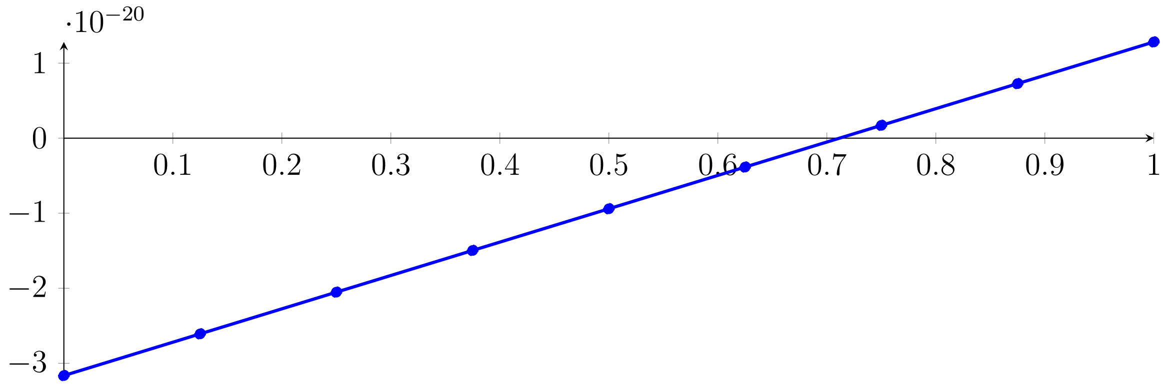
$$[0.235251622185842, 0.2352516255462581], \; [0.7656763954036629, 0.765676398764079]$$

Longest intersection interval: $3.360416099786671 \cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.50000002]$, interval 2: $[0.50000003, 0.500000030000$

62.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.5000000$

Normalized monomial und Bézier representations and the Bézier polygon:

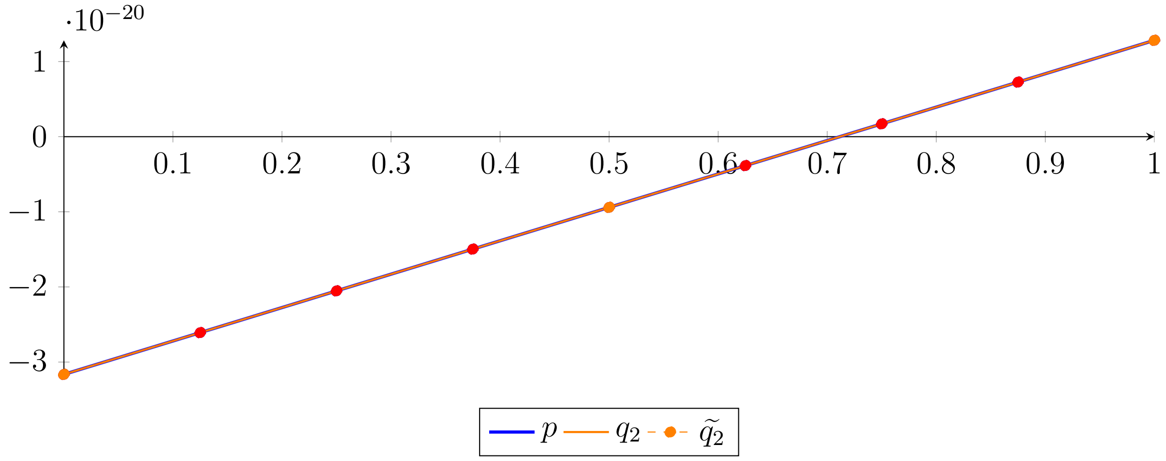
$$\begin{aligned} p = & -2.595099873436077 \cdot 10^{-130} X^8 - 1.59753148120066 \cdot 10^{-112} X^7 - 4.078240366489356 \cdot 10^{-95} X^6 \\ & - 5.52592084177806 \cdot 10^{-78} X^5 - 4.191391755953253 \cdot 10^{-61} X^4 - 1.687408749214622 \cdot 10^{-44} X^3 \\ & - 2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20} \\ = & -3.164099743671992 \cdot 10^{-20} B_{0,8}(X) - 2.608258017418989 \cdot 10^{-20} B_{1,8}(X) - 2.052416292172112 \\ & \cdot 10^{-20} B_{2,8}(X) - 1.49657456793136 \cdot 10^{-20} B_{3,8}(X) - 9.407328446967348 \cdot 10^{-21} B_{4,8}(X) \\ & - 3.848911224682354 \cdot 10^{-21} B_{5,8}(X) + 1.709505987541381 \cdot 10^{-21} B_{6,8}(X) \\ & + 7.267923189703856 \cdot 10^{-21} B_{7,8}(X) + 1.282634038180507 \cdot 10^{-20} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 & = -2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20} \\ & = -3.164099743671992 \cdot 10^{-20} B_{0,2} - 9.407328386599791 \cdot 10^{-21} B_{1,2} + 1.282634038180507 \cdot 10^{-20} B_{2,2} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 &= -2.04607227392422 \cdot 10^{-323} X^8 + 3.432164034855183 \cdot 10^{-323} X^7 + 6.927895597874022 \cdot 10^{-323} X^6 \\
&\quad - 2.227227667041084 \cdot 10^{-322} X^5 + 2.093778075420854 \cdot 10^{-322} X^4 - 7.695410174165002 \cdot 10^{-323} X^3 \\
&\quad - 2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20} \\
&= -3.164099743671992 \cdot 10^{-20} B_{0,8} - 2.608258017418989 \cdot 10^{-20} B_{1,8} - 2.052416292172112 \cdot 10^{-20} B_{2,8} \\
&\quad - 1.49657456793136 \cdot 10^{-20} B_{3,8} - 9.407328446967348 \cdot 10^{-21} B_{4,8} - 3.848911224682354 \cdot 10^{-21} B_{5,8} \\
&\quad + 1.709505987541381 \cdot 10^{-21} B_{6,8} + 7.267923189703856 \cdot 10^{-21} B_{7,8} + 1.282634038180507 \cdot 10^{-20} B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 8.437043746073111 \cdot 10^{-46}$.

Bounding polynomials M and m:

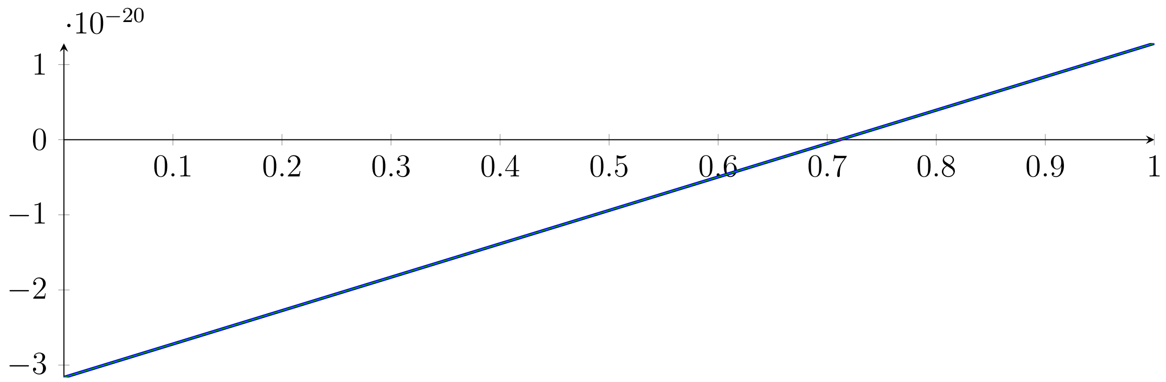
$$M = -2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20}$$

$$m = -2.817152660512407 \cdot 10^{-28} X^2 + 4.446733810024026 \cdot 10^{-20} X - 3.164099743671992 \cdot 10^{-20}$$

Root of M and m:

$N(M) = \{0.7115559179195557, 157844970.6438557\}$ $\quad N(m) = \{0.7115559179195557, 157844970.6438557\}$

Intersection intervals:



$$[0.7115559179195557, 0.7115559179195557]$$

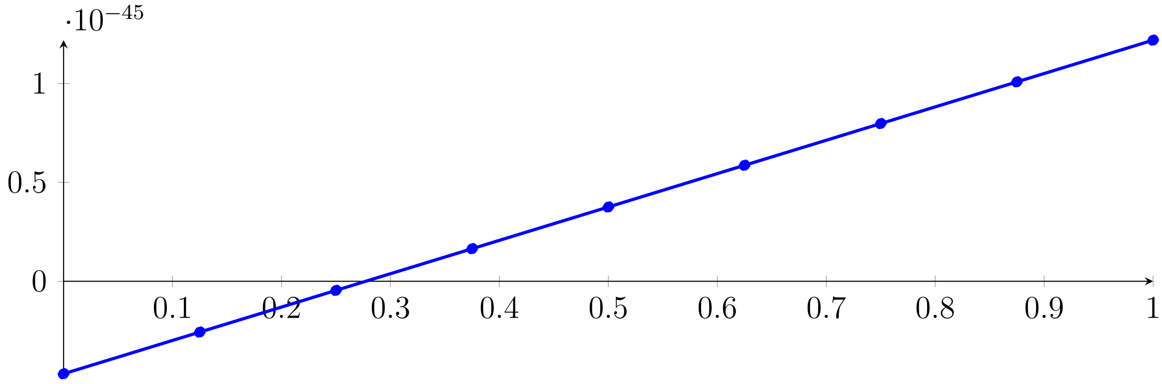
Longest intersection interval: $3.794715034716651 \cdot 10^{-26}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

62.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

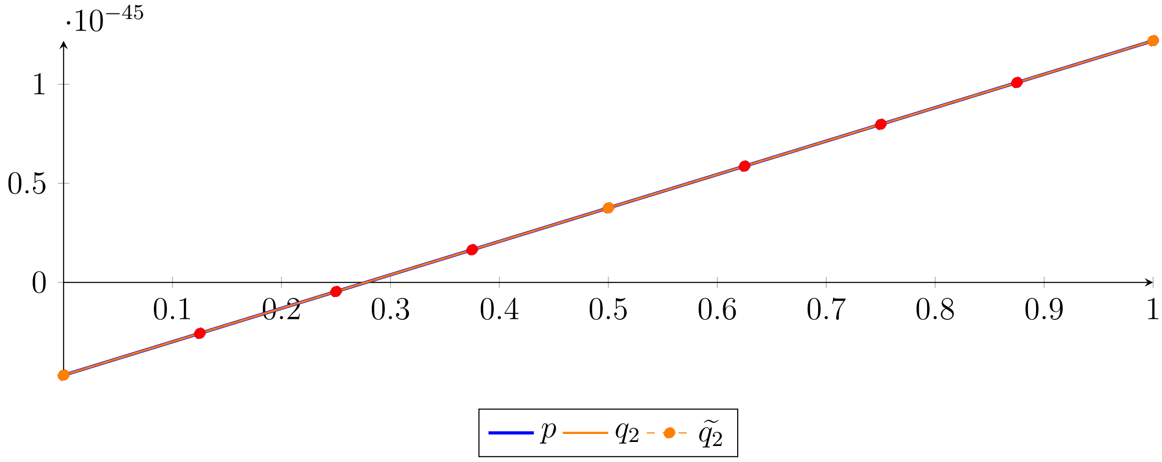
$$\begin{aligned} p = & -1.115802731757248\cdot 10^{-333}X^8 - 1.81010430406932\cdot 10^{-290}X^7 - 1.217721087493713\cdot 10^{-247}X^6 \\ & - 4.348109767863853\cdot 10^{-205}X^5 - 8.691103598298647\cdot 10^{-163}X^4 - 9.22057066454039\cdot 10^{-121}X^3 \\ & - 4.056661009282407\cdot 10^{-79}X^2 + 1.687408749214622\cdot 10^{-45}X - 4.680026339204605\cdot 10^{-46} \\ = & -4.680026339204605\cdot 10^{-46}B_{0,8}(X) - 2.570765402686327\cdot 10^{-46}B_{1,8}(X) - 4.615044661680489 \\ & \cdot 10^{-47}B_{2,8}(X) + 1.647756470350229\cdot 10^{-46}B_{3,8}(X) + 3.757017406868507\cdot 10^{-46}B_{4,8}(X) \\ & + 5.866278343386785\cdot 10^{-46}B_{5,8}(X) + 7.975539279905062\cdot 10^{-46}B_{6,8}(X) \\ & + 1.008480021642334\cdot 10^{-45}B_{7,8}(X) + 1.219406115294162\cdot 10^{-45}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -4.056661009282407\cdot 10^{-79}X^2 + 1.687408749214622\cdot 10^{-45}X - 4.680026339204605\cdot 10^{-46} \\ = & -4.680026339204605\cdot 10^{-46}B_{0,2} + 3.757017406868507\cdot 10^{-46}B_{1,2} + 1.219406115294162\cdot 10^{-45}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & \ 1.583039910757457\cdot 10^{-348}X^8 - 8.209830365232051\cdot 10^{-348}X^7 + 1.523964084824618\cdot 10^{-347}X^6 \\ & - 1.276185009399131\cdot 10^{-347}X^5 + 4.899925340940039\cdot 10^{-348}X^4 - 9.411348806295522\cdot 10^{-349}X^3 \\ & - 4.056661009282407\cdot 10^{-79}X^2 + 1.687408749214622\cdot 10^{-45}X - 4.680026339204605\cdot 10^{-46} \\ = & -4.680026339204605\cdot 10^{-46}B_{0,8} - 2.570765402686327\cdot 10^{-46}B_{1,8} - 4.615044661680489\cdot 10^{-47}B_{2,8} \\ & + 1.647756470350229\cdot 10^{-46}B_{3,8} + 3.757017406868507\cdot 10^{-46}B_{4,8} + 5.866278343386785\cdot 10^{-46}B_{5,8} \\ & + 7.975539279905062\cdot 10^{-46}B_{6,8} + 1.008480021642334\cdot 10^{-45}B_{7,8} + 1.219406115294162\cdot 10^{-45}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 4.610285332270195\cdot 10^{-122}$.

Bounding polynomials M and m:

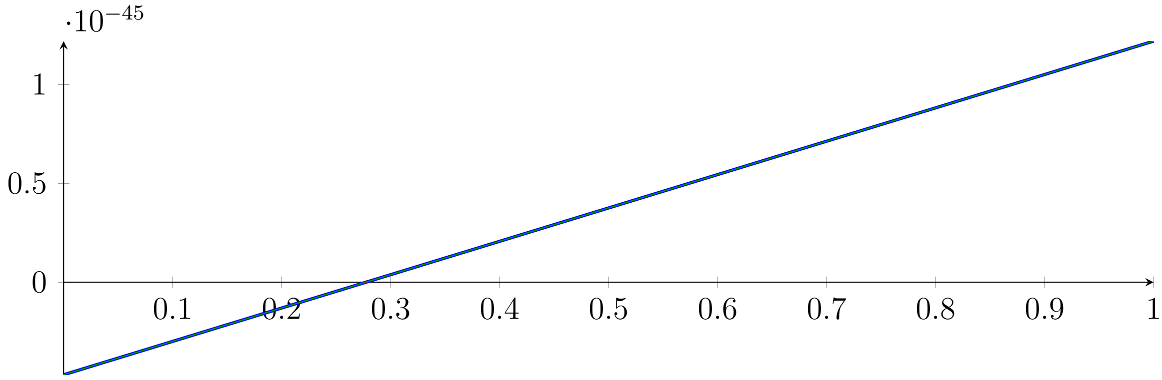
$$M = -4.056661009282407\cdot 10^{-79}X^2 + 1.687408749214622\cdot 10^{-45}X - 4.680026339204605\cdot 10^{-46}$$

$m = -4.056661009282407 \cdot 10^{-79} X^2 + 1.687408749214622 \cdot 10^{-45} X - 4.680026339204605 \cdot 10^{-46}$

Root of M and m:

$N(M) = \{0.2773498917427594, 4.159600088233924 \cdot 10^{33}\}$ $N(m) = \{0.2773498917427594, 4.15960008823392$

Intersection intervals:



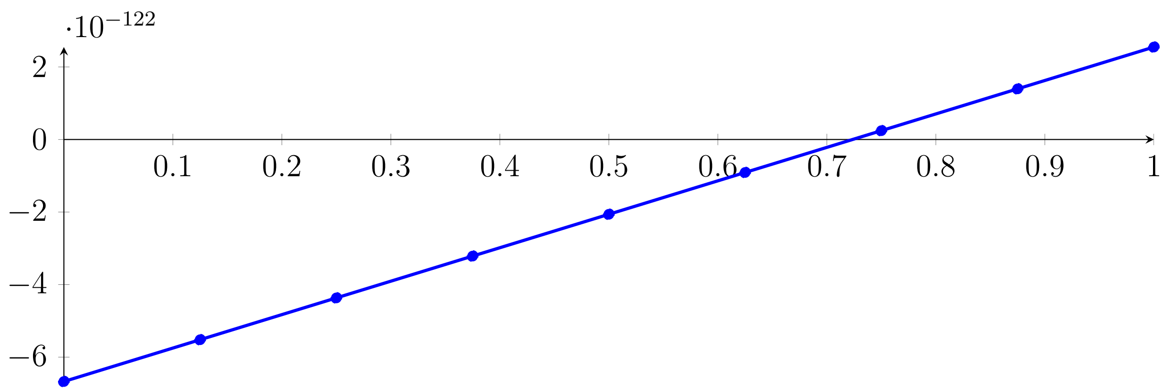
$[0.2773498917427594, 0.2773498917427594]$

Longest intersection interval: $5.464337356809344 \cdot 10^{-77}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

62.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Normalized monomial und Bézier representations and the Bézier polygon:

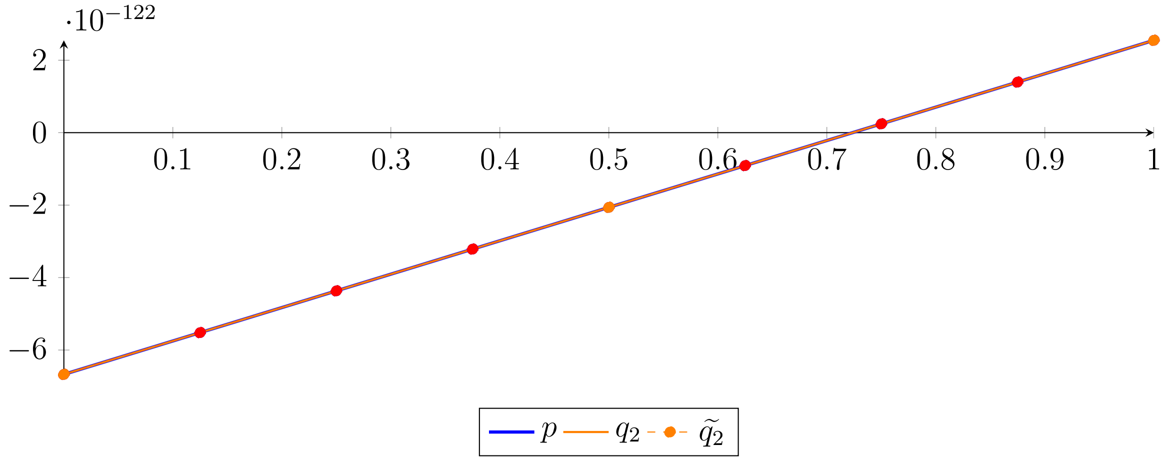
$$\begin{aligned} p = & -4.308401517198782 \cdot 10^{-429} X^8 + 1.723360606879513 \cdot 10^{-428} X^7 - 1.206352424815659 \cdot 10^{-427} X^6 \\ & + 2.412704849631318 \cdot 10^{-427} X^5 - 7.539702655097869 \cdot 10^{-428} X^4 - 1.504424205290455 \cdot 10^{-349} X^3 \\ & - 1.211277710947941 \cdot 10^{-231} X^2 + 9.22057066454039 \cdot 10^{-122} X - 6.672011081049187 \cdot 10^{-122} \\ = & -6.672011081049187 \cdot 10^{-122} B_{0,8}(X) - 5.519439747981638 \cdot 10^{-122} B_{1,8}(X) - 4.36686841491409 \\ & \cdot 10^{-122} B_{2,8}(X) - 3.214297081846541 \cdot 10^{-122} B_{3,8}(X) - 2.061725748778992 \\ & \cdot 10^{-122} B_{4,8}(X) - 9.091544157114434 \cdot 10^{-123} B_{5,8}(X) + 2.434169173561053 \\ & \cdot 10^{-123} B_{6,8}(X) + 1.395988250423654 \cdot 10^{-122} B_{7,8}(X) + 2.548559583491203 \cdot 10^{-122} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -1.211277710947941 \cdot 10^{-231} X^2 + 9.22057066454039 \cdot 10^{-122} X - 6.672011081049187 \cdot 10^{-122} \\ = & -6.672011081049187 \cdot 10^{-122} B_{0,2} - 2.061725748778992 \cdot 10^{-122} B_{1,2} + 2.548559583491203 \cdot 10^{-122} B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -4.59943403968556\cdot10^{-425}X^8 + 8.556485413156781\cdot10^{-425}X^7 + 1.246765231046984\cdot10^{-424}X^6\\ &\quad - 4.553980403679113\cdot10^{-424}X^5 + 4.388860915532469\cdot10^{-424}X^4 - 1.626163068651508\cdot10^{-424}X^3\\ &\quad - 1.211277710947941\cdot10^{-231}X^2 + 9.22057066454039\cdot10^{-122}X - 6.672011081049187\cdot10^{-122}\\ &= -6.672011081049187\cdot10^{-122}B_{0,8} - 5.519439747981638\cdot10^{-122}B_{1,8} - 4.36686841491409\cdot10^{-122}B_{2,8}\\ &\quad - 3.214297081846541\cdot10^{-122}B_{3,8} - 2.061725748778992\cdot10^{-122}B_{4,8} - 9.091544157114434\cdot10^{-123}B_{5,8}\\ &\quad + 2.434169173561053\cdot10^{-123}B_{6,8} + 1.395988250423654\cdot10^{-122}B_{7,8} + 2.548559583491203\cdot10^{-122}B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 7.522121026452274\cdot10^{-351}$.
Bounding polynomials M and m:

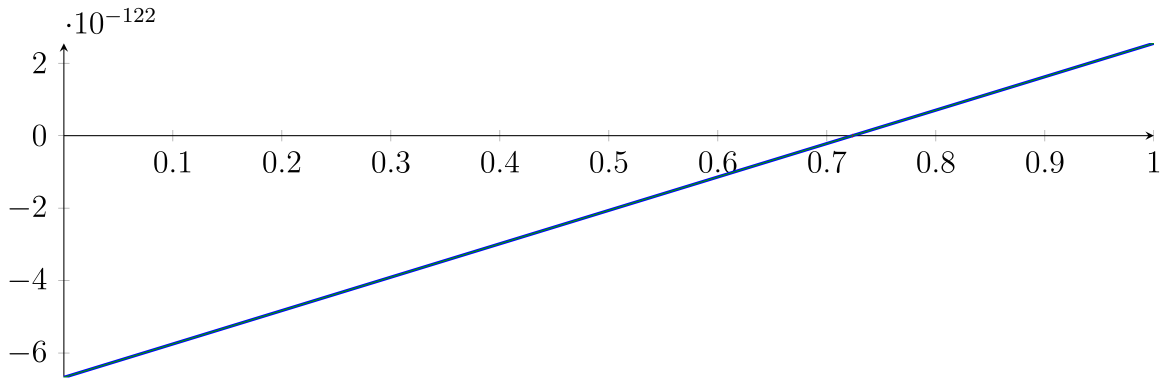
$$M = -1.211277710947941\cdot10^{-231}X^2 + 9.22057066454039\cdot10^{-122}X - 6.672011081049187\cdot10^{-122}$$

$$m = -1.211277710947941\cdot10^{-231}X^2 + 9.22057066454039\cdot10^{-122}X - 6.672011081049187\cdot10^{-122}$$

Root of M and m:

$N(M) = \{0.7236006667903739, 7.612268087823072\cdot10^{109}\}$ $\quad N(m) = \{0.7236006667903739, 7.61226808782307$

Intersection intervals:



$$[0.7236006667903739, 0.7236006667903739]$$

Longest intersection interval: $2.223066538673368\cdot10^{-199}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000002, 0.50000002]$,

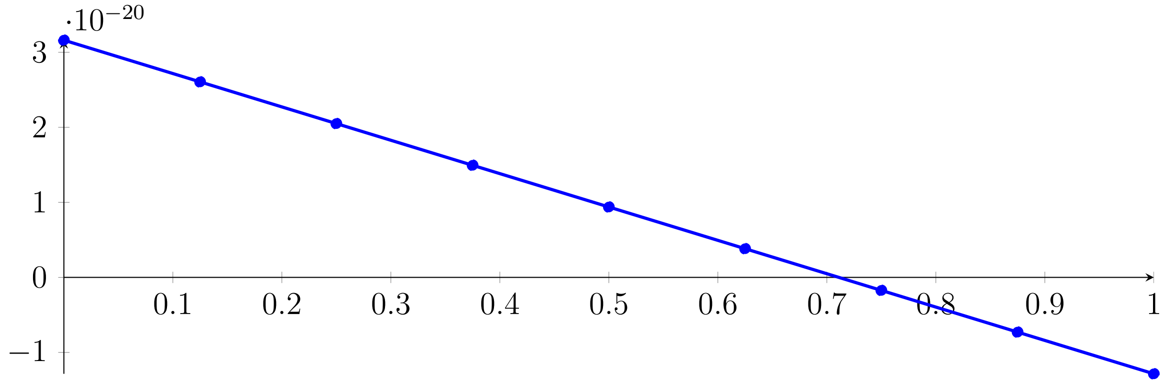
62.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.50000002, 0.50000002]$

Found root in interval $[0.50000002, 0.50000002]$ at recursion depth 11!

62.12 Recursion Branch 1 1 1 1 1 1 1 2 in Interval 2: $[0.50000003, 0.5000000300000$

Normalized monomial und Bézier representations and the Bézier polygon:

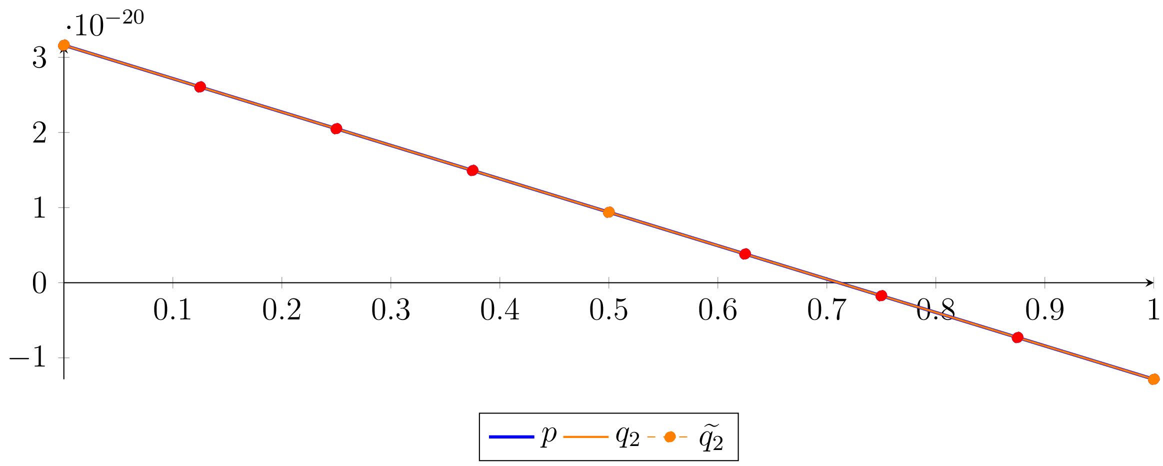
$$\begin{aligned} p = & -2.595099873436077\cdot 10^{-130}X^8 - 1.597531484477648\cdot 10^{-112}X^7 - 4.078240384140717\cdot 10^{-95}X^6 \\ & - 5.525920880401843\cdot 10^{-78}X^5 - 4.191391799565194\cdot 10^{-61}X^4 - 1.687408775678226\cdot 10^{-44}X^3 \\ & - 2.817152740417101\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20} \\ = & \, 3.161581953700541\cdot 10^{-20}B_{0,8}(X) + 2.605740232211127\cdot 10^{-20}B_{1,8}(X) + 2.049898509715588 \\ & \cdot 10^{-20}B_{2,8}(X) + 1.494056786213922\cdot 10^{-20}B_{3,8}(X) + 9.382150617061309\cdot 10^{-21}B_{4,8}(X) \\ & + 3.823733361922135\cdot 10^{-21}B_{5,8}(X) - 1.734683903278299\cdot 10^{-21}B_{6,8}(X) \\ & - 7.293101178539992\cdot 10^{-21}B_{7,8}(X) - 1.285151846386295\cdot 10^{-20}B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20} \\ = & \, 3.161581953700541\cdot 10^{-20}B_{0,2} + 9.382150677428867\cdot 10^{-21}B_{1,2} - 1.285151846386295\cdot 10^{-20}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & \, 2.039046071807459\cdot 10^{-323}X^8 - 3.404903576857106\cdot 10^{-323}X^7 - 6.962513967101668\cdot 10^{-323}X^6 \\ & + 2.228747497885225\cdot 10^{-322}X^5 - 2.092722637334645\cdot 10^{-322}X^4 + 7.6903440713512\cdot 10^{-323}X^3 \\ & - 2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20} \\ = & \, 3.161581953700541\cdot 10^{-20}B_{0,8} + 2.605740232211127\cdot 10^{-20}B_{1,8} + 2.049898509715588\cdot 10^{-20}B_{2,8} \\ & + 1.494056786213922\cdot 10^{-20}B_{3,8} + 9.382150617061309\cdot 10^{-21}B_{4,8} + 3.823733361922135\cdot 10^{-21}B_{5,8} \\ & - 1.734683903278299\cdot 10^{-21}B_{6,8} - 7.293101178539992\cdot 10^{-21}B_{7,8} - 1.285151846386295\cdot 10^{-20}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 8.437043878391132\cdot 10^{-46}$.

Bounding polynomials M and m:

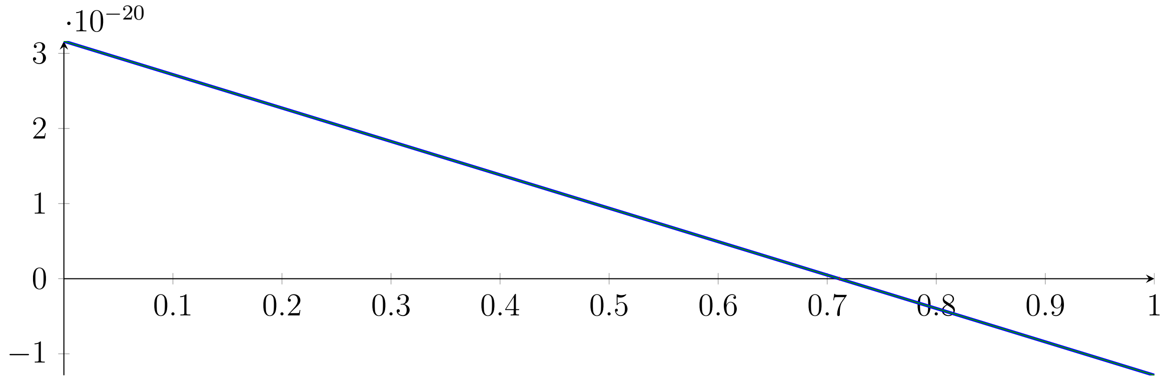
$$M = -2.817152740417102\cdot 10^{-28}X^2 - 4.446733771915308\cdot 10^{-20}X + 3.161581953700541\cdot 10^{-20}$$

$$m = -2.817152740417102 \cdot 10^{-28} X^2 - 4.446733771915308 \cdot 10^{-20} X + 3.161581953700541 \cdot 10^{-20}$$

Root of M and m:

$N(M) = \{-157844966.2366053, 0.7109897065184296\}$ $N(m) = \{-157844966.2366053, 0.7109897065184296\}$

Intersection intervals:



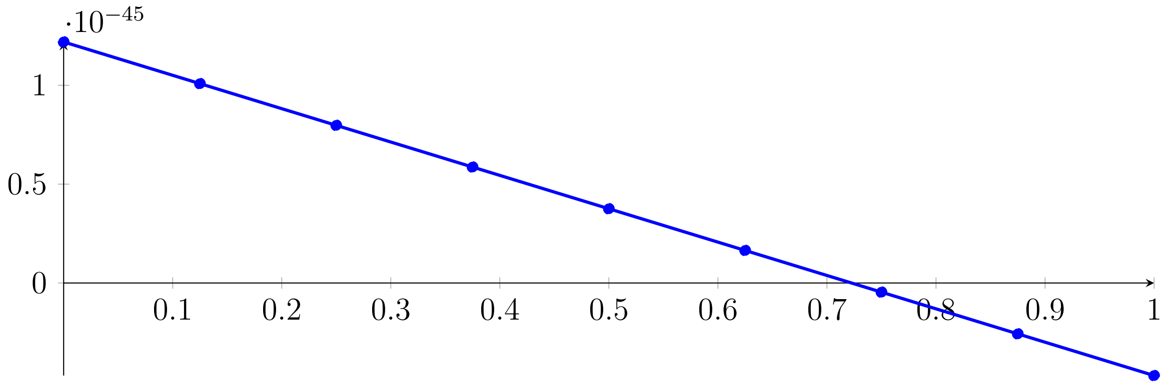
$$[0.7109897065184296, 0.7109897065184296]$$

Longest intersection interval: $3.794715058351799 \cdot 10^{-26}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

62.13 Recursion Branch 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

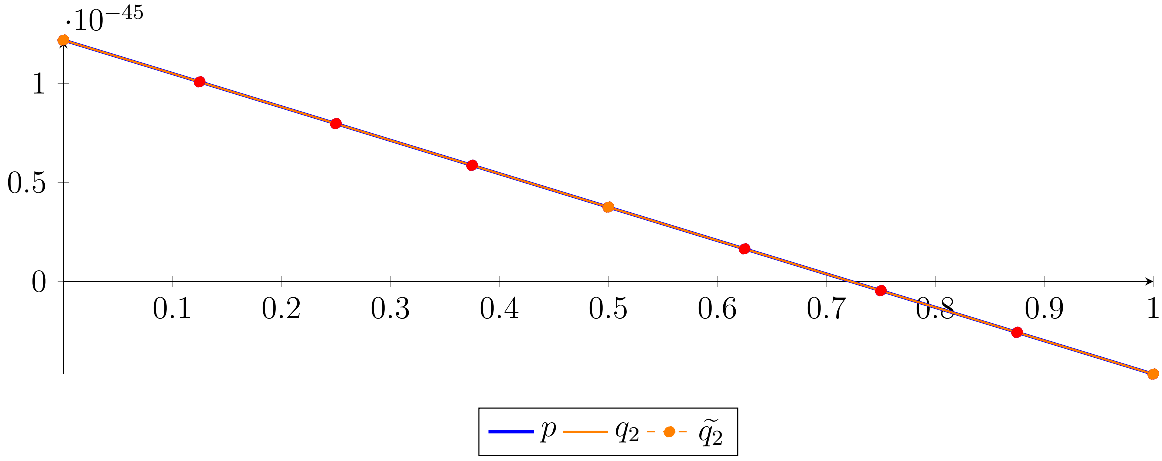
$$\begin{aligned} p = & -1.115802787354916 \cdot 10^{-333} X^8 - 1.810104386701217 \cdot 10^{-290} X^7 - 1.217721138271235 \cdot 10^{-247} X^6 \\ & - 4.348109933664918 \cdot 10^{-205} X^5 - 8.691103905258648 \cdot 10^{-163} X^4 - 9.220570981435719 \cdot 10^{-121} X^3 \\ & - 4.056661174877394 \cdot 10^{-79} X^2 - 1.687408775678226 \cdot 10^{-45} X + 1.219252393140717 \cdot 10^{-45} \\ = & \; 1.219252393140717 \cdot 10^{-45} B_{0,8}(X) + 1.008326296180938 \cdot 10^{-45} B_{1,8}(X) + 7.974001992211601 \\ & \cdot 10^{-46} B_{2,8}(X) + 5.864741022613818 \cdot 10^{-46} B_{3,8}(X) + 3.755480053016035 \cdot 10^{-46} B_{4,8}(X) \\ & + 1.646219083418252 \cdot 10^{-46} B_{5,8}(X) - 4.630418861795306 \cdot 10^{-47} B_{6,8}(X) \\ & - 2.572302855777314 \cdot 10^{-46} B_{7,8}(X) - 4.681563825375097 \cdot 10^{-46} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & -4.056661174877394 \cdot 10^{-79} X^2 - 1.687408775678226 \cdot 10^{-45} X + 1.219252393140717 \cdot 10^{-45} \\ = & \; 1.219252393140717 \cdot 10^{-45} B_{0,2} + 3.755480053016035 \cdot 10^{-46} B_{1,2} - 4.681563825375097 \cdot 10^{-46} B_{2,2} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 = {} & 8.361910453367177 \cdot 10^{-349} X^8 - 1.541597891866099 \cdot 10^{-348} X^7 - 2.313332235573428 \cdot 10^{-348} X^6 \\
& + 8.342313869995998 \cdot 10^{-348} X^5 - 8.027038050174711 \cdot 10^{-348} X^4 + 2.973130645625176 \cdot 10^{-348} X^3 \\
& - 4.056661174877394 \cdot 10^{-79} X^2 - 1.687408775678226 \cdot 10^{-45} X + 1.219252393140717 \cdot 10^{-45} \\
= {} & 1.219252393140717 \cdot 10^{-45} B_{0,8} + 1.008326296180938 \cdot 10^{-45} B_{1,8} + 7.974001992211601 \cdot 10^{-46} B_{2,8} \\
& + 5.864741022613818 \cdot 10^{-46} B_{3,8} + 3.755480053016035 \cdot 10^{-46} B_{4,8} + 1.646219083418252 \cdot 10^{-46} B_{5,8} \\
& - 4.630418861795306 \cdot 10^{-47} B_{6,8} - 2.572302855777314 \cdot 10^{-46} B_{7,8} - 4.681563825375097 \cdot 10^{-46} B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 4.61028549071786 \cdot 10^{-122}$.

Bounding polynomials M and m:

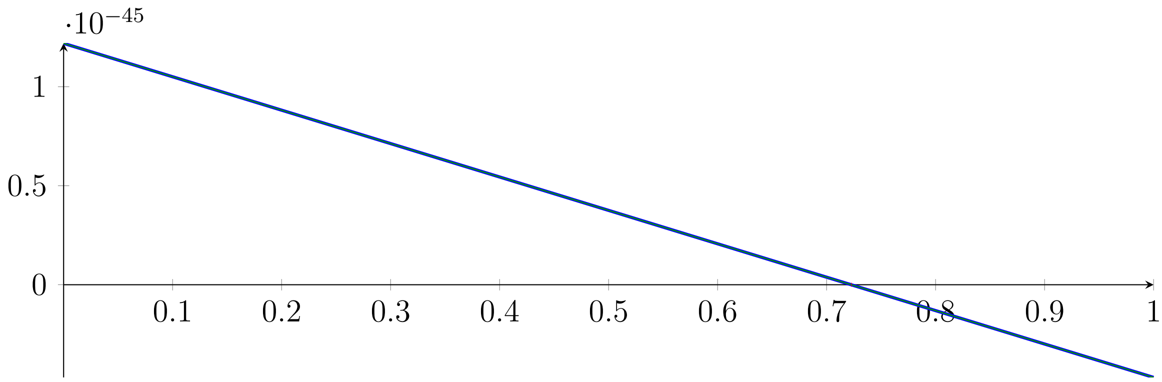
$$M = -4.056661174877394 \cdot 10^{-79} X^2 - 1.687408775678226 \cdot 10^{-45} X + 1.219252393140717 \cdot 10^{-45}$$

$$m = -4.056661174877394 \cdot 10^{-79} X^2 - 1.687408775678226 \cdot 10^{-45} X + 1.219252393140717 \cdot 10^{-45}$$

Root of M and m:

$N(M) = \{-4.159599983671857 \cdot 10^{33}, 0.7225589973897452\}$ $\quad N(m) = \{-4.159599983671857 \cdot 10^{33}, 0.72255899$

Intersection intervals:



$$[0.7225589973897452, 0.7225589973897452]$$

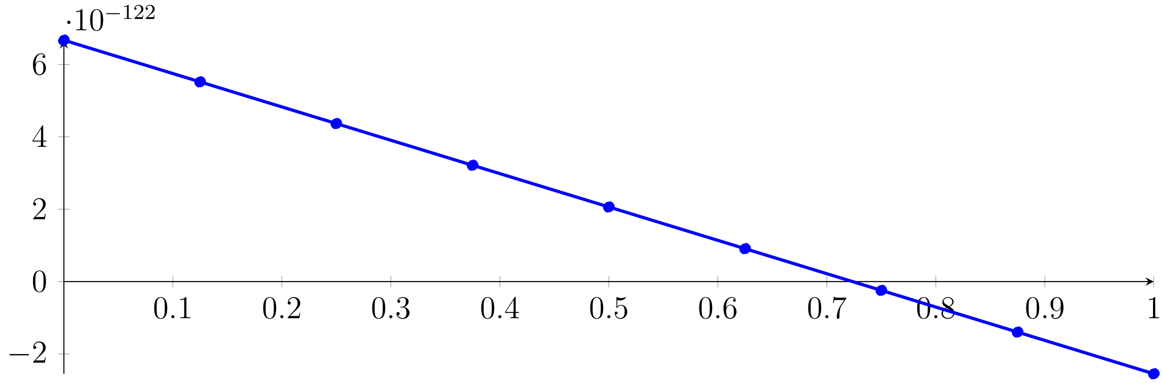
Longest intersection interval: $5.464337458912208 \cdot 10^{-77}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

62.14 Recursion Branch 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

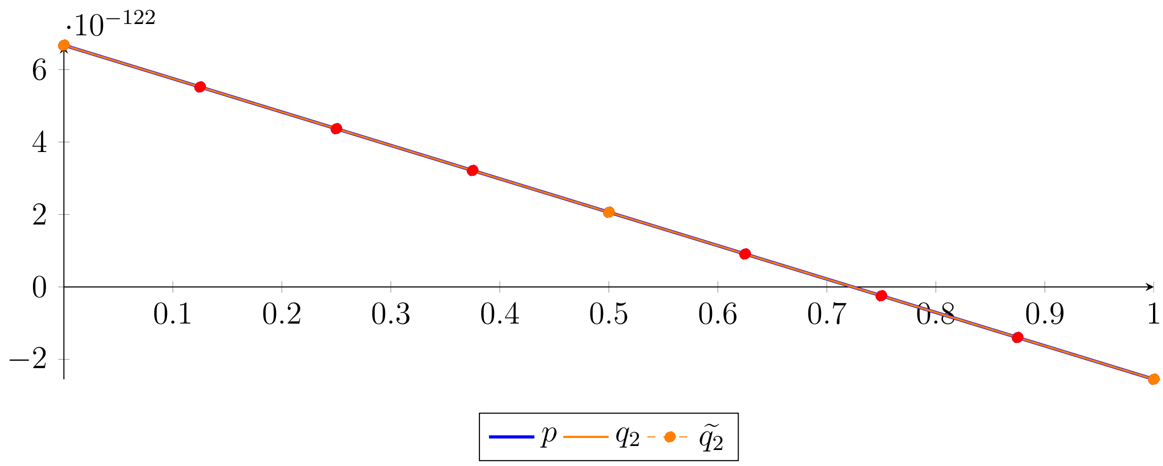
$$p = 1.723360606879513\cdot10^{-428}X^8 + 6.031762124078295\cdot10^{-428}X^6 - 1.206352424815659\cdot10^{-427}X^5 - 1.50442434132688\cdot10^{-349}X^3 - 1.211277805659132\cdot10^{-231}X^2 - 9.220570981435719\cdot10^{-122}X + 6.67200003933902\cdot10^{-122}$$
$$= 6.67200003933902\cdot10^{-122}B_{0,8}(X) + 5.519428666659555\cdot10^{-122}B_{1,8}(X) + 4.36685729398009\cdot10^{-122}B_{2,8}(X) + 3.214285921300625\cdot10^{-122}B_{3,8}(X) + 2.06171454862116\cdot10^{-122}B_{4,8}(X) + 9.091431759416951\cdot10^{-123}B_{5,8}(X) - 2.434281967377698\cdot10^{-123}B_{6,8}(X) - 1.395999569417235\cdot10^{-122}B_{7,8}(X) - 2.5485709420967\cdot10^{-122}B_{8,8}(X)$$



Degree reduction and raising:

$$q_2 = -1.211277805659132\cdot10^{-231}X^2 - 9.220570981435719\cdot10^{-122}X + 6.67200003933902\cdot10^{-122}$$
$$= 6.67200003933902\cdot10^{-122}B_{0,2} + 2.06171454862116\cdot10^{-122}B_{1,2} - 2.5485709420967\cdot10^{-122}B_{2,2}$$

$$\widetilde{q}_2 = 4.59975716979935\cdot10^{-425}X^8 - 8.552177011639582\cdot10^{-425}X^7 - 1.24646364294078\cdot10^{-424}X^6 + 4.553377227466705\cdot10^{-424}X^5 - 4.38735297500145\cdot10^{-424}X^4 + 1.6255598924391\cdot10^{-424}X^3 - 1.211277805659132\cdot10^{-231}X^2 - 9.220570981435719\cdot10^{-122}X + 6.67200003933902\cdot10^{-122}$$
$$= 6.67200003933902\cdot10^{-122}B_{0,8} + 5.519428666659555\cdot10^{-122}B_{1,8} + 4.36685729398009\cdot10^{-122}B_{2,8} + 3.214285921300625\cdot10^{-122}B_{3,8} + 2.06171454862116\cdot10^{-122}B_{4,8} + 9.091431759416951\cdot10^{-123}B_{5,8} - 2.434281967377698\cdot10^{-123}B_{6,8} - 1.395999569417235\cdot10^{-122}B_{7,8} - 2.5485709420967\cdot10^{-122}B_{8,8}$$



The maximum difference of the Bézier coefficients is $\delta = 7.522121706634401\cdot10^{-351}$.

Bounding polynomials M and m:

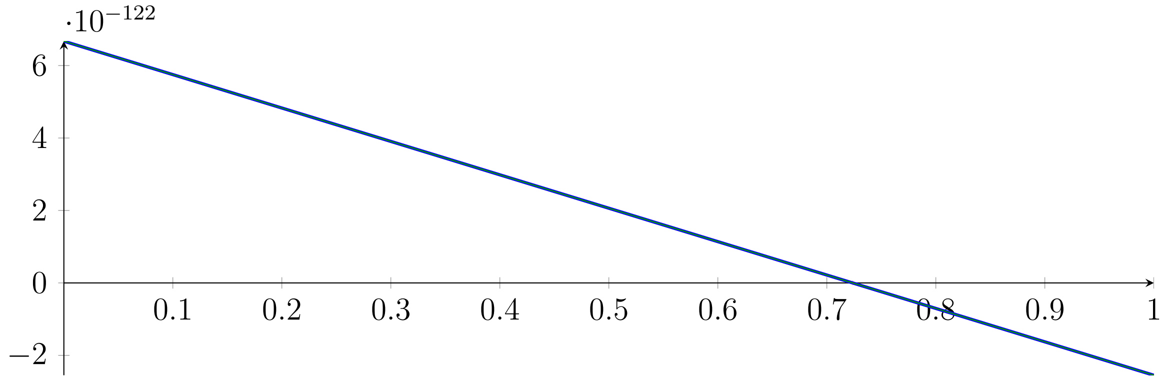
$$M = -1.211277805659132\cdot10^{-231}X^2 - 9.220570981435719\cdot10^{-122}X + 6.67200003933902\cdot10^{-122}$$

$m = -1.211277805659132 \cdot 10^{-231} X^2 - 9.220570981435719 \cdot 10^{-122} X + 6.67200003933902 \cdot 10^{-122}$

Root of M and m:

$N(M) = \{-7.612267754231844 \cdot 10^{109}, 0.723599444413163\}$ $N(m) = \{-7.612267754231844 \cdot 10^{109}, 0.7235994$

Intersection intervals:



$[0.723599444413163, 0.723599444413163]$

Longest intersection interval: $9.94152808147777 \cdot 10^{-200}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

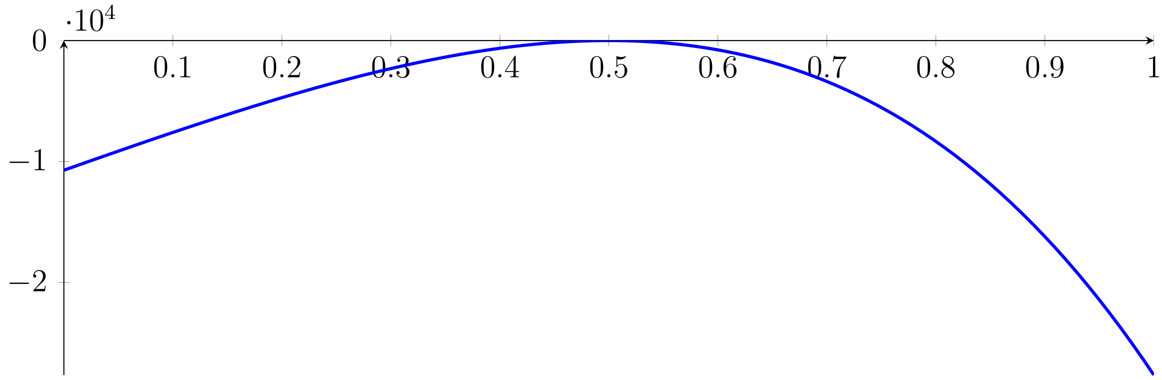
62.15 Recursion Branch 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 11!

62.16 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000002, 0.50000002],\ [0.50000003, 0.50000003]$$

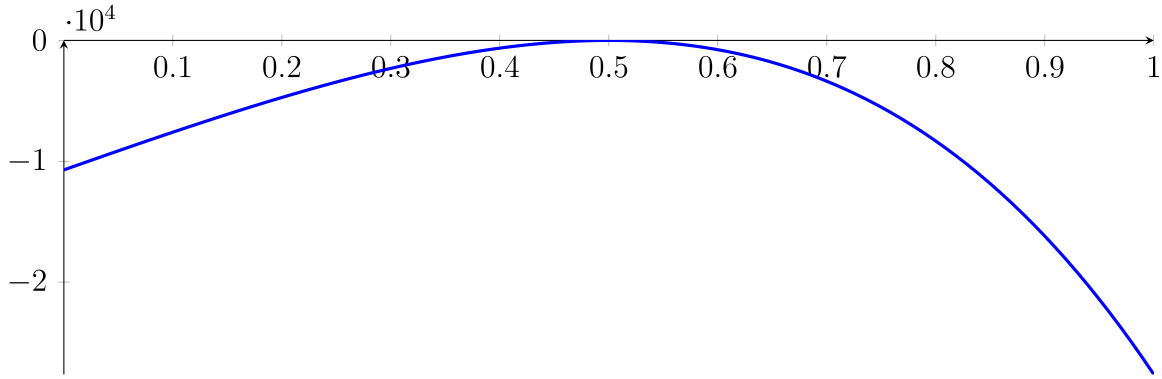
with precision $\varepsilon = 1 \cdot 10^{-128}$.

63 Running `CubeClip` on h_8 with epsilon 128

$$-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$

Called `CubeClip` with input polynomial on interval $[0,1]$:

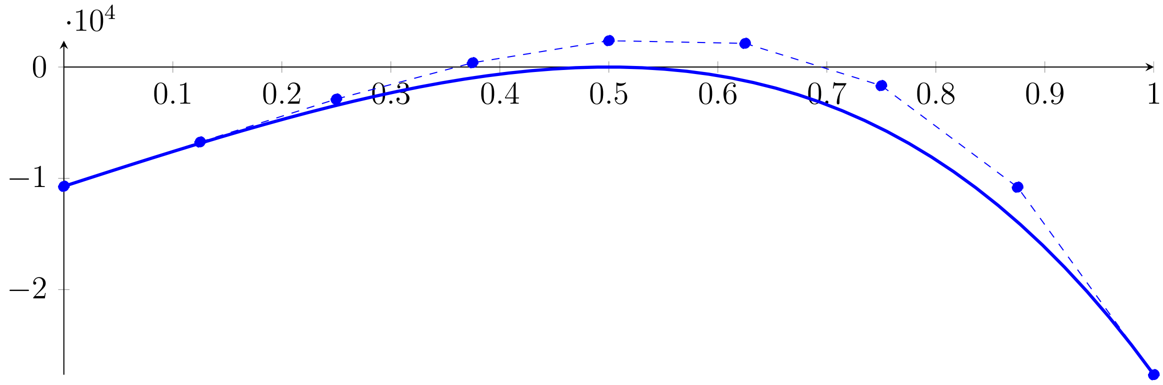
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \end{aligned}$$



63.1 Recursion Branch 1 for Input Interval $[0,1]$

Normalized monomial und Bézier representations and the Bézier polygon:

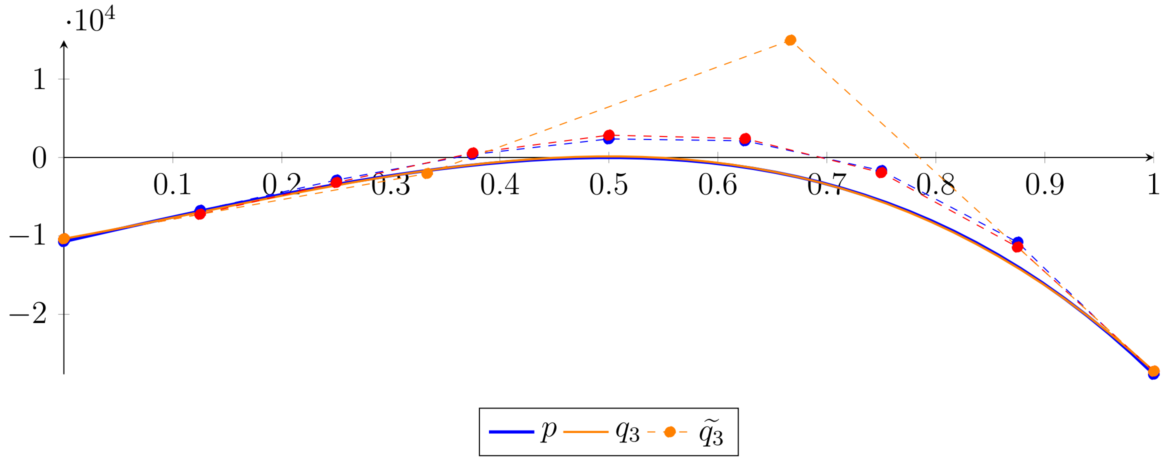
$$\begin{aligned} p = &-1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 \\ &- 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503 \\ = &-10718.75107187503B_{0,8}(X) - 6737.50094171878B_{1,8}(X) - 2880.313249593783B_{2,8}(X) \\ &+ 381.9727336704985B_{3,8}(X) + 2368.785597229958B_{4,8}(X) + 2123.393219260668B_{5,8}(X) \\ &- 1671.427589657195B_{6,8}(X) - 10799.99822880006B_{7,8}(X) - 27647.99723520006B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = &-67867.5491953649X^3 + 26008.32662806823X^2 + 24973.0963532161X - 10364.86272786554 \\ = &-10364.86272786554B_{0,3} - 2040.497276793509B_{1,3} \\ &+ 14953.3103836346B_{2,3} - 27250.98894194612B_{3,3} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_3 = & -5.703152099307112 \cdot 10^{-299} X^8 + 1.997854926822688 \cdot 10^{-298} X^7 - 2.620079279971631 \\
& \cdot 10^{-298} X^6 + 1.512761996617561 \cdot 10^{-298} X^5 - 2.636105166170616 \cdot 10^{-299} X^4 \\
& - 67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10364.86272786554 \\
= & -10364.86272786554 B_{0,8} - 7243.22568371353 B_{1,8} - 3192.719831416224 B_{2,8} \\
& + 574.7343076805747 B_{3,8} + 2847.216212231063 B_{4,8} + 2412.80536088944 B_{5,8} \\
& - 1940.418767690097 B_{6,8} - 11424.37669485335 B_{7,8} - 27250.98894194612 B_{8,8}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 624.3784660532909$.

Bounding polynomials M and m:

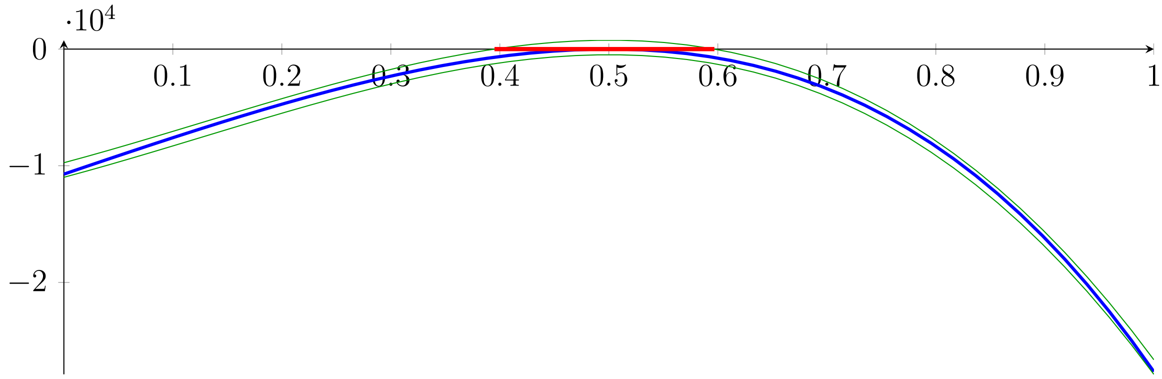
$$M = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 9740.484261812251$$

$$m = -67867.5491953649 X^3 + 26008.32662806823 X^2 + 24973.0963532161 X - 10989.24119391883$$

Root of M and m:

$N(M) = \{-0.6086847757398864, 0.3950604114095972, 0.5968461976869074\}$ $\quad N(m) = \{-0.623487965792376$

Intersection intervals:



$[0.3950604114095972, 0.5968461976869074]$

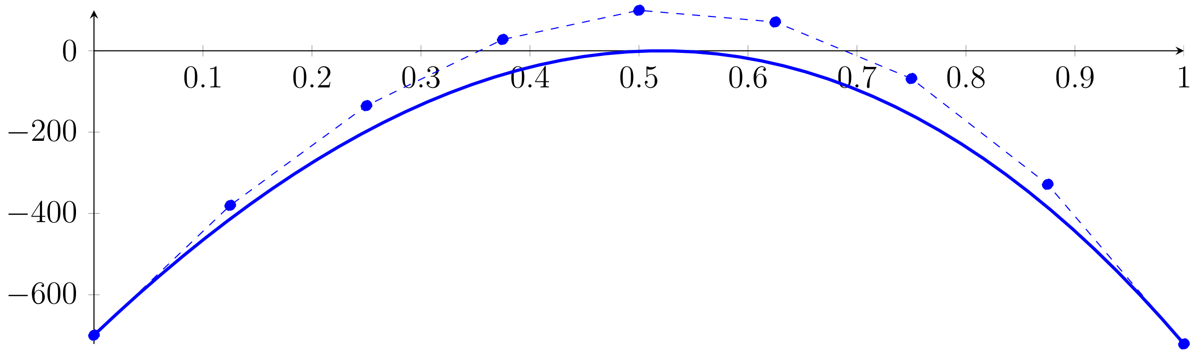
Longest intersection interval: 0.2017857862773101

$\Longrightarrow$ Selective recursion: interval 1: $[0.3950604114095972, 0.5968461976869074]$,

63.2 Recursion Branch 1 1 in Interval 1: $[0.3950604114095972, 0.5968461976869074]$

Normalized monomial und Bézier representations and the Bézier polygon:

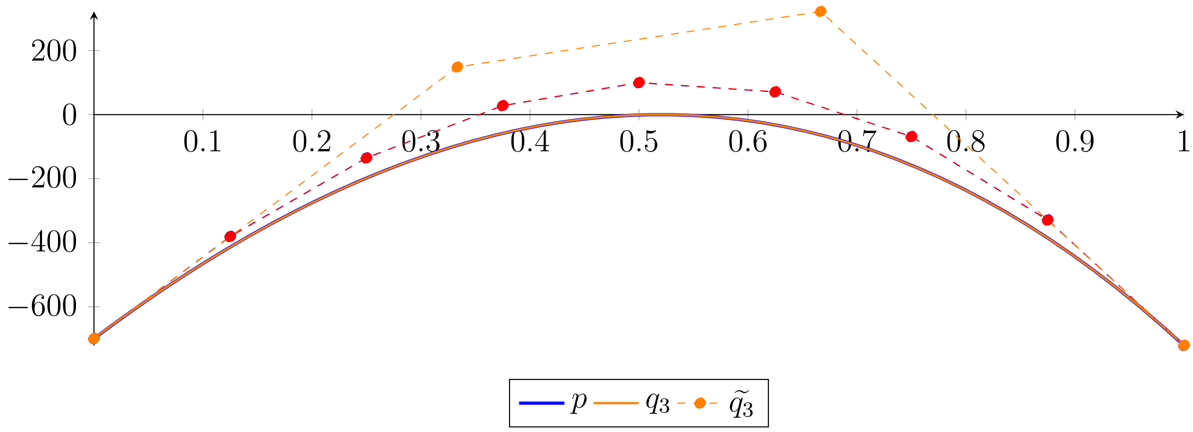
$$\begin{aligned}p = &-2.748682461629333\cdot 10^{-06}X^8 - 0.0005198138726602274X^7 - 0.04066637698425721X^6\\ &- 1.681520506726176X^5 - 38.59639865464246X^4 - 460.2831920090805X^3\\ &- 2074.780222477586X^2 + 2553.796807278985X - 699.3475151603227\\ = &-699.3475151603227B_{0,8}(X) - 380.1229142504495B_{1,8}(X) - 134.9976070004902B_{2,8}(X)\\ &+ 27.8090638751075B_{3,8}(X) + 99.52637853825791B_{4,8}(X) + 70.80221287533186B_{5,8}(X)\\ &- 68.32844102535583B_{6,8}(X) - 328.4764717433994B_{7,8}(X) - 720.9332304689123B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\ &= -698.7407882409323B_{0,3} + 148.5035278252892B_{1,3}\\ &\quad + 322.0745267793334B_{2,3} - 720.3121660793493B_{3,3}\end{aligned}$$

$$\begin{aligned}\tilde{q}_3 &= -2.24002635483637\cdot 10^{-300}X^8 + 8.593407551274415\cdot 10^{-300}X^7 - 1.289726574039331\\ &\quad \cdot 10^{-299}X^6 + 9.445830854385802\cdot 10^{-300}X^5 - 3.303106122422159\cdot 10^{-300}X^4\\ &\quad - 542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 698.7407882409323\\ &= -698.7407882409323B_{0,8} - 381.0241697160992B_{1,8} - 135.4868351675709B_{2,8}\\ &\quad + 28.18756585642868B_{3,8} + 100.3153838076753B_{4,8} + 71.21296913794494B_{5,8}\\ &\quad - 68.80332770098655B_{6,8} - 329.4171562573433B_{7,8} - 720.3121660793493B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.9406845139438908$.

Bounding polynomials M and m:

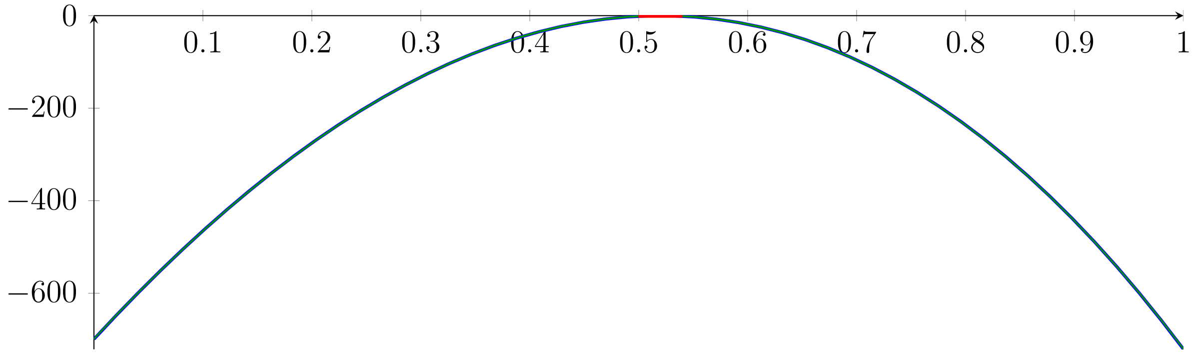
$$M = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 697.8001037269884$$

$$m = -542.2843747005496X^3 - 2021.019951336532X^2 + 2541.732948198665X - 699.6814727548762$$

Root of M and m:

$$N(M) = \{-4.766776582869526, 0.4997746332555131, 0.5401382492675976\} \quad N(m) = \{-4.76690070753073\}$$

Intersection intervals:



$$[0.4997746332555131, 0.5401382492675976]$$

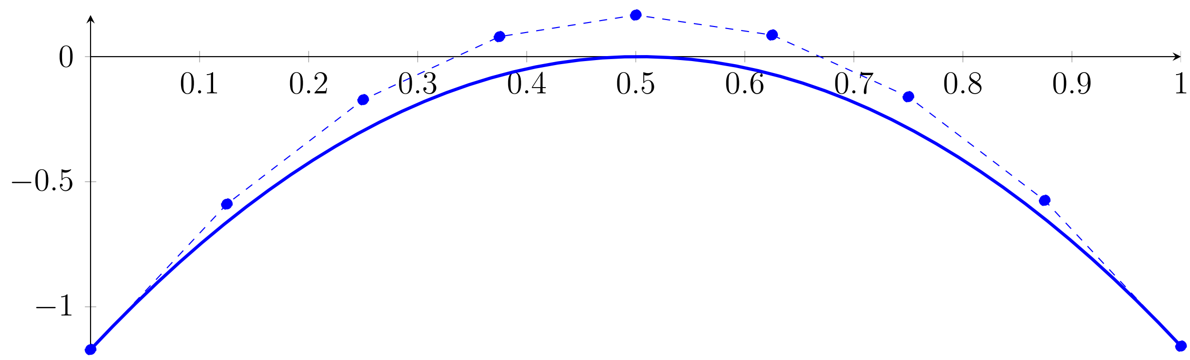
Longest intersection interval: 0.04036361601208447

$\Longrightarrow$ Selective recursion: interval 1: $[0.4959078287425153, 0.5040526327365092]$,

63.3 Recursion Branch 1 1 1 in Interval 1: $[0.4959078287425153, 0.504052632736509$

Normalized monomial und Bézier representations and the Bézier polygon:

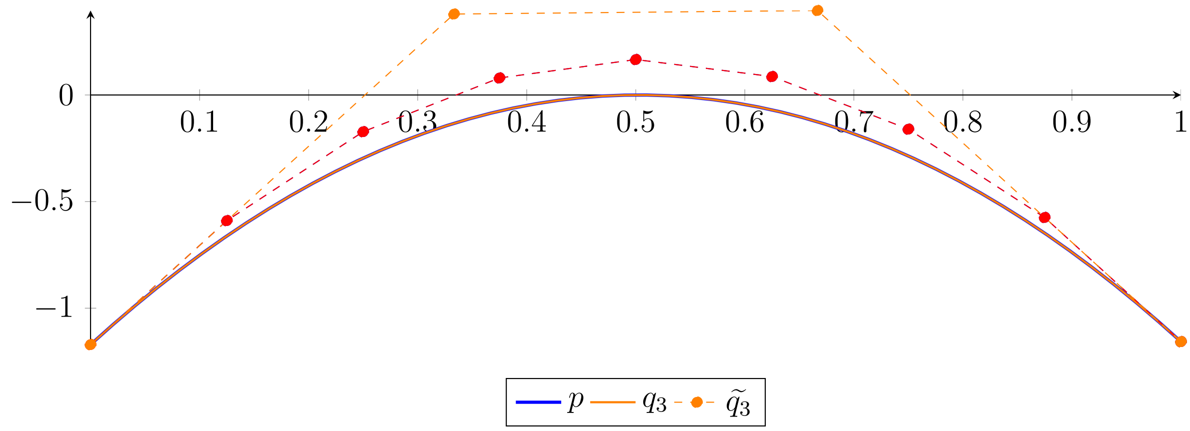
$$\begin{aligned}
p &= -1.936623061789877\cdot 10^{-17}X^8 - 9.265403983052604\cdot 10^{-14}X^7 - 1.838110077974511\cdot 10^{-10}X^6\\
&\quad - 1.935168004397404\cdot 10^{-07}X^5 - 0.0001140127070697086X^4 - 0.0356257658581906X^3\\
&\quad - 4.602344943530705X^2 + 4.651752639476855X - 1.170857231885768\\
&= -1.170857231885768B_{0,8}(X) - 0.5893881519511612B_{1,8}(X) - 0.172288534285508B_{2,8}(X)\\
&\quad + 0.07980544672086649B_{3,8}(X) + 0.1662559879246795B_{4,8}(X) + 0.08642365397403269B_{5,8}(X)\\
&\quad - 0.1603326261538093B_{6,8}(X) - 0.574655562621217B_{7,8}(X) - 1.157189508205582B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= -0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661\\
&= -1.170855596980661B_{0,3} + 0.3797177193764957B_{1,3}\\
&\quad + 0.3962250709273155B_{2,3} - 1.157187871760247B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -3.713970905558003\cdot 10^{-303}X^8 + 1.449155002864004\cdot 10^{-302}X^7 - 2.225710229630734\\
&\quad \cdot 10^{-302}X^6 + 1.689258272787799\cdot 10^{-302}X^5 - 6.343129302139404\cdot 10^{-303}X^4\\
&\quad - 0.03585432943204529X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170855596980661\\
&= -1.170855596980661B_{0,8} - 0.5893906033467271B_{1,8} - 0.1722898202277581B_{2,8}\\
&\quad + 0.07980649649353122B_{3,8} + 0.1662580909344257B_{4,8} + 0.08642470721221019B_{5,8}\\
&\quad - 0.1603339105558303B_{6,8} - 0.574658018252411B_{7,8} - 1.157187871760247B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.45563119394813 \cdot 10^{-06}$.

Bounding polynomials M and m:

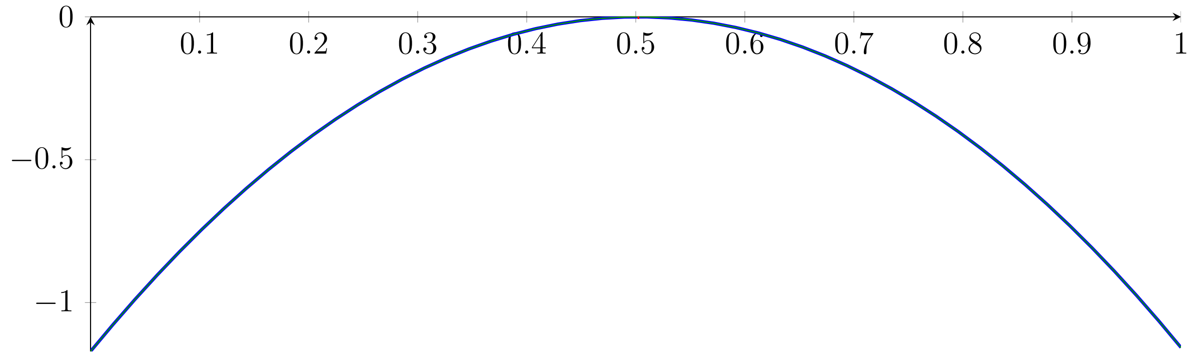
$$M = -0.0358543294320452 9X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170853141349467$$

$$m = -0.0358543294320452 9X^3 - 4.60219789441901X^2 + 4.65171994907147X - 1.170858052611855$$

Root of M and m:

$$N(M) = \{-129.3630802555855, 0.5016184355695172, 0.5032421204520033\} \quad N(m) = \{-129.3630802637075$$

Intersection intervals:



$$[0.5016184355695172, 0.5032421204520033]$$

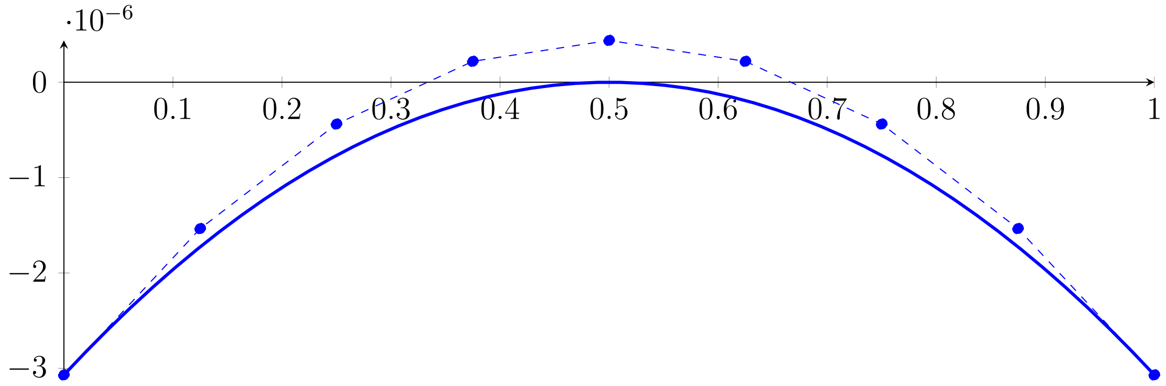
Longest intersection interval: 0.001623684882486142

$\Longrightarrow$ Selective recursion: interval 1: $[0.4999934125800028, 0.5000066371751187]$,

63.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.4999934125800028, 0.50000663717511$

Normalized monomial und Bézier representations and the Bézier polygon:

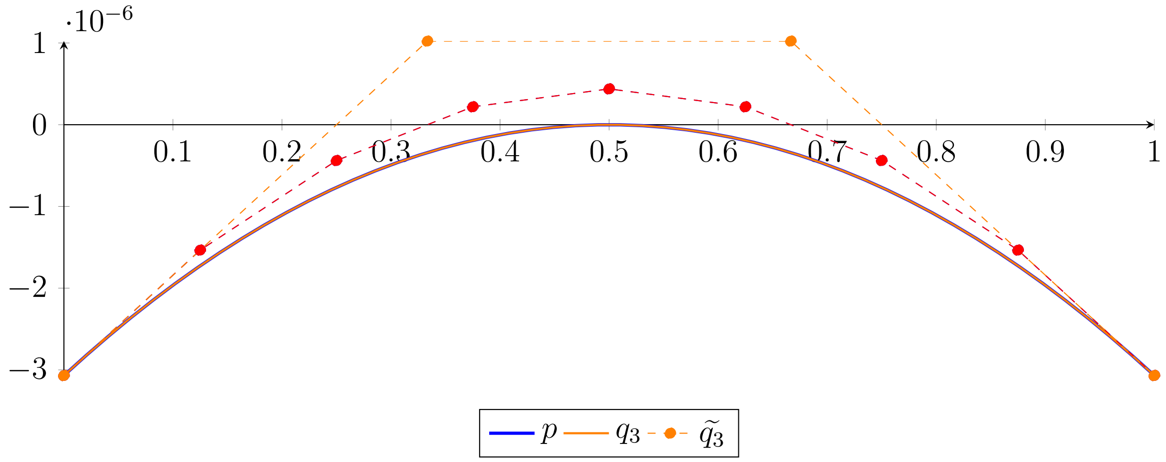
$$\begin{aligned} p = &-9.355329246270906 \cdot 10^{-40} X^8 - 2.758930189698976 \cdot 10^{-33} X^7 - 3.374040776758279 \cdot 10^{-27} X^6 \\ &- 2.190121853313308 \cdot 10^{-21} X^5 - 7.958070257617345 \cdot 10^{-16} X^4 - 1.534811947451426 \cdot 10^{-10} X^3 \\ &- 1.227519762250185 \cdot 10^{-05} X^2 + 1.227554003668957 \cdot 10^{-05} X - 3.068949673836091 \cdot 10^{-06} \\ = &-3.068949673836091 \cdot 10^{-06} B_{0,8}(X) - 1.534507169249895 \cdot 10^{-06} B_{1,8}(X) - 4.384645797530504 \\ &\cdot 10^{-07} B_{2,8}(X) + 2.191753539188221 \cdot 10^{-07} B_{3,8}(X) + 4.384098910187333 \cdot 10^{-07} B_{4,8}(X) \\ &+ 2.192362907883254 \cdot 10^{-07} B_{5,8}(X) - 4.383481875421282 \cdot 10^{-07} B_{6,8}(X) \\ &- 1.534346284753723 \cdot 10^{-06} B_{7,8}(X) - 3.068760741638923 \cdot 10^{-06} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -1.534827863652778\cdot 10^{-10} X^3 - 1.227519762147866\cdot 10^{-05} X^2 \\ &\quad + 1.22755400364622\cdot 10^{-05} X - 3.068949673824723\cdot 10^{-06} \\ &= -3.068949673824723\cdot 10^{-06} B_{0,3} + 1.022897004996009\cdot 10^{-06} B_{1,3} \\ &\quad + 1.023011143323854\cdot 10^{-06} B_{2,3} - 3.068760741627554\cdot 10^{-06} B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -9.792734715851906\cdot 10^{-309} X^8 + 3.822699025575463\cdot 10^{-308} X^7 - 5.87427002023018\cdot 10^{-308} X^6 \\ &\quad + 4.462726908063245\cdot 10^{-308} X^5 - 1.679538448591103\cdot 10^{-308} X^4 - 1.534827863652778\cdot 10^{-10} X^3 \\ &\quad - 1.227519762147866\cdot 10^{-05} X^2 + 1.22755400364622\cdot 10^{-05} X - 3.068949673824723\cdot 10^{-06} \\ &= -3.068949673824723\cdot 10^{-06} B_{0,8} - 1.534507169266948\cdot 10^{-06} B_{1,8} - 4.38464579761983\cdot 10^{-07} B_{2,8} \\ &\quad + 2.191753539261305\cdot 10^{-07} B_{3,8} + 4.384098910333502\cdot 10^{-07} B_{4,8} + 2.192362907956339\cdot 10^{-07} B_{5,8} \\ &\quad - 4.383481875510608\cdot 10^{-07} B_{6,8} - 1.534346284770776\cdot 10^{-06} B_{7,8} - 3.068760741627554\cdot 10^{-06} B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.705314892341177\cdot 10^{-17}$.

Bounding polynomials M and m:

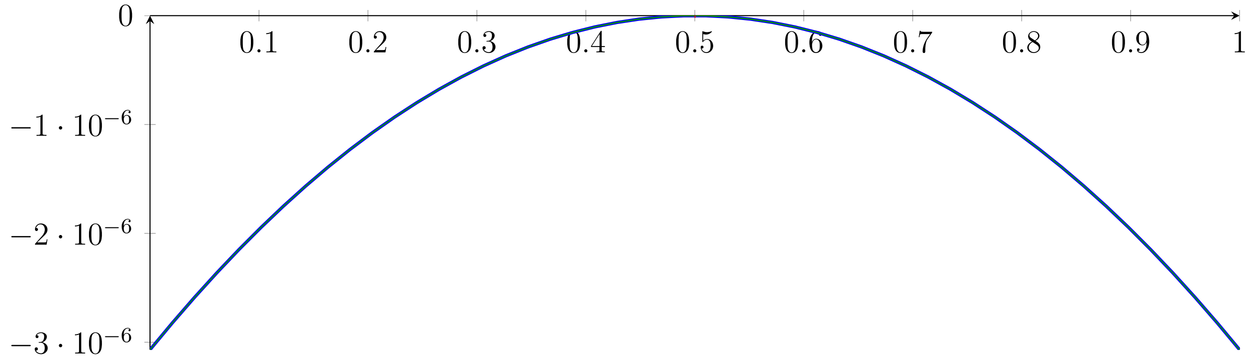
$$\begin{aligned} M &= -1.534827863652778\cdot 10^{-10} X^3 - 1.227519762147866\cdot 10^{-05} X^2 \\ &\quad + 1.22755400364622\cdot 10^{-05} X - 3.068949673807669\cdot 10^{-06} \end{aligned}$$

$$\begin{aligned} m &= -1.534827863652778\cdot 10^{-10} X^3 - 1.227519762147866\cdot 10^{-05} X^2 \\ &\quad + 1.22755400364622\cdot 10^{-05} X - 3.068949673841776\cdot 10^{-06} \end{aligned}$$

Root of M and m:

$N(M) = \{-79978.68293772447, 0.4996311727354866, 0.5003873441571286\}$ $\quad N(m) = \{-79978.68293772447,$

Intersection intervals:



$$[0.4996311727354866, 0.4996311764098305],\ [0.5003873404827848, 0.5003873441571286]$$

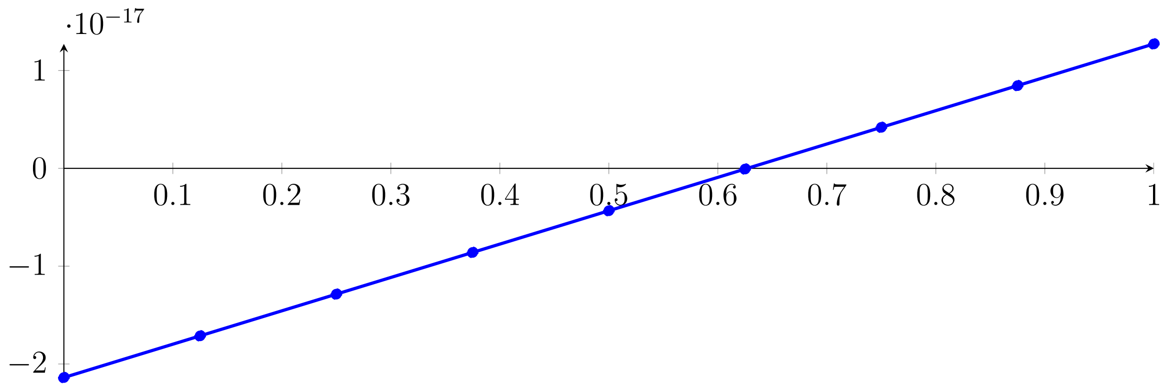
Longest intersection interval: $3.6743438794335088 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999695, 0.5000000200000181]$, interval 2: $[0.500000029999981$

63.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.5000000199999695, 0.500000020000$

Normalized monomial und Bézier representations and the Bézier polygon:

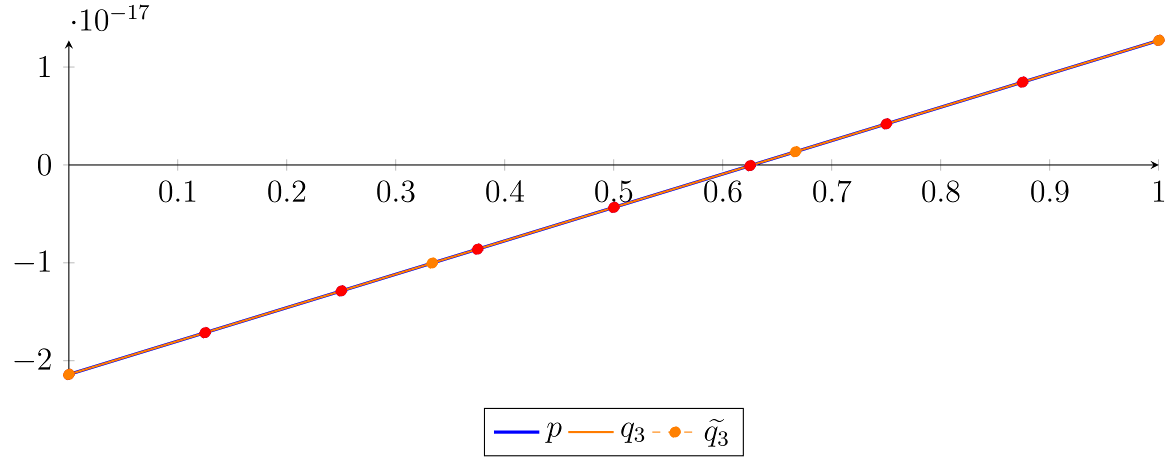
$$\begin{aligned}
p &= -3.10811779683933 \cdot 10^{-107} X^8 - 2.494594121391514 \cdot 10^{-92} X^7 - 8.302910645986526 \cdot 10^{-78} X^6 \\
&\quad - 1.466794525552917 \cdot 10^{-63} X^5 - 1.450540809134678 \cdot 10^{-49} X^4 - 7.613758006193666 \cdot 10^{-36} X^3 \\
&\quad - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
&= -2.138639484655546 \cdot 10^{-17} B_{0,8}(X) - 1.71230868147238 \cdot 10^{-17} B_{1,8}(X) - 1.285978470175393 \\
&\quad \cdot 10^{-17} B_{2,8}(X) - 8.596488507645843 \cdot 10^{-18} B_{3,8}(X) - 4.333198232399549 \\
&\quad \cdot 10^{-18} B_{4,8}(X) - 6.991387601504461 \cdot 10^{-20} B_{5,8}(X) + 4.19336456150767 \cdot 10^{-18} B_{6,8}(X) \\
&\quad + 8.456637080168595 \cdot 10^{-18} B_{7,8}(X) + 1.271990367996773 \cdot 10^{-17} B_{8,8}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 \\
&\quad + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
&= -2.138639484655546 \cdot 10^{-17} B_{0,3} - 1.001757342833769 \cdot 10^{-17} B_{1,3} \\
&\quad + 1.351192747170036 \cdot 10^{-18} B_{2,3} + 1.271990367996773 \cdot 10^{-17} B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= -3.883510371826523 \cdot 10^{-321} X^8 + 2.549502248952292 \cdot 10^{-320} X^7 - 4.188626787236358 \cdot 10^{-320} X^6 \\
&\quad + 1.055694768751284 \cdot 10^{-320} X^5 + 2.46739471443405 \cdot 10^{-320} X^4 - 7.613758006193956 \cdot 10^{-36} X^3 \\
&\quad - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17} \\
&= -2.138639484655546 \cdot 10^{-17} B_{0,8} - 1.71230868147238 \cdot 10^{-17} B_{1,8} - 1.285978470175393 \cdot 10^{-17} B_{2,8} \\
&\quad - 8.596488507645843 \cdot 10^{-18} B_{3,8} - 4.333198232399549 \cdot 10^{-18} B_{4,8} - 6.991387601504461 \cdot 10^{-20} B_{5,8} \\
&\quad + 4.19336456150767 \cdot 10^{-18} B_{6,8} + 8.456637080168595 \cdot 10^{-18} B_{7,8} + 1.271990367996773 \cdot 10^{-17} B_{8,8}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301733860119 \cdot 10^{-51}$.
Bounding polynomials M and m:

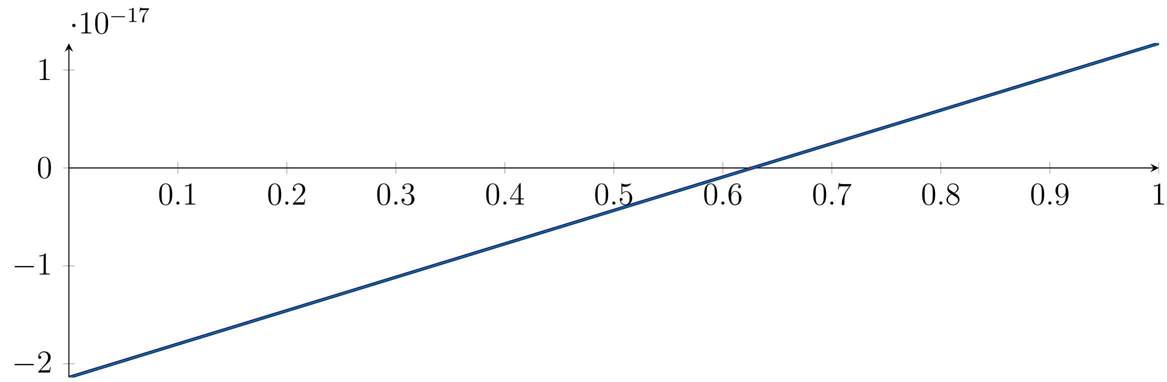
$$M = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

$$m = -7.613758006193956 \cdot 10^{-36} X^3 - 1.657281301066328 \cdot 10^{-22} X^2 + 3.41064642546533 \cdot 10^{-17} X - 2.138639484655546 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929227157.35, 0.6249980054152174, 205797.0483973646\}$ $N(m) = \{-21766929227157.35, 0$

Intersection intervals:



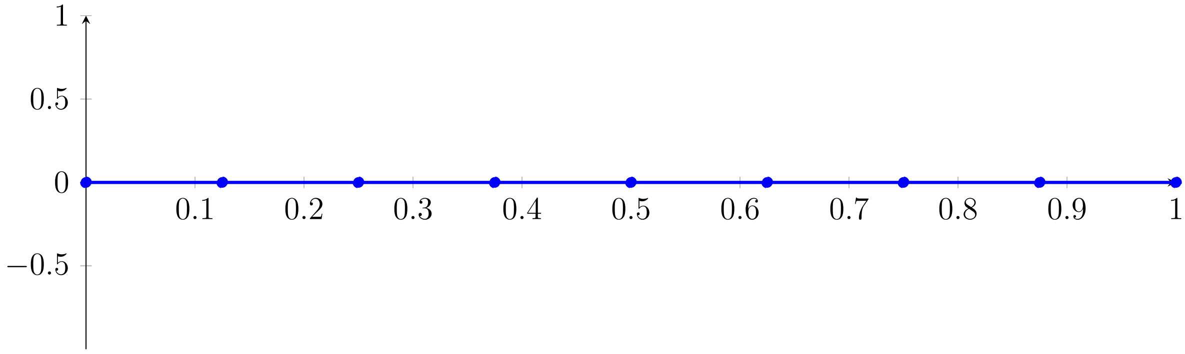
$$[0.6249980054152174, 0.6249980054152174]$$

Longest intersection interval: $1.822716451808538 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.5000000199999999, 0.5000000199999999]$,

63.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.5000000199999999, 0.50000001999$

Normalized monomial und Bézier representations and the Bézier polygon:

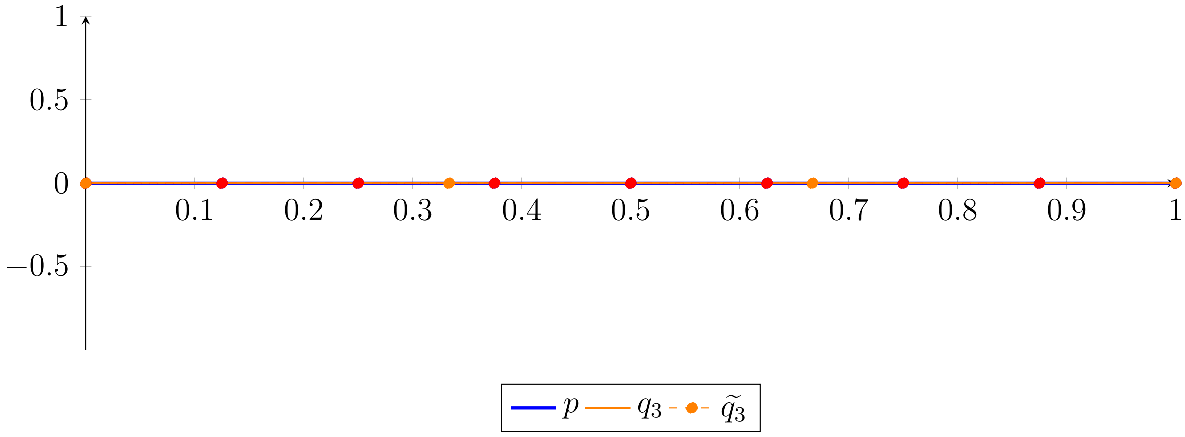
$$\begin{aligned} p &= -1.929943929067369 \cdot 10^{-326} X^8 + 3.859887858134739 \cdot 10^{-326} X^7 - 3.044703772619375 \cdot 10^{-280} X^6 \\ &\quad - 2.950970364376253 \cdot 10^{-232} X^5 - 1.601055569746729 \cdot 10^{-184} X^4 - 4.610588999829434 \cdot 10^{-137} X^3 \\ &\quad - 5.505977817140947 \cdot 10^{-90} X^2 + 6.216603591694483 \cdot 10^{-51} X - 6.998745276936609 \cdot 10^{-20} \\ &= -6.998745276936609 \cdot 10^{-20} B_{0,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{1,8}(X) - 6.998745276936609 \\ &\quad \cdot 10^{-20} B_{2,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{3,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{4,8}(X) \\ &\quad - 6.998745276936609 \cdot 10^{-20} B_{5,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{6,8}(X) \\ &\quad - 6.998745276936609 \cdot 10^{-20} B_{7,8}(X) - 6.998745276936609 \cdot 10^{-20} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -4.610588999829434{\cdot}10^{-137}X^3 - 5.505977817140947{\cdot}10^{-90}X^2 \\ &\quad + 6.216603591694483{\cdot}10^{-51}X - 6.998745276936609{\cdot}10^{-20} \\ &= -6.998745276936609{\cdot}10^{-20}B_{0,3} - 6.998745276936609{\cdot}10^{-20}B_{1,3} \\ &\quad - 6.998745276936609{\cdot}10^{-20}B_{2,3} - 6.998745276936609{\cdot}10^{-20}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= -3.734501813493143{\cdot}10^{-323}X^8 + 1.603059676081584{\cdot}10^{-322}X^7 - 4.947893748146468{\cdot}10^{-324}X^6 \\ &\quad - 5.631817628009717{\cdot}10^{-322}X^5 + 7.073546053770825{\cdot}10^{-322}X^4 - 4.610588999829434{\cdot}10^{-137}X^3 \\ &\quad - 5.505977817140947{\cdot}10^{-90}X^2 + 6.216603591694483{\cdot}10^{-51}X - 6.998745276936609{\cdot}10^{-20} \\ &= -6.998745276936609{\cdot}10^{-20}B_{0,8} - 6.998745276936609{\cdot}10^{-20}B_{1,8} - 6.998745276936609{\cdot}10^{-20}B_{2,8} \\ &\quad - 6.998745276936609{\cdot}10^{-20}B_{3,8} - 6.998745276936609{\cdot}10^{-20}B_{4,8} - 6.998745276936609{\cdot}10^{-20}B_{5,8} \\ &\quad - 6.998745276936609{\cdot}10^{-20}B_{6,8} - 6.998745276936609{\cdot}10^{-20}B_{7,8} - 6.998745276936609{\cdot}10^{-20}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.43083336374299{\cdot}10^{-186}$.
Bounding polynomials M and m:

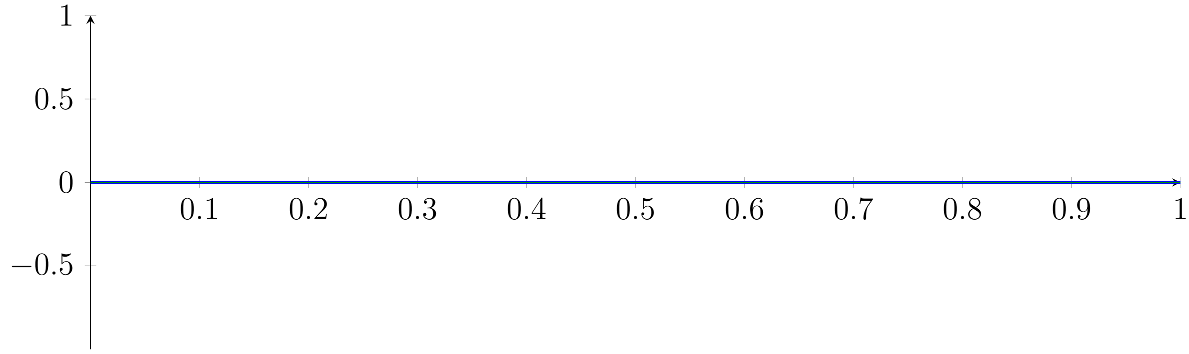
$$\begin{aligned} M &= -4.610588999829434{\cdot}10^{-137}X^3 - 5.505977817140947{\cdot}10^{-90}X^2 \\ &\quad + 6.216603591694483{\cdot}10^{-51}X - 6.998745276936609{\cdot}10^{-20} \end{aligned}$$

$$\begin{aligned} m &= -4.610588999829434{\cdot}10^{-137}X^3 - 5.505977817140947{\cdot}10^{-90}X^2 \\ &\quad + 6.216603591694483{\cdot}10^{-51}X - 6.998745276936609{\cdot}10^{-20} \end{aligned}$$

Root of M and m:

$N(M) = \{-1.19420270802736{\cdot}10^{47}, 1.07241027244706{\cdot}10^{17}, 1.129064387360764{\cdot}10^{39}\}$ $\quad N(m) = \{-1.1942027$

Intersection intervals:

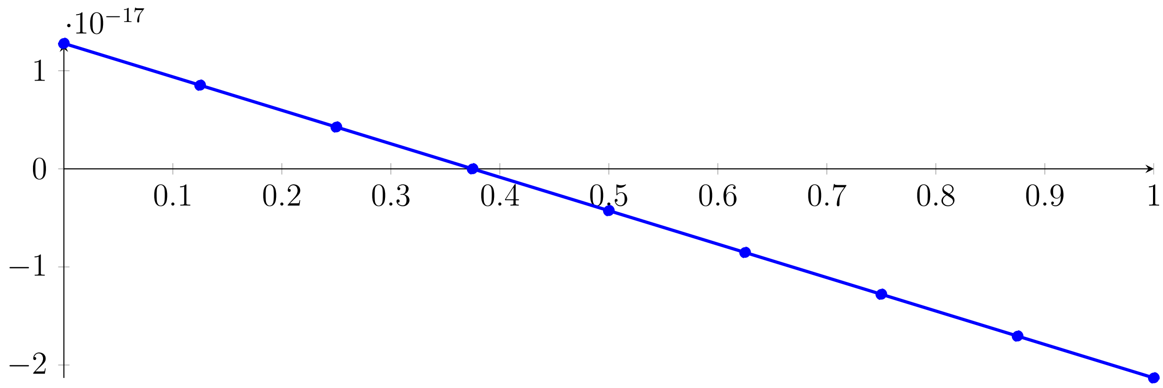


No intersection intervals with the x axis.

63.7 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.5000000299999818, 0.500000030000$

Normalized monomial und Bézier representations and the Bézier polygon:

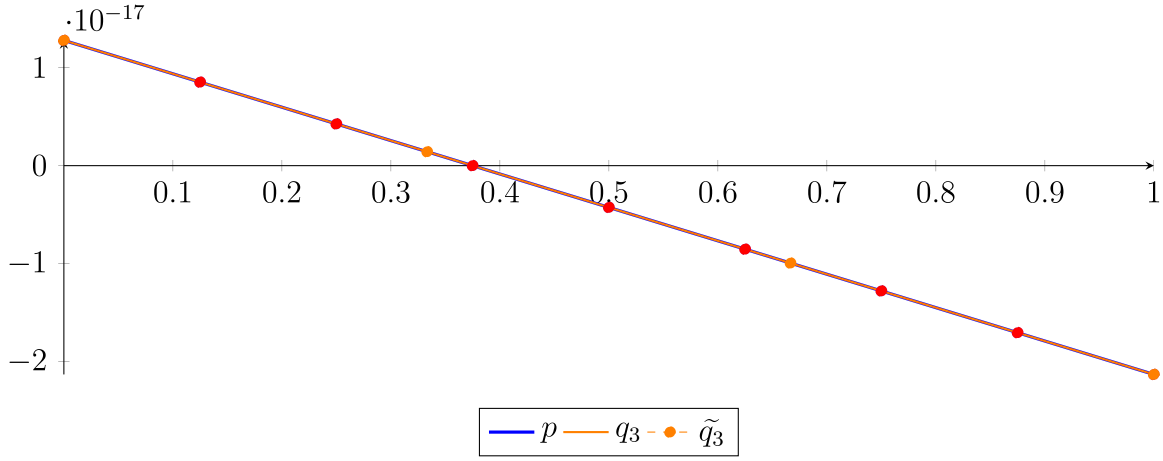
$$\begin{aligned}p &= -3.108117561752323 \cdot 10^{-107} X^8 - 2.494593961411665 \cdot 10^{-92} X^7 - 8.302910210921025 \cdot 10^{-78} X^6 \\ &\quad - 1.466794466465724 \cdot 10^{-63} X^5 - 1.450540769370867 \cdot 10^{-49} X^4 - 7.613757909646128 \cdot 10^{-36} X^3 \\ &\quad - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17} \\ &= 1.27898932402398 \cdot 10^{-17} B_{0,8}(X) + 8.526626725463981 \cdot 10^{-18} B_{1,8}(X) + 4.263354291826315 \\ &\quad \cdot 10^{-18} B_{2,8}(X) + 7.593932680254395 \cdot 10^{-23} B_{3,8}(X) - 4.263208332034555 \cdot 10^{-18} B_{4,8}(X) \\ &\quad - 8.526498522257758 \cdot 10^{-18} B_{5,8}(X) - 1.278979463134281 \cdot 10^{-17} B_{6,8}(X) \\ &\quad - 1.70530966592897 \cdot 10^{-17} B_{7,8}(X) - 2.131640460609844 \cdot 10^{-17} B_{8,8}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 \\ &\quad - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17} \\ &= 1.27898932402398 \cdot 10^{-17} B_{0,3} + 1.421182534170946 \cdot 10^{-18} B_{1,3} \\ &\quad - 9.947583414608468 \cdot 10^{-18} B_{2,3} - 2.131640460609844 \cdot 10^{-17} B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -6.944710234356022 \cdot 10^{-322} X^8 - 5.839855933843534 \cdot 10^{-321} X^7 + 4.132859127462027 \cdot 10^{-320} X^6 \\ &\quad - 7.976936884929846 \cdot 10^{-320} X^5 + 6.21874252599804 \cdot 10^{-320} X^4 - 7.613757909646418 \cdot 10^{-36} X^3 \\ &\quad - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17} \\ &= 1.27898932402398 \cdot 10^{-17} B_{0,8} + 8.526626725463981 \cdot 10^{-18} B_{1,8} + 4.263354291826315 \cdot 10^{-18} B_{2,8} \\ &\quad + 7.593932680254395 \cdot 10^{-23} B_{3,8} - 4.263208332034555 \cdot 10^{-18} B_{4,8} - 8.526498522257758 \cdot 10^{-18} B_{5,8} \\ &\quad - 1.278979463134281 \cdot 10^{-17} B_{6,8} - 1.70530966592897 \cdot 10^{-17} B_{7,8} - 2.131640460609844 \cdot 10^{-17} B_{8,8}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.108301648651952 \cdot 10^{-51}$.
Bounding polynomials M and m:

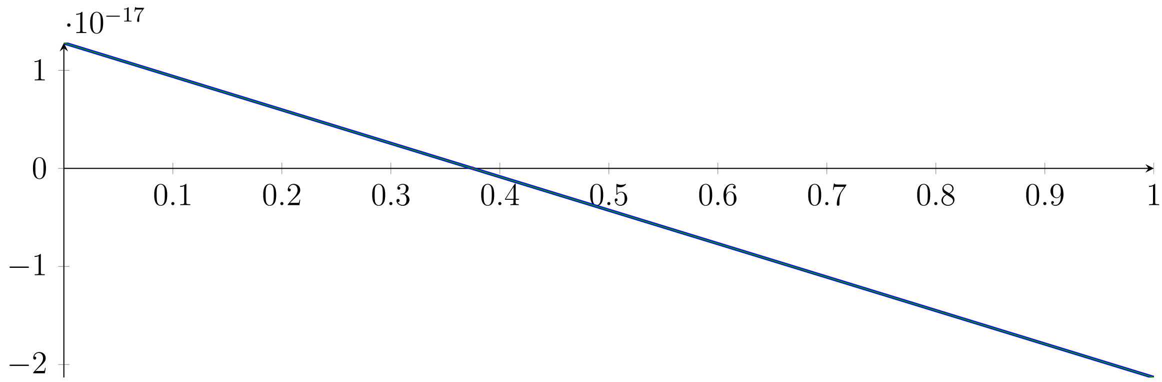
$$M = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

$$m = -7.613757909646418 \cdot 10^{-36} X^3 - 1.657281316735189 \cdot 10^{-22} X^2 - 3.410613211820657 \cdot 10^{-17} X + 1.27898932402398 \cdot 10^{-17}$$

Root of M and m:

$N(M) = \{-21766929297379.88, -205796.0503430093, 0.375002063855762\}$ $N(m) = \{-21766929297379.88,$

Intersection intervals:



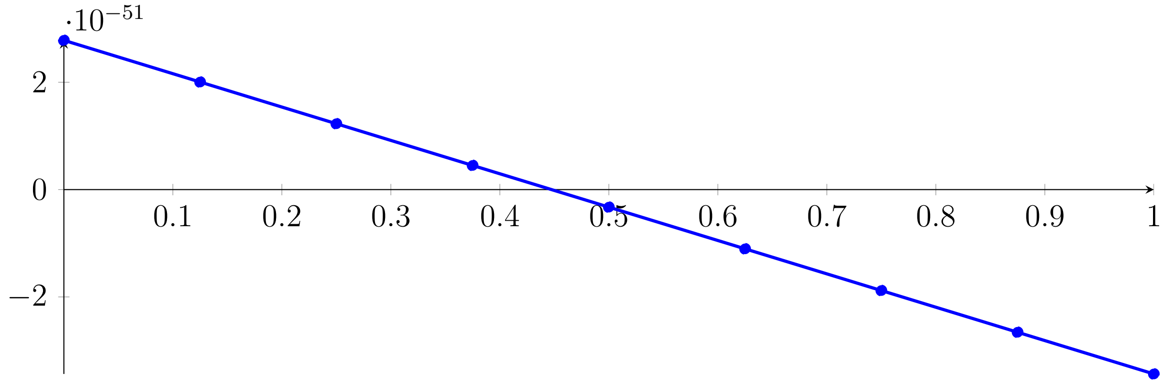
$$[0.375002063855762, 0.375002063855762]$$

Longest intersection interval: $1.822716401842262 \cdot 10^{-34}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

63.8 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.50000003, 0.50000003]$

Normalized monomial und Bézier representations and the Bézier polygon:

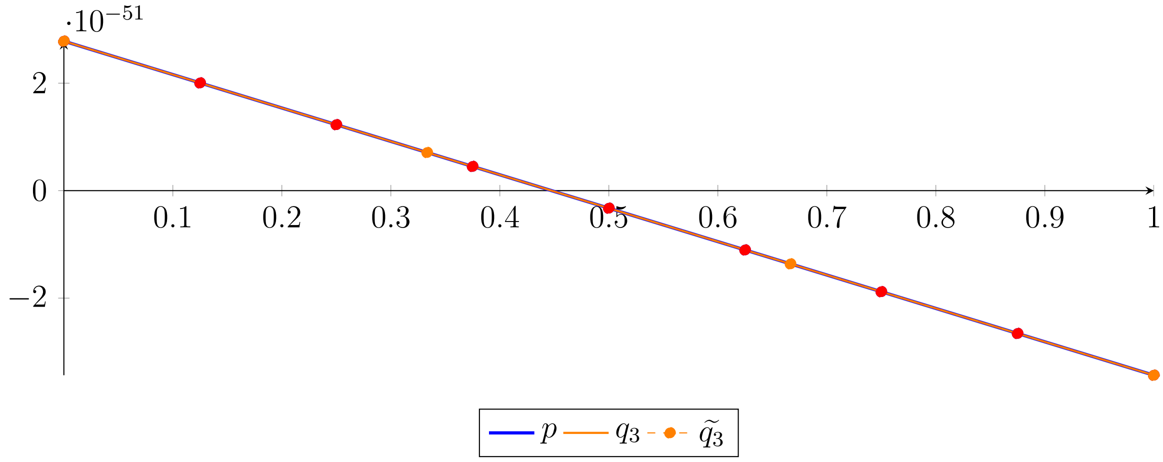
$$\begin{aligned} p &= 4.757679109093379 \cdot 10^{-358} X^8 - 1.667376123134876 \cdot 10^{-328} X^7 - 3.044703112291227 \cdot 10^{-280} X^6 \\ &\quad - 2.95096984102574 \cdot 10^{-232} X^5 - 1.601055350297363 \cdot 10^{-184} X^4 - 4.610588562192646 \cdot 10^{-137} X^3 \\ &\quad - 5.505977567325696 \cdot 10^{-90} X^2 - 6.216603297303905 \cdot 10^{-51} X + 2.781471077280957 \cdot 10^{-51} \\ &= 2.781471077280957 \cdot 10^{-51} B_{0,8}(X) + 2.004395665117968 \cdot 10^{-51} B_{1,8}(X) + 1.22732025295498 \\ &\quad \cdot 10^{-51} B_{2,8}(X) + 4.502448407919922 \cdot 10^{-52} B_{3,8}(X) - 3.268305713709959 \cdot 10^{-52} B_{4,8}(X) \\ &\quad - 1.103905983533984 \cdot 10^{-51} B_{5,8}(X) - 1.880981395696972 \cdot 10^{-51} B_{6,8}(X) \\ &\quad - 2.65805680785996 \cdot 10^{-51} B_{7,8}(X) - 3.435132220022948 \cdot 10^{-51} B_{8,8}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -4.610588562192646\cdot 10^{-137}X^3 - 5.505977567325696\cdot 10^{-90}X^2 \\ &\quad - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51} \\ &= 2.781471077280957\cdot 10^{-51}B_{0,3} + 7.092699781796549\cdot 10^{-52}B_{1,3} \\ &\quad - 1.362931120921647\cdot 10^{-51}B_{2,3} - 3.435132220022948\cdot 10^{-51}B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q_3} &= 1.121622849968764\cdot 10^{-355}X^8 - 2.091475736357449\cdot 10^{-354}X^7 + 7.553291353596648\cdot 10^{-354}X^6 \\ &\quad - 1.090697935759657\cdot 10^{-353}X^5 + 6.781476860123975\cdot 10^{-354}X^4 - 4.610588562192646\cdot 10^{-137}X^3 \\ &\quad - 5.505977567325696\cdot 10^{-90}X^2 - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51} \\ &= 2.781471077280957\cdot 10^{-51}B_{0,8} + 2.004395665117968\cdot 10^{-51}B_{1,8} + 1.22732025295498\cdot 10^{-51}B_{2,8} \\ &\quad + 4.502448407919922\cdot 10^{-52}B_{3,8} - 3.268305713709959\cdot 10^{-52}B_{4,8} - 1.103905983533984\cdot 10^{-51}B_{5,8} \\ &\quad - 1.880981395696972\cdot 10^{-51}B_{6,8} - 2.65805680785996\cdot 10^{-51}B_{7,8} - 3.435132220022948\cdot 10^{-51}B_{8,8} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.430832893494348\cdot 10^{-186}$.
Bounding polynomials M and m:

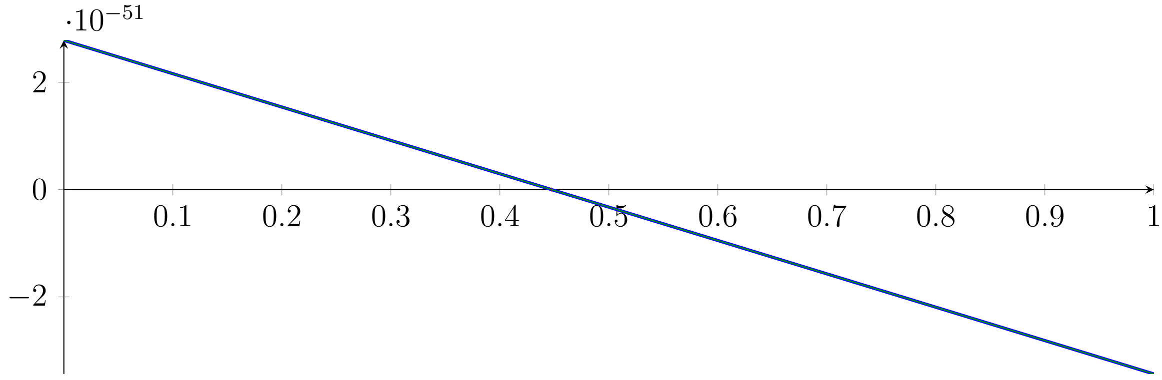
$$\begin{aligned} M &= -4.610588562192646\cdot 10^{-137}X^3 - 5.505977567325696\cdot 10^{-90}X^2 \\ &\quad - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51} \end{aligned}$$

$$\begin{aligned} m &= -4.610588562192646\cdot 10^{-137}X^3 - 5.505977567325696\cdot 10^{-90}X^2 \\ &\quad - 6.216603297303905\cdot 10^{-51}X + 2.781471077280957\cdot 10^{-51} \end{aligned}$$

Root of M and m:

$N(M) = \{-1.194202744616778\cdot 10^{47}, -1.129064428986703\cdot 10^{39}, 0.4474261818326513\}$ $N(m) = \{-1.194202$

Intersection intervals:



$$[0.4474261818326513, 0.4474261818326513]$$

Longest intersection interval: $1.103764460885666 \cdot 10^{-135}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.50000003, 0.50000003]$,

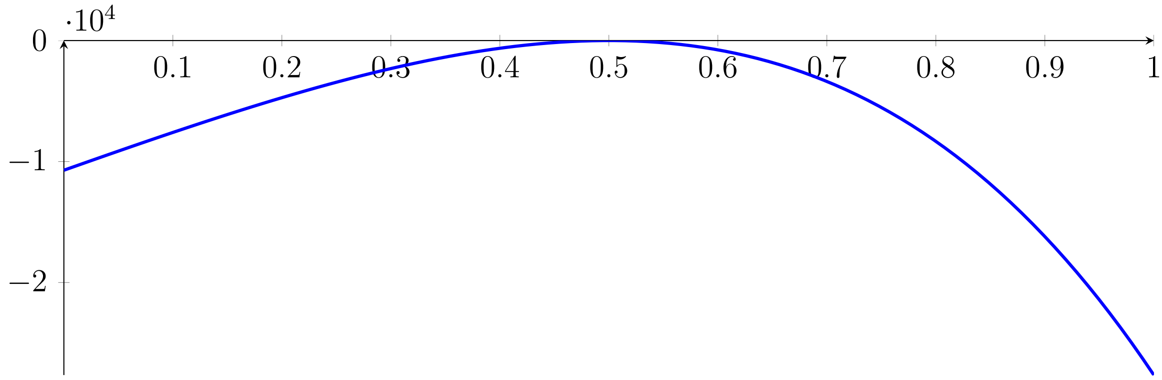
63.9 Recursion Branch 1 1 1 1 2 1 1 in Interval 1: $[0.50000003, 0.50000003]$

Found root in interval $[0.50000003, 0.50000003]$ at recursion depth 7!

63.10 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^8 - 34.99999995X^7 - 501.249998225X^6 - 3719.99997405X^5 - 14681.249801025X^4 - 26366.99916645X^3 - 3473.74826487501X^2 + 31850.00104124997X - 10718.75107187503$$



Result: Root Intervals

$$[0.50000003, 0.50000003]$$

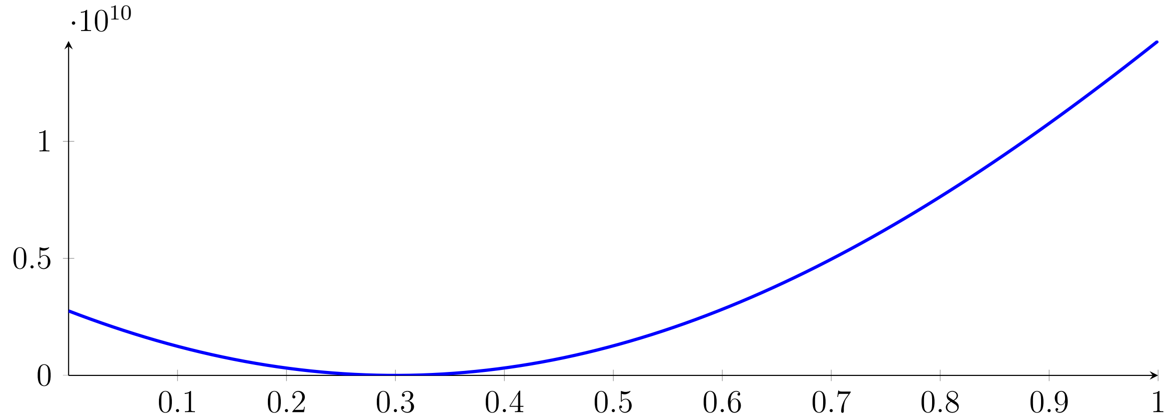
with precision $\varepsilon = 1 \cdot 10^{-128}$.

64 Running BezClip on h_{16} with epsilon 2

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called BezClip with input polynomial on interval $[0, 1]$:

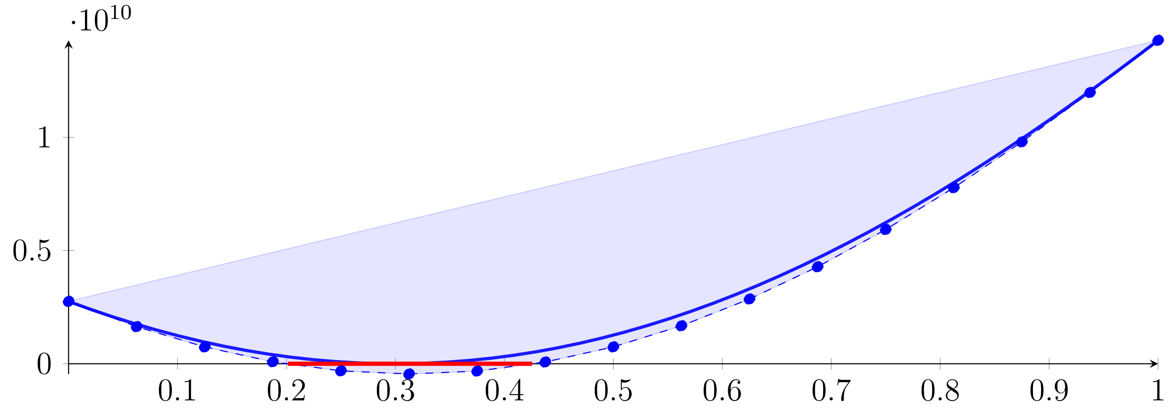
$$\begin{aligned} p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



64.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = & \, 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ & + 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ & - 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ & + 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ & + 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ & + 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

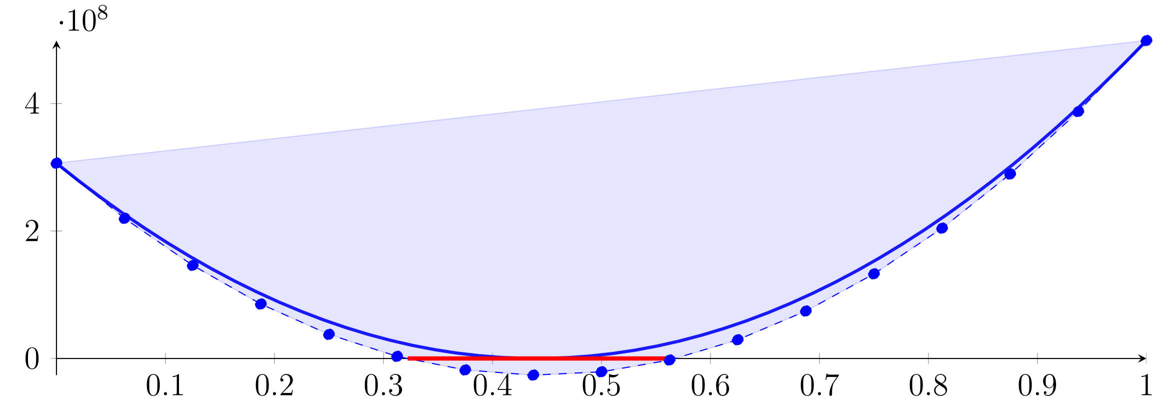
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

64.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = {} & -4.078702226246453\cdot 10^{-11} X^{16} + 4.329167405007062\cdot 10^{-10} X^{15} + 1.800827856148464\cdot 10^{-07} X^{14} \\
& - 1.7501300195896\cdot 10^{-06} X^{13} - 0.0003388881074603682 X^{12} + 0.002918727490860103 X^{11} \\
& + 0.353260574938002 X^{10} - 2.587751703352393 X^{9} - 220.7388291837263 X^{8} \\
& + 1298.537478880933 X^{7} + 82809.42227744751 X^{6} - 357003.6974521088 X^{5} - 17288824.7550166 X^{4} \\
& + 45617774.75376932 X^{3} + 1550181799.359372 X^{2} - 1385902556.54429 X + 306449533.9978484 \\
= {} & 306449533.9978484 B_{0,16}(X) + 219830624.2138303 B_{1,16}(X) + 146129896.0911402 B_{2,16}(X) \\
& + 85428809.9418386 B_{3,16}(X) + 37799326.72372467 B_{4,16}(X) + 3303826.308721026 B_{5,16}(X) \\
& - 18004963.90790522 B_{6,16}(X) - 26084029.94397575 B_{7,16}(X) - 20900121.61613177 B_{8,16}(X) \\
& - 2429792.29588269 B_{9,16}(X) + 29340572.02544693 B_{10,16}(X) + 74414734.40659503 B_{11,16}(X) \\
& + 132786599.3869978 B_{12,16}(X) + 204440216.186282 B_{13,16}(X) + 289349792.6628749 B_{14,16}(X) \\
& + 387479720.0042024 B_{15,16}(X) + 498784608.1032447 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

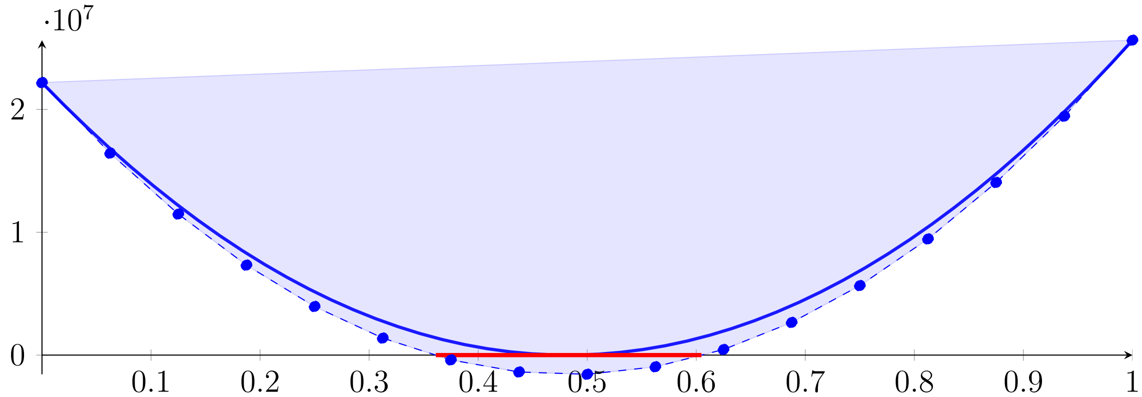
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: [0.2735013999129692, 0.328453288067671],

64.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -6.91388116995714 \cdot 10^{-21} X^{16} + 1.539971991909181 \cdot 10^{-19} X^{15} + 5.126557214158853 \\ & \cdot 10^{-16} X^{14} - 1.075268795442723 \cdot 10^{-14} X^{13} - 1.618466564965893 \cdot 10^{-11} X^{12} \\ & + 3.060191240732859 \cdot 10^{-10} X^{11} + 2.825398471644138 \cdot 10^{-07} X^{10} - 4.580412037193902 \\ & \cdot 10^{-06} X^{9} - 0.002949971804071592 X^{8} + 0.0383187988748973 X^{7} + 18.44286791446417 X^{6} \\ & - 172.012783850635 X^{5} - 63987.92092437637 X^{4} + 338934.074545568 X^{3} \\ & + 95113195.48898588 X^{2} - 91937923.73553449 X + 22182509.73328641 \\ = & \, 22182509.73328641 B_{0,16}(X) + 16436389.49981551 B_{1,16}(X) + 11482879.22875282 B_{2,16}(X) \\ & + 7322584.159517174 B_{3,16}(X) + 3956074.373329099 B_{4,16}(X) + 1383884.753830588 B_{5,16}(X) \\ & - 393485.0499920519 B_{6,16}(X) - 1375570.658578954 B_{7,16}(X) - 1561942.994257926 B_{8,16}(X) \\ & - 952208.311393262 B_{9,16}(X) + 453991.7757805191 B_{10,16}(X) + 2656980.267642443 B_{11,16}(X) \\ & + 5657044.755988558 B_{12,16}(X) + 9454437.403157084 B_{13,16}(X) + 14049374.92347699 B_{14,16}(X) \\ & + 19442038.56704145 B_{15,16}(X) + 25632574.10580758 B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

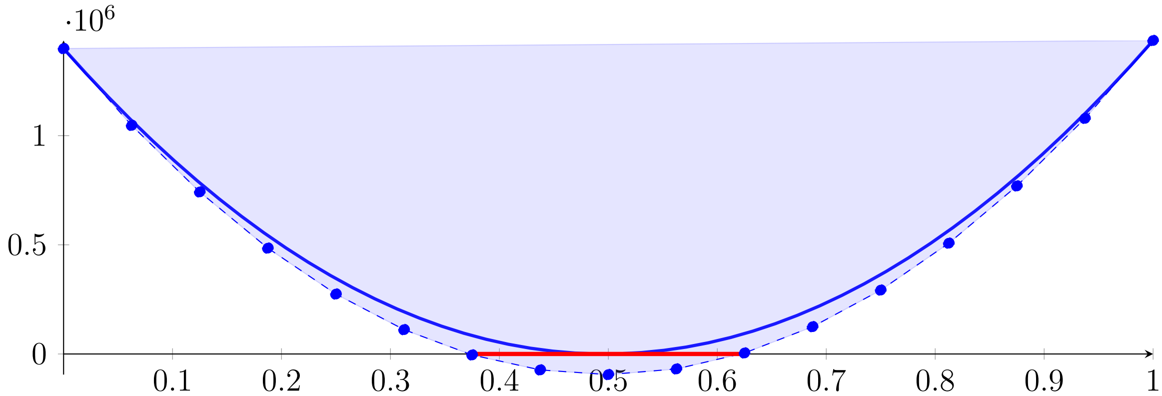
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: [0.2933480088018335, 0.3067375037518675],

64.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.067154088646128\cdot 10^{-30}X^{16} + 7.224340048814341\cdot 10^{-29}X^{15} + 1.334696477727887\\
&\cdot 10^{-24}X^{14} - 8.705175946298654\cdot 10^{-23}X^{13} - 7.106771480233297\cdot 10^{-19}X^{12}\\
&+ 4.237354105832321\cdot 10^{-17}X^{11} + 2.091927791167947\cdot 10^{-13}X^{10} - 1.077046671119229\\
&\cdot 10^{-11}X^{9} - 3.681404336048099\cdot 10^{-08}X^{8} + 1.51820465067154\cdot 10^{-06}X^{7}\\
&+ 0.003877397410843758X^{6} - 0.1133230374256462X^{5} - 226.5078434192974X^{4}\\
&+ 3562.753220436385X^{3} + 5665644.405911702X^{2} - 5632059.446508477X + 1399245.031427946\\
= &\ 1399245.031427946B_{0,16}(X) + 1047241.316021167B_{1,16}(X) + 742451.3039969843B_{2,16}(X)\\
&+ 484881.3574147218B_{3,16}(X) + 274537.7138788421B_{4,16}(X) + 111426.4865130056B_{5,16}(X)\\
&- 4446.336065390124B_{6,16}(X) - 73074.88977018565B_{7,16}(X) - 94453.43507095045B_{8,16}(X)\\
&- 68576.35701698946B_{9,16}(X) + 4561.834739135334B_{10,16}(X)\\
&+ 124966.505918568B_{11,16}(X) + 292642.897593607B_{12,16}(X) + 507596.1261656377B_{13,16}(X)\\
&+ 769831.1833435517B_{14,16}(X) + 1079352.93612265B_{15,16}(X) + 1436166.12676403B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

Longest intersection interval: 0.2484999837871829
$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

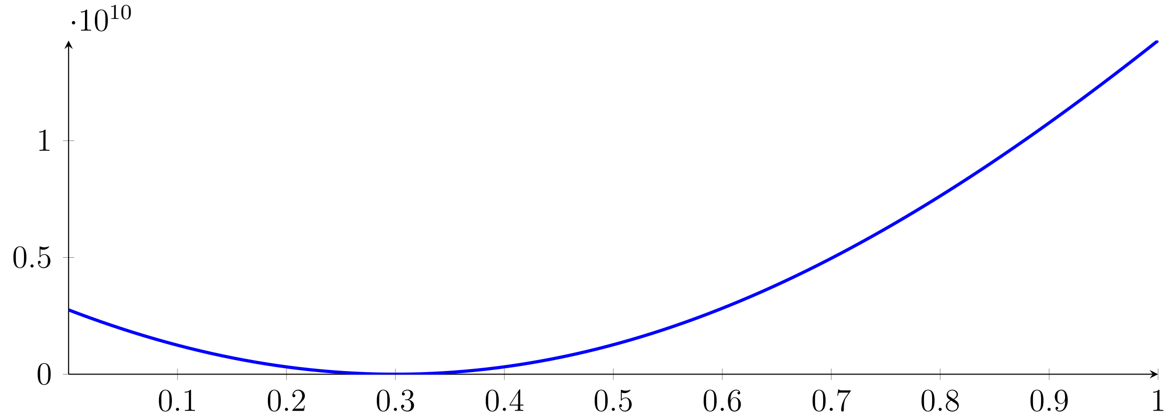
64.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Reached interval $[0.2983369575887709, 0.3016642468667729]$ **without sign change** at depth 5!

64.6 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

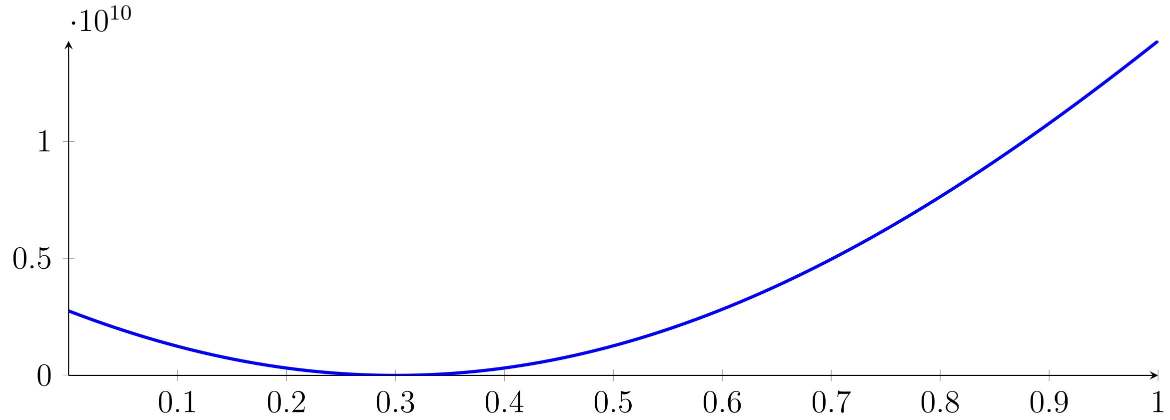
with precision $\varepsilon = 0.01$.

65 Running `QuadClip` on h_{16} with epsilon 2

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

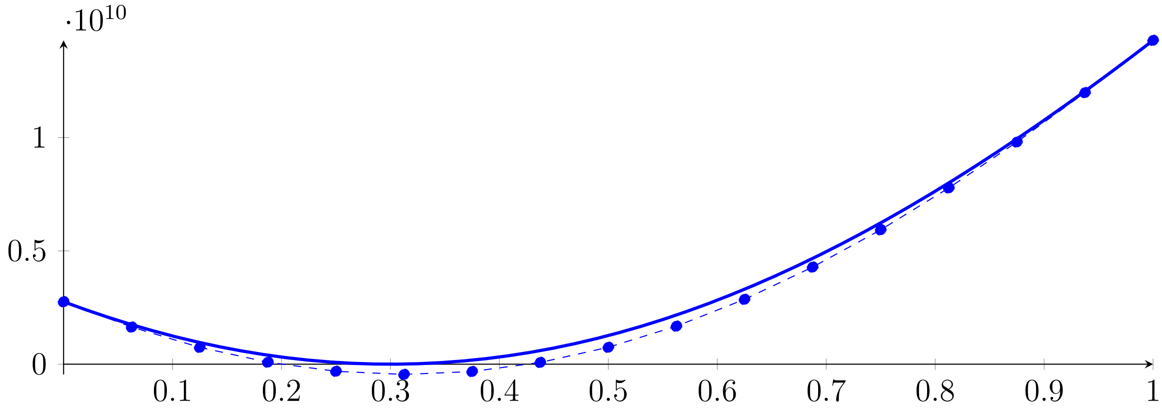
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



65.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

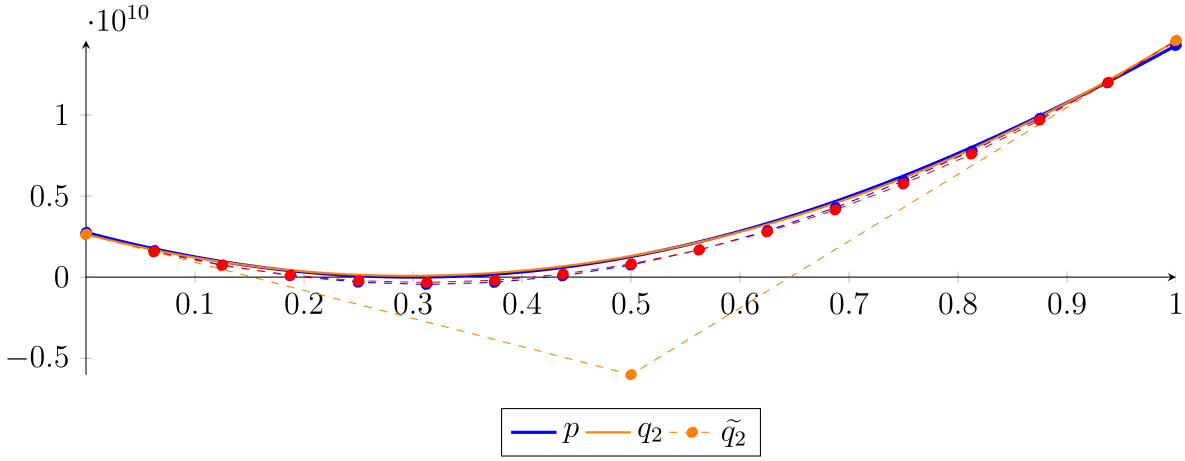
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328$$
$$= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2}$$

$$\widetilde{q}_2 = -4.590615853685512 \cdot 10^{-286}X^{16} + 3.626297739922361 \cdot 10^{-285}X^{15} - 1.337118562447102 \cdot 10^{-284}X^{14} + 3.057409950495903 \cdot 10^{-284}X^{13} - 4.816080313213711 \cdot 10^{-284}X^{12} + 5.445450819003583 \cdot 10^{-284}X^{11} - 4.453195910004646 \cdot 10^{-284}X^{10} + 2.601913123147853 \cdot 10^{-284}X^{9} - 1.060485581673177 \cdot 10^{-284}X^{8} + 2.936863540613122 \cdot 10^{-285}X^{7} - 5.487274374859613 \cdot 10^{-286}X^{6} + 7.23018920152705 \cdot 10^{-287}X^{5} - 7.254244946204871 \cdot 10^{-288}X^{4} + 6.659990111567495 \cdot 10^{-289}X^{3} + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328$$
$$= 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16} + 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16} - 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16} + 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16} + 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16} + 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.

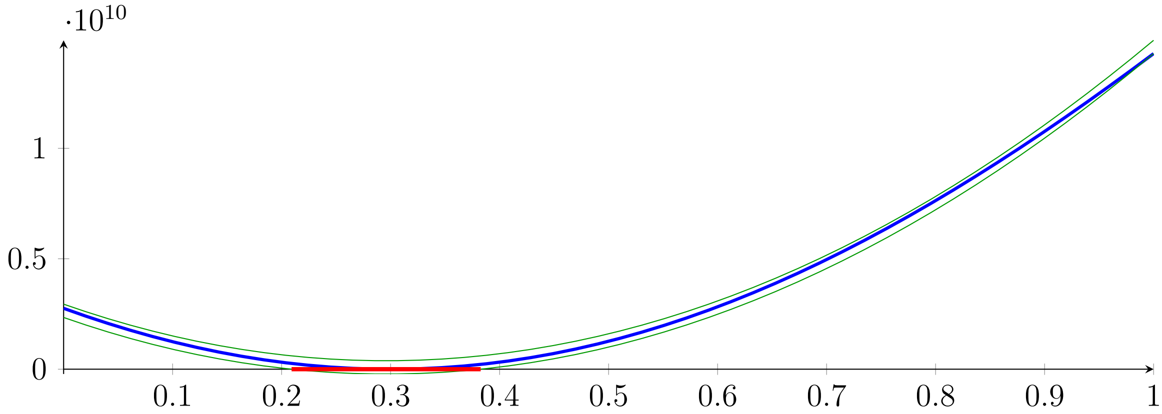
Bounding polynomials M and m:

$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$
$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



$$[0.2091580131065301, 0.3826391383239738]$$

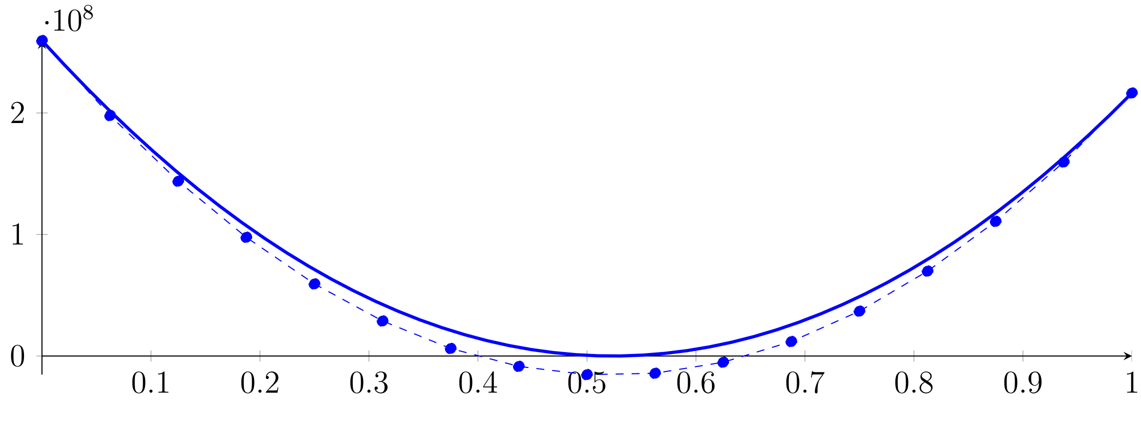
Longest intersection interval: 0.1734811252174437

$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

65.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

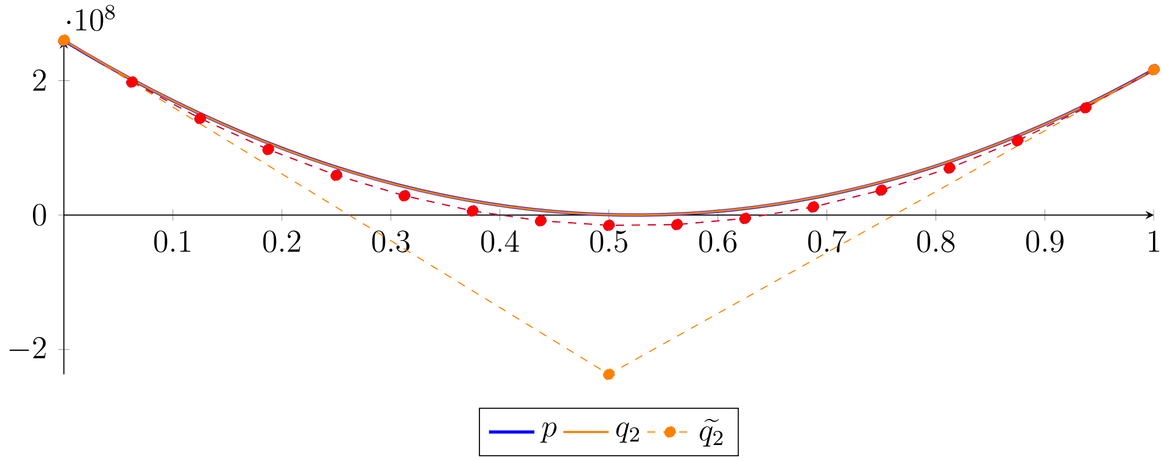
$$\begin{aligned}
p = &-6.730319568869656\cdot 10^{-13}X^{16} + 8.742499458235574\cdot 10^{-12}X^{15} + 4.969734366538428\cdot 10^{-09}X^{14} \\
&- 5.918022439742958\cdot 10^{-08}X^{13} - 1.563835677279211\cdot 10^{-05}X^{12} + 0.000165142833990623X^{11} \\
&+ 0.02725178204957372X^{10} - 0.2448183330655992X^{9} - 28.45820232077076X^{8} \\
&+ 205.241311077626X^{7} + 17835.39577074806X^{6} - 94141.21233496903X^{5} - 6218439.344794644X^{4} \\
&+ 20000843.19616221X^{3} + 930858984.7988209X^{2} - 987724621.0538478X + 259570777.0276934 \\
= &\ 259570777.0276934B_{0,16}(X) + 197837988.2118279B_{1,16}(X) + 143862357.6026193B_{2,16}(X) \\
&+ 97679600.99148916B_{3,16}(X) + 59322017.44494463B_{4,16}(X) + 28818467.75210257B_{5,16}(X) \\
&+ 6194355.099411763B_{6,16}(X) - 8528392.009487306B_{7,16}(X) - 15331334.57303514B_{8,16}(X) \\
&- 14199535.63666165B_{9,16}(X) - 5121570.552084079B_{10,16}(X) + 11910465.05798682B_{11,16}(X) \\
&+ 36900949.63643701B_{12,16}(X) + 69850732.33405701B_{13,16}(X) \\
&+ 110757131.9597003B_{14,16}(X) + 159613938.238137B_{15,16}(X) + 216411415.3731615B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= 950064117.9944898X^2 - 993959420.5216784X + 260029868.8092426 \\
&= 260029868.8092426B_{0,2} - 236949841.4515966B_{1,2} + 216134566.282054B_{2,2}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-1.41520176718356 \cdot 10^{-287} X^{16} + 1.139753408092347 \cdot 10^{-286} X^{15} - 4.276104932493763 \\ &\cdot 10^{-286} X^{14} + 9.896028733517596 \cdot 10^{-286} X^{13} - 1.566233430583374 \cdot 10^{-285} X^{12} \\ &+ 1.767044733566612 \cdot 10^{-285} X^{11} - 1.435814571471649 \cdot 10^{-285} X^{10} + 8.336078570237132 \\ &\cdot 10^{-286} X^{9} - 3.396584250429756 \cdot 10^{-286} X^{8} + 9.548198482934522 \cdot 10^{-287} X^{7} - 1.861944039074976 \\ &\cdot 10^{-287} X^{6} + 2.622613520689986 \cdot 10^{-288} X^{5} - 2.667693132942884 \cdot 10^{-289} X^{4} + 2.048626792988895 \\ &\cdot 10^{-290} X^{3} + 950064117.9944898 X^{2} - 993959420.5216784 X + 260029868.8092426 \\ = &\ 260029868.8092426 B_{0,16} + 197907405.0266377 B_{1,16} + 143702142.2273202 B_{2,16} \\ &+ 97414080.41129016 B_{3,16} + 59043219.5785475 B_{4,16} + 28589559.72909226 B_{5,16} \\ &+ 6053100.862924435 B_{6,16} - 8566157.019955976 B_{7,16} - 15268213.91954897 B_{8,16} \\ &- 14053069.83585455 B_{9,16} - 4920724.768872719 B_{10,16} + 12128821.28139653 B_{11,16} \\ &+ 37095568.31495319 B_{12,16} + 69979516.33179727 B_{13,16} \\ &+ 110780665.3319288 B_{14,16} + 159499015.3153477 B_{15,16} + 216134566.282054 B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

Bounding polynomials M and m:

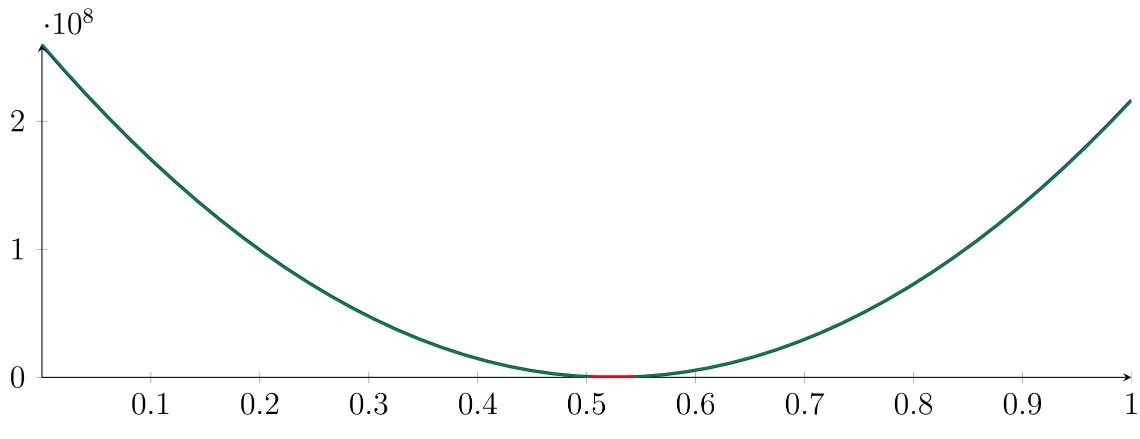
$$M = 950064117.9944898 X^2 - 993959420.5216784 X + 260488960.5907918$$

$$m = 950064117.9944898 X^2 - 993959420.5216784 X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

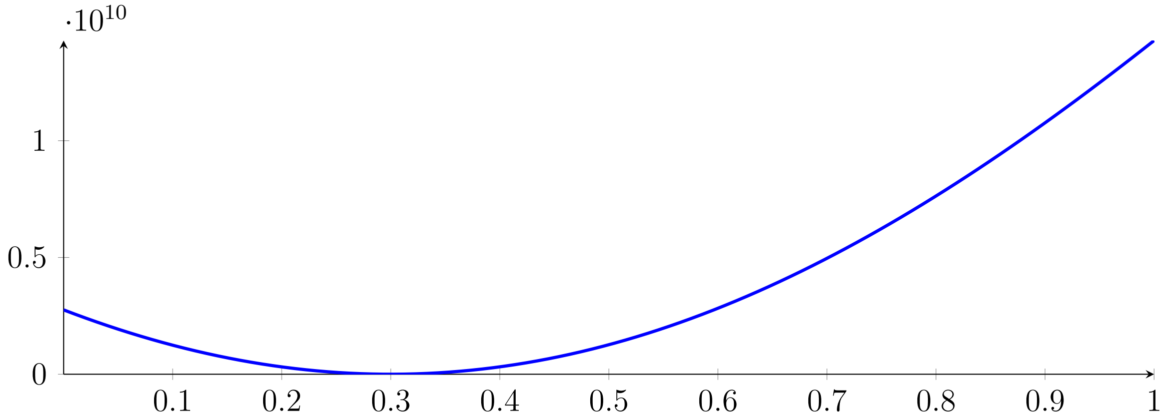
65.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Reached interval $[0.296346915178038, 0.3034654915704453]$ **without sign change** at depth 3!
p(0) = 422061.2284789393 - p(1) 379901.4776294016

65.4 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

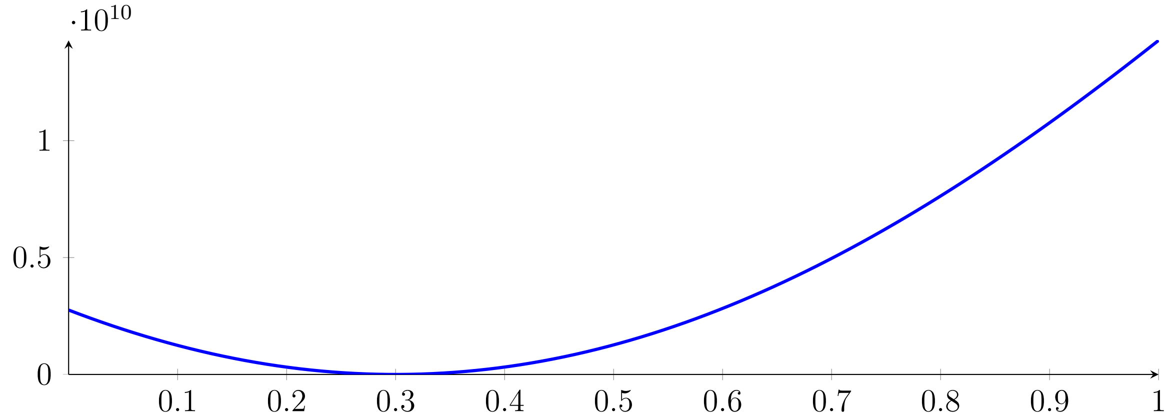
with precision $\varepsilon = 0.01$.

66 Running `CubeClip` on h_{16} with epsilon 2

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

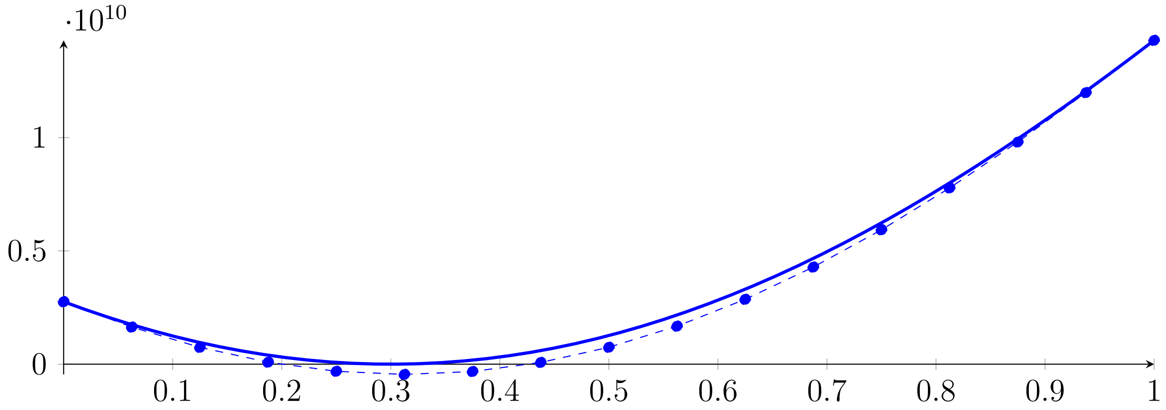
$$\begin{aligned} p = {} & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



66.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

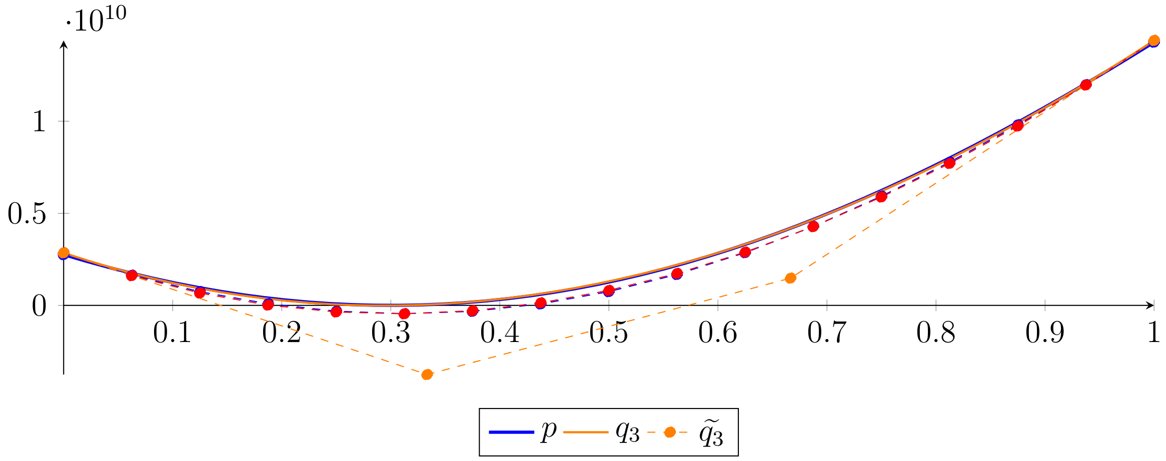
$$\begin{aligned} p = {} & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = {} & 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ & + 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ & - 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ & + 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ & + 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ & + 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353\\ &= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3}\\ &\quad + 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756\\ &\quad \cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12}\\ &\quad - 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948\\ &\quad \cdot 10^{-285}X^{9} + 6.984121997801746\cdot 10^{-286}X^{8} - 2.400829205648148\cdot 10^{-286}X^{7}\\ &\quad + 9.243609934541251\cdot 10^{-287}X^{6} - 2.7161427533585\cdot 10^{-287}X^{5} + 4.31464378661806\cdot 10^{-288}X^{4}\\ &\quad - 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353\\ &= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16}\\ &\quad + 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16}\\ &\quad - 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16}\\ &\quad + 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16}\\ &\quad + 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16}\\ &\quad + 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

Bounding polynomials M and m:

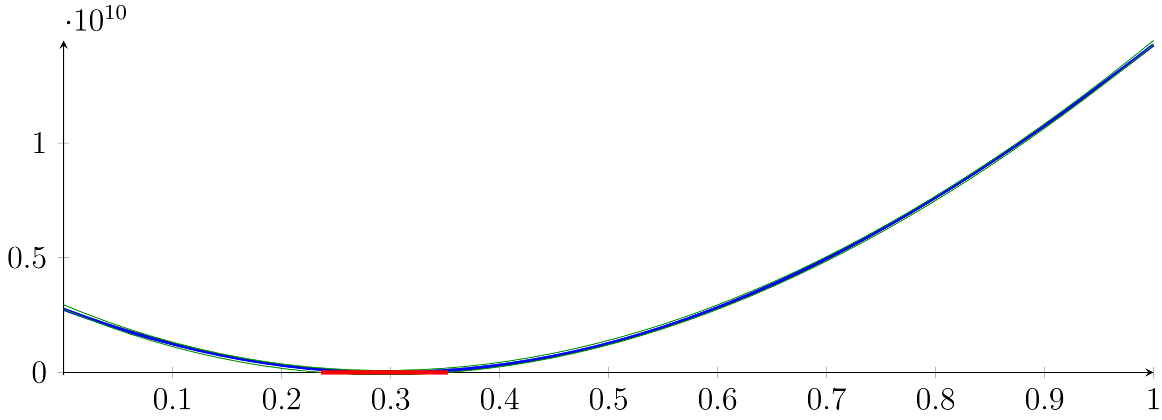
$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$

$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



$[0.2362027155340351, 0.3527550629571673]$

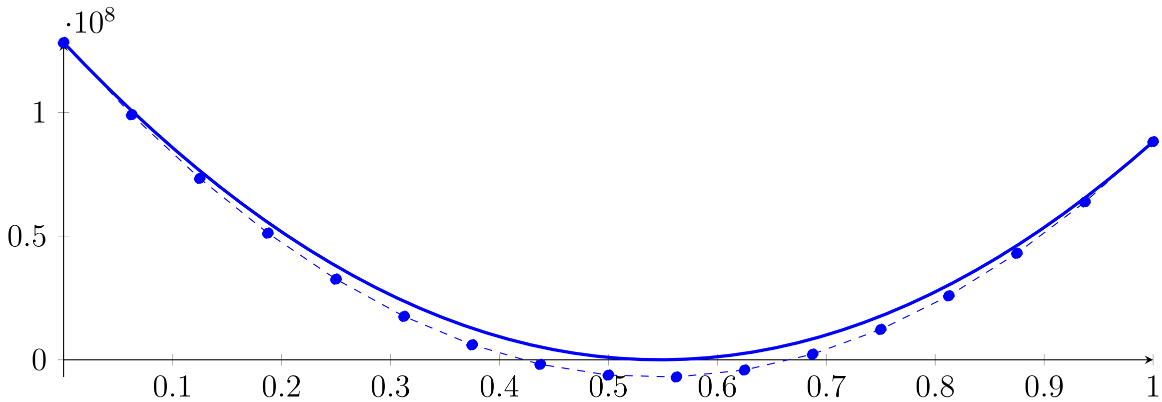
Longest intersection interval: 0.1165523474231321

$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

66.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

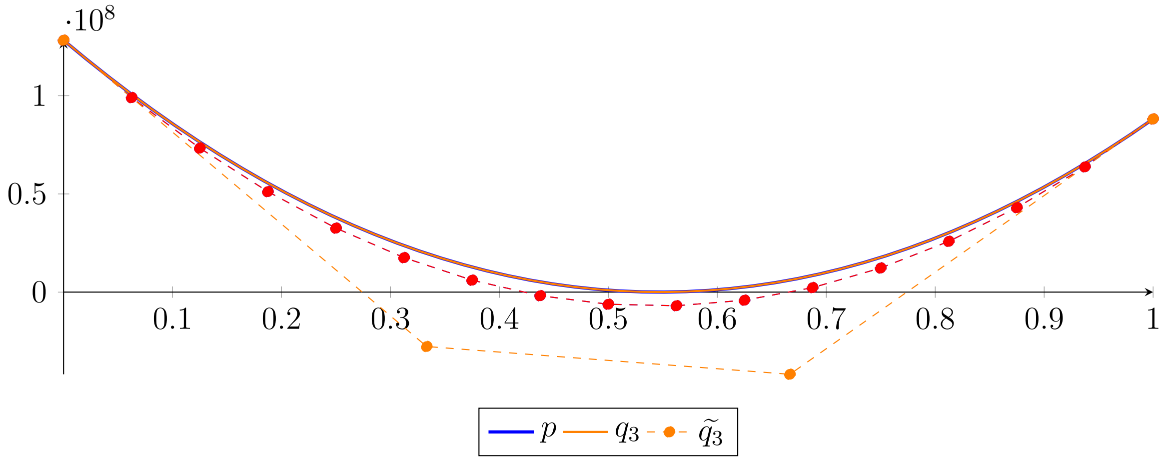
$$\begin{aligned} p = &-1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14} \\ &- 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321 \\ &\cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8} \\ &+ 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4} \\ &+ 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316 \\ = &\ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X) \\ &+ 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X) \\ &+ 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X) \\ &- 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X) \\ &+ 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X) \\ &+ 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128303546.2426166 \\ &= 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3} \\ &\quad - 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3} \end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= 1.029539679280458\cdot 10^{-288}X^{16} - 8.072523277872609\cdot 10^{-288}X^{15} + 2.888405037194055\\
&\quad\cdot 10^{-287}X^{14} - 6.260623433882179\cdot 10^{-287}X^{13} + 9.183807049175627\cdot 10^{-287}X^{12}\\
&\quad - 9.610199535206024\cdot 10^{-287}X^{11} + 7.321003139036714\cdot 10^{-287}X^{10} - 4.031610440228076\\
&\quad\cdot 10^{-287}X^{9} + 1.537754605851679\cdot 10^{-287}X^{8} - 3.606689105481158\cdot 10^{-288}X^{7}\\
&\quad + 3.223581314526531\cdot 10^{-289}X^{6} + 5.410058694892297\cdot 10^{-290}X^{5} - 1.24173721337985\cdot 10^{-290}X^{4}\\
&\quad + 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166\\
&= 128303546.2426166B_{0,16} + 99049684.91666918B_{1,16} + 73342858.56410606B_{2,16}\\
&\quad + 51187170.59978822B_{3,16} + 32586724.4385766B_{4,16} + 17545623.49533216B_{5,16}\\
&\quad + 6067971.184915845B_{6,16} - 1842129.077811386B_{7,16} - 6180573.877988583B_{8,16}\\
&\quad - 6943259.800754793B_{9,16} - 4126083.431249065B_{10,16} + 2275058.645389556B_{11,16}\\
&\quad + 12264269.84402202B_{12,16} + 25845653.57950928B_{13,16} + 43023313.26671229B_{14,16}\\
&\quad + 63801352.320492B_{15,16} + 88183874.15570936B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

Bounding polynomials M and m:

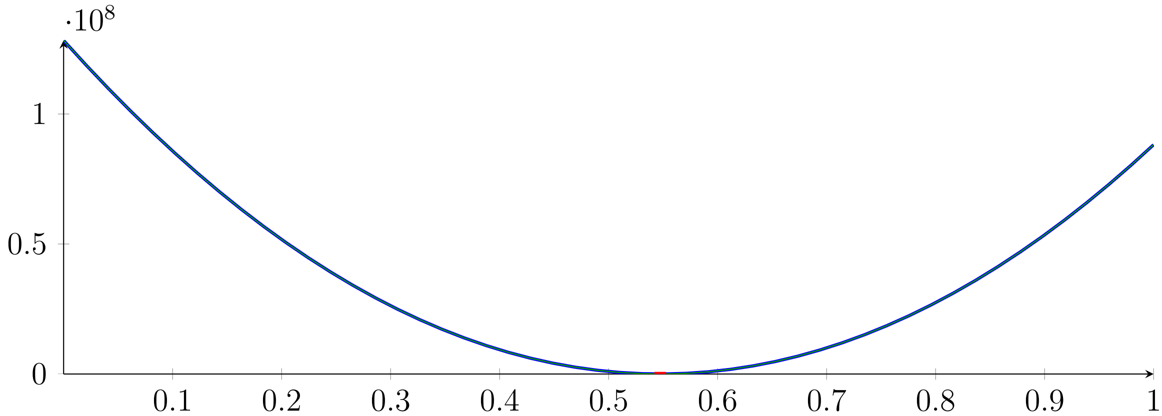
$$M = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128322140.7478178$$

$$m = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$[0.5420611331202266, 0.552740643902378]$

Longest intersection interval: 0.01067951078215144

$\Longrightarrow$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

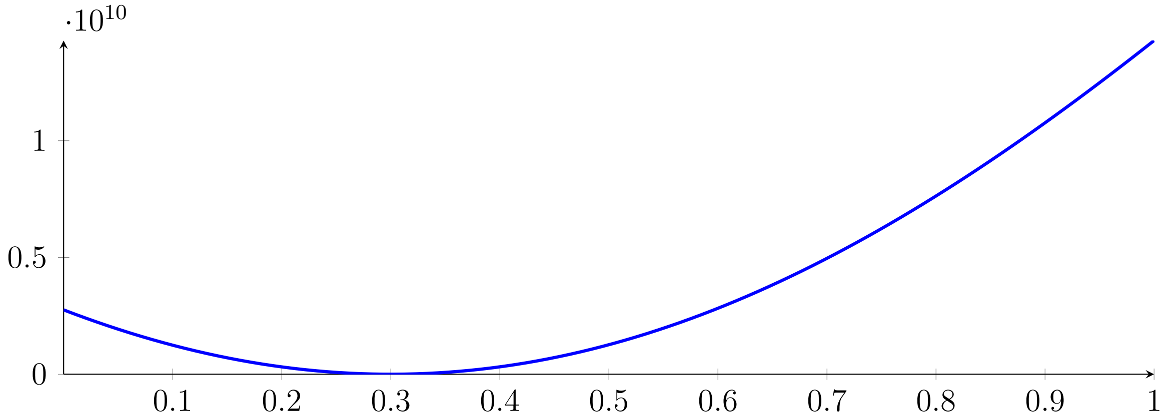
66.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.300625935097030$

Reached interval $[0.2993812130460404, 0.3006259350970308]$ **without sign change** at depth 3!
p(0) = 12114.14082112651 - p(1) 12389.50114837561

66.4 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$
\begin{aligned}
p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\
& + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\
& - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\
& - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\
& + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822
\end{aligned}
$$



Result: Root Intervals

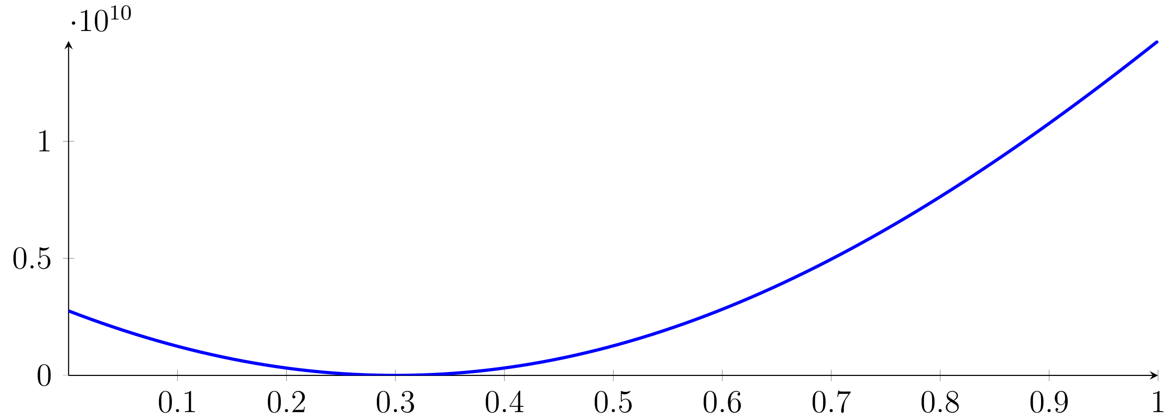
with precision $\varepsilon = 0.01$.

67 Running BezClip on h_{16} with epsilon 4

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called BezClip with input polynomial on interval $[0, 1]$:

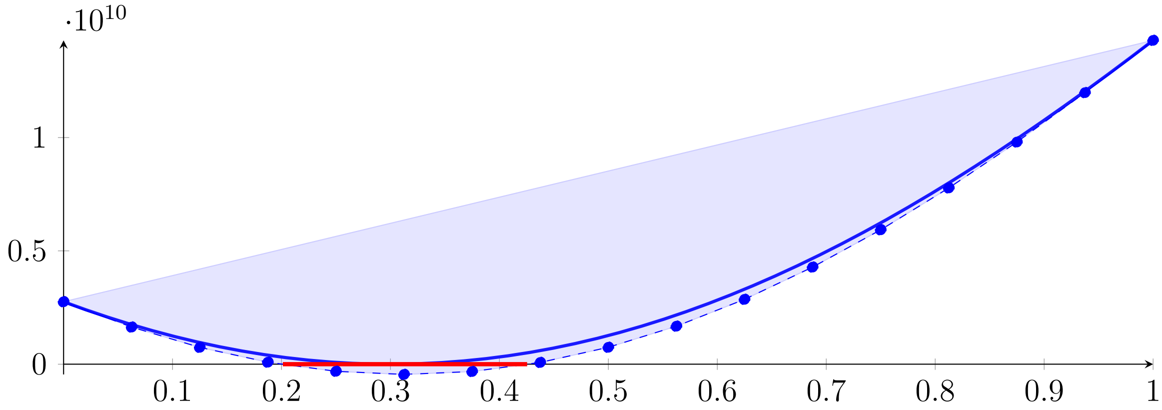
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



67.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X)\\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X)\\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X)\\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X)\\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X)\\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

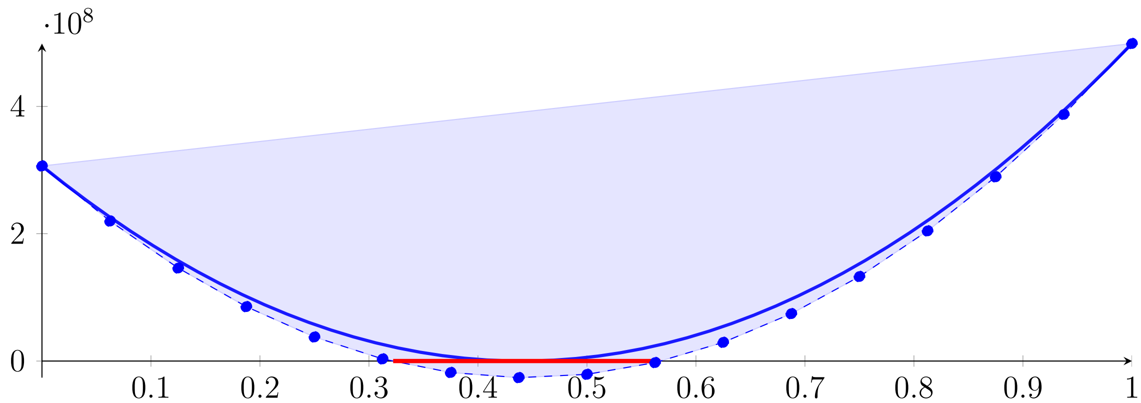
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

67.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -4.078702226246453 \cdot 10^{-11} X^{16} + 4.329167405007062 \cdot 10^{-10} X^{15} + 1.800827856148464 \cdot 10^{-07} X^{14} \\
& - 1.7501300195896 \cdot 10^{-06} X^{13} - 0.0003388881074603682 X^{12} + 0.002918727490860103 X^{11} \\
& + 0.353260574938002 X^{10} - 2.587751703352393 X^{9} - 220.7388291837263 X^{8} \\
& + 1298.537478880933 X^{7} + 82809.42227744751 X^{6} - 357003.6974521088 X^{5} - 17288824.7550166 X^{4} \\
& + 45617774.75376932 X^{3} + 1550181799.359372 X^{2} - 1385902556.54429 X + 306449533.9978484 \\
= & \, 306449533.9978484 B_{0,16}(X) + 219830624.2138303 B_{1,16}(X) + 146129896.0911402 B_{2,16}(X) \\
& + 85428809.9418386 B_{3,16}(X) + 37799326.72372467 B_{4,16}(X) + 3303826.308721026 B_{5,16}(X) \\
& - 18004963.90790522 B_{6,16}(X) - 26084029.94397575 B_{7,16}(X) - 20900121.61613177 B_{8,16}(X) \\
& - 2429792.29588269 B_{9,16}(X) + 29340572.02544693 B_{10,16}(X) + 74414734.40659503 B_{11,16}(X) \\
& + 132786599.3869978 B_{12,16}(X) + 204440216.186282 B_{13,16}(X) + 289349792.6628749 B_{14,16}(X) \\
& + 387479720.0042024 B_{15,16}(X) + 498784608.1032447 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

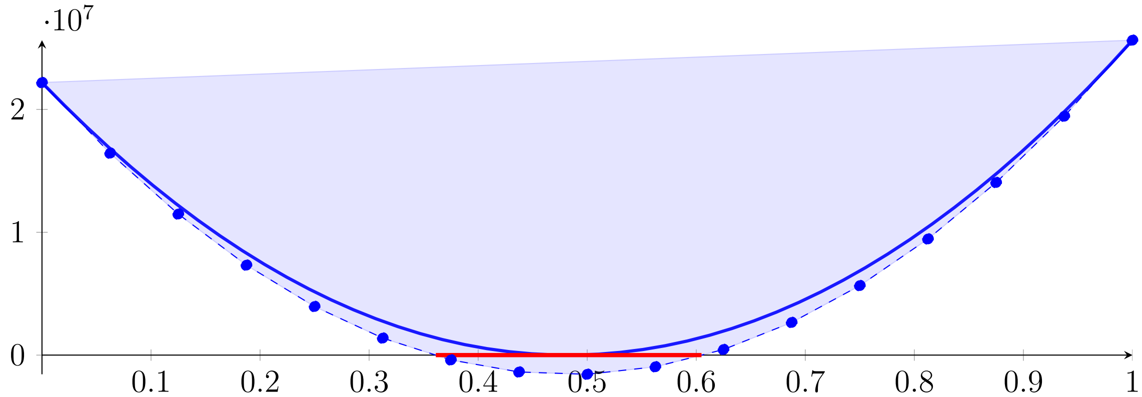
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: $[0.2735013999129692, 0.328453288067671]$,

67.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-6.913881169957 14\cdot 10^{-21}X^{16} + 1.539971991909181\cdot 10^{-19}X^{15} + 5.126557214158853\\ &\cdot 10^{-16}X^{14} - 1.075268795442723\cdot 10^{-14}X^{13} - 1.618466564965893\cdot 10^{-11}X^{12}\\ &+ 3.060191240732859\cdot 10^{-10}X^{11} + 2.825398471644138\cdot 10^{-07}X^{10} - 4.580412037193902\\ &\cdot 10^{-06}X^{9} - 0.002949971804071592X^{8} + 0.0383187988748973X^{7} + 18.44286791446417X^{6}\\ &- 172.012783850635X^{5} - 63987.92092437637X^{4} + 338934.074545568X^{3}\\ &+ 95113195.48898588X^{2} - 91937923.73553449X + 22182509.73328641\\ = &\ 22182509.73328641B_{0,16}(X) + 16436389.49981551B_{1,16}(X) + 11482879.22875282B_{2,16}(X)\\ &+ 7322584.159517174B_{3,16}(X) + 3956074.373329099B_{4,16}(X) + 1383884.753830588B_{5,16}(X)\\ &- 393485.0499920519B_{6,16}(X) - 1375570.658578954B_{7,16}(X) - 1561942.994257926B_{8,16}(X)\\ &- 952208.311393262B_{9,16}(X) + 453991.7757805191B_{10,16}(X) + 2656980.267642443B_{11,16}(X)\\ &+ 5657044.755988558B_{12,16}(X) + 9454437.403157084B_{13,16}(X) + 14049374.92347699B_{14,16}(X)\\ &+ 19442038.56704145B_{15,16}(X) + 25632574.10580758B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

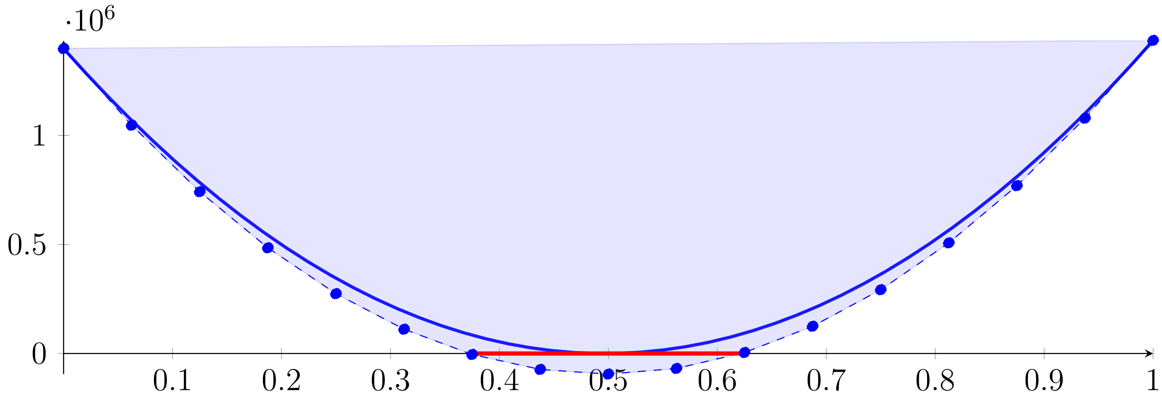
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: $[0.2933480088018335, 0.3067375037518675]$,

67.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.067154088646128\cdot10^{-30}X^{16} + 7.224340048814341\cdot10^{-29}X^{15} + 1.334696477727887\\ &\cdot10^{-24}X^{14} - 8.705175946298654\cdot10^{-23}X^{13} - 7.106771480233297\cdot10^{-19}X^{12}\\ &+ 4.237354105832321\cdot10^{-17}X^{11} + 2.091927791167947\cdot10^{-13}X^{10} - 1.077046671119229\\ &\cdot10^{-11}X^{9} - 3.681404336048099\cdot10^{-08}X^{8} + 1.51820465067154\cdot10^{-06}X^{7}\\ &+ 0.003877397410843758X^{6} - 0.1133230374256462X^{5} - 226.5078434192974X^{4}\\ &+ 3562.753220436385X^{3} + 5665644.405911702X^{2} - 5632059.446508477X + 1399245.031427946\\ = &\,1399245.031427946B_{0,16}(X) + 1047241.316021167B_{1,16}(X) + 742451.3039969843B_{2,16}(X)\\ &+ 484881.3574147218B_{3,16}(X) + 274537.7138788421B_{4,16}(X) + 111426.4865130056B_{5,16}(X)\\ &- 4446.336065390124B_{6,16}(X) - 73074.88977018565B_{7,16}(X) - 94453.43507095045B_{8,16}(X)\\ &- 68576.35701698946B_{9,16}(X) + 4561.834739135334B_{10,16}(X)\\ &+ 124966.505918568B_{11,16}(X) + 292642.897593607B_{12,16}(X) + 507596.1261656377B_{13,16}(X)\\ &+ 769831.1833435517B_{14,16}(X) + 1079352.93612265B_{15,16}(X) + 1436166.12676403B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

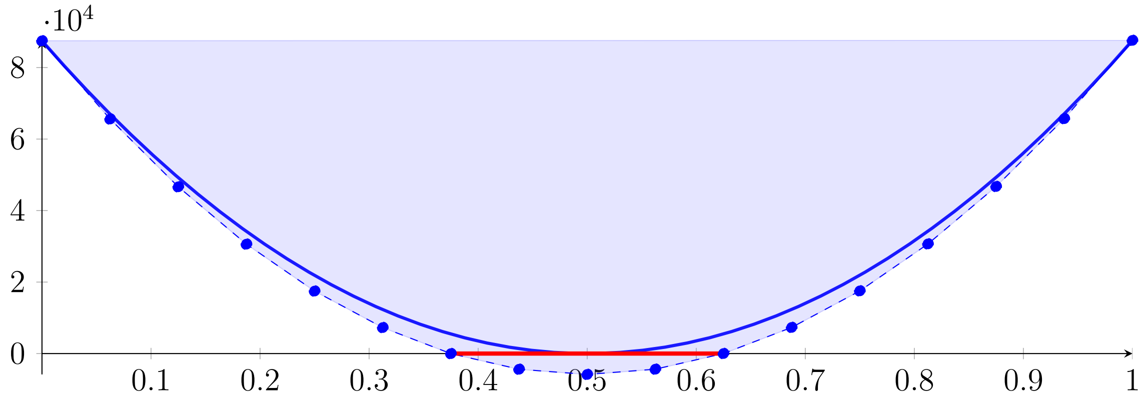
Longest intersection interval: 0.2484999837871829

$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

67.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.256570822524655\cdot10^{-40}X^{16} + 5.606069245850654\cdot10^{-38}X^{15} + 4.571694482208246\\ &\cdot10^{-33}X^{14} - 1.103601619609778\cdot10^{-30}X^{13} - 3.943084008767946\cdot10^{-26}X^{12}\\ &+ 8.746232232432483\cdot10^{-24}X^{11} + 1.879997320990413\cdot10^{-19}X^{10} - 3.610214442217249\\ &\cdot10^{-17}X^{9} - 5.35839719625663\cdot10^{-13}X^{8} + 8.241677187819251\cdot10^{-11}X^{7} + 9.139570434264648\\ &\cdot10^{-07}X^{6} - 9.91682351975725\cdot10^{-05}X^{5} - 0.864525557836322X^{4} + 49.4891850901473X^{3}\\ &+ 350100.5152669434X^{2} - 350028.3308017828X + 87482.91293301892\\ = &\,87482.91293301892B_{0,16}(X) + 65606.1422579075B_{1,16}(X) + 46646.87587668727B_{2,16}(X)\\ &+ 30605.20216290304B_{3,16}(X) + 17481.20901508557B_{4,16}(X) + 7274.983856728878B_{5,16}(X)\\ &- 13.38636373236149B_{6,16}(X) - 4383.815172945278B_{7,16}(X) - 5836.216572661172B_{8,16}(X)\\ &- 4370.505039757766B_{9,16}(X) + 13.40447373867094B_{10,16}(X) + 7315.596540631298B_{11,16}(X)\\ &+ 17536.1552585308B_{12,16}(X) + 30675.1642498336B_{13,16}(X) + 46732.70666170018B_{14,16}(X)\\ &+ 65708.86516603353B_{15,16}(X) + 87603.72195945767B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748852078437339, 0.6248088966923045\}$$

Intersection intervals with the x axis:

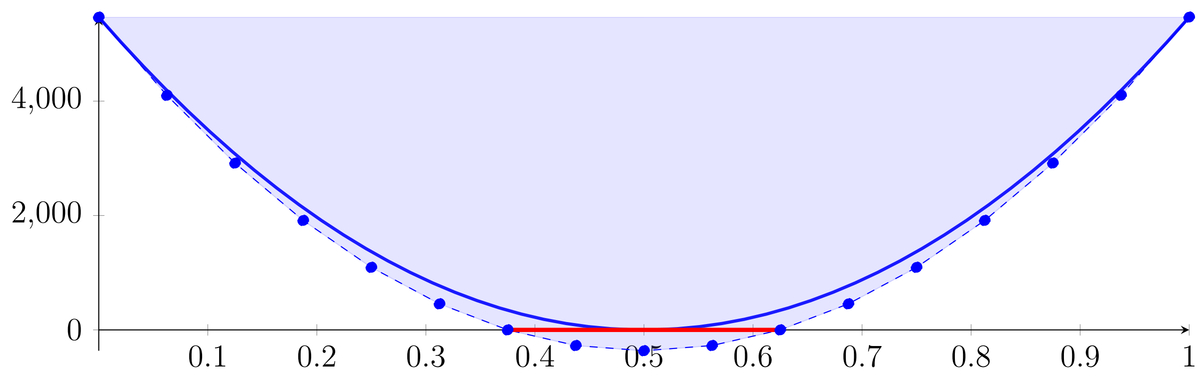
$$[0.3748852078437339, 0.6248088966923045]$$

Longest intersection interval: 0.2499236888485706
$\implies$ Selective recursion: interval 1: $[0.2995843091213109, 0.3004158775315354]$,

67.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.2995843091213109, 0.30041587753$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -5.228387005637929\cdot 10^{-50}X^{16} + 5.071723175262542\cdot 10^{-47}X^{15} + 1.695940836337518\\
& \cdot 10^{-41}X^{14} - 1.602367332110541\cdot 10^{-38}X^{13} - 2.341982733873566\cdot 10^{-33}X^{12}\\
& + 2.036120445711374\cdot 10^{-30}X^{11} + 1.787779362171969\cdot 10^{-25}X^{10} - 1.346594590666257\\
& \cdot 10^{-22}X^{9} - 8.158156485257833\cdot 10^{-18}X^{8} + 4.921695226295225\cdot 10^{-15}X^{7}\\
& + 2.227778844339431\cdot 10^{-10}X^{6} - 9.46914992513699\cdot 10^{-08}X^{5} - 0.003373649242827386X^{4}\\
& + 0.7523208927331248X^{3} + 21871.35693828986X^{2} - 21871.48147756637X + 5467.807676357308\\
= & \, 5467.807676357308B_{0,16}(X) + 4100.84008400941B_{1,16}(X) + 2916.133799480594B_{2,16}(X)\\
& + 1913.690166201026B_{3,16}(X) + 1093.510525747217B_{4,16}(X) + 455.5962178420057B_{5,16}(X)\\
& - 0.05141964546838 65B_{6,16}(X) - 273.4310506997833B_{7,16}(X) - 364.541341159257B_{8,16}(X)\\
& - 273.3809587159691B_{9,16}(X) + 0.05142708421757425B_{10,16}(X)\\
& + 455.7571448416359B_{11,16}(X) + 1093.737521302792B_{12,16}(X) + 1913.993881360346B_{13,16}(X)\\
& + 2916.527548053087B_{14,16}(X) + 4101.339842565916B_{15,16}(X) + 5468.43208422982B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749929469011197, 0.6249882450180355\}$$

Intersection intervals with the x axis:

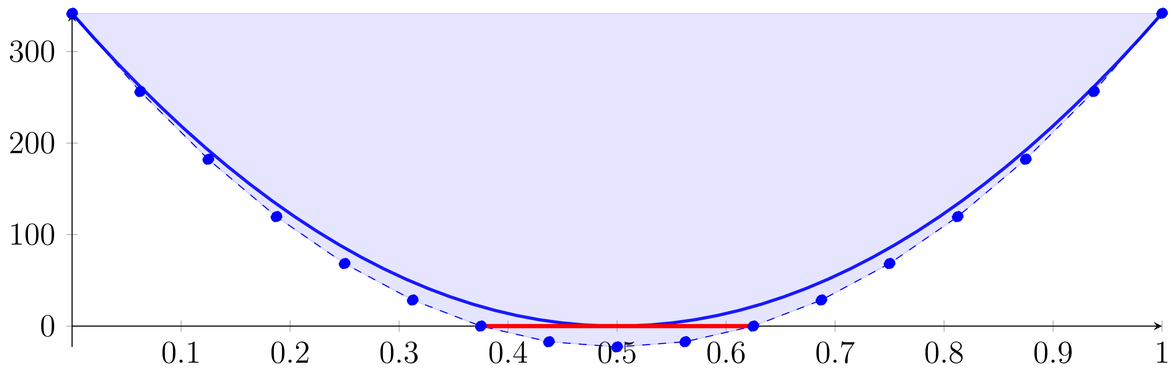
$$[0.3749929469011197, 0.6249882450180355]$$

Longest intersection interval: 0.2499952981169158
$\Longrightarrow$ Selective recursion: interval 1: $[0.2998961414100109, 0.3001040296026296]$,

67.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.2998961414100109, 0.300104029$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.216962444275584\cdot10^{-59}X^{16} + 4.692870826723641\cdot10^{-56}X^{15} + 6.316314535763934\\ &\cdot10^{-50}X^{14} - 2.373865516252114\cdot10^{-46}X^{13} - 1.395661893422859\cdot10^{-40}X^{12}\\ &+ 4.828363722587868\cdot10^{-37}X^{11} + 1.70471850273246\cdot10^{-31}X^{10} - 5.110411085982617\cdot10^{-28}X^{9}\\ &- 1.244717766622564\cdot10^{-22}X^{8} + 2.988632715177156\cdot10^{-19}X^{7} + 5.438614058423278\cdot10^{-14}X^{6}\\ &- 9.197401057829169\cdot10^{-11}X^{5} - 1.317801761681953\cdot10^{-05}X^{4} + 0.01167528468045732X^{3}\\ &+ 1366.961107526186X^{2} - 1366.963198735845X + 341.7398631167735\\ = &\ 341.7398631167735B_{0,16}(X) + 256.3046631957831B_{1,16}(X) + 182.260805837511B_{2,16}(X)\\ &+ 119.6083118906798B_{3,16}(X) + 68.34720219677136B_{4,16}(X) + 28.47749759002708B_{5,16}(X)\\ &- 0.0007811025525137025B_{6,16}(X) - 17.08761306120757B_{7,16}(X) - 22.782977473419B_{8,16}(X)\\ &- 17.08685353390849B_{9,16}(X) + 0.0007795553614777027B_{10,16}(X)\\ &+ 28.4799425851876B_{11,16}(X) + 68.35065633912575B_{12,16}(X) + 119.6129415934909B_{13,16}(X)\\ &+ 182.2668191173573B_{14,16}(X) + 256.312309672558B_{15,16}(X) + 341.7494340136854B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749982857492878, 0.6249971486858456\}$$

Intersection intervals with the x axis:

$$[0.3749982857492878, 0.6249971486858456]$$

Longest intersection interval: 0.2499988629365579
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999740991258704, 0.300026070937643]$,

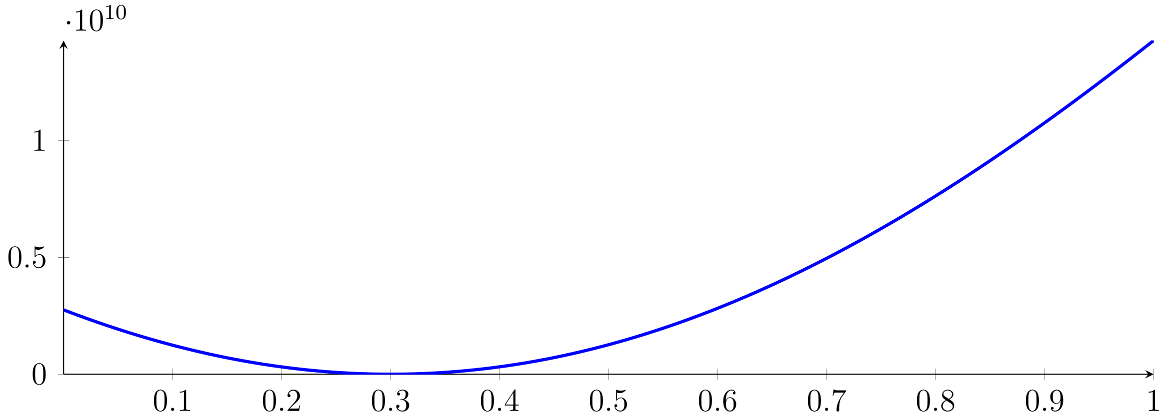
67.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999740991258704, 0.3000260$

Reached interval $[0.2999740991258704, 0.300026070937643]$ **without sign change** at depth 8!

67.9 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



Result: Root Intervals

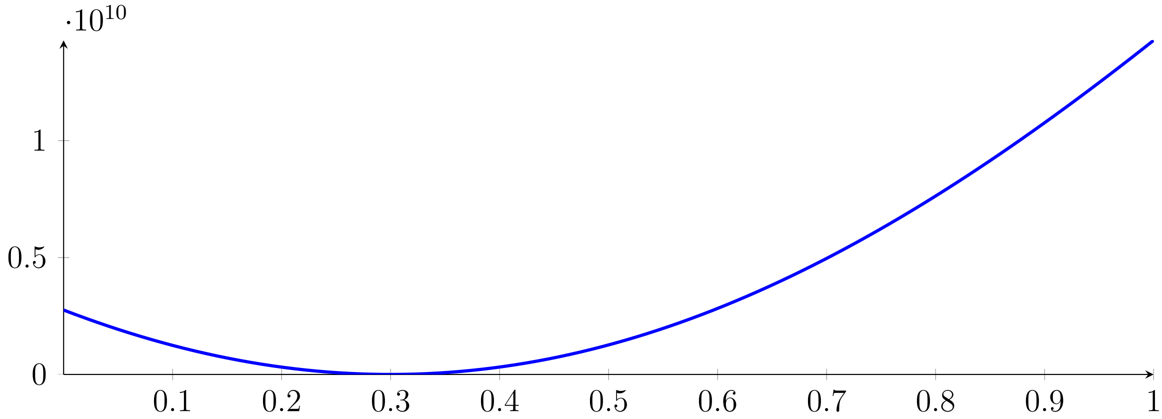
with precision $\varepsilon = 0.0001$.

68 Running QuadClip on h_{16} with epsilon 4

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called QuadClip with input polynomial on interval $[0, 1]$:

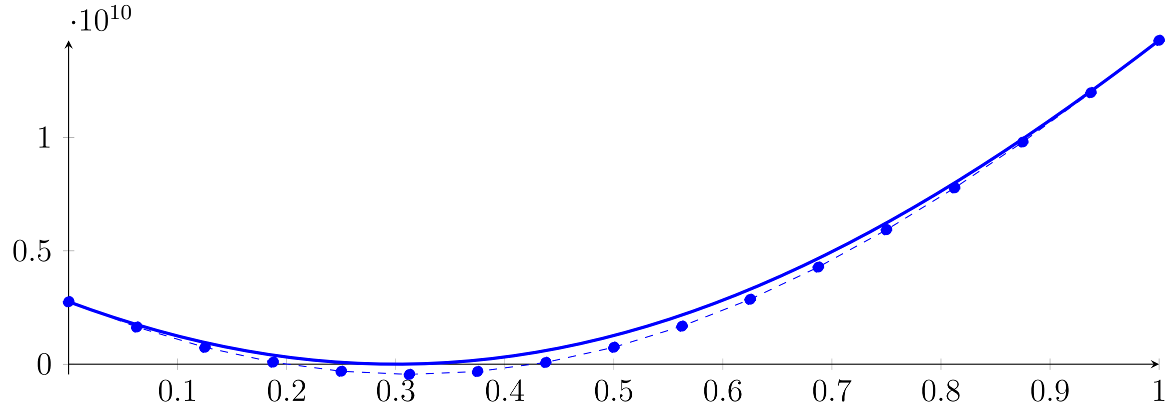
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



68.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

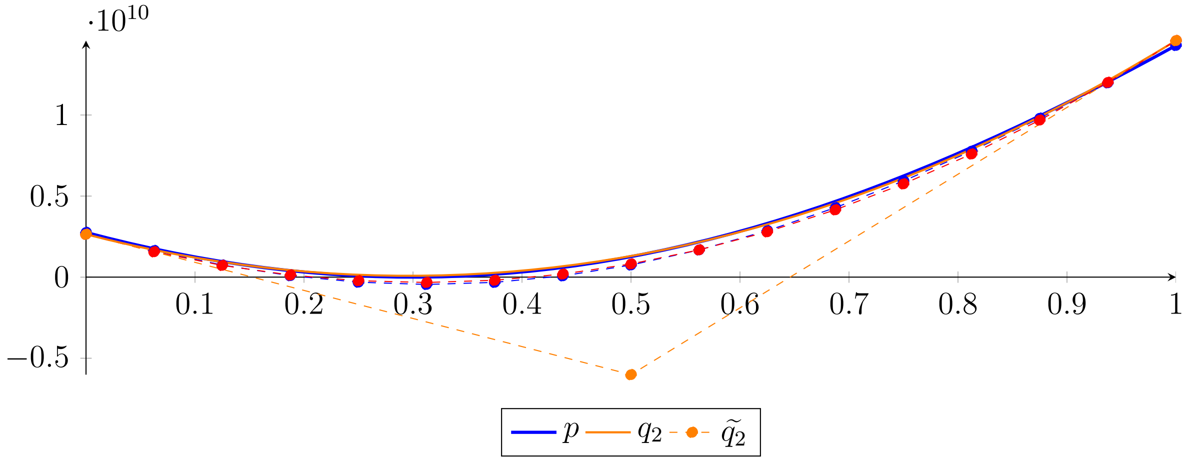
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\,2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328 \\
&= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q_2} &= -4.590615853685512\cdot 10^{-286}X^{16} + 3.626297739922361\cdot 10^{-285}X^{15} - 1.337118562447102 \\
&\quad \cdot 10^{-284}X^{14} + 3.057409950495903\cdot 10^{-284}X^{13} - 4.816080313213711\cdot 10^{-284}X^{12} \\
&\quad + 5.445450819003583\cdot 10^{-284}X^{11} - 4.453195910004646\cdot 10^{-284}X^{10} + 2.601913123147853 \\
&\quad \cdot 10^{-284}X^{9} - 1.060485581673177\cdot 10^{-284}X^{8} + 2.936863540613122\cdot 10^{-285}X^{7} - 5.487274374859613 \\
&\quad \cdot 10^{-286}X^{6} + 7.23018920152705\cdot 10^{-287}X^{5} - 7.254244946204871\cdot 10^{-288}X^{4} + 6.659990111567495 \\
&\quad \cdot 10^{-289}X^{3} + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328 \\
&= 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16} \\
&\quad + 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16} \\
&\quad - 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16} \\
&\quad + 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16} \\
&\quad + 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16} \\
&\quad + 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.

Bounding polynomials M and m:

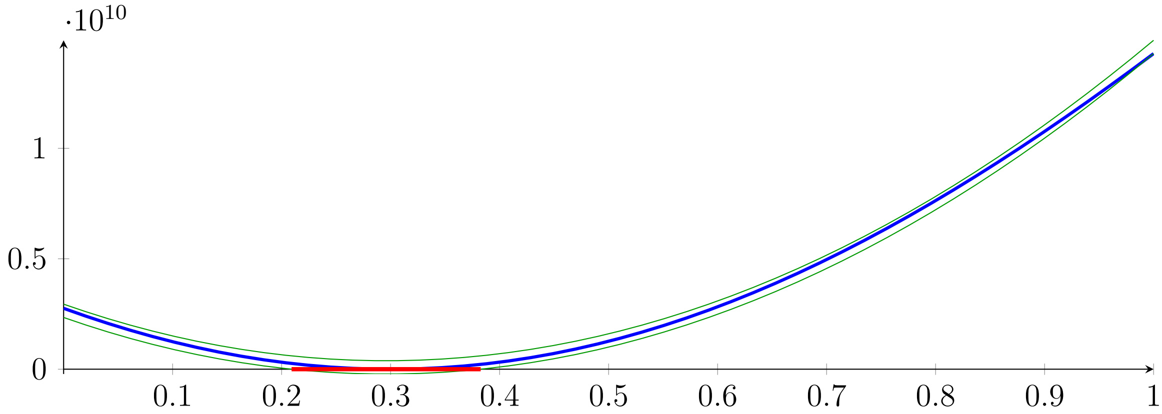
$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$

$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



$[0.2091580131065301, 0.3826391383239738]$

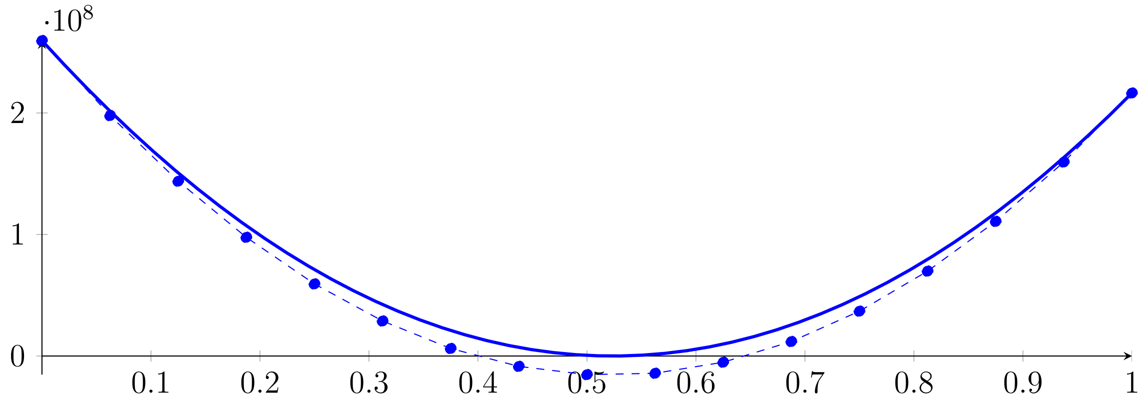
Longest intersection interval: 0.1734811252174437

$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

68.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

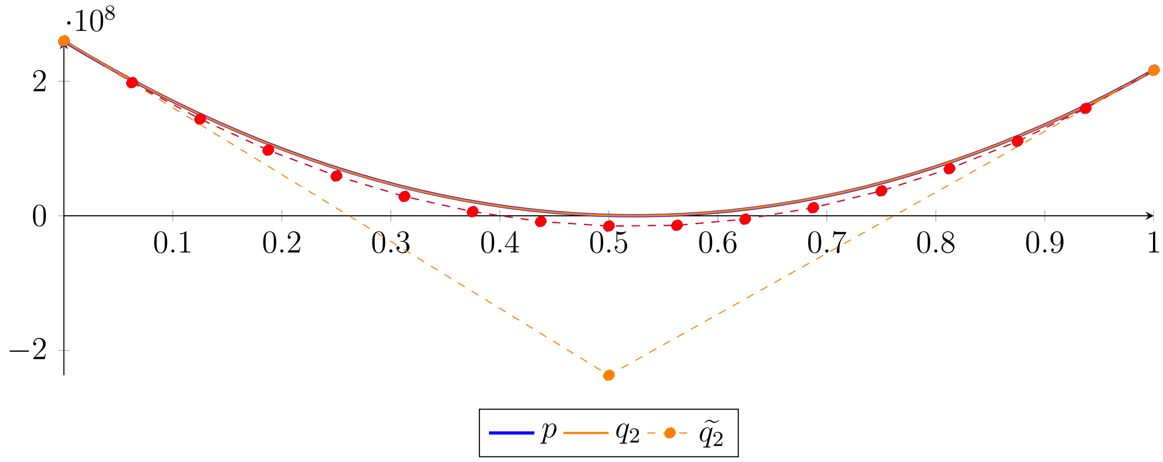
$$
\begin{aligned}
p = & -6.730319568869656\cdot 10^{-13}X^{16} + 8.742499458235574\cdot 10^{-12}X^{15} + 4.969734366538428\cdot 10^{-09}X^{14} \\
& - 5.918022439742958\cdot 10^{-08}X^{13} - 1.563835677279211\cdot 10^{-05}X^{12} + 0.000165142833990623X^{11} \\
& + 0.02725178204957372X^{10} - 0.2448183330655992X^{9} - 28.45820232077076X^{8} \\
& + 205.241311077626X^{7} + 17835.39577074806X^{6} - 94141.21233496903X^{5} - 6218439.344794644X^{4} \\
& + 20000843.19616221X^{3} + 930858984.7988209X^{2} - 987724621.0538478X + 259570777.0276934 \\
= & \, 259570777.0276934B_{0,16}(X) + 197837988.2118279B_{1,16}(X) + 143862357.6026193B_{2,16}(X) \\
& + 97679600.99148916B_{3,16}(X) + 59322017.44494463B_{4,16}(X) + 28818467.75210257B_{5,16}(X) \\
& + 6194355.099411763B_{6,16}(X) - 8528392.009487306B_{7,16}(X) - 15331334.57303514B_{8,16}(X) \\
& - 14199535.63666165B_{9,16}(X) - 5121570.552084079B_{10,16}(X) + 11910465.05798682B_{11,16}(X) \\
& + 36900949.63643701B_{12,16}(X) + 69850732.33405701B_{13,16}(X) \\
& + 110757131.9597003B_{14,16}(X) + 159613938.238137B_{15,16}(X) + 216411415.3731615B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 = & \, 950064117.9944898X^{2} - 993959420.5216784X + 260029868.8092426 \\
= & \, 260029868.8092426B_{0,2} - 236949841.4515966B_{1,2} + 216134566.282054B_{2,2}
\end{aligned}
$$

$$\begin{aligned}\widetilde{q}_2 &= -1.41520176718356\cdot 10^{-287}X^{16} + 1.139753408092347\cdot 10^{-286}X^{15} - 4.276104932493763\\ &\quad\cdot 10^{-286}X^{14} + 9.896028733517596\cdot 10^{-286}X^{13} - 1.566233430583374\cdot 10^{-285}X^{12}\\ &\quad + 1.767044733566612\cdot 10^{-285}X^{11} - 1.435814571471649\cdot 10^{-285}X^{10} + 8.336078570237132\\ &\quad\cdot 10^{-286}X^{9} - 3.396584250429756\cdot 10^{-286}X^{8} + 9.548198482934522\cdot 10^{-287}X^{7} - 1.861944039074976\\ &\quad\cdot 10^{-287}X^{6} + 2.622613520689986\cdot 10^{-288}X^{5} - 2.667693132942884\cdot 10^{-289}X^{4} + 2.048626792988895\\ &\quad\cdot 10^{-290}X^{3} + 950064117.9944898X^{2} - 993959420.5216784X + 260029868.8092426\\ &= 260029868.8092426B_{0,16} + 197907405.0266377B_{1,16} + 143702142.2273202B_{2,16}\\ &\quad + 97414080.41129016B_{3,16} + 59043219.5785475B_{4,16} + 28589559.72909226B_{5,16}\\ &\quad + 6053100.862924435B_{6,16} - 8566157.019955976B_{7,16} - 15268213.91954897B_{8,16}\\ &\quad - 14053069.83585455B_{9,16} - 4920724.768872719B_{10,16} + 12128821.28139653B_{11,16}\\ &\quad + 37095568.31495319B_{12,16} + 69979516.33179727B_{13,16}\\ &\quad + 110780665.3319288B_{14,16} + 159499015.3153477B_{15,16} + 216134566.282054B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

Bounding polynomials M and m:

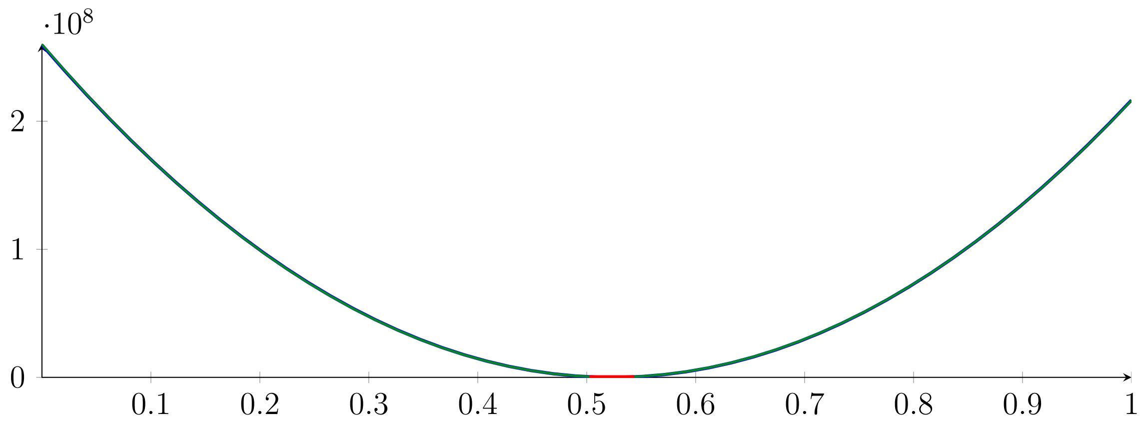
$$M = 950064117.9944898X^2 - 993959420.5216784X + 260488960.5907918$$

$$m = 950064117.9944898X^2 - 993959420.5216784X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

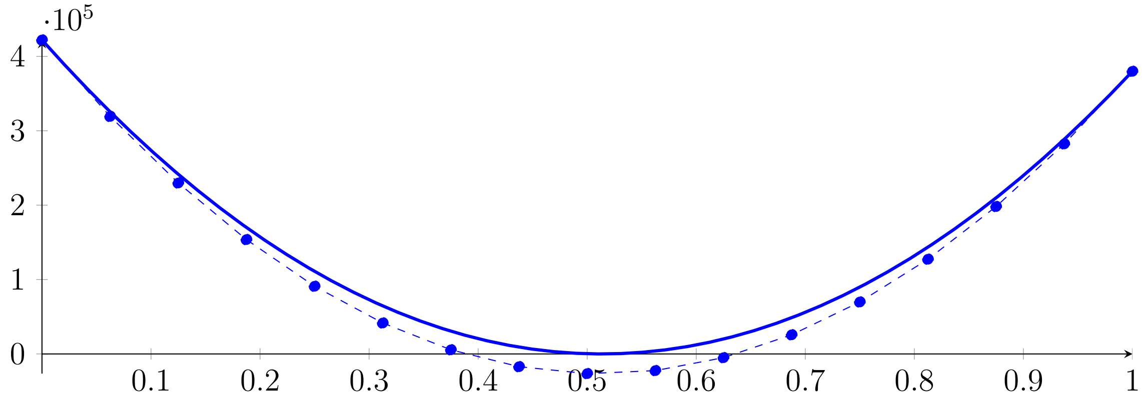
Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

68.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Normalized monomial und Bézier representations and the Bézier polygon:

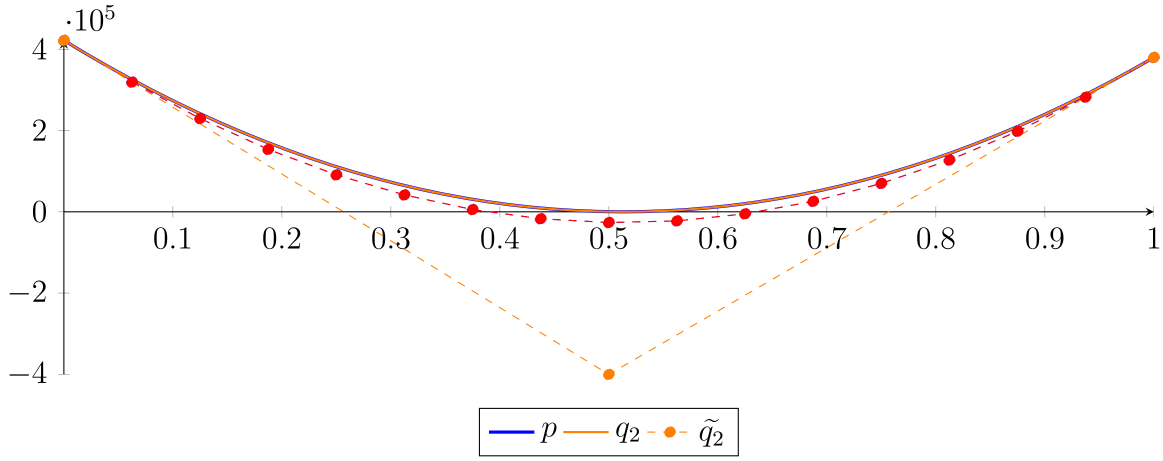
$$p = -4.348008654913047 \cdot 10^{-35} X^{16} + 5.243388239594399 \cdot 10^{-33} X^{15} + 1.924264426249969 \cdot 10^{-28} X^{14} - 2.246746658841888 \cdot 10^{-26} X^{13} - 3.625525335135332 \cdot 10^{-22} X^{12} + 3.881306066232412 \cdot 10^{-20} X^{11} + 3.776139474106064 \cdot 10^{-16} X^{10} - 3.496022725744 \cdot 10^{-14} X^{9} - 2.35123279599996 \cdot 10^{-10} X^{8} + 1.7435554055155 \cdot 10^{-08} X^{7} + 8.761425585865998 \cdot 10^{-05} X^{6} - 0.004592127899352905 X^{5} - 18.10659249096443 X^{4} + 504.889471438631 X^{3} + 1602084.653024796 X^{2} - 1644731.182248784 X + 422061.2284789393$$
$$= 422061.2284789393 B_{0,16}(X) + 319265.5295883902 B_{1,16}(X) + 229820.5361397145 B_{2,16}(X) + 153727.1497212539 B_{3,16}(X) + 90986.26197267314 B_{4,16}(X) + 41598.75458390835 B_{5,16}(X) + 5565.499294126806 B_{6,16}(X) - 17112.6421093025 B_{7,16}(X) - 26434.81779182738 B_{8,16}(X) - 22400.18587271997 B_{9,16}(X) - 5007.914426073306 B_{10,16}(X) + 25742.81851821299 B_{11,16}(X) + 69852.82497345804 B_{12,16}(X) + 127322.906995236 B_{13,16}(X) + 198153.8566804232 B_{14,16}(X) + 282346.4561662563 B_{15,16}(X) + 379901.4776294016 B_{16,16}(X)$$



Degree reduction and raising:

$$q_2 = 1602810.939315368 X^2 - 1645017.556512812 X + 422084.9204772878$$
$$= 422084.9204772878 B_{0,2} - 400423.8577791184 B_{1,2} + 379878.3032798431 B_{2,2}$$

$$\widetilde{q}_2 = -2.394754881528157 \cdot 10^{-290} X^{16} + 1.927007748221533 \cdot 10^{-289} X^{15} - 7.222560694913459 \cdot 10^{-289} X^{14} + 1.669823679170844 \cdot 10^{-288} X^{13} - 2.640537385502685 \cdot 10^{-288} X^{12} + 2.977245352423787 \cdot 10^{-288} X^{11} - 2.4183464566446 \cdot 10^{-288} X^{10} + 1.403945843258626 \cdot 10^{-288} X^{9} - 5.721369736991339 \cdot 10^{-289} X^{8} + 1.608948017596564 \cdot 10^{-289} X^{7} - 3.139550473135121 \cdot 10^{-290} X^{6} + 4.427570920272867 \cdot 10^{-291} X^{5} - 4.516145205270189 \cdot 10^{-292} X^{4} + 3.478035897244226 \cdot 10^{-293} X^{3} + 1602810.939315368 X^{2} - 1645017.556512812 X + 422084.9204772878$$
$$= 422084.9204772878 B_{0,16} + 319271.323195237 B_{1,16} + 229814.4837408143 B_{2,16} + 153714.4021140197 B_{3,16} + 90971.07831485311 B_{4,16} + 41584.51234331459 B_{5,16} + 5554.704199404135 B_{6,16} - 17118.34611687825 B_{7,16} - 26434.63860553258 B_{8,16} - 22394.17326655884 B_{9,16} - 4996.95009995704 B_{10,16} + 25757.03089427283 B_{11,16} + 69867.76971613076 B_{12,16} + 127335.2663656168 B_{13,16} + 198159.5208427308 B_{14,16} + 282340.5331474729 B_{15,16} + 379878.3032798431 B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 23.69199834856345$.
Bounding polynomials M and m:

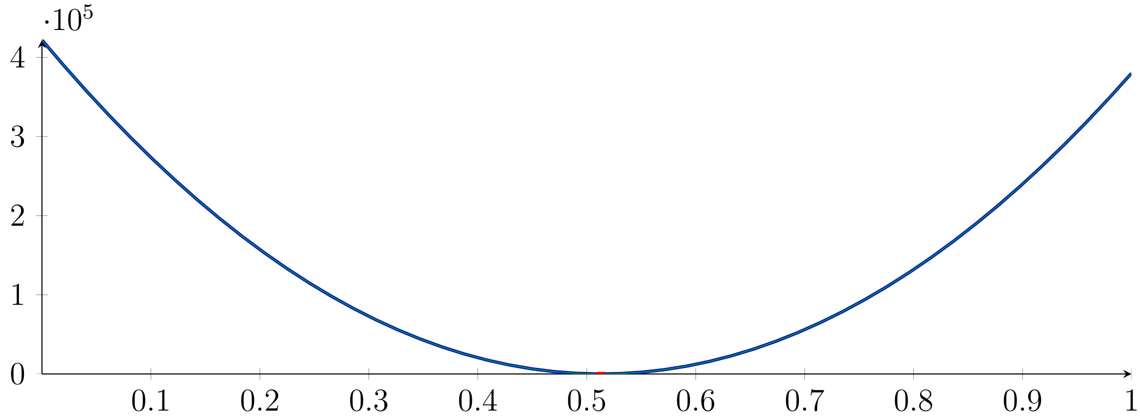
$$M = 1602810.939315368X^2 - 1645017.556512812X + 422108.6124756364$$

$$m = 1602810.939315368X^2 - 1645017.556512812X + 422061.2284789393$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.509405571919099, 0.5169273012614853\}$$

Intersection intervals:



$$[0.509405571919099, 0.5169273012614853]$$

Longest intersection interval: 0.007521729342386311
$\Longrightarrow$ Selective recursion: interval 1: $[0.299973157656462, 0.3000267016613888]$,

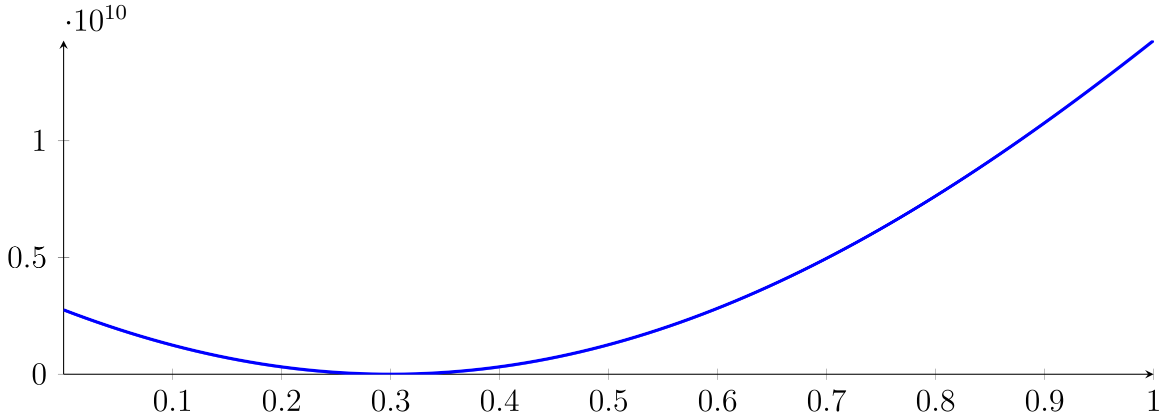
68.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.299973157656462, 0.3000267016613888]$

Reached interval $[0.299973157656462, 0.3000267016613888]$ **without sign change** at depth 4!
p(0) = 22.93446410530317 - p(1) 22.4083400664027

68.5 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

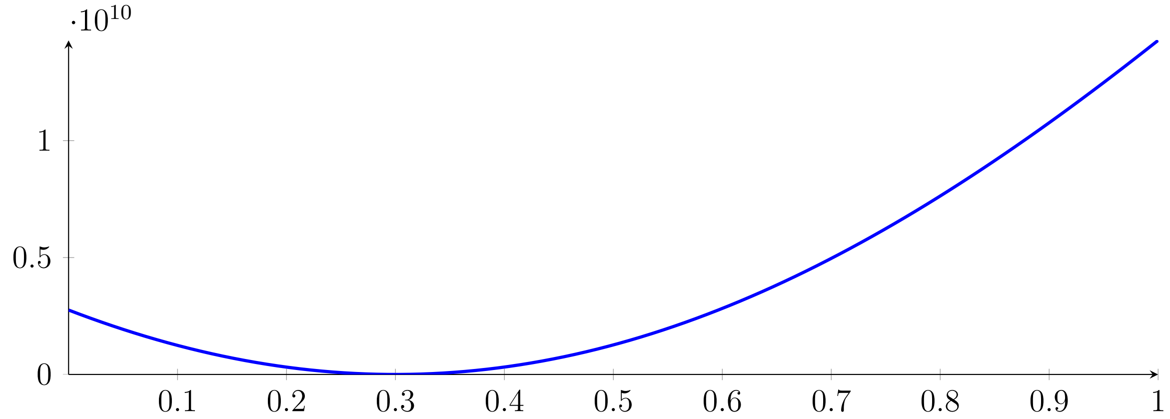
with precision $\varepsilon = 0.0001$.

69 Running `CubeClip` on h_{16} with epsilon 4

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

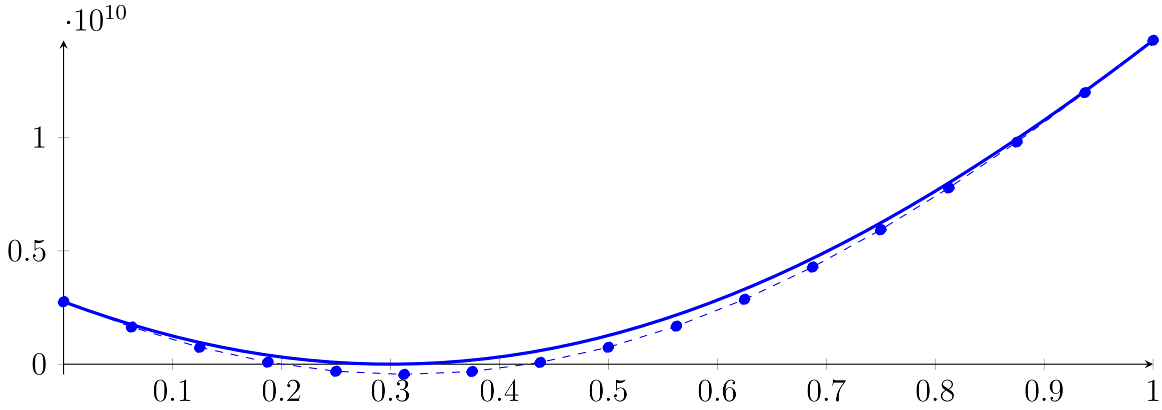
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



69.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

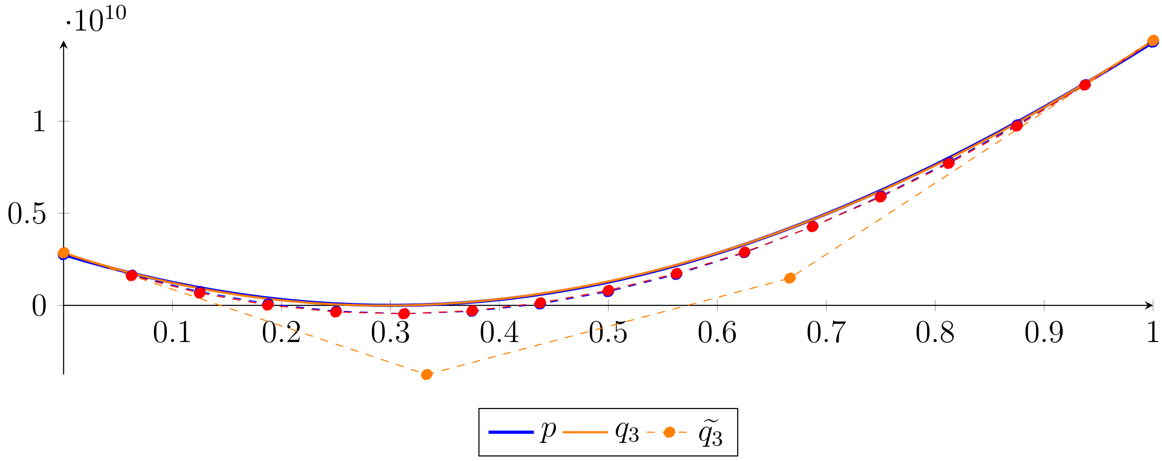
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3} + 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3}$$

$$\widetilde{q}_3 = 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756\cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12} - 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948\cdot 10^{-285}X^{9} + 6.984121997801746\cdot 10^{-286}X^{8} - 2.400829205648148\cdot 10^{-286}X^{7} + 9.243609934541251\cdot 10^{-287}X^{6} - 2.7161427533585\cdot 10^{-287}X^{5} + 4.31464378661806\cdot 10^{-288}X^{4} - 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16} + 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16} - 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16} + 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16} + 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16} + 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

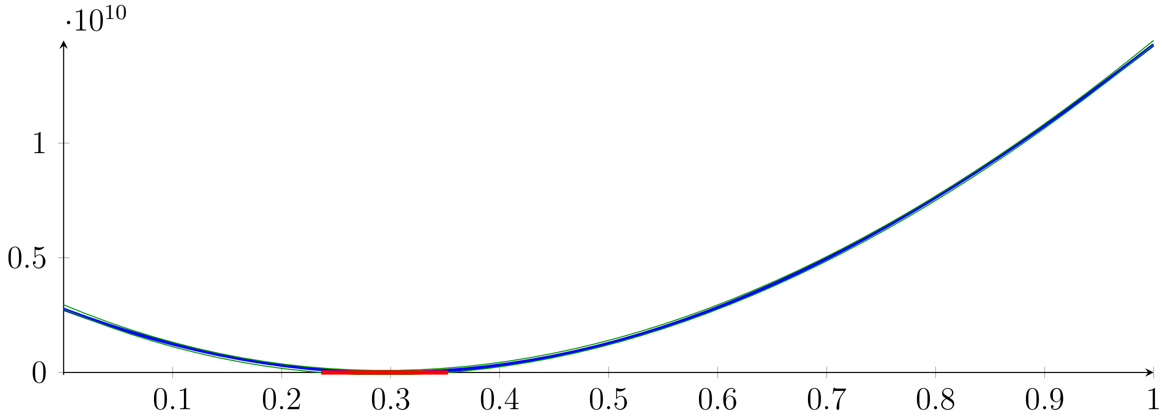
Bounding polynomials M and m:

$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$
$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



$[0.2362027155340351, 0.3527550629571673]$

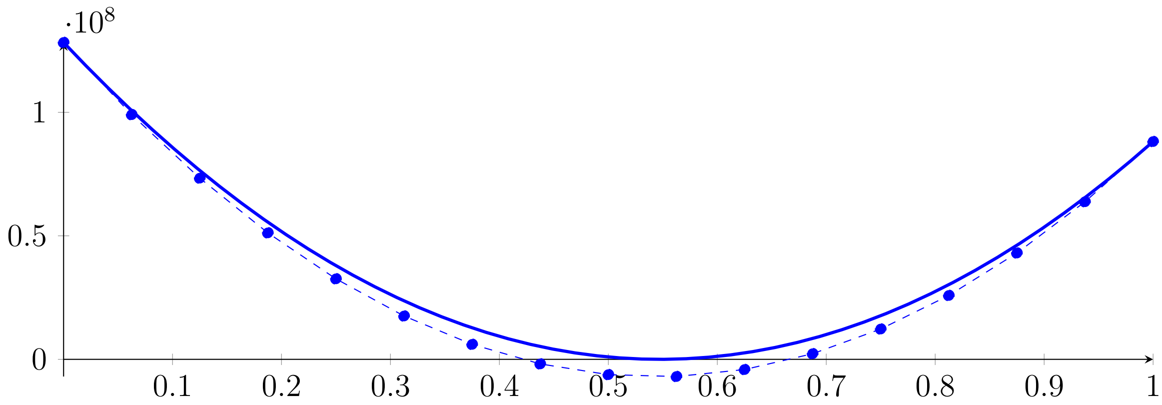
Longest intersection interval: 0.1165523474231321

$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

69.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

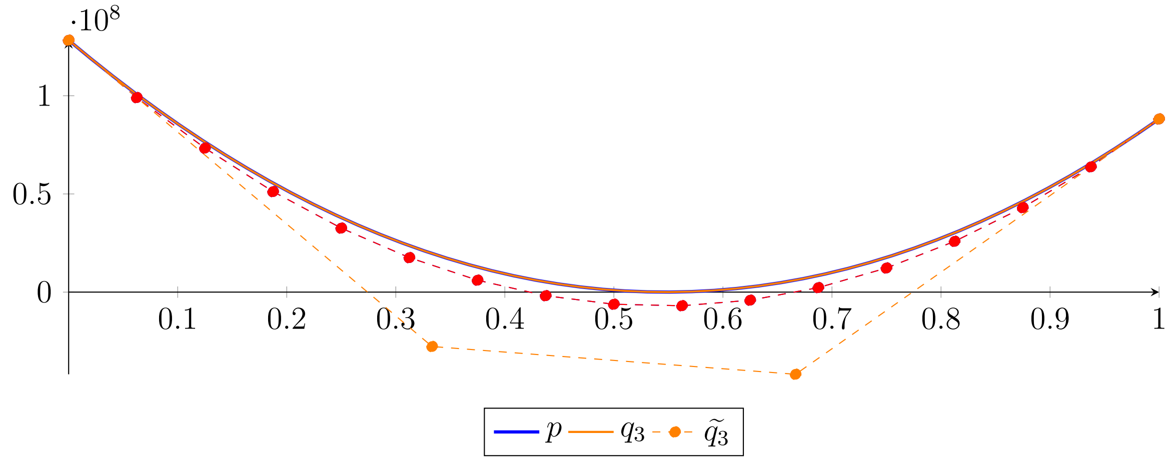
$$\begin{aligned}
p = &-1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14} \\
&- 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321 \\
&\cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8} \\
&+ 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4} \\
&+ 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316 \\
= &\ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X) \\
&+ 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X) \\
&+ 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X) \\
&- 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X) \\
&+ 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X) \\
&+ 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &\ 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166 \\
= &\ 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3} \\
&- 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= 1.029539679280458\cdot 10^{-288}X^{16} - 8.072523277872609\cdot 10^{-288}X^{15} + 2.888405037194055\\ &\quad\cdot 10^{-287}X^{14} - 6.260623433882179\cdot 10^{-287}X^{13} + 9.183807049175627\cdot 10^{-287}X^{12}\\ &\quad - 9.610199535206024\cdot 10^{-287}X^{11} + 7.321003139036714\cdot 10^{-287}X^{10} - 4.031610440228076\\ &\quad\cdot 10^{-287}X^{9} + 1.537754605851679\cdot 10^{-287}X^{8} - 3.606689105481158\cdot 10^{-288}X^{7}\\ &\quad + 3.223581314526531\cdot 10^{-289}X^{6} + 5.410058694892297\cdot 10^{-290}X^{5} - 1.24173721337985\cdot 10^{-290}X^{4}\\ &\quad + 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166\\ &= 128303546.2426166B_{0,16} + 99049684.91666918B_{1,16} + 73342858.56410606B_{2,16}\\ &\quad + 51187170.59978822B_{3,16} + 32586724.4385766B_{4,16} + 17545623.49533216B_{5,16}\\ &\quad + 6067971.184915845B_{6,16} - 1842129.077811386B_{7,16} - 6180573.877988583B_{8,16}\\ &\quad - 6943259.800754793B_{9,16} - 4126083.431249065B_{10,16} + 2275058.645389556B_{11,16}\\ &\quad + 12264269.84402202B_{12,16} + 25845653.57950928B_{13,16} + 43023313.26671229B_{14,16}\\ &\quad + 63801352.320492B_{15,16} + 88183874.15570936B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

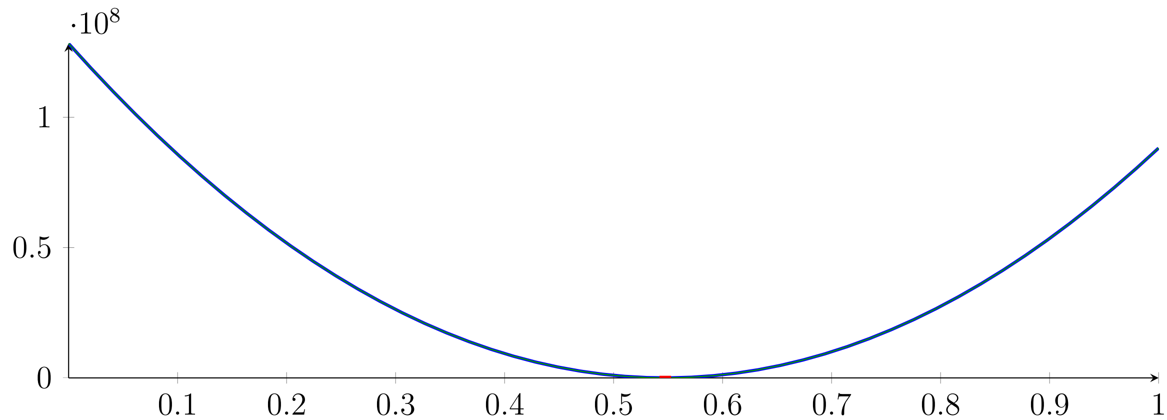
Bounding polynomials M and m:

$$M = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128322140.7478178$$
$$m = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$$[0.5420611331202266, 0.552740643902378]$$

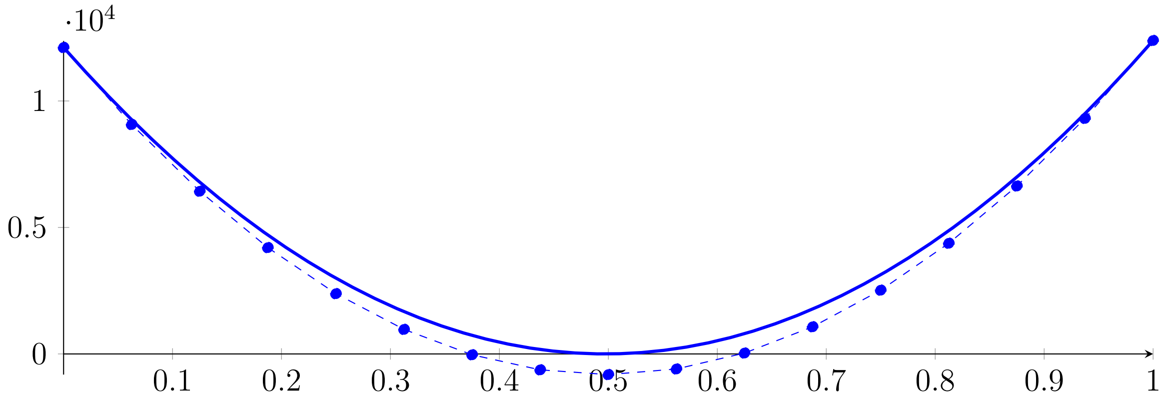
Longest intersection interval: 0.01067951078215144

$\Longrightarrow$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

69.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.300625935097030$

Normalized monomial und Bézier representations and the Bézier polygon:

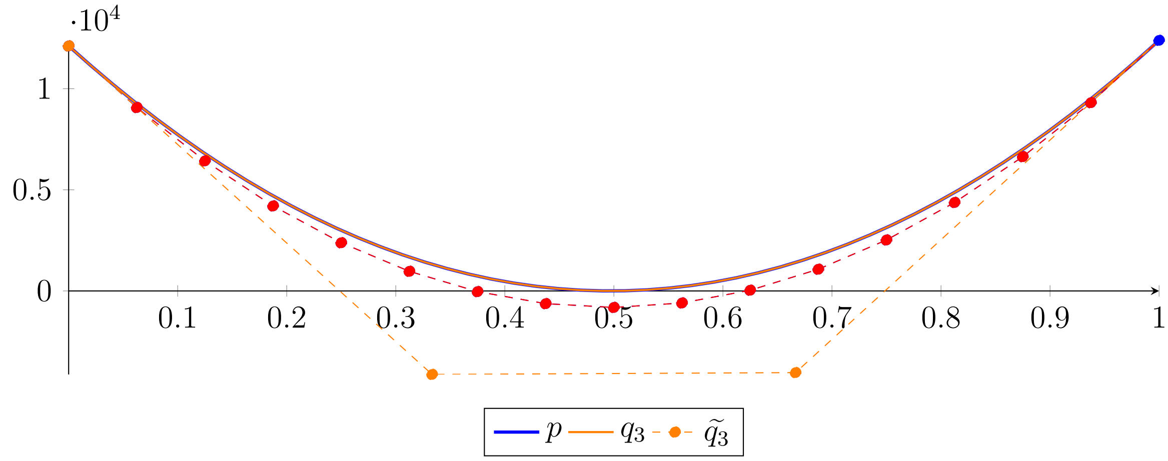
$$p = -3.320153282917824\cdot 10^{-47}X^{16} + 2.160317373028451\cdot 10^{-44}X^{15} + 4.806705749215272 \cdot 10^{-39}X^{14} - 3.045075915978909\cdot 10^{-36}X^{13} - 2.96255923979525\cdot 10^{-31}X^{12} + 1.726566936989629\cdot 10^{-28}X^{11} + 1.009356568315285\cdot 10^{-23}X^{10} - 5.095799973170639 \cdot 10^{-21}X^{9} - 2.055755460332604\cdot 10^{-16}X^{8} + 8.312655296209706\cdot 10^{-14}X^{7} + 2.505544083540367\cdot 10^{-09}X^{6} - 7.139662163470987\cdot 10^{-07}X^{5} - 0.01693489925927299X^{4} + 2.534105689487521X^{3} + 49001.97220419171X^{2} - 48729.12904702137X + 12114.14082112651$$
$$= 12114.14082112651B_{0,16}(X) + 9068.570255687672B_{1,16}(X) + 6431.349458617101B_{2,16}(X) + 4202.482955103525B_{3,16}(X) + 2381.975261030786B_{4,16}(X) + 969.830882977672B_{5,16}(X) - 33.94568178224417B_{6,16}(X) - 629.34994528077B_{7,16}(X) - 816.3774288552541B_{8,16}(X) - 595.0236631487491B_{9,16}(X) + 34.71581188982676B_{10,16}(X) + 1072.845447005528B_{11,16}(X) + 2519.36968363722B_{12,16}(X) + 4374.29295391742B_{13,16}(X) + 6637.619680672133B_{14,16}(X) + 9309.354277420696B_{15,16}(X) + 12389.50114837561B_{16,16}(X)$$



Degree reduction and raising:

$$q_3 = 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619$$
$$= 12114.14106307619B_{0,3} - 4128.903565586097B_{1,3} - 4037.950201139886B_{2,3} + 12389.5013903309B_{3,3}$$

$$\widetilde{q}_3 = 1.099600357934752\cdot 10^{-292}X^{16} - 8.377614657428995\cdot 10^{-292}X^{15} + 2.905985967667976 \cdot 10^{-291}X^{14} - 6.091421693711012\cdot 10^{-291}X^{13} + 8.615207440059339\cdot 10^{-291}X^{12} - 8.654253929665817\cdot 10^{-291}X^{11} + 6.288120919613368\cdot 10^{-291}X^{10} - 3.270943230058189 \cdot 10^{-291}X^{9} + 1.159364872956392\cdot 10^{-291}X^{8} - 2.426897197847659\cdot 10^{-292}X^{7} + 1.434105777365326\cdot 10^{-293}X^{6} + 4.843078869312239\cdot 10^{-294}X^{5} - 7.632080003106212\cdot 10^{-295}X^{4} + 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619$$
$$= 12114.14106307619B_{0,16} + 9068.570195202008B_{1,16} + 6431.349277155543B_{2,16} + 4202.482773640211B_{3,16} + 2381.975149359434B_{4,16} + 969.8308690166351B_{5,16} - 33.94560268476556B_{6,16} - 629.349801041346B_{7,16} - 816.3772613496847B_{8,16} - 595.0235189063601B_{9,16} + 34.71589099204953B_{10,16} + 1072.845433048966B_{11,16} + 2519.36957196781B_{12,16} + 4374.292772452004B_{13,16} + 6637.619499204969B_{14,16} + 9309.354216930127B_{15,16} + 12389.5013903309B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00024195528529181 86$.
Bounding polynomials M and m:

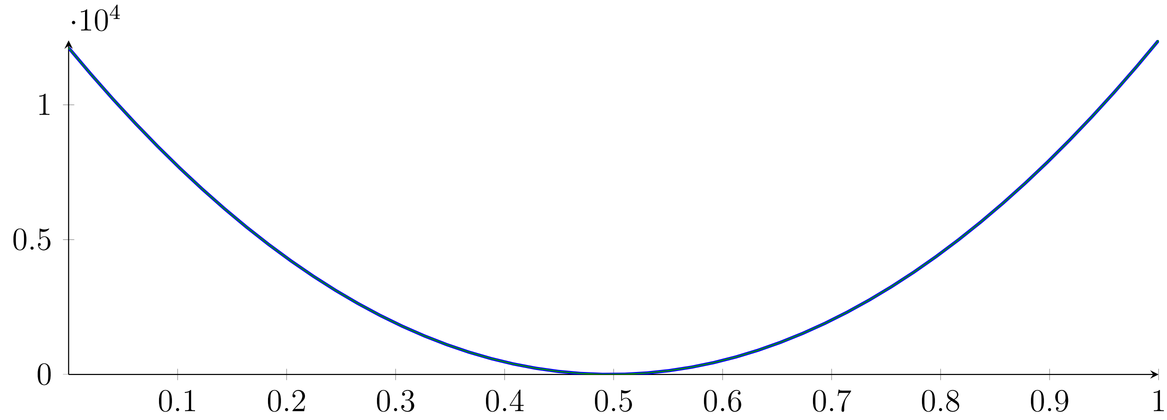
$$M = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.14130503147$$

$$m = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.1408211209$$

Root of M and m:

$$N(M) = \{-19599.95818041899\} \quad N(m) = \{-19599.95818041899, 0.4971412064138645, 0.4972526073815481\}$$

Intersection intervals:



$$[0.4971412064138645, 0.4972526073815481]$$

Longest intersection interval: 0.0001114009676835966
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000156681198, 0.3000001543313607]$,

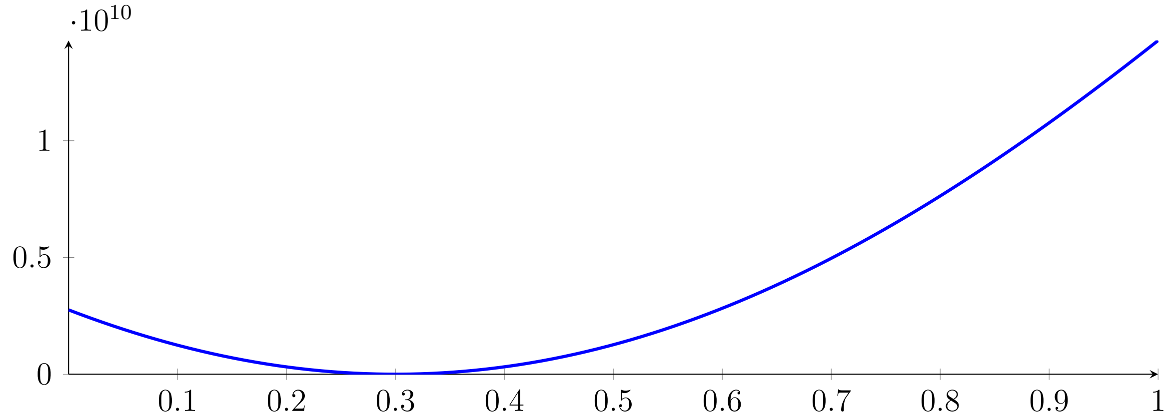
69.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3000000156681198, 0.3000001543313607]$

Reached interval $[0.3000000156681198, 0.3000001543313607]$ **without sign change** at depth 4!
p(0) = 0.0001512528027912101 - p(1) 0.0001512505250838 23

69.5 Result: 0 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

with precision $\varepsilon = 0.0001$.

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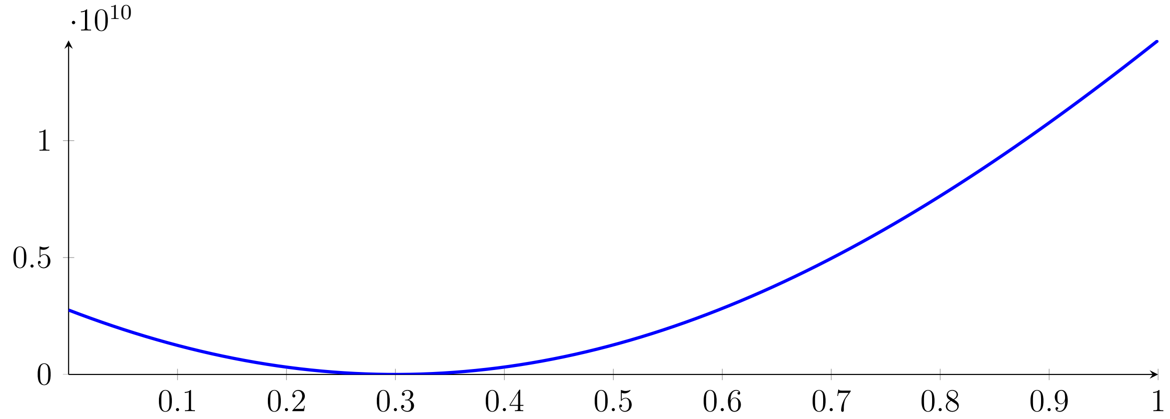
$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called BezClip with input polynomial on interval $[0, 1]$:

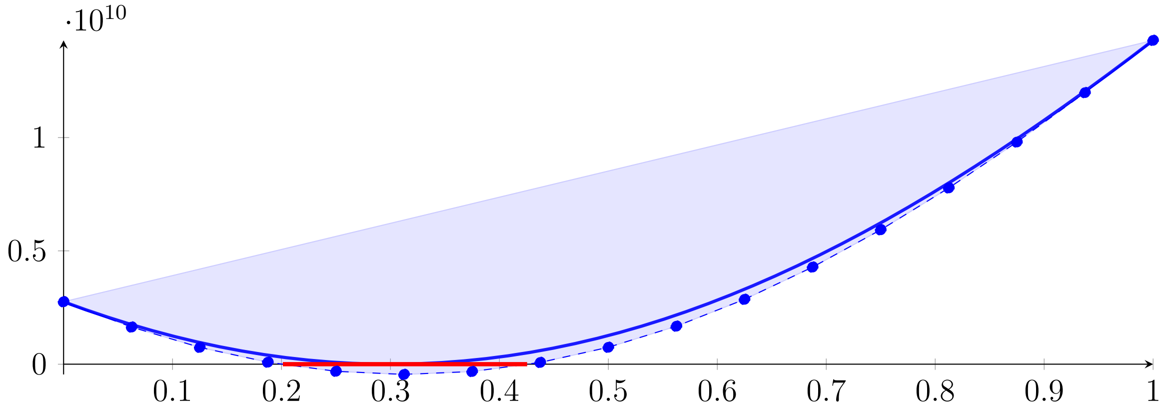
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\&+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\&- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\&- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\&+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



70.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\&+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\&- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\&- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\&+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\\= &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X)\\&+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X)\\&- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X)\\&+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X)\\&+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X)\\&+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

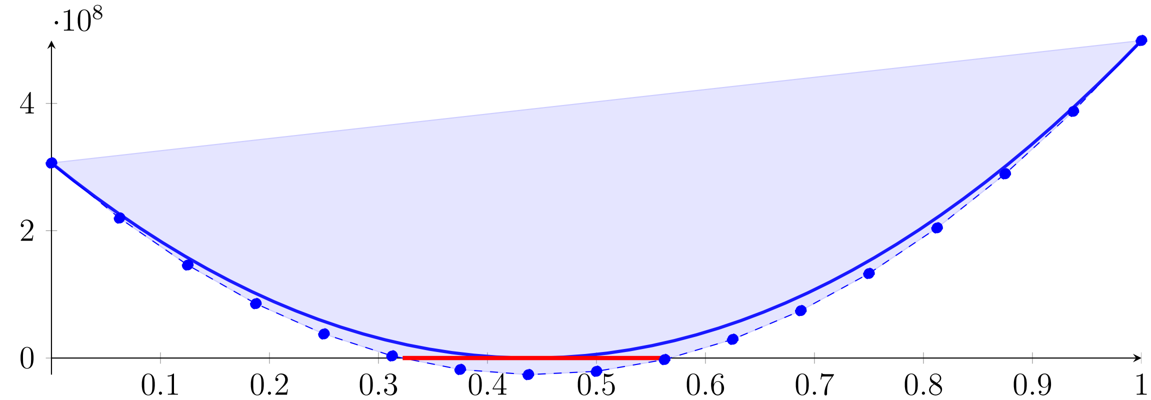
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

70.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -4.078702226246453 \cdot 10^{-11} X^{16} + 4.329167405007062 \cdot 10^{-10} X^{15} + 1.800827856148464 \cdot 10^{-07} X^{14} \\
& - 1.7501300195896 \cdot 10^{-06} X^{13} - 0.0003388881074603682 X^{12} + 0.002918727490860103 X^{11} \\
& + 0.353260574938002 X^{10} - 2.587751703352393 X^{9} - 220.7388291837263 X^{8} \\
& + 1298.537478880933 X^{7} + 82809.42227744751 X^{6} - 357003.6974521088 X^{5} - 17288824.7550166 X^{4} \\
& + 45617774.75376932 X^{3} + 1550181799.359372 X^{2} - 1385902556.54429 X + 306449533.9978484 \\
= & \ 306449533.9978484 B_{0,16}(X) + 219830624.2138303 B_{1,16}(X) + 146129896.0911402 B_{2,16}(X) \\
& + 85428809.9418386 B_{3,16}(X) + 37799326.72372467 B_{4,16}(X) + 3303826.308721026 B_{5,16}(X) \\
& - 18004963.90790522 B_{6,16}(X) - 26084029.94397575 B_{7,16}(X) - 20900121.61613177 B_{8,16}(X) \\
& - 2429792.29588269 B_{9,16}(X) + 29340572.02544693 B_{10,16}(X) + 74414734.40659503 B_{11,16}(X) \\
& + 132786599.3869978 B_{12,16}(X) + 204440216.186282 B_{13,16}(X) + 289349792.6628749 B_{14,16}(X) \\
& + 387479720.0042024 B_{15,16}(X) + 498784608.1032447 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

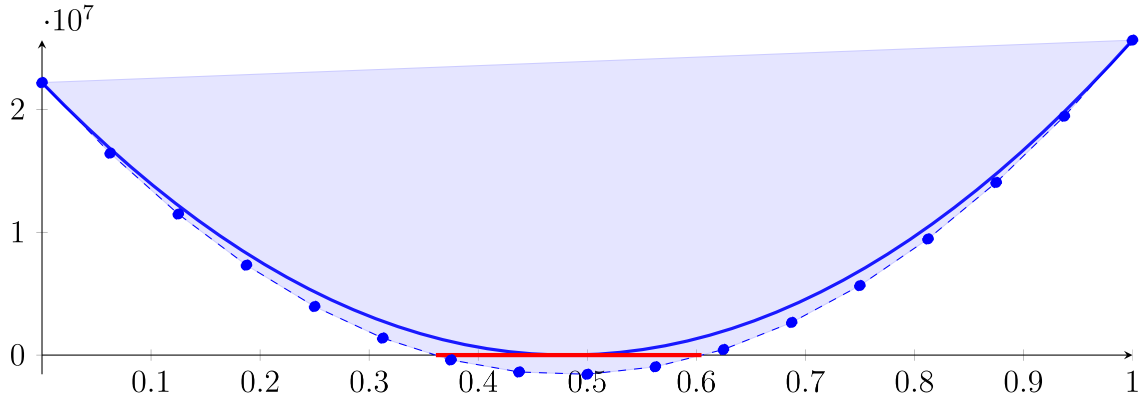
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: $[0.2735013999129692, 0.328453288067671]$,

70.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-6.91388116995714 \cdot 10^{-21} X^{16} + 1.539971991909181 \cdot 10^{-19} X^{15} + 5.126557214158853 \\ &\cdot 10^{-16} X^{14} - 1.075268795442723 \cdot 10^{-14} X^{13} - 1.618466564965893 \cdot 10^{-11} X^{12} \\ &+ 3.060191240732859 \cdot 10^{-10} X^{11} + 2.825398471644138 \cdot 10^{-07} X^{10} - 4.580412037193902 \\ &\cdot 10^{-06} X^{9} - 0.002949971804071592 X^{8} + 0.0383187988748973 X^{7} + 18.44286791446417 X^{6} \\ &- 172.012783850635 X^{5} - 63987.92092437637 X^{4} + 338934.074545568 X^{3} \\ &+ 95113195.48898588 X^{2} - 91937923.73553449 X + 22182509.73328641 \\ = &\ 22182509.73328641 B_{0,16}(X) + 16436389.49981551 B_{1,16}(X) + 11482879.22875282 B_{2,16}(X) \\ &+ 7322584.159517174 B_{3,16}(X) + 3956074.373329099 B_{4,16}(X) + 1383884.753830588 B_{5,16}(X) \\ &- 393485.0499920519 B_{6,16}(X) - 1375570.658578954 B_{7,16}(X) - 1561942.994257926 B_{8,16}(X) \\ &- 952208.311393262 B_{9,16}(X) + 453991.7757805191 B_{10,16}(X) + 2656980.267642443 B_{11,16}(X) \\ &+ 5657044.755988558 B_{12,16}(X) + 9454437.403157084 B_{13,16}(X) + 14049374.92347699 B_{14,16}(X) \\ &+ 19442038.56704145 B_{15,16}(X) + 25632574.10580758 B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

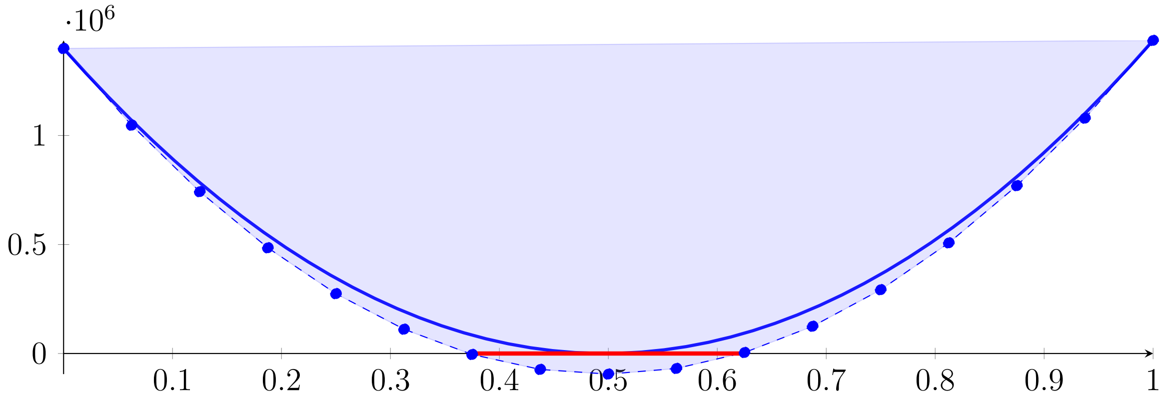
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: $[0.2933480088018335, 0.3067375037518675]$,

70.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.067154088646128\cdot 10^{-30}X^{16} + 7.224340048814341\cdot 10^{-29}X^{15} + 1.334696477727887 \\
& \cdot 10^{-24}X^{14} - 8.705175946298654\cdot 10^{-23}X^{13} - 7.106771480233297\cdot 10^{-19}X^{12} \\
& + 4.237354105832321\cdot 10^{-17}X^{11} + 2.091927791167947\cdot 10^{-13}X^{10} - 1.077046671119229 \\
& \cdot 10^{-11}X^{9} - 3.681404336048099\cdot 10^{-08}X^{8} + 1.51820465067154\cdot 10^{-06}X^{7} \\
& + 0.003877397410843758X^{6} - 0.1133230374256462X^{5} - 226.5078434192974X^{4} \\
& + 3562.753220436385X^{3} + 5665644.405911702X^{2} - 5632059.446508477X + 1399245.031427946 \\
= & \; 1399245.031427946B_{0,16}(X) + 1047241.316021167B_{1,16}(X) + 742451.3039969843B_{2,16}(X) \\
& + 484881.3574147218B_{3,16}(X) + 274537.7138788421B_{4,16}(X) + 111426.4865130056B_{5,16}(X) \\
& - 4446.336065390124B_{6,16}(X) - 73074.88977018565B_{7,16}(X) - 94453.43507095045B_{8,16}(X) \\
& - 68576.35701698946B_{9,16}(X) + 4561.834739135334B_{10,16}(X) \\
& + 124966.505918568B_{11,16}(X) + 292642.897593607B_{12,16}(X) + 507596.1261656377B_{13,16}(X) \\
& + 769831.1833435517B_{14,16}(X) + 1079352.93612265B_{15,16}(X) + 1436166.12676403B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

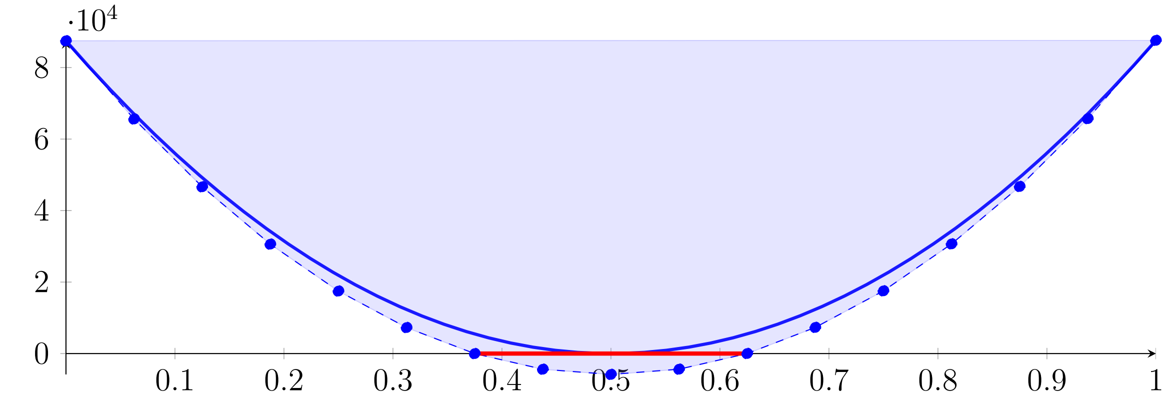
Longest intersection interval: 0.2484999837871829

$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

70.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -2.256570822524655\cdot 10^{-40}X^{16} + 5.606069245850654\cdot 10^{-38}X^{15} + 4.571694482208246 \\
& \cdot 10^{-33}X^{14} - 1.103601619609778\cdot 10^{-30}X^{13} - 3.943084008767946\cdot 10^{-26}X^{12} \\
& + 8.746232232432483\cdot 10^{-24}X^{11} + 1.879997320990413\cdot 10^{-19}X^{10} - 3.610214442217249 \\
& \cdot 10^{-17}X^{9} - 5.35839719625663\cdot 10^{-13}X^{8} + 8.241677187819251\cdot 10^{-11}X^{7} + 9.139570434264648 \\
& \cdot 10^{-07}X^{6} - 9.91682351975725\cdot 10^{-05}X^{5} - 0.864525557836322X^{4} + 49.4891850901473X^{3} \\
& + 350100.5152669434X^{2} - 350028.3308017828X + 87482.91293301892 \\
= & \; 87482.91293301892B_{0,16}(X) + 65606.1422579075B_{1,16}(X) + 46646.87587668727B_{2,16}(X) \\
& + 30605.20216290304B_{3,16}(X) + 17481.20901508557B_{4,16}(X) + 7274.983856728878B_{5,16}(X) \\
& - 13.38636373236149B_{6,16}(X) - 4383.815172945278B_{7,16}(X) - 5836.216572661172B_{8,16}(X) \\
& - 4370.505039757766B_{9,16}(X) + 13.40447373867094B_{10,16}(X) + 7315.596540631298B_{11,16}(X) \\
& + 17536.1552585308B_{12,16}(X) + 30675.1642498336B_{13,16}(X) + 46732.70666170018B_{14,16}(X) \\
& + 65708.86516603353B_{15,16}(X) + 87603.72195945767B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748852078437339, 0.6248088966923045\}$$

Intersection intervals with the x axis:

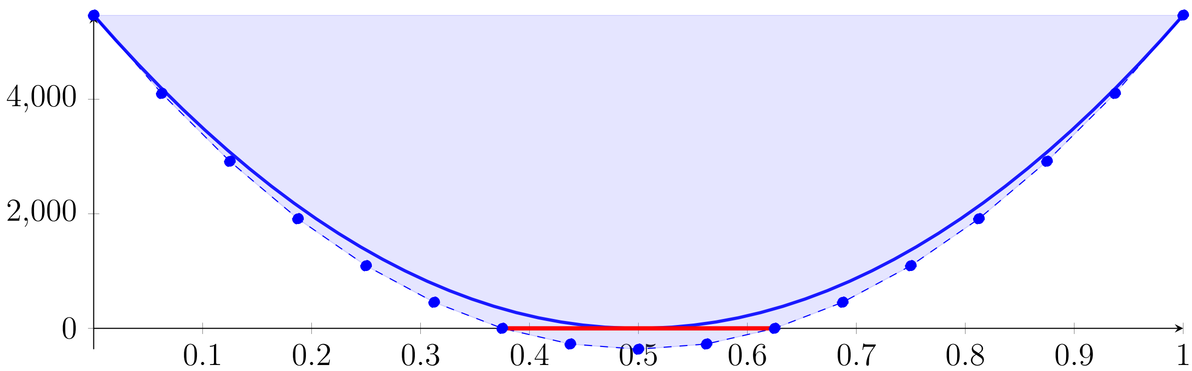
$$[0.3748852078437339, 0.6248088966923045]$$

Longest intersection interval: 0.2499236888485706
$\implies$ Selective recursion: interval 1: $[0.2995843091213109, 0.3004158775315354]$,

70.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.2995843091213109, 0.30041587753$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -5.228387005637929\cdot 10^{-50}X^{16} + 5.071723175262542\cdot 10^{-47}X^{15} + 1.695940836337518 \\
& \cdot 10^{-41}X^{14} - 1.602367332110541\cdot 10^{-38}X^{13} - 2.341982733873566\cdot 10^{-33}X^{12} \\
& + 2.036120445711374\cdot 10^{-30}X^{11} + 1.787779362171969\cdot 10^{-25}X^{10} - 1.346594590666257 \\
& \cdot 10^{-22}X^{9} - 8.158156485257833\cdot 10^{-18}X^{8} + 4.921695226295225\cdot 10^{-15}X^{7} \\
& + 2.227778844339431\cdot 10^{-10}X^{6} - 9.46914992513699\cdot 10^{-08}X^{5} - 0.003373649242827386X^{4} \\
& + 0.7523208927331248X^{3} + 21871.35693828986X^{2} - 21871.48147756637X + 5467.807676357308 \\
= & \; 5467.807676357308B_{0,16}(X) + 4100.84008400941B_{1,16}(X) + 2916.133799480594B_{2,16}(X) \\
& + 1913.690166201026B_{3,16}(X) + 1093.510525747217B_{4,16}(X) + 455.5962178420057B_{5,16}(X) \\
& - 0.05141964546838 65B_{6,16}(X) - 273.4310506997833B_{7,16}(X) - 364.541341159257B_{8,16}(X) \\
& - 273.3809587159691B_{9,16}(X) + 0.05142708421757425B_{10,16}(X) \\
& + 455.7571448416359B_{11,16}(X) + 1093.737521302792B_{12,16}(X) + 1913.993881360346B_{13,16}(X) \\
& + 2916.527548053087B_{14,16}(X) + 4101.339842565916B_{15,16}(X) + 5468.43208422982B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749929469011197, 0.6249882450180355\}$$

Intersection intervals with the x axis:

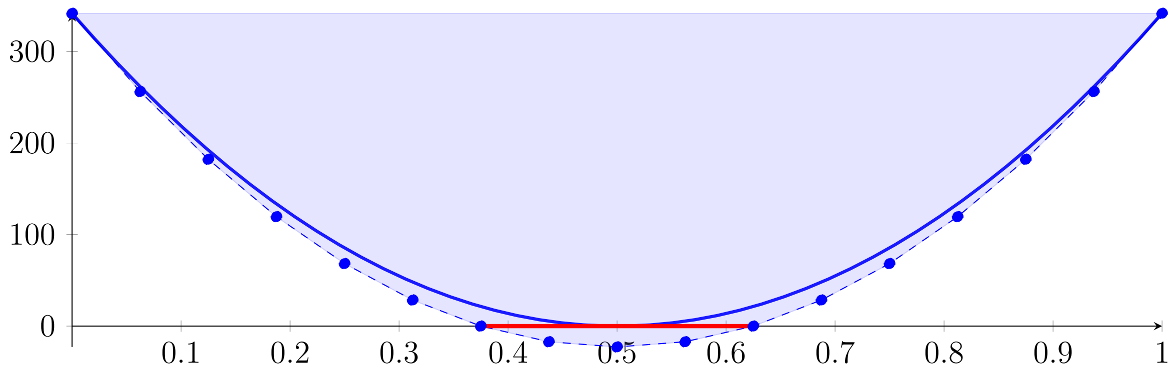
$$[0.3749929469011197, 0.6249882450180355]$$

Longest intersection interval: 0.2499952981169158
$\Longrightarrow$ Selective recursion: interval 1: $[0.2998961414100109, 0.3001040296026296]$,

70.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.2998961414100109, 0.300104029$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.216962444275584 \cdot 10^{-59} X^{16} + 4.692870826723641 \cdot 10^{-56} X^{15} + 6.316314535763934\\ &\cdot 10^{-50} X^{14} - 2.373865516252114 \cdot 10^{-46} X^{13} - 1.395661893422859 \cdot 10^{-40} X^{12}\\ &+ 4.828363722587868 \cdot 10^{-37} X^{11} + 1.70471850273246 \cdot 10^{-31} X^{10} - 5.110411085982617 \cdot 10^{-28} X^{9}\\ &- 1.244717766622564 \cdot 10^{-22} X^{8} + 2.988632715177156 \cdot 10^{-19} X^{7} + 5.438614058423278 \cdot 10^{-14} X^{6}\\ &- 9.197401057829169 \cdot 10^{-11} X^{5} - 1.317801761681953 \cdot 10^{-05} X^{4} + 0.01167528468045732 X^{3}\\ &+ 1366.961107526186 X^{2} - 1366.963198735845 X + 341.7398631167735\\ = &\ 341.7398631167735 B_{0,16}(X) + 256.3046631957831 B_{1,16}(X) + 182.260805837511 B_{2,16}(X)\\ &+ 119.6083118906798 B_{3,16}(X) + 68.34720219677136 B_{4,16}(X) + 28.47749759002708 B_{5,16}(X)\\ &- 0.0007811025525137025 B_{6,16}(X) - 17.08761306120757 B_{7,16}(X) - 22.782977473419 B_{8,16}(X)\\ &- 17.08685353390849 B_{9,16}(X) + 0.0007795553614777027 B_{10,16}(X)\\ &+ 28.4799425851876 B_{11,16}(X) + 68.35065633912575 B_{12,16}(X) + 119.6129415934909 B_{13,16}(X)\\ &+ 182.2668191173573 B_{14,16}(X) + 256.312309672558 B_{15,16}(X) + 341.7494340136854 B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749982857492878, 0.6249971486858456\}$$

Intersection intervals with the x axis:

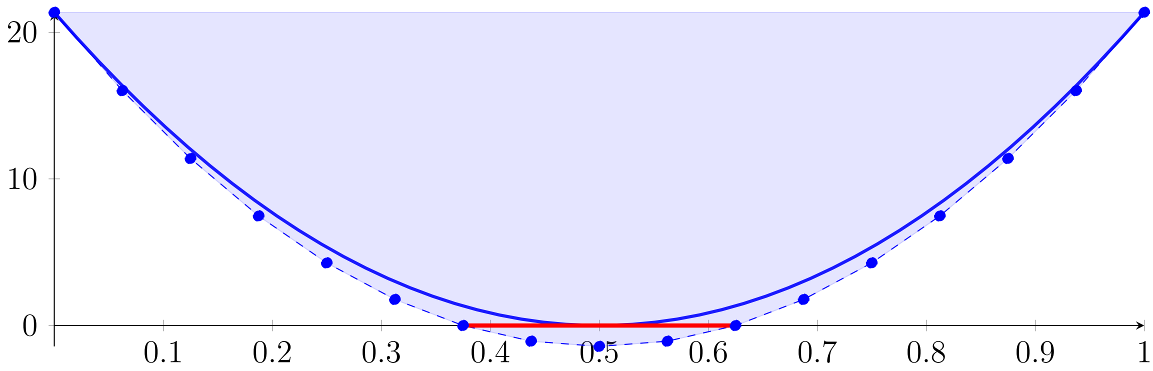
$$[0.3749982857492878, 0.6249971486858456]$$

Longest intersection interval: 0.2499988629365579
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999740991258704, 0.300026070937643]$,

70.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999740991258704, 0.3000260$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.833255302235776 \cdot 10^{-69} X^{16} + 4.363478560238502 \cdot 10^{-65} X^{15} + 2.35287052687137 \cdot 10^{-58} X^{14} - 3.532185105967992 \cdot 10^{-54} X^{13} - 8.318408043755505 \cdot 10^{-48} X^{12} + 1.149616718196078 \cdot 10^{-43} X^{11} + 1.625691292234855 \cdot 10^{-37} X^{10} - 1.946948798353601 \cdot 10^{-33} X^{9} - 1.899245778120626 \cdot 10^{-27} X^{8} + 1.821779493317236 \cdot 10^{-23} X^{7} + 1.327769541755276 \cdot 10^{-17} X^{6} - 8.969682874540076 \cdot 10^{-14} X^{5} - 5.147636798128115 \cdot 10^{-08} X^{4} + 0.000182114977798148 X^{3} + 85.43511227371199 X^{2} - 85.43514442584667 X + 21.35877059261705$$
$$= 21.35877059261705 B_{0,16}(X) + 16.01907406600164 B_{1,16}(X) + 11.39133680833382 B_{2,16}(X) + 7.475559144818918 B_{3,16}(X) + 4.271741400633969 B_{4,16}(X) + 1.779883900927722 B_{5,16}(X) - 1.302917935804413 \cdot 10^{-05} B_{6,16}(X) - 1.067949064595088 B_{7,16}(X) - 1.423923880255569 B_{8,16}(X) - 1.067937151125187 B_{9,16}(X) + 1.144780338983322 \cdot 10^{-05} B_{10,16}(X) + 1.779922241509208 B_{11,16}(X) + 4.271795554943032 B_{12,16}(X) + 7.47563171302734 B_{13,16}(X) + 11.39143104065633 B_{14,16}(X) + 16.01919386269591 B_{15,16}(X) + 21.35892050398371 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3749995424882778, 0.6249993300354412\}$$

Intersection intervals with the x axis:

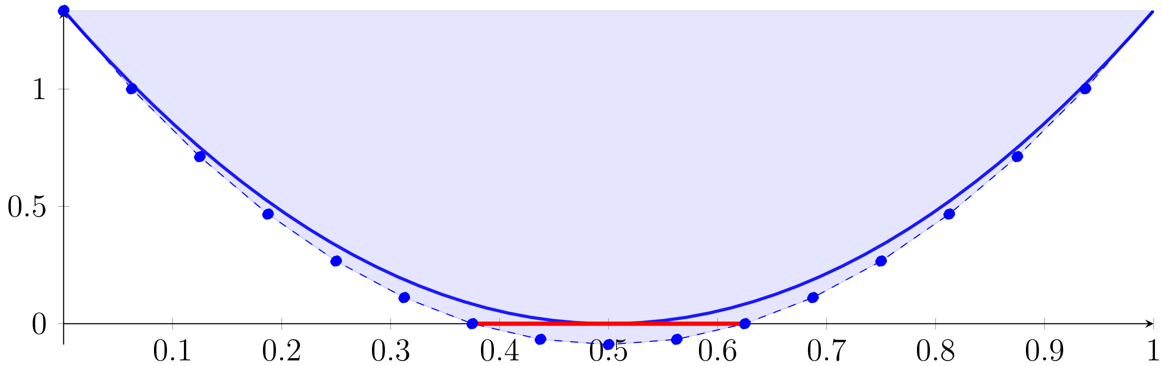
$$[0.3749995424882778, 0.6249993300354412]$$

Longest intersection interval: 0.2499997875471635
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999935885315074, 0.300006581473409]$,

70.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999935885315074, 0.30000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -6.596596862099299 \cdot 10^{-79} X^{16} + 4.062171102620318 \cdot 10^{-74} X^{15} + 8.765030607683244 \cdot 10^{-67} X^{14} - 5.261467161278336 \cdot 10^{-62} X^{13} - 4.958117265353456 \cdot 10^{-55} X^{12} + 2.739981763166457 \cdot 10^{-50} X^{11} + 1.550371378259149 \cdot 10^{-43} X^{10} - 7.424637900674377 \cdot 10^{-39} X^{9} - 2.898009393348535 \cdot 10^{-32} X^{8} + 1.111571585405421 \cdot 10^{-27} X^{7} + 3.241620002541647 \cdot 10^{-21} X^{6} - 8.756501261136649 \cdot 10^{-17} X^{5} - 2.010795357557322 \cdot 10^{-10} X^{4} + 2.844332798771609 \cdot 10^{-06} X^{3} + 5.339698243852309 X^{2} - 5.339698356505613 X + 1.33492347100565$$
$$= 1.33492347100565 B_{0,16}(X) + 1.00119232372405 B_{1,16}(X) + 0.7119586618078846 B_{2,16}(X) + 0.4672224903363214 B_{3,16}(X) + 0.266983814388415 B_{4,16}(X) + 0.1112426390431101 B_{5,16}(X) - 1.030620758846691 \cdot 10^{-06} B_{6,16}(X) - 0.066747189524468 19 B_{7,16}(X) - 0.0889958325894046 B_{8,16}(X) - 0.06674695473706526 B_{9,16}(X) - 5.508890578613216 \cdot 10^{-07} B_{10,16}(X) + 0.1112433840328995 B_{11,16}(X) + 0.2669848551069781 B_{12,16}(X) + 0.4672238674112388 B_{13,16}(X) + 0.7119604260236321 B_{14,16}(X) + 1.001194536021998 B_{15,16}(X) + 1.334926202484066 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3749994209666256, 0.6250003095051082\}$$

Intersection intervals with the x axis:

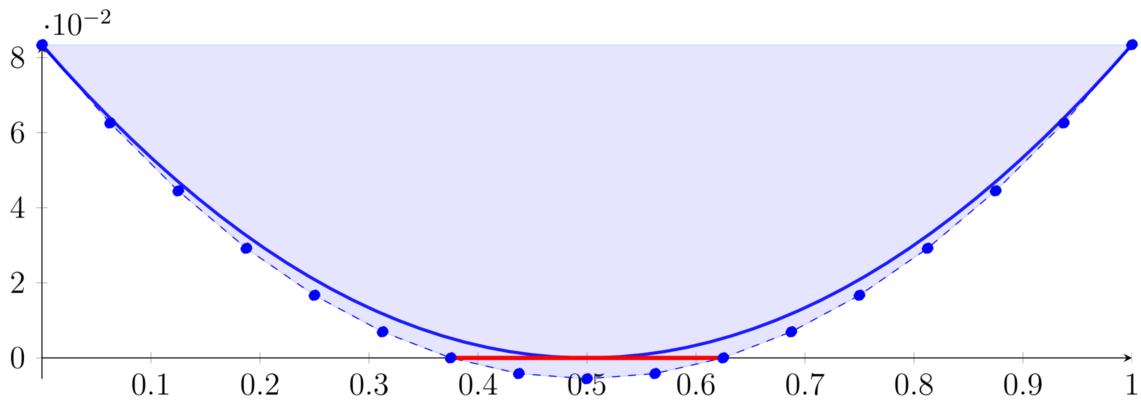
$$[0.3749994209666256, 0.6250003095051082]$$

Longest intersection interval: 0.2500008885384826
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999984608771972, 0.3000017091242173]$,

70.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999984608771972, 0.300$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.5359772362815 \cdot 10^{-88} X^{16} + 3.783024712863664 \cdot 10^{-83} X^{15} + 3.265391675561962 \cdot 10^{-75} X^{14} \\
&- 7.83987355616511 \cdot 10^{-70} X^{13} - 2.955395756595885 \cdot 10^{-62} X^{12} + 6.532349021061568 \\
&\cdot 10^{-57} X^{11} + 1.478602992973913 \cdot 10^{-49} X^{10} - 2.832143398905245 \cdot 10^{-44} X^{9} \\
&- 4.422140960586434 \cdot 10^{-37} X^{8} + 6.784132679612849 \cdot 10^{-32} X^{7} + 7.914287227230788 \cdot 10^{-25} X^{6} \\
&- 8.550710444673422 \cdot 10^{-20} X^{5} - 7.854787446136848 \cdot 10^{-13} X^{4} + 4.443846100456459 \\
&\cdot 10^{-08} X^{3} + 0.3337337124913315 X^{2} - 0.3337336004955191 X + 0.08343257581494635 \\
= &\ 0.08343257581494635 B_{0,16}(X) + 0.06257422578397641 B_{1,16}(X) + 0.04449699002376757 B_{2,16}(X) \\
&+ 0.02920086861367421 B_{3,16}(X) + 0.01668586163305031 B_{4,16}(X) \\
&+ 0.006951969161249391 B_{5,16}(X) - 8.08722375440678 \cdot 10^{-07} B_{6,16}(X) \\
&- 0.004172471938471519 B_{7,16}(X) - 0.005563020407686608 B_{8,16}(X) \\
&- 0.004172454050668901 B_{9,16}(X) - 7.727880670249089 \cdot 10^{-07} B_{10,16}(X) \\
&+ 0.006952023459469962 B_{11,16}(X) + 0.01668593477129257 B_{12,16}(X) \\
&+ 0.02920096122675088 B_{13,16}(X) + 0.04449710290519453 B_{14,16}(X) \\
&+ 0.06257435988597275 B_{15,16}(X) + 0.08343273224843432 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749927302224649, 0.6250069467380417\}$$

Intersection intervals with the x axis:

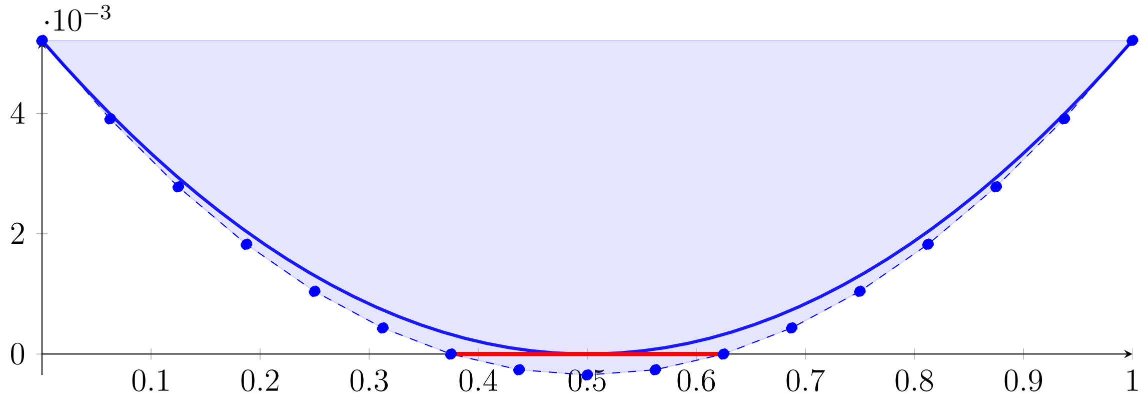
$$[0.3749927302224649, 0.6250069467380417]$$

Longest intersection interval: 0.2500142165155768
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999996789462157, 0.3000004910541495]$,

70.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999996789462157, 0.3$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.579480927696053\cdot 10^{-98}X^{16} + 3.526136882584451\cdot 10^{-92}X^{15} + 1.217422078989368 \\
& \cdot 10^{-83}X^{14} - 1.169070552755016\cdot 10^{-77}X^{13} - 1.762755817503918\cdot 10^{-69}X^{12} \\
& + 1.55837613779483\cdot 10^{-63}X^{11} + 1.410908032488726\cdot 10^{-55}X^{10} - 1.080908862805813\cdot 10^{-49}X^{9} \\
& - 6.750723381404434\cdot 10^{-42}X^{8} + 4.142273519440928\cdot 10^{-36}X^{7} + 1.932858818028869\cdot 10^{-28}X^{6} \\
& - 8.352503738960762\cdot 10^{-23}X^{5} - 3.06897495525372\cdot 10^{-15}X^{4} + 6.944510025023856\cdot 10^{-10}X^{3} \\
& + 0.02086073248825264X^{2} - 0.0208607236297874X + 0.00521438785078781 8 \\
= & \, 0.005214387850787818B_{0,16}(X) + 0.003910592623926106B_{1,16}(X) \\
& + 0.002780636834466498B_{2,16}(X) + 0.001824520483649087B_{3,16}(X) \\
& + 0.001042243572713962B_{4,16}(X) + 0.0004338061029012 11B_{5,16}(X) - 7.919245490808786 \\
& \cdot 10^{-07}B_{6,16}(X) - 0.0002615505083968289B_{7,16}(X) - 0.0003484696474019504B_{8,16}(X) \\
& - 0.0002615493403243644B_{9,16}(X) - 7.895859239916112\cdot 10^{-07}B_{10,16}(X) \\
& + 0.0004338096170392455B_{11,16}(X) + 0.001042248269805423B_{12,16}(X) \\
& + 0.001824526373614615B_{13,16}(X) + 0.002780643929706894B_{14,16}(X) \\
& + 0.00391060093932233B_{15,16}(X) + 0.005214397403700994B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748861124966261, 0.6251135508761015\}$$

Intersection intervals with the x axis:

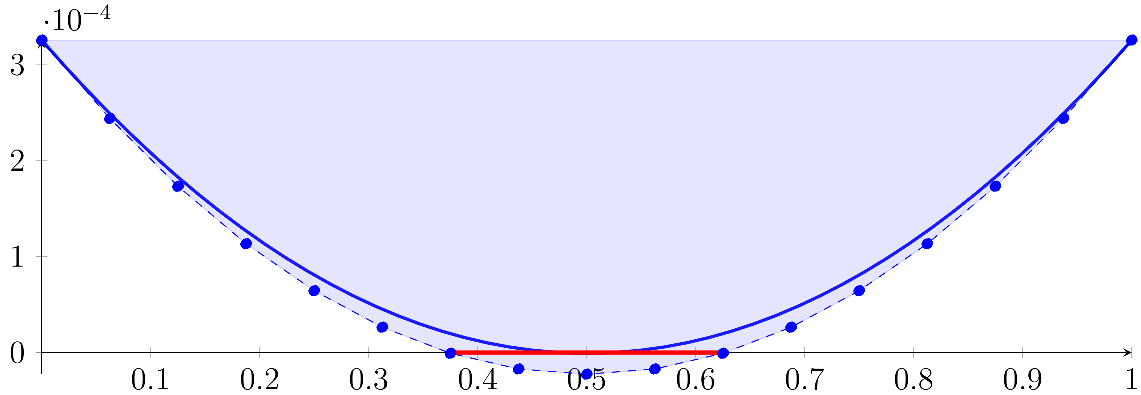
$$[0.3748861124966261, 0.6251135508761015]$$

Longest intersection interval: 0.2502274383794754
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999999833942019, 0.3000001866058899]$,

70.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999999833942019, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = &-8.456271781717561\cdot10^{-108}X^{16} + 3.329051186798107\cdot10^{-101}X^{15} + 4.593357002546748\\
&\cdot10^{-92}X^{14} - 1.76275695811523\cdot10^{-85}X^{13} - 1.062212310975265\cdot10^{-76}X^{12}\\
&+ 3.752790404631579\cdot10^{-70}X^{11} + 1.357838209904933\cdot10^{-61}X^{10} - 4.157203866707784\\
&\cdot10^{-55}X^{9} - 1.037599553128416\cdot10^{-46}X^{8} + 2.544375234544489\cdot10^{-40}X^{7} + 4.744710700950638\\
&\cdot10^{-32}X^{6} - 8.193869976131749\cdot10^{-26}X^{5} - 1.203186876925119\cdot10^{-17}X^{4} + 1.088036641260801\\
&\cdot10^{-11}X^{3} + 0.001306169174102627X^{2} - 0.001306168592474289X + 0.0003257512461139167\\
= &\,0.0003257512461139167B_{0,16}(X) + 0.0002441157090842736B_{1,16}(X)\\
&+ 0.0001733649151721524B_{2,16}(X) + 0.0001134988643969824B_{3,16}(X)\\
&+ 6.451755677819264\cdot10^{-05}B_{4,16}(X) + 2.642099233521247\cdot10^{-05}B_{5,16}(X)\\
&- 7.908289125289409\cdot10^{-07}B_{6,16}(X) - 1.71179069456024\cdot10^{-05}B_{7,16}(X) - 2.25602417445787\\
&\cdot10^{-05}B_{8,16}(X) - 1.711783329002867\cdot10^{-05}B_{9,16}(X) - 7.906815625231293\\
&\cdot10^{-07}B_{10,16}(X) + 2.64212134573671\cdot10^{-05}B_{11,16}(X) + 6.451785178907119\\
&\cdot10^{-05}B_{12,16}(X) + 0.0001134992334520183B_{13,16}(X) + 0.0001733653584656376B_{14,16}(X)\\
&+ 0.0002441162268493581B_{15,16}(X) + 0.0003257518386226092B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3731836273807965, 0.626816029262996\}$$

Intersection intervals with the x axis:

$$[0.3731836273807965, 0.626816029262996]$$

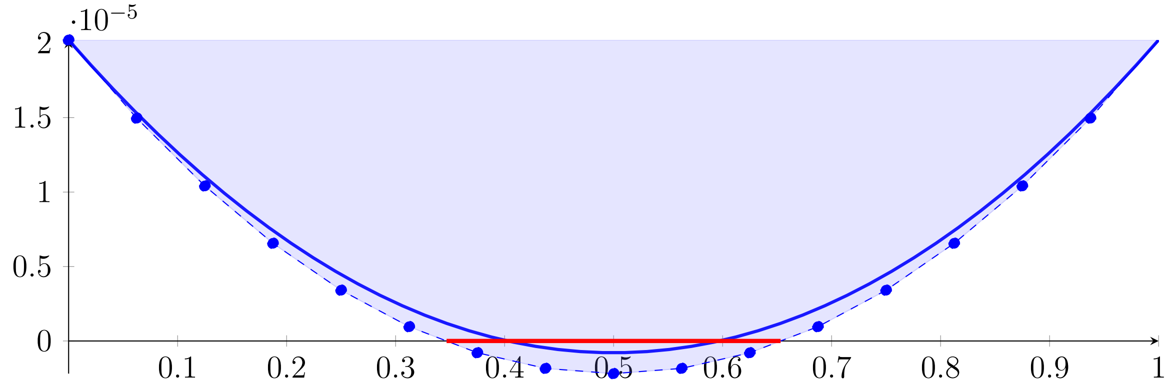
Longest intersection interval: 0.2536324018821995

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000592294767, 0.3000001107705452]$,

70.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000592294767

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.480017149294215\cdot 10^{-117}X^{16} + 3.84938040332832\cdot 10^{-110}X^{15} + 2.094095506848417\\&\cdot 10^{-100}X^{14} - 3.168497686663236\cdot 10^{-93}X^{13} - 7.527801307502411\cdot 10^{-84}X^{12}\\&+ 1.048590305184717\cdot 10^{-76}X^{11} + 1.495875245120848\cdot 10^{-67}X^{10} - 1.80569087496117\cdot 10^{-60}X^{9}\\&- 1.776919108083797\cdot 10^{-51}X^{8} + 1.717962865912528\cdot 10^{-44}X^{7} + 1.263101207166261\cdot 10^{-35}X^{6}\\&- 8.600271811104809\cdot 10^{-29}X^{5} - 4.979113541149949\cdot 10^{-20}X^{4} + 1.775239724826758\cdot 10^{-13}X^{3}\\&+ 8.402507389292435\cdot 10^{-05}X^{2} - 8.402503818595556\cdot 10^{-05}X + 2.02154953599674\cdot 10^{-05}\\= &\ 2.02154953599674\cdot 10^{-05}B_{0,16}(X) + 1.496393047334518\cdot 10^{-05}B_{1,16}(X) + 1.041257453583066\\&\cdot 10^{-05}B_{2,16}(X) + 6.561427547740848\cdot 10^{-06}B_{3,16}(X) + 3.410489509392755\cdot 10^{-06}B_{4,16}(X)\\&+ 9.59760421103387\cdot 10^{-07}B_{5,16}(X) - 7.907597168102504\cdot 10^{-07}B_{6,16}(X) - 1.84107090403115\\&\cdot 10^{-06}B_{7,16}(X) - 2.191173140242304\cdot 10^{-06}B_{8,16}(X) - 1.841066425126706\cdot 10^{-06}B_{9,16}(X)\\&- 7.907507583673494\cdot 10^{-07}B_{10,16}(X) + 9.59773860352773\cdot 10^{-07}B_{11,16}(X) + 3.410507431350668\\&\cdot 10^{-06}B_{12,16}(X) + 6.561449954943343\cdot 10^{-06}B_{13,16}(X) + 1.04126014314478\cdot 10^{-05}B_{14,16}(X)\\&+ 1.496396186118106\cdot 10^{-05}B_{15,16}(X) + 2.021553124446011\cdot 10^{-05}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3467669730097793, 0.6532326348738207\}$$

Intersection intervals with the x axis:

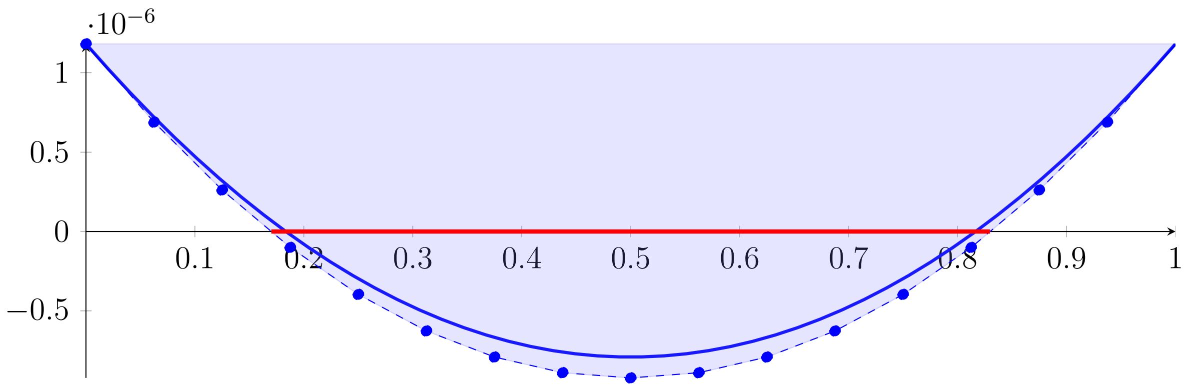
$$[0.3467669730097793, 0.6532326348738207]$$

Longest intersection interval: 0.3064656618640414
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000771022171, 0.3000000928977847]$,

70.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000771022171, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.50163304400299 \cdot 10^{-125} X^{16} + 7.605328677353438 \cdot 10^{-118} X^{15} + 1.35002551430803 \\ &\cdot 10^{-107} X^{14} - 6.665257392422306 \cdot 10^{-100} X^{13} - 5.167137254241996 \cdot 10^{-90} X^{12} \\ &+ 2.348582048403403 \cdot 10^{-82} X^{11} + 1.093235143477049 \cdot 10^{-72} X^{10} - 4.306055922184639 \cdot 10^{-65} X^{9} \\ &- 1.382681732768409 \cdot 10^{-55} X^{8} + 4.362007350883115 \cdot 10^{-48} X^{7} + 1.04647552702743 \cdot 10^{-38} X^{6} \\ &- 2.324990317134255 \cdot 10^{-31} X^{5} - 4.392171749769965 \cdot 10^{-22} X^{4} + 5.109781163448487 \cdot 10^{-15} X^{3} \\ &+ 7.891735947253277 \cdot 10^{-06} X^{2} - 7.891735064633028 \cdot 10^{-06} X + 1.182178307271875 \cdot 10^{-06} \\ = &\ 1.182178307271875 \cdot 10^{-06} B_{0,16}(X) + 6.889448657323109 \cdot 10^{-07} B_{1,16}(X) + 2.614758904198573 \\ &\cdot 10^{-07} B_{2,16}(X) - 1.00228618656361 \cdot 10^{-07} B_{3,16}(X) - 3.961686614872195 \cdot 10^{-07} B_{4,16}(X) \\ &- 6.263442380635935 \cdot 10^{-07} B_{5,16}(X) - 7.907553483763585 \cdot 10^{-07} B_{6,16}(X) - 8.894019924163897 \\ &\cdot 10^{-07} B_{7,16}(X) - 9.222841701745627 \cdot 10^{-07} B_{8,16}(X) - 8.894018816417527 \cdot 10^{-07} B_{9,16}(X) \\ &- 7.907551268088353 \cdot 10^{-07} B_{10,16}(X) - 6.263439056666857 \cdot 10^{-07} B_{11,16}(X) \\ &- 3.961682182061795 \cdot 10^{-07} B_{12,16}(X) - 1.002280644181919 \cdot 10^{-07} B_{13,16}(X) + 2.614765557064017 \\ &\cdot 10^{-07} B_{14,16}(X) + 6.889456421767258 \cdot 10^{-07} B_{15,16}(X) + 1.182179195001905 \cdot 10^{-06} B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1701811983018366, 0.8298187006126137\}$$

Intersection intervals with the x axis:

$$[0.1701811983018366, 0.8298187006126137]$$

Longest intersection interval: 0.6596375023107771
$\Longrightarrow$ Bisection: first half $[0.3000000771022171, 0.3000000850000009]$ und second half $[0.3000000850000009, 0.30$

70.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3000000771022171, 0.3000000850000009]$

Found root in interval $[0.3000000771022171, 0.3000000850000009]$ at recursion depth 15!

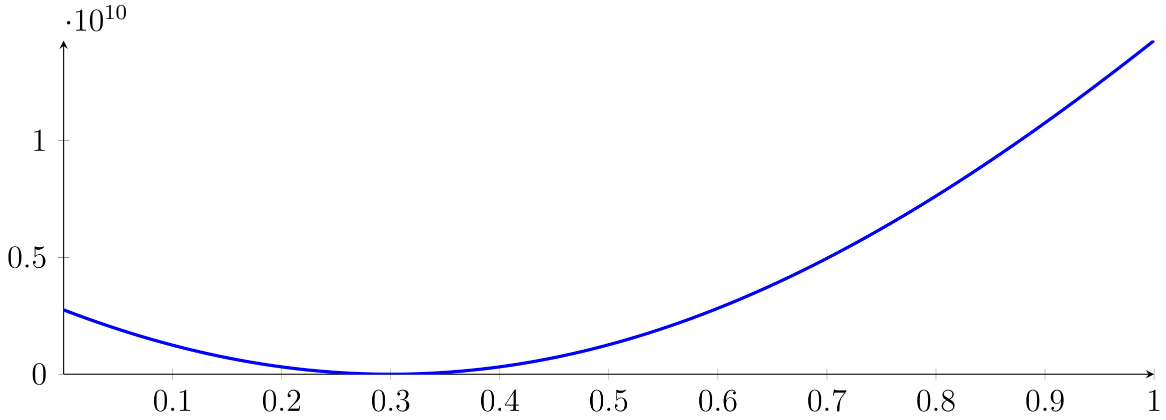
70.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.3000000850000009, 0.300000092897847]$

Found root in interval $[0.3000000850000009, 0.300000092897847]$ at recursion depth 15!

70.17 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.3000000771022171, 0.3000000850000009], [0.3000000850000009, 0.3000000928977847]$$

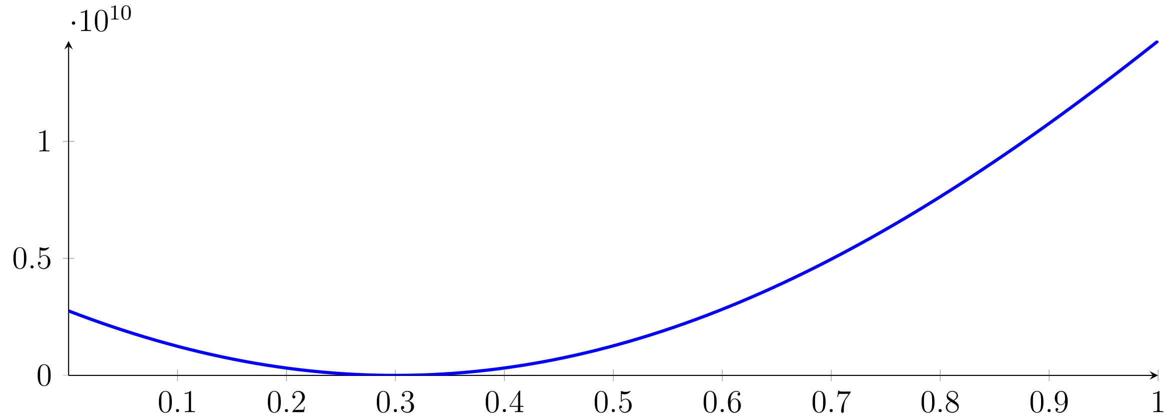
with precision $\varepsilon = 1 \cdot 10^{-08}$.

71 Running `QuadClip` on h_{16} with epsilon 8

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

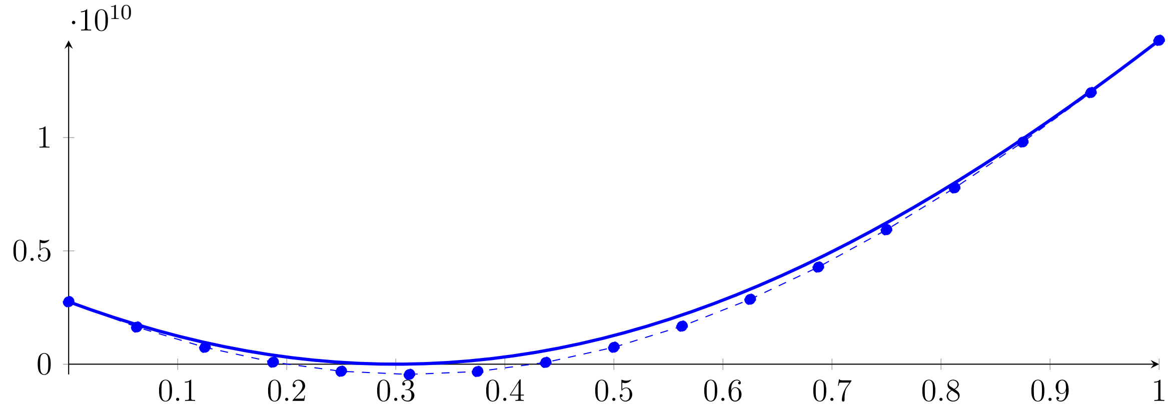
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



71.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

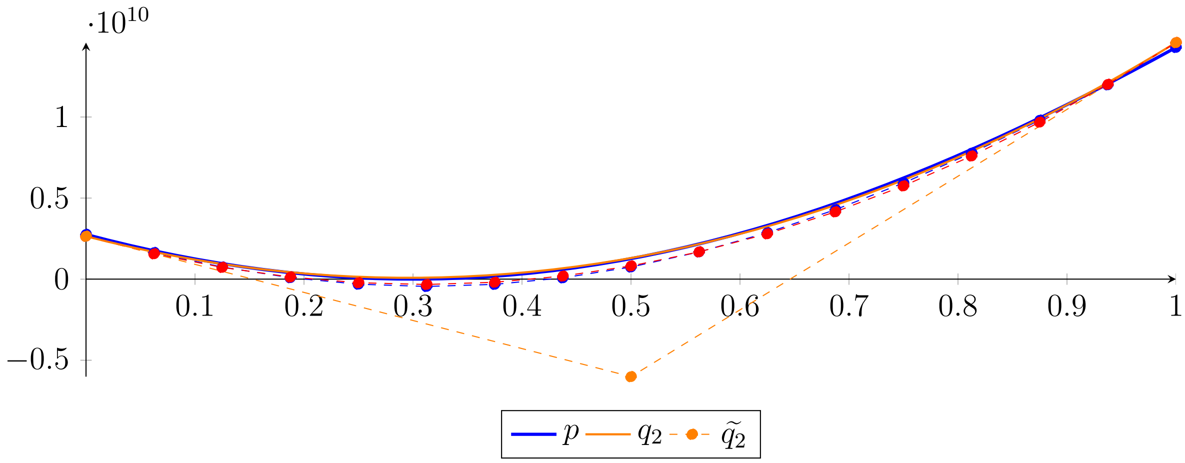
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328\\
&= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = &-4.590615853685512\cdot 10^{-286}X^{16} + 3.626297739922361\cdot 10^{-285}X^{15} - 1.337118562447102\\
&\cdot 10^{-284}X^{14} + 3.057409950495903\cdot 10^{-284}X^{13} - 4.816080313213711\cdot 10^{-284}X^{12}\\
&+ 5.445450819003583\cdot 10^{-284}X^{11} - 4.453195910004646\cdot 10^{-284}X^{10} + 2.601913123147853\\
&\cdot 10^{-284}X^{9} - 1.060485581673177\cdot 10^{-284}X^{8} + 2.936863540613122\cdot 10^{-285}X^{7} - 5.487274374859613\\
&\cdot 10^{-286}X^{6} + 7.23018920152705\cdot 10^{-287}X^{5} - 7.254244946204871\cdot 10^{-288}X^{4} + 6.659990111567495\\
&\cdot 10^{-289}X^{3} + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328\\
= &\ 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16}\\
&+ 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16}\\
&- 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16}\\
&+ 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16}\\
&+ 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16}\\
&+ 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.

Bounding polynomials M and m:

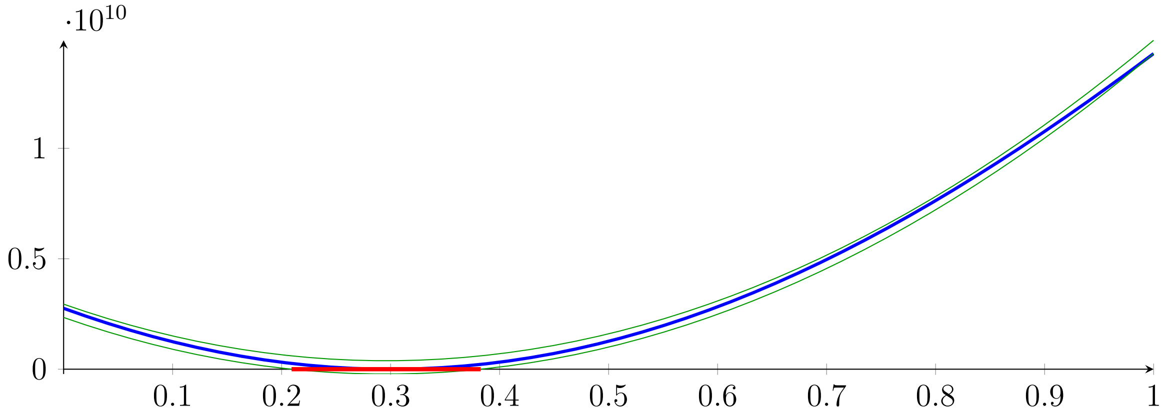
$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$

$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



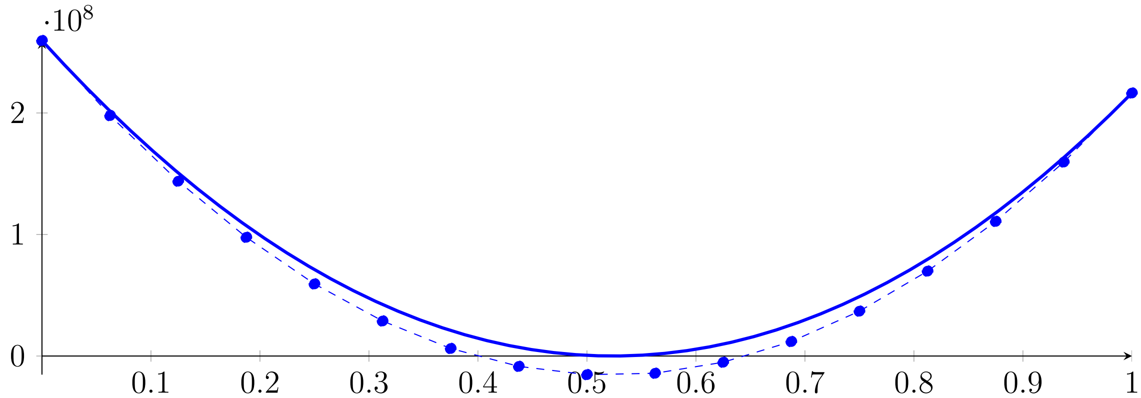
$[0.2091580131065301, 0.3826391383239738]$

Longest intersection interval: 0.1734811252174437
$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

71.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

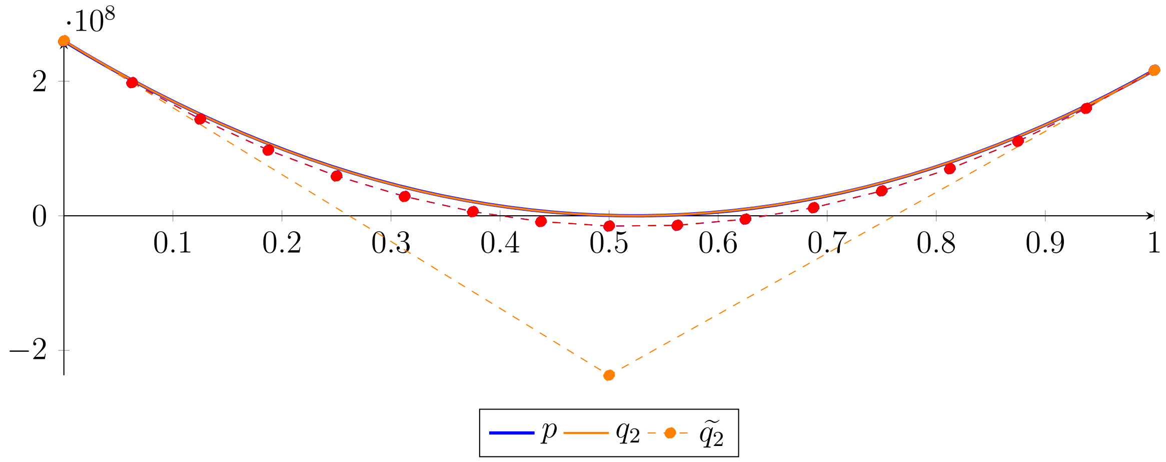
$$\begin{aligned}
p = & -6.730319568869656\cdot 10^{-13}X^{16} + 8.742499458235574\cdot 10^{-12}X^{15} + 4.969734366538428\cdot 10^{-09}X^{14} \\
& - 5.918022439742958\cdot 10^{-08}X^{13} - 1.563835677279211\cdot 10^{-05}X^{12} + 0.000165142833990623X^{11} \\
& + 0.02725178204957372X^{10} - 0.2448183330655992X^{9} - 28.45820232077076X^{8} \\
& + 205.241311077626X^{7} + 17835.39577074806X^{6} - 94141.21233496903X^{5} - 6218439.344794644X^{4} \\
& + 20000843.19616221X^{3} + 930858984.7988209X^{2} - 987724621.0538478X + 259570777.0276934 \\
= & \, 259570777.0276934B_{0,16}(X) + 197837988.2118279B_{1,16}(X) + 143862357.6026193B_{2,16}(X) \\
& + 97679600.99148916B_{3,16}(X) + 59322017.44494463B_{4,16}(X) + 28818467.75210257B_{5,16}(X) \\
& + 6194355.099411763B_{6,16}(X) - 8528392.009487306B_{7,16}(X) - 15331334.57303514B_{8,16}(X) \\
& - 14199535.63666165B_{9,16}(X) - 5121570.552084079B_{10,16}(X) + 11910465.05798682B_{11,16}(X) \\
& + 36900949.63643701B_{12,16}(X) + 69850732.33405701B_{13,16}(X) \\
& + 110757131.9597003B_{14,16}(X) + 159613938.238137B_{15,16}(X) + 216411415.3731615B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 = & \, 950064117.9944898X^2 - 993959420.5216784X + 260029868.8092426 \\
= & \, 260029868.8092426B_{0,2} - 236949841.4515966B_{1,2} + 216134566.282054B_{2,2}
\end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_2 &= -1.41520176718356{\cdot}10^{-287}X^{16} + 1.139753408092347{\cdot}10^{-286}X^{15} - 4.276104932493763 \\
&\quad {\cdot}10^{-286}X^{14} + 9.896028733517596{\cdot}10^{-286}X^{13} - 1.566233430583374{\cdot}10^{-285}X^{12} \\
&\quad + 1.767044733566612{\cdot}10^{-285}X^{11} - 1.435814571471649{\cdot}10^{-285}X^{10} + 8.336078570237132 \\
&\quad {\cdot}10^{-286}X^{9} - 3.396584250429756{\cdot}10^{-286}X^{8} + 9.548198482934522{\cdot}10^{-287}X^{7} - 1.861944039074976 \\
&\quad {\cdot}10^{-287}X^{6} + 2.622613520689986{\cdot}10^{-288}X^{5} - 2.667693132942884{\cdot}10^{-289}X^{4} + 2.048626792988895 \\
&\quad {\cdot}10^{-290}X^{3} + 950064117.9944898X^{2} - 993959420.5216784X + 260029868.8092426 \\
&= 260029868.8092426B_{0,16} + 197907405.0266377B_{1,16} + 143702142.2273202B_{2,16} \\
&\quad + 97414080.41129016B_{3,16} + 59043219.5785475B_{4,16} + 28589559.72909226B_{5,16} \\
&\quad + 6053100.862924435B_{6,16} - 8566157.019955976B_{7,16} - 15268213.91954897B_{8,16} \\
&\quad - 14053069.83585455B_{9,16} - 4920724.768872719B_{10,16} + 12128821.28139653B_{11,16} \\
&\quad + 37095568.31495319B_{12,16} + 69979516.33179727B_{13,16} \\
&\quad + 110780665.3319288B_{14,16} + 159499015.3153477B_{15,16} + 216134566.282054B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

Bounding polynomials M and m:

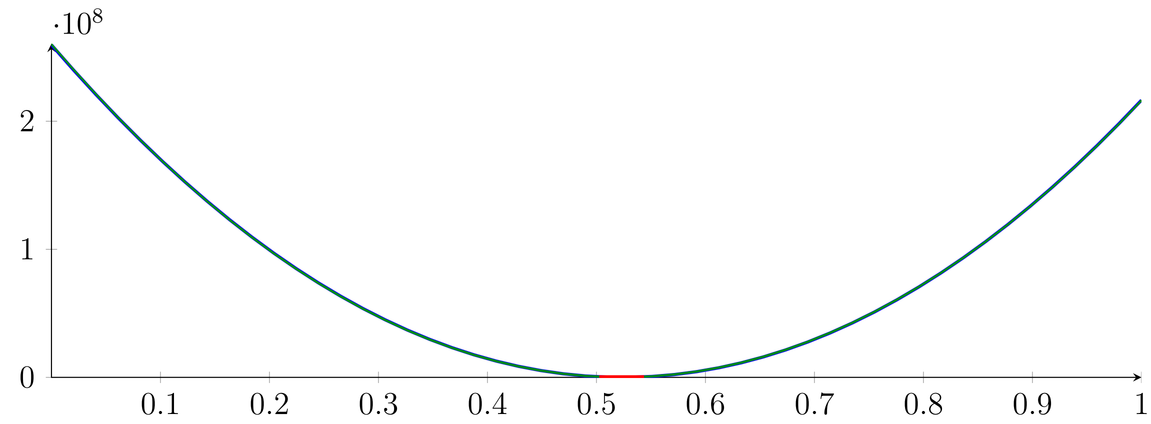
$$M = 950064117.9944898X^2 - 993959420.5216784X + 260488960.5907918$$

$$m = 950064117.9944898X^2 - 993959420.5216784X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

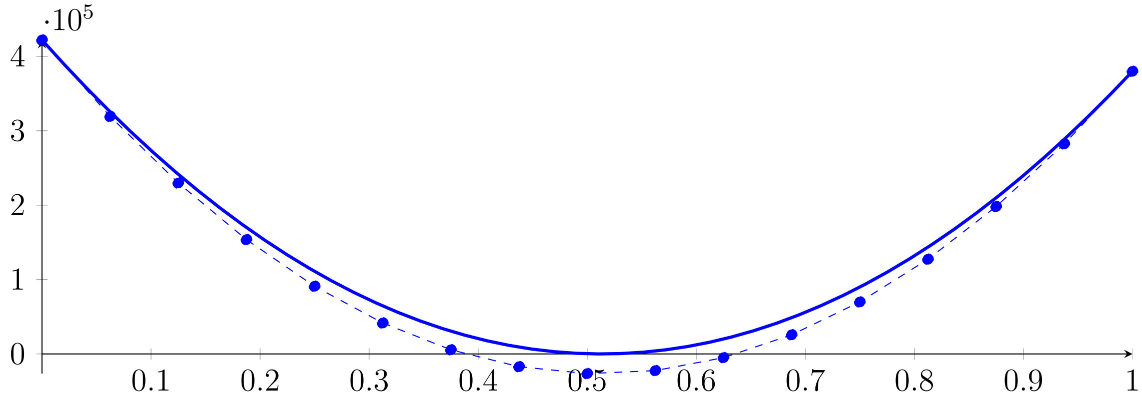
Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

71.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Normalized monomial und Bézier representations and the Bézier polygon:

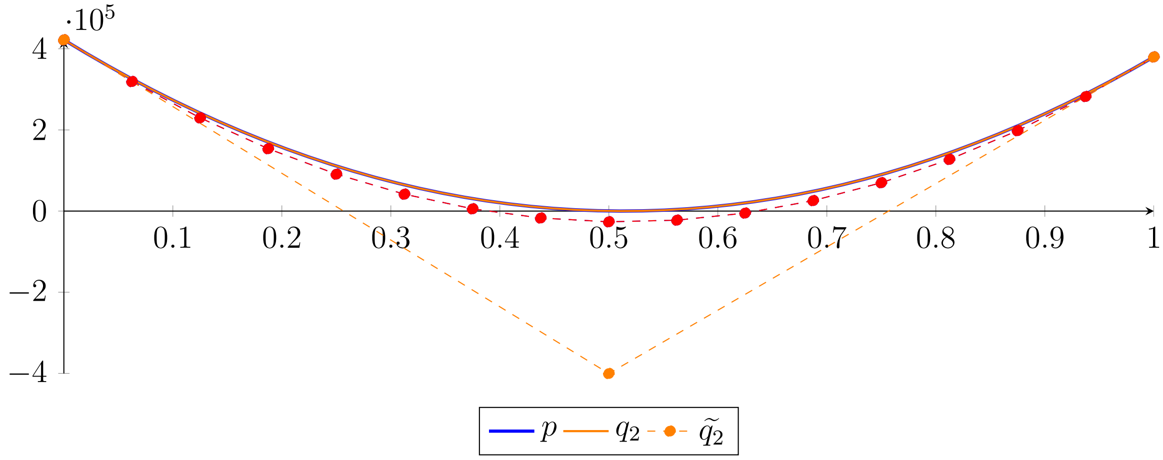
$$\begin{aligned}p = & -4.348008654913047\cdot10^{-35}X^{16} + 5.243388239594399\cdot10^{-33}X^{15} + 1.924264426249969\\ & \cdot10^{-28}X^{14} - 2.246746658841888\cdot10^{-26}X^{13} - 3.625525335135332\cdot10^{-22}X^{12}\\ & + 3.881306066232412\cdot10^{-20}X^{11} + 3.776139474106064\cdot10^{-16}X^{10} - 3.496022725744\\ & \cdot10^{-14}X^{9} - 2.35123279599996\cdot10^{-10}X^{8} + 1.7435554055155\cdot10^{-08}X^{7} + 8.761425585865998\\ & \cdot10^{-05}X^{6} - 0.004592127899352905X^{5} - 18.10659249096443X^{4} + 504.889471438631X^{3}\\ & + 1602084.653024796X^{2} - 1644731.182248784X + 422061.2284789393\\ = & \, 422061.2284789393B_{0,16}(X) + 319265.5295883902B_{1,16}(X) + 229820.5361397145B_{2,16}(X)\\ & + 153727.1497212539B_{3,16}(X) + 90986.26197267314B_{4,16}(X) + 41598.75458390835B_{5,16}(X)\\ & + 5565.499294126806B_{6,16}(X) - 17112.6421093025B_{7,16}(X) - 26434.81779182738B_{8,16}(X)\\ & - 22400.18587271997B_{9,16}(X) - 5007.914426073306B_{10,16}(X) + 25742.81851821299B_{11,16}(X)\\ & + 69852.82497345804B_{12,16}(X) + 127322.906995236B_{13,16}(X) + 198153.8566804232B_{14,16}(X)\\ & + 282346.4561662563B_{15,16}(X) + 379901.4776294016B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 = & \, 1602810.939315368X^2 - 1645017.556512812X + 422084.9204772878\\ = & \, 422084.9204772878B_{0,2} - 400423.8577791184B_{1,2} + 379878.3032798431B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = & -2.394754881528157\cdot10^{-290}X^{16} + 1.927007748221533\cdot10^{-289}X^{15} - 7.222560694913459\\ & \cdot10^{-289}X^{14} + 1.669823679170844\cdot10^{-288}X^{13} - 2.640537385502685\cdot10^{-288}X^{12}\\ & + 2.977245352423787\cdot10^{-288}X^{11} - 2.4183464566446\cdot10^{-288}X^{10} + 1.403945843258626\cdot10^{-288}X^{9}\\ & - 5.721369736991339\cdot10^{-289}X^{8} + 1.608948017596564\cdot10^{-289}X^{7} - 3.139550473135121\cdot10^{-290}X^{6}\\ & + 4.427570920272867\cdot10^{-291}X^{5} - 4.516145205270189\cdot10^{-292}X^{4} + 3.478035897244226\\ & \cdot10^{-293}X^{3} + 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\ = & \, 422084.9204772878B_{0,16} + 319271.323195237B_{1,16} + 229814.4837408143B_{2,16}\\ & + 153714.4021140197B_{3,16} + 90971.07831485311B_{4,16} + 41584.51234331459B_{5,16}\\ & + 5554.704199404135B_{6,16} - 17118.34611687825B_{7,16} - 26434.63860553258B_{8,16}\\ & - 22394.17326655884B_{9,16} - 4996.95009995704B_{10,16} + 25757.03089427283B_{11,16}\\ & + 69867.76971613076B_{12,16} + 127335.2663656168B_{13,16} + 198159.5208427308B_{14,16}\\ & + 282340.5331474729B_{15,16} + 379878.3032798431B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 23.69199834856345$.
Bounding polynomials M and m:

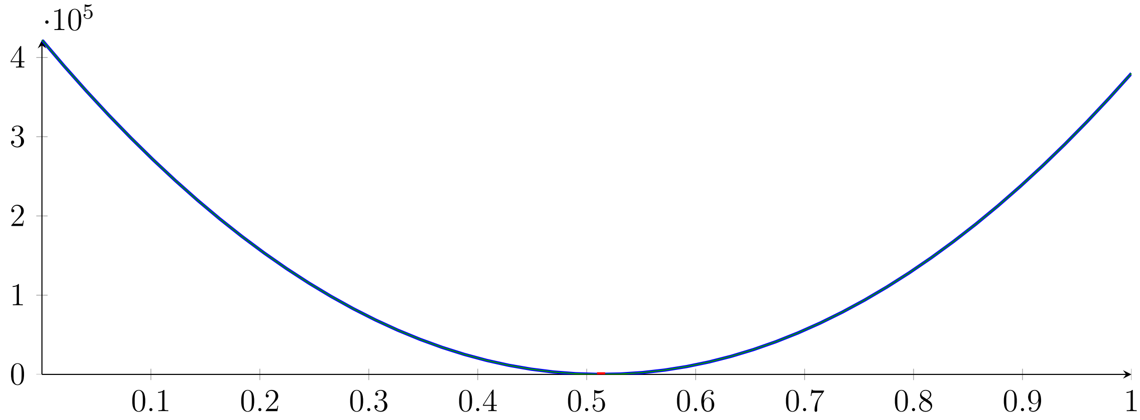
$$M = 1602810.939315368X^2 - 1645017.556512812X + 422108.6124756364$$

$$m = 1602810.939315368X^2 - 1645017.556512812X + 422061.2284789393$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.509405571919099, 0.5169273012614853\}$$

Intersection intervals:



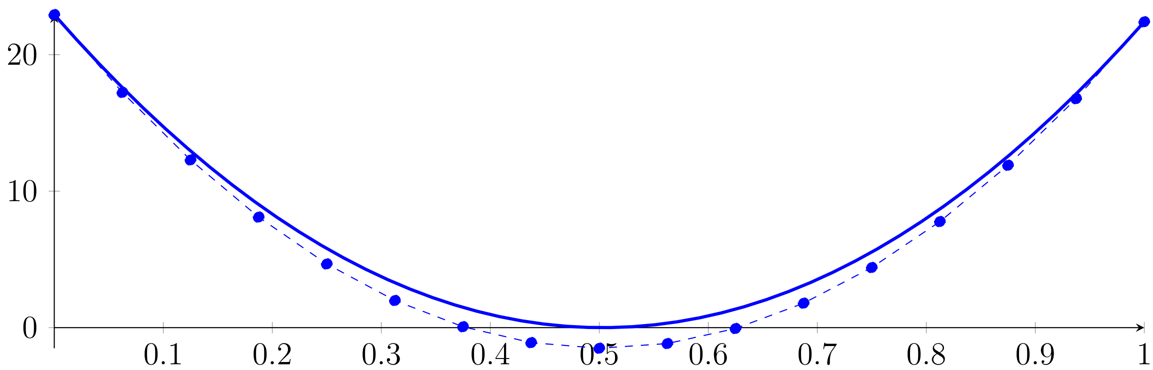
$$[0.509405571919099, 0.5169273012614853]$$

Longest intersection interval: 0.007521729342386311
$\implies$ Selective recursion: interval 1: $[0.299973157656462, 0.3000267016613888]$,

71.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.299973157656462, 0.30002670166138$

Normalized monomial und Bézier representations and the Bézier polygon:

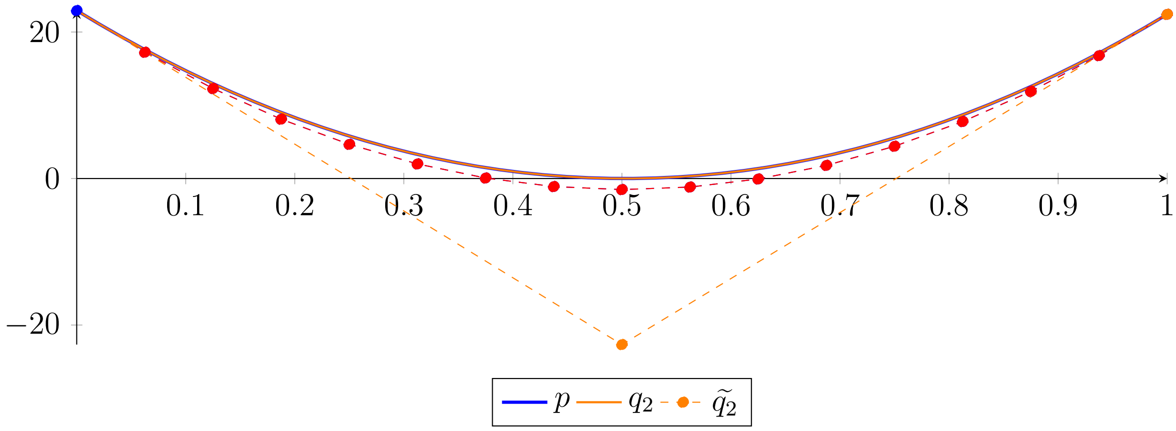
$$p = -4.564294214760651\cdot 10^{-69}X^{16} + 6.823166131086062\cdot 10^{-65}X^{15} + 3.571082953880241\cdot 10^{-58}X^{14} - 5.203669764436854\cdot 10^{-54}X^{13} - 1.189477187926378\cdot 10^{-47}X^{12} + 1.595632405197401\cdot 10^{-43}X^{11} + 2.190120291778079\cdot 10^{-37}X^{10} - 2.545939358967532\cdot 10^{-33}X^{9} - 2.410599475027015\cdot 10^{-27}X^{8} + 2.244415896590929\cdot 10^{-23}X^{7} + 1.587744426227316\cdot 10^{-17}X^{6} - 1.041115833763881\cdot 10^{-13}X^{5} - 5.799356524856294\cdot 10^{-08}X^{4} + 0.0001991514832499812X^{3} + 90.68225982541923X^{2} - 91.20858295780928X + 22.93446410530317$$
$$= 22.93446410530317B_{0,16}(X) + 17.23392767044009B_{1,16}(X) + 12.28907673412217B_{2,16}(X) + 8.099911651977055B_{3,16}(X) + 4.666432779600538B_{4,16}(X) + 1.988640472556532B_{5,16}(X) + 0.06653508637709412B_{6,16}(X) - 1.099883023437587B_{7,16}(X) - 1.510613501419185B_{8,16}(X) - 1.165655992131239B_{9,16}(X) - 0.0650101401691539B_{10,16}(X) + 1.791324409839802B_{11,16}(X) + 4.403348013236495B_{12,16}(X) + 7.771061025329927B_{13,16}(X) + 11.89446380139723B_{14,16}(X) + 16.77355669668369B_{15,16}(X) + 22.4083400664027B_{16,16}(X)$$



Degree reduction and raising:

$$q_2 = 90.68255845322638X^2 - 91.20870239567643X + 22.93447405790644$$
$$= 22.93447405790644B_{0,2} - 22.66987713993177B_{1,2} + 22.40833011545639B_{2,2}$$

$$\widetilde{q}_2 = -1.359001790090183\cdot 10^{-294}X^{16} + 1.092598333426672\cdot 10^{-293}X^{15} - 4.09106565145402\cdot 10^{-293}X^{14} + 9.449139850665356\cdot 10^{-293}X^{13} - 1.493001927703419\cdot 10^{-292}X^{12} + 1.682443127957849\cdot 10^{-292}X^{11} - 1.366228291670638\cdot 10^{-292}X^{10} + 7.931222707714909\cdot 10^{-293}X^{9} - 3.23261546930867\cdot 10^{-293}X^{8} + 9.093117628361849\cdot 10^{-294}X^{7} - 1.775022991079505\cdot 10^{-294}X^{6} + 2.505263643899121\cdot 10^{-295}X^{5} - 2.56169827233206\cdot 10^{-296}X^{4} + 1.979687487137548\cdot 10^{-297}X^{3} + 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644$$
$$= 22.93447405790644B_{0,16} + 17.23393015817666B_{1,16} + 12.28907424555711B_{2,16} + 8.099906320047769B_{3,16} + 4.666426381648652B_{4,16} + 1.988634430359754B_{5,16} + 0.06653046618107656B_{6,16} - 1.099885510887381B_{7,16} - 1.510613500845619B_{8,16} - 1.165653503693637B_{9,16} - 0.06500551943143578B_{10,16} + 1.791330451940986B_{11,16} + 4.403354410423627B_{12,16} + 7.771066356016488B_{13,16} + 11.89446628871957B_{14,16} + 16.77355420853287B_{15,16} + 22.40833011545639B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 9.952603274324834 \cdot 10^{-06}$.
Bounding polynomials M and m:

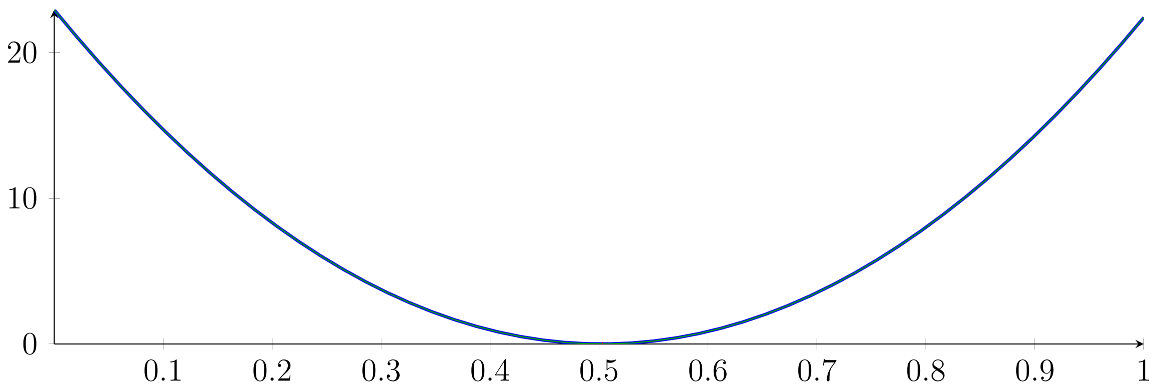
$$M = 90.68255845322638X^2 - 91.20870239567643X + 22.93448401050971$$

$$m = 90.68255845322638X^2 - 91.20870239567643X + 22.93446410530317$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025582179560734, 0.5032438232679998\}$$

Intersection intervals:



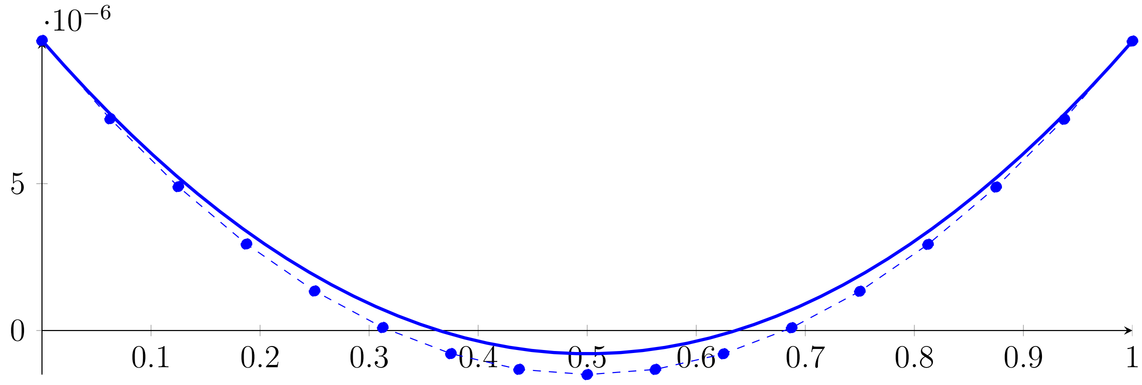
$$[0.5025582179560734, 0.5032438232679998]$$

Longest intersection interval: 0.0006856053119264748
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000666361603, 0.3000001033462145]$,

71.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000666361603, 0.300000103346$

Normalized monomial und Bézier representations and the Bézier polygon:

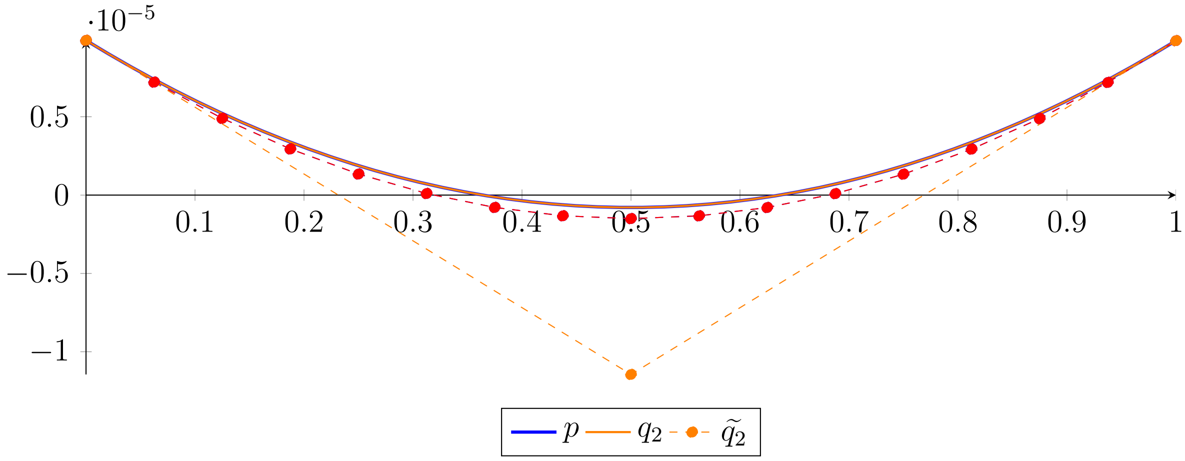
$$\begin{aligned}
p = &-1.087828446441416\cdot 10^{-119}X^{16} + 2.370636059353804\cdot 10^{-112}X^{15} + 1.810668709595505\\
&\cdot 10^{-102}X^{14} - 3.846486744130667\cdot 10^{-95}X^{13} - 1.283061410949319\cdot 10^{-85}X^{12}\\
&+ 2.509305114999168\cdot 10^{-78}X^{11} + 5.0258723654725\cdot 10^{-69}X^{10} - 8.517807546222726\cdot 10^{-62}X^{9}\\
&- 1.17684840157971\cdot 10^{-52}X^{8} + 1.597478332578791\cdot 10^{-45}X^{7} + 1.649027121563264\cdot 10^{-36}X^{6}\\
&- 1.576413560673164\cdot 10^{-29}X^{5} - 1.281381537279639\cdot 10^{-20}X^{4} + 6.414336823836069\cdot 10^{-14}X^{3}\\
&+ 4.262575843118847\cdot 10^{-05}X^{2} - 4.264622395354259\cdot 10^{-05}X + 9.875919579833914\cdot 10^{-06}\\
= &\ 9.875919579833914\cdot 10^{-06}B_{0,16}(X) + 7.210530582737503\cdot 10^{-06}B_{1,16}(X) + 4.900356239234328\\
&\cdot 10^{-06}B_{2,16}(X) + 2.945396549438933\cdot 10^{-06}B_{3,16}(X) + 1.345651513465858\cdot 10^{-06}B_{4,16}(X)\\
&+ 1.01121131429646\cdot 10^{-07}B_{5,16}(X) - 7.881945965551621\cdot 10^{-07}B_{6,16}(X) - 1.322295670374024\\
&\cdot 10^{-06}B_{7,16}(X) - 1.501182089912399\cdot 10^{-06}B_{8,16}(X) - 1.324853855055745\cdot 10^{-06}B_{9,16}(X)\\
&- 7.933109656895192\cdot 10^{-07}B_{10,16}(X) + 9.344657830081879\cdot 10^{-08}B_{11,16}(X)\\
&+ 1.335418777029811\cdot 10^{-06}B_{12,16}(X) + 2.932605630611999\cdot 10^{-06}B_{13,16}(X) + 4.885007139161925\\
&\cdot 10^{-06}B_{14,16}(X) + 7.192623302794129\cdot 10^{-06}B_{15,16}(X) + 9.855454121623155\cdot 10^{-06}B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 = &\ 4.26257585274035\cdot 10^{-05}X^{2} - 4.264622399202859\cdot 10^{-05}X + 9.875919583041081\cdot 10^{-06}\\
= &\ 9.875919583041081\cdot 10^{-06}B_{0,2} - 1.144719241297322\cdot 10^{-05}B_{1,2} + 9.855454118415988\cdot 10^{-06}B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 = &-6.428697903766316\cdot 10^{-301}X^{16} + 5.168486014171287\cdot 10^{-300}X^{15} - 1.931525871486767\\
&\cdot 10^{-299}X^{14} + 4.443748026156872\cdot 10^{-299}X^{13} - 6.983462617461767\cdot 10^{-299}X^{12}\\
&+ 7.823576270761152\cdot 10^{-299}X^{11} - 6.321034979462039\cdot 10^{-299}X^{10} + 3.658988671435299\cdot 10^{-299}X^{9}\\
&- 1.493155040297548\cdot 10^{-299}X^{8} + 4.235526792785023\cdot 10^{-300}X^{7} - 8.432417338595836\cdot 10^{-301}X^{6}\\
&+ 1.226905231702478\cdot 10^{-301}X^{5} - 1.286361467975875\cdot 10^{-302}X^{4} + 9.604810259325032\cdot 10^{-304}X^{3}\\
&+ 4.26257585274035\cdot 10^{-05}X^{2} - 4.264622399202859\cdot 10^{-05}X + 9.875919583041081\cdot 10^{-06}\\
= &\ 9.875919583041081\cdot 10^{-06}B_{0,16} + 7.210530583539294\cdot 10^{-06}B_{1,16} + 4.900356238432536\cdot 10^{-06}B_{2,16}\\
&+ 2.945396547720807\cdot 10^{-06}B_{3,16} + 1.345651511404108\cdot 10^{-06}B_{4,16} + 1.011211294824374\cdot 10^{-07}B_{5,16}\\
&- 7.881945980442039\cdot 10^{-07}B_{6,16} - 1.322295671175816\cdot 10^{-06}B_{7,16} - 1.501182089912399\cdot 10^{-06}B_{8,16}\\
&- 1.324853854253953\cdot 10^{-06}B_{9,16} - 7.933109642004772\cdot 10^{-07}B_{10,16} + 9.34465802480274\cdot 10^{-08}B_{11,16}\\
&+ 1.335418779091561\cdot 10^{-06}B_{12,16} + 2.932605632330124\cdot 10^{-06}B_{13,16} + 4.885007139963716\\
&\cdot 10^{-06}B_{14,16} + 7.192623301992338\cdot 10^{-06}B_{15,16} + 9.855454118415988\cdot 10^{-06}B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.207167313591001 \cdot 10^{-15}$.
Bounding polynomials M and m:

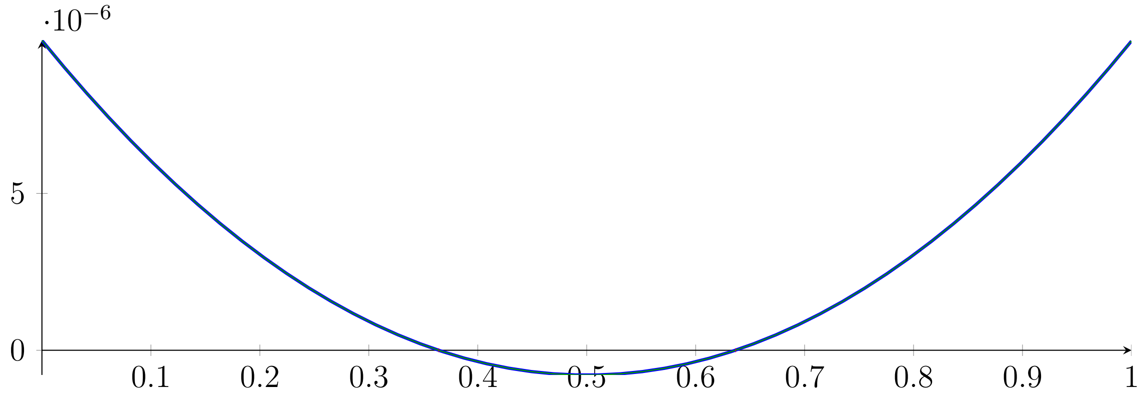
$$M = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919586248249 \cdot 10^{-06}$$

$$m = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06}$$

Root of M and m:

$N(M) = \{0.3640375916978796, 0.6364425279605644\}$ $N(m) = \{0.3640375911454658, 0.6364425285129782\}$

Intersection intervals:



$$[0.3640375911454658, 0.3640375916978796],\ [0.6364425279605644, 0.6364425285129782]$$

Longest intersection interval: $5.524137987858897 \cdot 10^{-10}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$, interval 2: $[0.30000009, 0.30000009]$,

71.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 6!

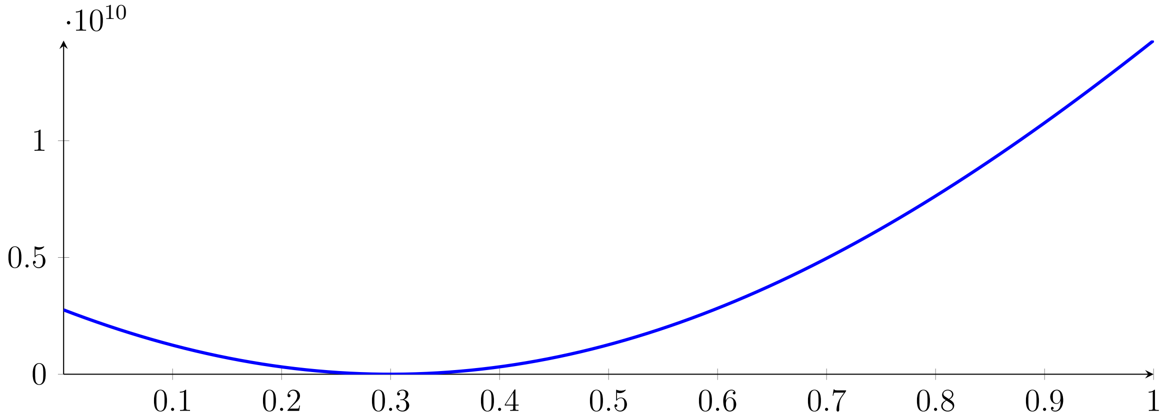
71.7 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 6!

71.8 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

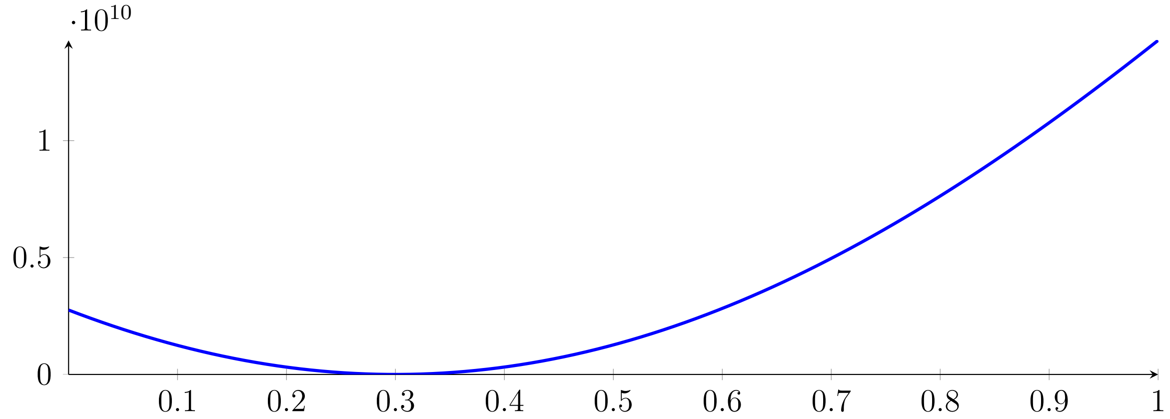
with precision $\varepsilon = 1 \cdot 10^{-08}$.

72 Running CubeClip on h_{16} with epsilon 8

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called CubeClip with input polynomial on interval $[0, 1]$:

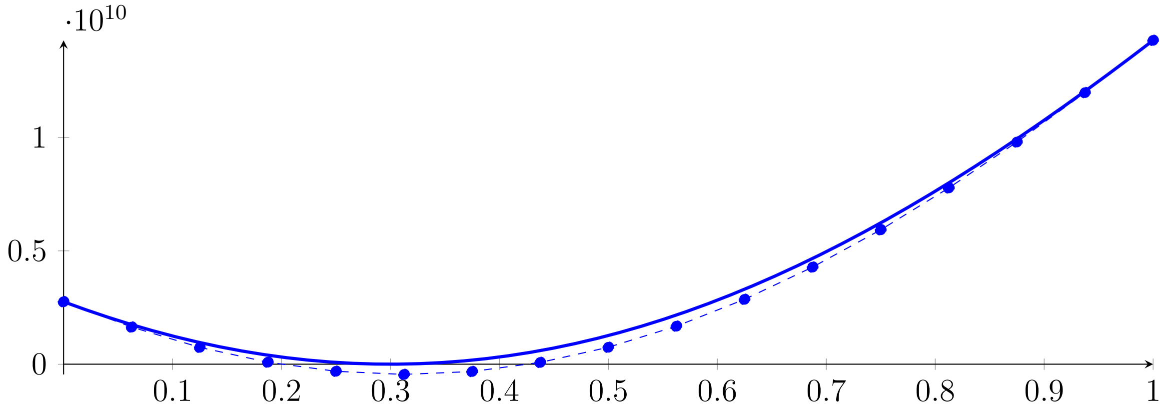
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



72.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

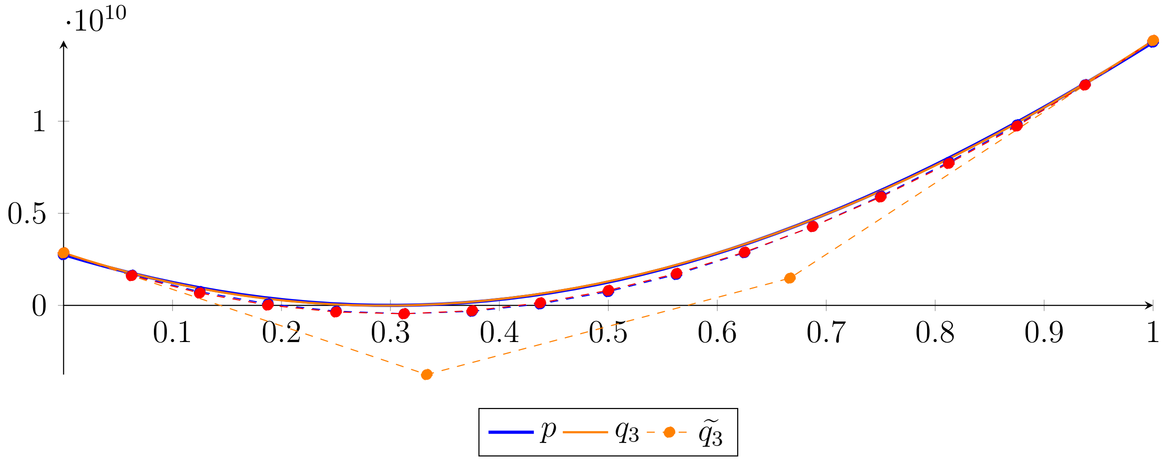
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 &= -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353 \\
&= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3} \\
&\quad + 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 &= 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756 \\
&\quad \cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12} \\
&\quad - 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948 \\
&\quad \cdot 10^{-285}X^{9} + 6.984121997801746\cdot 10^{-286}X^{8} - 2.400829205648148\cdot 10^{-286}X^{7} \\
&\quad + 9.243609934541251\cdot 10^{-287}X^{6} - 2.7161427533585\cdot 10^{-287}X^{5} + 4.31464378661806\cdot 10^{-288}X^{4} \\
&\quad - 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353 \\
&= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16} \\
&\quad + 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16} \\
&\quad - 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16} \\
&\quad + 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16} \\
&\quad + 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16} \\
&\quad + 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

Bounding polynomials M and m:

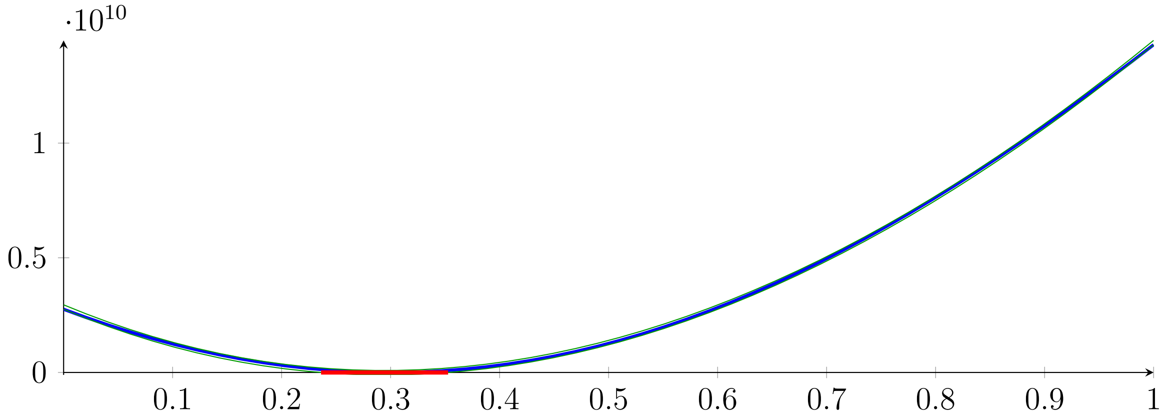
$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$

$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



$$[0.2362027155340351, 0.3527550629571673]$$

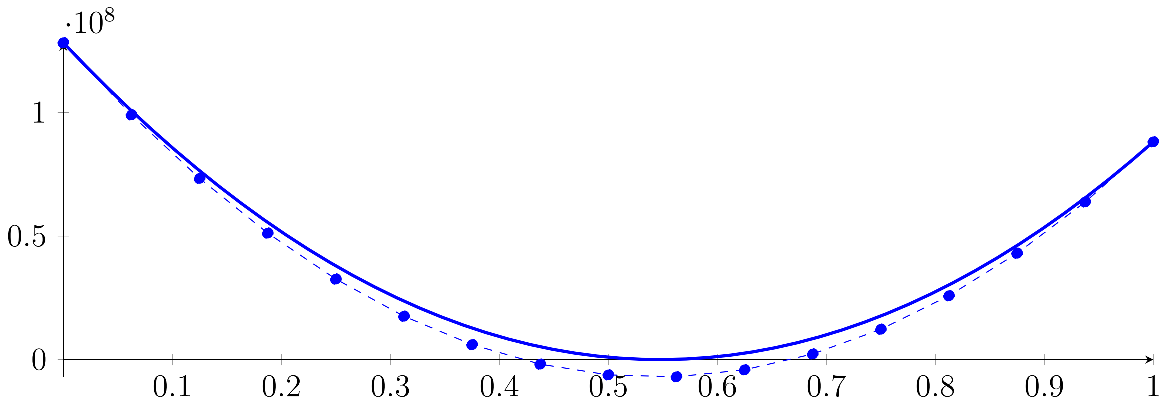
Longest intersection interval: 0.1165523474231321

$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

72.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

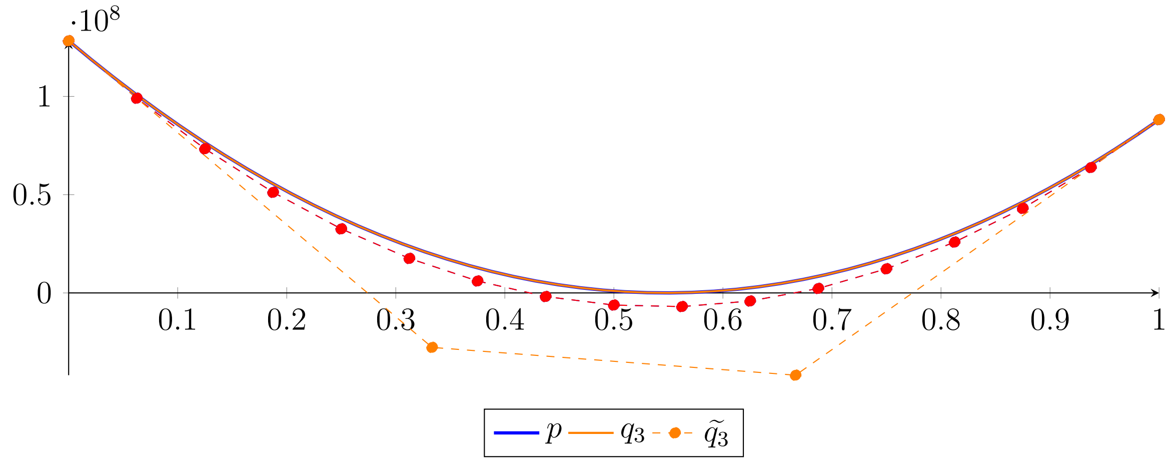
$$\begin{aligned}
p = &-1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14}\\
&- 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321\\
&\cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8}\\
&+ 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4}\\
&+ 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316\\
= &\ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X)\\
&+ 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X)\\
&+ 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X)\\
&- 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X)\\
&+ 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X)\\
&+ 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &\ 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166\\
= &\ 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3}\\
&- 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3}
\end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_3 = {} & 1.029539679280458 \cdot 10^{-288} X^{16} - 8.072523277872609 \cdot 10^{-288} X^{15} + 2.888405037194055 \\
& \cdot 10^{-287} X^{14} - 6.260623433882179 \cdot 10^{-287} X^{13} + 9.183807049175627 \cdot 10^{-287} X^{12} \\
& - 9.610199535206024 \cdot 10^{-287} X^{11} + 7.321003139036714 \cdot 10^{-287} X^{10} - 4.031610440228076 \\
& \cdot 10^{-287} X^{9} + 1.537754605851679 \cdot 10^{-287} X^{8} - 3.606689105481158 \cdot 10^{-288} X^{7} \\
& + 3.223581314526531 \cdot 10^{-289} X^{6} + 5.410058694892297 \cdot 10^{-290} X^{5} - 1.24173721337985 \cdot 10^{-290} X^{4} \\
& + 2297912.322133328 X^{3} + 425644196.8061184 X^{2} - 468061781.215159 X + 128303546.2426166 \\
= {} & 128303546.2426166 B_{0,16} + 99049684.91666918 B_{1,16} + 73342858.56410606 B_{2,16} \\
& + 51187170.59978822 B_{3,16} + 32586724.4385766 B_{4,16} + 17545623.49533216 B_{5,16} \\
& + 6067971.184915845 B_{6,16} - 1842129.077811386 B_{7,16} - 6180573.877988583 B_{8,16} \\
& - 6943259.800754793 B_{9,16} - 4126083.431249065 B_{10,16} + 2275058.645389556 B_{11,16} \\
& + 12264269.84402202 B_{12,16} + 25845653.57950928 B_{13,16} + 43023313.26671229 B_{14,16} \\
& + 63801352.320492 B_{15,16} + 88183874.15570936 B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

Bounding polynomials M and m:

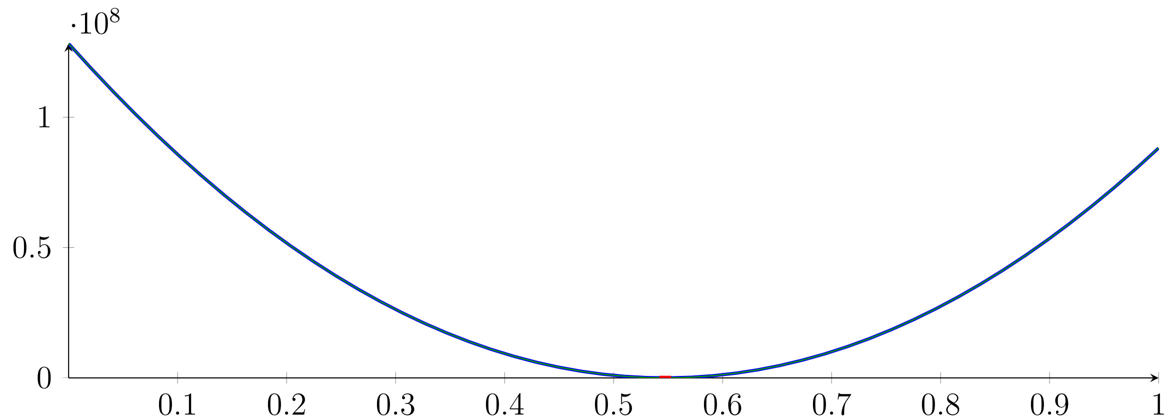
$$M = 2297912.322133328 X^3 + 425644196.8061184 X^2 - 468061781.215159 X + 128322140.7478178$$

$$m = 2297912.322133328 X^3 + 425644196.8061184 X^2 - 468061781.215159 X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$$[0.5420611331202266, 0.552740643902378]$$

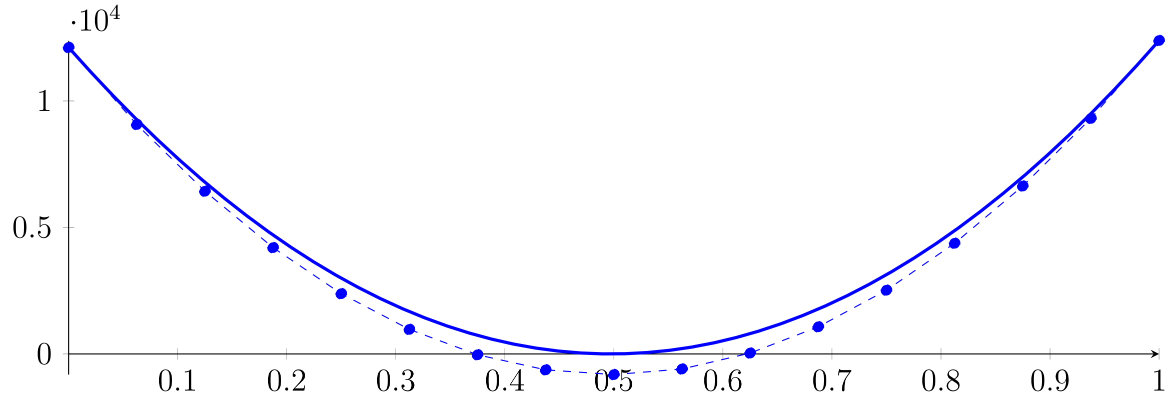
Longest intersection interval: 0.01067951078215144

$\Longrightarrow$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

72.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.300625935097030$

Normalized monomial und Bézier representations and the Bézier polygon:

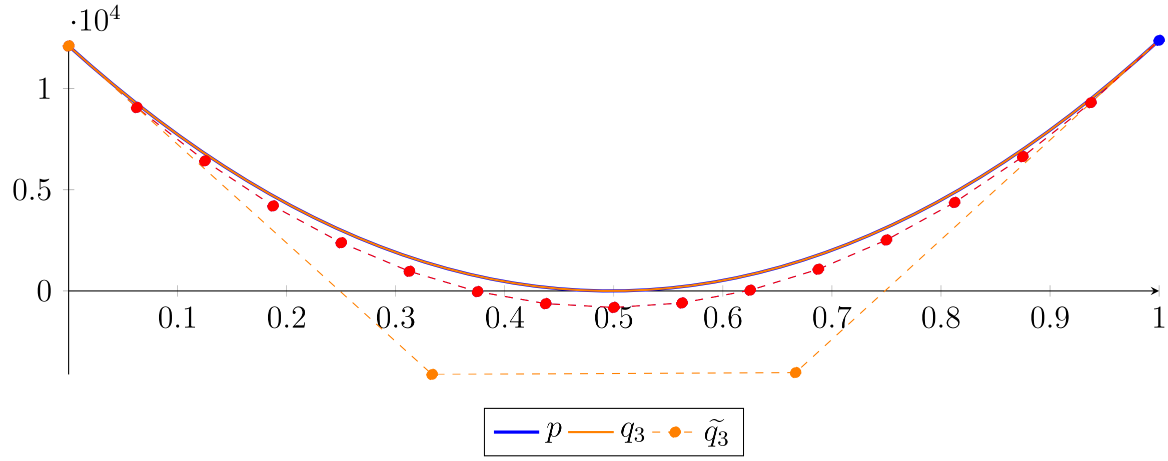
$$p = -3.320153282917824\cdot 10^{-47}X^{16} + 2.160317373028451\cdot 10^{-44}X^{15} + 4.806705749215272\cdot 10^{-39}X^{14} - 3.045075915978909\cdot 10^{-36}X^{13} - 2.96255923979525\cdot 10^{-31}X^{12} + 1.726566936989629\cdot 10^{-28}X^{11} + 1.009356568315285\cdot 10^{-23}X^{10} - 5.095799973170639\cdot 10^{-21}X^{9} - 2.055755460332604\cdot 10^{-16}X^{8} + 8.312655296209706\cdot 10^{-14}X^{7} + 2.505544083540367\cdot 10^{-09}X^{6} - 7.139662163470987\cdot 10^{-07}X^{5} - 0.01693489925927299X^{4} + 2.534105689487521X^{3} + 49001.97220419171X^{2} - 48729.12904702137X + 12114.14082112651$$
$$= 12114.14082112651B_{0,16}(X) + 9068.570255687672B_{1,16}(X) + 6431.349458617101B_{2,16}(X) + 4202.482955103525B_{3,16}(X) + 2381.975261030786B_{4,16}(X) + 969.830882977672B_{5,16}(X) - 33.94568178224417B_{6,16}(X) - 629.34994528077B_{7,16}(X) - 816.3774288552541B_{8,16}(X) - 595.0236631487491B_{9,16}(X) + 34.71581188982676B_{10,16}(X) + 1072.845447005528B_{11,16}(X) + 2519.36968363722B_{12,16}(X) + 4374.29295391742B_{13,16}(X) + 6637.619680672133B_{14,16}(X) + 9309.354277420696B_{15,16}(X) + 12389.50114837561B_{16,16}(X)$$



Degree reduction and raising:

$$q_3 = 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619$$
$$= 12114.14106307619B_{0,3} - 4128.903565586097B_{1,3} - 4037.950201139886B_{2,3} + 12389.5013903309B_{3,3}$$

$$\tilde{q}_3 = 1.099600357934752\cdot 10^{-292}X^{16} - 8.377614657428995\cdot 10^{-292}X^{15} + 2.905985967667976\cdot 10^{-291}X^{14} - 6.091421693711012\cdot 10^{-291}X^{13} + 8.615207440059339\cdot 10^{-291}X^{12} - 8.654253929665817\cdot 10^{-291}X^{11} + 6.288120919613368\cdot 10^{-291}X^{10} - 3.270943230058189\cdot 10^{-291}X^{9} + 1.159364872956392\cdot 10^{-291}X^{8} - 2.426897197847659\cdot 10^{-292}X^{7} + 1.434105777365326\cdot 10^{-293}X^{6} + 4.843078869312239\cdot 10^{-294}X^{5} - 7.632080003106212\cdot 10^{-295}X^{4} + 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619$$
$$= 12114.14106307619B_{0,16} + 9068.570195202008B_{1,16} + 6431.349277155543B_{2,16} + 4202.482773640211B_{3,16} + 2381.975149359434B_{4,16} + 969.8308690166351B_{5,16} - 33.94560268476556B_{6,16} - 629.349801041346B_{7,16} - 816.3772613496847B_{8,16} - 595.0235189063601B_{9,16} + 34.71589099204953B_{10,16} + 1072.845433048966B_{11,16} + 2519.36957196781B_{12,16} + 4374.292772452004B_{13,16} + 6637.619499204969B_{14,16} + 9309.354216930127B_{15,16} + 12389.5013903309B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00024195528529 18186$.

Bounding polynomials M and m:

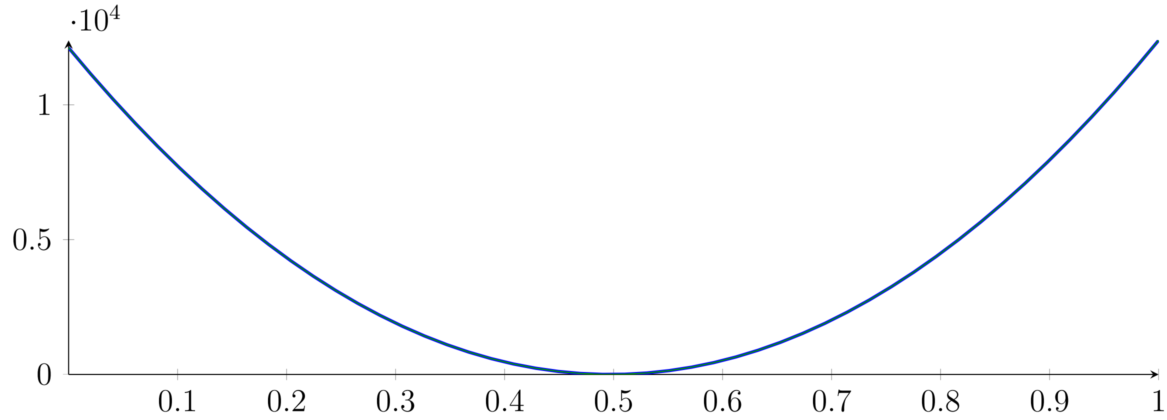
$$M = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.14130503147$$

$$m = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.1408211209$$

Root of M and m:

$$N(M) = \{-19599.95818041899\} \quad N(m) = \{-19599.95818041899, 0.4971412064138645, 0.4972526073815481\}$$

Intersection intervals:



$$[0.4971412064138645, 0.4972526073815481]$$

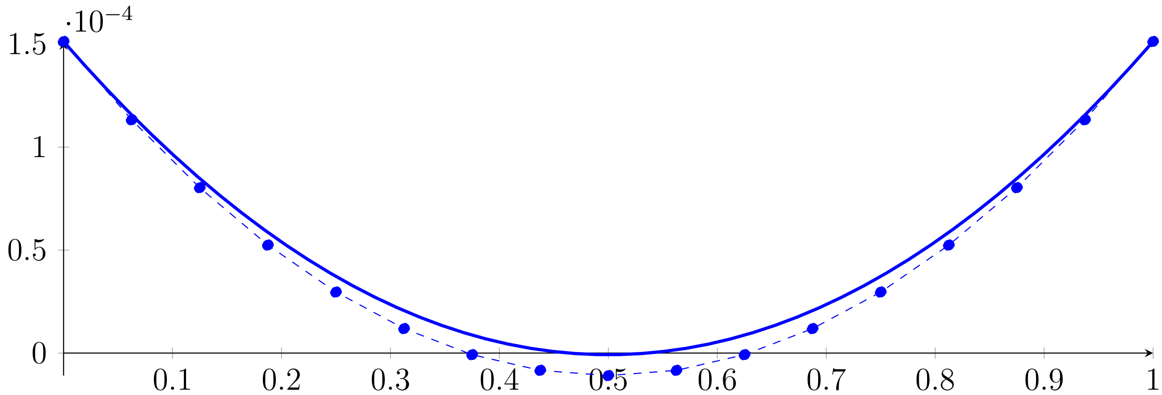
Longest intersection interval: 0.0001114009676835966

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000156681198, 0.3000001543313607]$,

72.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3000000156681198, 0.30000015433136$

Normalized monomial und Bézier representations and the Bézier polygon:

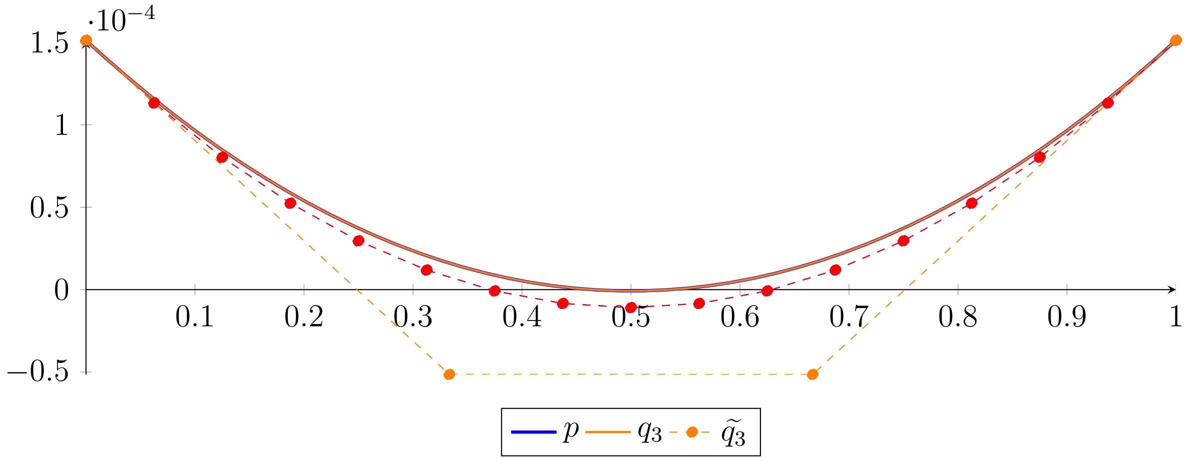
$$p = -1.868020113181866\cdot 10^{-110}X^{16} + 1.077730427531422\cdot 10^{-103}X^{15} + 2.179252184416492 \cdot 10^{-94}X^{14} - 1.225622742394903\cdot 10^{-87}X^{13} - 1.082338964896179\cdot 10^{-78}X^{12} + 5.603938665480187\cdot 10^{-72}X^{11} + 2.971492663890366\cdot 10^{-63}X^{10} - 1.333260671370819 \cdot 10^{-56}X^{9} - 4.876756665473299\cdot 10^{-48}X^{8} + 1.752546617551927\cdot 10^{-41}X^{7} + 4.789450848811937 \cdot 10^{-33}X^{6} - 1.212138221580256\cdot 10^{-26}X^{5} - 2.608457365264229\cdot 10^{-18}X^{4} + 3.456855344293596 \cdot 10^{-12}X^{3} + 0.0006081696715066215X^{2} - 0.0006081719526708613X + 0.0001512528027912101$$
$$= 0.0001512528027912101B_{0,16}(X) + 0.0001132420557492813B_{1,16}(X) + 8.029938930324099 \cdot 10^{-05}B_{2,16}(X) + 5.242480345926214\cdot 10^{-05}B_{3,16}(X) + 2.961829822351771\cdot 10^{-05}B_{4,16}(X) + 1.187987360218066\cdot 10^{-05}B_{5,16}(X) - 7.904703985760706\cdot 10^{-07}B_{6,16}(X) - 8.392733772579519 \cdot 10^{-06}B_{7,16}(X) - 1.092691651365674\cdot 10^{-05}B_{8,16}(X) - 8.393018615634788\cdot 10^{-06}B_{9,16}(X) - 7.910400723407154\cdot 10^{-07}B_{10,16}(X) + 1.187901912239842\cdot 10^{-05}B_{11,16}(X) + 2.961715897475557\cdot 10^{-05}B_{12,16}(X) + 5.242337949090366\cdot 10^{-05}B_{13,16}(X) + 8.029768067701564 \cdot 10^{-05}B_{14,16}(X) + 0.0001132400625392645B_{15,16}(X) + 0.0001512505250838231B_{16,16}(X)$$



Degree reduction and raising:

$$q_3 = 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,3} - 5.147118143241051\cdot 10^{-05}B_{1,3} - 5.147194182048959\cdot 10^{-05}B_{2,3} + 0.0001512505250838231B_{3,3}$$

$$\widetilde{q_3} = 1.301341971574797\cdot 10^{-300}X^{16} - 9.903701542140468\cdot 10^{-300}X^{15} + 3.435393711081806 \cdot 10^{-299}X^{14} - 7.213305599572634\cdot 10^{-299}X^{13} + 1.023930542727086\cdot 10^{-298}X^{12} - 1.03405078726607\cdot 10^{-298}X^{11} + 7.556175284428161\cdot 10^{-299}X^{10} - 3.94396828718345\cdot 10^{-299}X^{9} + 1.391742955019986\cdot 10^{-299}X^{8} - 2.822114462221502\cdot 10^{-300}X^{7} + 1.110580839880333 \cdot 10^{-301}X^{6} + 7.65504767714651\cdot 10^{-302}X^{5} - 1.17007493272161\cdot 10^{-302}X^{4} + 3.456850127378832 \cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,16} + 0.0001132420557492813B_{1,16} + 8.029938930324096\cdot 10^{-05}B_{2,16} + 5.242480345926211\cdot 10^{-05}B_{3,16} + 2.961829822351769\cdot 10^{-05}B_{4,16} + 1.187987360218065\cdot 10^{-05}B_{5,16} - 7.904703985760584\cdot 10^{-07}B_{6,16} - 8.392733772579497\cdot 10^{-06}B_{7,16} - 1.092691651365671\cdot 10^{-05}B_{8,16} - 8.393018615634766\cdot 10^{-06}B_{9,16} - 7.910400723407032\cdot 10^{-07}B_{10,16} + 1.187901912239842 \cdot 10^{-05}B_{11,16} + 2.961715897475555\cdot 10^{-05}B_{12,16} + 5.242337949090363\cdot 10^{-05}B_{13,16} + 8.029768067701561\cdot 10^{-05}B_{14,16} + 0.0001132400625392644B_{15,16} + 0.0001512505250838231B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 3.726367712763873 \cdot 10^{-20}$.
Bounding polynomials M and m:

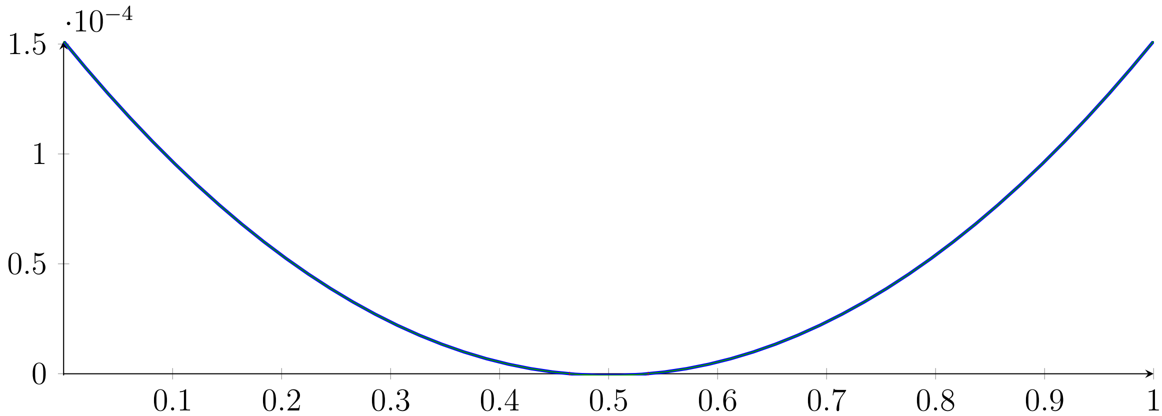
$$M = 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ - 0.0006081719526708621 X + 0.0001512528027912102$$

$$m = 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ - 0.0006081719526708621 X + 0.0001512528027912101$$

Root of M and m:

$N(M) = \{-175931744.9566824, 0.4639432736155652, 0.5360604563964476\}$ $N(m) = \{-175931744.9566824,$

Intersection intervals:



$$[0.4639432736155635, 0.4639432736155652],\ [0.5360604563964476, 0.5360604563964493]$$

Longest intersection interval: $1.699230475659398 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999977, 0.3000000799999977]$, interval 2: $[0.30000009, 0.3000$

72.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000799999977, 0.300000079999$

Found root in interval $[0.3000000799999977, 0.3000000799999977]$ at recursion depth 5!

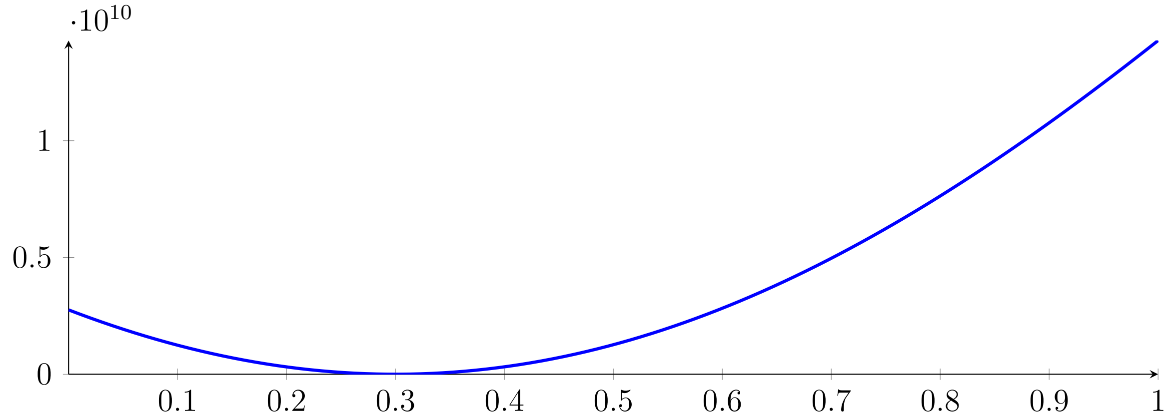
72.6 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 5!

72.7 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\&+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\&- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\&- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\&+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



Result: Root Intervals

$$[0.3000000799999977, 0.3000000799999977],\ [0.30000009, 0.30000009]$$

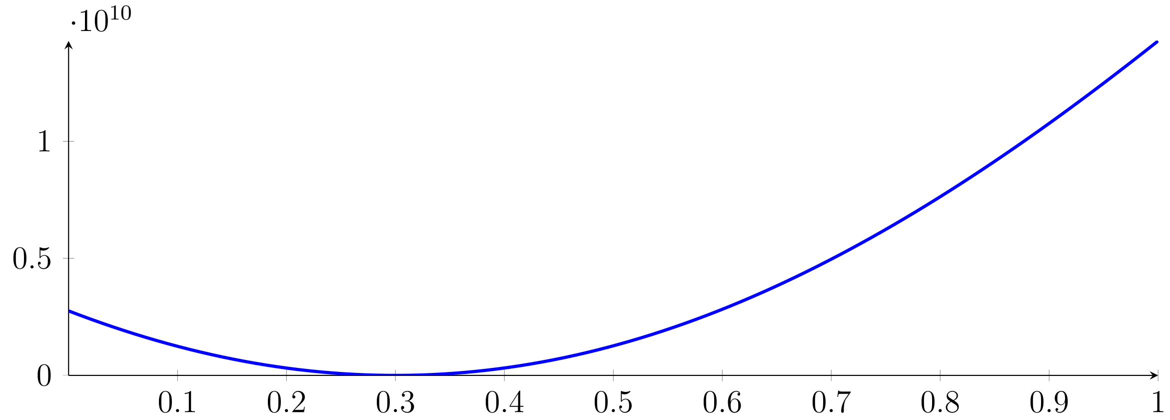
with precision $\varepsilon = 1{\cdot}10^{-08}$.

73 Running BezClip on h_{16} with epsilon 16

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called BezClip with input polynomial on interval $[0, 1]$:

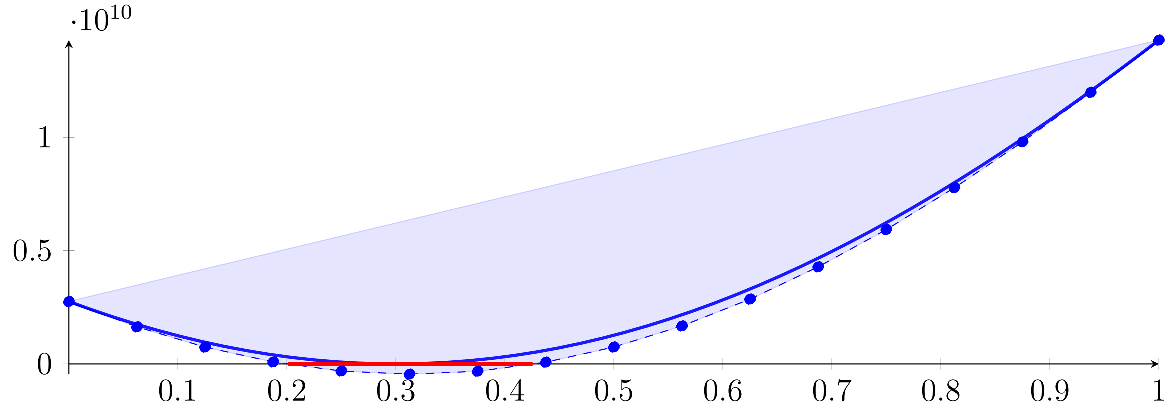
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



73.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X)\\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X)\\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X)\\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X)\\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X)\\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

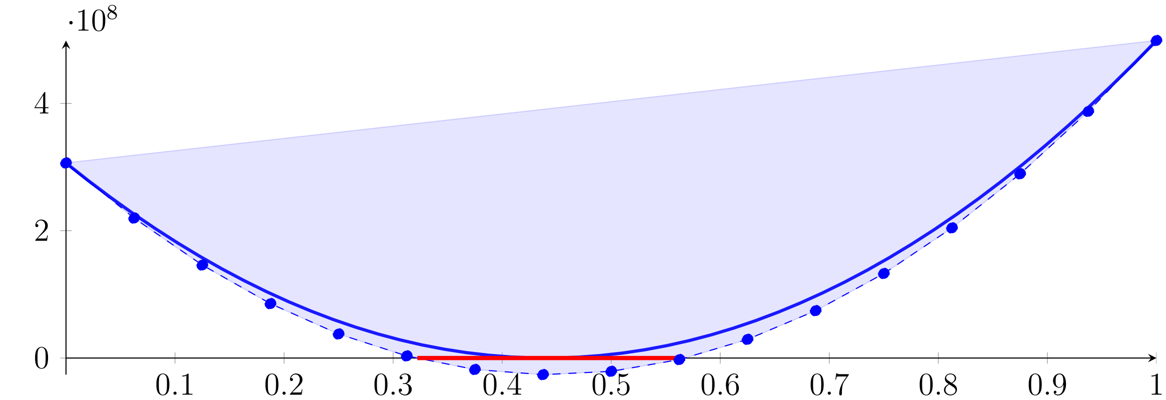
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

73.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-4.078702226246453\cdot 10^{-11}X^{16} + 4.329167405007062\cdot 10^{-10}X^{15} + 1.800827856148464\cdot 10^{-07}X^{14}\\
&- 1.7501300195896\cdot 10^{-06}X^{13} - 0.0003388881074603682X^{12} + 0.002918727490860103X^{11}\\
&+ 0.353260574938002X^{10} - 2.587751703352393X^{9} - 220.7388291837263X^{8}\\
&+ 1298.537478880933X^{7} + 82809.42227744751X^{6} - 357003.6974521088X^{5} - 17288824.7550166X^{4}\\
&+ 45617774.75376932X^{3} + 1550181799.359372X^{2} - 1385902556.54429X + 306449533.9978484\\
= &\ 306449533.9978484B_{0,16}(X) + 219830624.2138303B_{1,16}(X) + 146129896.0911402B_{2,16}(X)\\
&+ 85428809.9418386B_{3,16}(X) + 37799326.72372467B_{4,16}(X) + 3303826.308721026B_{5,16}(X)\\
&- 18004963.90790522B_{6,16}(X) - 26084029.94397575B_{7,16}(X) - 20900121.61613177B_{8,16}(X)\\
&- 2429792.29588269B_{9,16}(X) + 29340572.02544693B_{10,16}(X) + 74414734.40659503B_{11,16}(X)\\
&+ 132786599.3869978B_{12,16}(X) + 204440216.186282B_{13,16}(X) + 289349792.6628749B_{14,16}(X)\\
&+ 387479720.0042024B_{15,16}(X) + 498784608.1032447B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

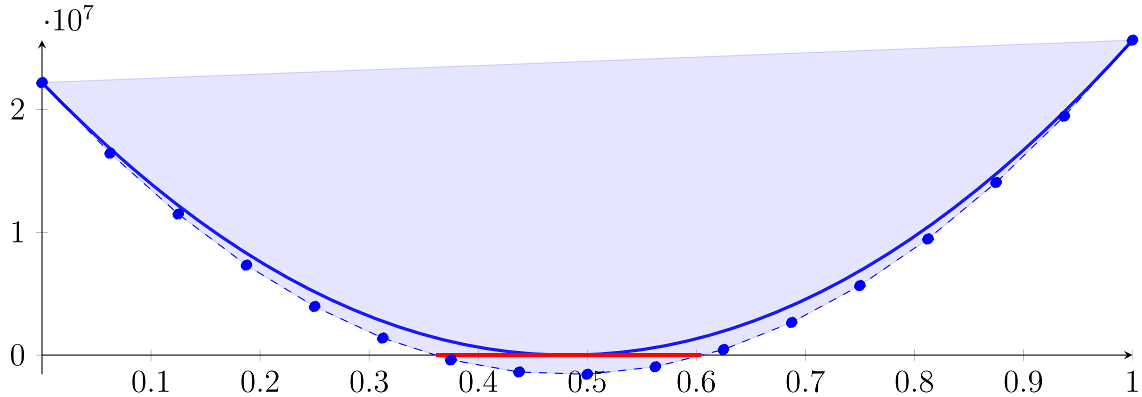
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: $[0.2735013999129692, 0.328453288067671]$,

73.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-6.91388116995714\cdot 10^{-21}X^{16} + 1.539971991909181\cdot 10^{-19}X^{15} + 5.126557214158853\\ &\cdot 10^{-16}X^{14} - 1.075268795442723\cdot 10^{-14}X^{13} - 1.618466564965893\cdot 10^{-11}X^{12}\\ &+ 3.060191240732859\cdot 10^{-10}X^{11} + 2.825398471644138\cdot 10^{-07}X^{10} - 4.580412037193902\\ &\cdot 10^{-06}X^{9} - 0.002949971804071592X^{8} + 0.0383187988748973X^{7} + 18.44286791446417X^{6}\\ &- 172.012783850635X^{5} - 63987.92092437637X^{4} + 338934.074545568X^{3}\\ &+ 95113195.48898588X^{2} - 91937923.73553449X + 22182509.73328641\\ = &\ 22182509.73328641B_{0,16}(X) + 16436389.49981551B_{1,16}(X) + 11482879.22875282B_{2,16}(X)\\ &+ 7322584.159517174B_{3,16}(X) + 3956074.373329099B_{4,16}(X) + 1383884.753830588B_{5,16}(X)\\ &- 393485.0499920519B_{6,16}(X) - 1375570.658578954B_{7,16}(X) - 1561942.994257926B_{8,16}(X)\\ &- 952208.311393262B_{9,16}(X) + 453991.7757805191B_{10,16}(X) + 2656980.267642443B_{11,16}(X)\\ &+ 5657044.755988558B_{12,16}(X) + 9454437.403157084B_{13,16}(X) + 14049374.92347699B_{14,16}(X)\\ &+ 19442038.56704145B_{15,16}(X) + 25632574.10580758B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

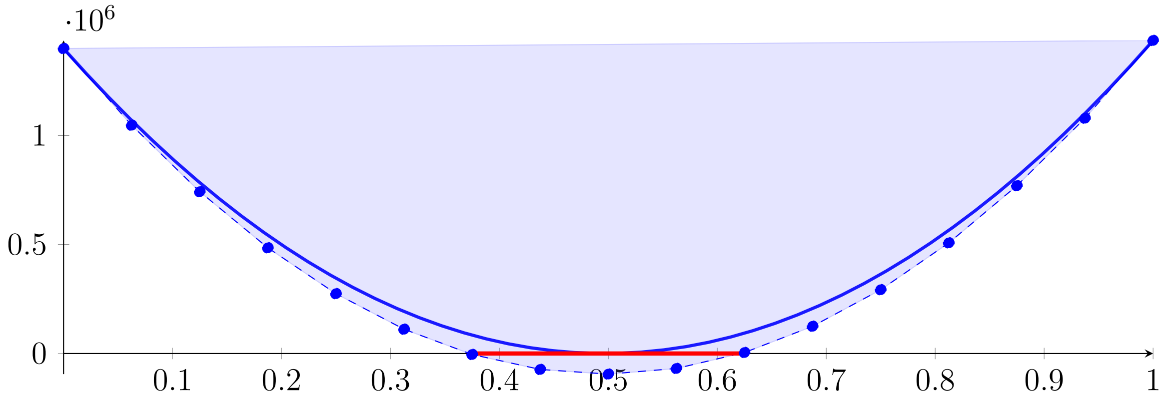
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: $[0.2933480088018335, 0.3067375037518675]$,

73.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.067154088646128 \cdot 10^{-30} X^{16} + 7.224340048814341 \cdot 10^{-29} X^{15} + 1.334696477727887 \\
& \cdot 10^{-24} X^{14} - 8.705175946298654 \cdot 10^{-23} X^{13} - 7.106771480233297 \cdot 10^{-19} X^{12} \\
& + 4.237354105832321 \cdot 10^{-17} X^{11} + 2.091927791167947 \cdot 10^{-13} X^{10} - 1.077046671119229 \\
& \cdot 10^{-11} X^{9} - 3.681404336048099 \cdot 10^{-08} X^{8} + 1.51820465067154 \cdot 10^{-06} X^{7} \\
& + 0.003877397410843758 X^{6} - 0.1133230374256462 X^{5} - 226.5078434192974 X^{4} \\
& + 3562.753220436385 X^{3} + 5665644.405911702 X^{2} - 5632059.446508477 X + 1399245.031427946 \\
= & \; 1399245.031427946 B_{0,16}(X) + 1047241.316021167 B_{1,16}(X) + 742451.3039969843 B_{2,16}(X) \\
& + 484881.3574147218 B_{3,16}(X) + 274537.7138788421 B_{4,16}(X) + 111426.4865130056 B_{5,16}(X) \\
& - 4446.336065390124 B_{6,16}(X) - 73074.88977018565 B_{7,16}(X) - 94453.43507095045 B_{8,16}(X) \\
& - 68576.35701698946 B_{9,16}(X) + 4561.834739135334 B_{10,16}(X) \\
& + 124966.505918568 B_{11,16}(X) + 292642.897593607 B_{12,16}(X) + 507596.1261656377 B_{13,16}(X) \\
& + 769831.1833435517 B_{14,16}(X) + 1079352.93612265 B_{15,16}(X) + 1436166.12676403 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

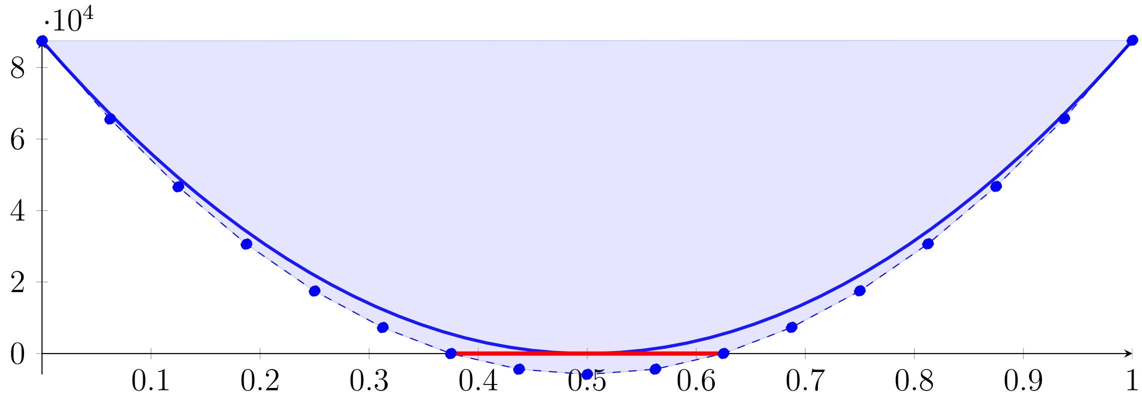
Longest intersection interval: 0.2484999837871829

$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

73.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -2.256570822524655 \cdot 10^{-40} X^{16} + 5.606069245850654 \cdot 10^{-38} X^{15} + 4.571694482208246 \\
& \cdot 10^{-33} X^{14} - 1.103601619609778 \cdot 10^{-30} X^{13} - 3.943084008767946 \cdot 10^{-26} X^{12} \\
& + 8.746232232432483 \cdot 10^{-24} X^{11} + 1.879997320990413 \cdot 10^{-19} X^{10} - 3.610214442217249 \\
& \cdot 10^{-17} X^{9} - 5.35839719625663 \cdot 10^{-13} X^{8} + 8.241677187819251 \cdot 10^{-11} X^{7} + 9.139570434264648 \\
& \cdot 10^{-07} X^{6} - 9.91682351975725 \cdot 10^{-05} X^{5} - 0.864525557836322 X^{4} + 49.4891850901473 X^{3} \\
& + 350100.5152669434 X^{2} - 350028.3308017828 X + 87482.91293301892 \\
= & \; 87482.91293301892 B_{0,16}(X) + 65606.1422579075 B_{1,16}(X) + 46646.87587668727 B_{2,16}(X) \\
& + 30605.20216290304 B_{3,16}(X) + 17481.20901508557 B_{4,16}(X) + 7274.983856728878 B_{5,16}(X) \\
& - 13.38636373236149 B_{6,16}(X) - 4383.815172945278 B_{7,16}(X) - 5836.216572661172 B_{8,16}(X) \\
& - 4370.505039757766 B_{9,16}(X) + 13.40447373867094 B_{10,16}(X) + 7315.596540631298 B_{11,16}(X) \\
& + 17536.1552585308 B_{12,16}(X) + 30675.1642498336 B_{13,16}(X) + 46732.70666170018 B_{14,16}(X) \\
& + 65708.86516603353 B_{15,16}(X) + 87603.72195945767 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748852078437339, 0.6248088966923045\}$$

Intersection intervals with the x axis:

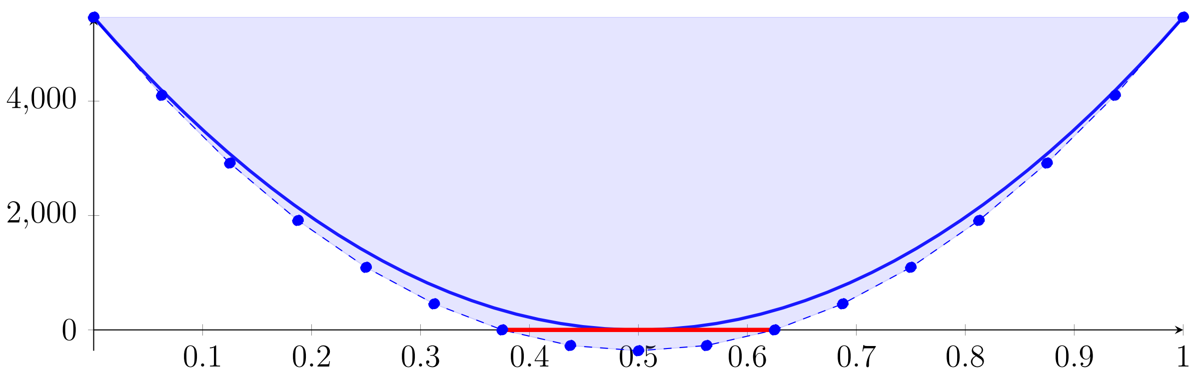
$$[0.3748852078437339, 0.6248088966923045]$$

Longest intersection interval: 0.2499236888485706
$\Longrightarrow$ Selective recursion: interval 1: $[0.2995843091213109, 0.3004158775315354]$,

73.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.2995843091213109, 0.30041587753$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-5.228387005637929 \cdot 10^{-50} X^{16} + 5.071723175262542 \cdot 10^{-47} X^{15} + 1.695940836337518 \\
&\cdot 10^{-41} X^{14} - 1.602367332110541 \cdot 10^{-38} X^{13} - 2.341982733873566 \cdot 10^{-33} X^{12} \\
&+ 2.036120445711374 \cdot 10^{-30} X^{11} + 1.787779362171969 \cdot 10^{-25} X^{10} - 1.346594590666257 \\
&\cdot 10^{-22} X^{9} - 8.158156485257833 \cdot 10^{-18} X^{8} + 4.921695226295225 \cdot 10^{-15} X^{7} \\
&+ 2.227778844339431 \cdot 10^{-10} X^{6} - 9.46914992513699 \cdot 10^{-08} X^{5} - 0.003373649242827386 X^{4} \\
&+ 0.7523208927331248 X^{3} + 21871.35693828986 X^{2} - 21871.48147756637 X + 5467.807676357308 \\
= &\, 5467.807676357308 B_{0,16}(X) + 4100.84008400941 B_{1,16}(X) + 2916.133799480594 B_{2,16}(X) \\
&+ 1913.690166201026 B_{3,16}(X) + 1093.510525747217 B_{4,16}(X) + 455.5962178420057 B_{5,16}(X) \\
&- 0.05141964546683865 B_{6,16}(X) - 273.4310506997833 B_{7,16}(X) - 364.541341159257 B_{8,16}(X) \\
&- 273.3809587159691 B_{9,16}(X) + 0.05142708421757425 B_{10,16}(X) \\
&+ 455.7571448416359 B_{11,16}(X) + 1093.737521302792 B_{12,16}(X) + 1913.993881360346 B_{13,16}(X) \\
&+ 2916.527548053087 B_{14,16}(X) + 4101.339842565916 B_{15,16}(X) + 5468.43208422982 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749929469011197, 0.6249882450180355\}$$

Intersection intervals with the x axis:

$$[0.3749929469011197, 0.6249882450180355]$$

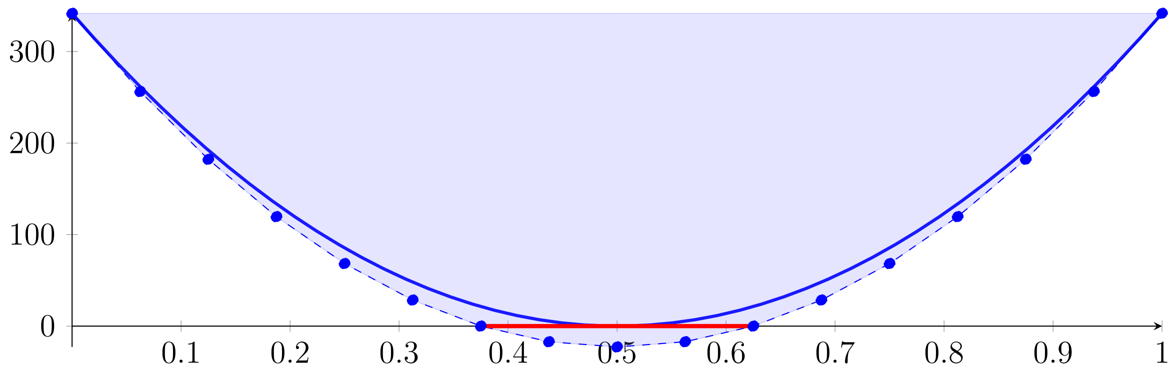
Longest intersection interval: 0.2499952981169158

$\Longrightarrow$ Selective recursion: interval 1: $[0.2998961414100109, 0.3001040296026296]$,

73.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.2998961414100109, 0.300104029$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.216962444275584\cdot 10^{-59}X^{16} + 4.692870826723641\cdot 10^{-56}X^{15} + 6.316314535763934\\
&\cdot 10^{-50}X^{14} - 2.373865516252114\cdot 10^{-46}X^{13} - 1.395661893422859\cdot 10^{-40}X^{12}\\
&+ 4.828363722587868\cdot 10^{-37}X^{11} + 1.70471850273246\cdot 10^{-31}X^{10} - 5.110411085982617\cdot 10^{-28}X^{9}\\
&- 1.244717766622564\cdot 10^{-22}X^{8} + 2.988632715177156\cdot 10^{-19}X^{7} + 5.438614058423278\cdot 10^{-14}X^{6}\\
&- 9.197401057829169\cdot 10^{-11}X^{5} - 1.317801761681953\cdot 10^{-05}X^{4} + 0.01167528468045732X^{3}\\
&+ 1366.961107526186X^{2} - 1366.963198735845X + 341.7398631167735\\
= &\,341.7398631167735B_{0,16}(X) + 256.3046631957831B_{1,16}(X) + 182.260805837511B_{2,16}(X)\\
&+ 119.6083118906798B_{3,16}(X) + 68.34720219677136B_{4,16}(X) + 28.47749759002708B_{5,16}(X)\\
&- 0.0007811025525137025B_{6,16}(X) - 17.08761306120757B_{7,16}(X) - 22.782977473419B_{8,16}(X)\\
&- 17.08685353390849B_{9,16}(X) + 0.0007795553614777027B_{10,16}(X)\\
&+ 28.4799425851876B_{11,16}(X) + 68.35065633912575B_{12,16}(X) + 119.6129415934909B_{13,16}(X)\\
&+ 182.2668191173573B_{14,16}(X) + 256.312309672558B_{15,16}(X) + 341.7494340136854B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749982857492878, 0.6249971486858456\}$$

Intersection intervals with the x axis:

$$[0.3749982857492878, 0.6249971486858456]$$

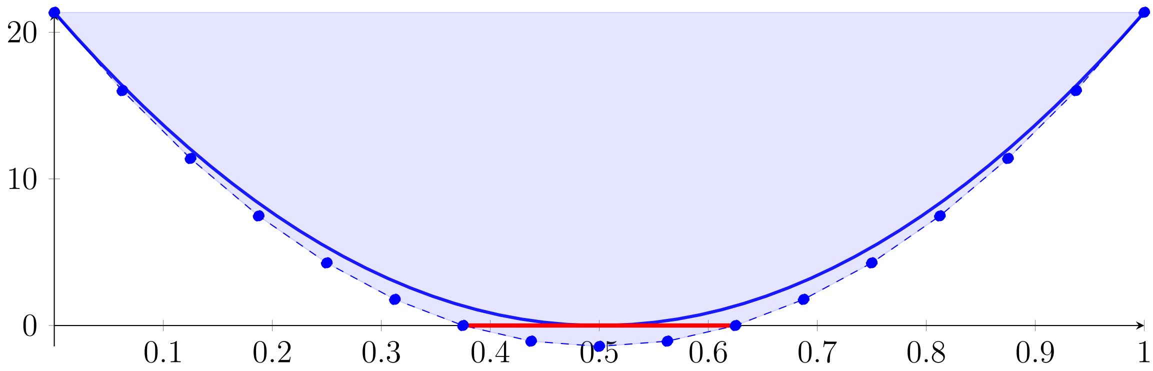
Longest intersection interval: 0.2499988629365579

$\Longrightarrow$ Selective recursion: interval 1: $[0.2999740991258704, 0.300026070937643]$,

73.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999740991258704, 0.3000260$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -2.833255302235776\cdot 10^{-69}X^{16} + 4.363478560238502\cdot 10^{-65}X^{15} + 2.35287052687137 \\
& \cdot 10^{-58}X^{14} - 3.532185105967992\cdot 10^{-54}X^{13} - 8.318408043755505\cdot 10^{-48}X^{12} \\
& + 1.149616718196078\cdot 10^{-43}X^{11} + 1.625691292234855\cdot 10^{-37}X^{10} - 1.946948798353601\cdot 10^{-33}X^{9} \\
& - 1.899245778120626\cdot 10^{-27}X^{8} + 1.821779493317236\cdot 10^{-23}X^{7} + 1.327769541755276\cdot 10^{-17}X^{6} \\
& - 8.969682874540076\cdot 10^{-14}X^{5} - 5.147636798128115\cdot 10^{-08}X^{4} + 0.000182114977798148X^{3} \\
& + 85.43511227371199X^{2} - 85.43514442584667X + 21.35877059261705 \\
= & \, 21.35877059261705B_{0,16}(X) + 16.01907406600164B_{1,16}(X) + 11.39133680833382B_{2,16}(X) \\
& + 7.475559144818918B_{3,16}(X) + 4.271741400633969B_{4,16}(X) + 1.779883900927722B_{5,16}(X) \\
& - 1.302917935804413\cdot 10^{-05}B_{6,16}(X) - 1.067949064595088B_{7,16}(X) - 1.423923880255569B_{8,16}(X) \\
& - 1.067937151125187B_{9,16}(X) + 1.144780338983322\cdot 10^{-05}B_{10,16}(X) \\
& + 1.779922241509208B_{11,16}(X) + 4.271795554943032B_{12,16}(X) + 7.47563171302734B_{13,16}(X) \\
& + 11.39143104065633B_{14,16}(X) + 16.01919386269591B_{15,16}(X) + 21.35892050398371B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3749995424882778, 0.6249993300354412\}$$

Intersection intervals with the x axis:

$$[0.3749995424882778, 0.6249993300354412]$$

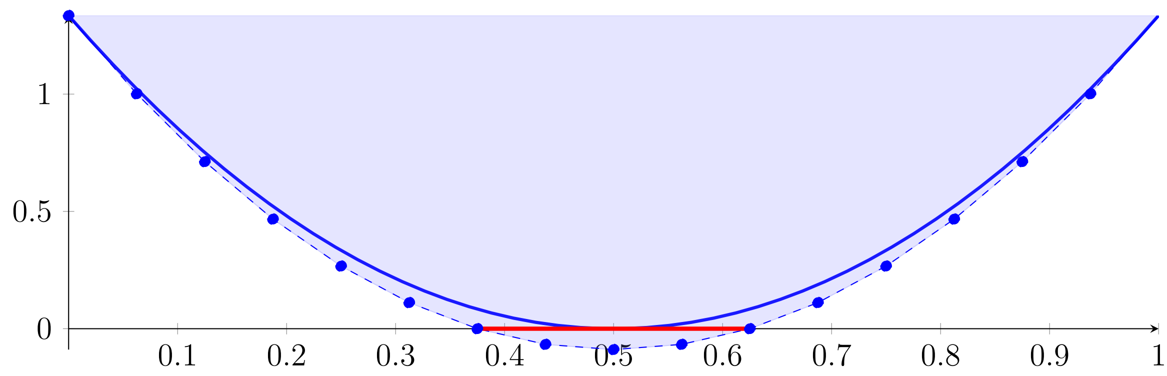
Longest intersection interval: 0.2499997875471635

$\Longrightarrow$ Selective recursion: interval 1: $[0.2999935885315074, 0.300006581473409]$,

73.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999935885315074, 0.30000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -6.596596862099299\cdot 10^{-79}X^{16} + 4.062171102620318\cdot 10^{-74}X^{15} + 8.765030607683244 \\
& \cdot 10^{-67}X^{14} - 5.261467161278336\cdot 10^{-62}X^{13} - 4.958117265353456\cdot 10^{-55}X^{12} \\
& + 2.739981763166457\cdot 10^{-50}X^{11} + 1.550371378259149\cdot 10^{-43}X^{10} - 7.424637900674377 \\
& \cdot 10^{-39}X^{9} - 2.898009393348535\cdot 10^{-32}X^{8} + 1.111571585405421\cdot 10^{-27}X^{7} + 3.241620002541647 \\
& \cdot 10^{-21}X^{6} - 8.756501261136649\cdot 10^{-17}X^{5} - 2.010795357557322\cdot 10^{-10}X^{4} + 2.844332798771609 \\
& \cdot 10^{-06}X^{3} + 5.339698243852309X^{2} - 5.339698356505613X + 1.33492347100565 \\
= & \, 1.33492347100565B_{0,16}(X) + 1.00119232372405B_{1,16}(X) + 0.7119586618078846B_{2,16}(X) \\
& + 0.4672224903363214B_{3,16}(X) + 0.266983814388415B_{4,16}(X) + 0.1112426390431101B_{5,16}(X) \\
& - 1.030620758846691\cdot 10^{-06}B_{6,16}(X) - 0.066747189524468196B_{7,16}(X) \\
& - 0.0889958325894046B_{8,16}(X) - 0.06674695473706526B_{9,16}(X) - 5.508890578613216 \\
& \cdot 10^{-07}B_{10,16}(X) + 0.1112433840328995B_{11,16}(X) + 0.2669848551069781B_{12,16}(X) \\
& + 0.4672238674112388B_{13,16}(X) + 0.7119604260236321B_{14,16}(X) \\
& + 1.001194536021998B_{15,16}(X) + 1.334926202484066B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3749994209666256, 0.6250003095051082\}$$

Intersection intervals with the x axis:

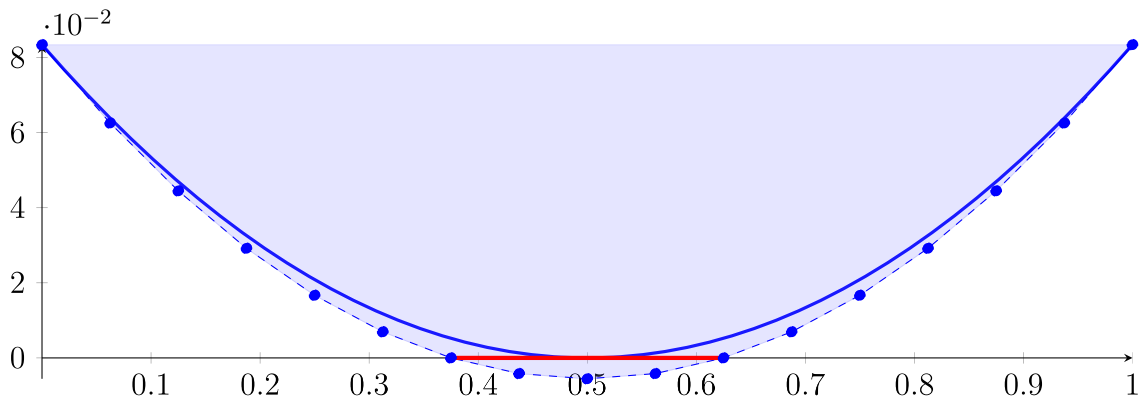
$$[0.3749994209666256, 0.6250003095051082]$$

Longest intersection interval: 0.2500008885384826
$\Longrightarrow$ Selective recursion: interval 1: [0.2999984608771972, 0.3000017091242173],

73.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.2999984608771972, 0.300

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.5359772362815\cdot10^{-88}X^{16} + 3.783024712863664\cdot10^{-83}X^{15} + 3.265391675561962\cdot10^{-75}X^{14}\\ &- 7.83987355616511\cdot10^{-70}X^{13} - 2.955395756595885\cdot10^{-62}X^{12} + 6.532349021061568\\ &\cdot10^{-57}X^{11} + 1.478602992973913\cdot10^{-49}X^{10} - 2.832143398905245\cdot10^{-44}X^{9}\\ &- 4.422140960586434\cdot10^{-37}X^{8} + 6.784132679612849\cdot10^{-32}X^{7} + 7.914287227230788\cdot10^{-25}X^{6}\\ &- 8.550710444673422\cdot10^{-20}X^{5} - 7.854787446136848\cdot10^{-13}X^{4} + 4.443846100456459\\ &\cdot10^{-08}X^{3} + 0.3337337124913315X^{2} - 0.3337336004955191X + 0.08343257581494635\\ = &\ 0.08343257581494635B_{0,16}(X) + 0.06257422578397641B_{1,16}(X) + 0.04449699002376757B_{2,16}(X)\\ &+ 0.02920086861367421B_{3,16}(X) + 0.01668586163305031B_{4,16}(X)\\ &+ 0.006951969161249391B_{5,16}(X) - 8.08722375440678\cdot10^{-07}B_{6,16}(X)\\ &- 0.004172471938471519B_{7,16}(X) - 0.005563020407686608B_{8,16}(X)\\ &- 0.004172454050668901B_{9,16}(X) - 7.727880670249089\cdot10^{-07}B_{10,16}(X)\\ &+ 0.006952023459469962B_{11,16}(X) + 0.01668593477129257B_{12,16}(X)\\ &+ 0.02920096122675088B_{13,16}(X) + 0.04449710290519453B_{14,16}(X)\\ &+ 0.06257435988597275B_{15,16}(X) + 0.08343273224843432B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749927302224649, 0.6250069467380417\}$$

Intersection intervals with the x axis:

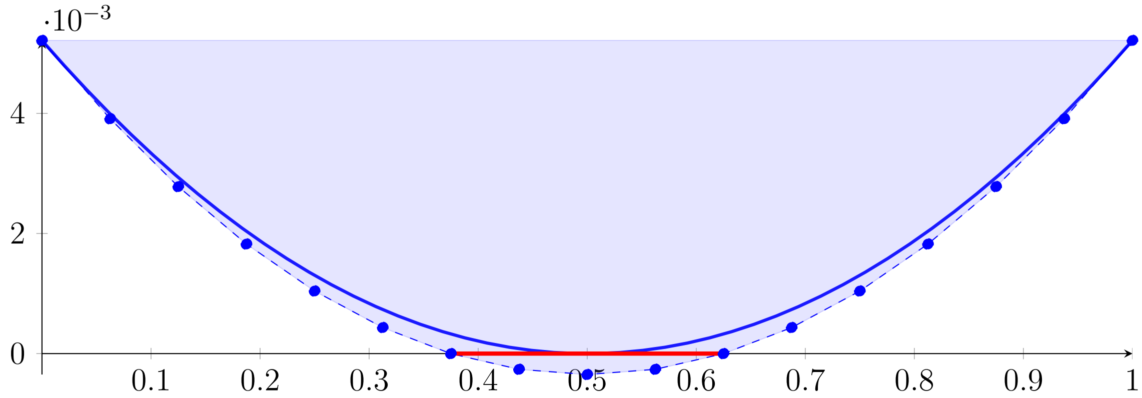
$$[0.3749927302224649, 0.6250069467380417]$$

Longest intersection interval: 0.2500142165155768
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999996789462157, 0.3000004910541495]$,

73.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999996789462157, 0.3$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.579480927696053 \cdot 10^{-98} X^{16} + 3.526136882584451 \cdot 10^{-92} X^{15} + 1.217422078989368 \\
& \cdot 10^{-83} X^{14} - 1.169070552755016 \cdot 10^{-77} X^{13} - 1.762755817503918 \cdot 10^{-69} X^{12} \\
& + 1.55837613779483 \cdot 10^{-63} X^{11} + 1.410908032488726 \cdot 10^{-55} X^{10} - 1.080908862805813 \cdot 10^{-49} X^{9} \\
& - 6.750723381404434 \cdot 10^{-42} X^{8} + 4.142273519440928 \cdot 10^{-36} X^{7} + 1.932858818028869 \cdot 10^{-28} X^{6} \\
& - 8.352503738960762 \cdot 10^{-23} X^{5} - 3.06897495525372 \cdot 10^{-15} X^{4} + 6.944510025023856 \cdot 10^{-10} X^{3} \\
& + 0.02086073248825264 X^{2} - 0.0208607236297874 X + 0.005214387850787818 \\
= & \; 0.005214387850787818 B_{0,16}(X) + 0.003910592623926106 B_{1,16}(X) \\
& + 0.002780636834466498 B_{2,16}(X) + 0.001824520483649087 B_{3,16}(X) \\
& + 0.001042243572713962 B_{4,16}(X) + 0.000433806102901211 B_{5,16}(X) - 7.919245490808786 \\
& \cdot 10^{-07} B_{6,16}(X) - 0.0002615505083968289 B_{7,16}(X) - 0.0003484696474019504 B_{8,16}(X) \\
& - 0.0002615493403243644 B_{9,16}(X) - 7.895859239916112 \cdot 10^{-07} B_{10,16}(X) \\
& + 0.0004338096170392455 B_{11,16}(X) + 0.001042248269805423 B_{12,16}(X) \\
& + 0.001824526373614615 B_{13,16}(X) + 0.002780643929706894 B_{14,16}(X) \\
& + 0.00391060093932233 B_{15,16}(X) + 0.005214397403700994 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748861124966261, 0.6251135508761015\}$$

Intersection intervals with the x axis:

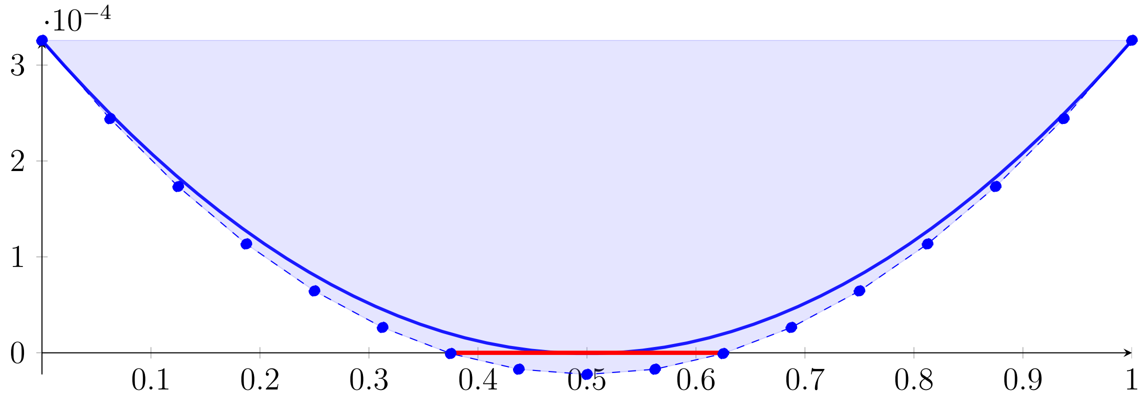
$$[0.3748861124966261, 0.6251135508761015]$$

Longest intersection interval: 0.2502274383794754
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999999833942019, 0.3000001866058899]$,

73.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999999833942019, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.456271781717561 \cdot 10^{-108} X^{16} + 3.329051186798107 \cdot 10^{-101} X^{15} + 4.593357002546748 \cdot 10^{-92} X^{14} - 1.76275695811523 \cdot 10^{-85} X^{13} - 1.062212310975265 \cdot 10^{-76} X^{12} + 3.752790404631579 \cdot 10^{-70} X^{11} + 1.357838209904933 \cdot 10^{-61} X^{10} - 4.157203866707784 \cdot 10^{-55} X^{9} - 1.037599553128416 \cdot 10^{-46} X^{8} + 2.544375234544489 \cdot 10^{-40} X^{7} + 4.744710700950638 \cdot 10^{-32} X^{6} - 8.193869976131749 \cdot 10^{-26} X^{5} - 1.203186876925119 \cdot 10^{-17} X^{4} + 1.088036641260801 \cdot 10^{-11} X^{3} + 0.001306169174102627 X^{2} - 0.001306168592474289 X + 0.0003257512461139167$$
$$= 0.0003257512461139167 B_{0,16}(X) + 0.0002441157090842736 B_{1,16}(X) + 0.0001733649151721524 B_{2,16}(X) + 0.0001134988643969824 B_{3,16}(X) + 6.451755677819264 \cdot 10^{-05} B_{4,16}(X) + 2.642099233521247 \cdot 10^{-05} B_{5,16}(X) - 7.908289125289409 \cdot 10^{-07} B_{6,16}(X) - 1.71179069456024 \cdot 10^{-05} B_{7,16}(X) - 2.25602417445787 \cdot 10^{-05} B_{8,16}(X) - 1.711783329002867 \cdot 10^{-05} B_{9,16}(X) - 7.906815625231293 \cdot 10^{-07} B_{10,16}(X) + 2.64212134573671 \cdot 10^{-05} B_{11,16}(X) + 6.451785178907119 \cdot 10^{-05} B_{12,16}(X) + 0.0001134992334520183 B_{13,16}(X) + 0.0001733653584656376 B_{14,16}(X) + 0.0002441162268493581 B_{15,16}(X) + 0.0003257518386226092 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3731836273807965, 0.626816029262996\}$$

Intersection intervals with the x axis:

$$[0.3731836273807965, 0.626816029262996]$$

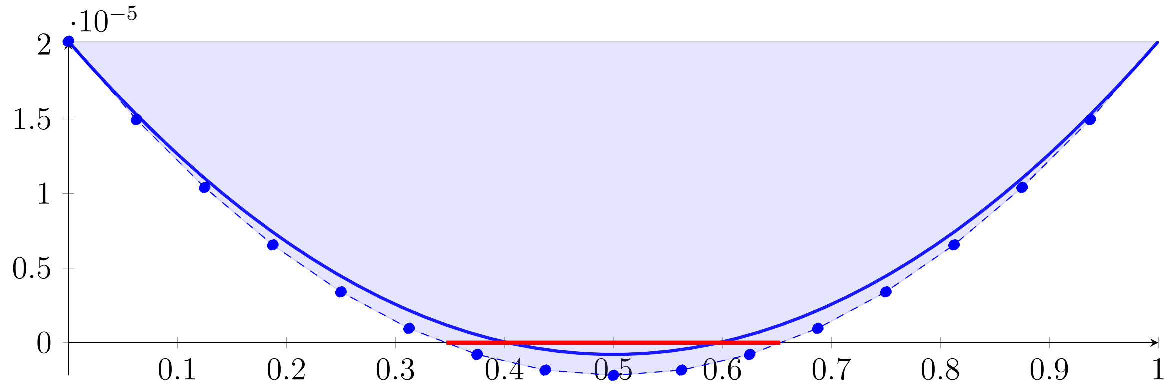
Longest intersection interval: 0.2536324018821995

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000592294767, 0.3000001107705452]$,

73.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000592294767

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.480017149294215 \cdot 10^{-117} X^{16} + 3.84938040332832 \cdot 10^{-110} X^{15} + 2.094095506848417 \cdot 10^{-100} X^{14} - 3.168497686663236 \cdot 10^{-93} X^{13} - 7.527801307502411 \cdot 10^{-84} X^{12} + 1.048590305184717 \cdot 10^{-76} X^{11} + 1.495875245120848 \cdot 10^{-67} X^{10} - 1.80569087496117 \cdot 10^{-60} X^{9} - 1.776919108083797 \cdot 10^{-51} X^{8} + 1.717962865912528 \cdot 10^{-44} X^{7} + 1.263101207166261 \cdot 10^{-35} X^{6} - 8.600271811104809 \cdot 10^{-29} X^{5} - 4.979113541149949 \cdot 10^{-20} X^{4} + 1.775239724826758 \cdot 10^{-13} X^{3} + 8.402507389292435 \cdot 10^{-05} X^{2} - 8.402503818595556 \cdot 10^{-05} X + 2.02154953599674 \cdot 10^{-05}$$
$$= 2.02154953599674 \cdot 10^{-05} B_{0,16}(X) + 1.496393047334518 \cdot 10^{-05} B_{1,16}(X) + 1.041257453583066 \cdot 10^{-05} B_{2,16}(X) + 6.561427547740848 \cdot 10^{-06} B_{3,16}(X) + 3.410489509392755 \cdot 10^{-06} B_{4,16}(X) + 9.59760421103387 \cdot 10^{-07} B_{5,16}(X) - 7.907597168102504 \cdot 10^{-07} B_{6,16}(X) - 1.84107090403115 \cdot 10^{-06} B_{7,16}(X) - 2.191173140242304 \cdot 10^{-06} B_{8,16}(X) - 1.841066425126706 \cdot 10^{-06} B_{9,16}(X) - 7.907507583673494 \cdot 10^{-07} B_{10,16}(X) + 9.59773860352773 \cdot 10^{-07} B_{11,16}(X) + 3.410507431350668 \cdot 10^{-06} B_{12,16}(X) + 6.561449954943343 \cdot 10^{-06} B_{13,16}(X) + 1.04126014314478 \cdot 10^{-05} B_{14,16}(X) + 1.496396186118106 \cdot 10^{-05} B_{15,16}(X) + 2.021553124446011 \cdot 10^{-05} B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3467669730097793, 0.6532326348738207\}$$

Intersection intervals with the x axis:

$$[0.3467669730097793, 0.6532326348738207]$$

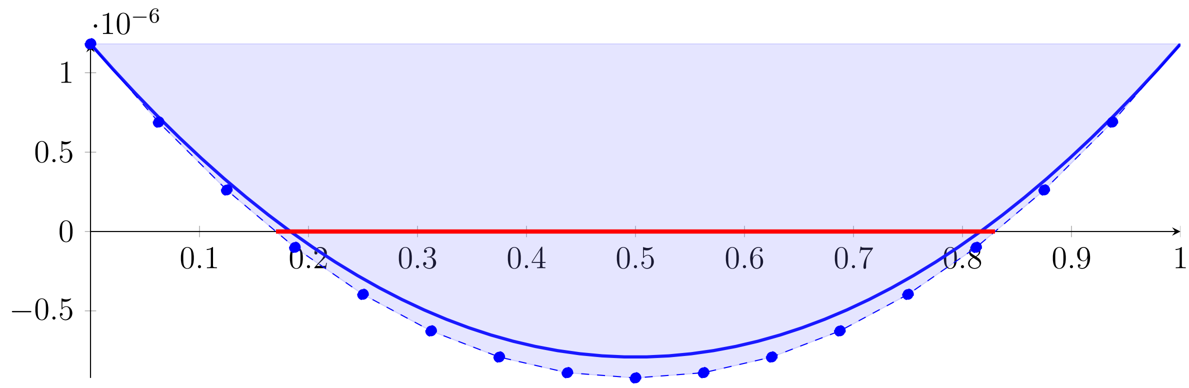
Longest intersection interval: 0.3064656618640414

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000771022171, 0.3000000928977847]$,

73.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000771022171, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.50163304400299\cdot 10^{-125}X^{16} + 7.605328677353438\cdot 10^{-118}X^{15} + 1.35002551430803\\
&\cdot 10^{-107}X^{14} - 6.665257392422306\cdot 10^{-100}X^{13} - 5.167137254241996\cdot 10^{-90}X^{12}\\
&+ 2.348582048403403\cdot 10^{-82}X^{11} + 1.093235143477049\cdot 10^{-72}X^{10} - 4.306055922184639\cdot 10^{-65}X^{9}\\
&- 1.382681732768409\cdot 10^{-55}X^{8} + 4.362007350883115\cdot 10^{-48}X^{7} + 1.04647552702743\cdot 10^{-38}X^{6}\\
&- 2.324990317134255\cdot 10^{-31}X^{5} - 4.392171749769965\cdot 10^{-22}X^{4} + 5.109781163448487\cdot 10^{-15}X^{3}\\
&+ 7.891735947253277\cdot 10^{-06}X^{2} - 7.891735064633028\cdot 10^{-06}X + 1.182178307271875\cdot 10^{-06}\\
= &\ 1.182178307271875\cdot 10^{-06}B_{0,16}(X) + 6.889448657323109\cdot 10^{-07}B_{1,16}(X) + 2.614758904198573\\
&\cdot 10^{-07}B_{2,16}(X) - 1.00228618656361\cdot 10^{-07}B_{3,16}(X) - 3.961686614872195\cdot 10^{-07}B_{4,16}(X)\\
&- 6.263442380635935\cdot 10^{-07}B_{5,16}(X) - 7.907553483763585\cdot 10^{-07}B_{6,16}(X) - 8.894019924163897\\
&\cdot 10^{-07}B_{7,16}(X) - 9.222841701745627\cdot 10^{-07}B_{8,16}(X) - 8.894018816417527\cdot 10^{-07}B_{9,16}(X)\\
&- 7.907551268088353\cdot 10^{-07}B_{10,16}(X) - 6.263439056666857\cdot 10^{-07}B_{11,16}(X)\\
&- 3.961682182061795\cdot 10^{-07}B_{12,16}(X) - 1.002280644181919\cdot 10^{-07}B_{13,16}(X) + 2.614765557064017\\
&\cdot 10^{-07}B_{14,16}(X) + 6.889456421767258\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1701811983018366, 0.8298187006126137\}$$

Intersection intervals with the x axis:

$$[0.1701811983018366, 0.8298187006126137]$$

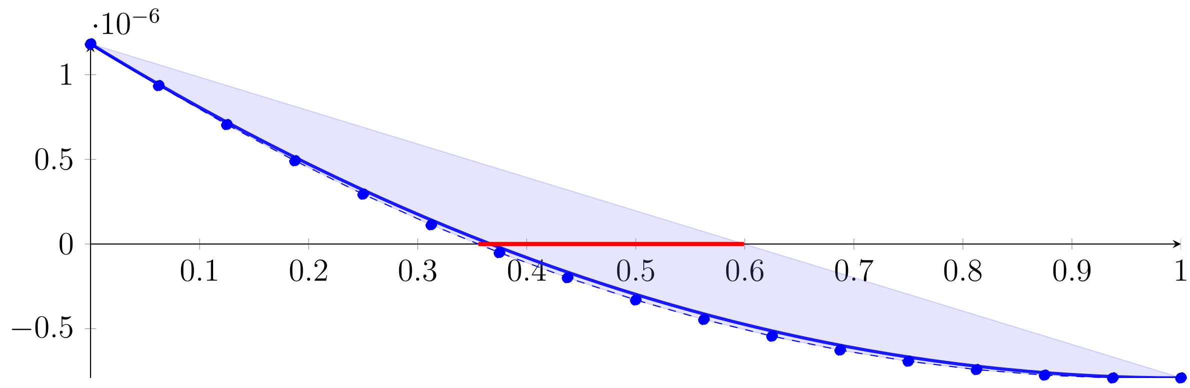
Longest intersection interval: 0.6596375023107771

$\Longrightarrow$ Bisection: first half $[0.3000000771022171, 0.3000000850000009]$ und second half $[0.3000000850000009, 0.30$

73.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3000000771022171, 0.3000000850000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -2.291310186772141\cdot 10^{-130}X^{16} + 2.320962120774364\cdot 10^{-122}X^{15} + 8.239901820727723 \\
& \cdot 10^{-112}X^{14} - 8.13630052785926\cdot 10^{-104}X^{13} - 1.261508118711425\cdot 10^{-93}X^{12} \\
& + 1.146768578321974\cdot 10^{-85}X^{11} + 1.067612444801806\cdot 10^{-75}X^{10} - 8.410265473016874\cdot 10^{-68}X^{9} \\
& - 5.401100518626597\cdot 10^{-58}X^{8} + 3.407818242877434\cdot 10^{-50}X^{7} + 1.635118010980359\cdot 10^{-40}X^{6} \\
& - 7.265594741044545\cdot 10^{-33}X^{5} - 2.745107343606228\cdot 10^{-23}X^{4} + 6.387226454310609\cdot 10^{-16}X^{3} \\
& + 1.972933986813319\cdot 10^{-06}X^{2} - 3.945867532316514\cdot 10^{-06}X + 1.182178307271875\cdot 10^{-06} \\
= & \; 1.182178307271875\cdot 10^{-06}B_{0,16}(X) + 9.35561586502093\cdot 10^{-07}B_{1,16}(X) + 7.053859822890886 \\
& \cdot 10^{-07}B_{2,16}(X) + 4.916514946340023\cdot 10^{-07}B_{3,16}(X) + 2.943581235379749\cdot 10^{-07}B_{4,16}(X) \\
& + 1.135058690021469\cdot 10^{-07}B_{5,16}(X) - 5.090526897234114\cdot 10^{-08}B_{6,16}(X) - 1.988752903843486 \\
& \cdot 10^{-07}B_{7,16}(X) - 3.30404195232735\cdot 10^{-07}B_{8,16}(X) - 4.454919835163597\cdot 10^{-07}B_{9,16}(X) \\
& - 5.441386552340822\cdot 10^{-07}B_{10,16}(X) - 6.263442103847618\cdot 10^{-07}B_{11,16}(X) - 6.921086489672579 \\
& \cdot 10^{-07}B_{12,16}(X) - 7.414319709804301\cdot 10^{-07}B_{13,16}(X) - 7.743141764231377\cdot 10^{-07}B_{14,16}(X) \\
& - 7.907552652942402\cdot 10^{-07}B_{15,16}(X) - 7.907552375925969\cdot 10^{-07}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.355648638833307, 0.599198239772989\}$$

Intersection intervals with the x axis:

$$[0.355648638833307, 0.599198239772989]$$

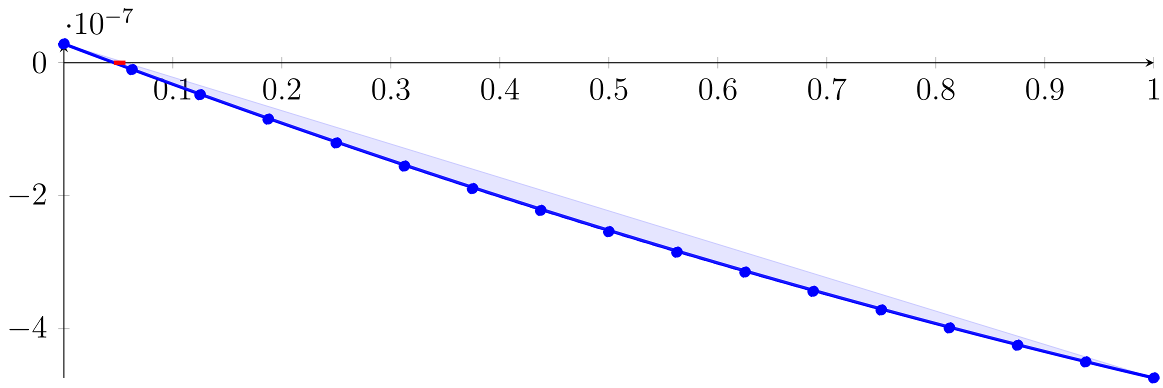
Longest intersection interval: 0.2435496009396821

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799110531, 0.3000000818345552]$,

73.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000799110531, 0.3000000818345552]

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -3.511418903935676\cdot 10^{-140}X^{16} + 1.460425350169266\cdot 10^{-131}X^{15} + 2.12885435080787 \\
& \cdot 10^{-120}X^{14} - 8.631046367525725\cdot 10^{-112}X^{13} - 5.494638409481287\cdot 10^{-101}X^{12} \\
& + 2.050866587664798\cdot 10^{-92}X^{11} + 7.839490728884222\cdot 10^{-82}X^{10} - 2.535691914683604\cdot 10^{-73}X^{9} \\
& - 6.686235030771963\cdot 10^{-63}X^{8} + 1.732161414466012\cdot 10^{-54}X^{7} + 3.412507672068927\cdot 10^{-44}X^{6} \\
& - 6.225987736384381\cdot 10^{-36}X^{5} - 9.658485252841958\cdot 10^{-26}X^{4} + 9.227298165814739\cdot 10^{-18}X^{3} \\
& + 1.170273575918748\cdot 10^{-07}X^{2} - 6.192309389580206\cdot 10^{-07}X + 2.838432851654629\cdot 10^{-08} \\
= & \; 2.838432851654629\cdot 10^{-08}B_{0,16}(X) - 1.031760516832999\cdot 10^{-08}B_{1,16}(X) - 4.804431087327399 \\
& \cdot 10^{-08}B_{2,16}(X) - 8.479578859826921\cdot 10^{-08}B_{3,16}(X) - 1.205720383432992\cdot 10^{-07}B_{4,16}(X) \\
& - 1.553730601083475\cdot 10^{-07}B_{5,16}(X) - 1.891988538933975\cdot 10^{-07}B_{6,16}(X) - 2.220494196984329 \\
& \cdot 10^{-07}B_{7,16}(X) - 2.539247575234371\cdot 10^{-07}B_{8,16}(X) - 2.848248673683937\cdot 10^{-07}B_{9,16}(X) \\
& - 3.147497492332862\cdot 10^{-07}B_{10,16}(X) - 3.436994031180981\cdot 10^{-07}B_{11,16}(X) - 3.71673829022813 \\
& \cdot 10^{-07}B_{12,16}(X) - 3.986730269474143\cdot 10^{-07}B_{13,16}(X) - 4.246969968918855\cdot 10^{-07}B_{14,16}(X) \\
& - 4.497457388562103\cdot 10^{-07}B_{15,16}(X) - 4.738192528403721\cdot 10^{-07}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.04583803348764934, 0.05651956610873593\}$$

Intersection intervals with the x axis:

$$[0.04583803348764934, 0.05651956610873593]$$

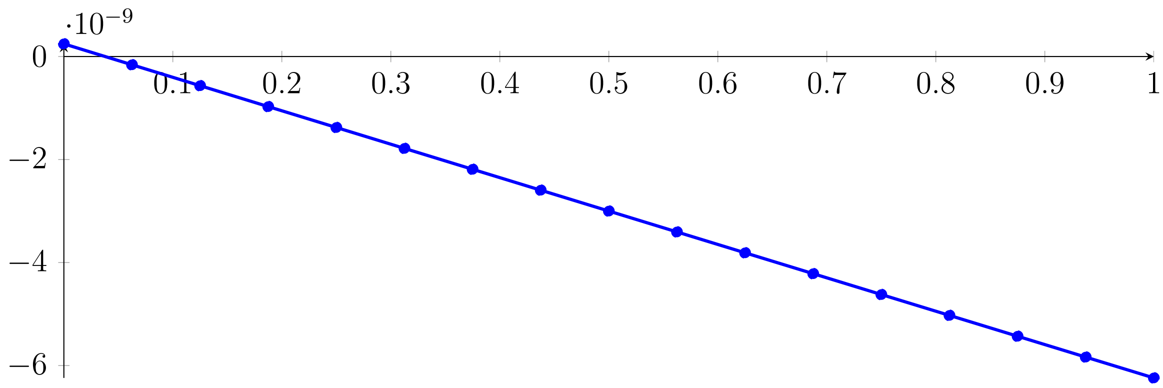
Longest intersection interval: 0.01068153262108659

$\Longrightarrow$ Selective recursion: interval 1: [0.3000000799992227, 0.3000000800197686],

73.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799992227, 0.3000000800197686]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.0083696859672\cdot10^{-171}X^{16} + 3.926295021503001\cdot10^{-161}X^{15} + 5.358163301918743\\ &\cdot10^{-148}X^{14} - 2.033760808321352\cdot10^{-137}X^{13} - 1.212109824997298\cdot10^{-124}X^{12}\\ &+ 4.235519560164542\cdot10^{-114}X^{11} + 1.515735988691705\cdot10^{-101}X^{10} - 4.589851404660334\cdot10^{-91}X^{9}\\ &- 1.133052955101212\cdot10^{-78}X^{8} + 2.748041945976198\cdot10^{-68}X^{7} + 5.068448795464027\cdot10^{-56}X^{6}\\ &- 8.657173392624336\cdot10^{-46}X^{5} - 1.257312709564581\cdot10^{-33}X^{4} + 1.124540929737694\cdot10^{-23}X^{3}\\ &+ 1.335225264723055\cdot10^{-11}X^{2} - 6.499737499496223\cdot10^{-09}X + 2.458891434694464\cdot10^{-10}\\ = &\ 2.458891434694464\cdot10^{-10}B_{0,16}(X) - 1.603444502490676\cdot10^{-10}B_{1,16}(X) - 5.664667751955213\\ &\cdot10^{-10}B_{2,16}(X) - 9.724778313699147\cdot10^{-10}B_{3,16}(X) - 1.378377618772248\cdot10^{-09}B_{4,16}(X)\\ &- 1.784166137402521\cdot10^{-09}B_{5,16}(X) - 2.189843387260733\cdot10^{-09}B_{6,16}(X) - 2.595409368346885\\ &\cdot10^{-09}B_{7,16}(X) - 3.000864080660977\cdot10^{-09}B_{8,16}(X) - 3.406207524203008\cdot10^{-09}B_{9,16}(X)\\ &- 3.811439698972979\cdot10^{-09}B_{10,16}(X) - 4.21656060497089\cdot10^{-09}B_{11,16}(X) - 4.62157024219674\\ &\cdot10^{-09}B_{12,16}(X) - 5.026468610650529\cdot10^{-09}B_{13,16}(X) - 5.431255710332258\cdot10^{-09}B_{14,16}(X)\\ &- 5.835931541241927\cdot10^{-09}B_{15,16}(X) - 6.240496103379535\cdot10^{-09}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03783062677354348, 0.03790850128565779\}$$

Intersection intervals with the x axis:

$$[0.03783062677354348, 0.03790850128565779]$$

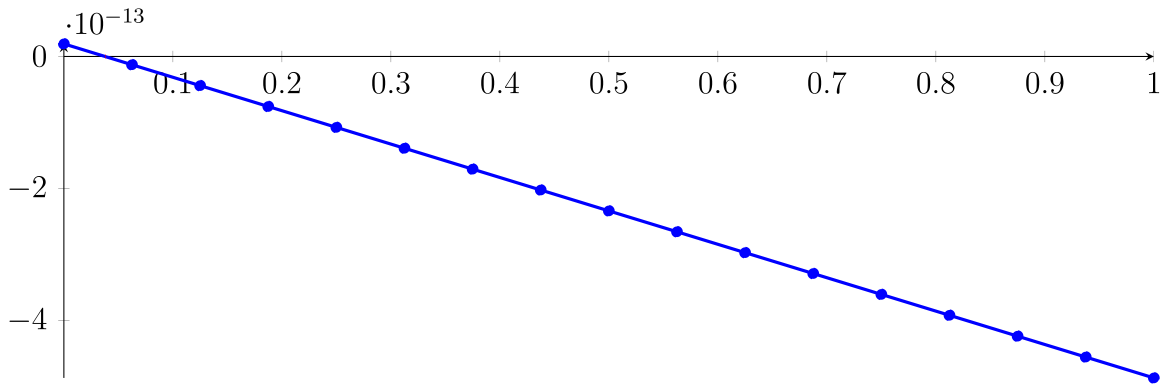
Longest intersection interval: $7.787451211431613\cdot10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999999, 0.3000000800000015]$,

73.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000799999999, 0.3000000800000015]

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = & -1.844782636201088\cdot 10^{-237}X^{16} + 9.223866560433282\cdot 10^{-223}X^{15} + 1.616406874872468\\ & \cdot 10^{-205}X^{14} - 7.878422704947929\cdot 10^{-191}X^{13} - 6.029565776386014\cdot 10^{-174}X^{12}\\ & + 2.705549092213795\cdot 10^{-159}X^{11} + 1.243302910922026\cdot 10^{-142}X^{10} - 4.83455688747521\cdot 10^{-128}X^{9}\\ & - 1.5325438805125\cdot 10^{-111}X^{8} + 4.772992249567202\cdot 10^{-97}X^{7} + 1.130438906312722\cdot 10^{-80}X^{6}\\ & - 2.479435471431183\cdot 10^{-66}X^{5} - 4.624072778948578\cdot 10^{-50}X^{4} + 5.310816347750779\cdot 10^{-36}X^{3}\\ & + 8.097393019768196\cdot 10^{-20}X^{2} - 5.060852140608093\cdot 10^{-13}X + 1.910916079008261\cdot 10^{-14}\\ = & \ 1.910916079008261\cdot 10^{-14}B_{0,16}(X) - 1.252116508871797\cdot 10^{-14}B_{1,16}(X) - 4.41514902927358\\ & \cdot 10^{-14}B_{2,16}(X) - 7.578181482197087\cdot 10^{-14}B_{3,16}(X) - 1.074121386764232\cdot 10^{-13}B_{4,16}(X)\\ & - 1.390424618560928\cdot 10^{-13}B_{5,16}(X) - 1.706727843609796\cdot 10^{-13}B_{6,16}(X) - 2.023031061910837\\ & \cdot 10^{-13}B_{7,16}(X) - 2.33933427346405\cdot 10^{-13}B_{8,16}(X) - 2.655637478269435\cdot 10^{-13}B_{9,16}(X)\\ & - 2.971940676326994\cdot 10^{-13}B_{10,16}(X) - 3.288243867636724\cdot 10^{-13}B_{11,16}(X) - 3.604547052198627\\ & \cdot 10^{-13}B_{12,16}(X) - 3.920850230012703\cdot 10^{-13}B_{13,16}(X) - 4.237153401078951\cdot 10^{-13}B_{14,16}(X)\\ & - 4.553456565397371\cdot 10^{-13}B_{15,16}(X) - 4.869759722967965\cdot 10^{-13}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775878104944304, 0.03775878709087106\}$$

Intersection intervals with the x axis:

$$[0.03775878104944304, 0.03775878709087106]$$

Longest intersection interval: $6.041428015081178\cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

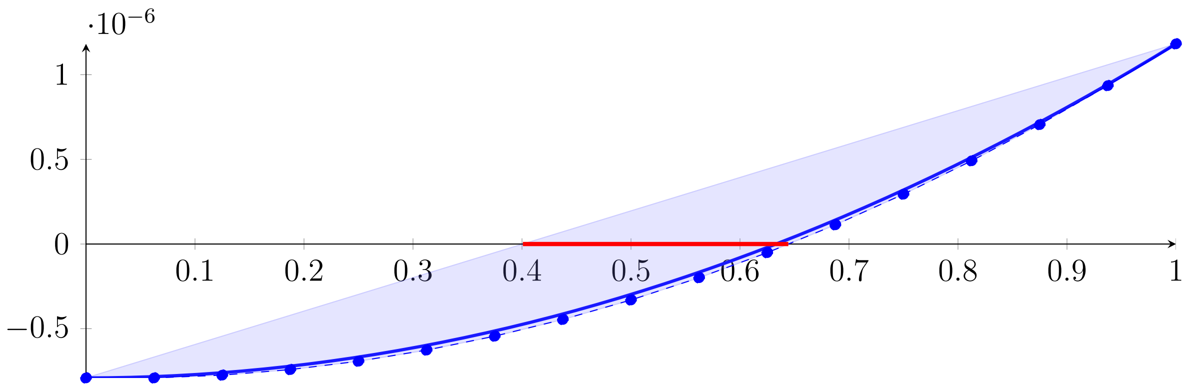
73.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.30000008, 0.30000008]

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 19!

73.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.3000000850000009, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -2.291310186772141\cdot 10^{-130}X^{16} + 2.320961754164735\cdot 10^{-122}X^{15} + 8.239901824209166 \\
& \cdot 10^{-112}X^{14} - 8.136299374273005\cdot 10^{-104}X^{13} - 1.261508119769144\cdot 10^{-93}X^{12} \\
& + 1.146768426941\cdot 10^{-85}X^{11} + 1.067612446063251\cdot 10^{-75}X^{10} - 8.410264405404428\cdot 10^{-68}X^{9} \\
& - 5.401100526195836\cdot 10^{-58}X^{8} + 3.407817810789392\cdot 10^{-50}X^{7} + 1.635118013365832\cdot 10^{-40}X^{6} \\
& - 7.265593759973738\cdot 10^{-33}X^{5} - 2.745107347239025\cdot 10^{-23}X^{4} + 6.387225356267671\cdot 10^{-16}X^{3} \\
& + 1.972933988729487\cdot 10^{-06}X^{2} + 4.432262923936659\cdot 10^{-13}X - 7.907552375925969\cdot 10^{-07} \\
= & -7.907552375925969\cdot 10^{-07}B_{0,16}(X) - 7.907552098909536\cdot 10^{-07}B_{1,16}(X) - 7.743140656165646 \\
& \cdot 10^{-07}B_{2,16}(X) - 7.414318047682893\cdot 10^{-07}B_{3,16}(X) - 6.921084273449871\cdot 10^{-07}B_{4,16}(X) \\
& - 6.263439333455175\cdot 10^{-07}B_{5,16}(X) - 5.441383227687398\cdot 10^{-07}B_{6,16}(X) - 4.454915956135136 \\
& \cdot 10^{-07}B_{7,16}(X) - 3.304037518786981\cdot 10^{-07}B_{8,16}(X) - 1.988747915631529\cdot 10^{-07}B_{9,16}(X) \\
& - 5.090471466573741\cdot 10^{-08}B_{10,16}(X) + 1.13506478814689\cdot 10^{-07}B_{11,16}(X) + 2.943587888792669 \\
& \cdot 10^{-07}B_{12,16}(X) + 4.916522155291369\cdot 10^{-07}B_{13,16}(X) + 7.053867587654395\cdot 10^{-07}B_{14,16}(X) \\
& + 9.355624185893154\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4008015798845968, 0.644351143917019\}$$

Intersection intervals with the x axis:

$$[0.4008015798845968, 0.644351143917019]$$

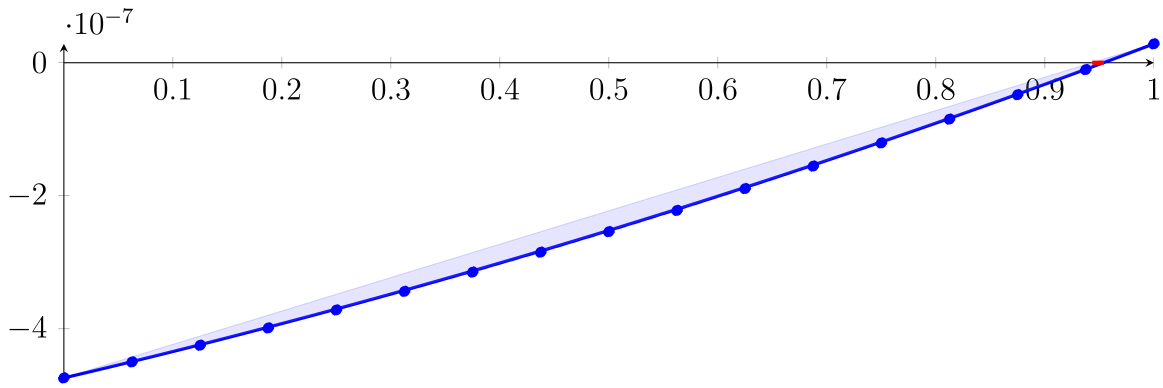
Longest intersection interval: 0.2435495640324222

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000881654451, 0.3000000900889469]$,

73.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.3000000881654451, 0.3000000900889469]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.511410390075519\cdot 10^{-140}X^{16} + 1.460421789403856\cdot 10^{-131}X^{15} + 2.12884983529034\\
&\cdot 10^{-120}X^{14} - 8.631028085335548\cdot 10^{-112}X^{13} - 5.494628422481359\cdot 10^{-101}X^{12}\\
&+ 2.050862886067143\cdot 10^{-92}X^{11} + 7.839478858688502\cdot 10^{-82}X^{10} - 2.535688119961682\cdot 10^{-73}X^{9}\\
&- 6.686226934767321\cdot 10^{-63}X^{8} + 1.732159347494186\cdot 10^{-54}X^{7} + 3.412504574505657\cdot 10^{-44}X^{6}\\
&- 6.225982140334265\cdot 10^{-36}X^{5} - 9.658479411653884\cdot 10^{-26}X^{4} + 9.227292313018015\cdot 10^{-18}X^{3}\\
&+ 1.170273222422553\cdot 10^{-07}X^{2} + 3.851762081107905\cdot 10^{-07}X - 4.738191826798638\cdot 10^{-07}\\
= & -4.738191826798638\cdot 10^{-07}B_{0,16}(X) - 4.497456696729394\cdot 10^{-07}B_{1,16}(X) - 4.246969289806629\\
&\cdot 10^{-07}B_{2,16}(X) - 3.986729606030177\cdot 10^{-07}B_{3,16}(X) - 3.716737645399875\cdot 10^{-07}B_{4,16}(X)\\
&- 3.436993407915557\cdot 10^{-07}B_{5,16}(X) - 3.147496893577059\cdot 10^{-07}B_{6,16}(X) - 2.848248102384216\\
&\cdot 10^{-07}B_{7,16}(X) - 2.539247034336863\cdot 10^{-07}B_{8,16}(X) - 2.220493689434835\cdot 10^{-07}B_{9,16}(X)\\
&- 1.891988067677968\cdot 10^{-07}B_{10,16}(X) - 1.553730169066096\cdot 10^{-07}B_{11,16}(X) - 1.205719993599056\\
&\cdot 10^{-07}B_{12,16}(X) - 8.479575412766813\cdot 10^{-08}B_{13,16}(X) - 4.804428120988084\cdot 10^{-08}B_{14,16}(X)\\
&- 1.031758060652722\cdot 10^{-08}B_{15,16}(X) + 2.838434768240921\cdot 10^{-08}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9434803899886293, 0.9541619291703946\}$$

Intersection intervals with the x axis:

$$[0.9434803899886293, 0.9541619291703946]$$

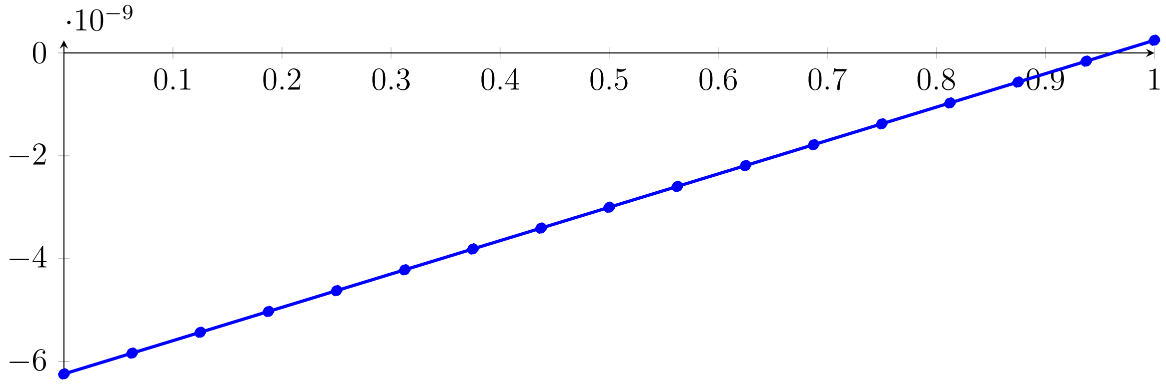
Longest intersection interval: 0.01068153918176529

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899802314, 0.3000000900007773]$,

73.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.3000000899802314, 0.3000000900007773]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -1.008377150647939 \cdot 10^{-171} X^{16} + 3.926321486414625 \cdot 10^{-161} X^{15} + 5.358198011634222 \\
& \cdot 10^{-148} X^{14} - 2.033772676384319 \cdot 10^{-137} X^{13} - 1.212116555959755 \cdot 10^{-124} X^{12} \\
& + 4.235540409672438 \cdot 10^{-114} X^{11} + 1.515743003803166 \cdot 10^{-101} X^{10} - 4.58986978058239 \cdot 10^{-91} X^{9} \\
& - 1.133057150938366 \cdot 10^{-78} X^{8} + 2.748050405674693 \cdot 10^{-68} X^{7} + 5.068462874895849 \cdot 10^{-56} X^{6} \\
& - 8.65719194232841 \cdot 10^{-46} X^{5} - 1.257315038545109 \cdot 10^{-33} X^{4} + 1.12454224629143 \cdot 10^{-23} X^{3} \\
& + 1.335226501895653 \cdot 10^{-11} X^{2} + 6.473035980688146 \cdot 10^{-09} X - 6.240498775828017 \cdot 10^{-09} \\
= & -6.240498775828017 \cdot 10^{-09} B_{0,16}(X) - 5.835934027035008 \cdot 10^{-09} B_{1,16}(X) - 5.43125800936684 \\
& \cdot 10^{-09} B_{2,16}(X) - 5.026470722823515 \cdot 10^{-09} B_{3,16}(X) - 4.621572167405032 \cdot 10^{-09} B_{4,16}(X) \\
& - 4.216562343111391 \cdot 10^{-09} B_{5,16}(X) - 3.811441249942592 \cdot 10^{-09} B_{6,16}(X) - 3.406208887898635 \\
& \cdot 10^{-09} B_{7,16}(X) - 3.000865256979519 \cdot 10^{-09} B_{8,16}(X) - 2.595410357185246 \cdot 10^{-09} B_{9,16}(X) \\
& - 2.189844188515814 \cdot 10^{-09} B_{10,16}(X) - 1.784166750971225 \cdot 10^{-09} B_{11,16}(X) \\
& - 1.378378044551477 \cdot 10^{-09} B_{12,16}(X) - 9.724780692565705 \cdot 10^{-10} B_{13,16}(X) - 5.664668250865062 \\
& \cdot 10^{-10} B_{14,16}(X) - 1.603443120412836 \cdot 10^{-10} B_{15,16}(X) + 2.458894698790972 \cdot 10^{-10} B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.9620914659183662, 0.9621693405339302\}$$

Intersection intervals with the x axis:

$$[0.9620914659183662, 0.9621693405339302]$$

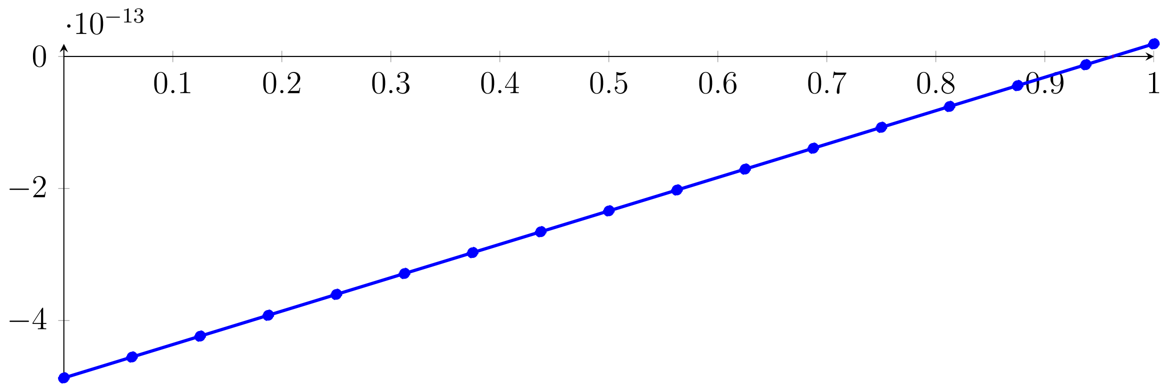
Longest intersection interval: $7.787461556406489 \cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899999985, 0.3000000900000001]$,

73.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.3000000899999985, 0.3000000900000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = {} & -1.844835503516189\cdot 10^{-237}X^{16} + 9.224112529673191\cdot 10^{-223}X^{15} + 1.616447407922287\\ & \cdot 10^{-205}X^{14} - 7.878604734573916\cdot 10^{-191}X^{13} - 6.029695377653573\cdot 10^{-174}X^{12}\\ & + 2.705601945023742\cdot 10^{-159}X^{11} + 1.243325181575566\cdot 10^{-142}X^{10} - 4.834634042887648\cdot 10^{-128}X^{9}\\ & - 1.532565842702471\cdot 10^{-111}X^{8} + 4.773051325459324\cdot 10^{-97}X^{7} + 1.130451056729432\cdot 10^{-80}X^{6}\\ & - 2.479457251994378\cdot 10^{-66}X^{5} - 4.624105915218427\cdot 10^{-50}X^{4} + 5.31084372815498\cdot 10^{-36}X^{3}\\ & + 8.097422035992938\cdot 10^{-20}X^{2} + 5.060859587615554\cdot 10^{-13}X - 4.869768282121624\cdot 10^{-13}\\ = {} & -4.869768282121624\cdot 10^{-13}B_{0,16}(X) - 4.553464557895652\cdot 10^{-13}B_{1,16}(X) - 4.237160826921828\\ & \cdot 10^{-13}B_{2,16}(X) - 3.920857089200152\cdot 10^{-13}B_{3,16}(X) - 3.604553344730625\cdot 10^{-13}B_{4,16}(X)\\ & - 3.288249593513246\cdot 10^{-13}B_{5,16}(X) - 2.971945835548016\cdot 10^{-13}B_{6,16}(X) - 2.655642070834933\\ & \cdot 10^{-13}B_{7,16}(X) - 2.339338299374\cdot 10^{-13}B_{8,16}(X) - 2.023034521165214\cdot 10^{-13}B_{9,16}(X)\\ & - 1.706730736208576\cdot 10^{-13}B_{10,16}(X) - 1.390426944504087\cdot 10^{-13}B_{11,16}(X)\\ & - 1.074123146051747\cdot 10^{-13}B_{12,16}(X) - 7.578193408515542\cdot 10^{-14}B_{13,16}(X) - 4.4151552890351\\ & \cdot 10^{-14}B_{14,16}(X) - 1.252117102076142\cdot 10^{-14}B_{15,16}(X) + 1.910921152361334\cdot 10^{-14}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411803068306, 0.9622411863482746\}$$

Intersection intervals with the x axis:

$$[0.9622411803068306, 0.9622411863482746]$$

Longest intersection interval: $6.041444057142176\cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

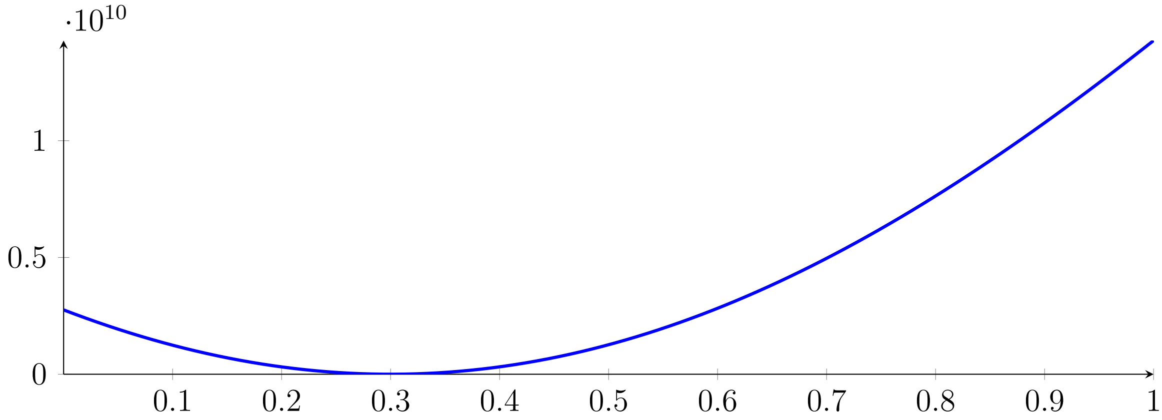
73.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 19!

73.25 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

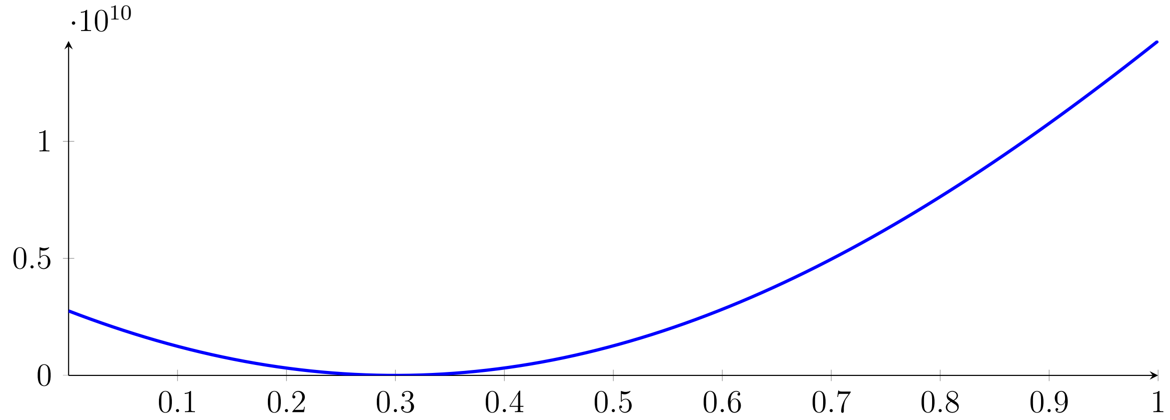
with precision $\varepsilon = 1 \cdot 10^{-16}$.

74 Running QuadClip on h_{16} with epsilon 16

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called QuadClip with input polynomial on interval $[0, 1]$:

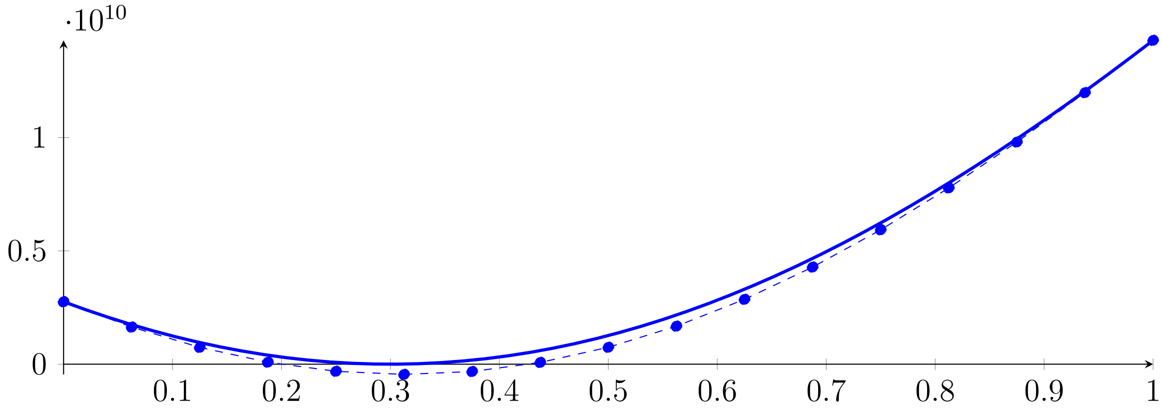
$$\begin{aligned} p = {} & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



74.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

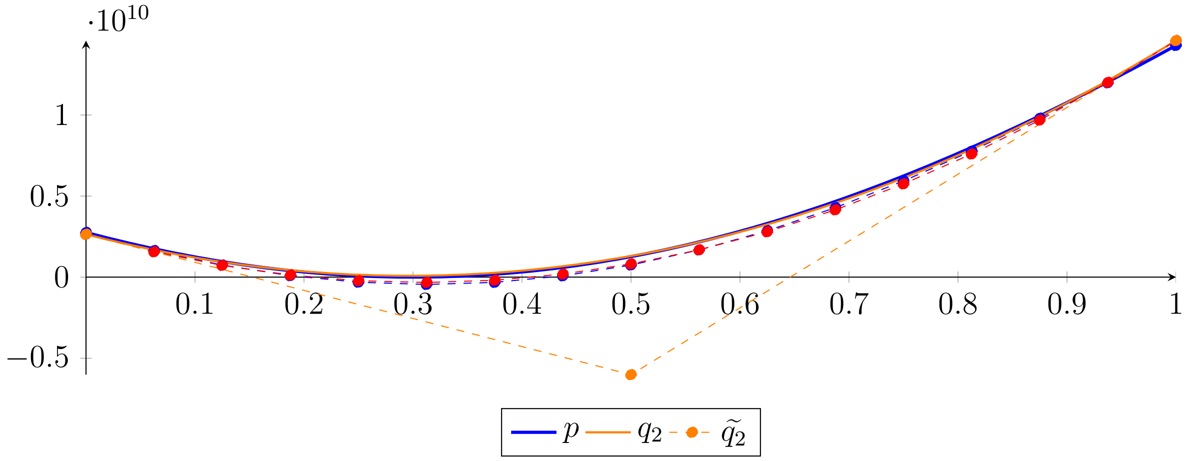
$$\begin{aligned} p = {} & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = {} & 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ & + 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ & - 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ & + 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ & + 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ & + 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 &= 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328 \\ &= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 &= -4.590615853685512\cdot 10^{-286}X^{16} + 3.626297739922361\cdot 10^{-285}X^{15} - 1.337118562447102 \\ &\quad \cdot 10^{-284}X^{14} + 3.057409950495903\cdot 10^{-284}X^{13} - 4.816080313213711\cdot 10^{-284}X^{12} \\ &\quad + 5.445450819003583\cdot 10^{-284}X^{11} - 4.453195910004646\cdot 10^{-284}X^{10} + 2.601913123147853 \\ &\quad \cdot 10^{-284}X^{9} - 1.060485581673177\cdot 10^{-284}X^{8} + 2.936863540613122\cdot 10^{-285}X^{7} - 5.487274374859613 \\ &\quad \cdot 10^{-286}X^{6} + 7.23018920152705\cdot 10^{-287}X^{5} - 7.254244946204871\cdot 10^{-288}X^{4} + 6.659990111567495 \\ &\quad \cdot 10^{-289}X^{3} + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328 \\ &= 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16} \\ &\quad + 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16} \\ &\quad - 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16} \\ &\quad + 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16} \\ &\quad + 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16} \\ &\quad + 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.

Bounding polynomials M and m:

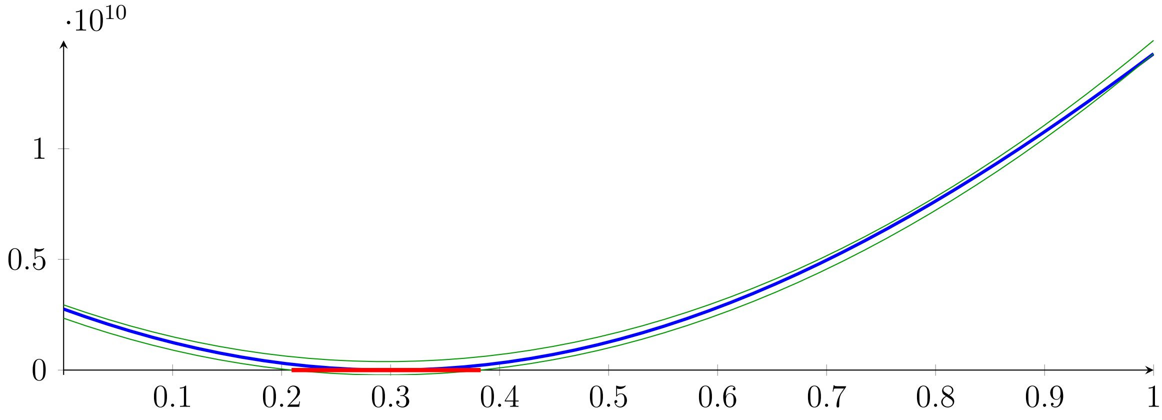
$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$

$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



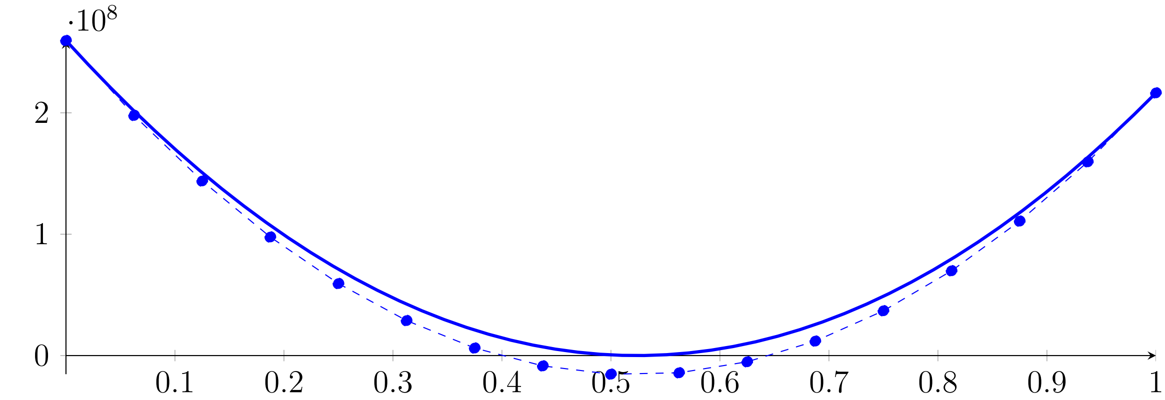
$[0.2091580131065301, 0.3826391383239738]$

Longest intersection interval: 0.1734811252174437
$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

74.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

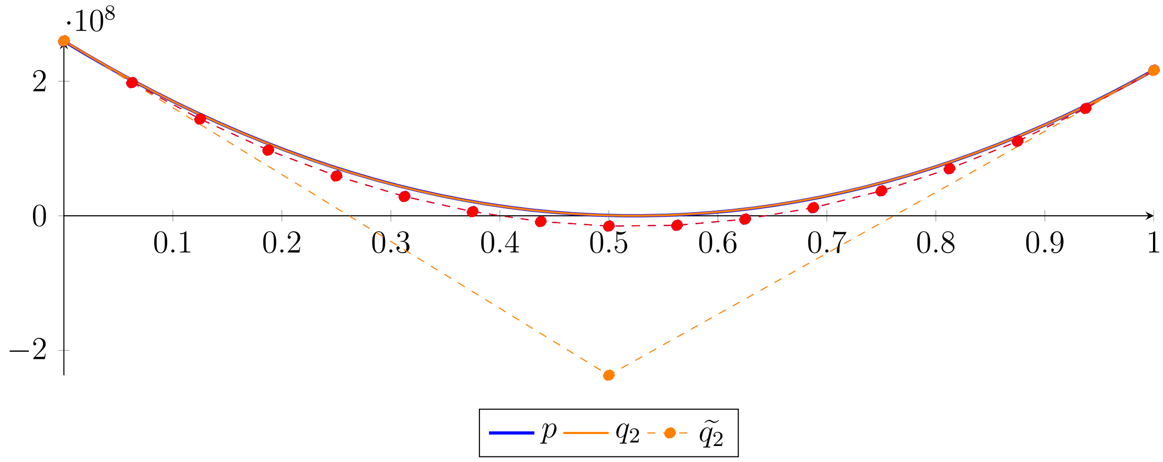
$$
\begin{aligned}
p = & -6.730319568869656 \cdot 10^{-13} X^{16} + 8.742499458235574 \cdot 10^{-12} X^{15} + 4.969734366538428 \cdot 10^{-09} X^{14} \\
& - 5.918022439742958 \cdot 10^{-08} X^{13} - 1.563835677279211 \cdot 10^{-05} X^{12} + 0.000165142833990623 X^{11} \\
& + 0.02725178204957372 X^{10} - 0.2448183330655992 X^{9} - 28.45820232077076 X^{8} \\
& + 205.241311077626 X^{7} + 17835.39577074806 X^{6} - 94141.21233496903 X^{5} - 6218439.344794644 X^{4} \\
& + 20000843.19616221 X^{3} + 930858984.7988209 X^{2} - 987724621.0538478 X + 259570777.0276934 \\
= & \, 259570777.0276934 B_{0,16}(X) + 197837988.2118279 B_{1,16}(X) + 143862357.6026193 B_{2,16}(X) \\
& + 97679600.99148916 B_{3,16}(X) + 59322017.44494463 B_{4,16}(X) + 28818467.75210257 B_{5,16}(X) \\
& + 6194355.099411763 B_{6,16}(X) - 8528392.009487306 B_{7,16}(X) - 15331334.57303514 B_{8,16}(X) \\
& - 14199535.63666165 B_{9,16}(X) - 5121570.552084079 B_{10,16}(X) + 11910465.05798682 B_{11,16}(X) \\
& + 36900949.63643701 B_{12,16}(X) + 69850732.33405701 B_{13,16}(X) \\
& + 110757131.9597003 B_{14,16}(X) + 159613938.238137 B_{15,16}(X) + 216411415.3731615 B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 950064117.9944898 X^2 - 993959420.5216784 X + 260029868.8092426 \\
&= 260029868.8092426 B_{0,2} - 236949841.4515966 B_{1,2} + 216134566.282054 B_{2,2}
\end{aligned}
$$

$$\begin{aligned}\widetilde{q}_2 &= -1.41520176718356\cdot 10^{-287}X^{16} + 1.139753408092347\cdot 10^{-286}X^{15} - 4.276104932493763\\&\quad\cdot 10^{-286}X^{14} + 9.896028733517596\cdot 10^{-286}X^{13} - 1.566233430583374\cdot 10^{-285}X^{12}\\&\quad + 1.767044733566612\cdot 10^{-285}X^{11} - 1.435814571471649\cdot 10^{-285}X^{10} + 8.336078570237132\\&\quad\cdot 10^{-286}X^{9} - 3.396584250429756\cdot 10^{-286}X^{8} + 9.548198482934522\cdot 10^{-287}X^{7} - 1.861944039074976\\&\quad\cdot 10^{-287}X^{6} + 2.622613520689986\cdot 10^{-288}X^{5} - 2.667693132942884\cdot 10^{-289}X^{4} + 2.048626792988895\\&\quad\cdot 10^{-290}X^{3} + 950064117.9944898X^{2} - 993959420.5216784X + 260029868.8092426\\&= 260029868.8092426B_{0,16} + 197907405.0266377B_{1,16} + 143702142.2273202B_{2,16}\\&\quad + 97414080.41129016B_{3,16} + 59043219.5785475B_{4,16} + 28589559.72909226B_{5,16}\\&\quad + 6053100.862924435B_{6,16} - 8566157.019955976B_{7,16} - 15268213.91954897B_{8,16}\\&\quad - 14053069.83585455B_{9,16} - 4920724.768872719B_{10,16} + 12128821.28139653B_{11,16}\\&\quad + 37095568.31495319B_{12,16} + 69979516.33179727B_{13,16}\\&\quad + 110780665.3319288B_{14,16} + 159499015.3153477B_{15,16} + 216134566.282054B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

Bounding polynomials M and m:

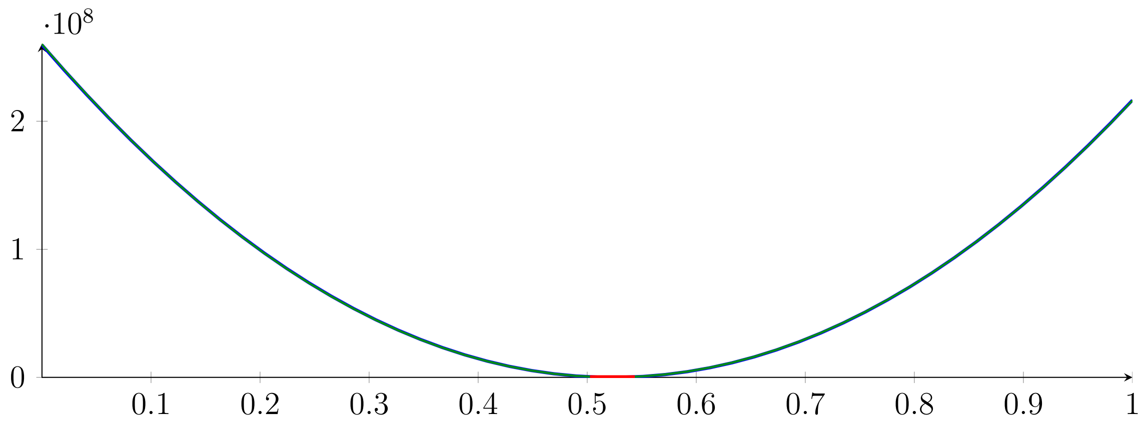
$$M = 950064117.9944898X^2 - 993959420.5216784X + 260488960.5907918$$

$$m = 950064117.9944898X^2 - 993959420.5216784X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

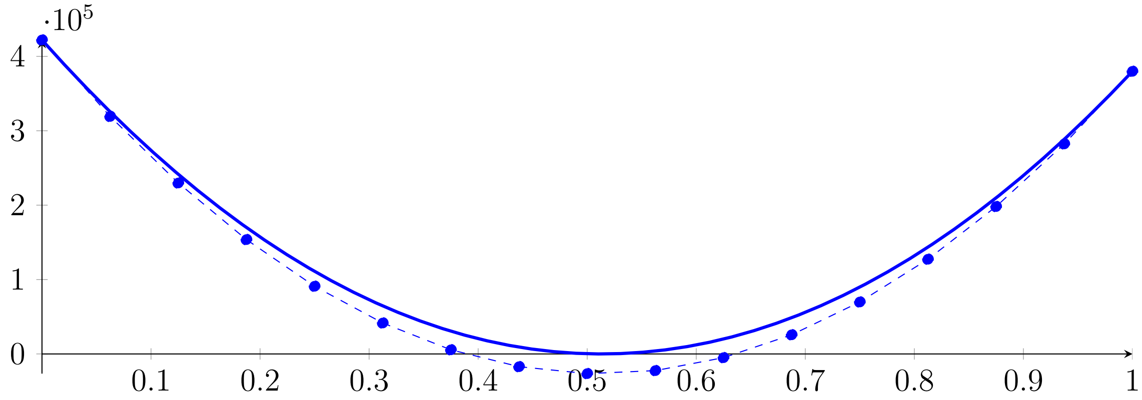
Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

74.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Normalized monomial und Bézier representations and the Bézier polygon:

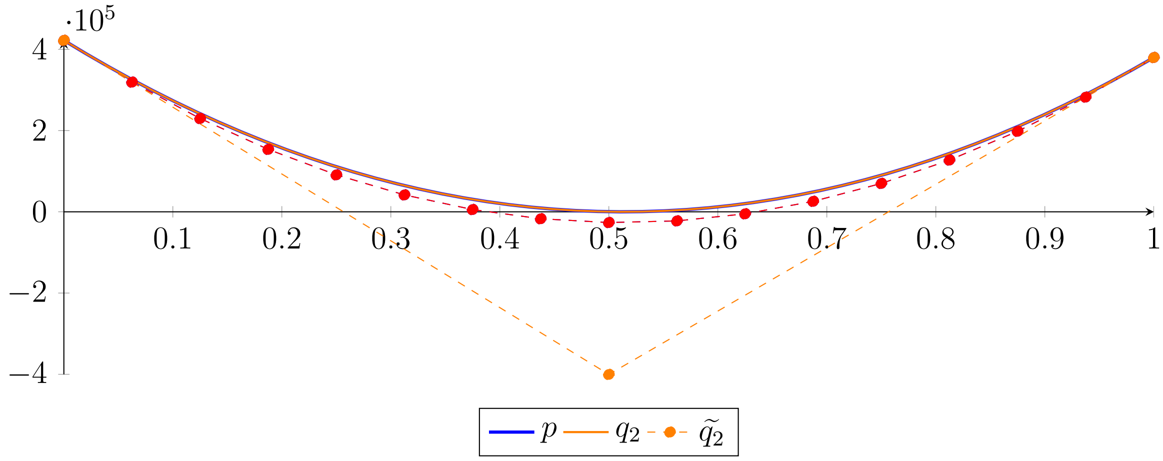
$$\begin{aligned}p &= -4.348008654913047\cdot10^{-35}X^{16} + 5.243388239594399\cdot10^{-33}X^{15} + 1.924264426249969\\&\quad\cdot10^{-28}X^{14} - 2.246746658841888\cdot10^{-26}X^{13} - 3.625525335135332\cdot10^{-22}X^{12}\\&\quad + 3.881306066232412\cdot10^{-20}X^{11} + 3.776139474106064\cdot10^{-16}X^{10} - 3.496022725744\\&\quad\cdot10^{-14}X^{9} - 2.35123279599996\cdot10^{-10}X^{8} + 1.7435554055155\cdot10^{-08}X^{7} + 8.761425585865998\\&\quad\cdot10^{-05}X^{6} - 0.004592127899352905X^{5} - 18.10659249096443X^{4} + 504.889471438631X^{3}\\&\quad + 1602084.653024796X^{2} - 1644731.182248784X + 422061.2284789393\\&= 422061.2284789393B_{0,16}(X) + 319265.5295883902B_{1,16}(X) + 229820.5361397145B_{2,16}(X)\\&\quad + 153727.1497212539B_{3,16}(X) + 90986.26197267314B_{4,16}(X) + 41598.75458390835B_{5,16}(X)\\&\quad + 5565.499294126806B_{6,16}(X) - 17112.6421093025B_{7,16}(X) - 26434.81779182738B_{8,16}(X)\\&\quad - 22400.18587271997B_{9,16}(X) - 5007.914426073306B_{10,16}(X) + 25742.81851821299B_{11,16}(X)\\&\quad + 69852.82497345804B_{12,16}(X) + 127322.906995236B_{13,16}(X) + 198153.8566804232B_{14,16}(X)\\&\quad + 282346.4561662563B_{15,16}(X) + 379901.4776294016B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\&= 422084.9204772878B_{0,2} - 400423.8577791184B_{1,2} + 379878.3032798431B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -2.394754881528157\cdot10^{-290}X^{16} + 1.927007748221533\cdot10^{-289}X^{15} - 7.222560694913459\\&\quad\cdot10^{-289}X^{14} + 1.669823679170844\cdot10^{-288}X^{13} - 2.640537385502685\cdot10^{-288}X^{12}\\&\quad + 2.977245352423787\cdot10^{-288}X^{11} - 2.4183464566446\cdot10^{-288}X^{10} + 1.403945843258626\cdot10^{-288}X^{9}\\&\quad - 5.721369736991339\cdot10^{-289}X^{8} + 1.608948017596564\cdot10^{-289}X^{7} - 3.139550473135121\cdot10^{-290}X^{6}\\&\quad + 4.427570920272867\cdot10^{-291}X^{5} - 4.516145205270189\cdot10^{-292}X^{4} + 3.478035897244226\\&\quad\cdot10^{-293}X^{3} + 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\&= 422084.9204772878B_{0,16} + 319271.323195237B_{1,16} + 229814.4837408143B_{2,16}\\&\quad + 153714.4021140197B_{3,16} + 90971.07831485311B_{4,16} + 41584.51234331459B_{5,16}\\&\quad + 5554.704199404135B_{6,16} - 17118.34611687825B_{7,16} - 26434.63860553258B_{8,16}\\&\quad - 22394.17326655884B_{9,16} - 4996.95009995704B_{10,16} + 25757.03089427283B_{11,16}\\&\quad + 69867.76971613076B_{12,16} + 127335.2663656168B_{13,16} + 198159.5208427308B_{14,16}\\&\quad + 282340.5331474729B_{15,16} + 379878.3032798431B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 23.69199834856345$.

Bounding polynomials M and m:

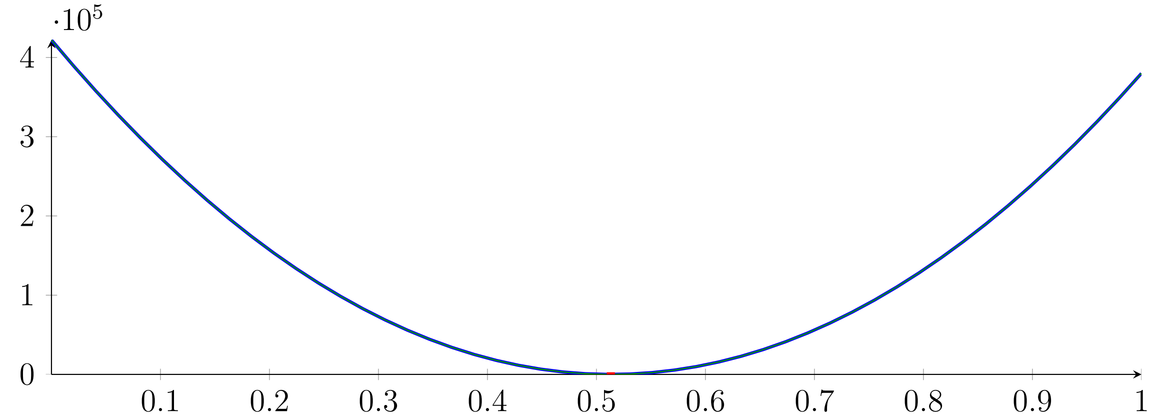
$$M = 1602810.939315368X^2 - 1645017.556512812X + 422108.6124756364$$

$$m = 1602810.939315368X^2 - 1645017.556512812X + 422061.2284789393$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.509405571919099, 0.5169273012614853\}$$

Intersection intervals:



$$[0.509405571919099, 0.5169273012614853]$$

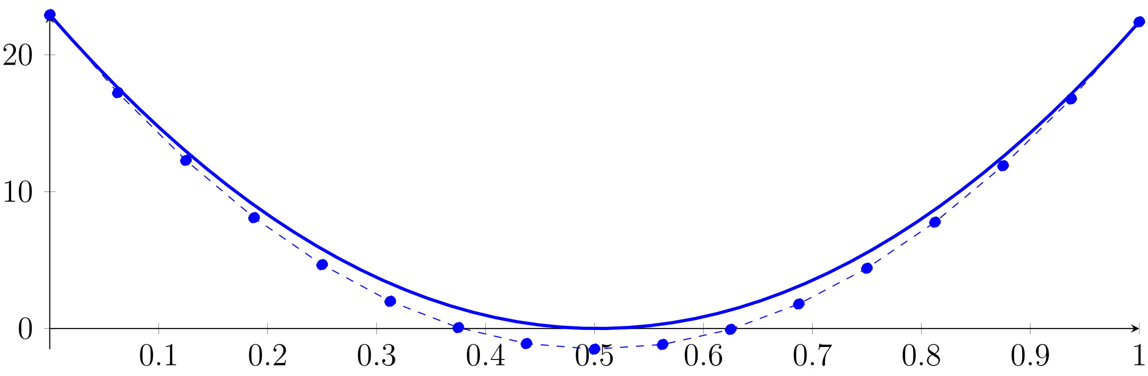
Longest intersection interval: 0.007521729342386311

$\Longrightarrow$ Selective recursion: interval 1: $[0.299973157656462, 0.3000267016613888]$,

74.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.299973157656462, 0.30002670166138$

Normalized monomial und Bézier representations and the Bézier polygon:

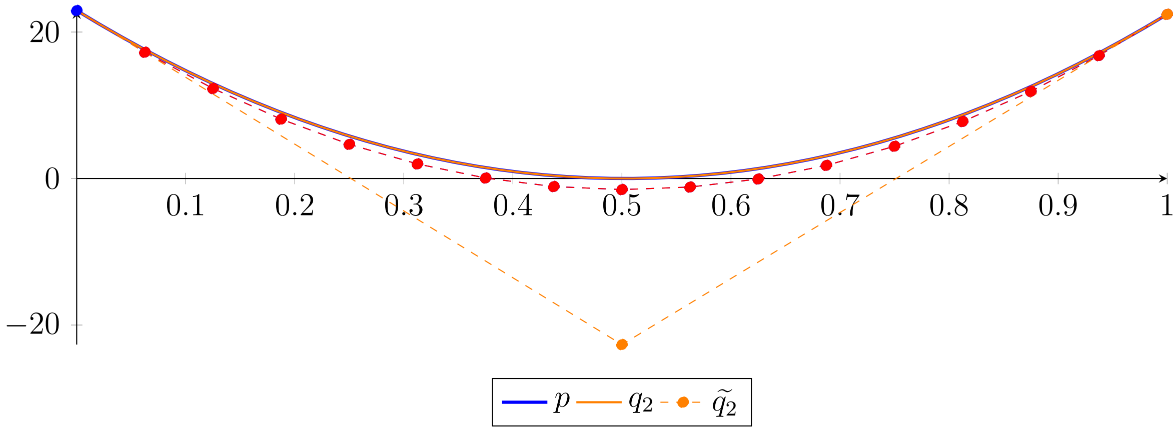
$$\begin{aligned}p = &-4.564294214760651\cdot 10^{-69}X^{16} + 6.823166131086062\cdot 10^{-65}X^{15} + 3.571082953880241\\ &\cdot 10^{-58}X^{14} - 5.203669764436854\cdot 10^{-54}X^{13} - 1.189477187926378\cdot 10^{-47}X^{12}\\ &+ 1.595632405197401\cdot 10^{-43}X^{11} + 2.190120291778079\cdot 10^{-37}X^{10} - 2.545939358967532\cdot 10^{-33}X^{9}\\ &- 2.410599475027015\cdot 10^{-27}X^{8} + 2.244415896590929\cdot 10^{-23}X^{7} + 1.587744426227316\cdot 10^{-17}X^{6}\\ &- 1.041115833763881\cdot 10^{-13}X^{5} - 5.799356524856294\cdot 10^{-08}X^{4} + 0.0001991514832499812X^{3}\\ &+ 90.68225982541923X^{2} - 91.20858295780928X + 22.93446410530317\\ = &\ 22.93446410530317B_{0,16}(X) + 17.23392767044009B_{1,16}(X) + 12.28907673412217B_{2,16}(X)\\ &+ 8.099911651977055B_{3,16}(X) + 4.666432779600538B_{4,16}(X) + 1.988640472556532B_{5,16}(X)\\ &+ 0.06653508637709412B_{6,16}(X) - 1.099883023437587B_{7,16}(X)\\ &- 1.510613501419185B_{8,16}(X) - 1.165655992131239B_{9,16}(X) - 0.0650101401691539B_{10,16}(X)\\ &+ 1.791324409839802B_{11,16}(X) + 4.403348013236495B_{12,16}(X) + 7.771061025329927B_{13,16}(X)\\ &+ 11.89446380139723B_{14,16}(X) + 16.77355669668369B_{15,16}(X) + 22.4083400664027B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\ &= 22.93447405790644B_{0,2} - 22.66987713993177B_{1,2} + 22.40833011545639B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-1.359001790090183\cdot 10^{-294}X^{16} + 1.092598333426672\cdot 10^{-293}X^{15} - 4.09106565145402\\ &\cdot 10^{-293}X^{14} + 9.449139850665356\cdot 10^{-293}X^{13} - 1.493001927703419\cdot 10^{-292}X^{12}\\ &+ 1.682443127957849\cdot 10^{-292}X^{11} - 1.366228291670638\cdot 10^{-292}X^{10} + 7.931222707714909\\ &\cdot 10^{-293}X^{9} - 3.23261546930867\cdot 10^{-293}X^{8} + 9.093117628361849\cdot 10^{-294}X^{7} - 1.775022991079505\\ &\cdot 10^{-294}X^{6} + 2.505263643899121\cdot 10^{-295}X^{5} - 2.56169827233206\cdot 10^{-296}X^{4} + 1.979687487137548\\ &\cdot 10^{-297}X^{3} + 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\ = &\ 22.93447405790644B_{0,16} + 17.23393015817666B_{1,16} + 12.28907424555711B_{2,16}\\ &+ 8.099906320047769B_{3,16} + 4.666426381648652B_{4,16} + 1.988634430359754B_{5,16}\\ &+ 0.06653046618107656B_{6,16} - 1.099885510887381B_{7,16} - 1.510613500845619B_{8,16}\\ &- 1.165653503693637B_{9,16} - 0.06500551943143578B_{10,16} + 1.791330451940986B_{11,16}\\ &+ 4.403354410423627B_{12,16} + 7.771066356016488B_{13,16} + 11.89446628871957B_{14,16}\\ &+ 16.77355420853287B_{15,16} + 22.40833011545639B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 9.952603274324834 \cdot 10^{-06}$.
Bounding polynomials M and m:

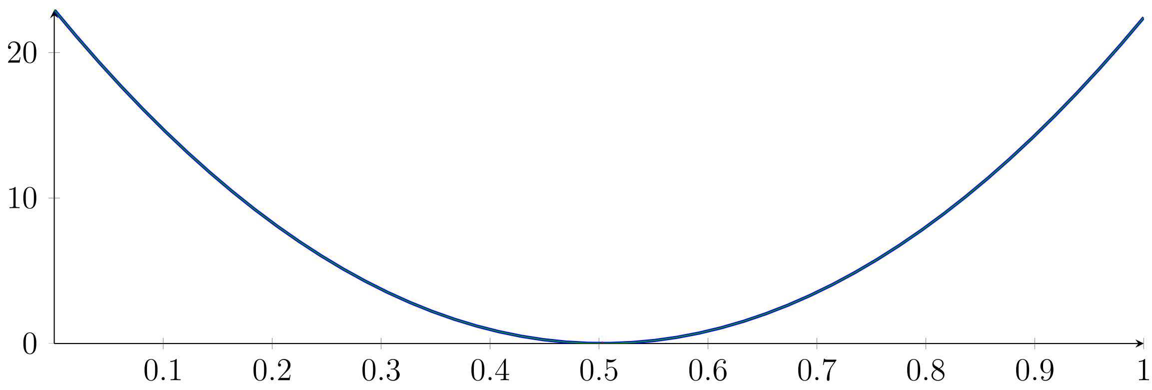
$$M = 90.68255845322638X^2 - 91.20870239567643X + 22.93448401050971$$

$$m = 90.68255845322638X^2 - 91.20870239567643X + 22.93446410530317$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025582179560734, 0.5032438232679998\}$$

Intersection intervals:



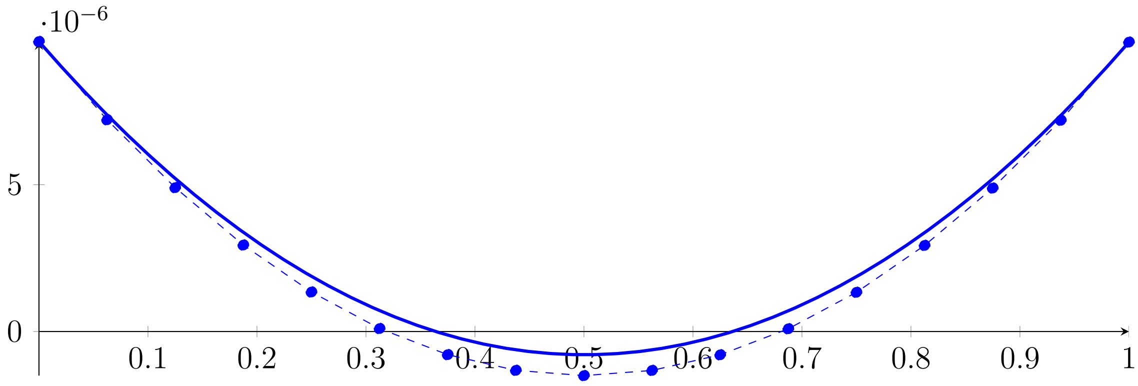
$$[0.5025582179560734, 0.5032438232679998]$$

Longest intersection interval: 0.0006856053119264748
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000666361603, 0.3000001033462145]$,

74.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000666361603, 0.300000103346$

Normalized monomial und Bézier representations and the Bézier polygon:

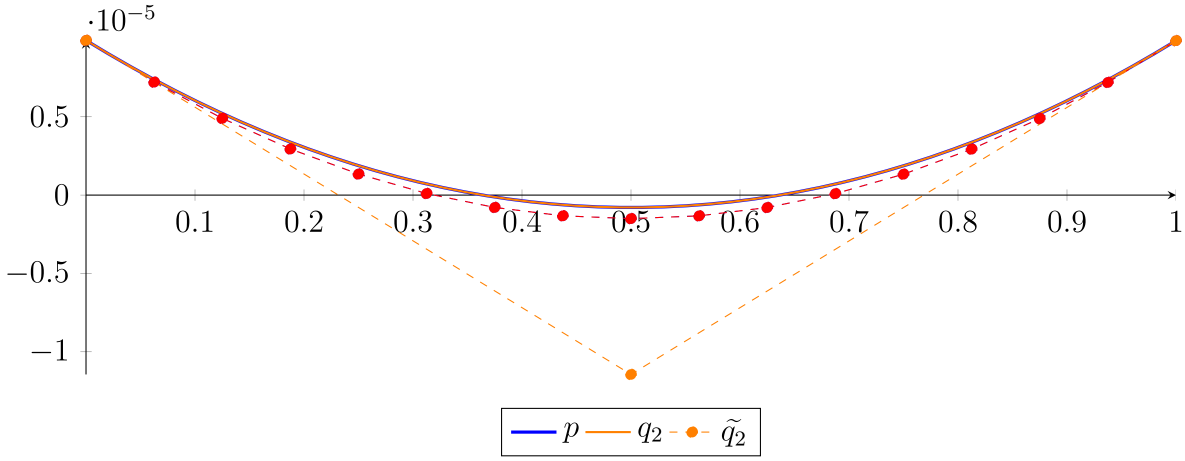
$$p = -1.087828446441416 \cdot 10^{-119} X^{16} + 2.370636059353804 \cdot 10^{-112} X^{15} + 1.810668709595505 \cdot 10^{-102} X^{14} - 3.846486744130667 \cdot 10^{-95} X^{13} - 1.283061410949319 \cdot 10^{-85} X^{12} + 2.509305114999168 \cdot 10^{-78} X^{11} + 5.0258723654725 \cdot 10^{-69} X^{10} - 8.517807546222726 \cdot 10^{-62} X^{9} - 1.17684840157971 \cdot 10^{-52} X^{8} + 1.597478332578791 \cdot 10^{-45} X^{7} + 1.649027121563264 \cdot 10^{-36} X^{6} - 1.576413560673164 \cdot 10^{-29} X^{5} - 1.281381537279639 \cdot 10^{-20} X^{4} + 6.414336823836069 \cdot 10^{-14} X^{3} + 4.262575843118847 \cdot 10^{-05} X^{2} - 4.264622395354259 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06}$$
$$= 9.875919579833914 \cdot 10^{-06} B_{0,16}(X) + 7.210530582737503 \cdot 10^{-06} B_{1,16}(X) + 4.900356239234328 \cdot 10^{-06} B_{2,16}(X) + 2.945396549438933 \cdot 10^{-06} B_{3,16}(X) + 1.345651513465858 \cdot 10^{-06} B_{4,16}(X) + 1.01121131429646 \cdot 10^{-07} B_{5,16}(X) - 7.881945965551621 \cdot 10^{-07} B_{6,16}(X) - 1.322295670374024 \cdot 10^{-06} B_{7,16}(X) - 1.501182089912399 \cdot 10^{-06} B_{8,16}(X) - 1.324853855055745 \cdot 10^{-06} B_{9,16}(X) - 7.933109656895192 \cdot 10^{-07} B_{10,16}(X) + 9.344657830081879 \cdot 10^{-08} B_{11,16}(X) + 1.335418777029811 \cdot 10^{-06} B_{12,16}(X) + 2.932605630611999 \cdot 10^{-06} B_{13,16}(X) + 4.885007139161925 \cdot 10^{-06} B_{14,16}(X) + 7.192623302794129 \cdot 10^{-06} B_{15,16}(X) + 9.855454121623155 \cdot 10^{-06} B_{16,16}(X)$$



Degree reduction and raising:

$$q_2 = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919583041081 \cdot 10^{-06}$$
$$= 9.875919583041081 \cdot 10^{-06} B_{0,2} - 1.144719241297322 \cdot 10^{-05} B_{1,2} + 9.855454118415988 \cdot 10^{-06} B_{2,2}$$

$$\widetilde{q}_2 = -6.428697903766316 \cdot 10^{-301} X^{16} + 5.168486014171287 \cdot 10^{-300} X^{15} - 1.931525871486767 \cdot 10^{-299} X^{14} + 4.443748026156872 \cdot 10^{-299} X^{13} - 6.983462617461767 \cdot 10^{-299} X^{12} + 7.823576270761152 \cdot 10^{-299} X^{11} - 6.321034979462039 \cdot 10^{-299} X^{10} + 3.658988671435299 \cdot 10^{-299} X^{9} - 1.493155040297548 \cdot 10^{-299} X^{8} + 4.235526792785023 \cdot 10^{-300} X^{7} - 8.432417338595836 \cdot 10^{-301} X^{6} + 1.226905231702478 \cdot 10^{-301} X^{5} - 1.286361467975875 \cdot 10^{-302} X^{4} + 9.604810259325032 \cdot 10^{-304} X^{3} + 4.26257585274035 \cdot 10^{-05} X^{2} - 4.264622399202859 \cdot 10^{-05} X + 9.875919583041081 \cdot 10^{-06}$$
$$= 9.875919583041081 \cdot 10^{-06} B_{0,16} + 7.210530583539294 \cdot 10^{-06} B_{1,16} + 4.900356238432536 \cdot 10^{-06} B_{2,16} + 2.945396547720807 \cdot 10^{-06} B_{3,16} + 1.345651511404108 \cdot 10^{-06} B_{4,16} + 1.011211294824374 \cdot 10^{-07} B_{5,16} - 7.881945980442039 \cdot 10^{-07} B_{6,16} - 1.322295671175816 \cdot 10^{-06} B_{7,16} - 1.501182089912399 \cdot 10^{-06} B_{8,16} - 1.324853854253953 \cdot 10^{-06} B_{9,16} - 7.933109642004772 \cdot 10^{-07} B_{10,16} + 9.34465802480274 \cdot 10^{-08} B_{11,16} + 1.335418779091561 \cdot 10^{-06} B_{12,16} + 2.932605632330124 \cdot 10^{-06} B_{13,16} + 4.885007139963716 \cdot 10^{-06} B_{14,16} + 7.192623301992338 \cdot 10^{-06} B_{15,16} + 9.855454118415988 \cdot 10^{-06} B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 3.207167313591001 \cdot 10^{-15}$.
Bounding polynomials M and m:

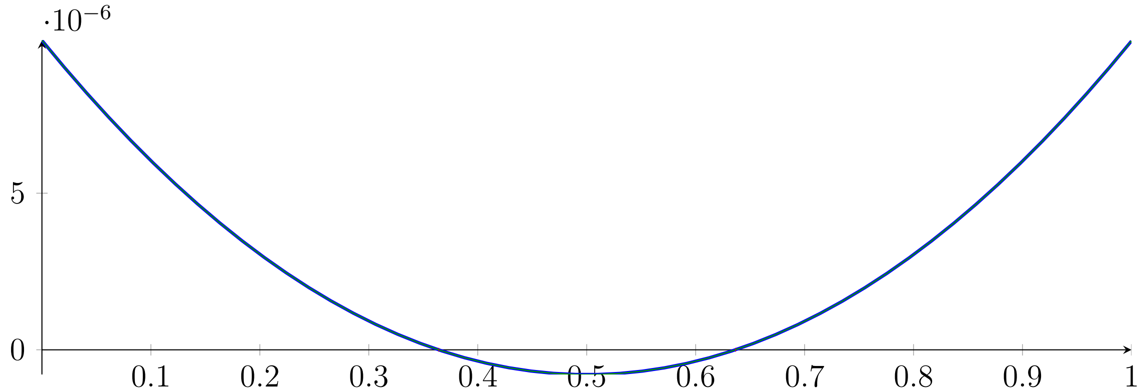
$$M = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919586248249 \cdot 10^{-06}$$

$$m = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06}$$

Root of M and m:

$N(M) = \{0.3640375916978796, 0.6364425279605644\}$ $N(m) = \{0.3640375911454658, 0.6364425285129782\}$

Intersection intervals:



$$[0.3640375911454658, 0.3640375916978796], [0.6364425279605644, 0.6364425285129782]$$

Longest intersection interval: $5.524137987858897 \cdot 10^{-10}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$, interval 2: $[0.30000009, 0.30000009]$,

74.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 6!

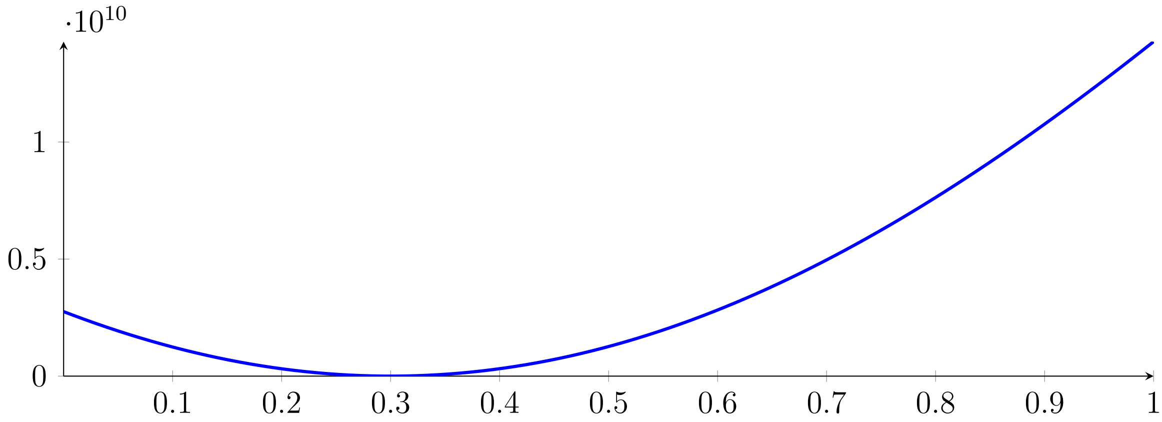
74.7 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 6!

74.8 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

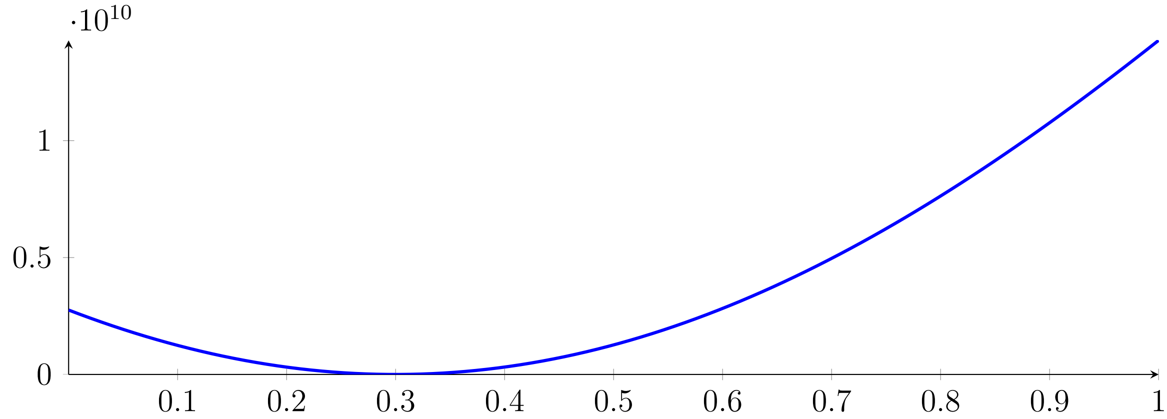
with precision $\varepsilon = 1 \cdot 10^{-16}$.

75 Running CubeClip on h_{16} with epsilon 16

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called CubeClip with input polynomial on interval $[0, 1]$:

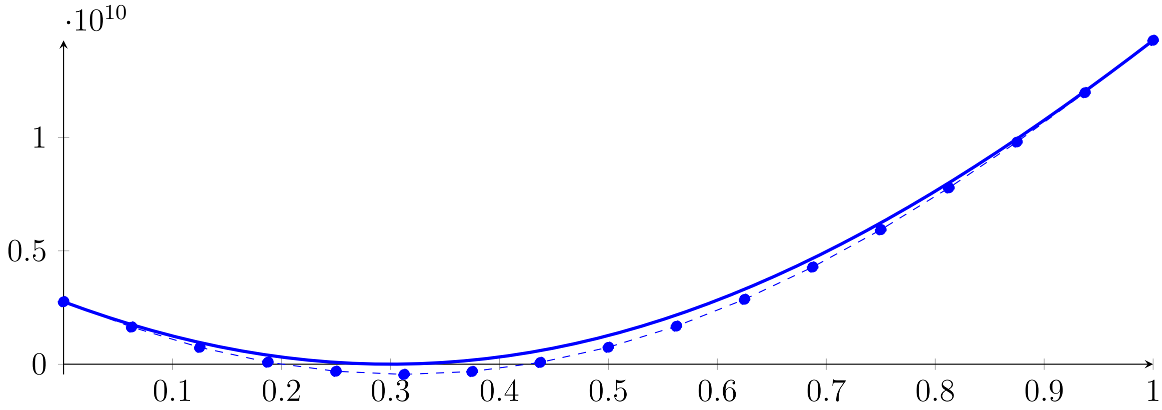
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



75.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

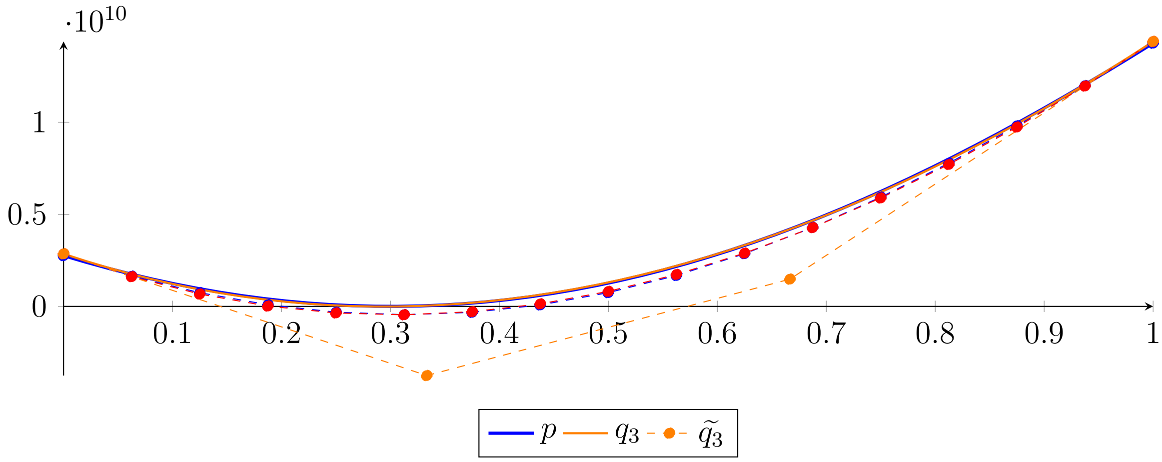
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 &= -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353 \\ &= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3} \\ &\quad + 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_3 &= 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756 \\ &\quad \cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12} \\ &\quad - 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948 \\ &\quad \cdot 10^{-285}X^{9} + 6.984121997801746\cdot 10^{-286}X^{8} - 2.400829205648148\cdot 10^{-286}X^{7} \\ &\quad + 9.243609934541251\cdot 10^{-287}X^{6} - 2.7161427533585\cdot 10^{-287}X^{5} + 4.31464378661806\cdot 10^{-288}X^{4} \\ &\quad - 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353 \\ &= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16} \\ &\quad + 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16} \\ &\quad - 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16} \\ &\quad + 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16} \\ &\quad + 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16} \\ &\quad + 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

Bounding polynomials M and m:

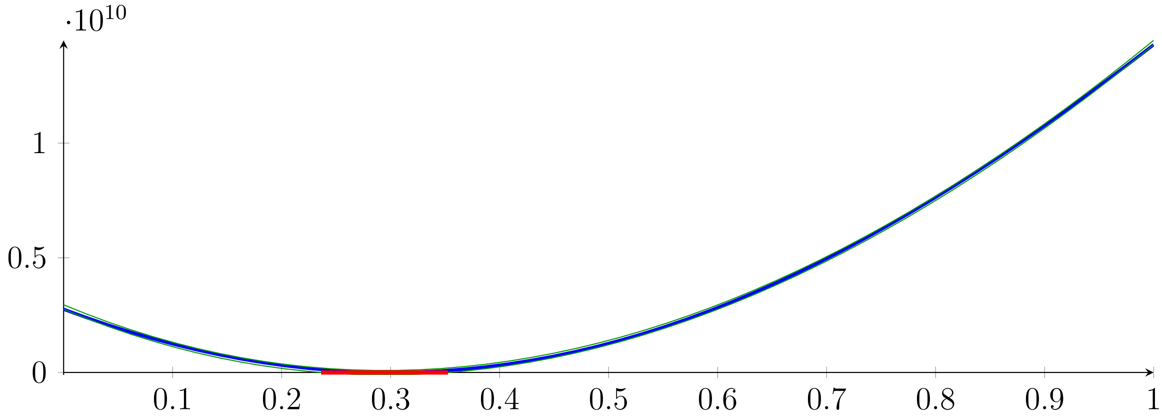
$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$

$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



$$[0.2362027155340351, 0.3527550629571673]$$

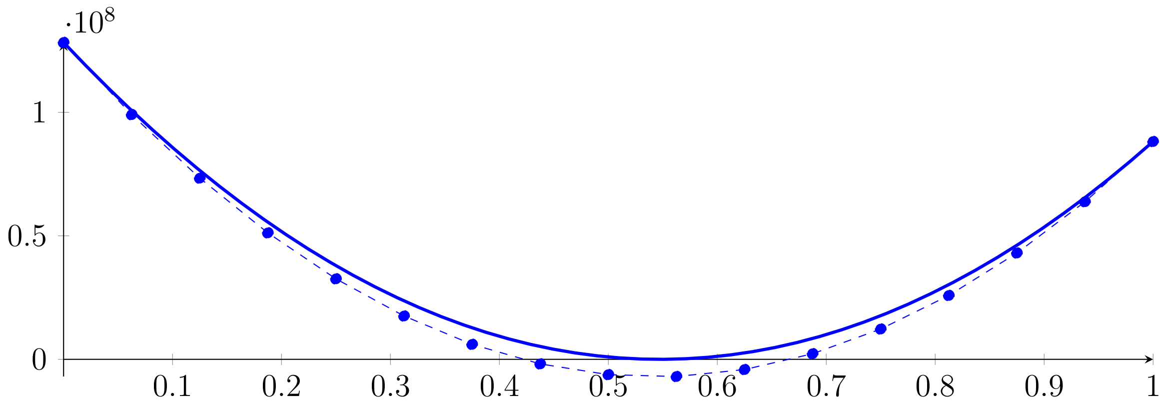
Longest intersection interval: 0.1165523474231321

$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

75.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

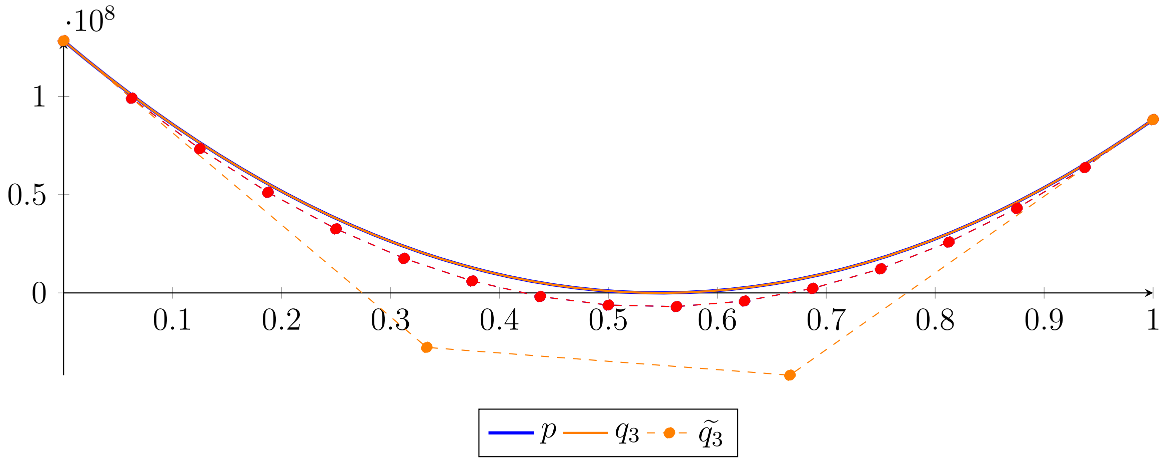
$$\begin{aligned}
p = &-1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14} \\
&- 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321 \\
&\cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8} \\
&+ 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4} \\
&+ 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316 \\
= &\ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X) \\
&+ 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X) \\
&+ 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X) \\
&- 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X) \\
&+ 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X) \\
&+ 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &\ 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166 \\
= &\ 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3} \\
&- 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3}
\end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_3 = {} & 1.029539679280458 \cdot 10^{-288} X^{16} - 8.072523277872609 \cdot 10^{-288} X^{15} + 2.888405037194055 \\
& \cdot 10^{-287} X^{14} - 6.260623433882179 \cdot 10^{-287} X^{13} + 9.183807049175627 \cdot 10^{-287} X^{12} \\
& - 9.610199535206024 \cdot 10^{-287} X^{11} + 7.321003139036714 \cdot 10^{-287} X^{10} - 4.031610440228076 \\
& \cdot 10^{-287} X^{9} + 1.537754605851679 \cdot 10^{-287} X^{8} - 3.606689105481158 \cdot 10^{-288} X^{7} \\
& + 3.223581314526531 \cdot 10^{-289} X^{6} + 5.410058694892297 \cdot 10^{-290} X^{5} - 1.24173721337985 \cdot 10^{-290} X^{4} \\
& + 2297912.322133328 X^{3} + 425644196.8061184 X^{2} - 468061781.215159 X + 128303546.2426166 \\
= {} & 128303546.2426166 B_{0,16} + 99049684.91666918 B_{1,16} + 73342858.56410606 B_{2,16} \\
& + 51187170.59978822 B_{3,16} + 32586724.4385766 B_{4,16} + 17545623.49533216 B_{5,16} \\
& + 6067971.184915845 B_{6,16} - 1842129.077811386 B_{7,16} - 6180573.877988583 B_{8,16} \\
& - 6943259.800754793 B_{9,16} - 4126083.431249065 B_{10,16} + 2275058.645389556 B_{11,16} \\
& + 12264269.84402202 B_{12,16} + 25845653.57950928 B_{13,16} + 43023313.26671229 B_{14,16} \\
& + 63801352.320492 B_{15,16} + 88183874.15570936 B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

Bounding polynomials M and m:

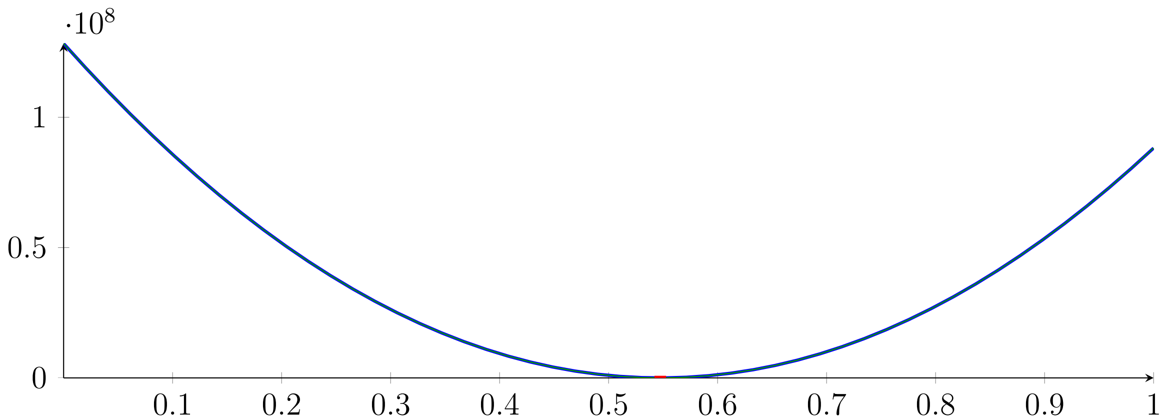
$$M = 2297912.322133328 X^3 + 425644196.8061184 X^2 - 468061781.215159 X + 128322140.7478178$$

$$m = 2297912.322133328 X^3 + 425644196.8061184 X^2 - 468061781.215159 X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$$[0.5420611331202266, 0.552740643902378]$$

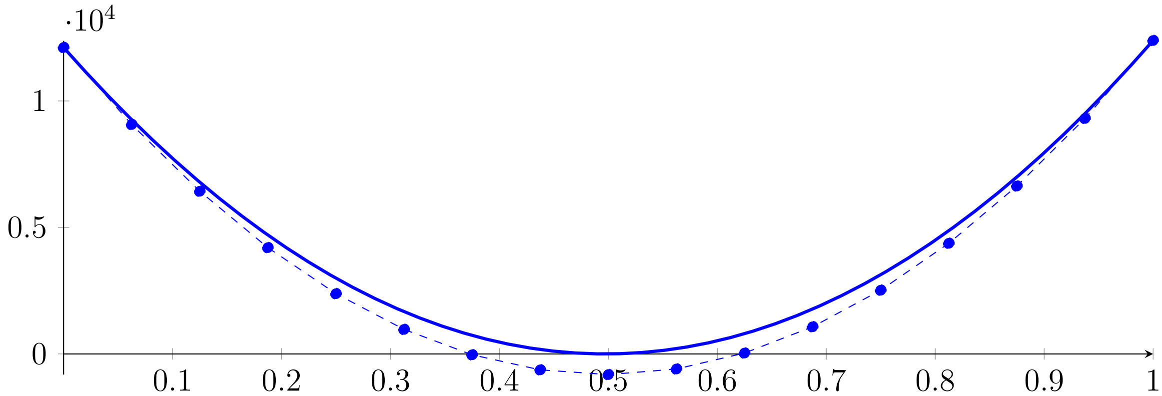
Longest intersection interval: 0.01067951078215144

$\implies$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

75.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.300625935097030$

Normalized monomial und Bézier representations and the Bézier polygon:

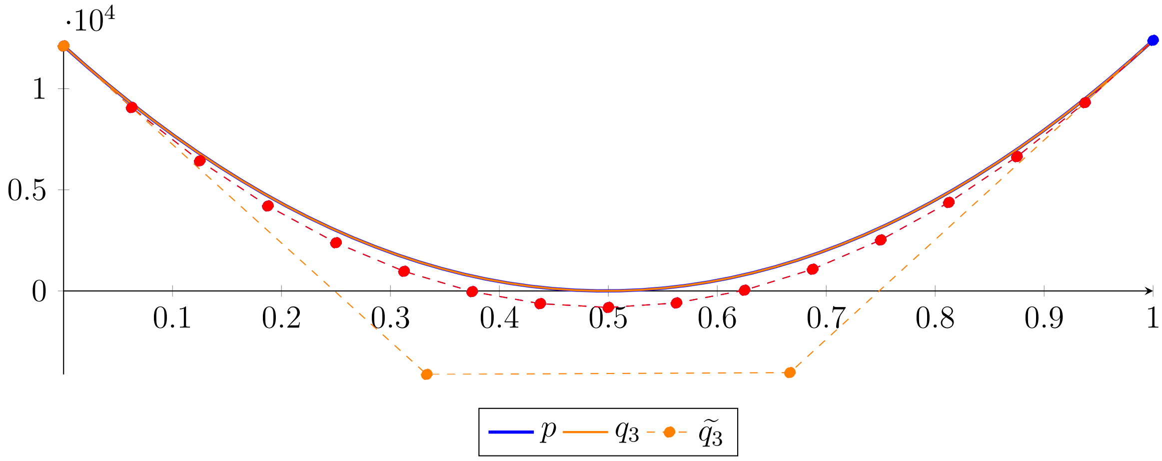
$$
\begin{aligned}
p = & -3.320153282917824 \cdot 10^{-47} X^{16} + 2.160317373028451 \cdot 10^{-44} X^{15} + 4.806705749215272 \\
& \cdot 10^{-39} X^{14} - 3.045075915978909 \cdot 10^{-36} X^{13} - 2.96255923979525 \cdot 10^{-31} X^{12} \\
& + 1.726566936989629 \cdot 10^{-28} X^{11} + 1.009356568315285 \cdot 10^{-23} X^{10} - 5.095799973170639 \\
& \cdot 10^{-21} X^{9} - 2.055755460332604 \cdot 10^{-16} X^{8} + 8.312655296209706 \cdot 10^{-14} X^{7} \\
& + 2.505544083540367 \cdot 10^{-09} X^{6} - 7.139662163470987 \cdot 10^{-07} X^{5} - 0.01693489925927299 X^{4} \\
& + 2.534105689487521 X^{3} + 49001.97220419171 X^{2} - 48729.12904702137 X + 12114.14082112651 \\
= & \, 12114.14082112651 B_{0,16}(X) + 9068.570255687672 B_{1,16}(X) + 6431.349458617101 B_{2,16}(X) \\
& + 4202.482955103525 B_{3,16}(X) + 2381.975261030786 B_{4,16}(X) + 969.830882977672 B_{5,16}(X) \\
& - 33.94568178224417 B_{6,16}(X) - 629.34994528077 B_{7,16}(X) - 816.3774288552541 B_{8,16}(X) \\
& - 595.0236631487491 B_{9,16}(X) + 34.71581188982676 B_{10,16}(X) + 1072.845447005528 B_{11,16}(X) \\
& + 2519.36968363722 B_{12,16}(X) + 4374.29295391742 B_{13,16}(X) + 6637.619680672133 B_{14,16}(X) \\
& + 9309.354277420696 B_{15,16}(X) + 12389.50114837561 B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = & \, 2.500233916081606 X^{3} + 49001.99397932548 X^{2} - 48729.13388598685 X + 12114.14106307619 \\
= & \, 12114.14106307619 B_{0,3} - 4128.903565586097 B_{1,3} \\
& - 4037.950201139886 B_{2,3} + 12389.5013903309 B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_3 = & \, 1.099600357934752 \cdot 10^{-292} X^{16} - 8.377614657428995 \cdot 10^{-292} X^{15} + 2.905985967667976 \\
& \cdot 10^{-291} X^{14} - 6.091421693711012 \cdot 10^{-291} X^{13} + 8.615207440059339 \cdot 10^{-291} X^{12} \\
& - 8.654253929665817 \cdot 10^{-291} X^{11} + 6.288120919613368 \cdot 10^{-291} X^{10} - 3.270943230058189 \\
& \cdot 10^{-291} X^{9} + 1.159364872956392 \cdot 10^{-291} X^{8} - 2.426897197847659 \cdot 10^{-292} X^{7} \\
& + 1.434105777365326 \cdot 10^{-293} X^{6} + 4.843078869312239 \cdot 10^{-294} X^{5} - 7.632080003106212 \cdot 10^{-295} X^{4} \\
& + 2.500233916081606 X^{3} + 49001.99397932548 X^{2} - 48729.13388598685 X + 12114.14106307619 \\
= & \, 12114.14106307619 B_{0,16} + 9068.570195202008 B_{1,16} + 6431.349277155543 B_{2,16} \\
& + 4202.482773640211 B_{3,16} + 2381.975149359434 B_{4,16} + 969.8308690166351 B_{5,16} \\
& - 33.94560268476556 B_{6,16} - 629.349801041346 B_{7,16} - 816.3772613496847 B_{8,16} \\
& - 595.0235189063601 B_{9,16} + 34.71589099204953 B_{10,16} \\
& + 1072.845433048966 B_{11,16} + 2519.36957196781 B_{12,16} + 4374.292772452004 B_{13,16} \\
& + 6637.619499204969 B_{14,16} + 9309.354216930127 B_{15,16} + 12389.5013903309 B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 0.00024195528529181 86$.

Bounding polynomials M and m:

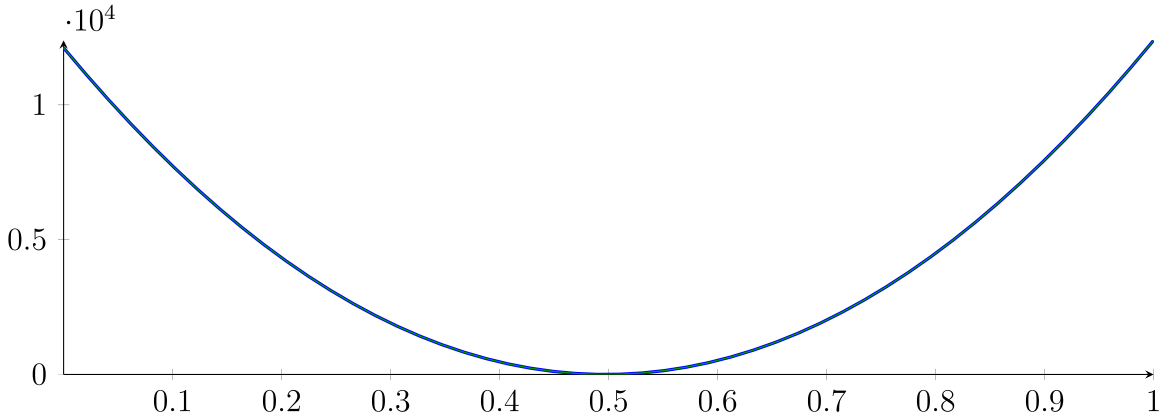
$$M = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.14130503147$$

$$m = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.1408211209$$

Root of M and m:

$$N(M) = \{-19599.95818041899\} \quad N(m) = \{-19599.95818041899, 0.4971412064138645, 0.4972526073815481\}$$

Intersection intervals:



$$[0.4971412064138645, 0.4972526073815481]$$

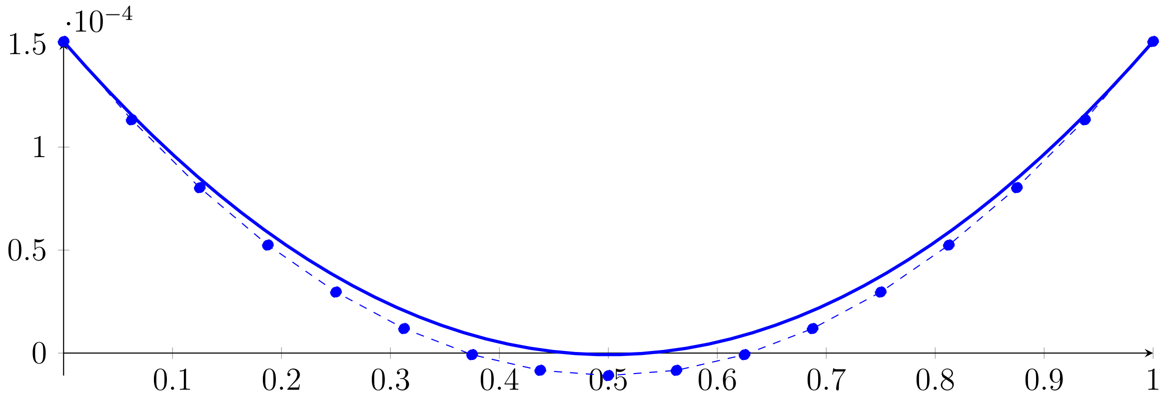
Longest intersection interval: 0.0001114009676835966

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000156681198, 0.3000001543313607]$,

75.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3000000156681198, 0.30000015433136$

Normalized monomial und Bézier representations and the Bézier polygon:

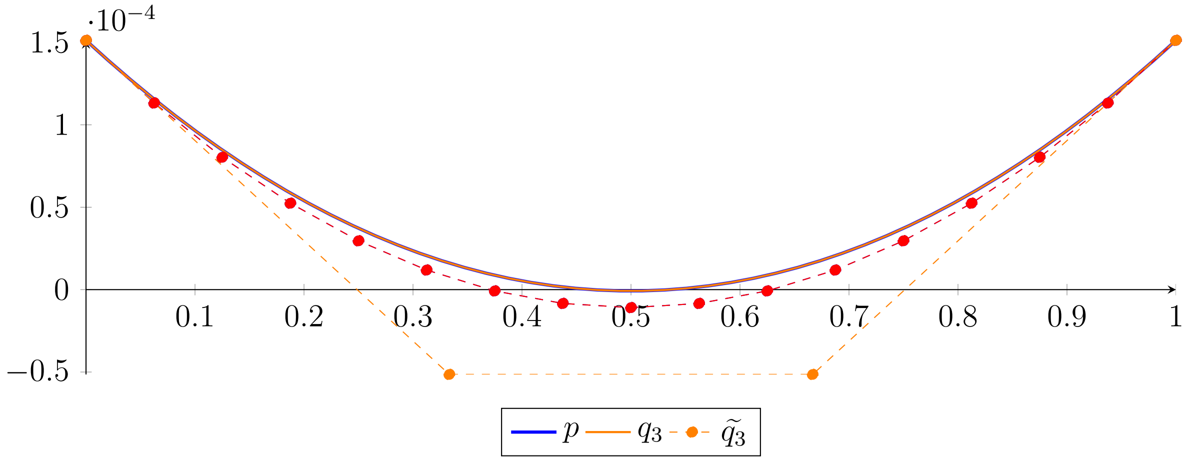
$$p = -1.868020113181866\cdot 10^{-110}X^{16} + 1.077730427531422\cdot 10^{-103}X^{15} + 2.179252184416492\cdot 10^{-94}X^{14} - 1.225622742394903\cdot 10^{-87}X^{13} - 1.082338964896179\cdot 10^{-78}X^{12} + 5.603938665480187\cdot 10^{-72}X^{11} + 2.971492663890366\cdot 10^{-63}X^{10} - 1.333260671370819\cdot 10^{-56}X^{9} - 4.876756665473299\cdot 10^{-48}X^{8} + 1.752546617551927\cdot 10^{-41}X^{7} + 4.789450848811937\cdot 10^{-33}X^{6} - 1.212138221580256\cdot 10^{-26}X^{5} - 2.608457365264229\cdot 10^{-18}X^{4} + 3.456855344293596\cdot 10^{-12}X^{3} + 0.0006081696715066215X^{2} - 0.0006081719526708613X + 0.0001512528027912101$$
$$= 0.0001512528027912101B_{0,16}(X) + 0.0001132420557492813B_{1,16}(X) + 8.029938930324099\cdot 10^{-05}B_{2,16}(X) + 5.242480345926214\cdot 10^{-05}B_{3,16}(X) + 2.961829822351771\cdot 10^{-05}B_{4,16}(X) + 1.187987360218066\cdot 10^{-05}B_{5,16}(X) - 7.904703985760706\cdot 10^{-07}B_{6,16}(X) - 8.392733772579519\cdot 10^{-06}B_{7,16}(X) - 1.092691651365674\cdot 10^{-05}B_{8,16}(X) - 8.393018615634788\cdot 10^{-06}B_{9,16}(X) - 7.910400723407154\cdot 10^{-07}B_{10,16}(X) + 1.187901912239842\cdot 10^{-05}B_{11,16}(X) + 2.961715897475557\cdot 10^{-05}B_{12,16}(X) + 5.242337949090366\cdot 10^{-05}B_{13,16}(X) + 8.029768067701564\cdot 10^{-05}B_{14,16}(X) + 0.0001132400625392645B_{15,16}(X) + 0.0001512505250838231B_{16,16}(X)$$



Degree reduction and raising:

$$q_3 = 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,3} - 5.147118143241051\cdot 10^{-05}B_{1,3} - 5.147194182048959\cdot 10^{-05}B_{2,3} + 0.0001512505250838231B_{3,3}$$

$$\widetilde{q_3} = 1.301341971574797\cdot 10^{-300}X^{16} - 9.903701542140468\cdot 10^{-300}X^{15} + 3.435393711081806\cdot 10^{-299}X^{14} - 7.213305599572634\cdot 10^{-299}X^{13} + 1.023930542727086\cdot 10^{-298}X^{12} - 1.03405078726607\cdot 10^{-298}X^{11} + 7.556175284428161\cdot 10^{-299}X^{10} - 3.94396828718345\cdot 10^{-299}X^{9} + 1.391742955019986\cdot 10^{-299}X^{8} - 2.822114462221502\cdot 10^{-300}X^{7} + 1.110580839880333\cdot 10^{-301}X^{6} + 7.65504767714651\cdot 10^{-302}X^{5} - 1.17007493272161\cdot 10^{-302}X^{4} + 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,16} + 0.0001132420557492813B_{1,16} + 8.029938930324096\cdot 10^{-05}B_{2,16} + 5.242480345926211\cdot 10^{-05}B_{3,16} + 2.961829822351769\cdot 10^{-05}B_{4,16} + 1.187987360218065\cdot 10^{-05}B_{5,16} - 7.904703985760584\cdot 10^{-07}B_{6,16} - 8.392733772579497\cdot 10^{-06}B_{7,16} - 1.092691651365671\cdot 10^{-05}B_{8,16} - 8.393018615634766\cdot 10^{-06}B_{9,16} - 7.910400723407032\cdot 10^{-07}B_{10,16} + 1.187901912239842\cdot 10^{-05}B_{11,16} + 2.961715897475555\cdot 10^{-05}B_{12,16} + 5.242337949090363\cdot 10^{-05}B_{13,16} + 8.029768067701561\cdot 10^{-05}B_{14,16} + 0.0001132400625392644B_{15,16} + 0.0001512505250838231B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 3.726367712763873 \cdot 10^{-20}$.
Bounding polynomials M and m:

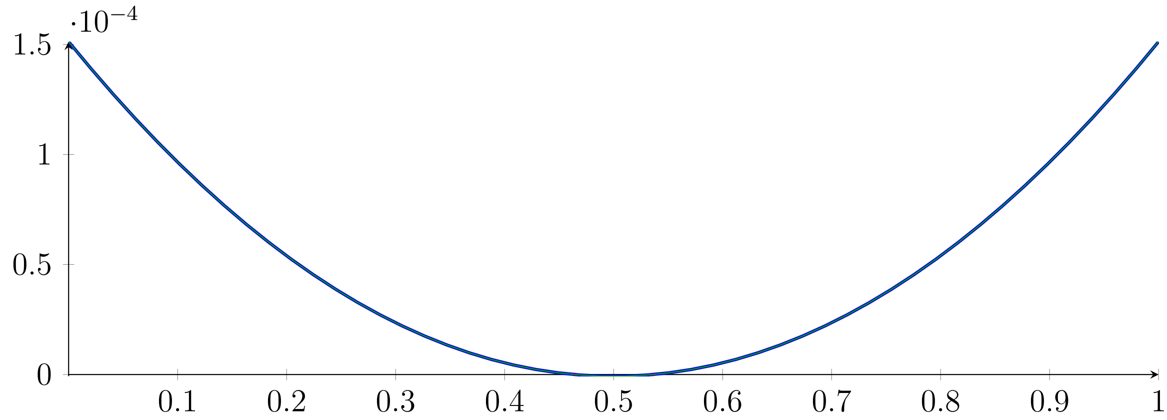
$$\begin{aligned} M &= 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ &\quad - 0.0006081719526708621 X + 0.0001512528027912102 \end{aligned}$$

$$\begin{aligned} m &= 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ &\quad - 0.0006081719526708621 X + 0.0001512528027912101 \end{aligned}$$

Root of M and m:

$N(M) = \{-175931744.9566824, 0.4639432736155652, 0.5360604563964476\}$ $\quad N(m) = \{-175931744.9566824,$

Intersection intervals:



$$[0.4639432736155635, 0.4639432736155652],\ [0.5360604563964476, 0.5360604563964493]$$

Longest intersection interval: $1.699230475659398 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999977, 0.3000000799999977]$, interval 2: $[0.30000009, 0.3000$

75.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000799999977, 0.300000079999$

Reached interval $[0.3000000799999977, 0.3000000799999977]$ **without sign change** at depth 5!
p(0) = 7.27439202738246e-13 - p(1) 7.274391282108574e-13

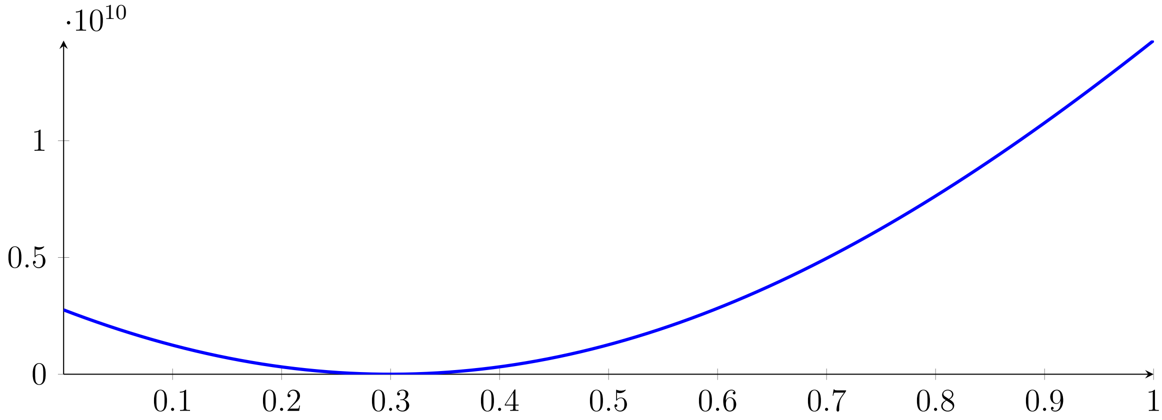
75.6 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 5!

75.7 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000009, 0.30000009]$$

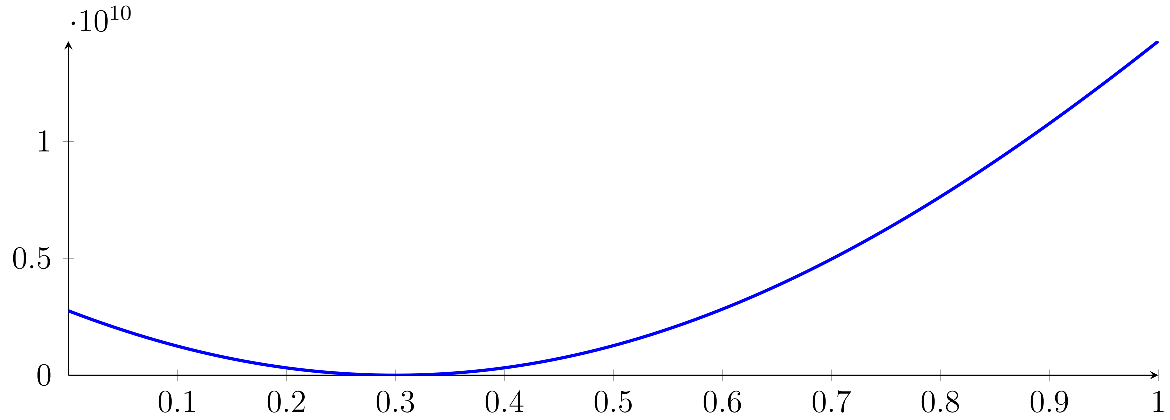
with precision $\varepsilon = 1 \cdot 10^{-16}$.

76 Running BezClip on h_{16} with epsilon 32

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

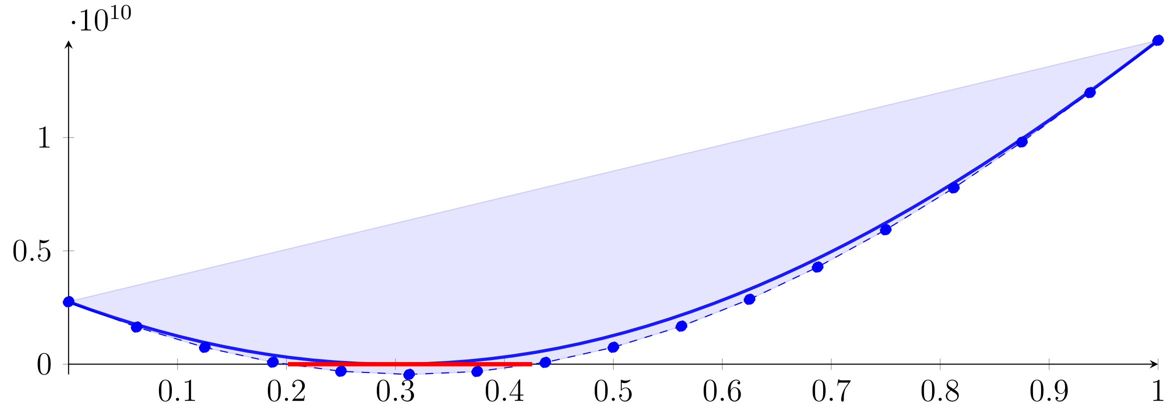
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



76.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

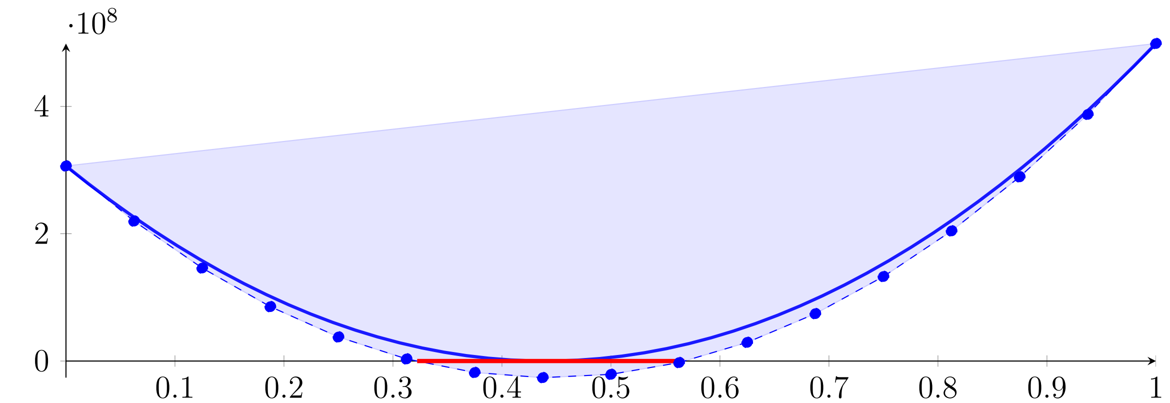
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

76.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-4.078702226246453\cdot 10^{-11}X^{16} + 4.329167405007062\cdot 10^{-10}X^{15} + 1.800827856148464\cdot 10^{-07}X^{14} \\
&- 1.7501300195896\cdot 10^{-06}X^{13} - 0.0003388881074603682X^{12} + 0.002918727490860103X^{11} \\
&+ 0.353260574938002X^{10} - 2.587751703352393X^{9} - 220.7388291837263X^{8} \\
&+ 1298.537478880933X^{7} + 82809.42227744751X^{6} - 357003.6974521088X^{5} - 17288824.7550166X^{4} \\
&+ 45617774.75376932X^{3} + 1550181799.359372X^{2} - 1385902556.54429X + 306449533.9978484 \\
= &\ 306449533.9978484B_{0,16}(X) + 219830624.2138303B_{1,16}(X) + 146129896.0911402B_{2,16}(X) \\
&+ 85428809.9418386B_{3,16}(X) + 37799326.72372467B_{4,16}(X) + 3303826.308721026B_{5,16}(X) \\
&- 18004963.90790522B_{6,16}(X) - 26084029.94397575B_{7,16}(X) - 20900121.61613177B_{8,16}(X) \\
&- 2429792.29588269B_{9,16}(X) + 29340572.02544693B_{10,16}(X) + 74414734.40659503B_{11,16}(X) \\
&+ 132786599.3869978B_{12,16}(X) + 204440216.186282B_{13,16}(X) + 289349792.6628749B_{14,16}(X) \\
&+ 387479720.0042024B_{15,16}(X) + 498784608.1032447B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

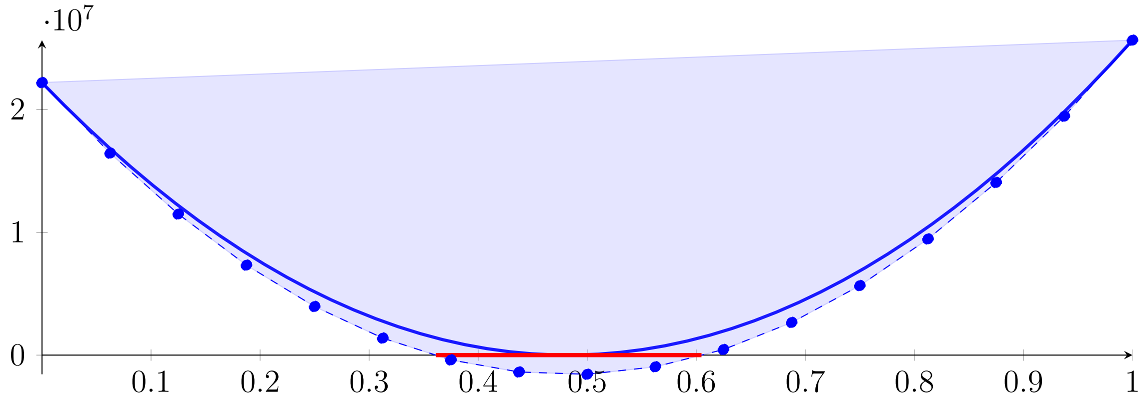
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: $[0.2735013999129692, 0.328453288067671]$,

76.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = & -6.91388116995714 \cdot 10^{-21} X^{16} + 1.539971991909181 \cdot 10^{-19} X^{15} + 5.126557214158853 \\ & \cdot 10^{-16} X^{14} - 1.075268795442723 \cdot 10^{-14} X^{13} - 1.618466564965893 \cdot 10^{-11} X^{12} \\ & + 3.060191240732859 \cdot 10^{-10} X^{11} + 2.825398471644138 \cdot 10^{-07} X^{10} - 4.580412037193902 \\ & \cdot 10^{-06} X^{9} - 0.002949971804071592 X^{8} + 0.0383187988748973 X^{7} + 18.44286791446417 X^{6} \\ & - 172.012783850635 X^{5} - 63987.92092437637 X^{4} + 338934.074545568 X^{3} \\ & + 95113195.48898588 X^{2} - 91937923.73553449 X + 22182509.73328641 \\ = & \; 22182509.73328641 B_{0,16}(X) + 16436389.49981551 B_{1,16}(X) + 11482879.22875282 B_{2,16}(X) \\ & + 7322584.159517174 B_{3,16}(X) + 3956074.373329099 B_{4,16}(X) + 1383884.753830588 B_{5,16}(X) \\ & - 393485.0499920519 B_{6,16}(X) - 1375570.658578954 B_{7,16}(X) - 1561942.994257926 B_{8,16}(X) \\ & - 952208.311393262 B_{9,16}(X) + 453991.7757805191 B_{10,16}(X) + 2656980.267642443 B_{11,16}(X) \\ & + 5657044.755988558 B_{12,16}(X) + 9454437.403157084 B_{13,16}(X) + 14049374.92347699 B_{14,16}(X) \\ & + 19442038.56704145 B_{15,16}(X) + 25632574.10580758 B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

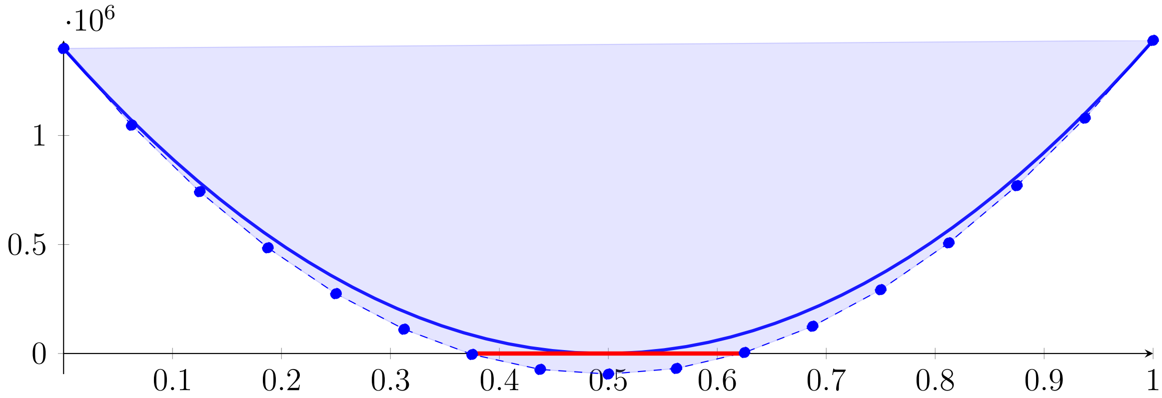
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: $[0.2933480088018335, 0.3067375037518675]$,

76.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -1.067154088646128\cdot 10^{-30}X^{16} + 7.224340048814341\cdot 10^{-29}X^{15} + 1.334696477727887\cdot 10^{-24}X^{14} - 8.705175946298654\cdot 10^{-23}X^{13} - 7.106771480233297\cdot 10^{-19}X^{12} + 4.237354105832321\cdot 10^{-17}X^{11} + 2.091927791167947\cdot 10^{-13}X^{10} - 1.077046671119229\cdot 10^{-11}X^{9} - 3.681404336048099\cdot 10^{-08}X^{8} + 1.51820465067154\cdot 10^{-06}X^{7} + 0.003877397410843758X^{6} - 0.1133230374256462X^{5} - 226.5078434192974X^{4} + 3562.753220436385X^{3} + 5665644.405911702X^{2} - 5632059.446508477X + 1399245.031427946$$
$$= 1399245.031427946B_{0,16}(X) + 1047241.316021167B_{1,16}(X) + 742451.3039969843B_{2,16}(X) + 484881.3574147218B_{3,16}(X) + 274537.7138788421B_{4,16}(X) + 111426.4865130056B_{5,16}(X) - 4446.336065390124B_{6,16}(X) - 73074.88977018565B_{7,16}(X) - 94453.43507095045B_{8,16}(X) - 68576.35701698946B_{9,16}(X) + 4561.834739135334B_{10,16}(X) + 124966.505918568B_{11,16}(X) + 292642.897593607B_{12,16}(X) + 507596.1261656377B_{13,16}(X) + 769831.1833435517B_{14,16}(X) + 1079352.93612265B_{15,16}(X) + 1436166.12676403B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

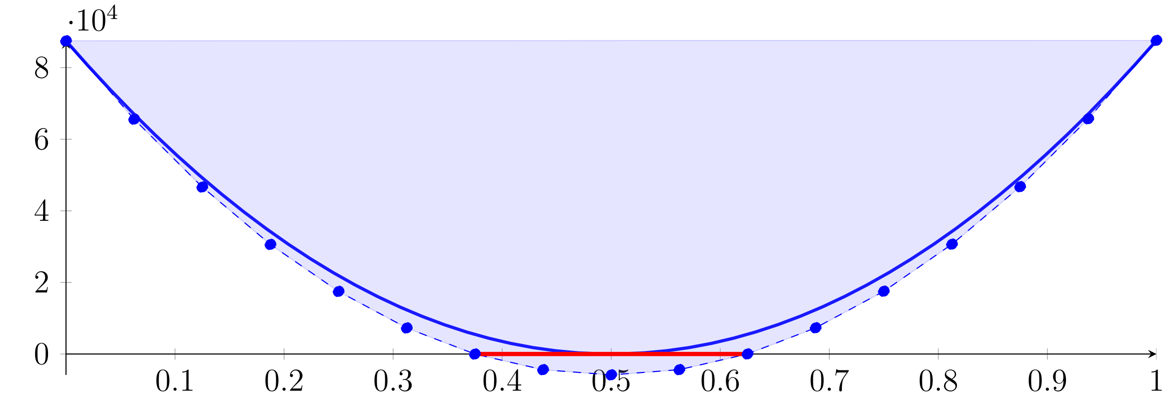
Longest intersection interval: 0.2484999837871829

$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

76.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.256570822524655\cdot 10^{-40}X^{16} + 5.606069245850654\cdot 10^{-38}X^{15} + 4.571694482208246\cdot 10^{-33}X^{14} - 1.103601619609778\cdot 10^{-30}X^{13} - 3.943084008767946\cdot 10^{-26}X^{12} + 8.746232232432483\cdot 10^{-24}X^{11} + 1.879997320990413\cdot 10^{-19}X^{10} - 3.610214442217249\cdot 10^{-17}X^{9} - 5.35839719625663\cdot 10^{-13}X^{8} + 8.241677187819251\cdot 10^{-11}X^{7} + 9.139570434264648\cdot 10^{-07}X^{6} - 9.91682351975725\cdot 10^{-05}X^{5} - 0.864525557836322X^{4} + 49.4891850901473X^{3} + 350100.5152669434X^{2} - 350028.3308017828X + 87482.91293301892$$
$$= 87482.91293301892B_{0,16}(X) + 65606.1422579075B_{1,16}(X) + 46646.87587668727B_{2,16}(X) + 30605.20216290304B_{3,16}(X) + 17481.20901508557B_{4,16}(X) + 7274.983856728878B_{5,16}(X) - 13.38636373236149B_{6,16}(X) - 4383.815172945278B_{7,16}(X) - 5836.216572661172B_{8,16}(X) - 4370.505039757766B_{9,16}(X) + 13.40447373867094B_{10,16}(X) + 7315.596540631298B_{11,16}(X) + 17536.1552585308B_{12,16}(X) + 30675.1642498336B_{13,16}(X) + 46732.70666170018B_{14,16}(X) + 65708.86516603353B_{15,16}(X) + 87603.72195945767B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3748852078437339, 0.6248088966923045\}$$

Intersection intervals with the x axis:

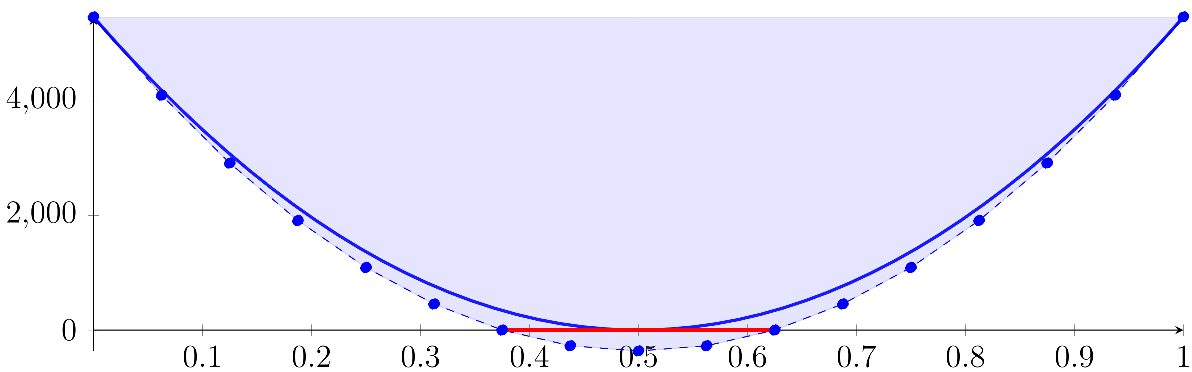
$$[0.3748852078437339, 0.6248088966923045]$$

Longest intersection interval: 0.2499236888485706
$\implies$ Selective recursion: interval 1: $[0.2995843091213109, 0.3004158775315354]$,

76.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.2995843091213109, 0.30041587753$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-5.228387005637929\cdot 10^{-50}X^{16} + 5.071723175262542\cdot 10^{-47}X^{15} + 1.695940836337518\\ &\cdot 10^{-41}X^{14} - 1.602367332110541\cdot 10^{-38}X^{13} - 2.341982733873566\cdot 10^{-33}X^{12}\\ &+ 2.036120445711374\cdot 10^{-30}X^{11} + 1.787779362171969\cdot 10^{-25}X^{10} - 1.346594590666257\\ &\cdot 10^{-22}X^{9} - 8.158156485257833\cdot 10^{-18}X^{8} + 4.921695226295225\cdot 10^{-15}X^{7}\\ &+ 2.227778844339431\cdot 10^{-10}X^{6} - 9.46914992513699\cdot 10^{-08}X^{5} - 0.003373649242827386X^{4}\\ &+ 0.7523208927331248X^{3} + 21871.35693828986X^{2} - 21871.48147756637X + 5467.807676357308\\ = &\,5467.807676357308B_{0,16}(X) + 4100.84008400941B_{1,16}(X) + 2916.133799480594B_{2,16}(X)\\ &+ 1913.690166201026B_{3,16}(X) + 1093.510525747217B_{4,16}(X) + 455.5962178420057B_{5,16}(X)\\ &- 0.05141964546833865B_{6,16}(X) - 273.4310506997833B_{7,16}(X) - 364.541341159257B_{8,16}(X)\\ &- 273.3809587159691B_{9,16}(X) + 0.05142708421757425B_{10,16}(X)\\ &+ 455.7571448416359B_{11,16}(X) + 1093.737521302792B_{12,16}(X) + 1913.993881360346B_{13,16}(X)\\ &+ 2916.527548053087B_{14,16}(X) + 4101.339842565916B_{15,16}(X) + 5468.43208422982B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749929469011197, 0.6249882450180355\}$$

Intersection intervals with the x axis:

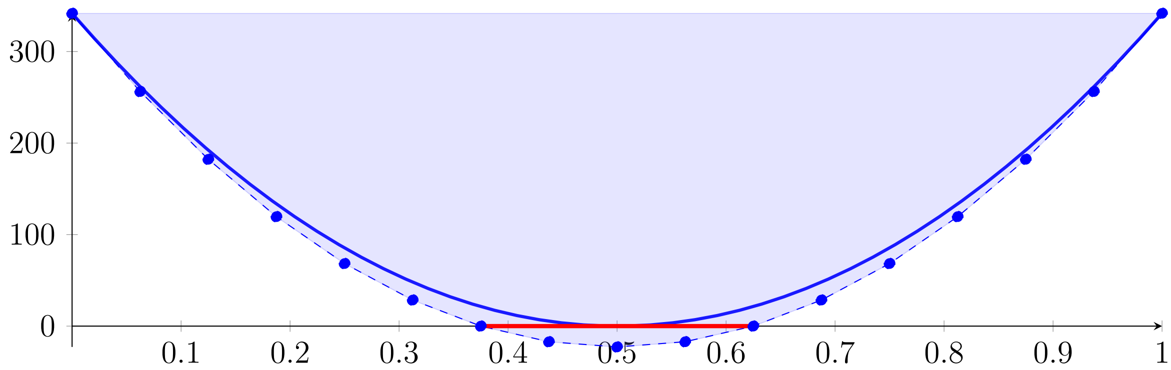
$$[0.3749929469011197, 0.6249882450180355]$$

Longest intersection interval: 0.2499952981169158
$\Longrightarrow$ Selective recursion: interval 1: $[0.2998961414100109, 0.3001040296026296]$,

76.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.2998961414100109, 0.300104029$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.216962444275584 \cdot 10^{-59} X^{16} + 4.692870826723641 \cdot 10^{-56} X^{15} + 6.316314535763934 \\
& \cdot 10^{-50} X^{14} - 2.373865516252114 \cdot 10^{-46} X^{13} - 1.395661893422859 \cdot 10^{-40} X^{12} \\
& + 4.828363722587868 \cdot 10^{-37} X^{11} + 1.70471850273246 \cdot 10^{-31} X^{10} - 5.110411085982617 \cdot 10^{-28} X^{9} \\
& - 1.244717766622564 \cdot 10^{-22} X^{8} + 2.988632715177156 \cdot 10^{-19} X^{7} + 5.438614058423278 \cdot 10^{-14} X^{6} \\
& - 9.197401057829169 \cdot 10^{-11} X^{5} - 1.317801761681953 \cdot 10^{-05} X^{4} + 0.01167528468045732 X^{3} \\
& + 1366.961107526186 X^{2} - 1366.963198735845 X + 341.7398631167735 \\
= & \; 341.7398631167735 B_{0,16}(X) + 256.3046631957831 B_{1,16}(X) + 182.260805837511 B_{2,16}(X) \\
& + 119.6083118906798 B_{3,16}(X) + 68.34720219677136 B_{4,16}(X) + 28.47749759002708 B_{5,16}(X) \\
& - 0.0007811025525137025 B_{6,16}(X) - 17.08761306120757 B_{7,16}(X) - 22.782977473419 B_{8,16}(X) \\
& - 17.08685353390849 B_{9,16}(X) + 0.0007795553614777027 B_{10,16}(X) \\
& + 28.4799425851876 B_{11,16}(X) + 68.35065633912575 B_{12,16}(X) + 119.6129415934909 B_{13,16}(X) \\
& + 182.2668191173573 B_{14,16}(X) + 256.312309672558 B_{15,16}(X) + 341.7494340136854 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749982857492878, 0.6249971486858456\}$$

Intersection intervals with the x axis:

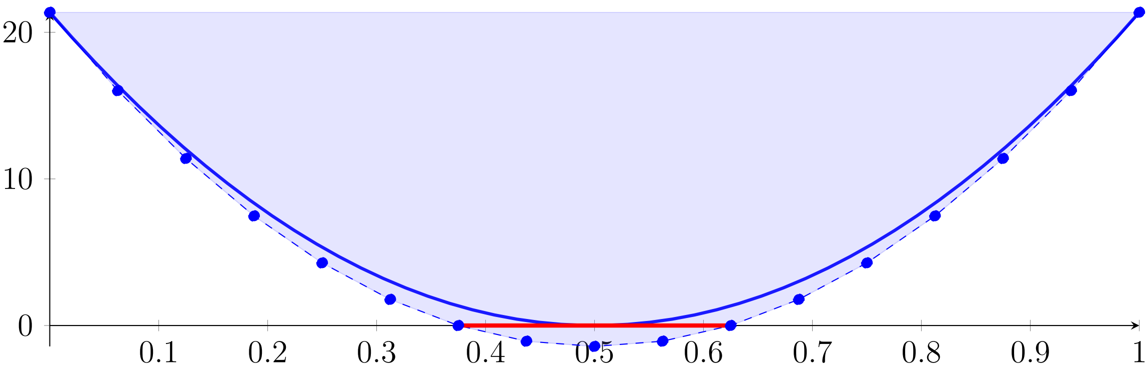
$$[0.3749982857492878, 0.6249971486858456]$$

Longest intersection interval: 0.2499988629365579
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999740991258704, 0.300026070937643]$,

76.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999740991258704, 0.3000260$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.833255302235776 \cdot 10^{-69} X^{16} + 4.363478560238502 \cdot 10^{-65} X^{15} + 2.35287052687137 \cdot 10^{-58} X^{14} - 3.532185105967992 \cdot 10^{-54} X^{13} - 8.318408043755505 \cdot 10^{-48} X^{12} + 1.149616718196078 \cdot 10^{-43} X^{11} + 1.625691292234855 \cdot 10^{-37} X^{10} - 1.946948798353601 \cdot 10^{-33} X^{9} - 1.899245778120626 \cdot 10^{-27} X^{8} + 1.821779493317236 \cdot 10^{-23} X^{7} + 1.327769541755276 \cdot 10^{-17} X^{6} - 8.969682874540076 \cdot 10^{-14} X^{5} - 5.147636798128115 \cdot 10^{-08} X^{4} + 0.000182114977798148 X^{3} + 85.43511227371199 X^{2} - 85.43514442584667 X + 21.35877059261705$$
$$= 21.35877059261705 B_{0,16}(X) + 16.01907406600164 B_{1,16}(X) + 11.39133680833382 B_{2,16}(X) + 7.475559144818918 B_{3,16}(X) + 4.271741400633969 B_{4,16}(X) + 1.779883900927722 B_{5,16}(X) - 1.302917935804413 \cdot 10^{-05} B_{6,16}(X) - 1.067949064595088 B_{7,16}(X) - 1.423923880255569 B_{8,16}(X) - 1.067937151125187 B_{9,16}(X) + 1.144780338983322 \cdot 10^{-05} B_{10,16}(X) + 1.779922241509208 B_{11,16}(X) + 4.271795554943032 B_{12,16}(X) + 7.47563171302734 B_{13,16}(X) + 11.39143104065633 B_{14,16}(X) + 16.01919386269591 B_{15,16}(X) + 21.35892050398371 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3749995424882778, 0.6249993300354412\}$$

Intersection intervals with the x axis:

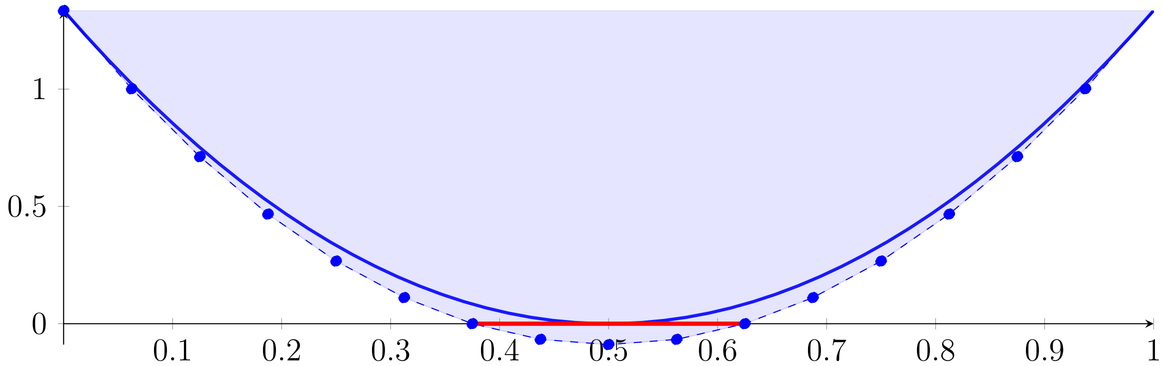
$$[0.3749995424882778, 0.6249993300354412]$$

Longest intersection interval: 0.2499997875471635
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999935885315074, 0.300006581473409]$,

76.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999935885315074, 0.30000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -6.596596862099299 \cdot 10^{-79} X^{16} + 4.062171102620318 \cdot 10^{-74} X^{15} + 8.765030607683244 \cdot 10^{-67} X^{14} - 5.261467161278336 \cdot 10^{-62} X^{13} - 4.958117265353456 \cdot 10^{-55} X^{12} + 2.739981763166457 \cdot 10^{-50} X^{11} + 1.550371378259149 \cdot 10^{-43} X^{10} - 7.424637900674377 \cdot 10^{-39} X^{9} - 2.898009393348535 \cdot 10^{-32} X^{8} + 1.111571585405421 \cdot 10^{-27} X^{7} + 3.241620002541647 \cdot 10^{-21} X^{6} - 8.756501261136649 \cdot 10^{-17} X^{5} - 2.010795357557322 \cdot 10^{-10} X^{4} + 2.844332798771609 \cdot 10^{-06} X^{3} + 5.339698243852309 X^{2} - 5.339698356505613 X + 1.33492347100565$$
$$= 1.33492347100565 B_{0,16}(X) + 1.00119232372405 B_{1,16}(X) + 0.7119586618078846 B_{2,16}(X) + 0.4672224903363214 B_{3,16}(X) + 0.266983814388415 B_{4,16}(X) + 0.1112426390431101 B_{5,16}(X) - 1.030620758846691 \cdot 10^{-06} B_{6,16}(X) - 0.06674718952446819 B_{7,16}(X) - 0.0889958325894046 B_{8,16}(X) - 0.06674695473706526 B_{9,16}(X) - 5.508890578613216 \cdot 10^{-07} B_{10,16}(X) + 0.1112433840328995 B_{11,16}(X) + 0.2669848551069781 B_{12,16}(X) + 0.4672238674112388 B_{13,16}(X) + 0.7119604260236321 B_{14,16}(X) + 1.001194536021998 B_{15,16}(X) + 1.334926202484066 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3749994209666256, 0.6250003095051082\}$$

Intersection intervals with the x axis:

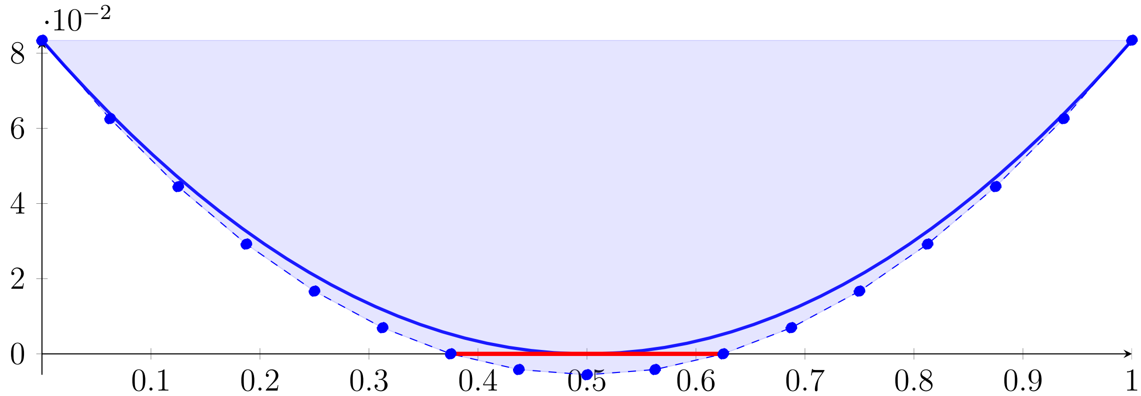
$$[0.3749994209666256, 0.6250003095051082]$$

Longest intersection interval: 0.2500008885384826
$\Longrightarrow$ Selective recursion: interval 1: [0.2999984608771972, 0.3000017091242173],

76.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.2999984608771972, 0.300

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = {} & -1.5359772362815\cdot 10^{-88}X^{16} + 3.783024712863664\cdot 10^{-83}X^{15} + 3.265391675561962\cdot 10^{-75}X^{14} \\ & - 7.83987355616511\cdot 10^{-70}X^{13} - 2.955395756595885\cdot 10^{-62}X^{12} + 6.532349021061568 \\ & \cdot 10^{-57}X^{11} + 1.478602992973913\cdot 10^{-49}X^{10} - 2.832143398905245\cdot 10^{-44}X^{9} \\ & - 4.422140960586434\cdot 10^{-37}X^{8} + 6.784132679612849\cdot 10^{-32}X^{7} + 7.914287227230788\cdot 10^{-25}X^{6} \\ & - 8.550710444673422\cdot 10^{-20}X^{5} - 7.854787446136848\cdot 10^{-13}X^{4} + 4.443846100456459 \\ & \cdot 10^{-08}X^{3} + 0.3337337124913315X^{2} - 0.3337336004955191X + 0.08343257581494635 \\ = {} & 0.08343257581494635B_{0,16}(X) + 0.06257422578397641B_{1,16}(X) + 0.04449699002376757B_{2,16}(X) \\ & + 0.02920086861367421B_{3,16}(X) + 0.01668586163305031B_{4,16}(X) \\ & + 0.006951969161249391B_{5,16}(X) - 8.08722375440678\cdot 10^{-07}B_{6,16}(X) \\ & - 0.004172471938471519B_{7,16}(X) - 0.005563020407686608B_{8,16}(X) \\ & - 0.004172454050668901B_{9,16}(X) - 7.727880670249089\cdot 10^{-07}B_{10,16}(X) \\ & + 0.006952023459469962B_{11,16}(X) + 0.01668593477129257B_{12,16}(X) \\ & + 0.02920096122675088B_{13,16}(X) + 0.04449710290519453B_{14,16}(X) \\ & + 0.06257435988597275B_{15,16}(X) + 0.08343273224843432B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749927302224649, 0.6250069467380417\}$$

Intersection intervals with the x axis:

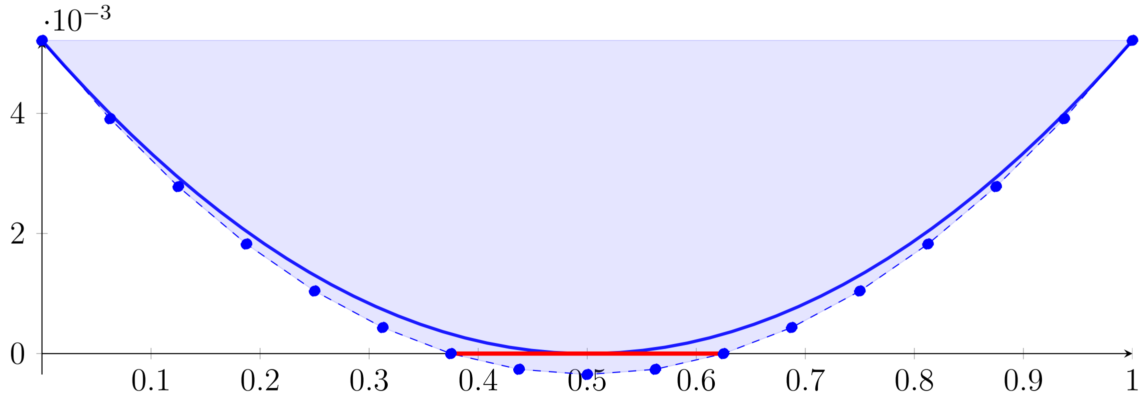
$$[0.3749927302224649, 0.6250069467380417]$$

Longest intersection interval: 0.2500142165155768
$\Longrightarrow$ Selective recursion: interval 1: [0.2999996789462157, 0.3000004910541495],

76.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.2999996789462157, 0.3

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.579480927696053\cdot 10^{-98}X^{16} + 3.526136882584451\cdot 10^{-92}X^{15} + 1.217422078989368 \\
& \cdot 10^{-83}X^{14} - 1.169070552755016\cdot 10^{-77}X^{13} - 1.762755817503918\cdot 10^{-69}X^{12} \\
& + 1.55837613779483\cdot 10^{-63}X^{11} + 1.410908032488726\cdot 10^{-55}X^{10} - 1.080908862805813\cdot 10^{-49}X^{9} \\
& - 6.750723381404434\cdot 10^{-42}X^{8} + 4.142273519440928\cdot 10^{-36}X^{7} + 1.932858818028869\cdot 10^{-28}X^{6} \\
& - 8.352503738960762\cdot 10^{-23}X^{5} - 3.06897495525372\cdot 10^{-15}X^{4} + 6.944510025023856\cdot 10^{-10}X^{3} \\
& + 0.02086073248825264X^{2} - 0.0208607236297874X + 0.005214387850787818 \\
= & \, 0.005214387850787818B_{0,16}(X) + 0.003910592623926106B_{1,16}(X) \\
& + 0.002780636834466498B_{2,16}(X) + 0.001824520483649087B_{3,16}(X) \\
& + 0.001042243572713962B_{4,16}(X) + 0.000433806102901211B_{5,16}(X) - 7.919245490808786 \\
& \cdot 10^{-07}B_{6,16}(X) - 0.0002615505083968289B_{7,16}(X) - 0.0003484696474019504B_{8,16}(X) \\
& - 0.0002615493403243644B_{9,16}(X) - 7.895859239916112\cdot 10^{-07}B_{10,16}(X) \\
& + 0.0004338096170392455B_{11,16}(X) + 0.001042248269805423B_{12,16}(X) \\
& + 0.001824526373614615B_{13,16}(X) + 0.002780643929706894B_{14,16}(X) \\
& + 0.00391060093932233B_{15,16}(X) + 0.005214397403700994B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748861124966261, 0.6251135508761015\}$$

Intersection intervals with the x axis:

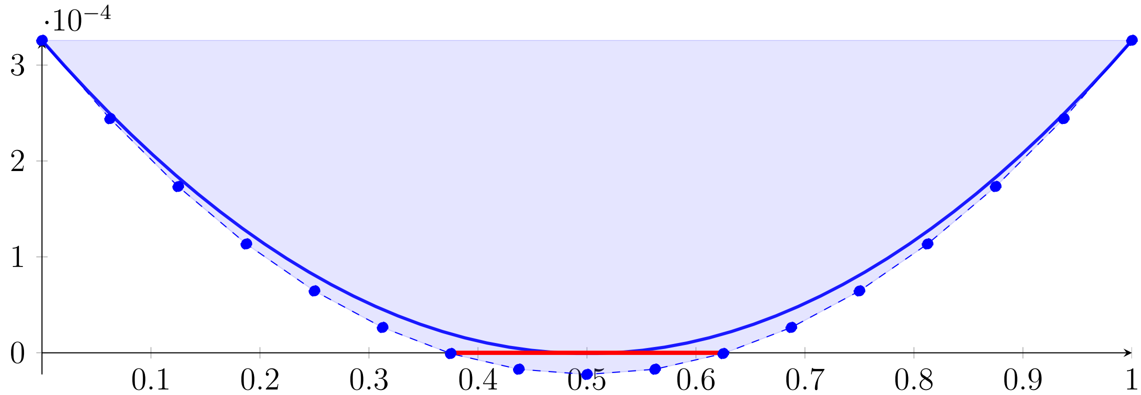
$$[0.3748861124966261, 0.6251135508761015]$$

Longest intersection interval: 0.2502274383794754
$\Longrightarrow$ Selective recursion: interval 1: [0.2999999833942019, 0.3000001866058899],

76.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999999833942019, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-8.456271781717561\cdot10^{-108}X^{16} + 3.329051186798107\cdot10^{-101}X^{15} + 4.593357002546748\\ &\cdot10^{-92}X^{14} - 1.76275695811523\cdot10^{-85}X^{13} - 1.062212310975265\cdot10^{-76}X^{12}\\ &+ 3.752790404631579\cdot10^{-70}X^{11} + 1.357838209904933\cdot10^{-61}X^{10} - 4.157203866707784\\ &\cdot10^{-55}X^{9} - 1.037599553128416\cdot10^{-46}X^{8} + 2.544375234544489\cdot10^{-40}X^{7} + 4.744710700950638\\ &\cdot10^{-32}X^{6} - 8.193869976131749\cdot10^{-26}X^{5} - 1.203186876925119\cdot10^{-17}X^{4} + 1.088036641260801\\ &\cdot10^{-11}X^{3} + 0.001306169174102627X^{2} - 0.001306168592474289X + 0.0003257512461139167\\ = &\,0.0003257512461139167B_{0,16}(X) + 0.0002441157090842736B_{1,16}(X)\\ &+ 0.0001733649151721524B_{2,16}(X) + 0.0001134988643969824B_{3,16}(X)\\ &+ 6.451755677819264\cdot10^{-05}B_{4,16}(X) + 2.642099233521247\cdot10^{-05}B_{5,16}(X)\\ &- 7.908289125289409\cdot10^{-07}B_{6,16}(X) - 1.71179069456024\cdot10^{-05}B_{7,16}(X) - 2.25602417445787\\ &\cdot10^{-05}B_{8,16}(X) - 1.711783329002867\cdot10^{-05}B_{9,16}(X) - 7.906815625231293\\ &\cdot10^{-07}B_{10,16}(X) + 2.64212134573671\cdot10^{-05}B_{11,16}(X) + 6.451785178907119\\ &\cdot10^{-05}B_{12,16}(X) + 0.0001134992334520183B_{13,16}(X) + 0.0001733653584656376B_{14,16}(X)\\ &+ 0.0002441162268493581B_{15,16}(X) + 0.0003257518386226092B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3731836273807965, 0.626816029262996\}$$

Intersection intervals with the x axis:

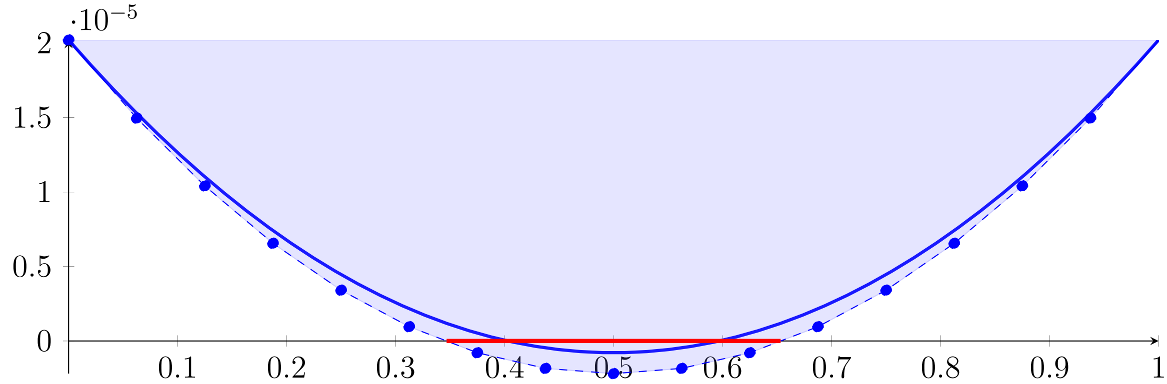
$$[0.3731836273807965, 0.626816029262996]$$

Longest intersection interval: 0.2536324018821995
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000592294767, 0.3000001107705452]$,

76.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000592294767

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.480017149294215 \cdot 10^{-117} X^{16} + 3.84938040332832 \cdot 10^{-110} X^{15} + 2.094095506848417 \cdot 10^{-100} X^{14} - 3.168497686663236 \cdot 10^{-93} X^{13} - 7.527801307502411 \cdot 10^{-84} X^{12} + 1.048590305184717 \cdot 10^{-76} X^{11} + 1.495875245120848 \cdot 10^{-67} X^{10} - 1.80569087496117 \cdot 10^{-60} X^{9} - 1.776919108083797 \cdot 10^{-51} X^{8} + 1.717962865912528 \cdot 10^{-44} X^{7} + 1.263101207166261 \cdot 10^{-35} X^{6} - 8.600271811104809 \cdot 10^{-29} X^{5} - 4.979113541149949 \cdot 10^{-20} X^{4} + 1.775239724826758 \cdot 10^{-13} X^{3} + 8.402507389292435 \cdot 10^{-05} X^{2} - 8.402503818595556 \cdot 10^{-05} X + 2.02154953599674 \cdot 10^{-05}$$
$$= 2.02154953599674 \cdot 10^{-05} B_{0,16}(X) + 1.496393047334518 \cdot 10^{-05} B_{1,16}(X) + 1.041257453583066 \cdot 10^{-05} B_{2,16}(X) + 6.561427547740848 \cdot 10^{-06} B_{3,16}(X) + 3.410489509392755 \cdot 10^{-06} B_{4,16}(X) + 9.59760421103387 \cdot 10^{-07} B_{5,16}(X) - 7.907597168102504 \cdot 10^{-07} B_{6,16}(X) - 1.84107090403115 \cdot 10^{-06} B_{7,16}(X) - 2.191173140242304 \cdot 10^{-06} B_{8,16}(X) - 1.841066425126706 \cdot 10^{-06} B_{9,16}(X) - 7.907507583673494 \cdot 10^{-07} B_{10,16}(X) + 9.59773860352773 \cdot 10^{-07} B_{11,16}(X) + 3.410507431350668 \cdot 10^{-06} B_{12,16}(X) + 6.561449954943343 \cdot 10^{-06} B_{13,16}(X) + 1.04126014314478 \cdot 10^{-05} B_{14,16}(X) + 1.496396186118106 \cdot 10^{-05} B_{15,16}(X) + 2.021553124446011 \cdot 10^{-05} B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3467669730097793, 0.6532326348738207\}$$

Intersection intervals with the x axis:

$$[0.3467669730097793, 0.6532326348738207]$$

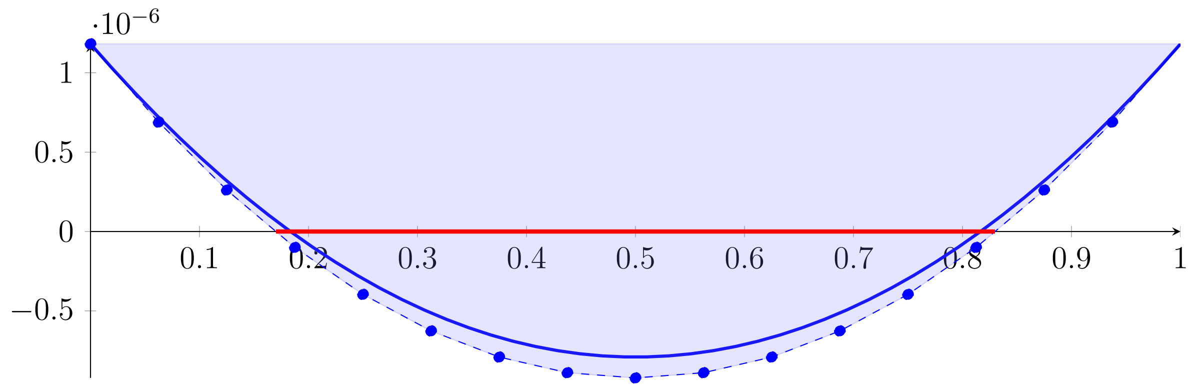
Longest intersection interval: 0.3064656618640414

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000771022171, 0.3000000928977847]$,

76.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000771022171, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -1.50163304400299\cdot 10^{-125}X^{16} + 7.605328677353438\cdot 10^{-118}X^{15} + 1.35002551430803 \\
& \cdot 10^{-107}X^{14} - 6.665257392422306\cdot 10^{-100}X^{13} - 5.167137254241996\cdot 10^{-90}X^{12} \\
& + 2.348582048403403\cdot 10^{-82}X^{11} + 1.093235143477049\cdot 10^{-72}X^{10} - 4.306055922184639\cdot 10^{-65}X^{9} \\
& - 1.382681732768409\cdot 10^{-55}X^{8} + 4.362007350883115\cdot 10^{-48}X^{7} + 1.04647552702743\cdot 10^{-38}X^{6} \\
& - 2.324990317134255\cdot 10^{-31}X^{5} - 4.392171749769965\cdot 10^{-22}X^{4} + 5.109781163448487\cdot 10^{-15}X^{3} \\
& + 7.891735947253277\cdot 10^{-06}X^{2} - 7.891735064633028\cdot 10^{-06}X + 1.182178307271875\cdot 10^{-06} \\
= & \ 1.182178307271875\cdot 10^{-06}B_{0,16}(X) + 6.889448657323109\cdot 10^{-07}B_{1,16}(X) + 2.614758904198573 \\
& \cdot 10^{-07}B_{2,16}(X) - 1.00228618656361\cdot 10^{-07}B_{3,16}(X) - 3.961686614872195\cdot 10^{-07}B_{4,16}(X) \\
& - 6.263442380635935\cdot 10^{-07}B_{5,16}(X) - 7.907553483763585\cdot 10^{-07}B_{6,16}(X) - 8.894019924163897 \\
& \cdot 10^{-07}B_{7,16}(X) - 9.222841701745627\cdot 10^{-07}B_{8,16}(X) - 8.894018816417527\cdot 10^{-07}B_{9,16}(X) \\
& - 7.907551268088353\cdot 10^{-07}B_{10,16}(X) - 6.263439056666857\cdot 10^{-07}B_{11,16}(X) \\
& - 3.961682182061795\cdot 10^{-07}B_{12,16}(X) - 1.002280644181919\cdot 10^{-07}B_{13,16}(X) + 2.614765557064017 \\
& \cdot 10^{-07}B_{14,16}(X) + 6.889456421767258\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.1701811983018366, 0.8298187006126137\}$$

Intersection intervals with the x axis:

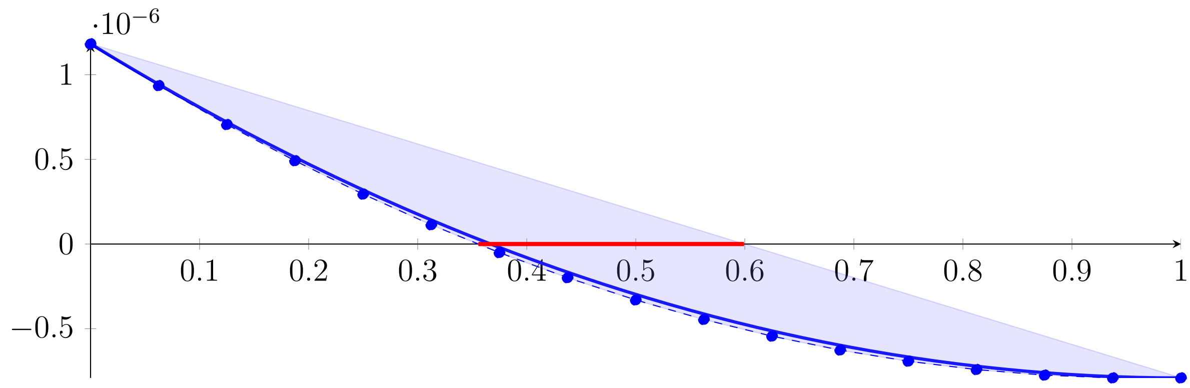
$$[0.1701811983018366, 0.8298187006126137]$$

Longest intersection interval: 0.6596375023107771
$\Longrightarrow$ Bisection: first half $[0.3000000771022171, 0.3000000850000009]$ und second half $[0.3000000850000009, 0.30$

76.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3000000771022171, 0.3000000850000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.291310186772141 \cdot 10^{-130} X^{16} + 2.320962120774364 \cdot 10^{-122} X^{15} + 8.239901820727723 \\ &\cdot 10^{-112} X^{14} - 8.13630052785926 \cdot 10^{-104} X^{13} - 1.261508118711425 \cdot 10^{-93} X^{12} \\ &+ 1.146768578321974 \cdot 10^{-85} X^{11} + 1.067612444801806 \cdot 10^{-75} X^{10} - 8.410265473016874 \cdot 10^{-68} X^{9} \\ &- 5.401100518626597 \cdot 10^{-58} X^{8} + 3.407818242877434 \cdot 10^{-50} X^{7} + 1.635118010980359 \cdot 10^{-40} X^{6} \\ &- 7.265594741044545 \cdot 10^{-33} X^{5} - 2.745107343606228 \cdot 10^{-23} X^{4} + 6.387226454310609 \cdot 10^{-16} X^{3} \\ &+ 1.972933986813319 \cdot 10^{-06} X^{2} - 3.945867532316514 \cdot 10^{-06} X + 1.182178307271875 \cdot 10^{-06} \\ = &\ 1.182178307271875 \cdot 10^{-06} B_{0,16}(X) + 9.35561586502093 \cdot 10^{-07} B_{1,16}(X) + 7.053859822890886 \\ &\cdot 10^{-07} B_{2,16}(X) + 4.916514946340023 \cdot 10^{-07} B_{3,16}(X) + 2.943581235379749 \cdot 10^{-07} B_{4,16}(X) \\ &+ 1.135058690021469 \cdot 10^{-07} B_{5,16}(X) - 5.090526897234114 \cdot 10^{-08} B_{6,16}(X) - 1.988752903843486 \\ &\cdot 10^{-07} B_{7,16}(X) - 3.30404195232735 \cdot 10^{-07} B_{8,16}(X) - 4.454919835163597 \cdot 10^{-07} B_{9,16}(X) \\ &- 5.441386552340822 \cdot 10^{-07} B_{10,16}(X) - 6.263442103847618 \cdot 10^{-07} B_{11,16}(X) - 6.921086489672579 \\ &\cdot 10^{-07} B_{12,16}(X) - 7.414319709804301 \cdot 10^{-07} B_{13,16}(X) - 7.743141764231377 \cdot 10^{-07} B_{14,16}(X) \\ &- 7.907552652942402 \cdot 10^{-07} B_{15,16}(X) - 7.907552375925969 \cdot 10^{-07} B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.355648638833307, 0.599198239772989\}$$

Intersection intervals with the x axis:

$$[0.355648638833307, 0.599198239772989]$$

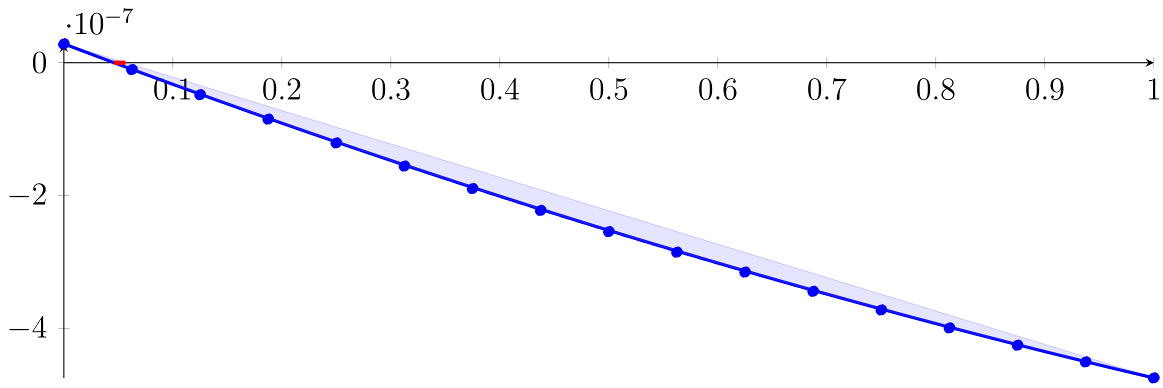
Longest intersection interval: 0.2435496009396821

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799110531, 0.3000000818345552]$,

76.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799110531, 0.3000000818345552]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -3.511418903935676\cdot 10^{-140}X^{16} + 1.460425350169266\cdot 10^{-131}X^{15} + 2.12885435080787 \\
& \cdot 10^{-120}X^{14} - 8.631046367525725\cdot 10^{-112}X^{13} - 5.494638409481287\cdot 10^{-101}X^{12} \\
& + 2.050866587664798\cdot 10^{-92}X^{11} + 7.839490728884222\cdot 10^{-82}X^{10} - 2.535691914683604\cdot 10^{-73}X^{9} \\
& - 6.686235030771963\cdot 10^{-63}X^{8} + 1.732161414466012\cdot 10^{-54}X^{7} + 3.412507672068927\cdot 10^{-44}X^{6} \\
& - 6.225987736384381\cdot 10^{-36}X^{5} - 9.658485252841958\cdot 10^{-26}X^{4} + 9.227298165814739\cdot 10^{-18}X^{3} \\
& + 1.170273575918748\cdot 10^{-07}X^{2} - 6.192309389580206\cdot 10^{-07}X + 2.838432851654629\cdot 10^{-08} \\
= & \; 2.838432851654629\cdot 10^{-08}B_{0,16}(X) - 1.031760516832999\cdot 10^{-08}B_{1,16}(X) - 4.804431087327399 \\
& \cdot 10^{-08}B_{2,16}(X) - 8.479578859826921\cdot 10^{-08}B_{3,16}(X) - 1.205720383432992\cdot 10^{-07}B_{4,16}(X) \\
& - 1.553730601083475\cdot 10^{-07}B_{5,16}(X) - 1.891988538933975\cdot 10^{-07}B_{6,16}(X) - 2.220494196984329 \\
& \cdot 10^{-07}B_{7,16}(X) - 2.539247575234371\cdot 10^{-07}B_{8,16}(X) - 2.848248673683937\cdot 10^{-07}B_{9,16}(X) \\
& - 3.147497492332862\cdot 10^{-07}B_{10,16}(X) - 3.436994031180981\cdot 10^{-07}B_{11,16}(X) - 3.71673829022813 \\
& \cdot 10^{-07}B_{12,16}(X) - 3.986730269474143\cdot 10^{-07}B_{13,16}(X) - 4.246969968918855\cdot 10^{-07}B_{14,16}(X) \\
& - 4.497457388562103\cdot 10^{-07}B_{15,16}(X) - 4.738192528403721\cdot 10^{-07}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.04583803348764934, 0.05651956610873593\}$$

Intersection intervals with the x axis:

$$[0.04583803348764934, 0.05651956610873593]$$

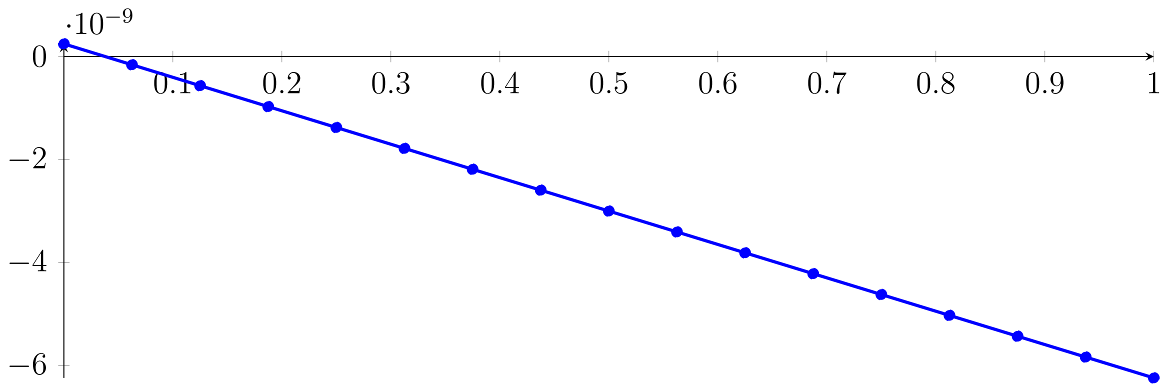
Longest intersection interval: 0.01068153262108659

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799992227, 0.3000000800197686]$,

76.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799992227, 0.3000000800197686]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.0083696859672\cdot10^{-171}X^{16} + 3.926295021503001\cdot10^{-161}X^{15} + 5.358163301918743\\ &\cdot10^{-148}X^{14} - 2.033760808321352\cdot10^{-137}X^{13} - 1.212109824997298\cdot10^{-124}X^{12}\\ &+ 4.235519560164542\cdot10^{-114}X^{11} + 1.515735988691705\cdot10^{-101}X^{10} - 4.589851404660334\cdot10^{-91}X^{9}\\ &- 1.133052955101212\cdot10^{-78}X^{8} + 2.748041945976198\cdot10^{-68}X^{7} + 5.068448795464027\cdot10^{-56}X^{6}\\ &- 8.657173392624336\cdot10^{-46}X^{5} - 1.257312709564581\cdot10^{-33}X^{4} + 1.124540929737694\cdot10^{-23}X^{3}\\ &+ 1.335225264723055\cdot10^{-11}X^{2} - 6.499737499496223\cdot10^{-09}X + 2.458891434694464\cdot10^{-10}\\ = &\ 2.458891434694464\cdot10^{-10}B_{0,16}(X) - 1.603444502490676\cdot10^{-10}B_{1,16}(X) - 5.664667751955213\\ &\cdot10^{-10}B_{2,16}(X) - 9.724778313699147\cdot10^{-10}B_{3,16}(X) - 1.378377618772248\cdot10^{-09}B_{4,16}(X)\\ &- 1.784166137402521\cdot10^{-09}B_{5,16}(X) - 2.189843387260733\cdot10^{-09}B_{6,16}(X) - 2.595409368346885\\ &\cdot10^{-09}B_{7,16}(X) - 3.000864080660977\cdot10^{-09}B_{8,16}(X) - 3.406207524203008\cdot10^{-09}B_{9,16}(X)\\ &- 3.811439698972979\cdot10^{-09}B_{10,16}(X) - 4.21656060497089\cdot10^{-09}B_{11,16}(X) - 4.62157024219674\\ &\cdot10^{-09}B_{12,16}(X) - 5.026468610650529\cdot10^{-09}B_{13,16}(X) - 5.431255710332258\cdot10^{-09}B_{14,16}(X)\\ &- 5.835931541241927\cdot10^{-09}B_{15,16}(X) - 6.240496103379535\cdot10^{-09}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03783062677354348, 0.03790850128565779\}$$

Intersection intervals with the x axis:

$$[0.03783062677354348, 0.03790850128565779]$$

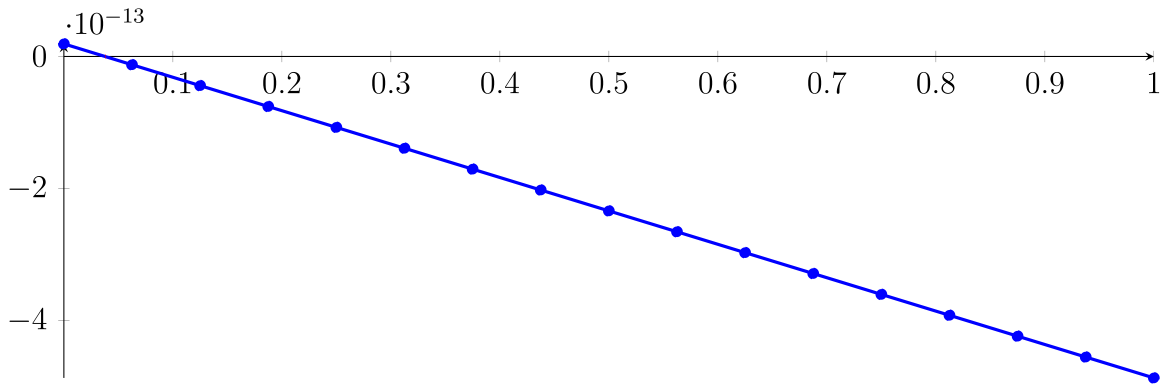
Longest intersection interval: $7.787451211431613\cdot10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999999, 0.3000000800000015]$,

76.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799999999, 0.3000000800000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.844782636201088\cdot 10^{-237}X^{16} + 9.223866560433282\cdot 10^{-223}X^{15} + 1.616406874872468\\ &\cdot 10^{-205}X^{14} - 7.878422704947929\cdot 10^{-191}X^{13} - 6.029565776386014\cdot 10^{-174}X^{12}\\ &+ 2.705549092213795\cdot 10^{-159}X^{11} + 1.243302910922026\cdot 10^{-142}X^{10} - 4.83455688747521\cdot 10^{-128}X^{9}\\ &- 1.5325438805125\cdot 10^{-111}X^{8} + 4.772992249567202\cdot 10^{-97}X^{7} + 1.130438906312722\cdot 10^{-80}X^{6}\\ &- 2.479435471431183\cdot 10^{-66}X^{5} - 4.624072778948578\cdot 10^{-50}X^{4} + 5.310816347750779\cdot 10^{-36}X^{3}\\ &+ 8.097393019768196\cdot 10^{-20}X^{2} - 5.060852140608093\cdot 10^{-13}X + 1.910916079008261\cdot 10^{-14}\\ = &\ 1.910916079008261\cdot 10^{-14}B_{0,16}(X) - 1.252116508871797\cdot 10^{-14}B_{1,16}(X) - 4.41514902927358\\ &\cdot 10^{-14}B_{2,16}(X) - 7.578181482197087\cdot 10^{-14}B_{3,16}(X) - 1.074121386764232\cdot 10^{-13}B_{4,16}(X)\\ &- 1.390424618560928\cdot 10^{-13}B_{5,16}(X) - 1.706727843609796\cdot 10^{-13}B_{6,16}(X) - 2.023031061910837\\ &\cdot 10^{-13}B_{7,16}(X) - 2.33933427346405\cdot 10^{-13}B_{8,16}(X) - 2.655637478269435\cdot 10^{-13}B_{9,16}(X)\\ &- 2.971940676326994\cdot 10^{-13}B_{10,16}(X) - 3.288243867636724\cdot 10^{-13}B_{11,16}(X) - 3.604547052198627\\ &\cdot 10^{-13}B_{12,16}(X) - 3.920850230012703\cdot 10^{-13}B_{13,16}(X) - 4.237153401078951\cdot 10^{-13}B_{14,16}(X)\\ &- 4.553456565397371\cdot 10^{-13}B_{15,16}(X) - 4.869759722967965\cdot 10^{-13}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775878104944304, 0.03775878709087106\}$$

Intersection intervals with the x axis:

$$[0.03775878104944304, 0.03775878709087106]$$

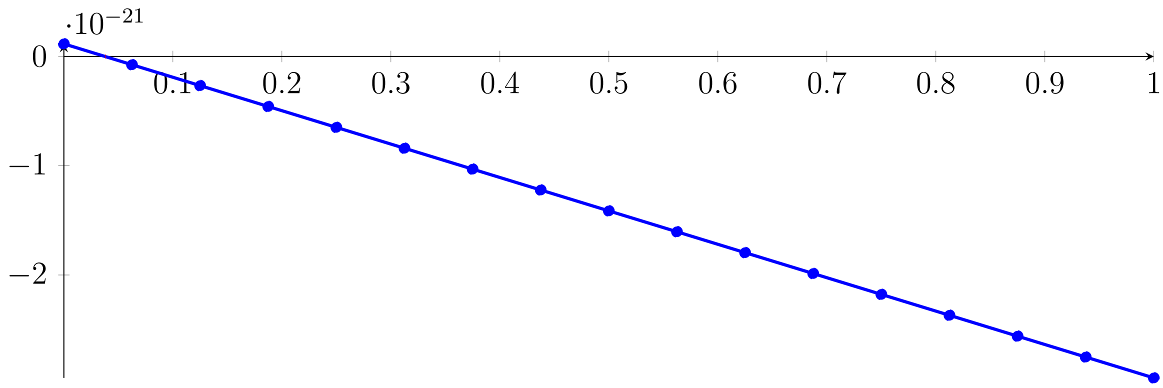
Longest intersection interval: $6.041428015081178\cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

76.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -9.719453947461351\cdot 10^{-326}X^{16} + 4.850491890476349\cdot 10^{-325}X^{15} + 1.39445463768969\cdot 10^{-320}X^{14} - 1.125263275537969\cdot 10^{-297}X^{13} - 1.425480559690934\cdot 10^{-272}X^{12} + 1.058744275447586\cdot 10^{-249}X^{11} + 8.053283759644375\cdot 10^{-225}X^{10} - 5.183380864729409\cdot 10^{-202}X^{9} - 2.7197548469932\cdot 10^{-177}X^{8} + 1.402064409388781\cdot 10^{-154}X^{7} + 5.496480949202085\cdot 10^{-130}X^{6} - 1.995495881284759\cdot 10^{-107}X^{5} - 6.160033605799806\cdot 10^{-83}X^{4} + 1.17106256664041\cdot 10^{-60}X^{3} + 2.955455531505519\cdot 10^{-36}X^{2} - 3.057477353302275\cdot 10^{-21}X + 1.154466008703692\cdot 10^{-22}$$
$$= 1.154466008703692\cdot 10^{-22}B_{0,16}(X) - 7.564573371102292\cdot 10^{-23}B_{1,16}(X) - 2.667380682924151\cdot 10^{-22}B_{2,16}(X) - 4.578304028738072\cdot 10^{-22}B_{3,16}(X) - 6.489227374551993\cdot 10^{-22}B_{4,16}(X) - 8.400150720365913\cdot 10^{-22}B_{5,16}(X) - 1.031107406617983\cdot 10^{-21}B_{6,16}(X) - 1.222199741199375\cdot 10^{-21}B_{7,16}(X) - 1.413292075780767\cdot 10^{-21}B_{8,16}(X) - 1.604384410362159\cdot 10^{-21}B_{9,16}(X) - 1.795476744943551\cdot 10^{-21}B_{10,16}(X) - 1.986569079524943\cdot 10^{-21}B_{11,16}(X) - 2.177661414106335\cdot 10^{-21}B_{12,16}(X) - 2.368753748687727\cdot 10^{-21}B_{13,16}(X) - 2.559846083269119\cdot 10^{-21}B_{14,16}(X) - 2.750938417850511\cdot 10^{-21}B_{15,16}(X) - 2.942030752431902\cdot 10^{-21}B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.03775877546424977, 0.0377587754642498\}$$

Intersection intervals with the x axis:

$$[0.03775877546424977, 0.0377587754642498]$$

Longest intersection interval: $3.649884166375354\cdot 10^{-17}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

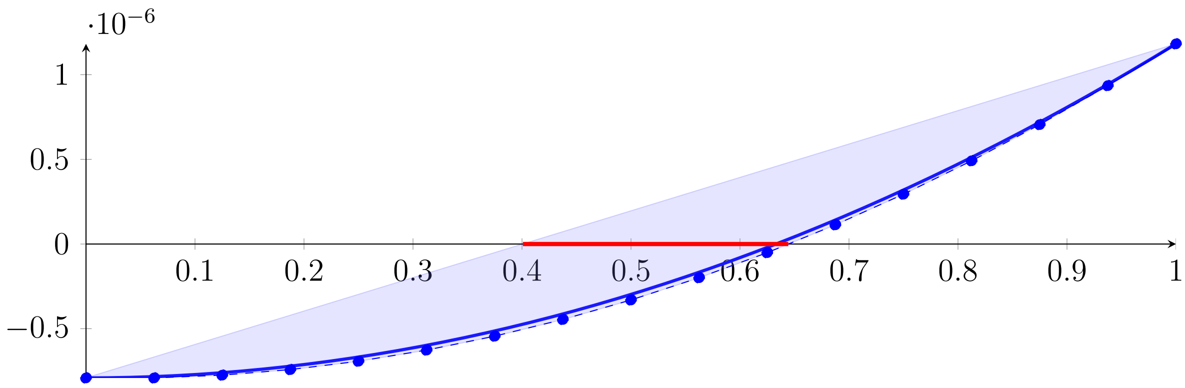
76.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 20!

76.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.3000000850000009, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-2.291310186772141\cdot 10^{-130}X^{16} + 2.320961754164735\cdot 10^{-122}X^{15} + 8.239901824209166 \\ &\cdot 10^{-112}X^{14} - 8.136299374273005\cdot 10^{-104}X^{13} - 1.261508119769144\cdot 10^{-93}X^{12} \\ &+ 1.146768426941\cdot 10^{-85}X^{11} + 1.067612446063251\cdot 10^{-75}X^{10} - 8.410264405404428\cdot 10^{-68}X^{9} \\ &- 5.401100526195836\cdot 10^{-58}X^{8} + 3.407817810789392\cdot 10^{-50}X^{7} + 1.635118013365832\cdot 10^{-40}X^{6} \\ &- 7.265593759973738\cdot 10^{-33}X^{5} - 2.745107347239025\cdot 10^{-23}X^{4} + 6.387225356267671\cdot 10^{-16}X^{3} \\ &+ 1.972933988729487\cdot 10^{-06}X^{2} + 4.432262923936659\cdot 10^{-13}X - 7.907552375925969\cdot 10^{-07} \\ = &-7.907552375925969\cdot 10^{-07}B_{0,16}(X) - 7.907552098909536\cdot 10^{-07}B_{1,16}(X) - 7.743140656165646 \\ &\cdot 10^{-07}B_{2,16}(X) - 7.414318047682893\cdot 10^{-07}B_{3,16}(X) - 6.921084273449871\cdot 10^{-07}B_{4,16}(X) \\ &- 6.263439333455175\cdot 10^{-07}B_{5,16}(X) - 5.441383227687398\cdot 10^{-07}B_{6,16}(X) - 4.454915956135136 \\ &\cdot 10^{-07}B_{7,16}(X) - 3.304037518786981\cdot 10^{-07}B_{8,16}(X) - 1.988747915631529\cdot 10^{-07}B_{9,16}(X) \\ &- 5.090471466573741\cdot 10^{-08}B_{10,16}(X) + 1.13506478814689\cdot 10^{-07}B_{11,16}(X) + 2.943587888792669 \\ &\cdot 10^{-07}B_{12,16}(X) + 4.916522155291369\cdot 10^{-07}B_{13,16}(X) + 7.053867587654395\cdot 10^{-07}B_{14,16}(X) \\ &+ 9.355624185893154\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4008015798845968, 0.644351143917019\}$$

Intersection intervals with the x axis:

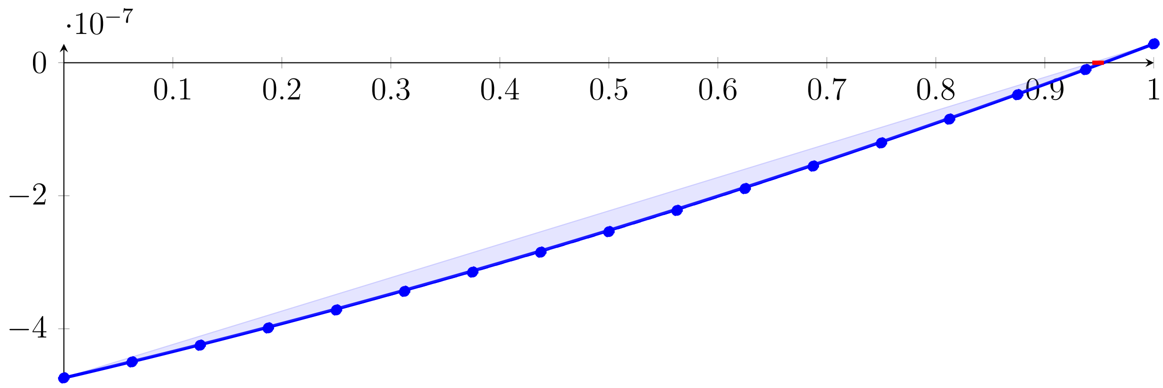
$$[0.4008015798845968, 0.644351143917019]$$

Longest intersection interval: 0.2435495640324222
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000881654451, 0.3000000900889469]$,

76.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.3000000881654451, 0.3000000900889469]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = & -3.511410390075519\cdot 10^{-140} X^{16} + 1.460421789403856\cdot 10^{-131} X^{15} + 2.12884983529034\\ & \cdot 10^{-120} X^{14} - 8.631028085335548\cdot 10^{-112} X^{13} - 5.494628422481359\cdot 10^{-101} X^{12}\\ & + 2.050862886067143\cdot 10^{-92} X^{11} + 7.839478858688502\cdot 10^{-82} X^{10} - 2.535688119961682\cdot 10^{-73} X^{9}\\ & - 6.686226934767321\cdot 10^{-63} X^{8} + 1.732159347494186\cdot 10^{-54} X^{7} + 3.412504574505657\cdot 10^{-44} X^{6}\\ & - 6.225982140334265\cdot 10^{-36} X^{5} - 9.658479411653884\cdot 10^{-26} X^{4} + 9.227292313018015\cdot 10^{-18} X^{3}\\ & + 1.170273222422553\cdot 10^{-07} X^{2} + 3.851762081107905\cdot 10^{-07} X - 4.738191826798638\cdot 10^{-07}\\ = & -4.738191826798638\cdot 10^{-07} B_{0,16}(X) - 4.497456696729394\cdot 10^{-07} B_{1,16}(X) - 4.246969289806629\\ & \cdot 10^{-07} B_{2,16}(X) - 3.986729606030177\cdot 10^{-07} B_{3,16}(X) - 3.716737645399875\cdot 10^{-07} B_{4,16}(X)\\ & - 3.436993407915557\cdot 10^{-07} B_{5,16}(X) - 3.147496893577059\cdot 10^{-07} B_{6,16}(X) - 2.848248102384216\\ & \cdot 10^{-07} B_{7,16}(X) - 2.539247034336863\cdot 10^{-07} B_{8,16}(X) - 2.220493689434835\cdot 10^{-07} B_{9,16}(X)\\ & - 1.891988067677968\cdot 10^{-07} B_{10,16}(X) - 1.553730169066096\cdot 10^{-07} B_{11,16}(X) - 1.205719993599056\\ & \cdot 10^{-07} B_{12,16}(X) - 8.479575412766813\cdot 10^{-08} B_{13,16}(X) - 4.804428120988084\cdot 10^{-08} B_{14,16}(X)\\ & - 1.031758060652722\cdot 10^{-08} B_{15,16}(X) + 2.838434768240921\cdot 10^{-08} B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9434803899886293, 0.9541619291703946\}$$

Intersection intervals with the x axis:

$$[0.9434803899886293, 0.9541619291703946]$$

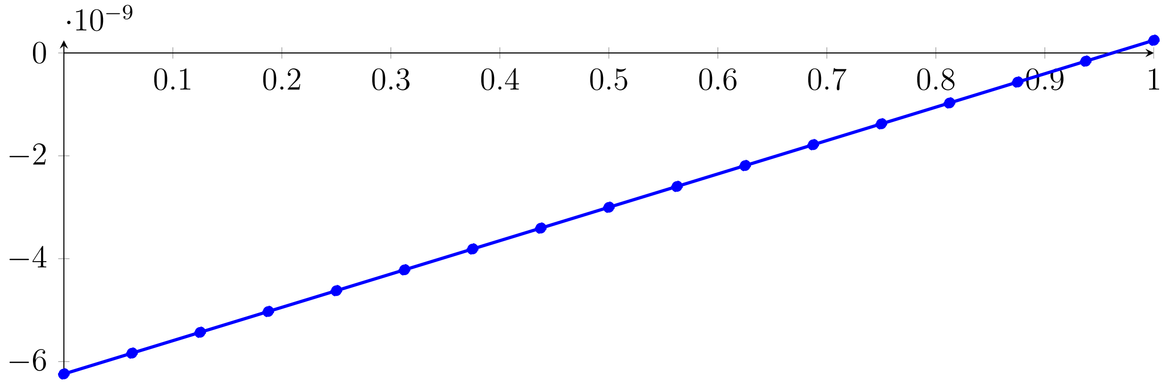
Longest intersection interval: 0.01068153918176529

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899802314, 0.3000000900007773]$,

76.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.3000000899802314, 0.3000000900007773]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -1.008377150647939 \cdot 10^{-171} X^{16} + 3.926321486414625 \cdot 10^{-161} X^{15} + 5.358198011634222 \\
& \cdot 10^{-148} X^{14} - 2.033772676384319 \cdot 10^{-137} X^{13} - 1.212116555959755 \cdot 10^{-124} X^{12} \\
& + 4.235540409672438 \cdot 10^{-114} X^{11} + 1.515743003803166 \cdot 10^{-101} X^{10} - 4.58986978058239 \cdot 10^{-91} X^{9} \\
& - 1.133057150938366 \cdot 10^{-78} X^{8} + 2.748050405674693 \cdot 10^{-68} X^{7} + 5.068462874895849 \cdot 10^{-56} X^{6} \\
& - 8.65719194232841 \cdot 10^{-46} X^{5} - 1.257315038545109 \cdot 10^{-33} X^{4} + 1.12454224629143 \cdot 10^{-23} X^{3} \\
& + 1.335226501895653 \cdot 10^{-11} X^{2} + 6.473035980688146 \cdot 10^{-09} X - 6.240498775828017 \cdot 10^{-09} \\
= & -6.240498775828017 \cdot 10^{-09} B_{0,16}(X) - 5.835934027035008 \cdot 10^{-09} B_{1,16}(X) - 5.43125800936684 \\
& \cdot 10^{-09} B_{2,16}(X) - 5.026470722823515 \cdot 10^{-09} B_{3,16}(X) - 4.621572167405032 \cdot 10^{-09} B_{4,16}(X) \\
& - 4.216562343111391 \cdot 10^{-09} B_{5,16}(X) - 3.811441249942592 \cdot 10^{-09} B_{6,16}(X) - 3.406208887898635 \\
& \cdot 10^{-09} B_{7,16}(X) - 3.000865256979519 \cdot 10^{-09} B_{8,16}(X) - 2.595410357185246 \cdot 10^{-09} B_{9,16}(X) \\
& - 2.189844188515814 \cdot 10^{-09} B_{10,16}(X) - 1.784166750971225 \cdot 10^{-09} B_{11,16}(X) \\
& - 1.378378044551477 \cdot 10^{-09} B_{12,16}(X) - 9.724780692565705 \cdot 10^{-10} B_{13,16}(X) - 5.664668250865062 \\
& \cdot 10^{-10} B_{14,16}(X) - 1.603443120412836 \cdot 10^{-10} B_{15,16}(X) + 2.458894698790972 \cdot 10^{-10} B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.9620914659183662, 0.9621693405339302\}$$

Intersection intervals with the x axis:

$$[0.9620914659183662, 0.9621693405339302]$$

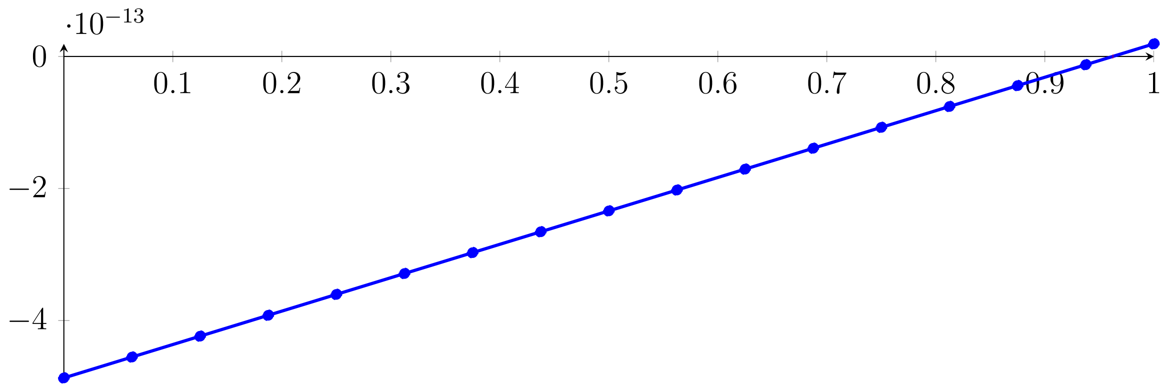
Longest intersection interval: $7.787461556406489 \cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899999985, 0.3000000900000001]$,

76.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.3000000899999985, 0.3000000900000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.844835503516189 \cdot 10^{-237} X^{16} + 9.224112529673191 \cdot 10^{-223} X^{15} + 1.616447407922287 \\
& \cdot 10^{-205} X^{14} - 7.878604734573916 \cdot 10^{-191} X^{13} - 6.029695377653573 \cdot 10^{-174} X^{12} \\
& + 2.705601945023742 \cdot 10^{-159} X^{11} + 1.243325181575566 \cdot 10^{-142} X^{10} - 4.834634042887648 \cdot 10^{-128} X^{9} \\
& - 1.532565842702471 \cdot 10^{-111} X^{8} + 4.773051325459324 \cdot 10^{-97} X^{7} + 1.130451056729432 \cdot 10^{-80} X^{6} \\
& - 2.479457251994378 \cdot 10^{-66} X^{5} - 4.624105915218427 \cdot 10^{-50} X^{4} + 5.31084372815498 \cdot 10^{-36} X^{3} \\
& + 8.097422035992938 \cdot 10^{-20} X^{2} + 5.060859587615554 \cdot 10^{-13} X - 4.869768282121624 \cdot 10^{-13} \\
= & -4.869768282121624 \cdot 10^{-13} B_{0,16}(X) - 4.553464557895652 \cdot 10^{-13} B_{1,16}(X) - 4.237160826921828 \\
& \cdot 10^{-13} B_{2,16}(X) - 3.920857089200152 \cdot 10^{-13} B_{3,16}(X) - 3.604553344730625 \cdot 10^{-13} B_{4,16}(X) \\
& - 3.288249593513246 \cdot 10^{-13} B_{5,16}(X) - 2.971945835548016 \cdot 10^{-13} B_{6,16}(X) - 2.655642070834933 \\
& \cdot 10^{-13} B_{7,16}(X) - 2.339338299374 \cdot 10^{-13} B_{8,16}(X) - 2.023034521165214 \cdot 10^{-13} B_{9,16}(X) \\
& - 1.706730736208576 \cdot 10^{-13} B_{10,16}(X) - 1.390426944504087 \cdot 10^{-13} B_{11,16}(X) \\
& - 1.074123146051747 \cdot 10^{-13} B_{12,16}(X) - 7.578193408515542 \cdot 10^{-14} B_{13,16}(X) - 4.4151552890351 \\
& \cdot 10^{-14} B_{14,16}(X) - 1.252117102076142 \cdot 10^{-14} B_{15,16}(X) + 1.910921152361334 \cdot 10^{-14} B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411803068306, 0.9622411863482746\}$$

Intersection intervals with the x axis:

$$[0.9622411803068306, 0.9622411863482746]$$

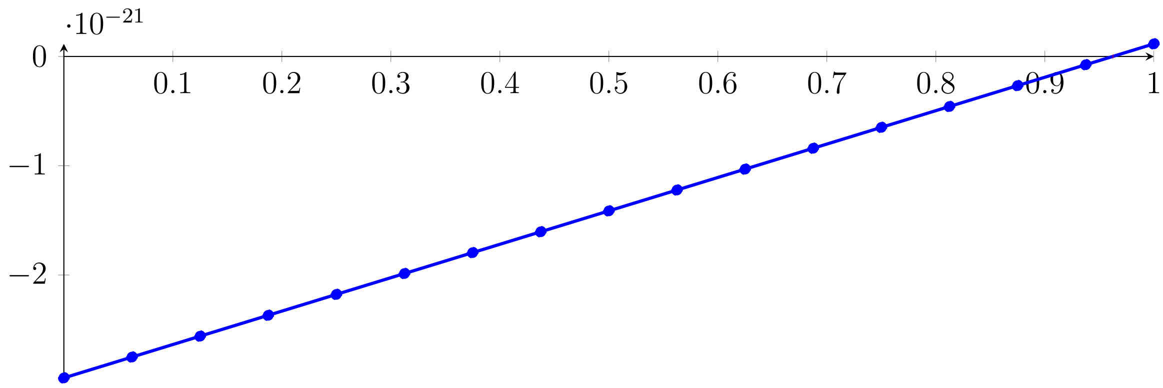
Longest intersection interval: $6.041444057142176 \cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

76.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 6.121540900010562\cdot10^{-326}X^{16} - 8.347007493216372\cdot10^{-325}X^{15} + 1.39439413847082\\&\quad\cdot10^{-320}X^{14} - 1.125328119545448\cdot10^{-297}X^{13} - 1.425556622701337\cdot10^{-272}X^{12}\\&\quad + 1.058795883612058\cdot10^{-249}X^{11} + 8.05364186268366\cdot10^{-225}X^{10} - 5.183587463234939\cdot10^{-202}X^{9}\\&\quad - 2.719851598975594\cdot10^{-177}X^{8} + 1.402107824165868\cdot10^{-154}X^{7} + 5.496627599374482\cdot10^{-130}X^{6}\\&\quad - 1.995539904687186\cdot10^{-107}X^{5} - 6.160143177546872\cdot10^{-83}X^{4} + 1.171077932951323\cdot10^{-60}X^{3}\\&\quad + 2.955481817665439\cdot10^{-36}X^{2} + 3.057490949422222\cdot10^{-21}X - 2.942043735497876\cdot10^{-21}\\&= -2.942043735497876\cdot10^{-21}B_{0,16}(X) - 2.750950551158987\cdot10^{-21}B_{1,16}(X) - 2.559857366820099\\&\quad\cdot10^{-21}B_{2,16}(X) - 2.36876418248121\cdot10^{-21}B_{3,16}(X) - 2.177670998142321\cdot10^{-21}B_{4,16}(X)\\&\quad - 1.986577813803432\cdot10^{-21}B_{5,16}(X) - 1.795484629464543\cdot10^{-21}B_{6,16}(X) - 1.604391445125654\\&\quad\cdot10^{-21}B_{7,16}(X) - 1.413298260786765\cdot10^{-21}B_{8,16}(X) - 1.222205076447875\cdot10^{-21}B_{9,16}(X)\\&\quad - 1.031111892108986\cdot10^{-21}B_{10,16}(X) - 8.400187077700972\cdot10^{-22}B_{11,16}(X) - 6.489255234312081\\&\quad\cdot10^{-22}B_{12,16}(X) - 4.578323390923189\cdot10^{-22}B_{13,16}(X) - 2.667391547534297\cdot10^{-22}B_{14,16}(X)\\&\quad - 7.564597041454045\cdot10^{-23}B_{15,16}(X) + 1.154472139243488\cdot10^{-22}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411919334823, 0.9622411919334823\}$$

Intersection intervals with the x axis:

$$[0.9622411919334823, 0.9622411919334823]$$

Longest intersection interval: $3.649903549784843\cdot10^{-17}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

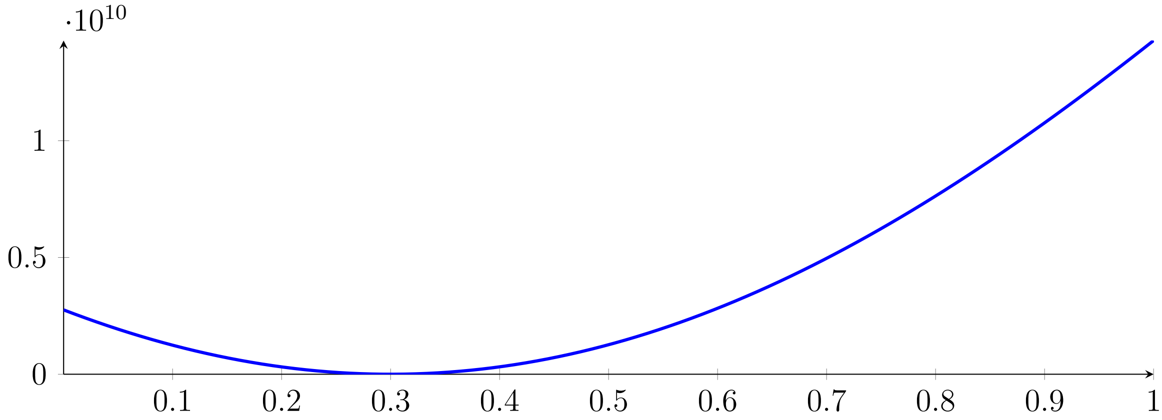
76.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 20!

76.27 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

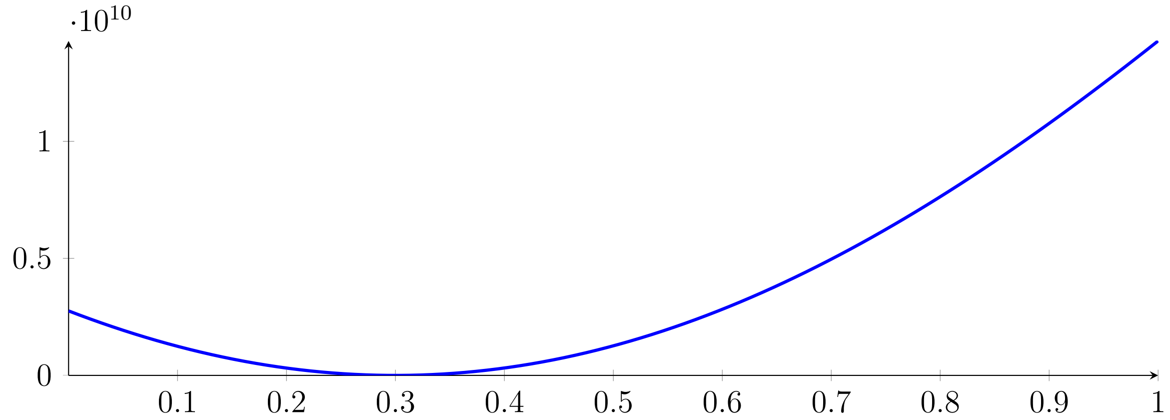
with precision $\varepsilon = 1 \cdot 10^{-32}$.

77 Running QuadClip on h_{16} with epsilon 32

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called QuadClip with input polynomial on interval $[0, 1]$:

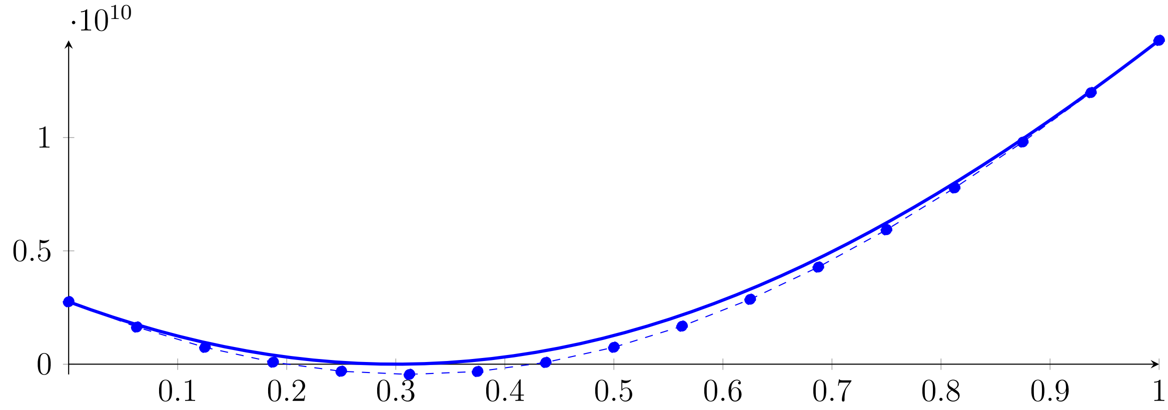
$$\begin{aligned} p = {} & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



77.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

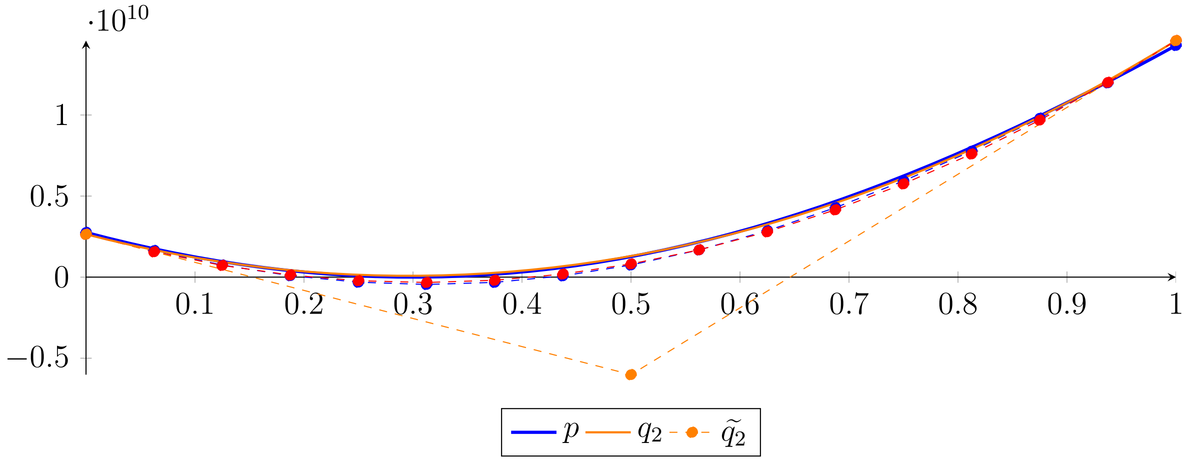
$$\begin{aligned} p = {} & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = {} & 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ & + 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ & - 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ & + 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ & + 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ & + 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328$$
$$= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2}$$

$$\widetilde{q}_2 = -4.590615853685512\cdot 10^{-286}X^{16} + 3.626297739922361\cdot 10^{-285}X^{15} - 1.337118562447102 \cdot 10^{-284}X^{14} + 3.057409950495903\cdot 10^{-284}X^{13} - 4.816080313213711\cdot 10^{-284}X^{12} + 5.445450819003583\cdot 10^{-284}X^{11} - 4.453195910004646\cdot 10^{-284}X^{10} + 2.601913123147853 \cdot 10^{-284}X^9 - 1.060485581673177\cdot 10^{-284}X^8 + 2.936863540613122\cdot 10^{-285}X^7 - 5.487274374859613 \cdot 10^{-286}X^6 + 7.23018920152705\cdot 10^{-287}X^5 - 7.254244946204871\cdot 10^{-288}X^4 + 6.659990111567495 \cdot 10^{-289}X^3 + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328$$
$$= 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16} + 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16} - 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16} + 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16} + 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16} + 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16}$$



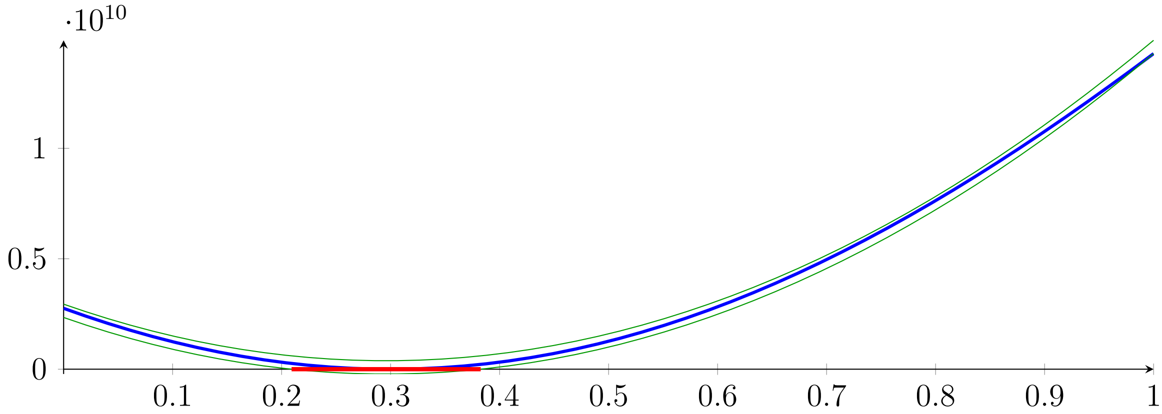
The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.
Bounding polynomials M and m:

$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$
$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



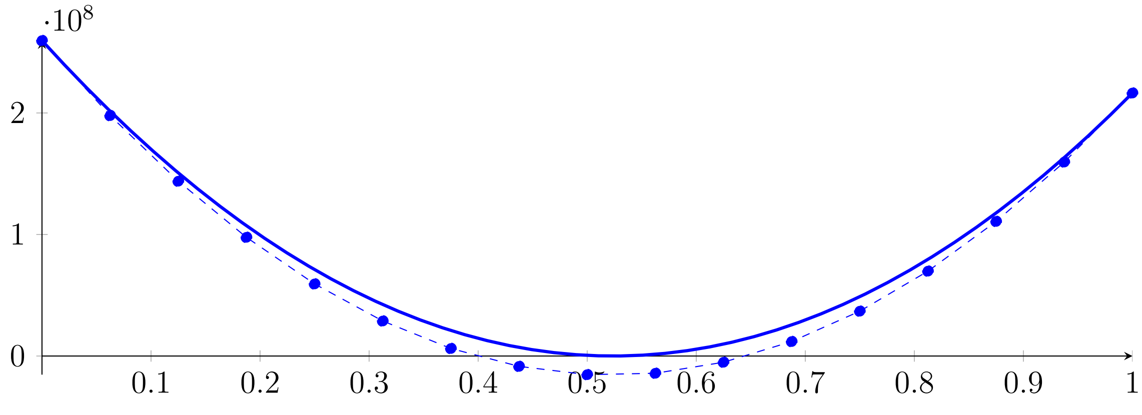
$[0.2091580131065301, 0.3826391383239738]$

Longest intersection interval: 0.1734811252174437
$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

77.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

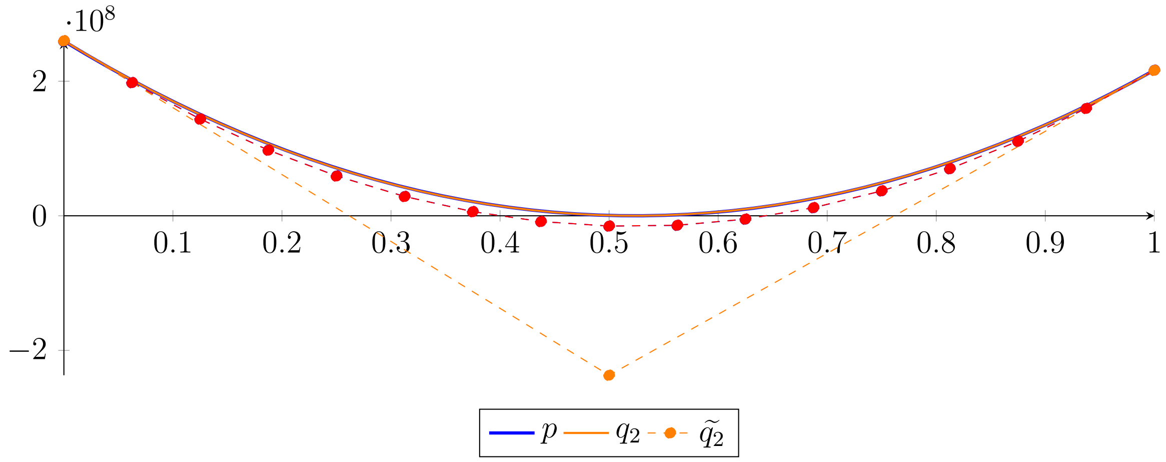
$$\begin{aligned} p = & -6.730319568869656 \cdot 10^{-13} X^{16} + 8.742499458235574 \cdot 10^{-12} X^{15} + 4.969734366538428 \cdot 10^{-09} X^{14} \\ & - 5.918022439742958 \cdot 10^{-08} X^{13} - 1.563835677279211 \cdot 10^{-05} X^{12} + 0.000165142833990623 X^{11} \\ & + 0.02725178204957372 X^{10} - 0.2448183330655992 X^{9} - 28.45820232077076 X^{8} \\ & + 205.241311077626 X^{7} + 17835.39577074806 X^{6} - 94141.21233496903 X^{5} - 6218439.344794644 X^{4} \\ & + 20000843.19616221 X^{3} + 930858984.7988209 X^{2} - 987724621.0538478 X + 259570777.0276934 \\ = & \, 259570777.0276934 B_{0,16}(X) + 197837988.2118279 B_{1,16}(X) + 143862357.6026193 B_{2,16}(X) \\ & + 97679600.99148916 B_{3,16}(X) + 59322017.44494463 B_{4,16}(X) + 28818467.75210257 B_{5,16}(X) \\ & + 6194355.099411763 B_{6,16}(X) - 8528392.009487306 B_{7,16}(X) - 15331334.57303514 B_{8,16}(X) \\ & - 14199535.63666165 B_{9,16}(X) - 5121570.552084079 B_{10,16}(X) + 11910465.05798682 B_{11,16}(X) \\ & + 36900949.63643701 B_{12,16}(X) + 69850732.33405701 B_{13,16}(X) \\ & + 110757131.9597003 B_{14,16}(X) + 159613938.238137 B_{15,16}(X) + 216411415.3731615 B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & \, 950064117.9944898 X^{2} - 993959420.5216784 X + 260029868.8092426 \\ = & \, 260029868.8092426 B_{0,2} - 236949841.4515966 B_{1,2} + 216134566.282054 B_{2,2} \end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-1.41520176718356\cdot10^{-287}X^{16} + 1.139753408092347\cdot10^{-286}X^{15} - 4.276104932493763\\ &\cdot10^{-286}X^{14} + 9.896028733517596\cdot10^{-286}X^{13} - 1.566233430583374\cdot10^{-285}X^{12}\\ &+ 1.767044733566612\cdot10^{-285}X^{11} - 1.435814571471649\cdot10^{-285}X^{10} + 8.336078570237132\\ &\cdot10^{-286}X^{9} - 3.396584250429756\cdot10^{-286}X^{8} + 9.548198482934522\cdot10^{-287}X^{7} - 1.861944039074976\\ &\cdot10^{-287}X^{6} + 2.622613520689986\cdot10^{-288}X^{5} - 2.667693132942884\cdot10^{-289}X^{4} + 2.048626792988895\\ &\cdot10^{-290}X^{3} + 950064117.9944898X^2 - 993959420.5216784X + 260029868.8092426\\ = &\,260029868.8092426B_{0,16} + 197907405.0266377B_{1,16} + 143702142.2273202B_{2,16}\\ &+ 97414080.41129016B_{3,16} + 59043219.5785475B_{4,16} + 28589559.72909226B_{5,16}\\ &+ 6053100.862924435B_{6,16} - 8566157.019955976B_{7,16} - 15268213.91954897B_{8,16}\\ &- 14053069.83585455B_{9,16} - 4920724.768872719B_{10,16} + 12128821.28139653B_{11,16}\\ &+ 37095568.31495319B_{12,16} + 69979516.33179727B_{13,16}\\ &+ 110780665.3319288B_{14,16} + 159499015.3153477B_{15,16} + 216134566.282054B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

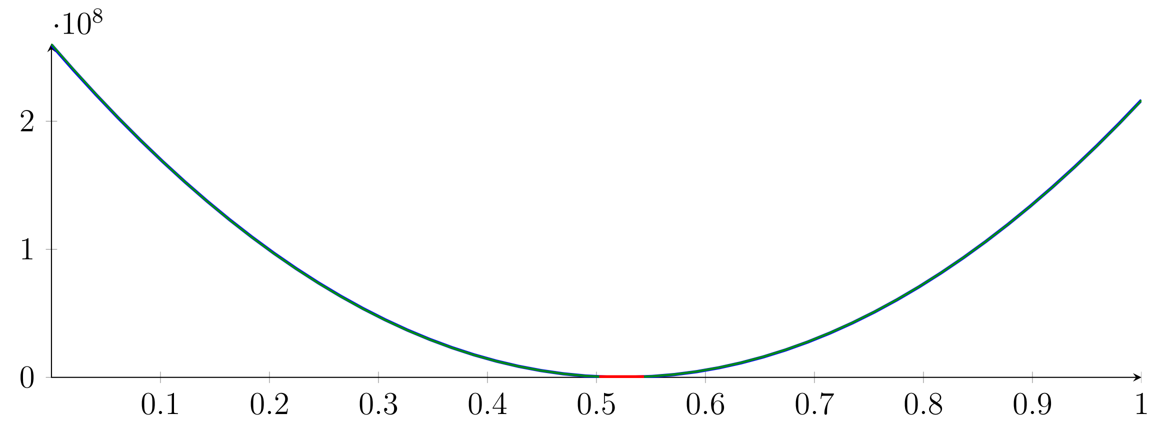
Bounding polynomials M and m:

$$M = 950064117.9944898X^2 - 993959420.5216784X + 260488960.5907918$$
$$m = 950064117.9944898X^2 - 993959420.5216784X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

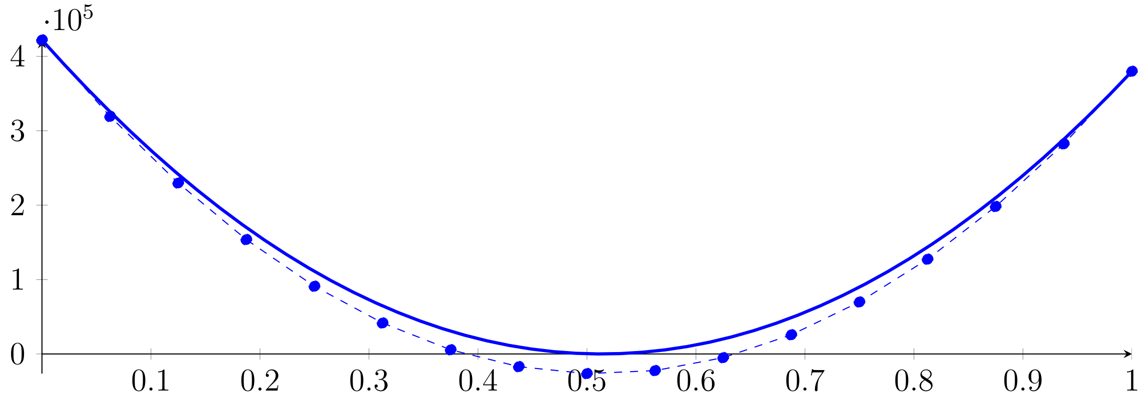
Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

77.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Normalized monomial und Bézier representations and the Bézier polygon:

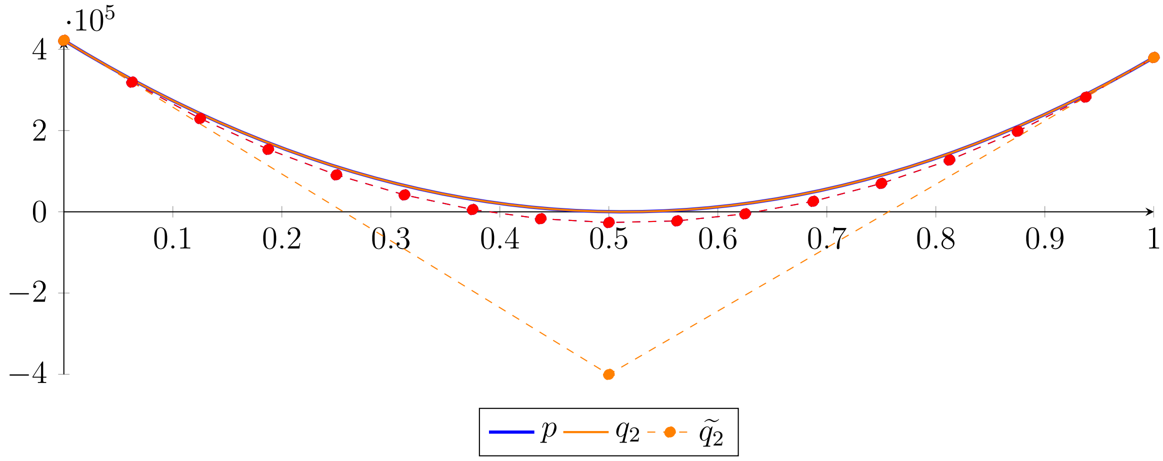
$$
\begin{aligned}
p = &-4.348008654913047\cdot 10^{-35}X^{16} + 5.243388239594399\cdot 10^{-33}X^{15} + 1.924264426249969\\
&\cdot 10^{-28}X^{14} - 2.246746658841888\cdot 10^{-26}X^{13} - 3.625525335135332\cdot 10^{-22}X^{12}\\
&+ 3.881306066232412\cdot 10^{-20}X^{11} + 3.776139474106064\cdot 10^{-16}X^{10} - 3.496022725744\\
&\cdot 10^{-14}X^{9} - 2.35123279599996\cdot 10^{-10}X^{8} + 1.7435554055155\cdot 10^{-08}X^{7} + 8.761425585865998\\
&\cdot 10^{-05}X^{6} - 0.004592127899352905X^{5} - 18.10659249096443X^{4} + 504.889471438631X^{3}\\
&+ 1602084.653024796X^{2} - 1644731.182248784X + 422061.2284789393\\
= &\ 422061.2284789393B_{0,16}(X) + 319265.5295883902B_{1,16}(X) + 229820.5361397145B_{2,16}(X)\\
&+ 153727.1497212539B_{3,16}(X) + 90986.26197267314B_{4,16}(X) + 41598.75458390835B_{5,16}(X)\\
&+ 5565.499294126806B_{6,16}(X) - 17112.6421093025B_{7,16}(X) - 26434.81779182738B_{8,16}(X)\\
&- 22400.18587271997B_{9,16}(X) - 5007.914426073306B_{10,16}(X) + 25742.81851821299B_{11,16}(X)\\
&+ 69852.82497345804B_{12,16}(X) + 127322.906995236B_{13,16}(X) + 198153.8566804232B_{14,16}(X)\\
&+ 282346.4561662563B_{15,16}(X) + 379901.4776294016B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 1602810.939315368X^2 - 1645017.556512812X + 422084.9204772878\\
&= 422084.9204772878B_{0,2} - 400423.8577791184B_{1,2} + 379878.3032798431B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = &-2.394754881528157\cdot 10^{-290}X^{16} + 1.927007748221533\cdot 10^{-289}X^{15} - 7.222560694913459\\
&\cdot 10^{-289}X^{14} + 1.669823679170844\cdot 10^{-288}X^{13} - 2.640537385502685\cdot 10^{-288}X^{12}\\
&+ 2.977245352423787\cdot 10^{-288}X^{11} - 2.4183464566446\cdot 10^{-288}X^{10} + 1.403945843258626\cdot 10^{-288}X^{9}\\
&- 5.721369736991339\cdot 10^{-289}X^{8} + 1.608948017596564\cdot 10^{-289}X^{7} - 3.139550473135121\cdot 10^{-290}X^{6}\\
&+ 4.427570920272867\cdot 10^{-291}X^{5} - 4.516145205270189\cdot 10^{-292}X^{4} + 3.478035897244226\\
&\cdot 10^{-293}X^{3} + 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\
= &\ 422084.9204772878B_{0,16} + 319271.323195237B_{1,16} + 229814.4837408143B_{2,16}\\
&+ 153714.4021140197B_{3,16} + 90971.07831485311B_{4,16} + 41584.51234331459B_{5,16}\\
&+ 5554.704199404135B_{6,16} - 17118.34611687825B_{7,16} - 26434.63860553258B_{8,16}\\
&- 22394.17326655884B_{9,16} - 4996.95009995704B_{10,16} + 25757.03089427283B_{11,16}\\
&+ 69867.76971613076B_{12,16} + 127335.2663656168B_{13,16} + 198159.5208427308B_{14,16}\\
&+ 282340.5331474729B_{15,16} + 379878.3032798431B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 23.69199834856345$.

Bounding polynomials M and m:

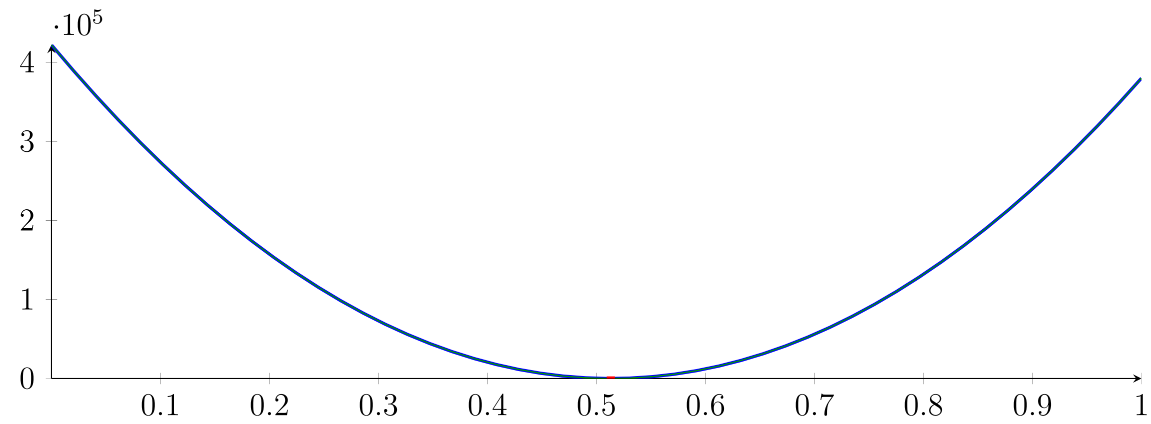
$$M = 1602810.939315368X^2 - 1645017.556512812X + 422108.6124756364$$

$$m = 1602810.939315368X^2 - 1645017.556512812X + 422061.2284789393$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.509405571919099, 0.5169273012614853\}$$

Intersection intervals:



$$[0.509405571919099, 0.5169273012614853]$$

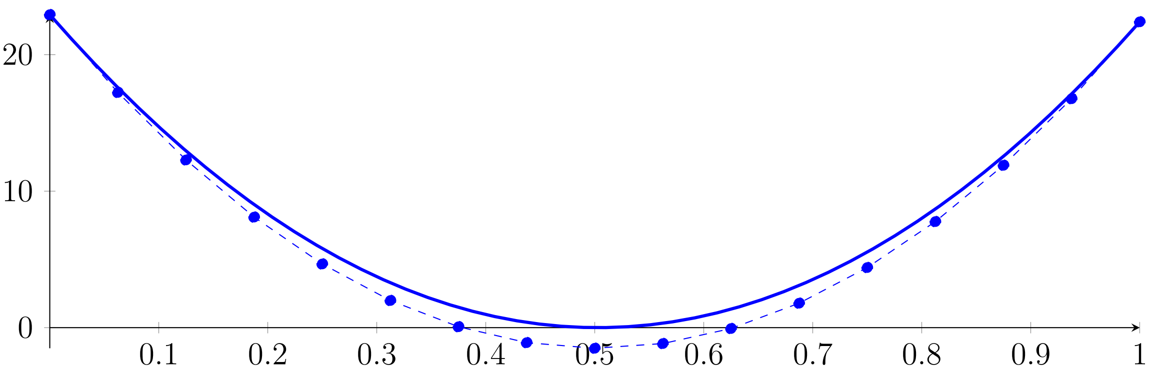
Longest intersection interval: 0.007521729342386311

$\Longrightarrow$ Selective recursion: interval 1: $[0.299973157656462, 0.3000267016613888]$,

77.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.299973157656462, 0.30002670166138$

Normalized monomial und Bézier representations and the Bézier polygon:

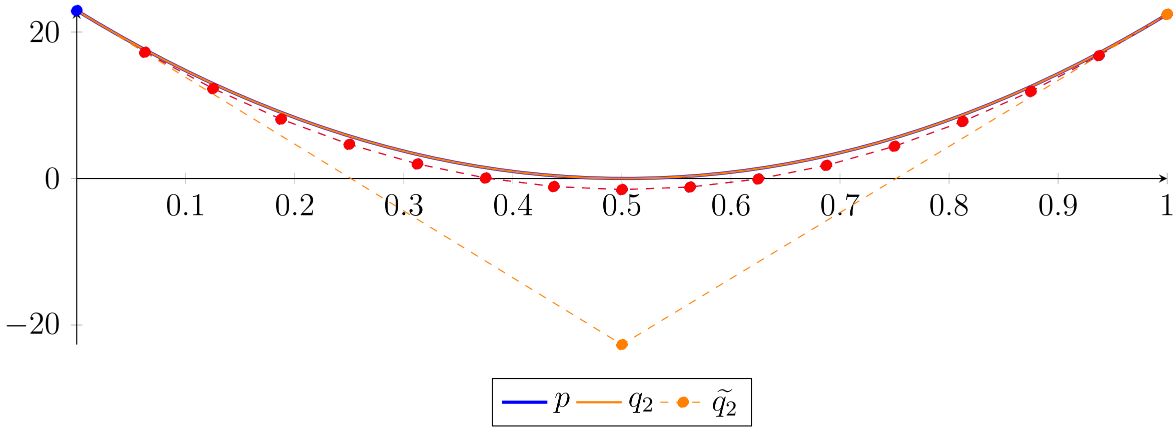
$$\begin{aligned}p = &-4.564294214760651\cdot 10^{-69}X^{16} + 6.823166131086062\cdot 10^{-65}X^{15} + 3.571082953880241\\ &\cdot 10^{-58}X^{14} - 5.203669764436854\cdot 10^{-54}X^{13} - 1.189477187926378\cdot 10^{-47}X^{12}\\ &+ 1.595632405197401\cdot 10^{-43}X^{11} + 2.190120291778079\cdot 10^{-37}X^{10} - 2.545939358967532\cdot 10^{-33}X^{9}\\ &- 2.410599475027015\cdot 10^{-27}X^{8} + 2.244415896590929\cdot 10^{-23}X^{7} + 1.587744426227316\cdot 10^{-17}X^{6}\\ &- 1.041115833763881\cdot 10^{-13}X^{5} - 5.799356524856294\cdot 10^{-08}X^{4} + 0.0001991514832499812X^{3}\\ &+ 90.68225982541923X^{2} - 91.20858295780928X + 22.93446410530317\\ = &\,22.93446410530317B_{0,16}(X) + 17.23392767044009B_{1,16}(X) + 12.28907673412217B_{2,16}(X)\\ &+ 8.099911651977055B_{3,16}(X) + 4.666432779600538B_{4,16}(X) + 1.988640472556532B_{5,16}(X)\\ &+ 0.06653508637709412B_{6,16}(X) - 1.099883023437587B_{7,16}(X)\\ &- 1.510613501419185B_{8,16}(X) - 1.165655992131239B_{9,16}(X) - 0.0650101401691539B_{10,16}(X)\\ &+ 1.791324409839802B_{11,16}(X) + 4.403348013236495B_{12,16}(X) + 7.771061025329927B_{13,16}(X)\\ &+ 11.89446380139723B_{14,16}(X) + 16.77355669668369B_{15,16}(X) + 22.4083400664027B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\ &= 22.93447405790644B_{0,2} - 22.66987713993177B_{1,2} + 22.40833011545639B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-1.359001790090183\cdot 10^{-294}X^{16} + 1.092598333426672\cdot 10^{-293}X^{15} - 4.09106565145402\\ &\cdot 10^{-293}X^{14} + 9.449139850665356\cdot 10^{-293}X^{13} - 1.493001927703419\cdot 10^{-292}X^{12}\\ &+ 1.682443127957849\cdot 10^{-292}X^{11} - 1.366228291670638\cdot 10^{-292}X^{10} + 7.931222707714909\\ &\cdot 10^{-293}X^{9} - 3.23261546930867\cdot 10^{-293}X^{8} + 9.093117628361849\cdot 10^{-294}X^{7} - 1.775022991079505\\ &\cdot 10^{-294}X^{6} + 2.505263643899121\cdot 10^{-295}X^{5} - 2.56169827233206\cdot 10^{-296}X^{4} + 1.979687487137548\\ &\cdot 10^{-297}X^{3} + 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\ = &\,22.93447405790644B_{0,16} + 17.23393015817666B_{1,16} + 12.28907424555711B_{2,16}\\ &+ 8.099906320047769B_{3,16} + 4.666426381648652B_{4,16} + 1.988634430359754B_{5,16}\\ &+ 0.06653046618107656B_{6,16} - 1.099885510887381B_{7,16} - 1.510613500845619B_{8,16}\\ &- 1.165653503693637B_{9,16} - 0.06500551943143578B_{10,16} + 1.791330451940986B_{11,16}\\ &+ 4.403354410423627B_{12,16} + 7.771066356016488B_{13,16} + 11.89446628871957B_{14,16}\\ &+ 16.77355420853287B_{15,16} + 22.40833011545639B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 9.952603274324834 \cdot 10^{-06}$.
Bounding polynomials M and m:

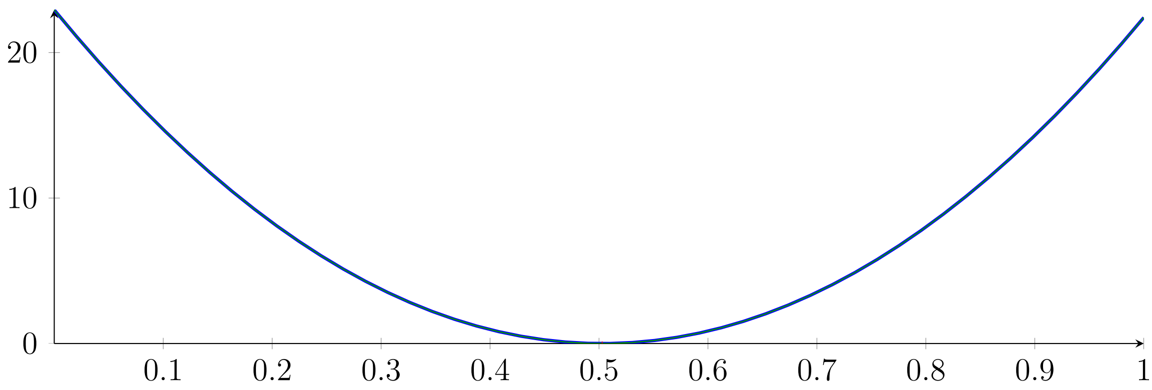
$$M = 90.68255845322638X^2 - 91.20870239567643X + 22.93448401050971$$

$$m = 90.68255845322638X^2 - 91.20870239567643X + 22.93446410530317$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025582179560734, 0.5032438232679998\}$$

Intersection intervals:



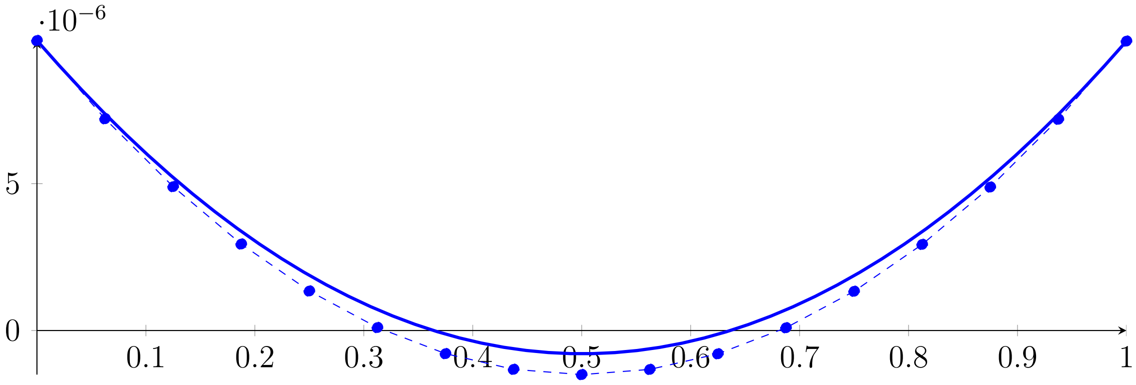
$$[0.5025582179560734, 0.5032438232679998]$$

Longest intersection interval: 0.0006856053119264748
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000666361603, 0.3000001033462145]$,

77.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000666361603, 0.300000103346$

Normalized monomial und Bézier representations and the Bézier polygon:

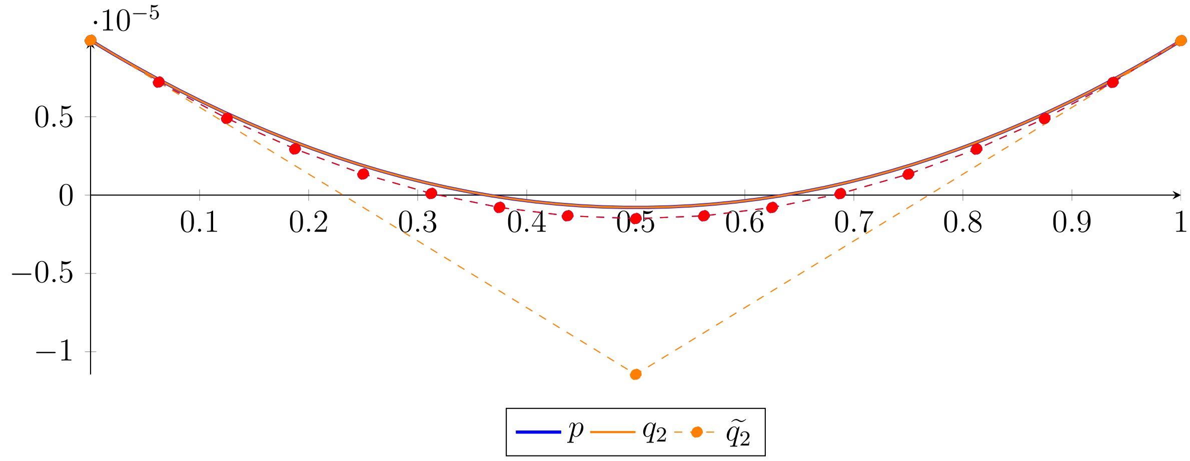
$$
\begin{aligned}
p = & -1.087828446441416\cdot 10^{-119}X^{16} + 2.370636059353804\cdot 10^{-112}X^{15} + 1.810668709595505\\
& \cdot 10^{-102}X^{14} - 3.846486744130667\cdot 10^{-95}X^{13} - 1.283061410949319\cdot 10^{-85}X^{12}\\
& + 2.509305114999168\cdot 10^{-78}X^{11} + 5.0258723654725\cdot 10^{-69}X^{10} - 8.517807546222726\cdot 10^{-62}X^{9}\\
& - 1.17684840157971\cdot 10^{-52}X^{8} + 1.597478332578791\cdot 10^{-45}X^{7} + 1.649027121563264\cdot 10^{-36}X^{6}\\
& - 1.576413560673164\cdot 10^{-29}X^{5} - 1.281381537279639\cdot 10^{-20}X^{4} + 6.414336823836069\cdot 10^{-14}X^{3}\\
& + 4.262575843118847\cdot 10^{-05}X^{2} - 4.264622395354259\cdot 10^{-05}X + 9.875919579833914\cdot 10^{-06}\\
= & \; 9.875919579833914\cdot 10^{-06}B_{0,16}(X) + 7.210530582737503\cdot 10^{-06}B_{1,16}(X) + 4.900356239234328\\
& \cdot 10^{-06}B_{2,16}(X) + 2.945396549438933\cdot 10^{-06}B_{3,16}(X) + 1.345651513465858\cdot 10^{-06}B_{4,16}(X)\\
& + 1.01121131429646\cdot 10^{-07}B_{5,16}(X) - 7.881945965551621\cdot 10^{-07}B_{6,16}(X) - 1.322295670374024\\
& \cdot 10^{-06}B_{7,16}(X) - 1.501182089912399\cdot 10^{-06}B_{8,16}(X) - 1.324853855055745\cdot 10^{-06}B_{9,16}(X)\\
& - 7.933109656895192\cdot 10^{-07}B_{10,16}(X) + 9.344657830081879\cdot 10^{-08}B_{11,16}(X)\\
& + 1.335418777029811\cdot 10^{-06}B_{12,16}(X) + 2.932605630611999\cdot 10^{-06}B_{13,16}(X) + 4.885007139161925\\
& \cdot 10^{-06}B_{14,16}(X) + 7.192623302794129\cdot 10^{-06}B_{15,16}(X) + 9.855454121623155\cdot 10^{-06}B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 = & \; 4.26257585274035\cdot 10^{-05}X^{2} - 4.264622399202859\cdot 10^{-05}X + 9.875919583041081\cdot 10^{-06}\\
= & \; 9.875919583041081\cdot 10^{-06}B_{0,2} - 1.144719241297322\cdot 10^{-05}B_{1,2} + 9.855454118415988\cdot 10^{-06}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = & -6.428697903766316\cdot 10^{-301}X^{16} + 5.168486014171287\cdot 10^{-300}X^{15} - 1.931525871486767\\
& \cdot 10^{-299}X^{14} + 4.443748026156872\cdot 10^{-299}X^{13} - 6.983462617461767\cdot 10^{-299}X^{12}\\
& + 7.823576270761152\cdot 10^{-299}X^{11} - 6.321034979462039\cdot 10^{-299}X^{10} + 3.658988671435299\cdot 10^{-299}X^{9}\\
& - 1.493155040297548\cdot 10^{-299}X^{8} + 4.235526792785023\cdot 10^{-300}X^{7} - 8.432417338595836\cdot 10^{-301}X^{6}\\
& + 1.226905231702478\cdot 10^{-301}X^{5} - 1.286361467975875\cdot 10^{-302}X^{4} + 9.604810259325032\cdot 10^{-304}X^{3}\\
& + 4.26257585274035\cdot 10^{-05}X^{2} - 4.264622399202859\cdot 10^{-05}X + 9.875919583041081\cdot 10^{-06}\\
= & \; 9.875919583041081\cdot 10^{-06}B_{0,16} + 7.210530583539294\cdot 10^{-06}B_{1,16} + 4.900356238432536\cdot 10^{-06}B_{2,16}\\
& + 2.945396547720807\cdot 10^{-06}B_{3,16} + 1.345651511404108\cdot 10^{-06}B_{4,16} + 1.011211294824374\cdot 10^{-07}B_{5,16}\\
& - 7.881945980442039\cdot 10^{-07}B_{6,16} - 1.322295671175816\cdot 10^{-06}B_{7,16} - 1.501182089912399\cdot 10^{-06}B_{8,16}\\
& - 1.324853854253953\cdot 10^{-06}B_{9,16} - 7.933109642004772\cdot 10^{-07}B_{10,16} + 9.34465802480274\cdot 10^{-08}B_{11,16}\\
& + 1.335418779091561\cdot 10^{-06}B_{12,16} + 2.932605632330124\cdot 10^{-06}B_{13,16} + 4.885007139963716\\
& \cdot 10^{-06}B_{14,16} + 7.192623301992338\cdot 10^{-06}B_{15,16} + 9.855454118415988\cdot 10^{-06}B_{16,16}
\end{aligned}
$$



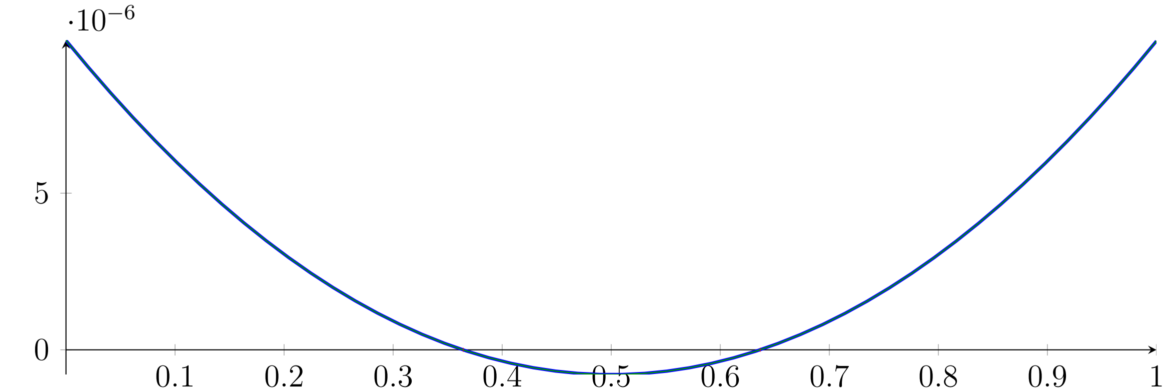
The maximum difference of the Bézier coefficients is $\delta = 3.207167313591001 \cdot 10^{-15}$.
Bounding polynomials M and m:

$$M = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919586248249 \cdot 10^{-06}$$
$$m = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06}$$

Root of M and m:

$N(M) = \{0.3640375916978796, 0.6364425279605644\}$ $\quad N(m) = \{0.3640375911454658, 0.6364425285129782\}$

Intersection intervals:



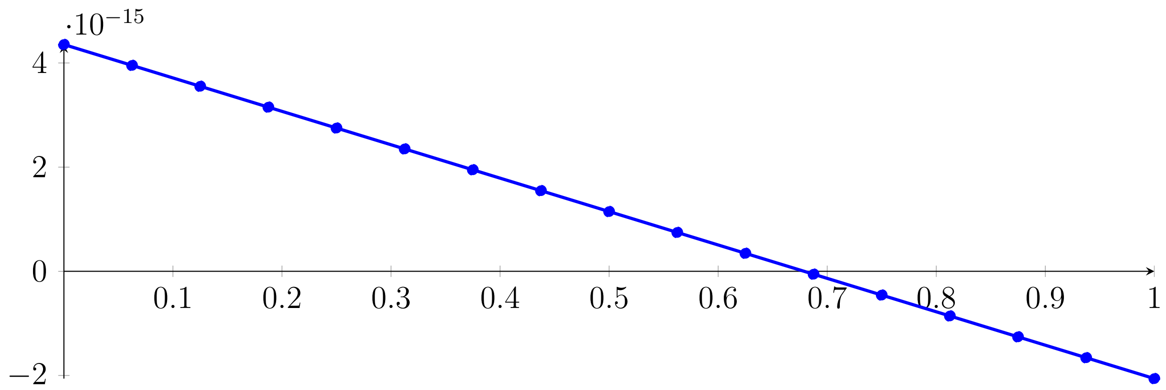
$$[0.3640375911454658, 0.3640375916978796],\ [0.6364425279605644, 0.6364425285129782]$$

Longest intersection interval: $5.524137987858897 \cdot 10^{-10}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$, interval 2: $[0.30000009, 0.30000009]$,

77.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

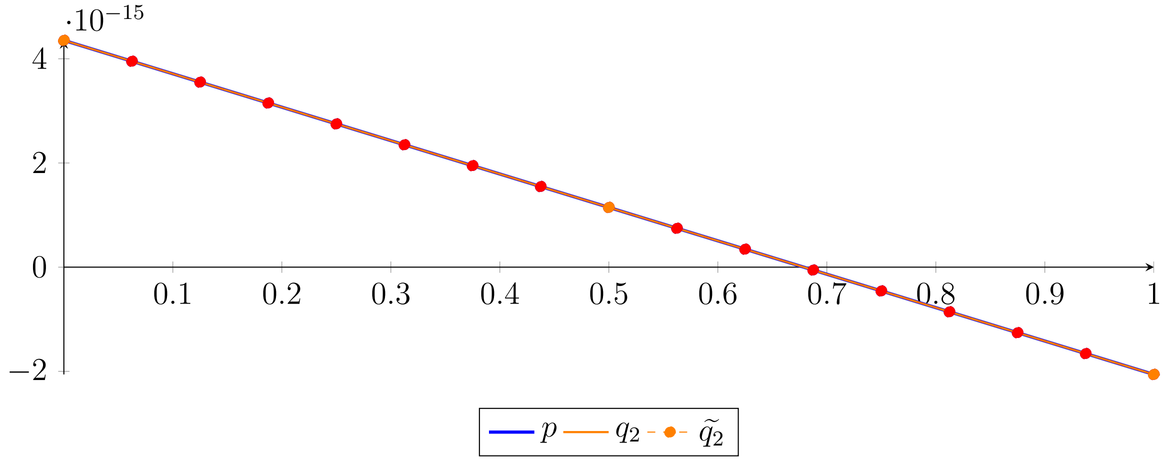
$$\begin{aligned}p = &-8.180742151883693\cdot10^{-268}X^{16} + 3.227249519181262\cdot10^{-251}X^{15} + 4.462130353465124\\&\cdot10^{-232}X^{14} - 1.715943065651804\cdot10^{-215}X^{13} - 1.03614769176239\cdot10^{-196}X^{12} + 3.668285054157079\\&\cdot10^{-180}X^{11} + 1.330015307195714\cdot10^{-161}X^{10} - 4.080453140608508\cdot10^{-145}X^{9}\\&- 1.02055559825224\cdot10^{-126}X^{8} + 2.507763120804761\cdot10^{-110}X^{7} + 4.686136227440808\cdot10^{-92}X^{6}\\&- 8.109473416022201\cdot10^{-76}X^{5} - 1.19326399448035\cdot10^{-57}X^{4} + 1.081297478088303\cdot10^{-41}X^{3}\\&+ 1.300771930692191\cdot10^{-23}X^{2} - 6.414334643539715\cdot10^{-15}X + 4.354113798188606\cdot10^{-15}\\= &\,4.354113798188606\cdot10^{-15}B_{0,16}(X) + 3.953217882967374\cdot10^{-15}B_{1,16}(X) + 3.552321967854539\\&\cdot10^{-15}B_{2,16}(X) + 3.151426052850102\cdot10^{-15}B_{3,16}(X) + 2.750530137954063\cdot10^{-15}B_{4,16}(X)\\&+ 2.349634223166422\cdot10^{-15}B_{5,16}(X) + 1.948738308487178\cdot10^{-15}B_{6,16}(X) + 1.547842393916332\\&\cdot10^{-15}B_{7,16}(X) + 1.146946479453883\cdot10^{-15}B_{8,16}(X) + 7.460505650998323\cdot10^{-16}B_{9,16}(X)\\&+ 3.451546508541791\cdot10^{-16}B_{10,16}(X) - 5.574126328307647\cdot10^{-17}B_{11,16}(X) - 4.566371773119344\\&\cdot10^{-16}B_{12,16}(X) - 8.575330912323946\cdot10^{-16}B_{13,16}(X) - 1.258429005044457\cdot10^{-15}B_{14,16}(X)\\&- 1.659324918748122\cdot10^{-15}B_{15,16}(X) - 2.060220832343389\cdot10^{-15}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.300771930692191\cdot10^{-23}X^{2} - 6.414334643539715\cdot10^{-15}X + 4.354113798188606\cdot10^{-15}\\&= 4.354113798188606\cdot10^{-15}B_{0,2} + 1.146946476418749\cdot10^{-15}B_{1,2} - 2.060220832343389\cdot10^{-15}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q_2} = &\,1.912651228969383\cdot10^{-311}X^{16} - 1.223741742808027\cdot10^{-310}X^{15} + 2.733522262624021\\&\cdot10^{-310}X^{14} - 8.679644486233328\cdot10^{-311}X^{13} - 7.862443329477107\cdot10^{-310}X^{12}\\&+ 1.83195702234036\cdot10^{-309}X^{11} - 2.057972005087252\cdot10^{-309}X^{10} + 1.349302407721643\cdot10^{-309}X^{9}\\&- 5.089946779834537\cdot10^{-310}X^{8} + 8.605298284676211\cdot10^{-311}X^{7} + 8.242572388969257\cdot10^{-312}X^{6}\\&- 6.927894069632637\cdot10^{-312}X^{5} + 1.356821367314693\cdot10^{-312}X^{4} - 8.054206617575827\cdot10^{-314}X^{3}\\&+ 1.300771930692191\cdot10^{-23}X^{2} - 6.414334643539715\cdot10^{-15}X + 4.354113798188606\cdot10^{-15}\\= &\,4.354113798188606\cdot10^{-15}B_{0,16} + 3.953217882967374\cdot10^{-15}B_{1,16} + 3.552321967854539\\&\cdot10^{-15}B_{2,16} + 3.151426052850102\cdot10^{-15}B_{3,16} + 2.750530137954063\cdot10^{-15}B_{4,16}\\&+ 2.349634223166422\cdot10^{-15}B_{5,16} + 1.948738308487178\cdot10^{-15}B_{6,16} + 1.547842393916332\\&\cdot10^{-15}B_{7,16} + 1.146946479453883\cdot10^{-15}B_{8,16} + 7.460505650998323\cdot10^{-16}B_{9,16}\\&+ 3.451546508541791\cdot10^{-16}B_{10,16} - 5.574126328307647\cdot10^{-17}B_{11,16} - 4.566371773119344\\&\cdot10^{-16}B_{12,16} - 8.575330912323946\cdot10^{-16}B_{13,16} - 1.258429005044457\cdot10^{-15}B_{14,16}\\&- 1.659324918748122\cdot10^{-15}B_{15,16} - 2.060220832343389\cdot10^{-15}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.406487390441514 \cdot 10^{-43}$.
Bounding polynomials M and m:

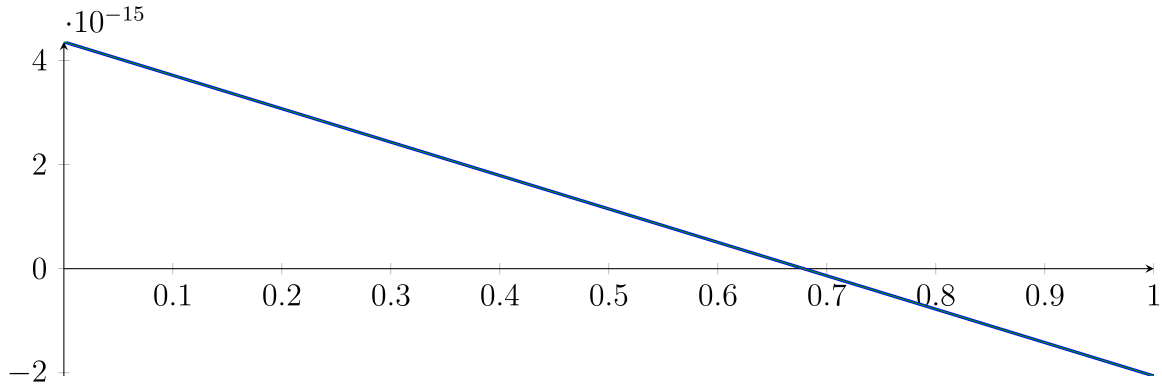
$$M = 1.300771930692191 \cdot 10^{-23} X^2 - 6.414334643539715 \cdot 10^{-15} X + 4.354113798188606 \cdot 10^{-15}$$

$$m = 1.300771930692191 \cdot 10^{-23} X^2 - 6.414334643539715 \cdot 10^{-15} X + 4.354113798188606 \cdot 10^{-15}$$

Root of M and m:

$N(M) = \{0.678809891617932, 493117546.8474808\}$ $N(m) = \{0.678809891617932, 493117546.8474808\}$

Intersection intervals:



$[0.678809891617932, 0.678809891617932]$

Longest intersection interval: $1.685751587897438 \cdot 10^{-28}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

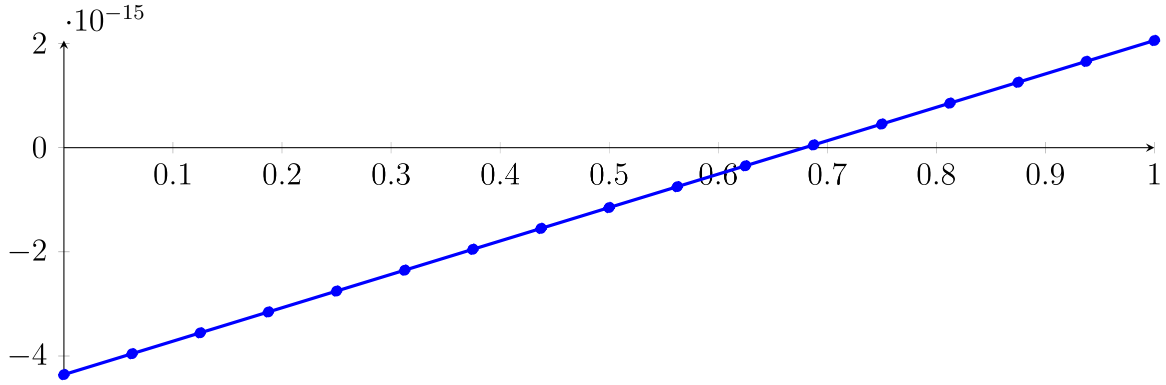
77.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 7!

77.8 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

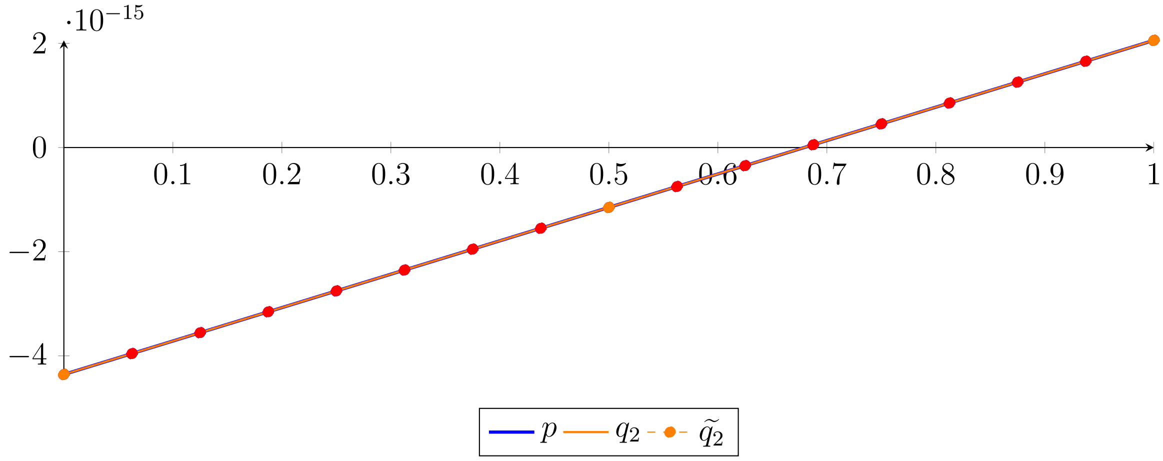
$$p = -8.180742151883693\cdot10^{-268}X^{16} + 3.227248873730463\cdot10^{-251}X^{15} + 4.462130355852244\cdot10^{-232}X^{14} - 1.715942757602136\cdot10^{-215}X^{13} - 1.0361476928624\cdot10^{-196}X^{12} + 3.66828444102595\cdot10^{-180}X^{11} + 1.330015309185499\cdot10^{-161}X^{10} - 4.080452484754623\cdot10^{-145}X^{9} - 1.020555600063168\cdot10^{-126}X^{8} + 2.507762718201662\cdot10^{-110}X^{7} + 4.686136236097161\cdot10^{-92}X^{6} - 8.109472029532602\cdot10^{-76}X^{5} - 1.193263996479812\cdot10^{-57}X^{4} + 1.081297242720538\cdot10^{-41}X^{3} + 1.300771932291811\cdot10^{-23}X^{2} + 6.414334610838192\cdot10^{-15}X - 4.357019382395145\cdot10^{-15}$$
$$= -4.357019382395145\cdot10^{-15}B_{0,16}(X) - 3.956123469217758\cdot10^{-15}B_{1,16}(X) - 3.555227555931974\cdot10^{-15}B_{2,16}(X) - 3.154331642537791\cdot10^{-15}B_{3,16}(X) - 2.753435729035212\cdot10^{-15}B_{4,16}(X) - 2.352539815424234\cdot10^{-15}B_{5,16}(X) - 1.951643901704859\cdot10^{-15}B_{6,16}(X) - 1.550747987877086\cdot10^{-15}B_{7,16}(X) - 1.149852073940915\cdot10^{-15}B_{8,16}(X) - 7.489561598963468\cdot10^{-16}B_{9,16}(X) - 3.480602457433808\cdot10^{-16}B_{10,16}(X) + 5.283566851798276\cdot10^{-17}B_{11,16}(X) + 4.53731582887744\cdot10^{-16}B_{12,16}(X) + 8.546274973659029\cdot10^{-16}B_{13,16}(X) + 1.255523411952459\cdot10^{-15}B_{14,16}(X) + 1.656419326647414\cdot10^{-15}B_{15,16}(X) + 2.057315241450766\cdot10^{-15}B_{16,16}(X)$$



Degree reduction and raising:

$$q_2 = 1.300771932291811\cdot10^{-23}X^{2} + 6.414334610838192\cdot10^{-15}X - 4.357019382395145\cdot10^{-15}$$
$$= -4.357019382395145\cdot10^{-15}B_{0,2} - 1.14985207697605\cdot10^{-15}B_{1,2} + 2.057315241450766\cdot10^{-15}B_{2,2}$$

$$\widetilde{q}_2 = -1.913962948891774\cdot10^{-311}X^{16} + 1.224839229220605\cdot10^{-310}X^{15} - 2.736439272631334\cdot10^{-310}X^{14} + 8.686727754411293\cdot10^{-311}X^{13} + 7.87470789022501\cdot10^{-310}X^{12} - 1.83498999880619\cdot10^{-309}X^{11} + 2.061599642193156\cdot10^{-309}X^{10} - 1.351793491536379\cdot10^{-309}X^{9} + 5.099450965546735\cdot10^{-310}X^{8} - 8.61911462129453\cdot10^{-311}X^{7} - 8.275840669030232\cdot10^{-312}X^{6} + 6.946029354848196\cdot10^{-312}X^{5} - 1.359818722878116\cdot10^{-312}X^{4} + 8.064687133307928\cdot10^{-314}X^{3} + 1.300771932291811\cdot10^{-23}X^{2} + 6.414334610838192\cdot10^{-15}X - 4.357019382395145\cdot10^{-15}$$
$$= -4.357019382395145\cdot10^{-15}B_{0,16} - 3.956123469217758\cdot10^{-15}B_{1,16} - 3.555227555931974\cdot10^{-15}B_{2,16} - 3.154331642537791\cdot10^{-15}B_{3,16} - 2.753435729035212\cdot10^{-15}B_{4,16} - 2.352539815424234\cdot10^{-15}B_{5,16} - 1.951643901704859\cdot10^{-15}B_{6,16} - 1.550747987877086\cdot10^{-15}B_{7,16} - 1.149852073940915\cdot10^{-15}B_{8,16} - 7.489561598963468\cdot10^{-16}B_{9,16} - 3.480602457433808\cdot10^{-16}B_{10,16} + 5.283566851798276\cdot10^{-17}B_{11,16} + 4.53731582887744\cdot10^{-16}B_{12,16} + 8.546274973659029\cdot10^{-16}B_{13,16} + 1.255523411952459\cdot10^{-15}B_{14,16} + 1.656419326647414\cdot10^{-15}B_{15,16} + 2.057315241450766\cdot10^{-15}B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 5.406486213602688 \cdot 10^{-43}$.
Bounding polynomials M and m:

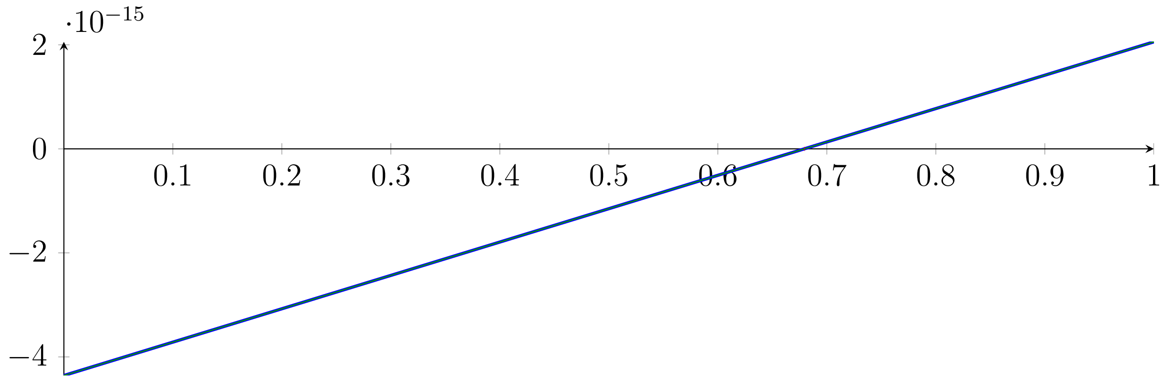
$$M = 1.300771932291811 \cdot 10^{-23} X^2 + 6.414334610838192 \cdot 10^{-15} X - 4.357019382395145 \cdot 10^{-15}$$

$$m = 1.300771932291811 \cdot 10^{-23} X^2 + 6.414334610838192 \cdot 10^{-15} X - 4.357019382395145 \cdot 10^{-15}$$

Root of M and m:

$N(M) = \{-493117545.0851349, 0.6792628761573223\} \quad N(m) = \{-493117545.0851349, 0.6792628761573223\}$

Intersection intervals:



$$[0.6792628761573223, 0.6792628761573223]$$

Longest intersection interval: $1.685751220266157 \cdot 10^{-28}$
$\implies$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

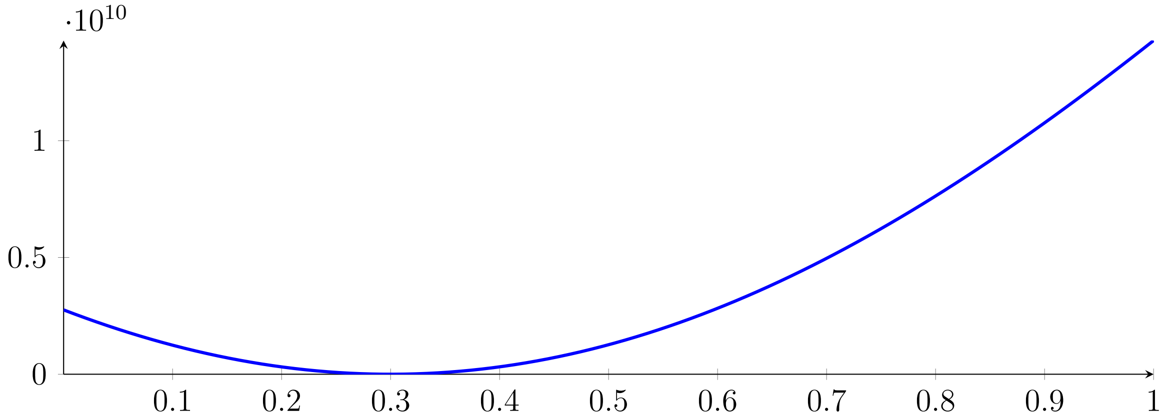
77.9 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 7!

77.10 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

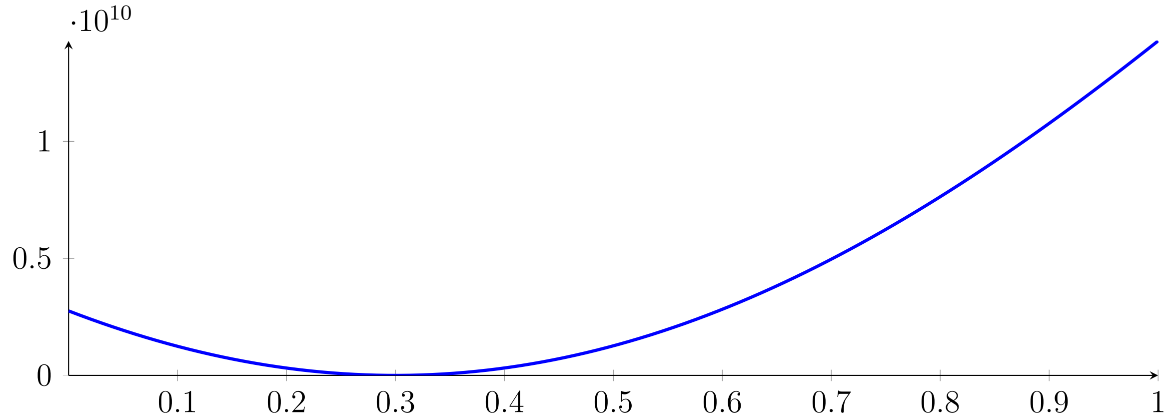
with precision $\varepsilon = 1 \cdot 10^{-32}$.

78 Running `CubeClip` on h_{16} with epsilon 32

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `CubeClip` with input polynomial on interval $[0, 1]$:

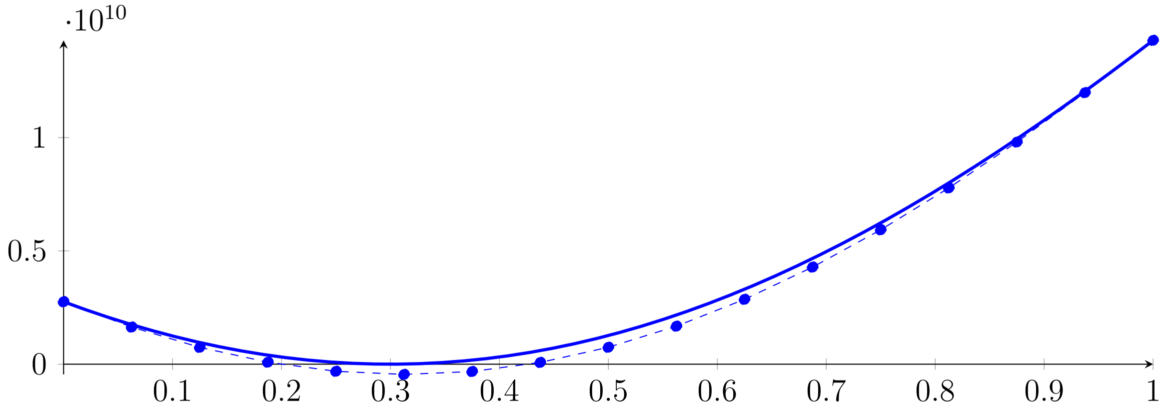
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



78.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

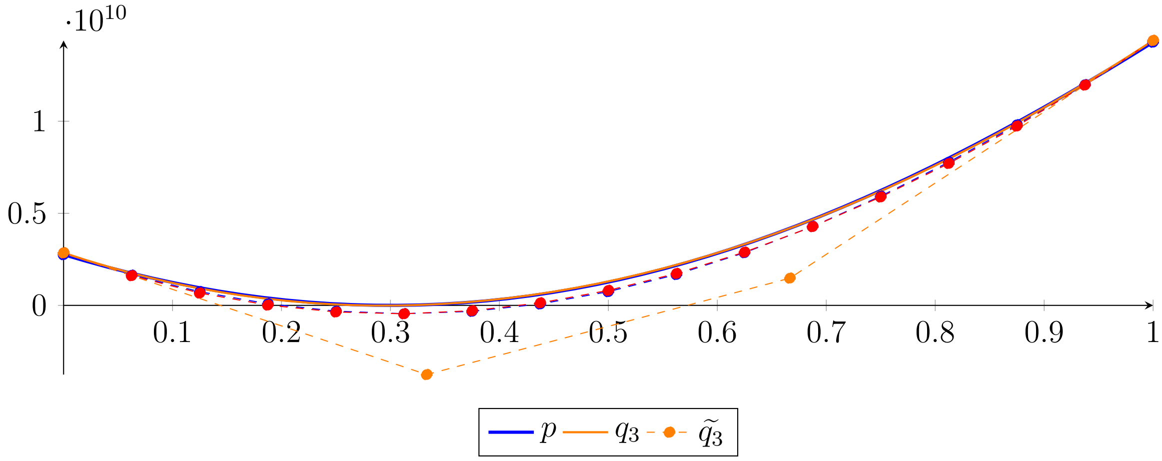
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X)\\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X)\\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X)\\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X)\\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X)\\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$q_3 = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3} + 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3}$$

$$\widetilde{q}_3 = 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756\cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12} - 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948\cdot 10^{-285}X^9 + 6.984121997801746\cdot 10^{-286}X^8 - 2.400829205648148\cdot 10^{-286}X^7 + 9.243609934541251\cdot 10^{-287}X^6 - 2.7161427533585\cdot 10^{-287}X^5 + 4.31464378661806\cdot 10^{-288}X^4 - 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16} + 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16} - 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16} + 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16} + 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16} + 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

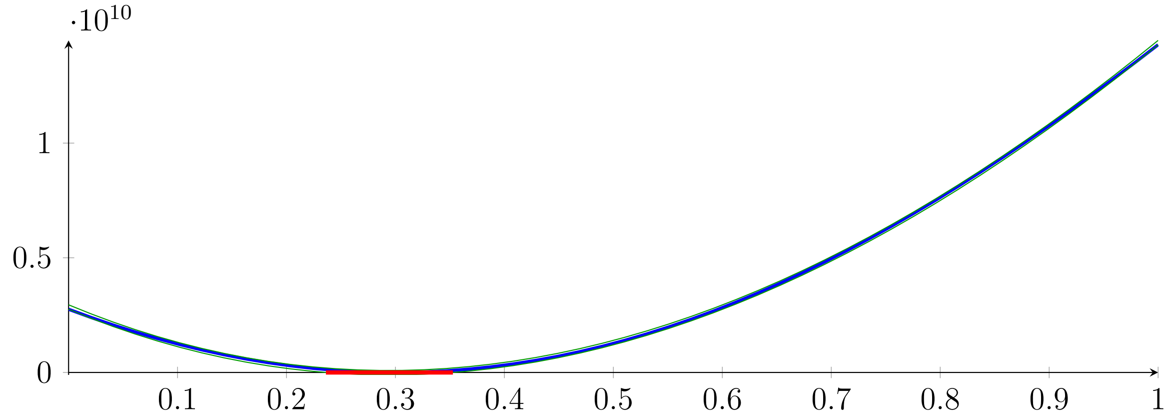
Bounding polynomials M and m:

$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$
$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



$$[0.2362027155340351, 0.3527550629571673]$$

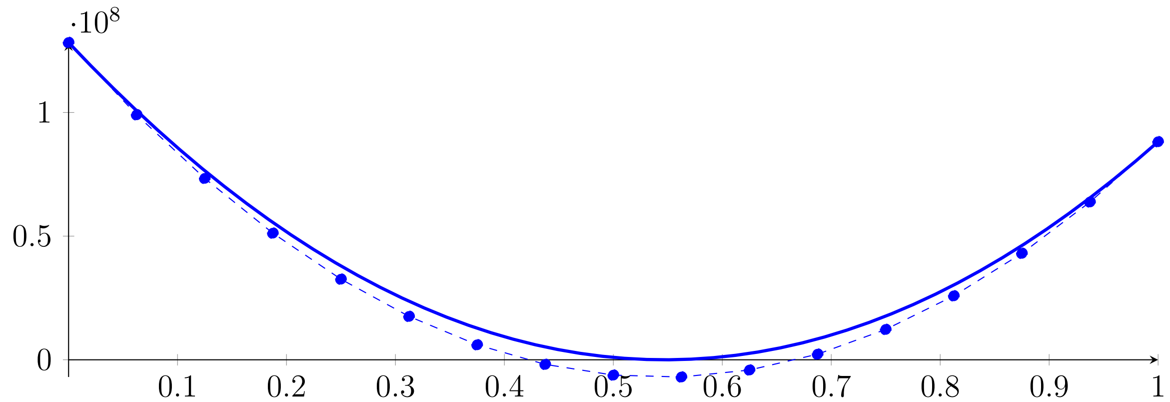
Longest intersection interval: 0.1165523474231321

$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

78.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

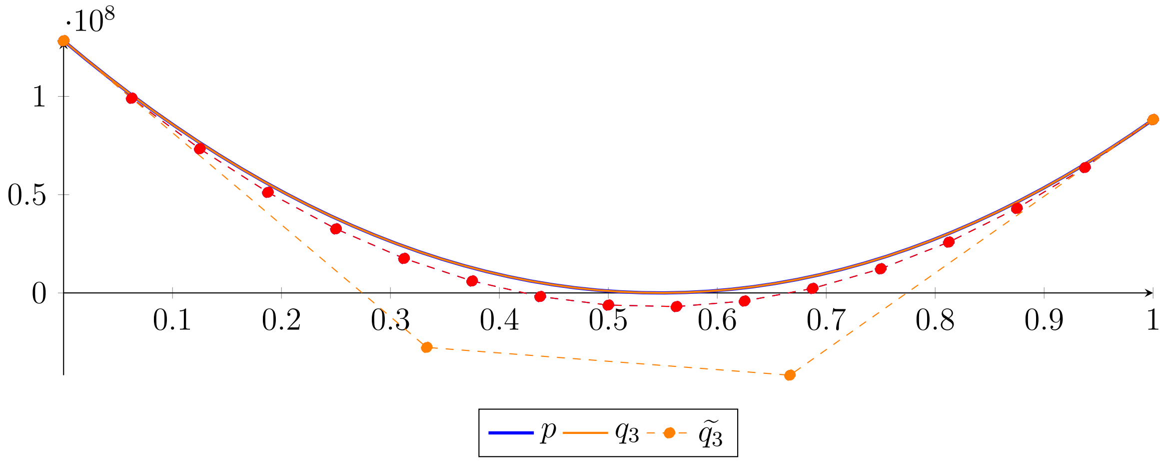
$$\begin{aligned}
p = &-1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14} \\
&- 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321 \\
&\cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8} \\
&+ 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4} \\
&+ 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316 \\
= &\ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X) \\
&+ 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X) \\
&+ 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X) \\
&- 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X) \\
&+ 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X) \\
&+ 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = &\ 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128303546.2426166 \\
= &\ 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3} \\
&- 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= 1.029539679280458\cdot 10^{-288}X^{16} - 8.072523277872609\cdot 10^{-288}X^{15} + 2.888405037194055 \\ &\quad \cdot 10^{-287}X^{14} - 6.260623433882179\cdot 10^{-287}X^{13} + 9.183807049175627\cdot 10^{-287}X^{12} \\ &\quad - 9.610199535206024\cdot 10^{-287}X^{11} + 7.321003139036714\cdot 10^{-287}X^{10} - 4.031610440228076 \\ &\quad \cdot 10^{-287}X^{9} + 1.537754605851679\cdot 10^{-287}X^{8} - 3.606689105481158\cdot 10^{-288}X^{7} \\ &\quad + 3.223581314526531\cdot 10^{-289}X^{6} + 5.410058694892297\cdot 10^{-290}X^{5} - 1.24173721337985\cdot 10^{-290}X^{4} \\ &\quad + 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166 \\ &= 128303546.2426166B_{0,16} + 99049684.91666918B_{1,16} + 73342858.56410606B_{2,16} \\ &\quad + 51187170.59978822B_{3,16} + 32586724.4385766B_{4,16} + 17545623.49533216B_{5,16} \\ &\quad + 6067971.184915845B_{6,16} - 1842129.077811386B_{7,16} - 6180573.877988583B_{8,16} \\ &\quad - 6943259.800754793B_{9,16} - 4126083.431249065B_{10,16} + 2275058.645389556B_{11,16} \\ &\quad + 12264269.84402202B_{12,16} + 25845653.57950928B_{13,16} + 43023313.26671229B_{14,16} \\ &\quad + 63801352.320492B_{15,16} + 88183874.15570936B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

Bounding polynomials M and m:

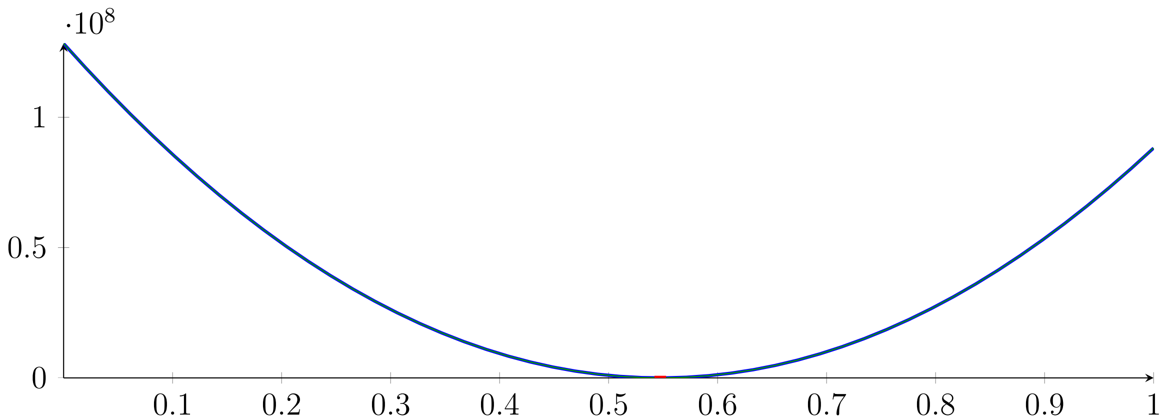
$$M = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128322140.7478178$$

$$m = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$$[0.5420611331202266, 0.552740643902378]$$

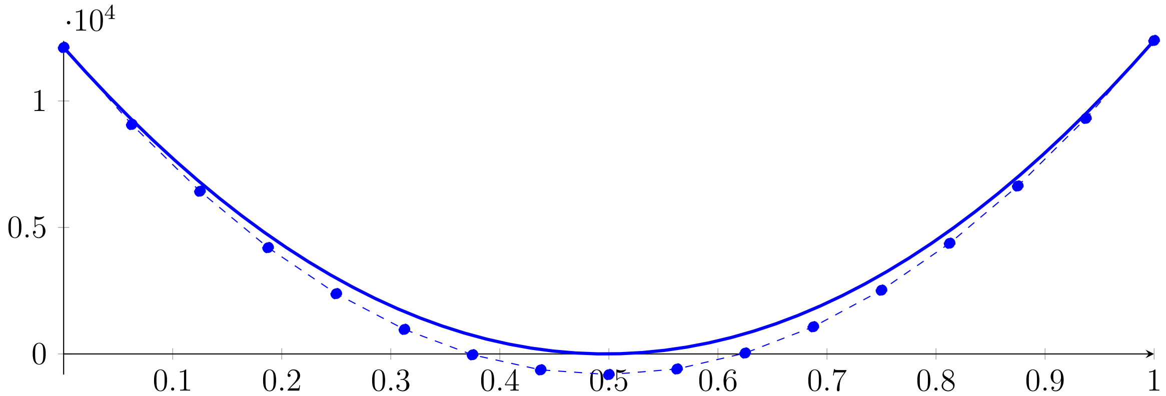
Longest intersection interval: 0.01067951078215144

$\implies$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

78.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.300625935097030$

Normalized monomial und Bézier representations and the Bézier polygon:

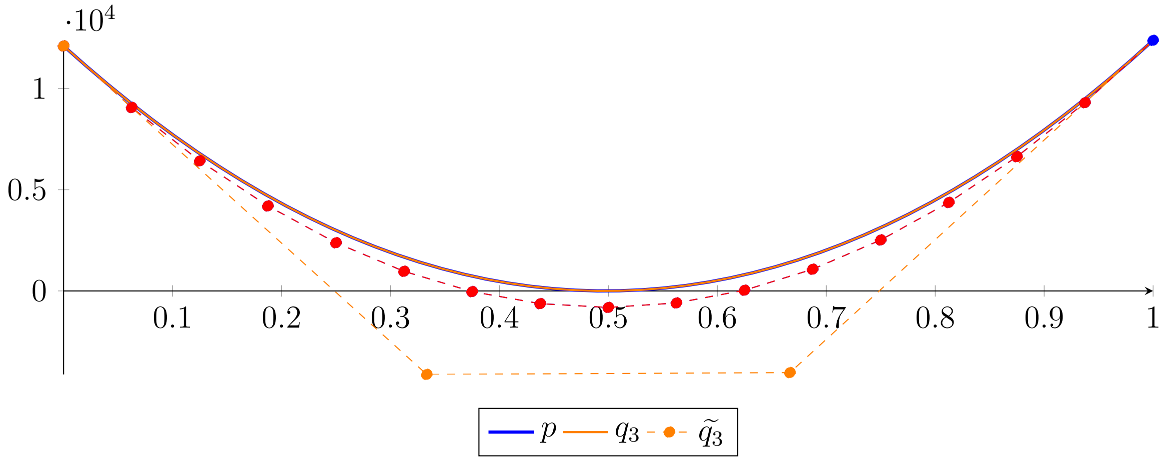
$$\begin{aligned}
p &= -3.320153282917824\cdot 10^{-47}X^{16} + 2.160317373028451\cdot 10^{-44}X^{15} + 4.806705749215272\\
&\quad \cdot 10^{-39}X^{14} - 3.045075915978909\cdot 10^{-36}X^{13} - 2.96255923979525\cdot 10^{-31}X^{12}\\
&\quad + 1.726566936989629\cdot 10^{-28}X^{11} + 1.009356568315285\cdot 10^{-23}X^{10} - 5.095799973170639\\
&\quad \cdot 10^{-21}X^{9} - 2.055755460332604\cdot 10^{-16}X^{8} + 8.312655296209706\cdot 10^{-14}X^{7}\\
&\quad + 2.505544083540367\cdot 10^{-09}X^{6} - 7.139662163470987\cdot 10^{-07}X^{5} - 0.01693489925927299X^{4}\\
&\quad + 2.534105689487521X^{3} + 49001.97220419171X^{2} - 48729.12904702137X + 12114.14082112651\\
&= 12114.14082112651B_{0,16}(X) + 9068.570255687672B_{1,16}(X) + 6431.349458617101B_{2,16}(X)\\
&\quad + 4202.482955103525B_{3,16}(X) + 2381.975261030786B_{4,16}(X) + 969.830882977672B_{5,16}(X)\\
&\quad - 33.94568178224417B_{6,16}(X) - 629.34994528077B_{7,16}(X) - 816.3774288552541B_{8,16}(X)\\
&\quad - 595.0236631487491B_{9,16}(X) + 34.71581188982676B_{10,16}(X) + 1072.845447005528B_{11,16}(X)\\
&\quad + 2519.36968363722B_{12,16}(X) + 4374.29295391742B_{13,16}(X) + 6637.619680672133B_{14,16}(X)\\
&\quad + 9309.354277420696B_{15,16}(X) + 12389.50114837561B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619\\
&= 12114.14106307619B_{0,3} - 4128.903565586097B_{1,3}\\
&\quad - 4037.950201139886B_{2,3} + 12389.5013903309B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= 1.099600357934752\cdot 10^{-292}X^{16} - 8.377614657428995\cdot 10^{-292}X^{15} + 2.905985967667976\\
&\quad \cdot 10^{-291}X^{14} - 6.091421693711012\cdot 10^{-291}X^{13} + 8.615207440059339\cdot 10^{-291}X^{12}\\
&\quad - 8.654253929665817\cdot 10^{-291}X^{11} + 6.288120919613368\cdot 10^{-291}X^{10} - 3.270943230058189\\
&\quad \cdot 10^{-291}X^{9} + 1.159364872956392\cdot 10^{-291}X^{8} - 2.426897197847659\cdot 10^{-292}X^{7}\\
&\quad + 1.434105777365326\cdot 10^{-293}X^{6} + 4.843078869312239\cdot 10^{-294}X^{5} - 7.632080003106212\cdot 10^{-295}X^{4}\\
&\quad + 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619\\
&= 12114.14106307619B_{0,16} + 9068.570195202008B_{1,16} + 6431.349277155543B_{2,16}\\
&\quad + 4202.482773640211B_{3,16} + 2381.975149359434B_{4,16} + 969.8308690166351B_{5,16}\\
&\quad - 33.94560268476556B_{6,16} - 629.349801041346B_{7,16} - 816.3772613496847B_{8,16}\\
&\quad - 595.0235189063601B_{9,16} + 34.71589099204953B_{10,16}\\
&\quad + 1072.845433048966B_{11,16} + 2519.36957196781B_{12,16} + 4374.292772452004B_{13,16}\\
&\quad + 6637.619499204969B_{14,16} + 9309.354216930127B_{15,16} + 12389.5013903309B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00024195528529 18186$.
Bounding polynomials M and m:

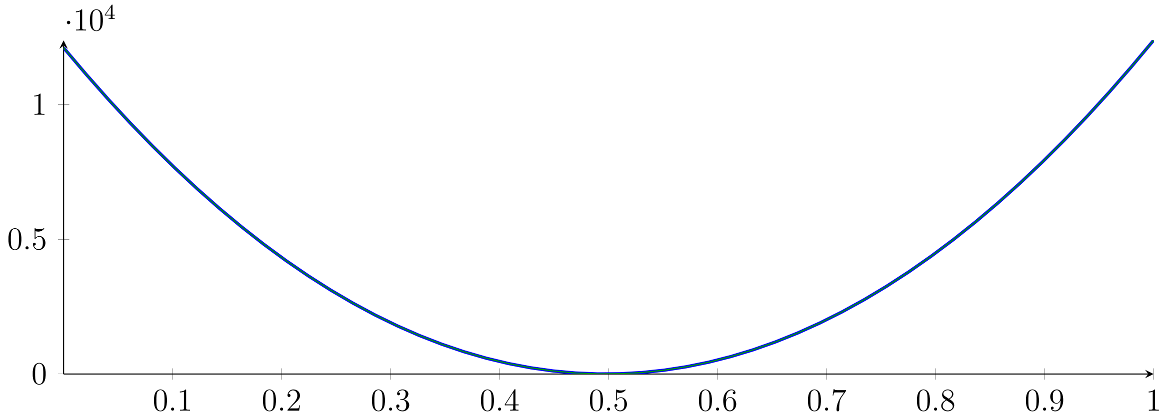
$$M = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.14130503147$$

$$m = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.1408211209$$

Root of M and m:

$$N(M) = \{-19599.95818041899\} \quad N(m) = \{-19599.95818041899, 0.4971412064138645, 0.4972526073815481\}$$

Intersection intervals:



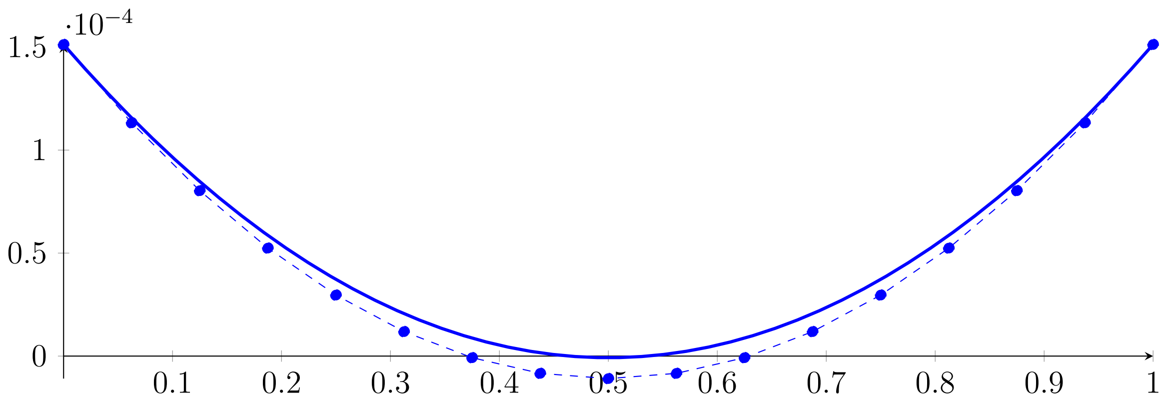
$$[0.4971412064138645, 0.4972526073815481]$$

Longest intersection interval: 0.0001114009676835966
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000156681198, 0.3000001543313607]$,

78.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3000000156681198, 0.30000015433136$

Normalized monomial und Bézier representations and the Bézier polygon:

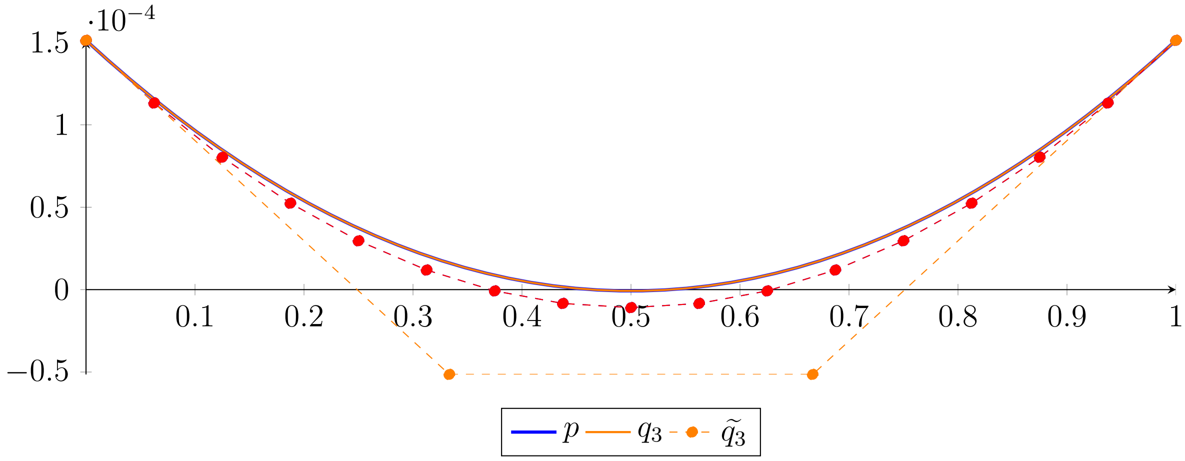
$$p = -1.868020113181866\cdot 10^{-110}X^{16} + 1.077730427531422\cdot 10^{-103}X^{15} + 2.179252184416492 \cdot 10^{-94}X^{14} - 1.225622742394903\cdot 10^{-87}X^{13} - 1.082338964896179\cdot 10^{-78}X^{12} + 5.603938665480187\cdot 10^{-72}X^{11} + 2.971492663890366\cdot 10^{-63}X^{10} - 1.333260671370819 \cdot 10^{-56}X^{9} - 4.876756665473299\cdot 10^{-48}X^{8} + 1.752546617551927\cdot 10^{-41}X^{7} + 4.789450848811937 \cdot 10^{-33}X^{6} - 1.212138221580256\cdot 10^{-26}X^{5} - 2.608457365264229\cdot 10^{-18}X^{4} + 3.456855344293596 \cdot 10^{-12}X^{3} + 0.0006081696715066215X^{2} - 0.0006081719526708613X + 0.0001512528027912101$$
$$= 0.0001512528027912101B_{0,16}(X) + 0.0001132420557492813B_{1,16}(X) + 8.029938930324099 \cdot 10^{-05}B_{2,16}(X) + 5.242480345926214\cdot 10^{-05}B_{3,16}(X) + 2.961829822351771\cdot 10^{-05}B_{4,16}(X) + 1.187987360218066\cdot 10^{-05}B_{5,16}(X) - 7.904703985760706\cdot 10^{-07}B_{6,16}(X) - 8.392733772579519 \cdot 10^{-06}B_{7,16}(X) - 1.092691651365674\cdot 10^{-05}B_{8,16}(X) - 8.393018615634788\cdot 10^{-06}B_{9,16}(X) - 7.910400723407154\cdot 10^{-07}B_{10,16}(X) + 1.187901912239842\cdot 10^{-05}B_{11,16}(X) + 2.961715897475557\cdot 10^{-05}B_{12,16}(X) + 5.242337949090366\cdot 10^{-05}B_{13,16}(X) + 8.029768067701564 \cdot 10^{-05}B_{14,16}(X) + 0.0001132400625392645B_{15,16}(X) + 0.0001512505250838231B_{16,16}(X)$$



Degree reduction and raising:

$$q_3 = 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,3} - 5.147118143241051\cdot 10^{-05}B_{1,3} - 5.147194182048959\cdot 10^{-05}B_{2,3} + 0.0001512505250838231B_{3,3}$$

$$\widetilde{q}_3 = 1.301341971574797\cdot 10^{-300}X^{16} - 9.903701542140468\cdot 10^{-300}X^{15} + 3.435393711081806 \cdot 10^{-299}X^{14} - 7.213305599572634\cdot 10^{-299}X^{13} + 1.023930542727086\cdot 10^{-298}X^{12} - 1.03405078726607\cdot 10^{-298}X^{11} + 7.556175284428161\cdot 10^{-299}X^{10} - 3.94396828718345\cdot 10^{-299}X^{9} + 1.391742955019986\cdot 10^{-299}X^{8} - 2.822114462221502\cdot 10^{-300}X^{7} + 1.110580839880333 \cdot 10^{-301}X^{6} + 7.65504767714651\cdot 10^{-302}X^{5} - 1.17007493272161\cdot 10^{-302}X^{4} + 3.456850127378832 \cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,16} + 0.0001132420557492813B_{1,16} + 8.029938930324096\cdot 10^{-05}B_{2,16} + 5.242480345926211\cdot 10^{-05}B_{3,16} + 2.961829822351769\cdot 10^{-05}B_{4,16} + 1.187987360218065\cdot 10^{-05}B_{5,16} - 7.904703985760584\cdot 10^{-07}B_{6,16} - 8.392733772579497\cdot 10^{-06}B_{7,16} - 1.092691651365671\cdot 10^{-05}B_{8,16} - 8.393018615634766\cdot 10^{-06}B_{9,16} - 7.910400723407032\cdot 10^{-07}B_{10,16} + 1.187901912239842 \cdot 10^{-05}B_{11,16} + 2.961715897475555\cdot 10^{-05}B_{12,16} + 5.242337949090363\cdot 10^{-05}B_{13,16} + 8.029768067701561\cdot 10^{-05}B_{14,16} + 0.0001132400625392644B_{15,16} + 0.0001512505250838231B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 3.726367712763873 \cdot 10^{-20}$.
Bounding polynomials M and m:

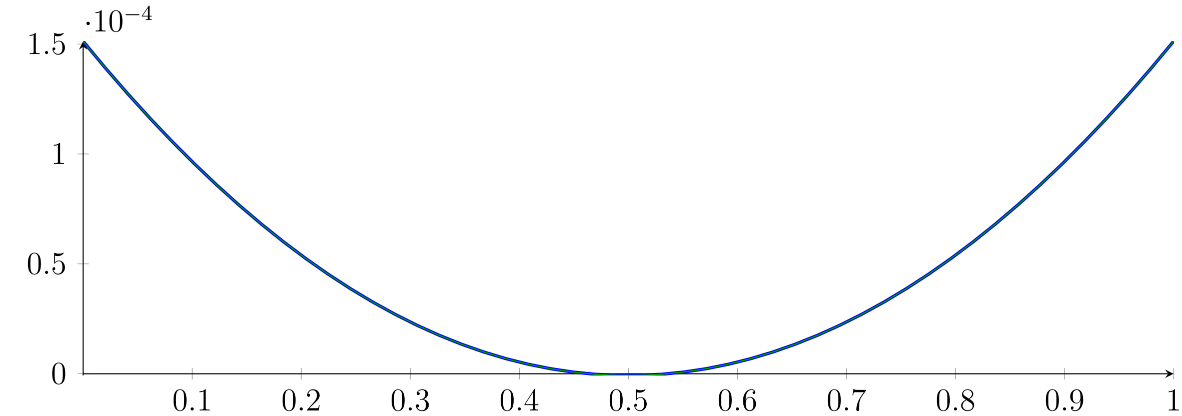
$$\begin{aligned} M = &\, 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ &- 0.0006081719526708621 X + 0.0001512528027912102 \end{aligned}$$

$$\begin{aligned} m = &\, 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ &- 0.0006081719526708621 X + 0.0001512528027912101 \end{aligned}$$

Root of M and m:

$N(M) = \{-175931744.9566824, 0.4639432736155652, 0.5360604563964476\}$ $\quad N(m) = \{-175931744.9566824,$

Intersection intervals:



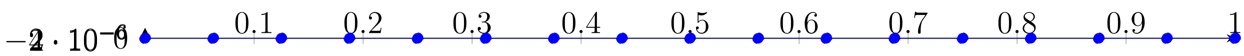
$$[0.4639432736155635, 0.4639432736155652],\ [0.5360604563964476, 0.5360604563964493]$$

Longest intersection interval: $1.699230475659398 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999977, 0.3000000799999977]$, interval 2: $[0.30000009, 0.3000$

78.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000799999977, 0.300000079999$

Normalized monomial und Bézier representations and the Bézier polygon:

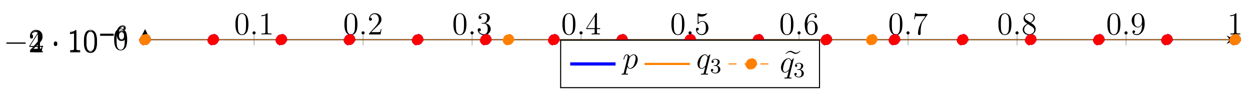
$$
\begin{aligned}
p = {} & 5.528728962819218 \cdot 10^{-317} X^{16} + 6.592381943367454 \cdot 10^{-316} X^{15} + 3.646191975443216 \\
& \cdot 10^{-301} X^{14} - 1.206802108791304 \cdot 10^{-279} X^{13} - 6.271779178309597 \cdot 10^{-256} X^{12} \\
& + 1.911032531933869 \cdot 10^{-234} X^{11} + 5.963449582578663 \cdot 10^{-211} X^{10} - 1.574654273552139 \\
& \cdot 10^{-189} X^{9} - 3.389607402559108 \cdot 10^{-166} X^{8} + 7.168613242382433 \cdot 10^{-145} X^{7} + 1.152920753962593 \\
& \cdot 10^{-121} X^{6} - 1.717169281481298 \cdot 10^{-100} X^{5} - 2.174667687267227 \cdot 10^{-77} X^{4} + 1.696045363009562 \\
& \cdot 10^{-56} X^{3} + 1.7560195200423 \cdot 10^{-33} X^{2} - 7.452738853523238 \cdot 10^{-20} X + 7.27439202738246 \cdot 10^{-13} \\
= {} & 7.27439202738246 \cdot 10^{-13} B_{0,16}(X) + 7.274391980802842 \cdot 10^{-13} B_{1,16}(X) + 7.274391934223224 \\
& \cdot 10^{-13} B_{2,16}(X) + 7.274391887643606 \cdot 10^{-13} B_{3,16}(X) + 7.274391841063988 \cdot 10^{-13} B_{4,16}(X) \\
& + 7.27439179448437 \cdot 10^{-13} B_{5,16}(X) + 7.274391747904753 \cdot 10^{-13} B_{6,16}(X) + 7.274391701325135 \\
& \cdot 10^{-13} B_{7,16}(X) + 7.274391654745517 \cdot 10^{-13} B_{8,16}(X) + 7.274391608165899 \cdot 10^{-13} B_{9,16}(X) \\
& + 7.274391561586281 \cdot 10^{-13} B_{10,16}(X) + 7.274391515006663 \cdot 10^{-13} B_{11,16}(X) + 7.274391468427046 \\
& \cdot 10^{-13} B_{12,16}(X) + 7.274391421847428 \cdot 10^{-13} B_{13,16}(X) + 7.27439137526781 \cdot 10^{-13} B_{14,16}(X) \\
& + 7.274391328688192 \cdot 10^{-13} B_{15,16}(X) + 7.274391282108574 \cdot 10^{-13} B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = {} & 1.696045363009562 \cdot 10^{-56} X^{3} + 1.7560195200423 \cdot 10^{-33} X^{2} \\
& - 7.452738853523238 \cdot 10^{-20} X + 7.27439202738246 \cdot 10^{-13} \\
= {} & 7.27439202738246 \cdot 10^{-13} B_{0,3} + 7.274391778957831 \cdot 10^{-13} B_{1,3} \\
& + 7.274391530533203 \cdot 10^{-13} B_{2,3} + 7.274391282108574 \cdot 10^{-13} B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q_3} = {} & 5.719971833076747 \cdot 10^{-308} X^{16} - 4.598890442375195 \cdot 10^{-307} X^{15} + 1.652591351302372 \\
& \cdot 10^{-306} X^{14} - 3.497346441216264 \cdot 10^{-306} X^{13} + 4.849354007960553 \cdot 10^{-306} X^{12} \\
& - 4.665113144336927 \cdot 10^{-306} X^{11} + 3.252638633884502 \cdot 10^{-306} X^{10} - 1.725306268115569 \\
& \cdot 10^{-306} X^{9} + 7.334647684203053 \cdot 10^{-307} X^{8} - 2.545384960198585 \cdot 10^{-307} X^{7} + 6.841253439376372 \\
& \cdot 10^{-308} X^{6} - 1.288279056767672 \cdot 10^{-308} X^{5} + 1.476493526426595 \cdot 10^{-309} X^{4} + 1.696045363009562 \\
& \cdot 10^{-56} X^{3} + 1.7560195200423 \cdot 10^{-33} X^{2} - 7.452738853523238 \cdot 10^{-20} X + 7.27439202738246 \cdot 10^{-13} \\
= {} & 7.27439202738246 \cdot 10^{-13} B_{0,16} + 7.274391980802842 \cdot 10^{-13} B_{1,16} + 7.274391934223224 \\
& \cdot 10^{-13} B_{2,16} + 7.274391887643606 \cdot 10^{-13} B_{3,16} + 7.274391841063988 \cdot 10^{-13} B_{4,16} \\
& + 7.27439179448437 \cdot 10^{-13} B_{5,16} + 7.274391747904753 \cdot 10^{-13} B_{6,16} + 7.274391701325135 \\
& \cdot 10^{-13} B_{7,16} + 7.274391654745517 \cdot 10^{-13} B_{8,16} + 7.274391608165899 \cdot 10^{-13} B_{9,16} \\
& + 7.274391561586281 \cdot 10^{-13} B_{10,16} + 7.274391515006663 \cdot 10^{-13} B_{11,16} + 7.274391468427046 \\
& \cdot 10^{-13} B_{12,16} + 7.274391421847428 \cdot 10^{-13} B_{13,16} + 7.27439137526781 \cdot 10^{-13} B_{14,16} \\
& + 7.274391328688192 \cdot 10^{-13} B_{15,16} + 7.274391282108574 \cdot 10^{-13} B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.106668124667468 \cdot 10^{-79}$.

Bounding polynomials M and m:

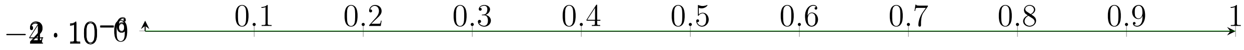
$$
\begin{aligned}
M = {} & 1.696045363009562 \cdot 10^{-56} X^{3} + 1.7560195200423 \cdot 10^{-33} X^{2} \\
& - 7.452738853523238 \cdot 10^{-20} X + 7.27439202738246 \cdot 10^{-13}
\end{aligned}
$$

$$m = 1.696045363009562{\cdot}10^{-56}X^3 + 1.7560195200423{\cdot}10^{-33}X^2 - 7.452738853523238{\cdot}10^{-20}X + 7.27439202738246{\cdot}10^{-13}$$

Root of M and m:

$N(M) = \{-1.035361175508972{\cdot}10^{23}, 1.384450244011441, 42441083663415.11\}$ $\quad N(m) = \{-1.0353611755089$

Intersection intervals:

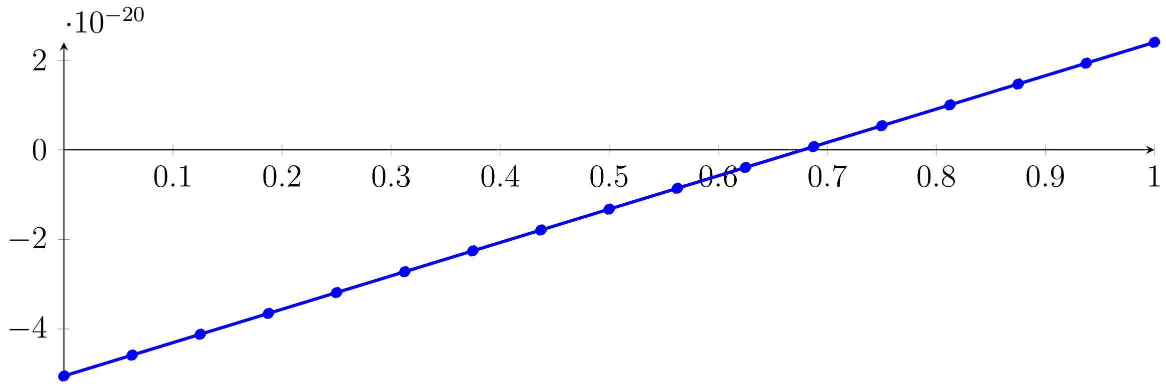


No intersection intervals with the x axis.

78.6 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

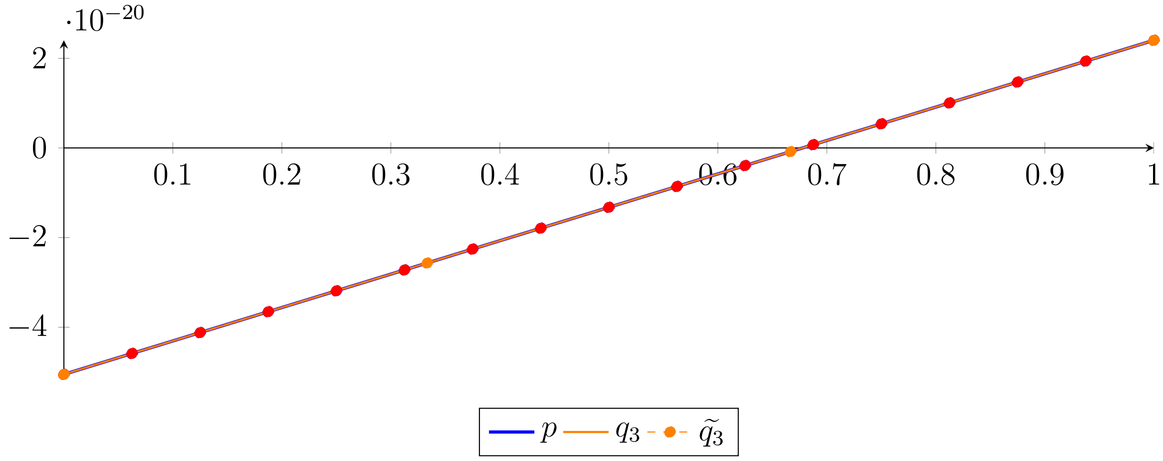
$$\begin{aligned} p = {} & 7.285538332229319{\cdot}10^{-325}X^{16} - 3.937085615297433{\cdot}10^{-324}X^{15} + 3.646191956469014 \\ & {\cdot}10^{-301}X^{14} - 1.206801885712699{\cdot}10^{-279}X^{13} - 6.271779154117127{\cdot}10^{-256}X^{12} \\ & + 1.911032203899561{\cdot}10^{-234}X^{11} + 5.963449567055239{\cdot}10^{-211}X^{10} - 1.574654014647597{\cdot}10^{-189}X^9 \\ & - 3.389607397458184{\cdot}10^{-166}X^8 + 7.168612070943855{\cdot}10^{-145}X^7 + 1.152920753256697{\cdot}10^{-121}X^6 \\ & - 1.717168984374588{\cdot}10^{-100}X^5 - 2.174667687345432{\cdot}10^{-77}X^4 + 1.696044991742842{\cdot}10^{-56}X^3 \\ & + 1.75601952076212{\cdot}10^{-33}X^2 + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20} \\ = {} & -5.051512577078811{\cdot}10^{-20}B_{0,16}(X) - 4.585716612983338{\cdot}10^{-20}B_{1,16}(X) - 4.119920648887863 \\ & {\cdot}10^{-20}B_{2,16}(X) - 3.654124684792386{\cdot}10^{-20}B_{3,16}(X) - 3.188328720696908{\cdot}10^{-20}B_{4,16}(X) \\ & - 2.722532756601429{\cdot}10^{-20}B_{5,16}(X) - 2.256736792505948{\cdot}10^{-20}B_{6,16}(X) - 1.790940828410466 \\ & {\cdot}10^{-20}B_{7,16}(X) - 1.325144864314982{\cdot}10^{-20}B_{8,16}(X) - 8.593489002194965{\cdot}10^{-21}B_{9,16}(X) \\ & - 3.935529361240098{\cdot}10^{-21}B_{10,16}(X) + 7.224302797147847{\cdot}10^{-22}B_{11,16}(X) \\ & + 5.380389920669682{\cdot}10^{-21}B_{12,16}(X) + 1.003834956162459{\cdot}10^{-20}B_{13,16}(X) + 1.469630920257952 \\ & {\cdot}10^{-20}B_{14,16}(X) + 1.935426884353446{\cdot}10^{-20}B_{15,16}(X) + 2.401222848448942{\cdot}10^{-20}B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_3 = {} & 1.696044991742842{\cdot}10^{-56}X^3 + 1.75601952076212{\cdot}10^{-33}X^2 \\ & + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20} \\ = {} & -5.051512577078811{\cdot}10^{-20}B_{0,3} - 2.567267435236286{\cdot}10^{-20}B_{1,3} \\ & - 8.302229339370134{\cdot}10^{-22}B_{2,3} + 2.401222848448942{\cdot}10^{-20}B_{3,3} \end{aligned}$$

$$
\begin{aligned}
\widetilde{q}_3 = {} & -1.026066912900859\cdot 10^{-315}X^{16} + 8.593538471070521\cdot 10^{-315}X^{15} - 3.223576101816875 \\
& \cdot 10^{-314}X^{14} + 7.163252241057264\cdot 10^{-314}X^{13} - 1.054054043306296\cdot 10^{-313}X^{12} \\
& + 1.091820578452099\cdot 10^{-313}X^{11} - 8.293103170105507\cdot 10^{-314}X^{10} + 4.748156916296184\cdot 10^{-314}X^{9} \\
& - 2.062106326583159\cdot 10^{-314}X^{8} + 6.57938063450856\cdot 10^{-315}X^{7} - 1.420328445432424\cdot 10^{-315}X^{6} \\
& + 1.834102728564758\cdot 10^{-316}X^{5} - 1.348970337656149\cdot 10^{-317}X^{4} + 1.696044991742842\cdot 10^{-56}X^{3} \\
& + 1.75601952076212\cdot 10^{-33}X^{2} + 7.452735425527577\cdot 10^{-20}X - 5.051512577078811\cdot 10^{-20} \\
= {} & -5.051512577078811\cdot 10^{-20}B_{0,16} - 4.585716612983338\cdot 10^{-20}B_{1,16} - 4.119920648887863 \\
& \cdot 10^{-20}B_{2,16} - 3.654124684792386\cdot 10^{-20}B_{3,16} - 3.188328720696908\cdot 10^{-20}B_{4,16} \\
& - 2.722532756601429\cdot 10^{-20}B_{5,16} - 2.256736792505948\cdot 10^{-20}B_{6,16} - 1.790940828410466 \\
& \cdot 10^{-20}B_{7,16} - 1.325144864314982\cdot 10^{-20}B_{8,16} - 8.593489002194965\cdot 10^{-21}B_{9,16} \\
& - 3.935529361240098\cdot 10^{-21}B_{10,16} + 7.224302797147847\cdot 10^{-22}B_{11,16} \\
& + 5.380389920669682\cdot 10^{-21}B_{12,16} + 1.003834956162459\cdot 10^{-20}B_{13,16} + 1.469630920257952 \\
& \cdot 10^{-20}B_{14,16} + 1.935426884353446\cdot 10^{-20}B_{15,16} + 2.401222848448942\cdot 10^{-20}B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.106668124779188\cdot 10^{-79}$.

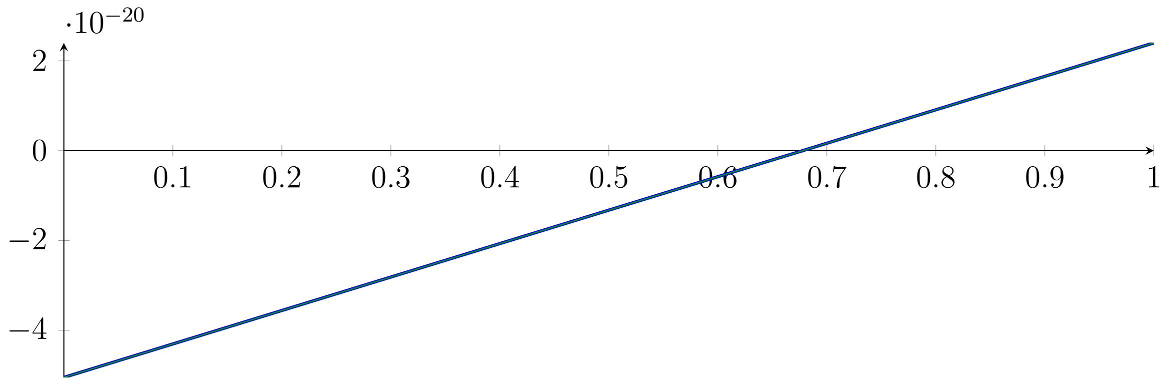
Bounding polynomials M and m:

$$
\begin{aligned}
M &= 1.696044991742842\cdot 10^{-56}X^{3} + 1.75601952076212\cdot 10^{-33}X^{2} \\
&\quad + 7.452735425527577\cdot 10^{-20}X - 5.051512577078811\cdot 10^{-20} \\
m &= 1.696044991742842\cdot 10^{-56}X^{3} + 1.75601952076212\cdot 10^{-33}X^{2} \\
&\quad + 7.452735425527577\cdot 10^{-20}X - 5.051512577078811\cdot 10^{-20}
\end{aligned}
$$

Root of M and m:

$N(M) = \{-1.03536140172663\cdot 10^{23}, -42441083680812.44, 0.6778065084366168\}$ $\quad N(m) = \{-1.035361401726$

Intersection intervals:



$$[0.6778065084366168, 0.6778065084366168]$$

Longest intersection interval: $8.336987555302001\cdot 10^{-60}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

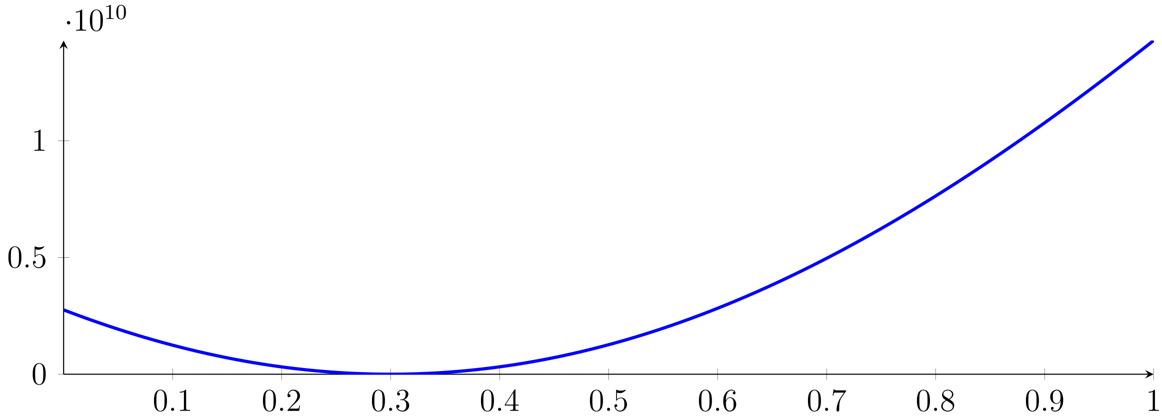
78.7 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 6!

78.8 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000009, 0.30000009]$$

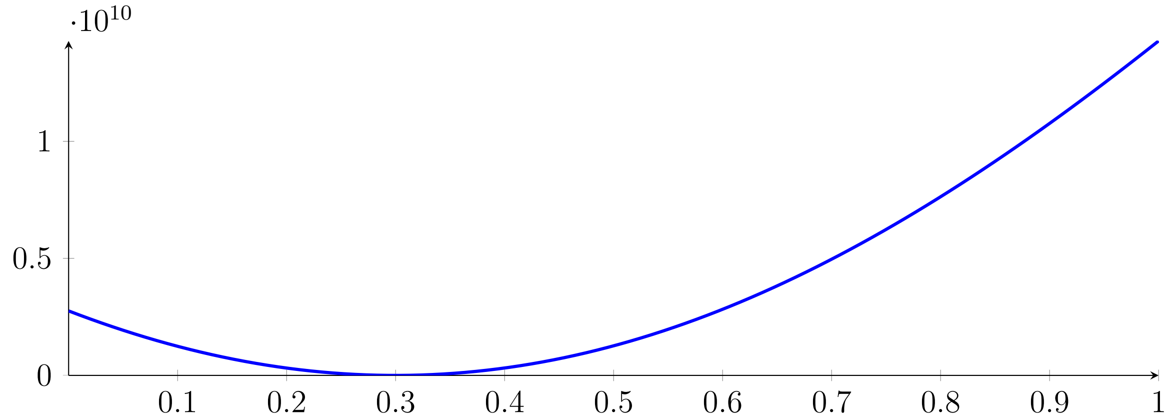
with precision $\varepsilon = 1 \cdot 10^{-32}$.

79 Running `BezClip` on h_{16} with epsilon 64

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

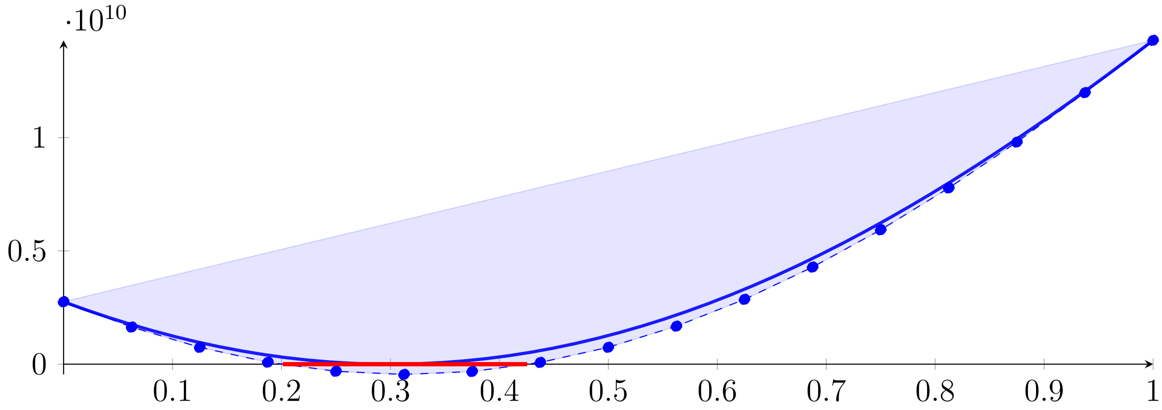
$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



79.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X)\\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X)\\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X)\\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X)\\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X)\\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

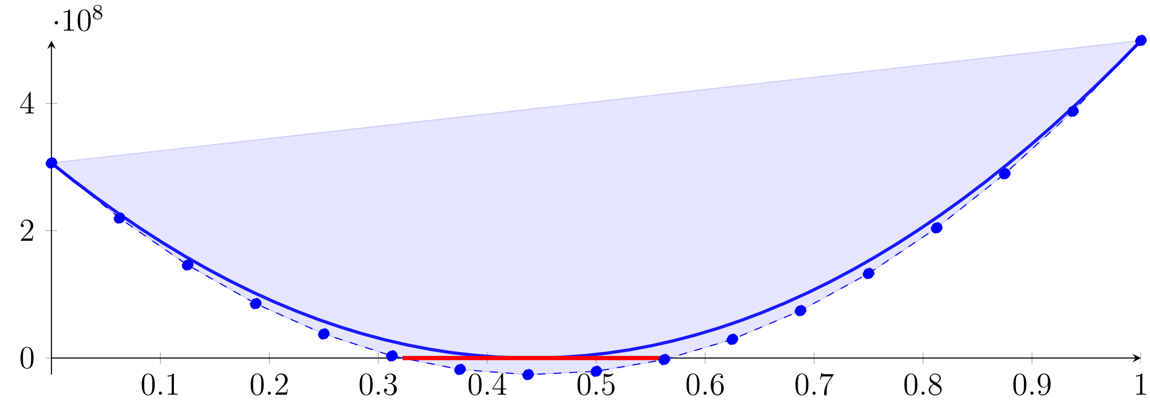
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

79.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -4.078702226246453 \cdot 10^{-11} X^{16} + 4.329167405007062 \cdot 10^{-10} X^{15} + 1.800827856148464 \cdot 10^{-07} X^{14} \\
& - 1.7501300195896 \cdot 10^{-06} X^{13} - 0.0003388881074603682 X^{12} + 0.002918727490860103 X^{11} \\
& + 0.353260574938002 X^{10} - 2.587751703352393 X^{9} - 220.7388291837263 X^{8} \\
& + 1298.537478880933 X^{7} + 82809.42227744751 X^{6} - 357003.6974521088 X^{5} - 17288824.7550166 X^{4} \\
& + 45617774.75376932 X^{3} + 1550181799.359372 X^{2} - 1385902556.54429 X + 306449533.9978484 \\
= & \, 306449533.9978484 B_{0,16}(X) + 219830624.2138303 B_{1,16}(X) + 146129896.0911402 B_{2,16}(X) \\
& + 85428809.9418386 B_{3,16}(X) + 37799326.72372467 B_{4,16}(X) + 3303826.308721026 B_{5,16}(X) \\
& - 18004963.90790522 B_{6,16}(X) - 26084029.94397575 B_{7,16}(X) - 20900121.61613177 B_{8,16}(X) \\
& - 2429792.29588269 B_{9,16}(X) + 29340572.02544693 B_{10,16}(X) + 74414734.40659503 B_{11,16}(X) \\
& + 132786599.3869978 B_{12,16}(X) + 204440216.186282 B_{13,16}(X) + 289349792.6628749 B_{14,16}(X) \\
& + 387479720.0042024 B_{15,16}(X) + 498784608.1032447 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

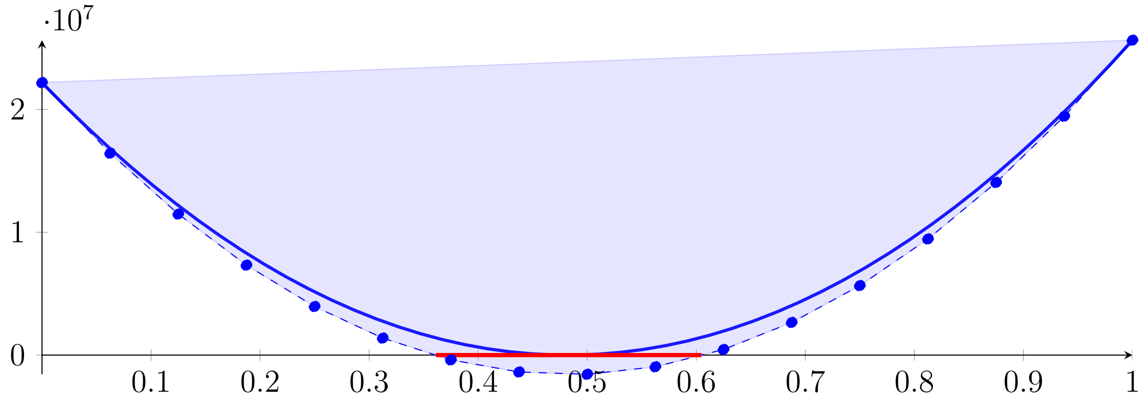
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: $[0.2735013999129692, 0.328453288067671]$,

79.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-6.91388116995714\cdot10^{-21}X^{16} + 1.539971991909181\cdot10^{-19}X^{15} + 5.126557214158853\\&\cdot10^{-16}X^{14} - 1.075268795442723\cdot10^{-14}X^{13} - 1.618466564965893\cdot10^{-11}X^{12}\\&+ 3.060191240732859\cdot10^{-10}X^{11} + 2.825398471644138\cdot10^{-07}X^{10} - 4.580412037193902\\&\cdot10^{-06}X^{9} - 0.002949971804071592X^{8} + 0.0383187988748973X^{7} + 18.44286791446417X^{6}\\&- 172.012783850635X^{5} - 63987.92092437637X^{4} + 338934.074545568X^{3}\\&+ 95113195.48898588X^{2} - 91937923.73553449X + 22182509.73328641\\= &\ 22182509.73328641B_{0,16}(X) + 16436389.49981551B_{1,16}(X) + 11482879.22875282B_{2,16}(X)\\&+ 7322584.159517174B_{3,16}(X) + 3956074.373329099B_{4,16}(X) + 1383884.753830588B_{5,16}(X)\\&- 393485.0499920519B_{6,16}(X) - 1375570.658578954B_{7,16}(X) - 1561942.994257926B_{8,16}(X)\\&- 952208.311393262B_{9,16}(X) + 453991.7757805191B_{10,16}(X) + 2656980.267642443B_{11,16}(X)\\&+ 5657044.755988558B_{12,16}(X) + 9454437.403157084B_{13,16}(X) + 14049374.92347699B_{14,16}(X)\\&+ 19442038.56704145B_{15,16}(X) + 25632574.10580758B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

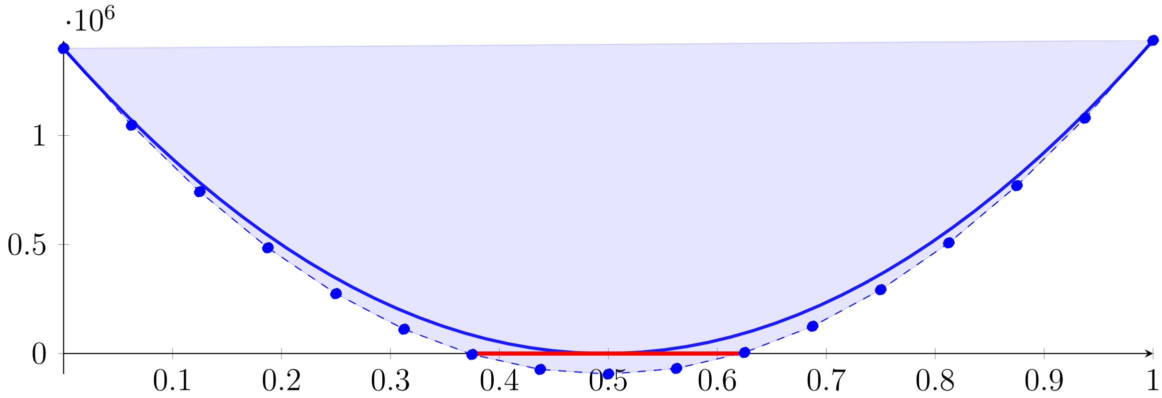
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: $[0.2933480088018335, 0.3067375037518675]$,

79.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & -1.067154088646128 \cdot 10^{-30} X^{16} + 7.224340048814341 \cdot 10^{-29} X^{15} + 1.334696477727887 \\
& \cdot 10^{-24} X^{14} - 8.705175946298654 \cdot 10^{-23} X^{13} - 7.106771480233297 \cdot 10^{-19} X^{12} \\
& + 4.237354105832321 \cdot 10^{-17} X^{11} + 2.091927791167947 \cdot 10^{-13} X^{10} - 1.077046671119229 \\
& \cdot 10^{-11} X^{9} - 3.681404336048099 \cdot 10^{-08} X^{8} + 1.51820465067154 \cdot 10^{-06} X^{7} \\
& + 0.003877397410843758 X^{6} - 0.1133230374256462 X^{5} - 226.5078434192974 X^{4} \\
& + 3562.753220436385 X^{3} + 5665644.405911702 X^{2} - 5632059.446508477 X + 1399245.031427946 \\
= {} & 1399245.031427946 B_{0,16}(X) + 1047241.316021167 B_{1,16}(X) + 742451.3039969843 B_{2,16}(X) \\
& + 484881.3574147218 B_{3,16}(X) + 274537.7138788421 B_{4,16}(X) + 111426.4865130056 B_{5,16}(X) \\
& - 4446.336065390124 B_{6,16}(X) - 73074.88977018565 B_{7,16}(X) - 94453.43507095045 B_{8,16}(X) \\
& - 68576.35701698946 B_{9,16}(X) + 4561.834739135334 B_{10,16}(X) \\
& + 124966.505918568 B_{11,16}(X) + 292642.897593607 B_{12,16}(X) + 507596.1261656377 B_{13,16}(X) \\
& + 769831.1833435517 B_{14,16}(X) + 1079352.93612265 B_{15,16}(X) + 1436166.12676403 B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

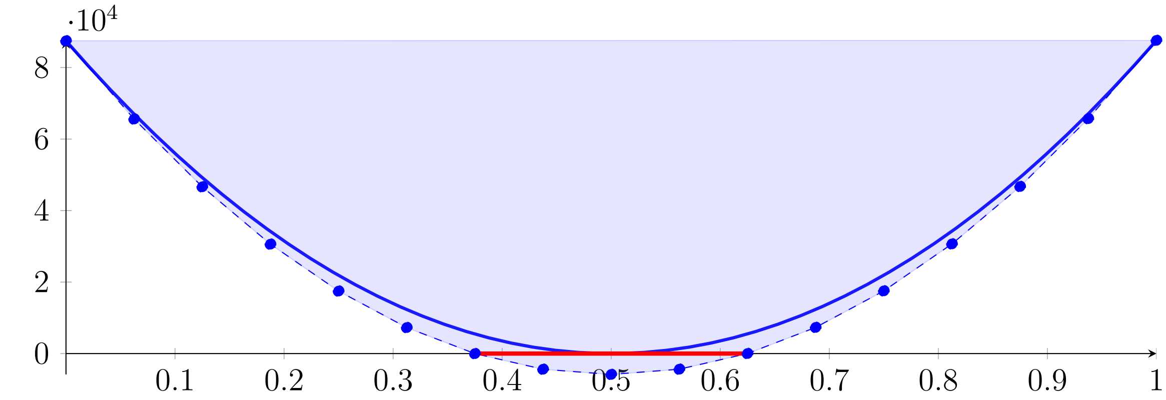
Longest intersection interval: 0.2484999837871829

$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

79.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & -2.256570822524655 \cdot 10^{-40} X^{16} + 5.606069245850654 \cdot 10^{-38} X^{15} + 4.571694482208246 \\
& \cdot 10^{-33} X^{14} - 1.103601619609778 \cdot 10^{-30} X^{13} - 3.943084008767946 \cdot 10^{-26} X^{12} \\
& + 8.746232232432483 \cdot 10^{-24} X^{11} + 1.879997320990413 \cdot 10^{-19} X^{10} - 3.610214442217249 \\
& \cdot 10^{-17} X^{9} - 5.35839719625663 \cdot 10^{-13} X^{8} + 8.241677187819251 \cdot 10^{-11} X^{7} + 9.139570434264648 \\
& \cdot 10^{-07} X^{6} - 9.91682351975725 \cdot 10^{-05} X^{5} - 0.864525557836322 X^{4} + 49.4891850901473 X^{3} \\
& + 350100.5152669434 X^{2} - 350028.3308017828 X + 87482.91293301892 \\
= {} & 87482.91293301892 B_{0,16}(X) + 65606.1422579075 B_{1,16}(X) + 46646.87587668727 B_{2,16}(X) \\
& + 30605.20216290304 B_{3,16}(X) + 17481.20901508557 B_{4,16}(X) + 7274.983856728878 B_{5,16}(X) \\
& - 13.38636373236149 B_{6,16}(X) - 4383.815172945278 B_{7,16}(X) - 5836.216572661172 B_{8,16}(X) \\
& - 4370.505039757766 B_{9,16}(X) + 13.40447373867094 B_{10,16}(X) + 7315.596540631298 B_{11,16}(X) \\
& + 17536.1552585308 B_{12,16}(X) + 30675.1642498336 B_{13,16}(X) + 46732.70666170018 B_{14,16}(X) \\
& + 65708.86516603353 B_{15,16}(X) + 87603.72195945767 B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3748852078437339, 0.6248088966923045\}$$

Intersection intervals with the x axis:

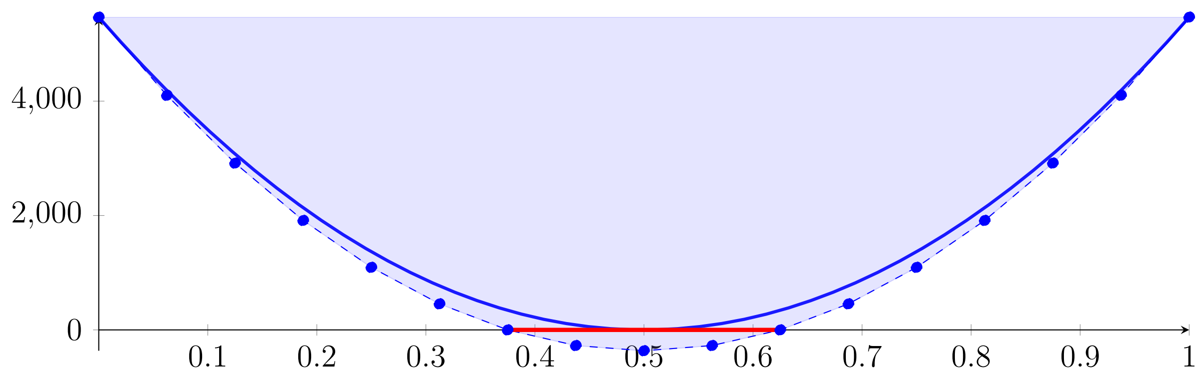
$$[0.3748852078437339, 0.6248088966923045]$$

Longest intersection interval: 0.2499236888485706
$\Longrightarrow$ Selective recursion: interval 1: $[0.2995843091213109, 0.3004158775315354]$,

79.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.2995843091213109, 0.30041587753$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -5.228387005637929 \cdot 10^{-50} X^{16} + 5.071723175262542 \cdot 10^{-47} X^{15} + 1.695940836337518 \\
& \cdot 10^{-41} X^{14} - 1.602367332110541 \cdot 10^{-38} X^{13} - 2.341982733873566 \cdot 10^{-33} X^{12} \\
& + 2.036120445711374 \cdot 10^{-30} X^{11} + 1.787779362171969 \cdot 10^{-25} X^{10} - 1.346594590666257 \\
& \cdot 10^{-22} X^{9} - 8.158156485257833 \cdot 10^{-18} X^{8} + 4.921695226295225 \cdot 10^{-15} X^{7} \\
& + 2.227778844339431 \cdot 10^{-10} X^{6} - 9.46914992513699 \cdot 10^{-08} X^{5} - 0.003373649242827386 X^{4} \\
& + 0.7523208927331248 X^{3} + 21871.35693828986 X^{2} - 21871.48147756637 X + 5467.807676357308 \\
= & \, 5467.807676357308 B_{0,16}(X) + 4100.84008400941 B_{1,16}(X) + 2916.133799480594 B_{2,16}(X) \\
& + 1913.690166201026 B_{3,16}(X) + 1093.510525747217 B_{4,16}(X) + 455.5962178420057 B_{5,16}(X) \\
& - 0.0514196454683865 B_{6,16}(X) - 273.4310506997833 B_{7,16}(X) - 364.541341159257 B_{8,16}(X) \\
& - 273.3809587159691 B_{9,16}(X) + 0.05142708421757425 B_{10,16}(X) \\
& + 455.7571448416359 B_{11,16}(X) + 1093.737521302792 B_{12,16}(X) + 1913.993881360346 B_{13,16}(X) \\
& + 2916.527548053087 B_{14,16}(X) + 4101.339842565916 B_{15,16}(X) + 5468.43208422982 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749929469011197, 0.6249882450180355\}$$

Intersection intervals with the x axis:

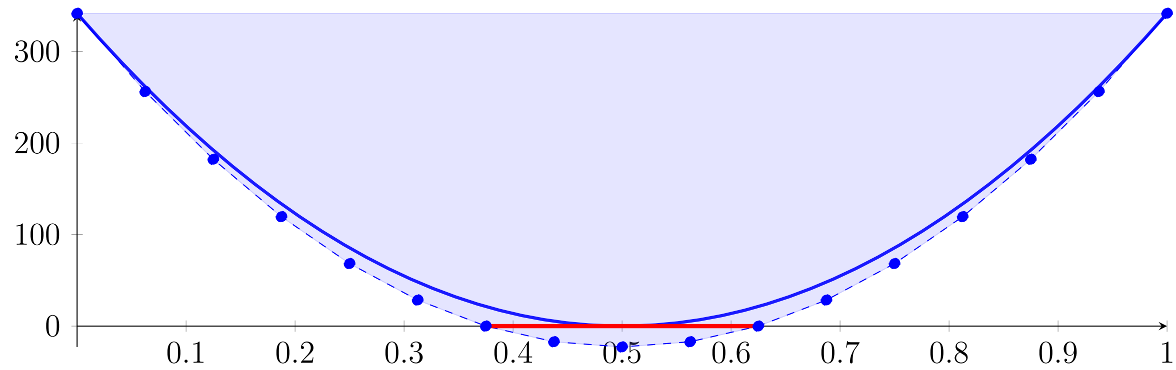
$$[0.3749929469011197, 0.6249882450180355]$$

Longest intersection interval: 0.2499952981169158
$\Longrightarrow$ Selective recursion: interval 1: $[0.2998961414100109, 0.3001040296026296]$,

79.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.2998961414100109, 0.300104029$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.216962444275584\cdot 10^{-59}X^{16} + 4.692870826723641\cdot 10^{-56}X^{15} + 6.316314535763934 \\
& \cdot 10^{-50}X^{14} - 2.373865516252114\cdot 10^{-46}X^{13} - 1.395661893422859\cdot 10^{-40}X^{12} \\
& + 4.828363722587868\cdot 10^{-37}X^{11} + 1.70471850273246\cdot 10^{-31}X^{10} - 5.110411085982617\cdot 10^{-28}X^{9} \\
& - 1.244717766622564\cdot 10^{-22}X^{8} + 2.988632715177156\cdot 10^{-19}X^{7} + 5.438614058423278\cdot 10^{-14}X^{6} \\
& - 9.197401057829169\cdot 10^{-11}X^{5} - 1.317801761681953\cdot 10^{-05}X^{4} + 0.01167528468045732X^{3} \\
& + 1366.961107526186X^{2} - 1366.963198735845X + 341.7398631167735 \\
= & \, 341.7398631167735B_{0,16}(X) + 256.3046631957831B_{1,16}(X) + 182.260805837511B_{2,16}(X) \\
& + 119.6083118906798B_{3,16}(X) + 68.34720219677136B_{4,16}(X) + 28.47749759002708B_{5,16}(X) \\
& - 0.0007811025525137025B_{6,16}(X) - 17.08761306120757B_{7,16}(X) - 22.782977473419B_{8,16}(X) \\
& - 17.08685353390849B_{9,16}(X) + 0.0007795553614777027B_{10,16}(X) \\
& + 28.4799425851876B_{11,16}(X) + 68.35065633912575B_{12,16}(X) + 119.6129415934909B_{13,16}(X) \\
& + 182.2668191173573B_{14,16}(X) + 256.312309672558B_{15,16}(X) + 341.7494340136854B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749982857492878, 0.6249971486858456\}$$

Intersection intervals with the x axis:

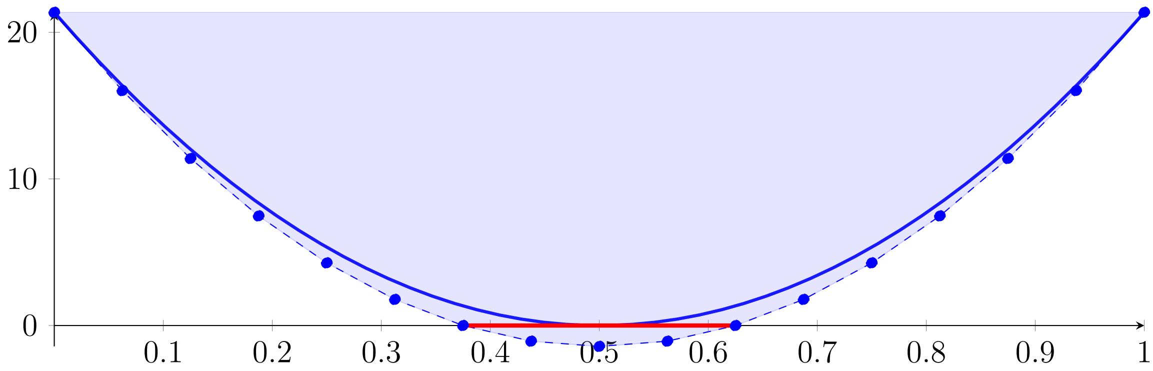
$$[0.3749982857492878, 0.6249971486858456]$$

Longest intersection interval: 0.2499988629365579
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999740991258704, 0.300026070937643]$,

79.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999740991258704, 0.3000260$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.833255302235776 \cdot 10^{-69} X^{16} + 4.363478560238502 \cdot 10^{-65} X^{15} + 2.35287052687137 \cdot 10^{-58} X^{14} - 3.532185105967992 \cdot 10^{-54} X^{13} - 8.318408043755505 \cdot 10^{-48} X^{12} + 1.149616718196078 \cdot 10^{-43} X^{11} + 1.625691292234855 \cdot 10^{-37} X^{10} - 1.946948798353601 \cdot 10^{-33} X^{9} - 1.899245778120626 \cdot 10^{-27} X^{8} + 1.821779493317236 \cdot 10^{-23} X^{7} + 1.327769541755276 \cdot 10^{-17} X^{6} - 8.969682874540076 \cdot 10^{-14} X^{5} - 5.147636798128115 \cdot 10^{-08} X^{4} + 0.000182114977798148 X^{3} + 85.43511227371199 X^{2} - 85.43514442584667 X + 21.35877059261705$$
$$= 21.35877059261705 B_{0,16}(X) + 16.01907406600164 B_{1,16}(X) + 11.39133680833382 B_{2,16}(X) + 7.475559144818918 B_{3,16}(X) + 4.271741400633969 B_{4,16}(X) + 1.779883900927722 B_{5,16}(X) - 1.302917935804413 \cdot 10^{-05} B_{6,16}(X) - 1.067949064595088 B_{7,16}(X) - 1.423923880255569 B_{8,16}(X) - 1.067937151125187 B_{9,16}(X) + 1.144780338983322 \cdot 10^{-05} B_{10,16}(X) + 1.779922241509208 B_{11,16}(X) + 4.271795554943032 B_{12,16}(X) + 7.47563171302734 B_{13,16}(X) + 11.39143104065633 B_{14,16}(X) + 16.01919386269591 B_{15,16}(X) + 21.35892050398371 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3749995424882778, 0.6249993300354412\}$$

Intersection intervals with the x axis:

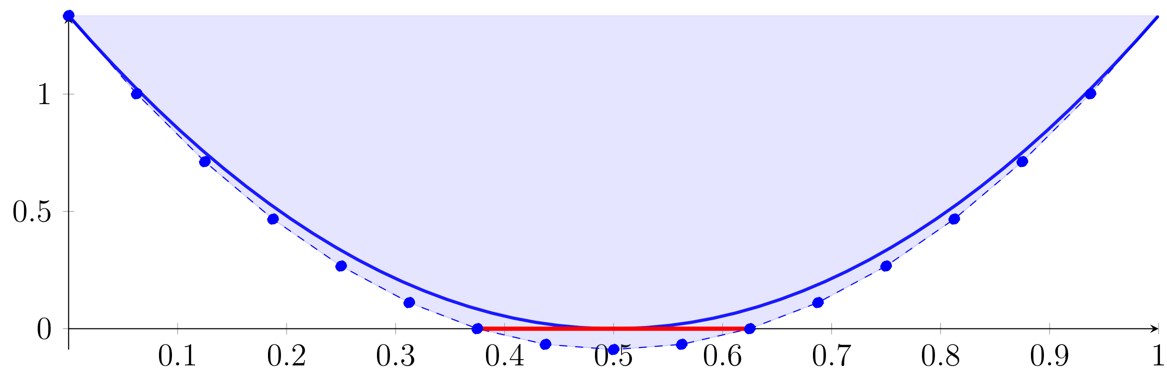
$$[0.3749995424882778, 0.6249993300354412]$$

Longest intersection interval: 0.2499997875471635
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999935885315074, 0.300006581473409]$,

79.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999935885315074, 0.30000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -6.596596862099299 \cdot 10^{-79} X^{16} + 4.062171102620318 \cdot 10^{-74} X^{15} + 8.765030607683244 \cdot 10^{-67} X^{14} - 5.261467161278336 \cdot 10^{-62} X^{13} - 4.958117265353456 \cdot 10^{-55} X^{12} + 2.739981763166457 \cdot 10^{-50} X^{11} + 1.550371378259149 \cdot 10^{-43} X^{10} - 7.424637900674377 \cdot 10^{-39} X^{9} - 2.898009393348535 \cdot 10^{-32} X^{8} + 1.111571585405421 \cdot 10^{-27} X^{7} + 3.241620002541647 \cdot 10^{-21} X^{6} - 8.756501261136649 \cdot 10^{-17} X^{5} - 2.010795357557322 \cdot 10^{-10} X^{4} + 2.844332798771609 \cdot 10^{-06} X^{3} + 5.339698243852309 X^{2} - 5.339698356505613 X + 1.33492347100565$$
$$= 1.33492347100565 B_{0,16}(X) + 1.00119232372405 B_{1,16}(X) + 0.7119586618078846 B_{2,16}(X) + 0.4672224903363214 B_{3,16}(X) + 0.266983814388415 B_{4,16}(X) + 0.1112426390431101 B_{5,16}(X) - 1.030620758846691 \cdot 10^{-06} B_{6,16}(X) - 0.06674718952446819 B_{7,16}(X) - 0.0889958325894046 B_{8,16}(X) - 0.06674695473706526 B_{9,16}(X) - 5.508890578613216 \cdot 10^{-07} B_{10,16}(X) + 0.1112433840328995 B_{11,16}(X) + 0.2669848551069781 B_{12,16}(X) + 0.4672238674112388 B_{13,16}(X) + 0.7119604260236321 B_{14,16}(X) + 1.001194536021998 B_{15,16}(X) + 1.334926202484066 B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3749994209666256, 0.6250003095051082\}$$

Intersection intervals with the x axis:

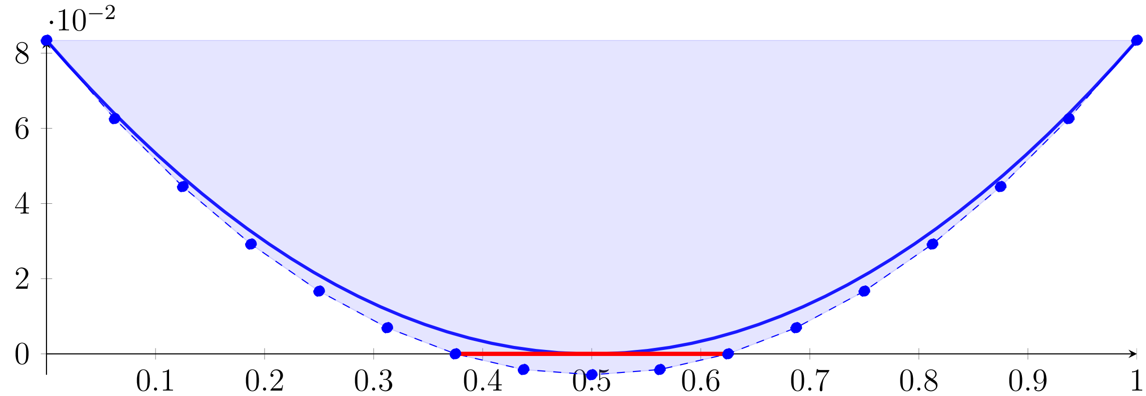
$$[0.3749994209666256, 0.6250003095051082]$$

Longest intersection interval: 0.2500008885384826
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999984608771972, 0.3000017091242173]$,

79.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999984608771972, 0.300$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.5359772362815\cdot 10^{-88}X^{16} + 3.783024712863664\cdot 10^{-83}X^{15} + 3.265391675561962\cdot 10^{-75}X^{14}\\
&- 7.83987355616511\cdot 10^{-70}X^{13} - 2.955395756595885\cdot 10^{-62}X^{12} + 6.532349021061568\\
&\cdot 10^{-57}X^{11} + 1.478602992973913\cdot 10^{-49}X^{10} - 2.832143398905245\cdot 10^{-44}X^{9}\\
&- 4.422140960586434\cdot 10^{-37}X^{8} + 6.784132679612849\cdot 10^{-32}X^{7} + 7.914287227230788\cdot 10^{-25}X^{6}\\
&- 8.550710444673422\cdot 10^{-20}X^{5} - 7.854787446136848\cdot 10^{-13}X^{4} + 4.443846100456459\\
&\cdot 10^{-08}X^{3} + 0.3337337124913315X^{2} - 0.3337336004955191X + 0.08343257581494635\\
= &\ 0.08343257581494635B_{0,16}(X) + 0.06257422578397641B_{1,16}(X) + 0.04449699002376757B_{2,16}(X)\\
&+ 0.02920086861367421B_{3,16}(X) + 0.01668586163305031B_{4,16}(X)\\
&+ 0.006951969161249391B_{5,16}(X) - 8.08722375440678\cdot 10^{-07}B_{6,16}(X)\\
&- 0.004172471938471519B_{7,16}(X) - 0.005563020407686608B_{8,16}(X)\\
&- 0.004172454050668901B_{9,16}(X) - 7.727880670249089\cdot 10^{-07}B_{10,16}(X)\\
&+ 0.006952023459469962B_{11,16}(X) + 0.01668593477129257B_{12,16}(X)\\
&+ 0.02920096122675088B_{13,16}(X) + 0.04449710290519453B_{14,16}(X)\\
&+ 0.06257435988597275B_{15,16}(X) + 0.08343273224843432B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749927302224649, 0.6250069467380417\}$$

Intersection intervals with the x axis:

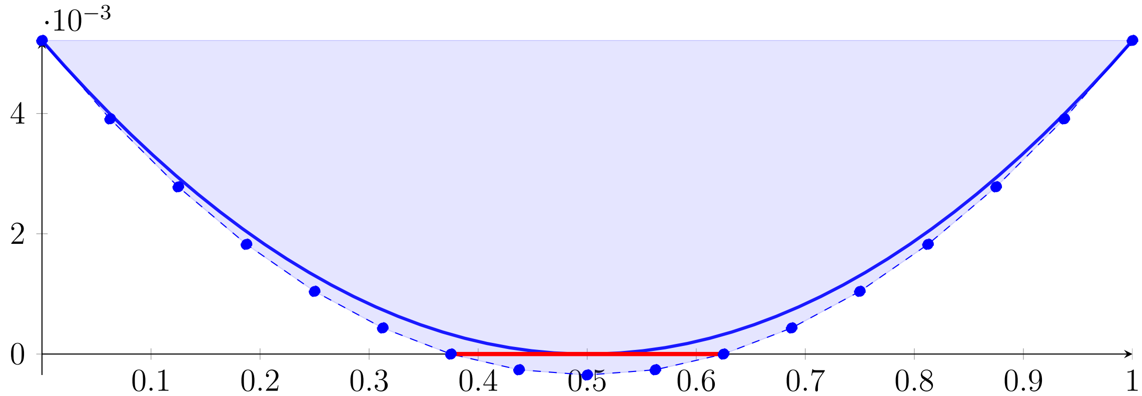
$$[0.3749927302224649, 0.6250069467380417]$$

Longest intersection interval: 0.2500142165155768
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999996789462157, 0.3000004910541495]$,

79.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999996789462157, 0.3$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.579480927696053\cdot 10^{-98}X^{16} + 3.526136882584451\cdot 10^{-92}X^{15} + 1.217422078989368\\&\cdot 10^{-83}X^{14} - 1.169070552755016\cdot 10^{-77}X^{13} - 1.762755817503918\cdot 10^{-69}X^{12}\\&+ 1.55837613779483\cdot 10^{-63}X^{11} + 1.410908032488726\cdot 10^{-55}X^{10} - 1.080908862805813\cdot 10^{-49}X^{9}\\&- 6.750723381404434\cdot 10^{-42}X^{8} + 4.142273519440928\cdot 10^{-36}X^{7} + 1.932858818028869\cdot 10^{-28}X^{6}\\&- 8.352503738960762\cdot 10^{-23}X^{5} - 3.06897495525372\cdot 10^{-15}X^{4} + 6.944510025023856\cdot 10^{-10}X^{3}\\&+ 0.02086073248825264X^{2} - 0.0208607236297874X + 0.005214387850787818\\= &\,0.005214387850787818B_{0,16}(X) + 0.003910592623926106B_{1,16}(X)\\&+ 0.002780636834466498B_{2,16}(X) + 0.001824520483649087B_{3,16}(X)\\&+ 0.001042243572713962B_{4,16}(X) + 0.000433806102901211B_{5,16}(X) - 7.919245490808786\\&\cdot 10^{-07}B_{6,16}(X) - 0.0002615505083968289B_{7,16}(X) - 0.0003484696474019504B_{8,16}(X)\\&- 0.0002615493403243644B_{9,16}(X) - 7.895859239916112\cdot 10^{-07}B_{10,16}(X)\\&+ 0.0004338096170392455B_{11,16}(X) + 0.001042248269805423B_{12,16}(X)\\&+ 0.001824526373614615B_{13,16}(X) + 0.002780643929706894B_{14,16}(X)\\&+ 0.00391060093932233B_{15,16}(X) + 0.005214397403700994B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748861124966261, 0.6251135508761015\}$$

Intersection intervals with the x axis:

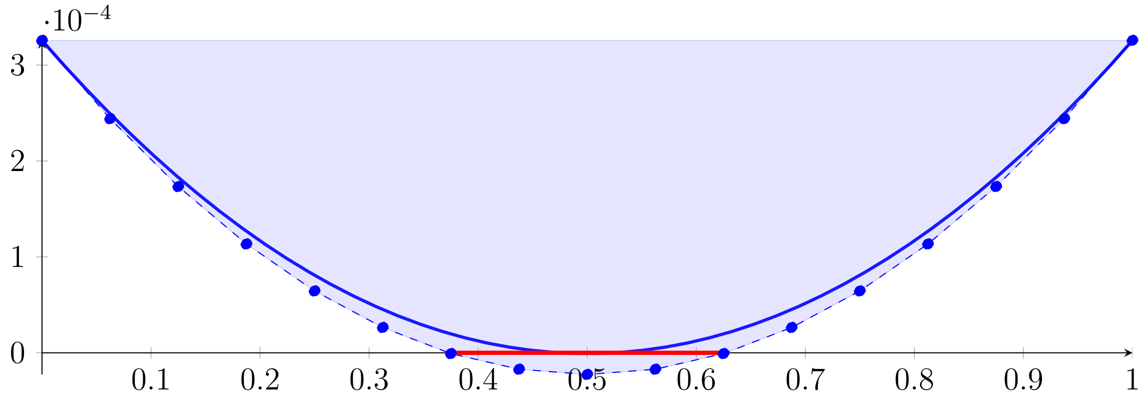
$$[0.3748861124966261, 0.6251135508761015]$$

Longest intersection interval: 0.2502274383794754
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999999833942019, 0.3000001866058899]$,

79.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999999833942019, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -8.456271781717561\cdot 10^{-108}X^{16} + 3.329051186798107\cdot 10^{-101}X^{15} + 4.593357002546748\cdot 10^{-92}X^{14} - 1.76275695811523\cdot 10^{-85}X^{13} - 1.062212310975265\cdot 10^{-76}X^{12} + 3.752790404631579\cdot 10^{-70}X^{11} + 1.357838209904933\cdot 10^{-61}X^{10} - 4.157203866707784\cdot 10^{-55}X^{9} - 1.037599553128416\cdot 10^{-46}X^{8} + 2.544375234544489\cdot 10^{-40}X^{7} + 4.744710700950638\cdot 10^{-32}X^{6} - 8.193869976131749\cdot 10^{-26}X^{5} - 1.203186876925119\cdot 10^{-17}X^{4} + 1.088036641260801\cdot 10^{-11}X^{3} + 0.001306169174102627X^{2} - 0.001306168592474289X + 0.0003257512461139167$$
$$= 0.0003257512461139167B_{0,16}(X) + 0.0002441157090842736B_{1,16}(X) + 0.0001733649151721524B_{2,16}(X) + 0.0001134988643969824B_{3,16}(X) + 6.451755677819264\cdot 10^{-05}B_{4,16}(X) + 2.642099233521247\cdot 10^{-05}B_{5,16}(X) - 7.908289125289409\cdot 10^{-07}B_{6,16}(X) - 1.71179069456024\cdot 10^{-05}B_{7,16}(X) - 2.25602417445787\cdot 10^{-05}B_{8,16}(X) - 1.711783329002867\cdot 10^{-05}B_{9,16}(X) - 7.906815625231293\cdot 10^{-07}B_{10,16}(X) + 2.64212134573671\cdot 10^{-05}B_{11,16}(X) + 6.451785178907119\cdot 10^{-05}B_{12,16}(X) + 0.0001134992334520183B_{13,16}(X) + 0.0001733653584656376B_{14,16}(X) + 0.0002441162268493581B_{15,16}(X) + 0.0003257518386226092B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3731836273807965, 0.626816029262996\}$$

Intersection intervals with the x axis:

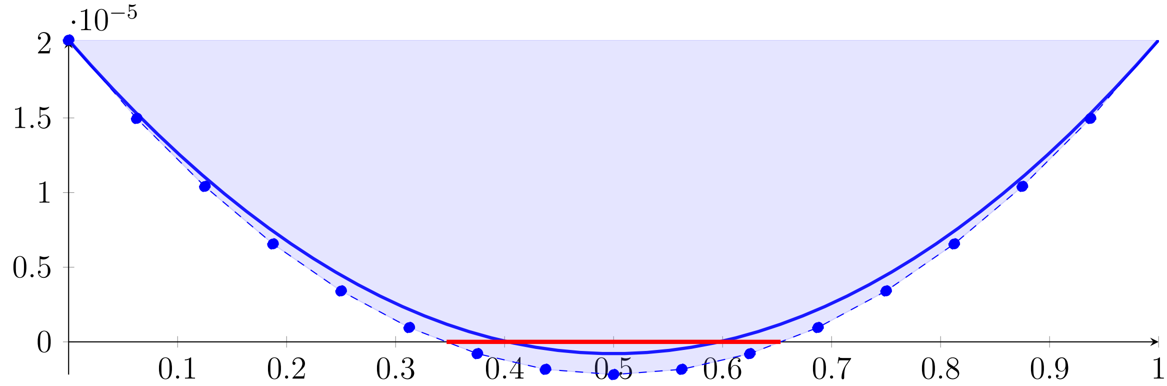
$$[0.3731836273807965, 0.626816029262996]$$

Longest intersection interval: 0.2536324018821995
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000592294767, 0.3000001107705452]$,

79.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000592294767

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.480017149294215\cdot10^{-117}X^{16} + 3.84938040332832\cdot10^{-110}X^{15} + 2.094095506848417\cdot10^{-100}X^{14} - 3.168497686663236\cdot10^{-93}X^{13} - 7.527801307502411\cdot10^{-84}X^{12} + 1.048590305184717\cdot10^{-76}X^{11} + 1.495875245120848\cdot10^{-67}X^{10} - 1.80569087496117\cdot10^{-60}X^{9} - 1.776919108083797\cdot10^{-51}X^{8} + 1.717962865912528\cdot10^{-44}X^{7} + 1.263101207166261\cdot10^{-35}X^{6} - 8.600271811104809\cdot10^{-29}X^{5} - 4.979113541149949\cdot10^{-20}X^{4} + 1.775239724826758\cdot10^{-13}X^{3} + 8.402507389292435\cdot10^{-05}X^{2} - 8.402503818595556\cdot10^{-05}X + 2.02154953599674\cdot10^{-05}$$
$$= 2.02154953599674\cdot10^{-05}B_{0,16}(X) + 1.496393047334518\cdot10^{-05}B_{1,16}(X) + 1.041257453583066\cdot10^{-05}B_{2,16}(X) + 6.561427547740848\cdot10^{-06}B_{3,16}(X) + 3.410489509392755\cdot10^{-06}B_{4,16}(X) + 9.59760421103387\cdot10^{-07}B_{5,16}(X) - 7.907597168102504\cdot10^{-07}B_{6,16}(X) - 1.84107090403115\cdot10^{-06}B_{7,16}(X) - 2.191173140242304\cdot10^{-06}B_{8,16}(X) - 1.841066425126706\cdot10^{-06}B_{9,16}(X) - 7.907507583673494\cdot10^{-07}B_{10,16}(X) + 9.59773860352773\cdot10^{-07}B_{11,16}(X) + 3.410507431350668\cdot10^{-06}B_{12,16}(X) + 6.561449954943343\cdot10^{-06}B_{13,16}(X) + 1.04126014314478\cdot10^{-05}B_{14,16}(X) + 1.496396186118106\cdot10^{-05}B_{15,16}(X) + 2.021553124446011\cdot10^{-05}B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3467669730097793, 0.6532326348738207\}$$

Intersection intervals with the x axis:

$$[0.3467669730097793, 0.6532326348738207]$$

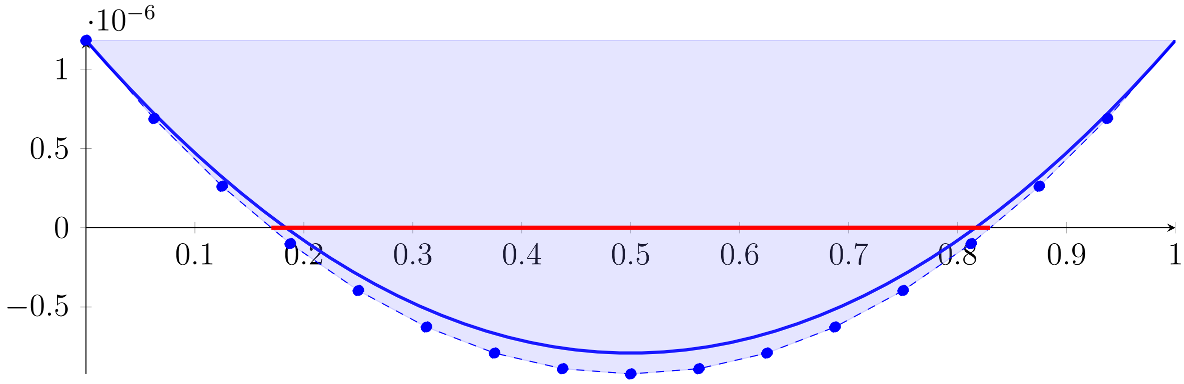
Longest intersection interval: 0.3064656618640414

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000771022171, 0.3000000928977847]$,

79.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000771022171, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = & -1.50163304400299\cdot 10^{-125}X^{16} + 7.605328677353438\cdot 10^{-118}X^{15} + 1.35002551430803\\ & \cdot 10^{-107}X^{14} - 6.665257392422306\cdot 10^{-100}X^{13} - 5.167137254241996\cdot 10^{-90}X^{12}\\ & + 2.348582048403403\cdot 10^{-82}X^{11} + 1.093235143477049\cdot 10^{-72}X^{10} - 4.306055922184639\cdot 10^{-65}X^{9}\\ & - 1.382681732768409\cdot 10^{-55}X^{8} + 4.362007350883115\cdot 10^{-48}X^{7} + 1.04647552702743\cdot 10^{-38}X^{6}\\ & - 2.324990317134255\cdot 10^{-31}X^{5} - 4.392171749769965\cdot 10^{-22}X^{4} + 5.109781163448487\cdot 10^{-15}X^{3}\\ & + 7.891735947253277\cdot 10^{-06}X^{2} - 7.891735064633028\cdot 10^{-06}X + 1.182178307271875\cdot 10^{-06}\\ = & \ 1.182178307271875\cdot 10^{-06}B_{0,16}(X) + 6.889448657323109\cdot 10^{-07}B_{1,16}(X) + 2.614758904198573\\ & \cdot 10^{-07}B_{2,16}(X) - 1.00228618656361\cdot 10^{-07}B_{3,16}(X) - 3.961686614872195\cdot 10^{-07}B_{4,16}(X)\\ & - 6.263442380635935\cdot 10^{-07}B_{5,16}(X) - 7.907553483763585\cdot 10^{-07}B_{6,16}(X) - 8.894019924163897\\ & \cdot 10^{-07}B_{7,16}(X) - 9.222841701745627\cdot 10^{-07}B_{8,16}(X) - 8.894018816417527\cdot 10^{-07}B_{9,16}(X)\\ & - 7.907551268088353\cdot 10^{-07}B_{10,16}(X) - 6.263439056666857\cdot 10^{-07}B_{11,16}(X)\\ & - 3.961682182061795\cdot 10^{-07}B_{12,16}(X) - 1.002280644181919\cdot 10^{-07}B_{13,16}(X) + 2.614765557064017\\ & \cdot 10^{-07}B_{14,16}(X) + 6.889456421767258\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1701811983018366, 0.8298187006126137\}$$

Intersection intervals with the x axis:

$$[0.1701811983018366, 0.8298187006126137]$$

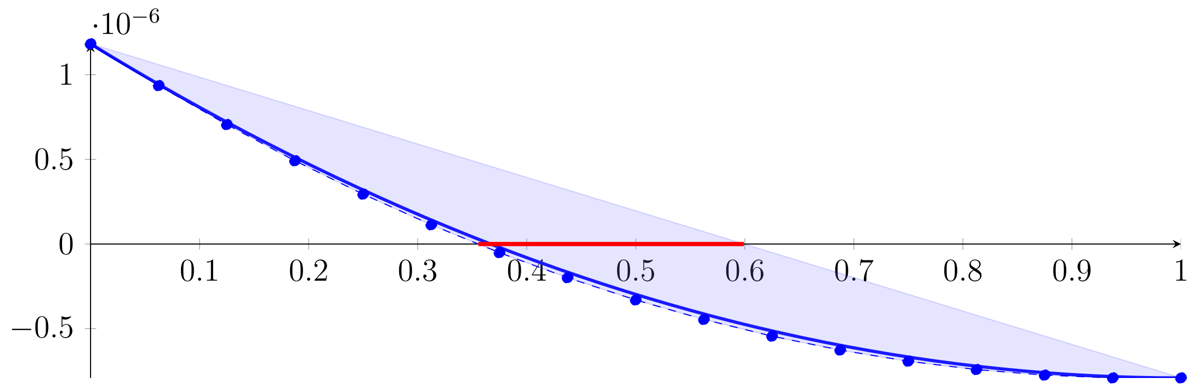
Longest intersection interval: 0.6596375023107771

$\Longrightarrow$ Bisection: first half $[0.3000000771022171, 0.3000000850000009]$ und second half $[0.3000000850000009, 0.30$

79.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3000000771022171, 0.3000000850000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.291310186772141\cdot10^{-130}X^{16} + 2.320962120774364\cdot10^{-122}X^{15} + 8.239901820727723\\ &\cdot10^{-112}X^{14} - 8.13630052785926\cdot10^{-104}X^{13} - 1.261508118711425\cdot10^{-93}X^{12}\\ &+ 1.146768578321974\cdot10^{-85}X^{11} + 1.067612444801806\cdot10^{-75}X^{10} - 8.410265473016874\cdot10^{-68}X^{9}\\ &- 5.401100518626597\cdot10^{-58}X^{8} + 3.407818242877434\cdot10^{-50}X^{7} + 1.635118010980359\cdot10^{-40}X^{6}\\ &- 7.265594741044545\cdot10^{-33}X^{5} - 2.745107343606228\cdot10^{-23}X^{4} + 6.387226454310609\cdot10^{-16}X^{3}\\ &+ 1.972933986813319\cdot10^{-06}X^{2} - 3.945867532316514\cdot10^{-06}X + 1.182178307271875\cdot10^{-06}\\ = &\ 1.182178307271875\cdot10^{-06}B_{0,16}(X) + 9.35561586502093\cdot10^{-07}B_{1,16}(X) + 7.053859822890886\\ &\cdot10^{-07}B_{2,16}(X) + 4.916514946340023\cdot10^{-07}B_{3,16}(X) + 2.943581235379749\cdot10^{-07}B_{4,16}(X)\\ &+ 1.135058690021469\cdot10^{-07}B_{5,16}(X) - 5.090526897234114\cdot10^{-08}B_{6,16}(X) - 1.988752903843486\\ &\cdot10^{-07}B_{7,16}(X) - 3.30404195232735\cdot10^{-07}B_{8,16}(X) - 4.454919835163597\cdot10^{-07}B_{9,16}(X)\\ &- 5.441386552340822\cdot10^{-07}B_{10,16}(X) - 6.263442103847618\cdot10^{-07}B_{11,16}(X) - 6.921086489672579\\ &\cdot10^{-07}B_{12,16}(X) - 7.414319709804301\cdot10^{-07}B_{13,16}(X) - 7.743141764231377\cdot10^{-07}B_{14,16}(X)\\ &- 7.907552652942402\cdot10^{-07}B_{15,16}(X) - 7.907552375925969\cdot10^{-07}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.355648638833307, 0.599198239772989\}$$

Intersection intervals with the x axis:

$$[0.355648638833307, 0.599198239772989]$$

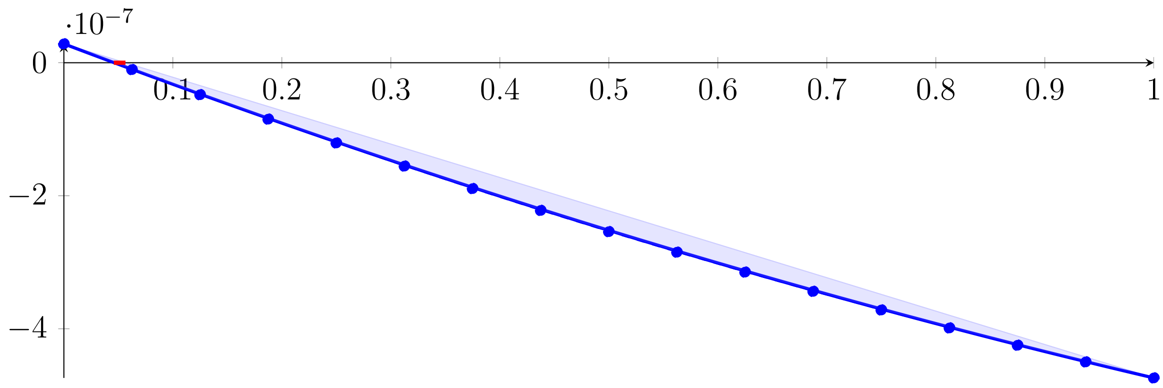
Longest intersection interval: 0.2435496009396821

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799110531, 0.3000000818345552]$,

79.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799110531, 0.3000000818345552]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -3.511418903935676\cdot 10^{-140}X^{16} + 1.460425350169266\cdot 10^{-131}X^{15} + 2.12885435080787 \\
& \cdot 10^{-120}X^{14} - 8.631046367525725\cdot 10^{-112}X^{13} - 5.494638409481287\cdot 10^{-101}X^{12} \\
& + 2.050866587664798\cdot 10^{-92}X^{11} + 7.839490728884222\cdot 10^{-82}X^{10} - 2.535691914683604\cdot 10^{-73}X^{9} \\
& - 6.686235030771963\cdot 10^{-63}X^{8} + 1.732161414466012\cdot 10^{-54}X^{7} + 3.412507672068927\cdot 10^{-44}X^{6} \\
& - 6.225987736384381\cdot 10^{-36}X^{5} - 9.658485252841958\cdot 10^{-26}X^{4} + 9.227298165814739\cdot 10^{-18}X^{3} \\
& + 1.170273575918748\cdot 10^{-07}X^{2} - 6.192309389580206\cdot 10^{-07}X + 2.838432851654629\cdot 10^{-08} \\
= & \; 2.838432851654629\cdot 10^{-08}B_{0,16}(X) - 1.031760516832999\cdot 10^{-08}B_{1,16}(X) - 4.804431087327399 \\
& \cdot 10^{-08}B_{2,16}(X) - 8.479578859826921\cdot 10^{-08}B_{3,16}(X) - 1.205720383432992\cdot 10^{-07}B_{4,16}(X) \\
& - 1.553730601083475\cdot 10^{-07}B_{5,16}(X) - 1.891988538933975\cdot 10^{-07}B_{6,16}(X) - 2.220494196984329 \\
& \cdot 10^{-07}B_{7,16}(X) - 2.539247575234371\cdot 10^{-07}B_{8,16}(X) - 2.848248673683937\cdot 10^{-07}B_{9,16}(X) \\
& - 3.147497492332862\cdot 10^{-07}B_{10,16}(X) - 3.436994031180981\cdot 10^{-07}B_{11,16}(X) - 3.71673829022813 \\
& \cdot 10^{-07}B_{12,16}(X) - 3.986730269474143\cdot 10^{-07}B_{13,16}(X) - 4.246969968918855\cdot 10^{-07}B_{14,16}(X) \\
& - 4.497457388562103\cdot 10^{-07}B_{15,16}(X) - 4.738192528403721\cdot 10^{-07}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.04583803348764934, 0.05651956610873593\}$$

Intersection intervals with the x axis:

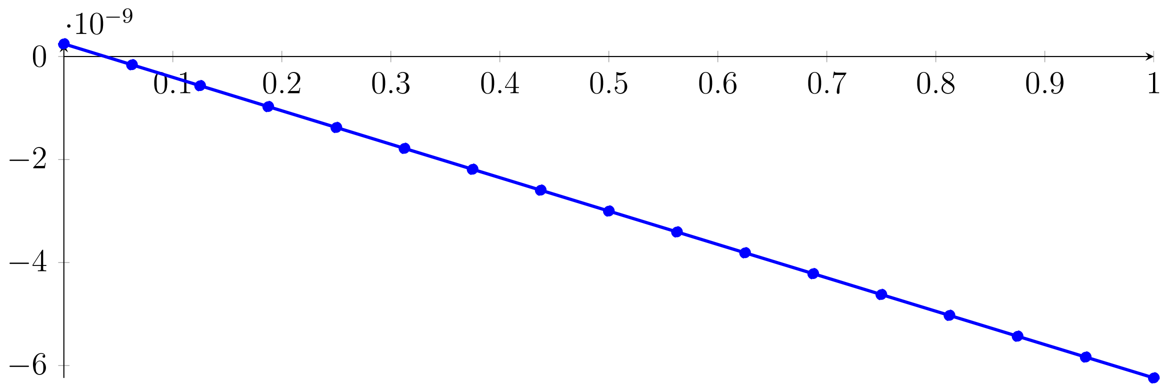
$$[0.04583803348764934, 0.05651956610873593]$$

Longest intersection interval: 0.01068153262108659
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799992227, 0.3000000800197686]$,

79.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799992227, 0.3000000800197686]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.0083696859672\cdot 10^{-171}X^{16} + 3.926295021503001\cdot 10^{-161}X^{15} + 5.358163301918743 \\
& \cdot 10^{-148}X^{14} - 2.033760808321352\cdot 10^{-137}X^{13} - 1.212109824997298\cdot 10^{-124}X^{12} \\
& + 4.235519560164542\cdot 10^{-114}X^{11} + 1.515735988691705\cdot 10^{-101}X^{10} - 4.589851404660334\cdot 10^{-91}X^{9} \\
& - 1.133052955101212\cdot 10^{-78}X^{8} + 2.748041945976198\cdot 10^{-68}X^{7} + 5.068448795464027\cdot 10^{-56}X^{6} \\
& - 8.657173392624336\cdot 10^{-46}X^{5} - 1.257312709564581\cdot 10^{-33}X^{4} + 1.124540929737694\cdot 10^{-23}X^{3} \\
& + 1.335225264723055\cdot 10^{-11}X^{2} - 6.499737499496223\cdot 10^{-09}X + 2.458891434694464\cdot 10^{-10} \\
= & \; 2.458891434694464\cdot 10^{-10}B_{0,16}(X) - 1.603444502490676\cdot 10^{-10}B_{1,16}(X) - 5.664667751955213 \\
& \cdot 10^{-10}B_{2,16}(X) - 9.724778313699147\cdot 10^{-10}B_{3,16}(X) - 1.378377618772248\cdot 10^{-09}B_{4,16}(X) \\
& - 1.784166137402521\cdot 10^{-09}B_{5,16}(X) - 2.189843387260733\cdot 10^{-09}B_{6,16}(X) - 2.595409368346885 \\
& \cdot 10^{-09}B_{7,16}(X) - 3.000864080660977\cdot 10^{-09}B_{8,16}(X) - 3.406207524203008\cdot 10^{-09}B_{9,16}(X) \\
& - 3.811439698972979\cdot 10^{-09}B_{10,16}(X) - 4.21656060497089\cdot 10^{-09}B_{11,16}(X) - 4.62157024219674 \\
& \cdot 10^{-09}B_{12,16}(X) - 5.026468610650529\cdot 10^{-09}B_{13,16}(X) - 5.431255710332258\cdot 10^{-09}B_{14,16}(X) \\
& - 5.835931541241927\cdot 10^{-09}B_{15,16}(X) - 6.240496103379535\cdot 10^{-09}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03783062677354348, 0.03790850128565779\}$$

Intersection intervals with the x axis:

$$[0.03783062677354348, 0.03790850128565779]$$

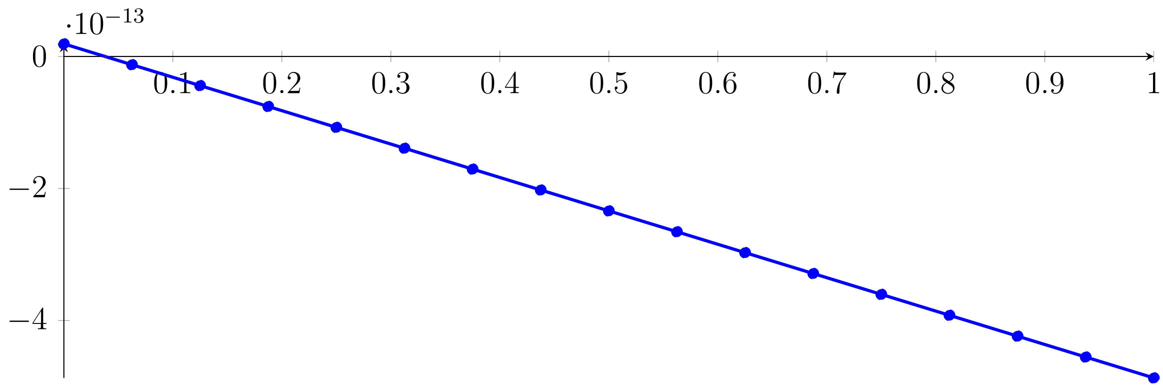
Longest intersection interval: $7.787451211431613\cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999999, 0.3000000800000015]$,

79.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799999999, 0.3000000800000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.844782636201088\cdot10^{-237}X^{16} + 9.223866560433282\cdot10^{-223}X^{15} + 1.616406874872468\\&\cdot10^{-205}X^{14} - 7.878422704947929\cdot10^{-191}X^{13} - 6.029565776386014\cdot10^{-174}X^{12}\\&+ 2.705549092213795\cdot10^{-159}X^{11} + 1.243302910922026\cdot10^{-142}X^{10} - 4.83455688747521\cdot10^{-128}X^{9}\\&- 1.5325438805125\cdot10^{-111}X^{8} + 4.772992249567202\cdot10^{-97}X^{7} + 1.130438906312722\cdot10^{-80}X^{6}\\&- 2.479435471431183\cdot10^{-66}X^{5} - 4.624072778948578\cdot10^{-50}X^{4} + 5.310816347750779\cdot10^{-36}X^{3}\\&+ 8.097393019768196\cdot10^{-20}X^{2} - 5.060852140608093\cdot10^{-13}X + 1.910916079008261\cdot10^{-14}\\= &\,1.910916079008261\cdot10^{-14}B_{0,16}(X) - 1.252116508871797\cdot10^{-14}B_{1,16}(X) - 4.41514902927358\\&\cdot10^{-14}B_{2,16}(X) - 7.578181482197087\cdot10^{-14}B_{3,16}(X) - 1.074121386764232\cdot10^{-13}B_{4,16}(X)\\&- 1.390424618560928\cdot10^{-13}B_{5,16}(X) - 1.706727843609796\cdot10^{-13}B_{6,16}(X) - 2.023031061910837\\&\cdot10^{-13}B_{7,16}(X) - 2.33933427346405\cdot10^{-13}B_{8,16}(X) - 2.655637478269435\cdot10^{-13}B_{9,16}(X)\\&- 2.971940676326994\cdot10^{-13}B_{10,16}(X) - 3.288243867636724\cdot10^{-13}B_{11,16}(X) - 3.604547052198627\\&\cdot10^{-13}B_{12,16}(X) - 3.920850230012703\cdot10^{-13}B_{13,16}(X) - 4.237153401078951\cdot10^{-13}B_{14,16}(X)\\&- 4.553456565397371\cdot10^{-13}B_{15,16}(X) - 4.869759722967965\cdot10^{-13}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775878104944304, 0.03775878709087106\}$$

Intersection intervals with the x axis:

$$[0.03775878104944304, 0.03775878709087106]$$

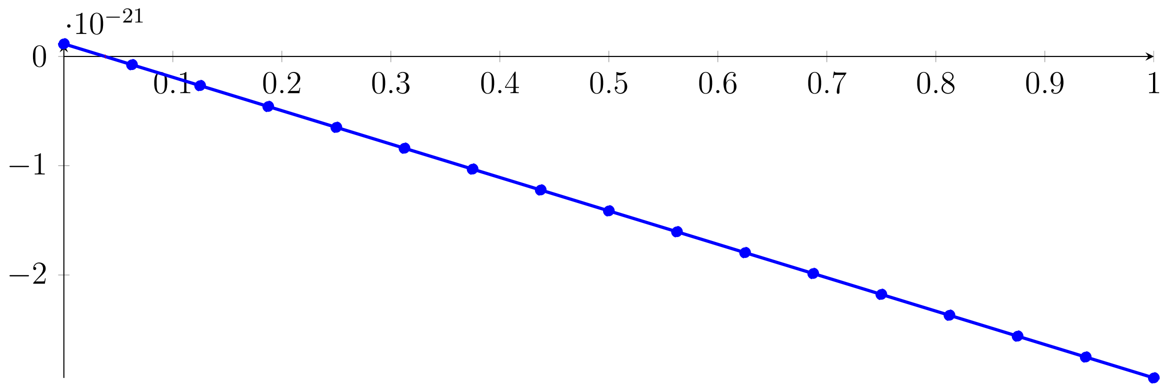
Longest intersection interval: $6.041428015081178\cdot10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

79.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-9.719453947461351\cdot 10^{-326}X^{16} + 4.850491890476349\cdot 10^{-325}X^{15} + 1.39445463768969\\&\cdot 10^{-320}X^{14} - 1.125263275537969\cdot 10^{-297}X^{13} - 1.425480559690934\cdot 10^{-272}X^{12}\\&+ 1.058744275447586\cdot 10^{-249}X^{11} + 8.053283759644375\cdot 10^{-225}X^{10} - 5.183380864729409\cdot 10^{-202}X^{9}\\&- 2.7197548469932\cdot 10^{-177}X^{8} + 1.402064409388781\cdot 10^{-154}X^{7} + 5.496480949202085\cdot 10^{-130}X^{6}\\&- 1.995495881284759\cdot 10^{-107}X^{5} - 6.160033605799806\cdot 10^{-83}X^{4} + 1.17106256664041\cdot 10^{-60}X^{3}\\&+ 2.955455531505519\cdot 10^{-36}X^{2} - 3.057477353302275\cdot 10^{-21}X + 1.154466008703692\cdot 10^{-22}\\= &\ 1.154466008703692\cdot 10^{-22}B_{0,16}(X) - 7.564573371102292\cdot 10^{-23}B_{1,16}(X) - 2.667380682924151\\&\cdot 10^{-22}B_{2,16}(X) - 4.578304028738072\cdot 10^{-22}B_{3,16}(X) - 6.489227374551993\cdot 10^{-22}B_{4,16}(X)\\&- 8.400150720365913\cdot 10^{-22}B_{5,16}(X) - 1.031107406617983\cdot 10^{-21}B_{6,16}(X) - 1.222199741199375\\&\cdot 10^{-21}B_{7,16}(X) - 1.413292075780767\cdot 10^{-21}B_{8,16}(X) - 1.604384410362159\cdot 10^{-21}B_{9,16}(X)\\&- 1.795476744943551\cdot 10^{-21}B_{10,16}(X) - 1.986569079524943\cdot 10^{-21}B_{11,16}(X) - 2.177661414106335\\&\cdot 10^{-21}B_{12,16}(X) - 2.368753748687727\cdot 10^{-21}B_{13,16}(X) - 2.559846083269119\cdot 10^{-21}B_{14,16}(X)\\&- 2.750938417850511\cdot 10^{-21}B_{15,16}(X) - 2.942030752431902\cdot 10^{-21}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775877546424977, 0.0377587754642498\}$$

Intersection intervals with the x axis:

$$[0.03775877546424977, 0.0377587754642498]$$

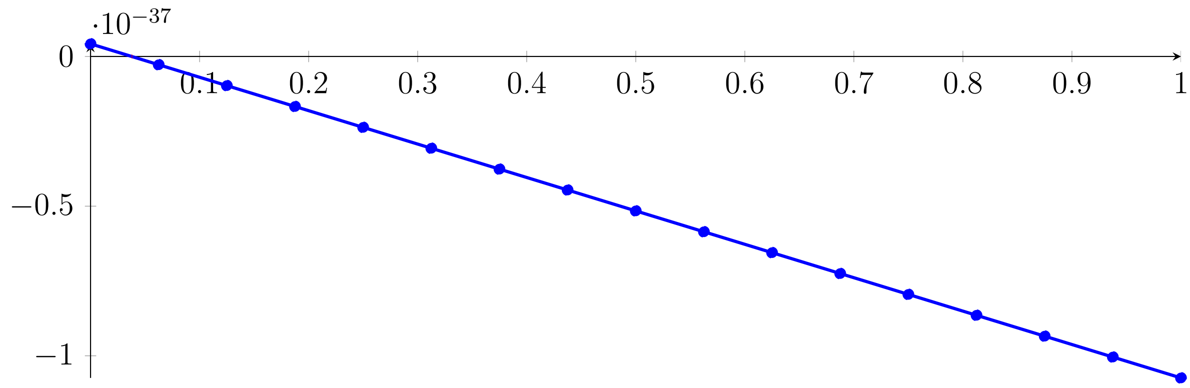
Longest intersection interval: $3.649884166375354\cdot 10^{-17}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

79.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = {} & 3.068577046082631\cdot10^{-342}X^{16} + 1.486894544116139\cdot10^{-341}X^{15} - 7.903181386330787 \\ & \cdot10^{-341}X^{14} - 4.482026616235252\cdot10^{-340}X^{13} + 6.545443281471007\cdot10^{-340}X^{12} \\ & + 6.169377812935582\cdot10^{-341}X^{11} - 3.267485286110327\cdot10^{-340}X^{10} + 1.705555506486159\cdot10^{-340}X^{9} \\ & - 8.565691202159373\cdot10^{-309}X^{8} + 1.20982216727754\cdot10^{-269}X^{7} + 1.299448935949168\cdot10^{-228}X^{6} \\ & - 1.292546704525185\cdot10^{-189}X^{5} - 1.093199460362076\cdot10^{-148}X^{4} + 5.693998445459821\cdot10^{-110}X^{3} \\ & + 3.937155726791203\cdot10^{-69}X^{2} - 1.11594381808692\cdot10^{-37}X + 4.213667205786155\cdot10^{-39} \\ = {} & 4.213667205786155\cdot10^{-39}B_{0,16}(X) - 2.760981657257093\cdot10^{-39}B_{1,16}(X) - 9.73563052030034 \\ & \cdot10^{-39}B_{2,16}(X) - 1.671027938334359\cdot10^{-38}B_{3,16}(X) - 2.368492824638684\cdot10^{-38}B_{4,16}(X) \\ & - 3.065957710943008\cdot10^{-38}B_{5,16}(X) - 3.763422597247333\cdot10^{-38}B_{6,16}(X) - 4.460887483551658 \\ & \cdot10^{-38}B_{7,16}(X) - 5.158352369855983\cdot10^{-38}B_{8,16}(X) - 5.855817256160307\cdot10^{-38}B_{9,16}(X) \\ & - 6.553282142464632\cdot10^{-38}B_{10,16}(X) - 7.250747028768957\cdot10^{-38}B_{11,16}(X) - 7.948211915073282 \\ & \cdot10^{-38}B_{12,16}(X) - 8.645676801377607\cdot10^{-38}B_{13,16}(X) - 9.343141687681931\cdot10^{-38}B_{14,16}(X) \\ & - 1.004060657398626\cdot10^{-37}B_{15,16}(X) - 1.073807146029058\cdot10^{-37}B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775877546424973, 0.03775877546424973\}$$

Intersection intervals with the x axis:

$$[0.03775877546424973, 0.03775877546424973]$$

Longest intersection interval: $1.332165442795749\cdot10^{-33}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

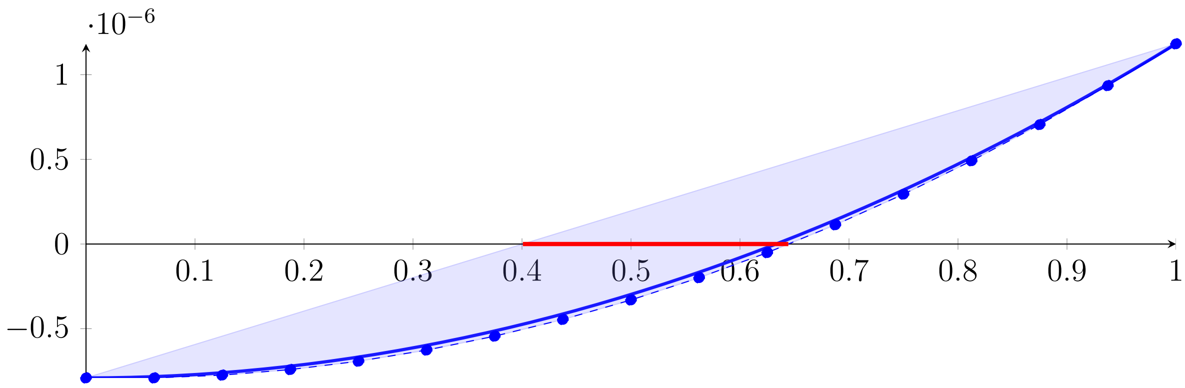
79.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 21!

79.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.3000000850000009, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-2.291310186772141\cdot 10^{-130}X^{16} + 2.320961754164735\cdot 10^{-122}X^{15} + 8.239901824209166\\&\cdot 10^{-112}X^{14} - 8.136299374273005\cdot 10^{-104}X^{13} - 1.261508119769144\cdot 10^{-93}X^{12}\\&+ 1.146768426941\cdot 10^{-85}X^{11} + 1.067612446063251\cdot 10^{-75}X^{10} - 8.410264405404428\cdot 10^{-68}X^{9}\\&- 5.401100526195836\cdot 10^{-58}X^{8} + 3.407817810789392\cdot 10^{-50}X^{7} + 1.635118013365832\cdot 10^{-40}X^{6}\\&- 7.265593759973738\cdot 10^{-33}X^{5} - 2.745107347239025\cdot 10^{-23}X^{4} + 6.387225356267671\cdot 10^{-16}X^{3}\\&+ 1.972933988729487\cdot 10^{-06}X^{2} + 4.432262923936659\cdot 10^{-13}X - 7.907552375925969\cdot 10^{-07}\\= &-7.907552375925969\cdot 10^{-07}B_{0,16}(X) - 7.907552098909536\cdot 10^{-07}B_{1,16}(X) - 7.743140656165646\\&\cdot 10^{-07}B_{2,16}(X) - 7.414318047682893\cdot 10^{-07}B_{3,16}(X) - 6.921084273449871\cdot 10^{-07}B_{4,16}(X)\\&- 6.263439333455175\cdot 10^{-07}B_{5,16}(X) - 5.441383227687398\cdot 10^{-07}B_{6,16}(X) - 4.454915956135136\\&\cdot 10^{-07}B_{7,16}(X) - 3.304037518786981\cdot 10^{-07}B_{8,16}(X) - 1.988747915631529\cdot 10^{-07}B_{9,16}(X)\\&- 5.090471466573741\cdot 10^{-08}B_{10,16}(X) + 1.13506478814689\cdot 10^{-07}B_{11,16}(X) + 2.943587888792669\\&\cdot 10^{-07}B_{12,16}(X) + 4.916522155291369\cdot 10^{-07}B_{13,16}(X) + 7.053867587654395\cdot 10^{-07}B_{14,16}(X)\\&+ 9.355624185893154\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4008015798845968, 0.644351143917019\}$$

Intersection intervals with the x axis:

$$[0.4008015798845968, 0.644351143917019]$$

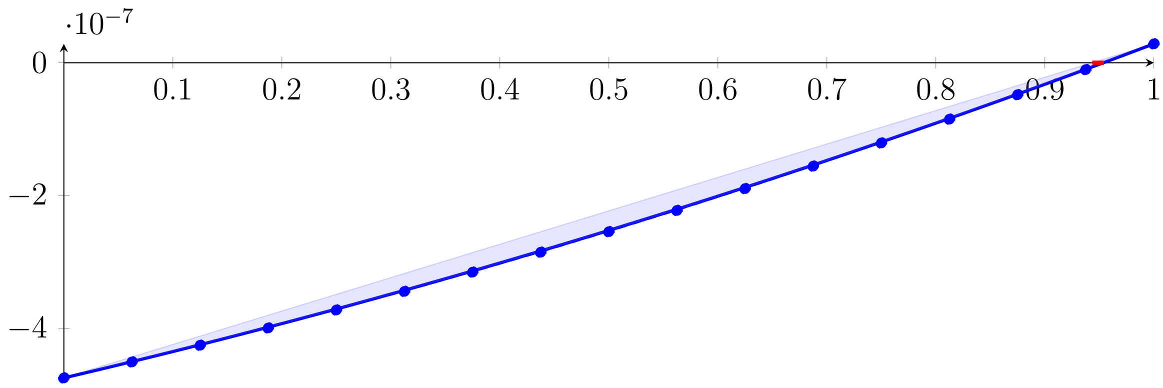
Longest intersection interval: 0.2435495640324222

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000881654451, 0.3000000900889469]$,

79.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.3000000881654451, 0.3000000900889469]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-3.511410390075519\cdot 10^{-140}X^{16} + 1.460421789403856\cdot 10^{-131}X^{15} + 2.12884983529034\\
&\cdot 10^{-120}X^{14} - 8.631028085335548\cdot 10^{-112}X^{13} - 5.494628422481359\cdot 10^{-101}X^{12}\\
&+ 2.050862886067143\cdot 10^{-92}X^{11} + 7.839478858688502\cdot 10^{-82}X^{10} - 2.535688119961682\cdot 10^{-73}X^{9}\\
&- 6.686226934767321\cdot 10^{-63}X^{8} + 1.732159347494186\cdot 10^{-54}X^{7} + 3.412504574505657\cdot 10^{-44}X^{6}\\
&- 6.225982140334265\cdot 10^{-36}X^{5} - 9.658479411653884\cdot 10^{-26}X^{4} + 9.227292313018015\cdot 10^{-18}X^{3}\\
&+ 1.170273222422553\cdot 10^{-07}X^{2} + 3.851762081107905\cdot 10^{-07}X - 4.738191826798638\cdot 10^{-07}\\
= &-4.738191826798638\cdot 10^{-07}B_{0,16}(X) - 4.497456696729394\cdot 10^{-07}B_{1,16}(X) - 4.246969289806629\\
&\cdot 10^{-07}B_{2,16}(X) - 3.986729606030177\cdot 10^{-07}B_{3,16}(X) - 3.716737645399875\cdot 10^{-07}B_{4,16}(X)\\
&- 3.436993407915557\cdot 10^{-07}B_{5,16}(X) - 3.147496893577059\cdot 10^{-07}B_{6,16}(X) - 2.848248102384216\\
&\cdot 10^{-07}B_{7,16}(X) - 2.539247034336863\cdot 10^{-07}B_{8,16}(X) - 2.220493689434835\cdot 10^{-07}B_{9,16}(X)\\
&- 1.891988067677968\cdot 10^{-07}B_{10,16}(X) - 1.553730169066096\cdot 10^{-07}B_{11,16}(X) - 1.205719993599056\\
&\cdot 10^{-07}B_{12,16}(X) - 8.479575412766813\cdot 10^{-08}B_{13,16}(X) - 4.804428120988084\cdot 10^{-08}B_{14,16}(X)\\
&- 1.031758060652722\cdot 10^{-08}B_{15,16}(X) + 2.838434768240921\cdot 10^{-08}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9434803899886293, 0.9541619291703946\}$$

Intersection intervals with the x axis:

$$[0.9434803899886293, 0.9541619291703946]$$

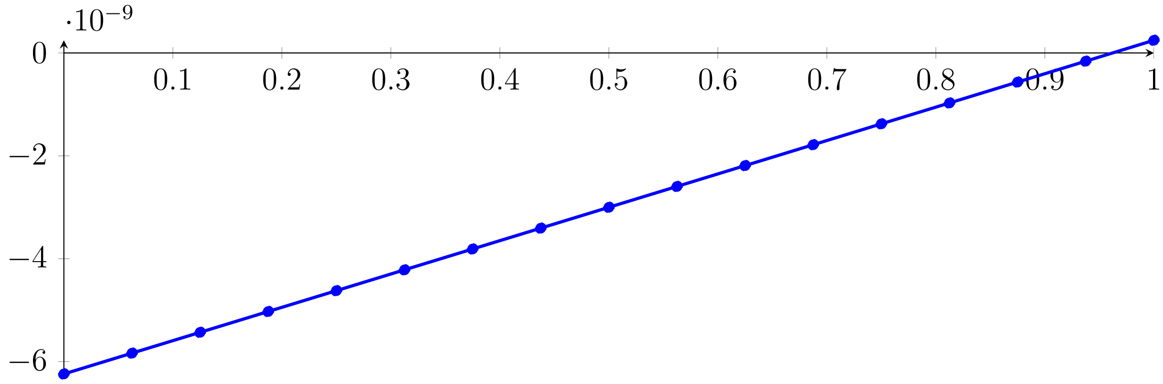
Longest intersection interval: 0.01068153918176529

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899802314, 0.3000000900007773]$,

79.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.3000000899802314, 0.3000000900007773]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.008377150647939\cdot 10^{-171}X^{16} + 3.926321486414625\cdot 10^{-161}X^{15} + 5.358198011634222\\ &\cdot 10^{-148}X^{14} - 2.033772676384319\cdot 10^{-137}X^{13} - 1.212116555959755\cdot 10^{-124}X^{12}\\ &+ 4.235540409672438\cdot 10^{-114}X^{11} + 1.515743003803166\cdot 10^{-101}X^{10} - 4.58986978058239\cdot 10^{-91}X^{9}\\ &- 1.133057150938366\cdot 10^{-78}X^{8} + 2.748050405674693\cdot 10^{-68}X^{7} + 5.068462874895849\cdot 10^{-56}X^{6}\\ &- 8.65719194232841\cdot 10^{-46}X^{5} - 1.257315038545109\cdot 10^{-33}X^{4} + 1.12454224629143\cdot 10^{-23}X^{3}\\ &+ 1.335226501895653\cdot 10^{-11}X^{2} + 6.473035980688146\cdot 10^{-09}X - 6.240498775828017\cdot 10^{-09}\\ = &-6.240498775828017\cdot 10^{-09}B_{0,16}(X) - 5.835934027035008\cdot 10^{-09}B_{1,16}(X) - 5.43125800936684\\ &\cdot 10^{-09}B_{2,16}(X) - 5.026470722823515\cdot 10^{-09}B_{3,16}(X) - 4.621572167405032\cdot 10^{-09}B_{4,16}(X)\\ &- 4.216562343111391\cdot 10^{-09}B_{5,16}(X) - 3.811441249942592\cdot 10^{-09}B_{6,16}(X) - 3.406208887898635\\ &\cdot 10^{-09}B_{7,16}(X) - 3.000865256979519\cdot 10^{-09}B_{8,16}(X) - 2.595410357185246\cdot 10^{-09}B_{9,16}(X)\\ &- 2.189844188515814\cdot 10^{-09}B_{10,16}(X) - 1.784166750971225\cdot 10^{-09}B_{11,16}(X)\\ &- 1.378378044551477\cdot 10^{-09}B_{12,16}(X) - 9.724780692565705\cdot 10^{-10}B_{13,16}(X) - 5.664668250865062\\ &\cdot 10^{-10}B_{14,16}(X) - 1.603443120412836\cdot 10^{-10}B_{15,16}(X) + 2.458894698790972\cdot 10^{-10}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9620914659183662, 0.9621693405339302\}$$

Intersection intervals with the x axis:

$$[0.9620914659183662, 0.9621693405339302]$$

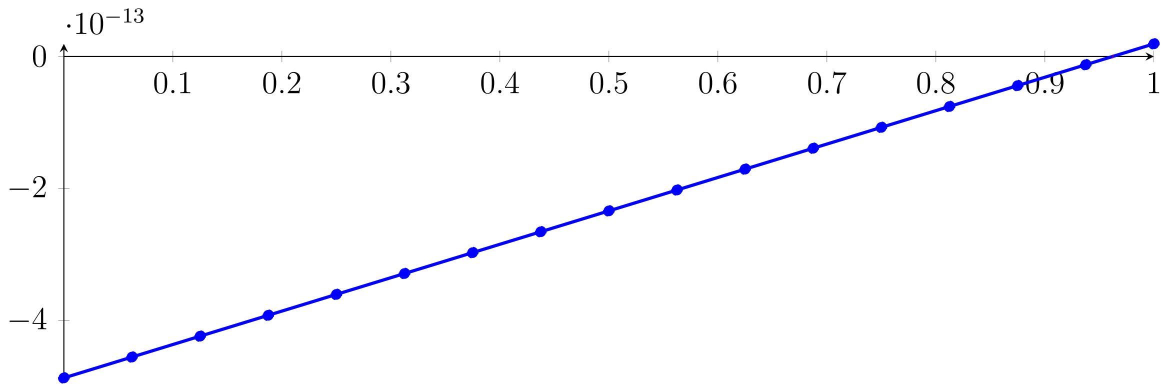
Longest intersection interval: $7.787461556406489\cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899999985, 0.3000000900000001]$,

79.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.3000000899999985, 0.3000000900000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.844835503516189\cdot 10^{-237}X^{16} + 9.224112529673191\cdot 10^{-223}X^{15} + 1.616447407922287 \\
& \cdot 10^{-205}X^{14} - 7.878604734573916\cdot 10^{-191}X^{13} - 6.029695377653573\cdot 10^{-174}X^{12} \\
& + 2.705601945023742\cdot 10^{-159}X^{11} + 1.243325181575566\cdot 10^{-142}X^{10} - 4.834634042887648\cdot 10^{-128}X^{9} \\
& - 1.532565842702471\cdot 10^{-111}X^{8} + 4.773051325459324\cdot 10^{-97}X^{7} + 1.130451056729432\cdot 10^{-80}X^{6} \\
& - 2.479457251994378\cdot 10^{-66}X^{5} - 4.624105915218427\cdot 10^{-50}X^{4} + 5.31084372815498\cdot 10^{-36}X^{3} \\
& + 8.097422035992938\cdot 10^{-20}X^{2} + 5.060859587615554\cdot 10^{-13}X - 4.869768282121624\cdot 10^{-13} \\
= & -4.869768282121624\cdot 10^{-13}B_{0,16}(X) - 4.553464557895652\cdot 10^{-13}B_{1,16}(X) - 4.237160826921828 \\
& \cdot 10^{-13}B_{2,16}(X) - 3.920857089200152\cdot 10^{-13}B_{3,16}(X) - 3.604553344730625\cdot 10^{-13}B_{4,16}(X) \\
& - 3.288249593513246\cdot 10^{-13}B_{5,16}(X) - 2.971945835548016\cdot 10^{-13}B_{6,16}(X) - 2.655642070834933 \\
& \cdot 10^{-13}B_{7,16}(X) - 2.339338299374\cdot 10^{-13}B_{8,16}(X) - 2.023034521165214\cdot 10^{-13}B_{9,16}(X) \\
& - 1.706730736208576\cdot 10^{-13}B_{10,16}(X) - 1.390426944504087\cdot 10^{-13}B_{11,16}(X) \\
& - 1.074123146051747\cdot 10^{-13}B_{12,16}(X) - 7.578193408515542\cdot 10^{-14}B_{13,16}(X) - 4.4151552890351 \\
& \cdot 10^{-14}B_{14,16}(X) - 1.252117102076142\cdot 10^{-14}B_{15,16}(X) + 1.910921152361334\cdot 10^{-14}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411803068306, 0.9622411863482746\}$$

Intersection intervals with the x axis:

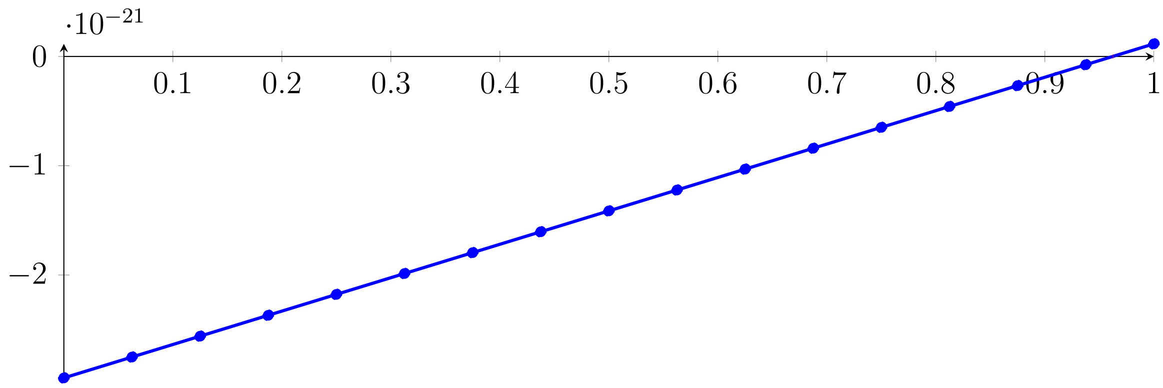
$$[0.9622411803068306, 0.9622411863482746]$$

Longest intersection interval: $6.041444057142176\cdot 10^{-09}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

79.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p &= 6.121540900010562\cdot 10^{-326} X^{16} - 8.347007493216372\cdot 10^{-325} X^{15} + 1.39439413847082 \\
&\quad \cdot 10^{-320} X^{14} - 1.125328119545448\cdot 10^{-297} X^{13} - 1.425556622701337\cdot 10^{-272} X^{12} \\
&\quad + 1.058795883612058\cdot 10^{-249} X^{11} + 8.05364186268366\cdot 10^{-225} X^{10} - 5.183587463234939\cdot 10^{-202} X^{9} \\
&\quad - 2.719851598975594\cdot 10^{-177} X^{8} + 1.402107824165868\cdot 10^{-154} X^{7} + 5.496627599374482\cdot 10^{-130} X^{6} \\
&\quad - 1.995539904687186\cdot 10^{-107} X^{5} - 6.160143177546872\cdot 10^{-83} X^{4} + 1.171077932951323\cdot 10^{-60} X^{3} \\
&\quad + 2.955481817665439\cdot 10^{-36} X^{2} + 3.057490949422222\cdot 10^{-21} X - 2.942043735497876\cdot 10^{-21} \\
&= -2.942043735497876\cdot 10^{-21} B_{0,16}(X) - 2.750950551158987\cdot 10^{-21} B_{1,16}(X) - 2.559857366820099 \\
&\quad \cdot 10^{-21} B_{2,16}(X) - 2.36876418248121\cdot 10^{-21} B_{3,16}(X) - 2.177670998142321\cdot 10^{-21} B_{4,16}(X) \\
&\quad - 1.986577813803432\cdot 10^{-21} B_{5,16}(X) - 1.795484629464543\cdot 10^{-21} B_{6,16}(X) - 1.604391445125654 \\
&\quad \cdot 10^{-21} B_{7,16}(X) - 1.413298260786765\cdot 10^{-21} B_{8,16}(X) - 1.222205076447875\cdot 10^{-21} B_{9,16}(X) \\
&\quad - 1.031111892108986\cdot 10^{-21} B_{10,16}(X) - 8.400187077700972\cdot 10^{-22} B_{11,16}(X) - 6.489255234312081 \\
&\quad \cdot 10^{-22} B_{12,16}(X) - 4.578323390923189\cdot 10^{-22} B_{13,16}(X) - 2.667391547534297\cdot 10^{-22} B_{14,16}(X) \\
&\quad - 7.564597041454045\cdot 10^{-23} B_{15,16}(X) + 1.154472139243488\cdot 10^{-22} B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.9622411919334823, 0.9622411919334823\}$$

Intersection intervals with the x axis:

$$[0.9622411919334823, 0.9622411919334823]$$

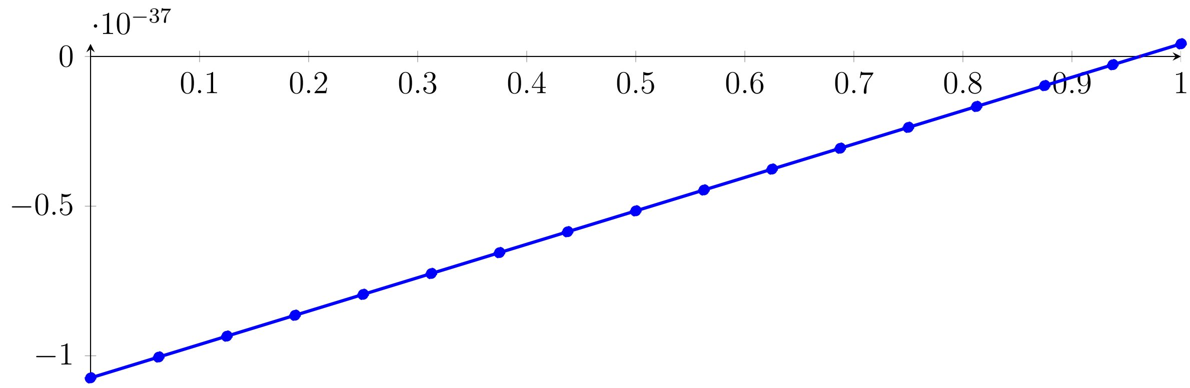
Longest intersection interval: $3.649903549784843\cdot 10^{-17}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

79.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= 6.227129416394075\cdot 10^{-342}X^{16} - 5.276320408729603\cdot 10^{-341}X^{15} - 2.711814423268387\\&\cdot 10^{-341}X^{14} + 1.499867730400293\cdot 10^{-340}X^{13} + 3.046606327375596\cdot 10^{-341}X^{12}\\&+ 1.0236597259982\cdot 10^{-339}X^{11} - 6.03228052820368\cdot 10^{-340}X^{10} + 5.745029074479695\cdot 10^{-340}X^{9}\\&- 8.566359853913263\cdot 10^{-309}X^{8} + 1.209904606350681\cdot 10^{-269}X^{7} + 1.299525013697219\cdot 10^{-228}X^{6}\\&- 1.292609542602263\cdot 10^{-189}X^{5} - 1.093242128853173\cdot 10^{-148}X^{4} + 5.69416387918096\cdot 10^{-110}X^{3}\\&+ 3.937232562828401\cdot 10^{-69}X^{2} + 1.115954706973122\cdot 10^{-37}X - 1.073817587381597\cdot 10^{-37}\\&= -1.073817587381597\cdot 10^{-37}B_{0,16}(X) - 1.004070418195777\cdot 10^{-37}B_{1,16}(X) - 9.343232490099568\\&\cdot 10^{-38}B_{2,16}(X) - 8.645760798241366\cdot 10^{-38}B_{3,16}(X) - 7.948289106383165\cdot 10^{-38}B_{4,16}(X)\\&- 7.250817414524964\cdot 10^{-38}B_{5,16}(X) - 6.553345722666762\cdot 10^{-38}B_{6,16}(X) - 5.855874030808561\\&\cdot 10^{-38}B_{7,16}(X) - 5.15840233895036\cdot 10^{-38}B_{8,16}(X) - 4.460930647092159\cdot 10^{-38}B_{9,16}(X)\\&- 3.763458955233957\cdot 10^{-38}B_{10,16}(X) - 3.065987263375756\cdot 10^{-38}B_{11,16}(X) - 2.368515571517555\\&\cdot 10^{-38}B_{12,16}(X) - 1.671043879659353\cdot 10^{-38}B_{13,16}(X) - 9.735721878011521\cdot 10^{-39}B_{14,16}(X)\\&- 2.761004959429508\cdot 10^{-39}B_{15,16}(X) + 4.213711959152505\cdot 10^{-39}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411919334824, 0.9622411919334824\}$$

Intersection intervals with the x axis:

$$[0.9622411919334824, 0.9622411919334824]$$

Longest intersection interval: $1.332179592273198\cdot 10^{-33}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

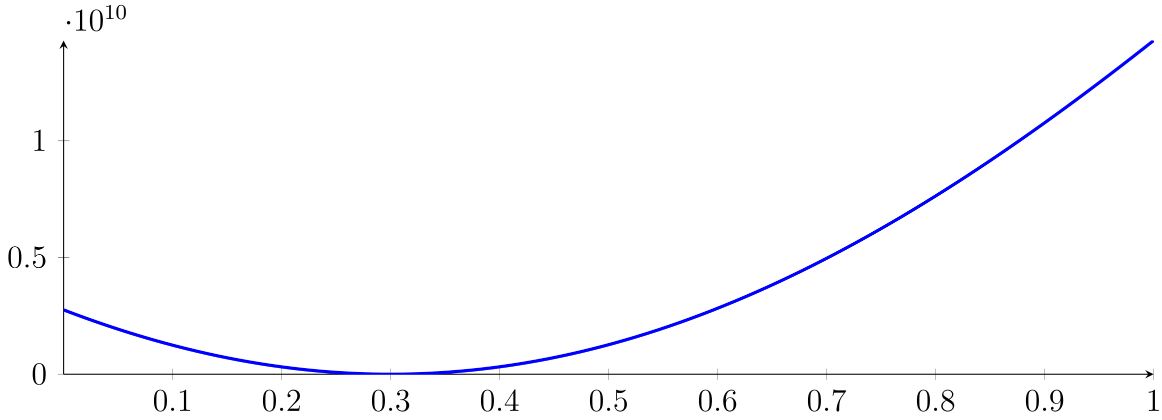
79.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 21!

79.29 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

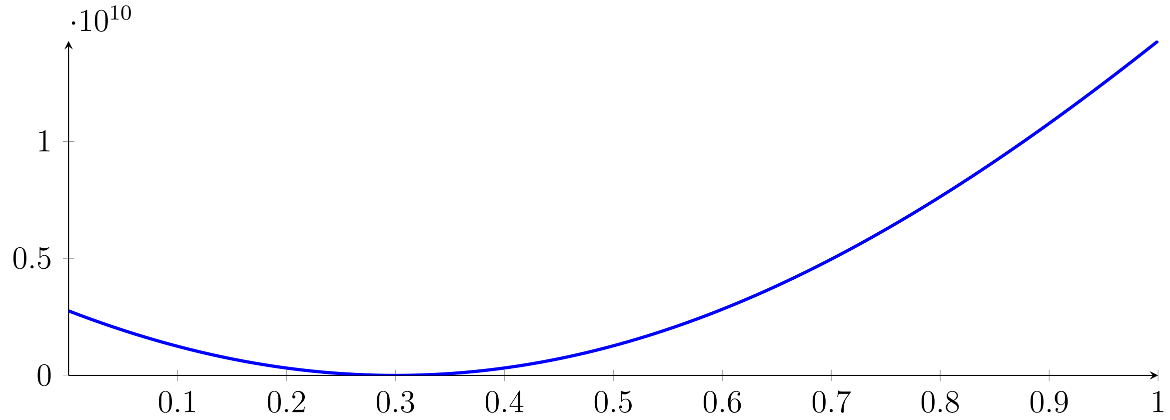
with precision $\varepsilon = 1 \cdot 10^{-64}$.

80 Running `QuadClip` on h_{16} with epsilon 64

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

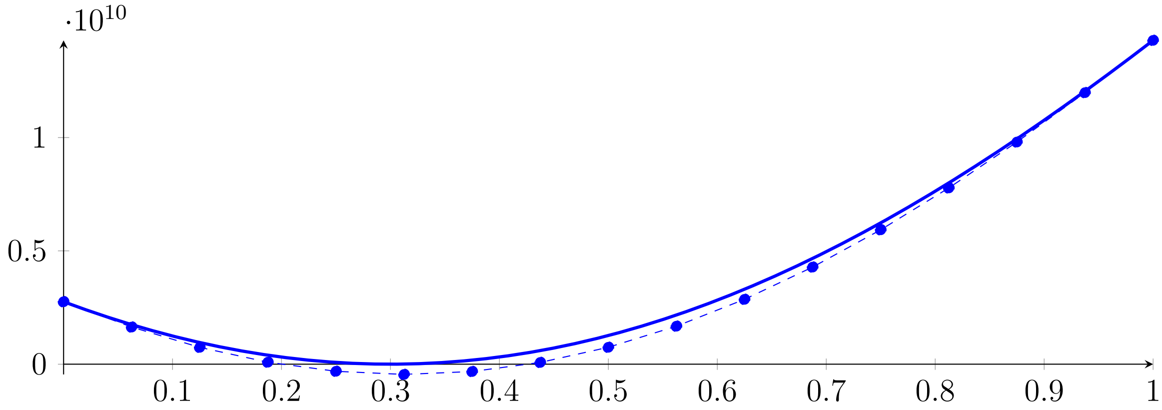
$$\begin{aligned} p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



80.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

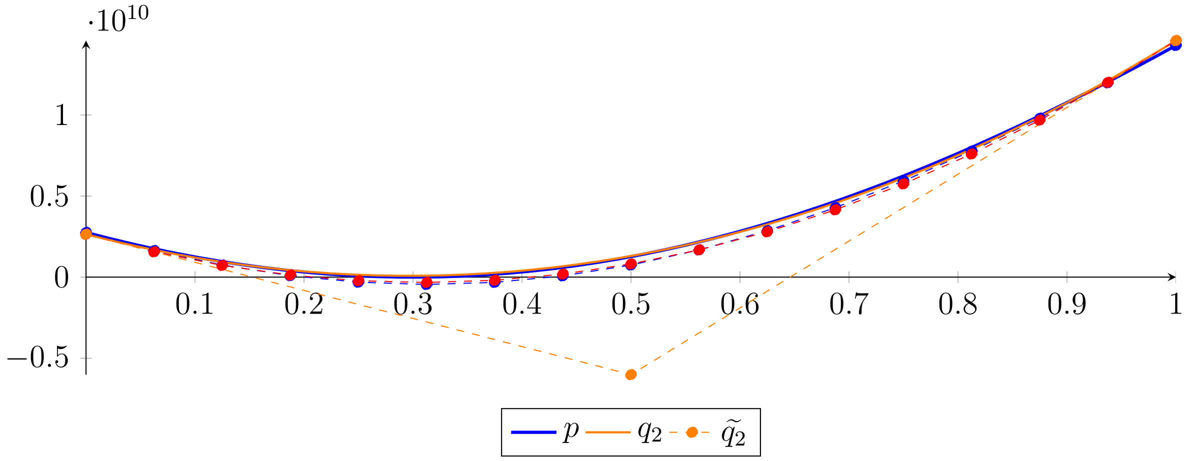
$$\begin{aligned} p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = & \ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ & + 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ & - 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ & + 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ & + 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ & + 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 &= 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328 \\
&= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 &= -4.590615853685512\cdot 10^{-286}X^{16} + 3.626297739922361\cdot 10^{-285}X^{15} - 1.337118562447102 \\
&\quad \cdot 10^{-284}X^{14} + 3.057409950495903\cdot 10^{-284}X^{13} - 4.816080313213711\cdot 10^{-284}X^{12} \\
&\quad + 5.445450819003583\cdot 10^{-284}X^{11} - 4.453195910004646\cdot 10^{-284}X^{10} + 2.601913123147853 \\
&\quad \cdot 10^{-284}X^{9} - 1.060485581673177\cdot 10^{-284}X^{8} + 2.936863540613122\cdot 10^{-285}X^{7} - 5.487274374859613 \\
&\quad \cdot 10^{-286}X^{6} + 7.23018920152705\cdot 10^{-287}X^{5} - 7.254244946204871\cdot 10^{-288}X^{4} + 6.659990111567495 \\
&\quad \cdot 10^{-289}X^{3} + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328 \\
&= 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16} \\
&\quad + 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16} \\
&\quad - 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16} \\
&\quad + 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16} \\
&\quad + 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16} \\
&\quad + 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.

Bounding polynomials M and m:

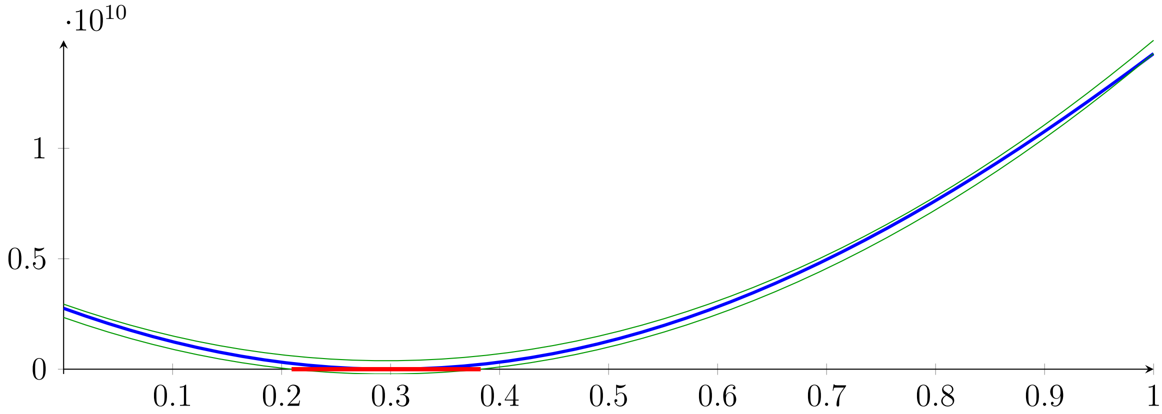
$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$

$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



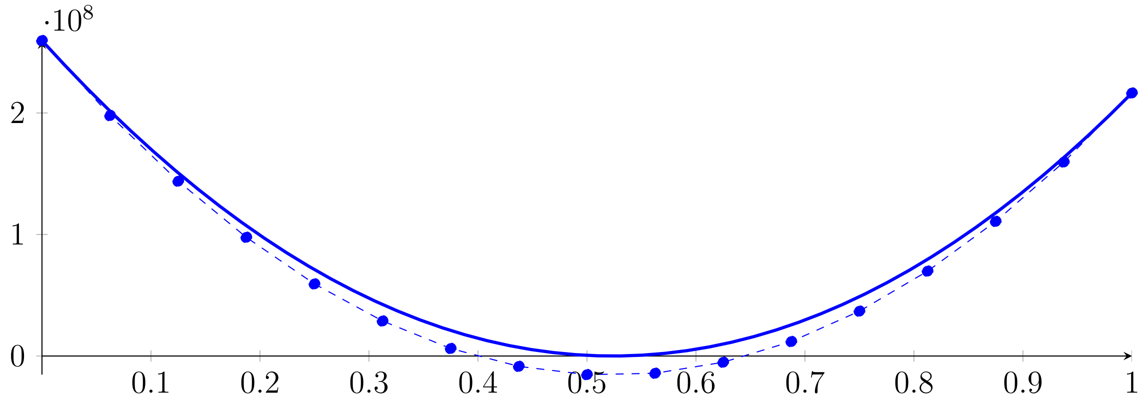
$[0.2091580131065301, 0.3826391383239738]$

Longest intersection interval: 0.1734811252174437
$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

80.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

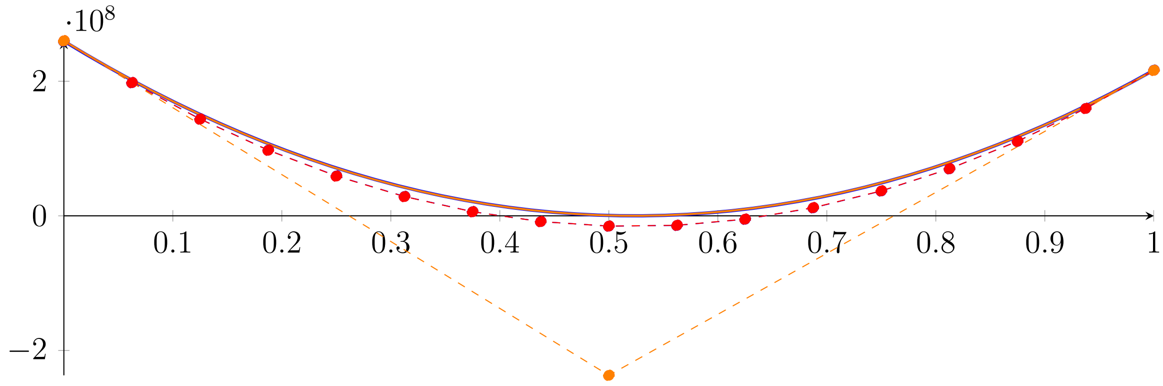
$$\begin{aligned}
p = & -6.730319568869656\cdot 10^{-13}X^{16} + 8.742499458235574\cdot 10^{-12}X^{15} + 4.969734366538428\cdot 10^{-09}X^{14} \\
& - 5.918022439742958\cdot 10^{-08}X^{13} - 1.563835677279211\cdot 10^{-05}X^{12} + 0.000165142833990623X^{11} \\
& + 0.02725178204957372X^{10} - 0.2448183330655992X^{9} - 28.45820232077076X^{8} \\
& + 205.241311077626X^{7} + 17835.39577074806X^{6} - 94141.21233496903X^{5} - 6218439.344794644X^{4} \\
& + 20000843.19616221X^{3} + 930858984.7988209X^{2} - 987724621.0538478X + 259570777.0276934 \\
= & \ 259570777.0276934B_{0,16}(X) + 197837988.2118279B_{1,16}(X) + 143862357.6026193B_{2,16}(X) \\
& + 97679600.99148916B_{3,16}(X) + 59322017.44494463B_{4,16}(X) + 28818467.75210257B_{5,16}(X) \\
& + 6194355.099411763B_{6,16}(X) - 8528392.009487306B_{7,16}(X) - 15331334.57303514B_{8,16}(X) \\
& - 14199535.63666165B_{9,16}(X) - 5121570.552084079B_{10,16}(X) + 11910465.05798682B_{11,16}(X) \\
& + 36900949.63643701B_{12,16}(X) + 69850732.33405701B_{13,16}(X) \\
& + 110757131.9597003B_{14,16}(X) + 159613938.238137B_{15,16}(X) + 216411415.3731615B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= 950064117.9944898X^2 - 993959420.5216784X + 260029868.8092426 \\
&= 260029868.8092426B_{0,2} - 236949841.4515966B_{1,2} + 216134566.282054B_{2,2}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-1.41520176718356\cdot 10^{-287}X^{16} + 1.139753408092347\cdot 10^{-286}X^{15} - 4.276104932493763\\&\cdot 10^{-286}X^{14} + 9.896028733517596\cdot 10^{-286}X^{13} - 1.566233430583374\cdot 10^{-285}X^{12}\\&+ 1.767044733566612\cdot 10^{-285}X^{11} - 1.435814571471649\cdot 10^{-285}X^{10} + 8.336078570237132\\&\cdot 10^{-286}X^{9} - 3.396584250429756\cdot 10^{-286}X^{8} + 9.548198482934522\cdot 10^{-287}X^{7} - 1.861944039074976\\&\cdot 10^{-287}X^{6} + 2.622613520689986\cdot 10^{-288}X^{5} - 2.667693132942884\cdot 10^{-289}X^{4} + 2.048626792988895\\&\cdot 10^{-290}X^{3} + 950064117.9944898X^2 - 993959420.5216784X + 260029868.8092426\\= &\ 260029868.8092426B_{0,16} + 197907405.0266377B_{1,16} + 143702142.2273202B_{2,16}\\&+ 97414080.41129016B_{3,16} + 59043219.5785475B_{4,16} + 28589559.72909226B_{5,16}\\&+ 6053100.862924435B_{6,16} - 8566157.019955976B_{7,16} - 15268213.91954897B_{8,16}\\&- 14053069.83585455B_{9,16} - 4920724.768872719B_{10,16} + 12128821.28139653B_{11,16}\\&+ 37095568.31495319B_{12,16} + 69979516.33179727B_{13,16}\\&+ 110780665.3319288B_{14,16} + 159499015.3153477B_{15,16} + 216134566.282054B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

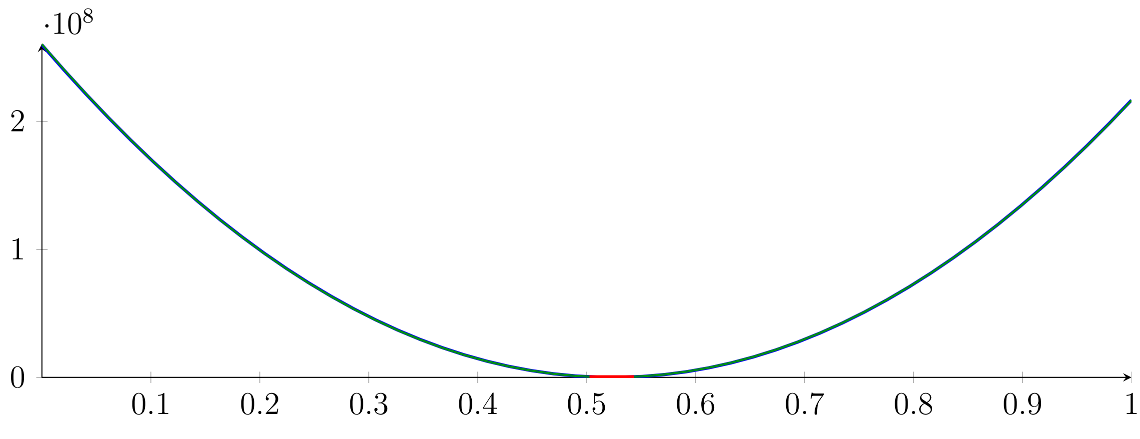
Bounding polynomials M and m:

$$M = 950064117.9944898X^2 - 993959420.5216784X + 260488960.5907918$$
$$m = 950064117.9944898X^2 - 993959420.5216784X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

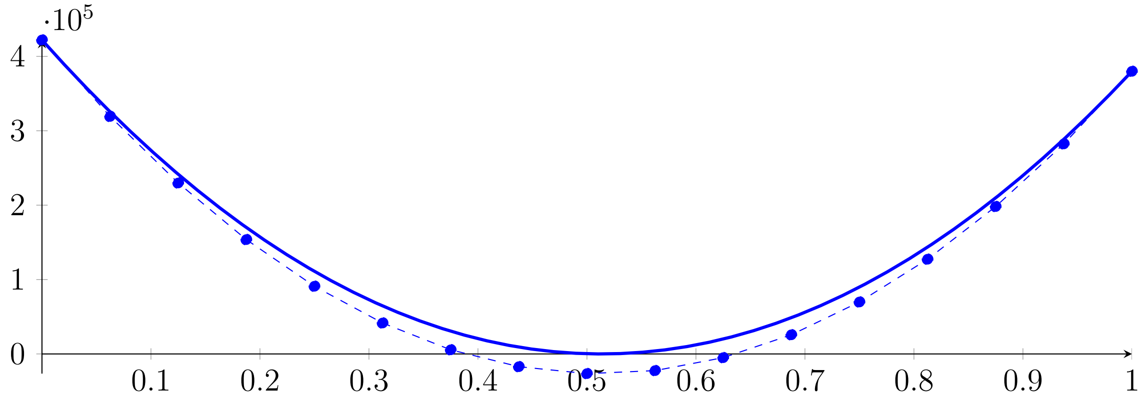
Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

80.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Normalized monomial und Bézier representations and the Bézier polygon:

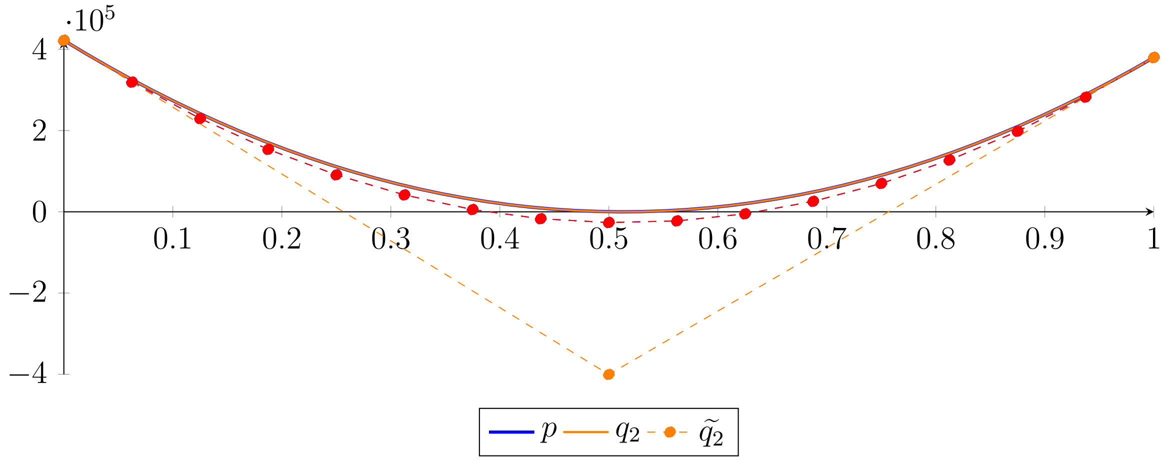
$$\begin{aligned}p &= -4.348008654913047\cdot10^{-35}X^{16} + 5.243388239594399\cdot10^{-33}X^{15} + 1.924264426249969\\ &\cdot10^{-28}X^{14} - 2.246746658841888\cdot10^{-26}X^{13} - 3.625525335135332\cdot10^{-22}X^{12}\\ &+ 3.881306066232412\cdot10^{-20}X^{11} + 3.776139474106064\cdot10^{-16}X^{10} - 3.496022725744\\ &\cdot10^{-14}X^{9} - 2.35123279599996\cdot10^{-10}X^{8} + 1.7435554055155\cdot10^{-08}X^{7} + 8.761425585865998\\ &\cdot10^{-05}X^{6} - 0.004592127899352905X^{5} - 18.10659249096443X^{4} + 504.889471438631X^{3}\\ &+ 1602084.653024796X^{2} - 1644731.182248784X + 422061.2284789393\\ &= 422061.2284789393B_{0,16}(X) + 319265.5295883902B_{1,16}(X) + 229820.5361397145B_{2,16}(X)\\ &+ 153727.1497212539B_{3,16}(X) + 90986.26197267314B_{4,16}(X) + 41598.75458390835B_{5,16}(X)\\ &+ 5565.499294126806B_{6,16}(X) - 17112.6421093025B_{7,16}(X) - 26434.81779182738B_{8,16}(X)\\ &- 22400.18587271997B_{9,16}(X) - 5007.914426073306B_{10,16}(X) + 25742.81851821299B_{11,16}(X)\\ &+ 69852.82497345804B_{12,16}(X) + 127322.906995236B_{13,16}(X) + 198153.8566804232B_{14,16}(X)\\ &+ 282346.4561662563B_{15,16}(X) + 379901.4776294016B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1602810.939315368X^2 - 1645017.556512812X + 422084.9204772878\\ &= 422084.9204772878B_{0,2} - 400423.8577791184B_{1,2} + 379878.3032798431B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -2.394754881528157\cdot10^{-290}X^{16} + 1.927007748221533\cdot10^{-289}X^{15} - 7.222560694913459\\ &\cdot10^{-289}X^{14} + 1.669823679170844\cdot10^{-288}X^{13} - 2.640537385502685\cdot10^{-288}X^{12}\\ &+ 2.977245352423787\cdot10^{-288}X^{11} - 2.4183464566446\cdot10^{-288}X^{10} + 1.403945843258626\cdot10^{-288}X^{9}\\ &- 5.721369736991339\cdot10^{-289}X^{8} + 1.608948017596564\cdot10^{-289}X^{7} - 3.139550473135121\cdot10^{-290}X^{6}\\ &+ 4.427570920272867\cdot10^{-291}X^{5} - 4.516145205270189\cdot10^{-292}X^{4} + 3.478035897244226\\ &\cdot10^{-293}X^{3} + 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\ &= 422084.9204772878B_{0,16} + 319271.323195237B_{1,16} + 229814.4837408143B_{2,16}\\ &+ 153714.4021140197B_{3,16} + 90971.07831485311B_{4,16} + 41584.51234331459B_{5,16}\\ &+ 5554.704199404135B_{6,16} - 17118.34611687825B_{7,16} - 26434.63860553258B_{8,16}\\ &- 22394.17326655884B_{9,16} - 4996.95009995704B_{10,16} + 25757.03089427283B_{11,16}\\ &+ 69867.76971613076B_{12,16} + 127335.2663656168B_{13,16} + 198159.5208427308B_{14,16}\\ &+ 282340.5331474729B_{15,16} + 379878.3032798431B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 23.69199834856345$.
Bounding polynomials M and m:

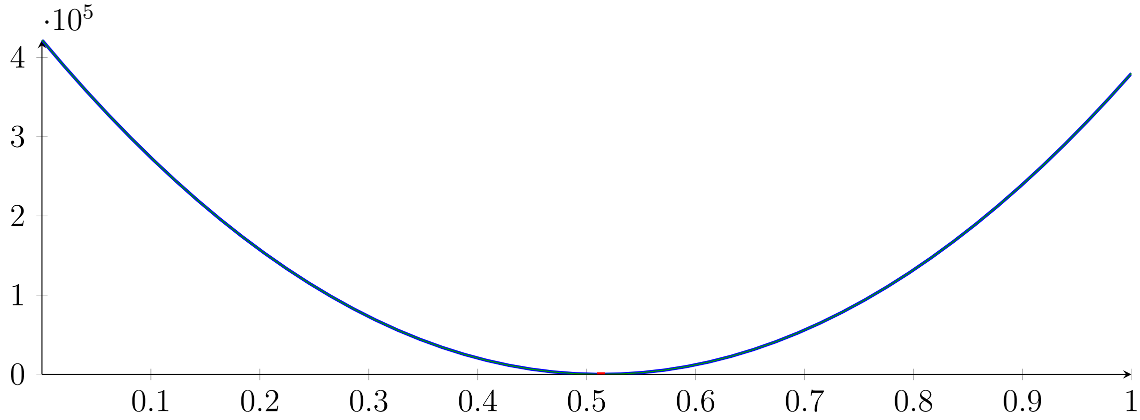
$$M = 1602810.939315368X^2 - 1645017.556512812X + 422108.6124756364$$

$$m = 1602810.939315368X^2 - 1645017.556512812X + 422061.2284789393$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.509405571919099, 0.5169273012614853\}$$

Intersection intervals:



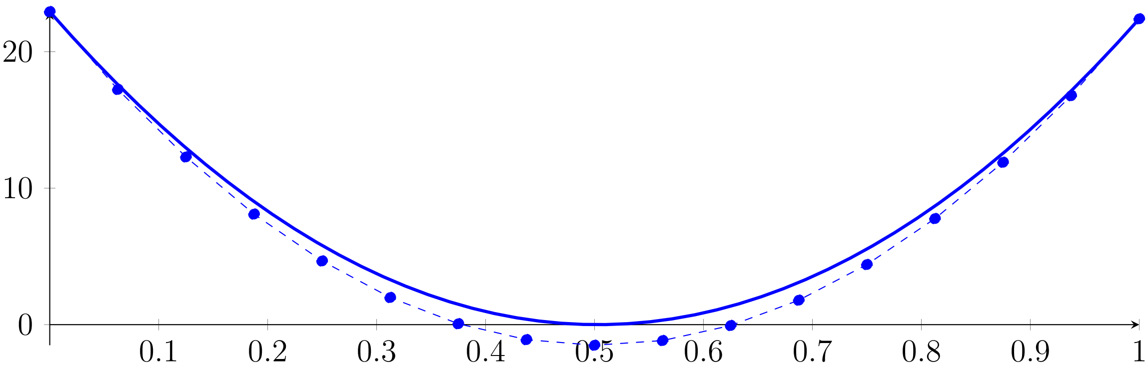
$$[0.509405571919099, 0.5169273012614853]$$

Longest intersection interval: 0.007521729342386311
$\implies$ Selective recursion: interval 1: $[0.299973157656462, 0.3000267016613888]$,

80.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.299973157656462, 0.300026701661388$

Normalized monomial und Bézier representations and the Bézier polygon:

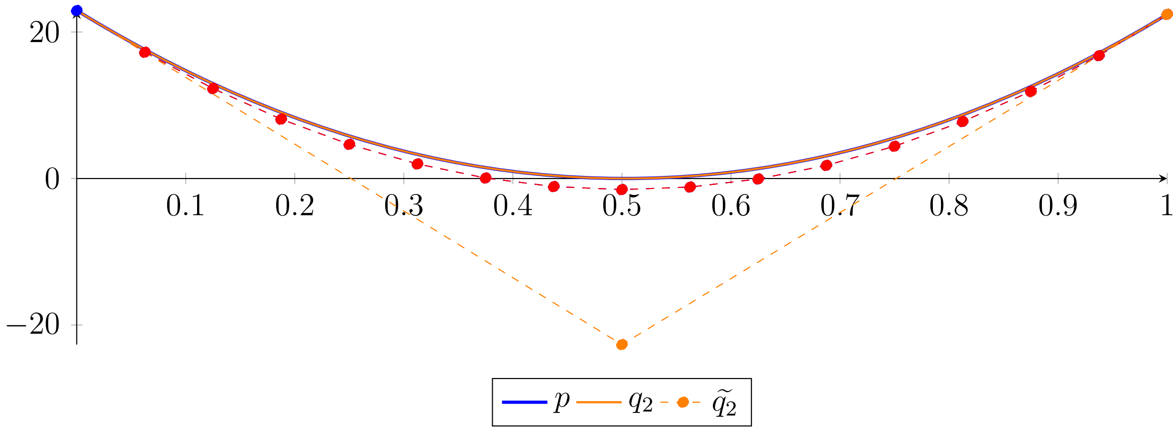
$$\begin{aligned}
p = &-4.564294214760651{\cdot}10^{-69}X^{16} + 6.823166131086062{\cdot}10^{-65}X^{15} + 3.571082953880241\\
&{\cdot}10^{-58}X^{14} - 5.203669764436854{\cdot}10^{-54}X^{13} - 1.189477187926378{\cdot}10^{-47}X^{12}\\
&+ 1.595632405197401{\cdot}10^{-43}X^{11} + 2.190120291778079{\cdot}10^{-37}X^{10} - 2.545939358967532{\cdot}10^{-33}X^{9}\\
&- 2.410599475027015{\cdot}10^{-27}X^{8} + 2.244415896590929{\cdot}10^{-23}X^{7} + 1.587744426227316{\cdot}10^{-17}X^{6}\\
&- 1.041115833763881{\cdot}10^{-13}X^{5} - 5.799356524856294{\cdot}10^{-08}X^{4} + 0.0001991514832499812X^{3}\\
&+ 90.68225982541923X^{2} - 91.20858295780928X + 22.93446410530317\\
= &\ 22.93446410530317B_{0,16}(X) + 17.23392767044009B_{1,16}(X) + 12.28907673412217B_{2,16}(X)\\
&+ 8.099911651977055B_{3,16}(X) + 4.666432779600538B_{4,16}(X) + 1.988640472556532B_{5,16}(X)\\
&+ 0.06653508637709412B_{6,16}(X) - 1.099883023437587B_{7,16}(X)\\
&- 1.510613501419185B_{8,16}(X) - 1.165655992131239B_{9,16}(X) - 0.0650101401691539B_{10,16}(X)\\
&+ 1.791324409839802B_{11,16}(X) + 4.403348013236495B_{12,16}(X) + 7.771061025329927B_{13,16}(X)\\
&+ 11.89446380139723B_{14,16}(X) + 16.77355669668369B_{15,16}(X) + 22.4083400664027B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 &= 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\
&= 22.93447405790644B_{0,2} - 22.66987713993177B_{1,2} + 22.40833011545639B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 = &-1.359001790090183{\cdot}10^{-294}X^{16} + 1.092598333426672{\cdot}10^{-293}X^{15} - 4.09106565145402\\
&{\cdot}10^{-293}X^{14} + 9.449139850665356{\cdot}10^{-293}X^{13} - 1.493001927703419{\cdot}10^{-292}X^{12}\\
&+ 1.682443127957849{\cdot}10^{-292}X^{11} - 1.366228291670638{\cdot}10^{-292}X^{10} + 7.931222707714909\\
&{\cdot}10^{-293}X^{9} - 3.23261546930867{\cdot}10^{-293}X^{8} + 9.093117628361849{\cdot}10^{-294}X^{7} - 1.775022991079505\\
&{\cdot}10^{-294}X^{6} + 2.505263643899121{\cdot}10^{-295}X^{5} - 2.56169827233206{\cdot}10^{-296}X^{4} + 1.979687487137548\\
&{\cdot}10^{-297}X^{3} + 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\
= &\ 22.93447405790644B_{0,16} + 17.23393015817666B_{1,16} + 12.28907424555711B_{2,16}\\
&+ 8.099906320047769B_{3,16} + 4.666426381648652B_{4,16} + 1.988634430359754B_{5,16}\\
&+ 0.06653046618107656B_{6,16} - 1.099885510887381B_{7,16} - 1.510613500845619B_{8,16}\\
&- 1.165653503693637B_{9,16} - 0.06500551943143578B_{10,16} + 1.791330451940986B_{11,16}\\
&+ 4.403354410423627B_{12,16} + 7.771066356016488B_{13,16} + 11.89446628871957B_{14,16}\\
&+ 16.77355420853287B_{15,16} + 22.40833011545639B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 9.952603274324834 \cdot 10^{-06}$.
Bounding polynomials M and m:

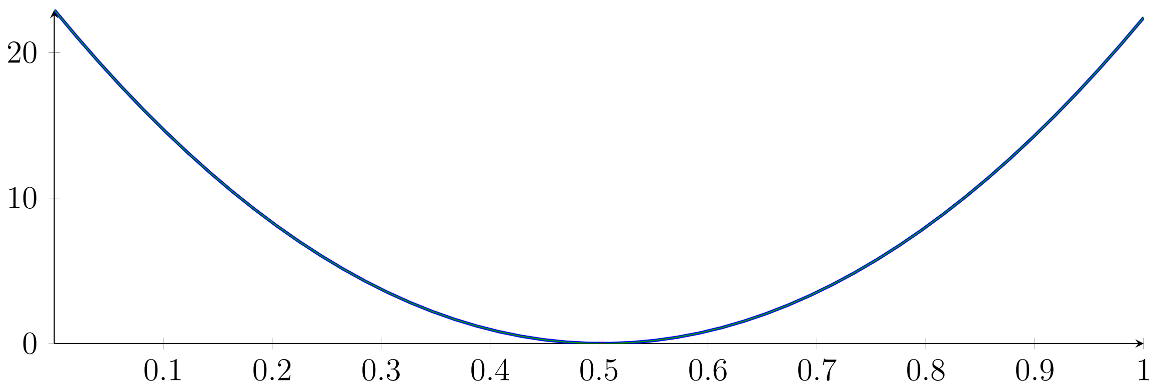
$$M = 90.68255845322638X^2 - 91.20870239567643X + 22.93448401050971$$

$$m = 90.68255845322638X^2 - 91.20870239567643X + 22.93446410530317$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025582179560734, 0.5032438232679998\}$$

Intersection intervals:



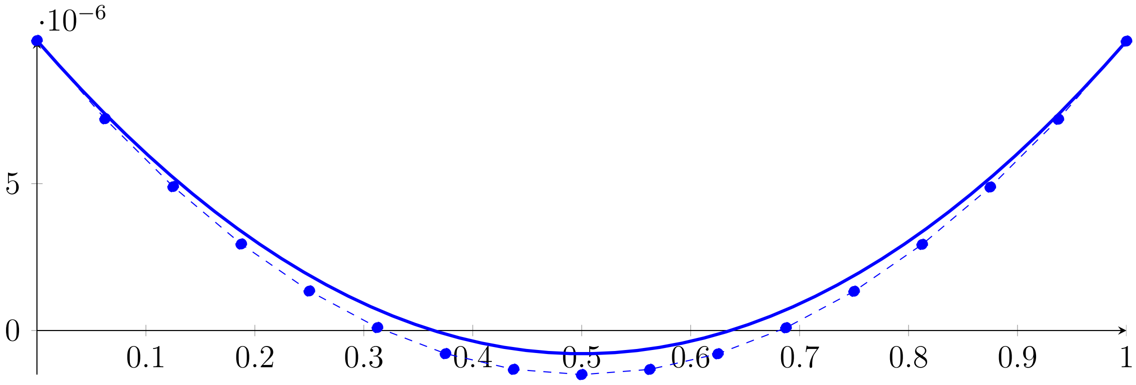
$$[0.5025582179560734, 0.5032438232679998]$$

Longest intersection interval: 0.0006856053119264748
$\implies$ Selective recursion: interval 1: $[0.3000000666361603, 0.3000001033462145]$,

80.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000666361603, 0.300000103346$

Normalized monomial und Bézier representations and the Bézier polygon:

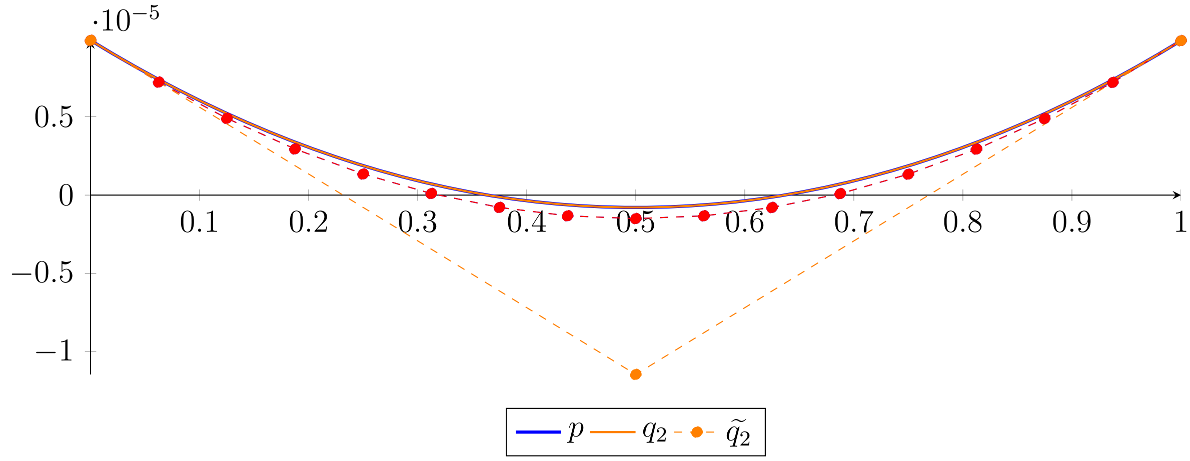
$$
\begin{aligned}
p = &-1.087828446441416 \cdot 10^{-119} X^{16} + 2.370636059353804 \cdot 10^{-112} X^{15} + 1.810668709595505 \\
&\cdot 10^{-102} X^{14} - 3.846486744130667 \cdot 10^{-95} X^{13} - 1.283061410949319 \cdot 10^{-85} X^{12} \\
&+ 2.509305114999168 \cdot 10^{-78} X^{11} + 5.0258723654725 \cdot 10^{-69} X^{10} - 8.517807546222726 \cdot 10^{-62} X^{9} \\
&- 1.17684840157971 \cdot 10^{-52} X^{8} + 1.597478332578791 \cdot 10^{-45} X^{7} + 1.649027121563264 \cdot 10^{-36} X^{6} \\
&- 1.576413560673164 \cdot 10^{-29} X^{5} - 1.281381537279639 \cdot 10^{-20} X^{4} + 6.414336823836069 \cdot 10^{-14} X^{3} \\
&+ 4.262575843118847 \cdot 10^{-05} X^{2} - 4.264622395354259 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06} \\
= &\ 9.875919579833914 \cdot 10^{-06} B_{0,16}(X) + 7.210530582737503 \cdot 10^{-06} B_{1,16}(X) + 4.900356239234328 \\
&\cdot 10^{-06} B_{2,16}(X) + 2.945396549438933 \cdot 10^{-06} B_{3,16}(X) + 1.345651513465858 \cdot 10^{-06} B_{4,16}(X) \\
&+ 1.01121131429646 \cdot 10^{-07} B_{5,16}(X) - 7.881945965551621 \cdot 10^{-07} B_{6,16}(X) - 1.322295670374024 \\
&\cdot 10^{-06} B_{7,16}(X) - 1.501182089912399 \cdot 10^{-06} B_{8,16}(X) - 1.324853855055745 \cdot 10^{-06} B_{9,16}(X) \\
&- 7.933109656895192 \cdot 10^{-07} B_{10,16}(X) + 9.344657830081879 \cdot 10^{-08} B_{11,16}(X) \\
&+ 1.335418777029811 \cdot 10^{-06} B_{12,16}(X) + 2.932605630611999 \cdot 10^{-06} B_{13,16}(X) + 4.885007139161925 \\
&\cdot 10^{-06} B_{14,16}(X) + 7.192623302794129 \cdot 10^{-06} B_{15,16}(X) + 9.855454121623155 \cdot 10^{-06} B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 = &\ 4.26257585274035 \cdot 10^{-05} X^{2} - 4.264622399202859 \cdot 10^{-05} X + 9.875919583041081 \cdot 10^{-06} \\
= &\ 9.875919583041081 \cdot 10^{-06} B_{0,2} - 1.144719241297322 \cdot 10^{-05} B_{1,2} + 9.855454118415988 \cdot 10^{-06} B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = &-6.428697903766316 \cdot 10^{-301} X^{16} + 5.168486014171287 \cdot 10^{-300} X^{15} - 1.931525871486767 \\
&\cdot 10^{-299} X^{14} + 4.443748026156872 \cdot 10^{-299} X^{13} - 6.983462617461767 \cdot 10^{-299} X^{12} \\
&+ 7.823576270761152 \cdot 10^{-299} X^{11} - 6.321034979462039 \cdot 10^{-299} X^{10} + 3.658988671435299 \cdot 10^{-299} X^{9} \\
&- 1.493155040297548 \cdot 10^{-299} X^{8} + 4.235526792785023 \cdot 10^{-300} X^{7} - 8.432417338595836 \cdot 10^{-301} X^{6} \\
&+ 1.226905231702478 \cdot 10^{-301} X^{5} - 1.286361467975875 \cdot 10^{-302} X^{4} + 9.604810259325032 \cdot 10^{-304} X^{3} \\
&+ 4.26257585274035 \cdot 10^{-05} X^{2} - 4.264622399202859 \cdot 10^{-05} X + 9.875919583041081 \cdot 10^{-06} \\
= &\ 9.875919583041081 \cdot 10^{-06} B_{0,16} + 7.210530583539294 \cdot 10^{-06} B_{1,16} + 4.900356238432536 \cdot 10^{-06} B_{2,16} \\
&+ 2.945396547720807 \cdot 10^{-06} B_{3,16} + 1.345651511404108 \cdot 10^{-06} B_{4,16} + 1.011211294824374 \cdot 10^{-07} B_{5,16} \\
&- 7.881945980442039 \cdot 10^{-07} B_{6,16} - 1.322295671175816 \cdot 10^{-06} B_{7,16} - 1.501182089912399 \cdot 10^{-06} B_{8,16} \\
&- 1.324853854253953 \cdot 10^{-06} B_{9,16} - 7.933109642004772 \cdot 10^{-07} B_{10,16} + 9.34465802480274 \cdot 10^{-08} B_{11,16} \\
&+ 1.335418779091561 \cdot 10^{-06} B_{12,16} + 2.932605632330124 \cdot 10^{-06} B_{13,16} + 4.885007139963716 \\
&\cdot 10^{-06} B_{14,16} + 7.192623301992338 \cdot 10^{-06} B_{15,16} + 9.855454118415988 \cdot 10^{-06} B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.207167313591001 \cdot 10^{-15}$.

Bounding polynomials M and m:

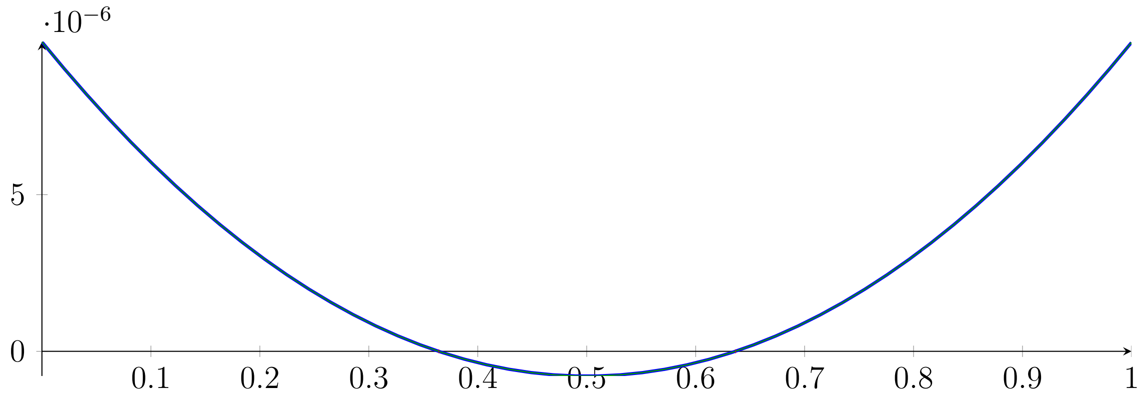
$$M = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919586248249 \cdot 10^{-06}$$

$$m = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06}$$

Root of M and m:

$N(M) = \{0.3640375916978796, 0.6364425279605644\}$ $\quad N(m) = \{0.3640375911454658, 0.6364425285129782\}$

Intersection intervals:



$$[0.3640375911454658, 0.3640375916978796],\ [0.6364425279605644, 0.6364425285129782]$$

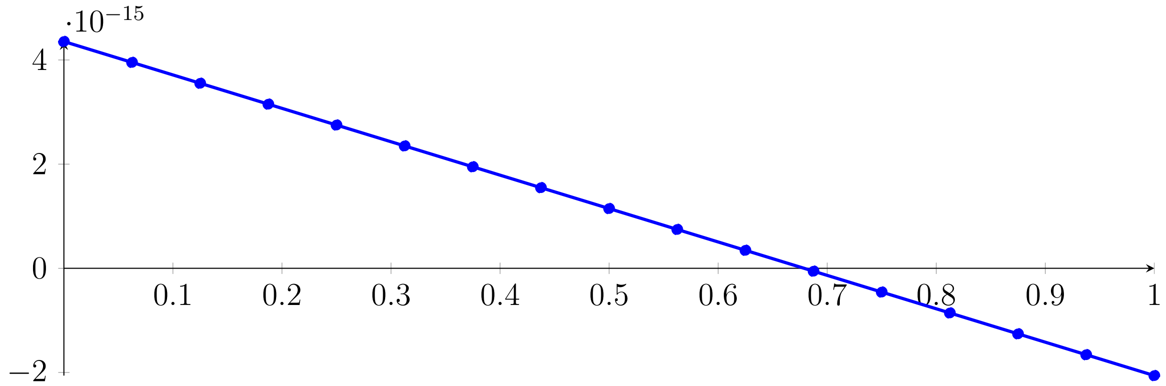
Longest intersection interval: $5.524137987858897 \cdot 10^{-10}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$, interval 2: $[0.30000009, 0.30000009]$,

80.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

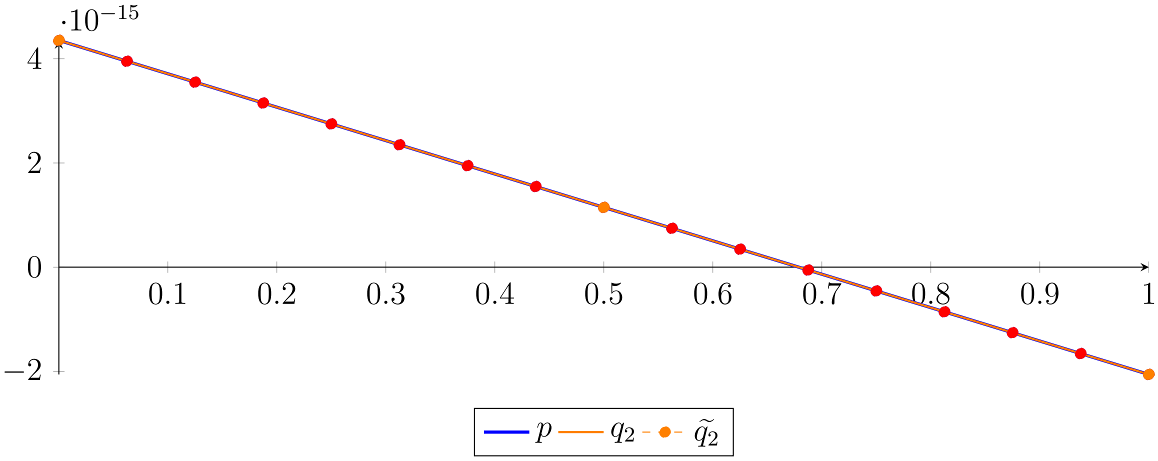
$$\begin{aligned}p = &-8.180742151883693\cdot10^{-268}X^{16} + 3.227249519181262\cdot10^{-251}X^{15} + 4.462130353465124\\ &\cdot10^{-232}X^{14} - 1.715943065651804\cdot10^{-215}X^{13} - 1.03614769176239\cdot10^{-196}X^{12} + 3.668285054157079\\ &\cdot10^{-180}X^{11} + 1.330015307195714\cdot10^{-161}X^{10} - 4.080453140608508\cdot10^{-145}X^{9}\\ &- 1.02055559825224\cdot10^{-126}X^{8} + 2.507763120804761\cdot10^{-110}X^{7} + 4.686136227440808\cdot10^{-92}X^{6}\\ &- 8.109473416022201\cdot10^{-76}X^{5} - 1.19326399448035\cdot10^{-57}X^{4} + 1.081297478088303\cdot10^{-41}X^{3}\\ &+ 1.300771930692191\cdot10^{-23}X^{2} - 6.414334643539715\cdot10^{-15}X + 4.354113798188606\cdot10^{-15}\\ = &\ 4.354113798188606\cdot10^{-15}B_{0,16}(X) + 3.953217882967374\cdot10^{-15}B_{1,16}(X) + 3.552321967854539\\ &\cdot10^{-15}B_{2,16}(X) + 3.151426052850102\cdot10^{-15}B_{3,16}(X) + 2.750530137954063\cdot10^{-15}B_{4,16}(X)\\ &+ 2.349634223166422\cdot10^{-15}B_{5,16}(X) + 1.948738308487178\cdot10^{-15}B_{6,16}(X) + 1.547842393916332\\ &\cdot10^{-15}B_{7,16}(X) + 1.146946479453883\cdot10^{-15}B_{8,16}(X) + 7.460505650998323\cdot10^{-16}B_{9,16}(X)\\ &+ 3.451546508541791\cdot10^{-16}B_{10,16}(X) - 5.574126328307647\cdot10^{-17}B_{11,16}(X) - 4.566371773119344\\ &\cdot10^{-16}B_{12,16}(X) - 8.575330912323946\cdot10^{-16}B_{13,16}(X) - 1.258429005044457\cdot10^{-15}B_{14,16}(X)\\ &- 1.659324918748122\cdot10^{-15}B_{15,16}(X) - 2.060220832343389\cdot10^{-15}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.300771930692191\cdot10^{-23}X^{2} - 6.414334643539715\cdot10^{-15}X + 4.354113798188606\cdot10^{-15}\\ &= 4.354113798188606\cdot10^{-15}B_{0,2} + 1.146946476418749\cdot10^{-15}B_{1,2} - 2.060220832343389\cdot10^{-15}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q_2} = &\ 1.912651228969383\cdot10^{-311}X^{16} - 1.223741742808027\cdot10^{-310}X^{15} + 2.733522262624021\\ &\cdot10^{-310}X^{14} - 8.679644486233328\cdot10^{-311}X^{13} - 7.862443329477107\cdot10^{-310}X^{12}\\ &+ 1.83195702234036\cdot10^{-309}X^{11} - 2.057972005087252\cdot10^{-309}X^{10} + 1.349302407721643\cdot10^{-309}X^{9}\\ &- 5.089946779834537\cdot10^{-310}X^{8} + 8.605298284676211\cdot10^{-311}X^{7} + 8.242572388969257\cdot10^{-312}X^{6}\\ &- 6.927894069632637\cdot10^{-312}X^{5} + 1.356821367314693\cdot10^{-312}X^{4} - 8.054206617575827\cdot10^{-314}X^{3}\\ &+ 1.300771930692191\cdot10^{-23}X^{2} - 6.414334643539715\cdot10^{-15}X + 4.354113798188606\cdot10^{-15}\\ = &\ 4.354113798188606\cdot10^{-15}B_{0,16} + 3.953217882967374\cdot10^{-15}B_{1,16} + 3.552321967854539\\ &\cdot10^{-15}B_{2,16} + 3.151426052850102\cdot10^{-15}B_{3,16} + 2.750530137954063\cdot10^{-15}B_{4,16}\\ &+ 2.349634223166422\cdot10^{-15}B_{5,16} + 1.948738308487178\cdot10^{-15}B_{6,16} + 1.547842393916332\\ &\cdot10^{-15}B_{7,16} + 1.146946479453883\cdot10^{-15}B_{8,16} + 7.460505650998323\cdot10^{-16}B_{9,16}\\ &+ 3.451546508541791\cdot10^{-16}B_{10,16} - 5.574126328307647\cdot10^{-17}B_{11,16} - 4.566371773119344\\ &\cdot10^{-16}B_{12,16} - 8.575330912323946\cdot10^{-16}B_{13,16} - 1.258429005044457\cdot10^{-15}B_{14,16}\\ &- 1.659324918748122\cdot10^{-15}B_{15,16} - 2.060220832343389\cdot10^{-15}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.406487390441514 \cdot 10^{-43}$.

Bounding polynomials M and m:

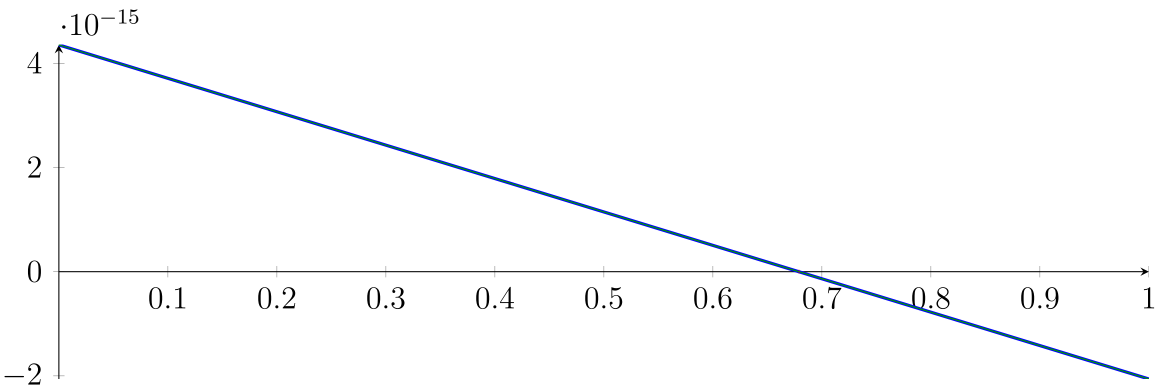
$$M = 1.300771930692191 \cdot 10^{-23} X^2 - 6.414334643539715 \cdot 10^{-15} X + 4.354113798188606 \cdot 10^{-15}$$

$$m = 1.300771930692191 \cdot 10^{-23} X^2 - 6.414334643539715 \cdot 10^{-15} X + 4.354113798188606 \cdot 10^{-15}$$

Root of M and m:

$N(M) = \{0.678809891617932, 493117546.8474808\}$ $\quad N(m) = \{0.678809891617932, 493117546.8474808\}$

Intersection intervals:



$$[0.678809891617932, 0.678809891617932]$$

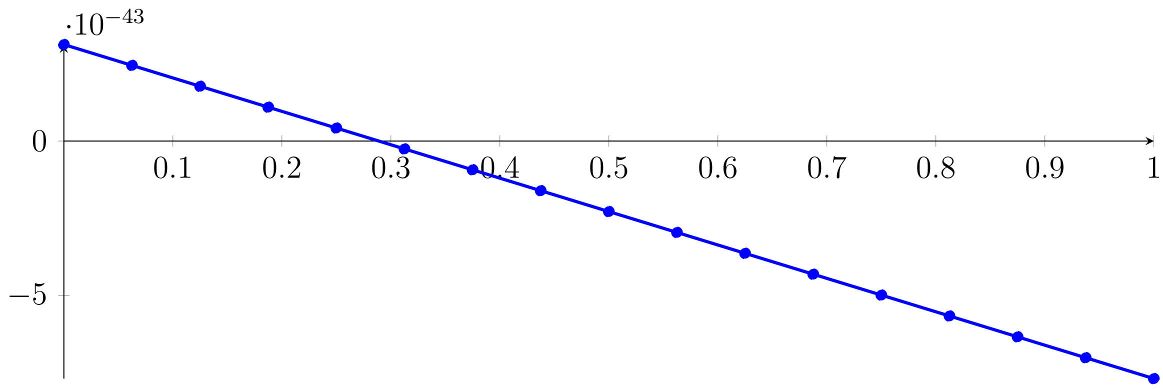
Longest intersection interval: $1.685751587897438 \cdot 10^{-28}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

80.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

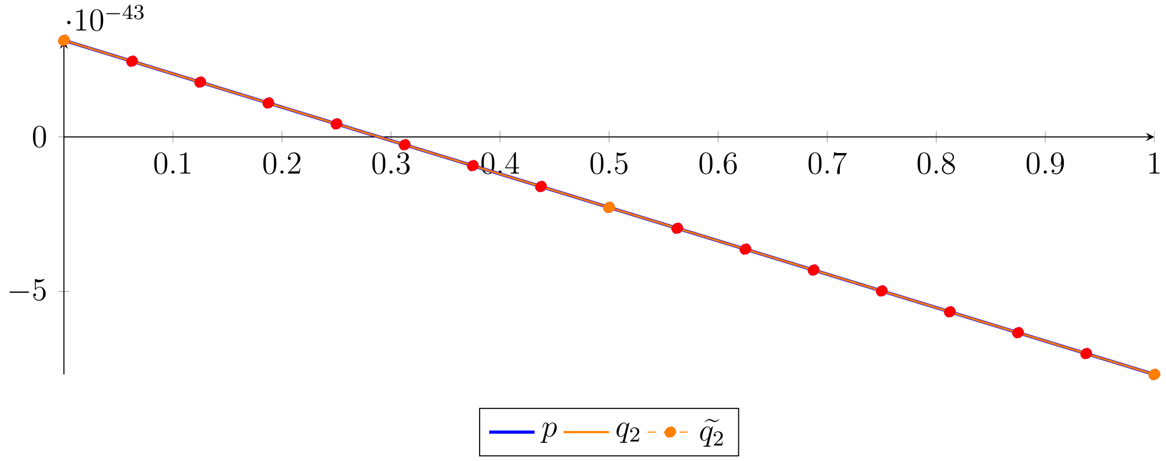
$$\begin{aligned}p &= -1.309058005179934\cdot 10^{-347}X^{16} + 1.062571961277761\cdot 10^{-346}X^{15} + 3.448250355108118\\ &\quad\cdot 10^{-347}X^{14} - 6.257935829640659\cdot 10^{-346}X^{13} - 1.627063315706571\cdot 10^{-345}X^{12}\\ &\quad - 2.789251398354122\cdot 10^{-346}X^{11} - 1.022725512729845\cdot 10^{-345}X^{10} + 1.278406890912306\cdot 10^{-345}X^{9}\\ &\quad - 4.109165006503841\cdot 10^{-346}X^{8} + 9.701559540649497\cdot 10^{-305}X^{7} + 1.075415704126648\cdot 10^{-258}X^{6}\\ &\quad - 1.103978312017599\cdot 10^{-214}X^{5} - 9.636311849715274\cdot 10^{-169}X^{4} + 5.1799542305243\cdot 10^{-125}X^{3}\\ &\quad + 3.696479581469533\cdot 10^{-79}X^{2} - 1.081297478088303\cdot 10^{-42}X + 3.12447403488504\cdot 10^{-43}\\ &= 3.12447403488504\cdot 10^{-43}B_{0,16}(X) + 2.448663111079851\cdot 10^{-43}B_{1,16}(X) + 1.772852187274661\\ &\quad\cdot 10^{-43}B_{2,16}(X) + 1.097041263469472\cdot 10^{-43}B_{3,16}(X) + 4.212303396642828\cdot 10^{-44}B_{4,16}(X)\\ &\quad - 2.545805841409065\cdot 10^{-44}B_{5,16}(X) - 9.303915079460958\cdot 10^{-44}B_{6,16}(X) - 1.606202431751285\\ &\quad\cdot 10^{-43}B_{7,16}(X) - 2.282013355556474\cdot 10^{-43}B_{8,16}(X) - 2.957824279361664\cdot 10^{-43}B_{9,16}(X)\\ &\quad - 3.633635203166853\cdot 10^{-43}B_{10,16}(X) - 4.309446126972042\cdot 10^{-43}B_{11,16}(X)\\ &\quad - 4.985257050777232\cdot 10^{-43}B_{12,16}(X) - 5.661067974582421\cdot 10^{-43}B_{13,16}(X) - 6.33687889838761\\ &\quad\cdot 10^{-43}B_{14,16}(X) - 7.0126898221928\cdot 10^{-43}B_{15,16}(X) - 7.688500745997989\cdot 10^{-43}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 3.696479581469533\cdot 10^{-79}X^{2} - 1.081297478088303\cdot 10^{-42}X + 3.12447403488504\cdot 10^{-43}\\ &= 3.12447403488504\cdot 10^{-43}B_{0,2} - 2.282013355556474\cdot 10^{-43}B_{1,2} - 7.688500745997989\cdot 10^{-43}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= 1.321235643657083\cdot 10^{-339}X^{16} - 4.707039676382068\cdot 10^{-339}X^{15} + 3.761577682517833\\ &\quad\cdot 10^{-339}X^{14} - 4.359613352194916\cdot 10^{-339}X^{13} + 4.54005469441269\cdot 10^{-338}X^{12}\\ &\quad - 1.312163583808635\cdot 10^{-337}X^{11} + 1.79382514864333\cdot 10^{-337}X^{10} - 1.339511978482456\cdot 10^{-337}X^{9}\\ &\quad + 5.208651878296088\cdot 10^{-338}X^{8} - 5.540207059869677\cdot 10^{-339}X^{7} - 3.436463990345086\cdot 10^{-339}X^{6}\\ &\quad + 1.463153300127776\cdot 10^{-339}X^{5} - 2.06132658730541\cdot 10^{-340}X^{4} + 1.627315868116839\cdot 10^{-342}X^{3}\\ &\quad + 3.696479581469533\cdot 10^{-79}X^{2} - 1.081297478088303\cdot 10^{-42}X + 3.12447403488504\cdot 10^{-43}\\ &= 3.12447403488504\cdot 10^{-43}B_{0,16} + 2.448663111079851\cdot 10^{-43}B_{1,16} + 1.772852187274661\cdot 10^{-43}B_{2,16}\\ &\quad + 1.097041263469472\cdot 10^{-43}B_{3,16} + 4.212303396642828\cdot 10^{-44}B_{4,16} - 2.545805841409065\cdot 10^{-44}B_{5,16}\\ &\quad - 9.303915079460958\cdot 10^{-44}B_{6,16} - 1.606202431751285\cdot 10^{-43}B_{7,16} - 2.282013355556474\cdot 10^{-43}B_{8,16}\\ &\quad - 2.957824279361664\cdot 10^{-43}B_{9,16} - 3.633635203166853\cdot 10^{-43}B_{10,16} - 4.309446126972042\\ &\quad\cdot 10^{-43}B_{11,16} - 4.985257050777232\cdot 10^{-43}B_{12,16} - 5.661067974582421\cdot 10^{-43}B_{13,16}\\ &\quad - 6.33687889838761\cdot 10^{-43}B_{14,16} - 7.0126898221928\cdot 10^{-43}B_{15,16} - 7.688500745997989\cdot 10^{-43}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.58997711526215 \cdot 10^{-126}$.
Bounding polynomials M and m:

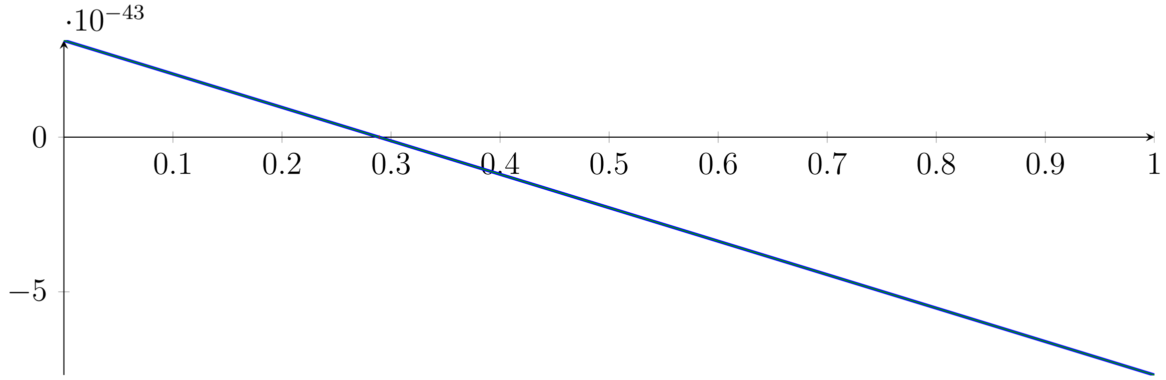
$$M = 3.696479581469533 \cdot 10^{-79} X^2 - 1.081297478088303 \cdot 10^{-42} X + 3.12447403488504 \cdot 10^{-43}$$

$$m = 3.696479581469533 \cdot 10^{-79} X^2 - 1.081297478088303 \cdot 10^{-42} X + 3.12447403488504 \cdot 10^{-43}$$

Root of M and m:

$N(M) = \{0.2889560087025269, 2.925208848735027 \cdot 10^{36}\}$ $\quad N(m) = \{0.2889560087025269, 2.92520884873502$

Intersection intervals:



$$[0.2889560087025269, 0.2889560087025269]$$

Longest intersection interval: $4.790498762359349 \cdot 10^{-84}$
$\implies$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

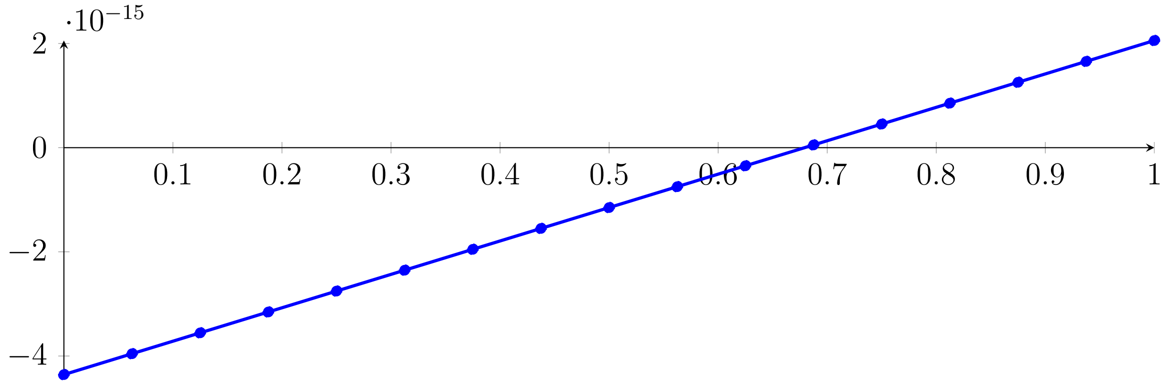
80.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 8!

80.9 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

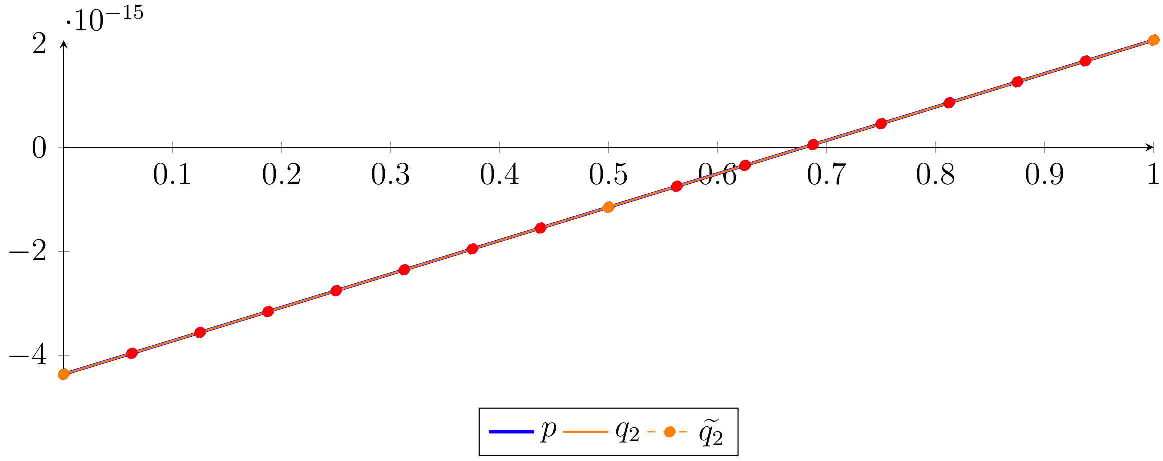
$$\begin{aligned} p = & -8.180742151883693\cdot 10^{-268}X^{16} + 3.227248873730463\cdot 10^{-251}X^{15} + 4.462130355852244 \\ & \cdot 10^{-232}X^{14} - 1.715942757602136\cdot 10^{-215}X^{13} - 1.0361476928624\cdot 10^{-196}X^{12} + 3.66828444102595 \\ & \cdot 10^{-180}X^{11} + 1.330015309185499\cdot 10^{-161}X^{10} - 4.080452484754623\cdot 10^{-145}X^{9} \\ & - 1.020555600063168\cdot 10^{-126}X^{8} + 2.507762718201662\cdot 10^{-110}X^{7} + 4.686136236097161\cdot 10^{-92}X^{6} \\ & - 8.109472029532602\cdot 10^{-76}X^{5} - 1.193263996479812\cdot 10^{-57}X^{4} + 1.081297242720538\cdot 10^{-41}X^{3} \\ & + 1.300771932291811\cdot 10^{-23}X^{2} + 6.414334610838192\cdot 10^{-15}X - 4.357019382395145\cdot 10^{-15} \\ = & -4.357019382395145\cdot 10^{-15}B_{0,16}(X) - 3.956123469217758\cdot 10^{-15}B_{1,16}(X) - 3.555227555931974 \\ & \cdot 10^{-15}B_{2,16}(X) - 3.154331642537791\cdot 10^{-15}B_{3,16}(X) - 2.753435729035212\cdot 10^{-15}B_{4,16}(X) \\ & - 2.352539815424234\cdot 10^{-15}B_{5,16}(X) - 1.951643901704859\cdot 10^{-15}B_{6,16}(X) - 1.550747987877086 \\ & \cdot 10^{-15}B_{7,16}(X) - 1.149852073940915\cdot 10^{-15}B_{8,16}(X) - 7.489561598963468\cdot 10^{-16}B_{9,16}(X) \\ & - 3.480602457433808\cdot 10^{-16}B_{10,16}(X) + 5.283566851798276\cdot 10^{-17}B_{11,16}(X) \\ & + 4.53731582887744\cdot 10^{-16}B_{12,16}(X) + 8.546274973659029\cdot 10^{-16}B_{13,16}(X) + 1.255523411952459 \\ & \cdot 10^{-15}B_{14,16}(X) + 1.656419326647414\cdot 10^{-15}B_{15,16}(X) + 2.057315241450766\cdot 10^{-15}B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$\begin{aligned} q_2 = & 1.300771932291811\cdot 10^{-23}X^{2} + 6.414334610838192\cdot 10^{-15}X - 4.357019382395145\cdot 10^{-15} \\ = & -4.357019382395145\cdot 10^{-15}B_{0,2} - 1.14985207697605\cdot 10^{-15}B_{1,2} + 2.057315241450766\cdot 10^{-15}B_{2,2} \end{aligned}$$

$$\begin{aligned} \widetilde{q}_2 = & -1.913962948891774\cdot 10^{-311}X^{16} + 1.224839229220605\cdot 10^{-310}X^{15} - 2.736439272631334 \\ & \cdot 10^{-310}X^{14} + 8.686727754411293\cdot 10^{-311}X^{13} + 7.87470789022501\cdot 10^{-310}X^{12} - 1.83498999880619 \\ & \cdot 10^{-309}X^{11} + 2.061599642193156\cdot 10^{-309}X^{10} - 1.351793491536379\cdot 10^{-309}X^{9} \\ & + 5.099450965546735\cdot 10^{-310}X^{8} - 8.61911462129453\cdot 10^{-311}X^{7} - 8.275840669030232\cdot 10^{-312}X^{6} \\ & + 6.946029354848196\cdot 10^{-312}X^{5} - 1.359818722878116\cdot 10^{-312}X^{4} + 8.064687133307928\cdot 10^{-314}X^{3} \\ & + 1.300771932291811\cdot 10^{-23}X^{2} + 6.414334610838192\cdot 10^{-15}X - 4.357019382395145\cdot 10^{-15} \\ = & -4.357019382395145\cdot 10^{-15}B_{0,16} - 3.956123469217758\cdot 10^{-15}B_{1,16} - 3.555227555931974 \\ & \cdot 10^{-15}B_{2,16} - 3.154331642537791\cdot 10^{-15}B_{3,16} - 2.753435729035212\cdot 10^{-15}B_{4,16} \\ & - 2.352539815424234\cdot 10^{-15}B_{5,16} - 1.951643901704859\cdot 10^{-15}B_{6,16} - 1.550747987877086 \\ & \cdot 10^{-15}B_{7,16} - 1.149852073940915\cdot 10^{-15}B_{8,16} - 7.489561598963468\cdot 10^{-16}B_{9,16} \\ & - 3.480602457433808\cdot 10^{-16}B_{10,16} + 5.283566851798276\cdot 10^{-17}B_{11,16} + 4.53731582887744 \\ & \cdot 10^{-16}B_{12,16} + 8.546274973659029\cdot 10^{-16}B_{13,16} + 1.255523411952459\cdot 10^{-15}B_{14,16} \\ & + 1.656419326647414\cdot 10^{-15}B_{15,16} + 2.057315241450766\cdot 10^{-15}B_{16,16} \end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.406486213602688 \cdot 10^{-43}$.
Bounding polynomials M and m:

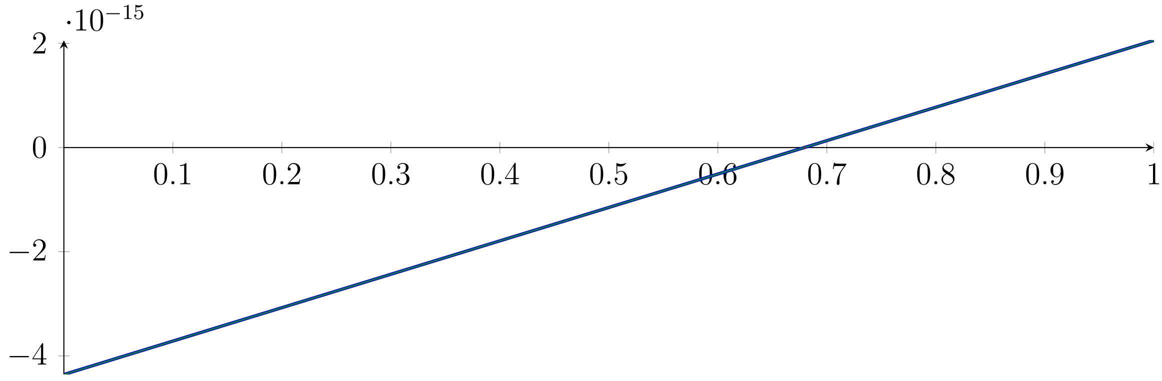
$$M = 1.300771932291811 \cdot 10^{-23} X^2 + 6.414334610838192 \cdot 10^{-15} X - 4.357019382395145 \cdot 10^{-15}$$

$$m = 1.300771932291811 \cdot 10^{-23} X^2 + 6.414334610838192 \cdot 10^{-15} X - 4.357019382395145 \cdot 10^{-15}$$

Root of M and m:

$N(M) = \{-493117545.0851349, 0.6792628761573223\}$ $N(m) = \{-493117545.0851349, 0.6792628761573223\}$

Intersection intervals:



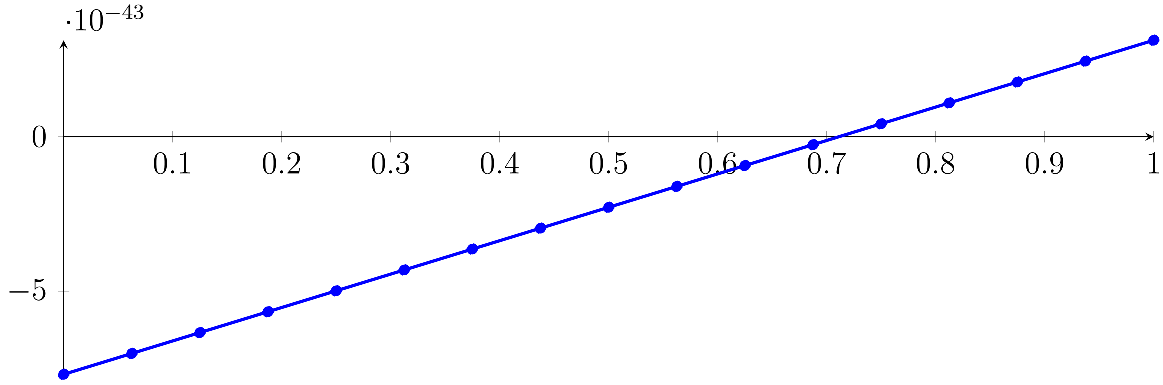
$$[0.6792628761573223, 0.6792628761573223]$$

Longest intersection interval: $1.685751220266157 \cdot 10^{-28}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

80.10 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

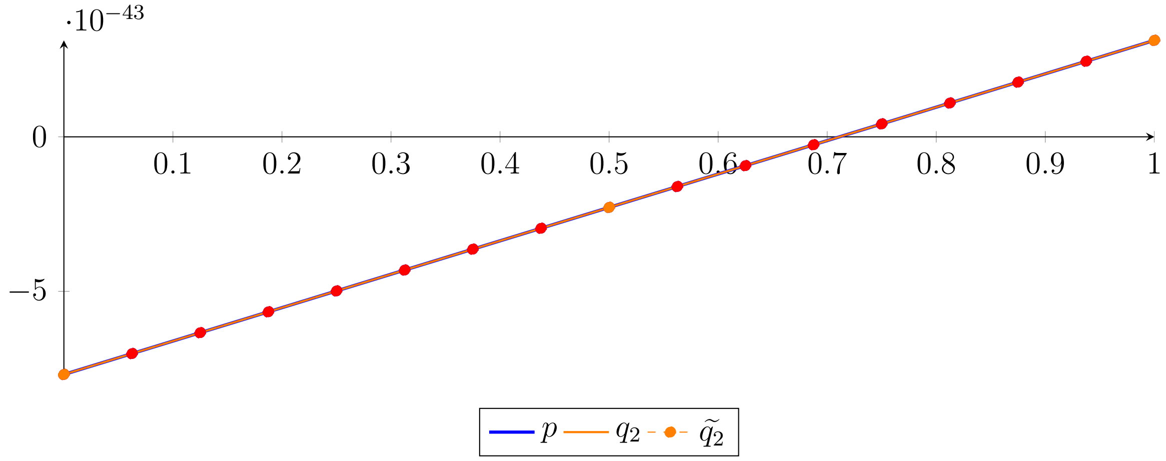
$$p = -6.385648805755774\cdot10^{-349}X^{16} + 2.55425952230231\cdot10^{-347}X^{15} + 5.44057278250392\cdot10^{-346}X^{14} - 1.394625699177061\cdot10^{-345}X^{12} - 1.394625699177061\cdot10^{-345}X^{11} - 1.022725512729845\cdot10^{-345}X^{10} - 2.922072893513842\cdot10^{-345}X^{9} - 2.465499003902304\cdot10^{-345}X^{8} + 9.701543173030387\cdot10^{-305}X^{7} + 1.075414298944151\cdot10^{-258}X^{6} - 1.103976919483014\cdot10^{-214}X^{5} - 9.636303459858408\cdot10^{-169}X^{4} + 5.179949714039793\cdot10^{-125}X^{3} + 3.696477973747613\cdot10^{-79}X^{2} + 1.081297242720538\cdot10^{-42}X - 7.691136110088861\cdot10^{-43}$$
$$= -7.691136110088861\cdot10^{-43}B_{0,16}(X) - 7.015325333388525\cdot10^{-43}B_{1,16}(X) - 6.33951455668189\cdot10^{-43}B_{2,16}(X) - 5.663703779987853\cdot10^{-43}B_{3,16}(X) - 4.987893003287517\cdot10^{-43}B_{4,16}(X) - 4.312082226587181\cdot10^{-43}B_{5,16}(X) - 3.636271449886845\cdot10^{-43}B_{6,16}(X) - 2.960460673186509\cdot10^{-43}B_{7,16}(X) - 2.284649896486173\cdot10^{-43}B_{8,16}(X) - 1.608839119785837\cdot10^{-43}B_{9,16}(X) - 9.330283430855006\cdot10^{-44}B_{10,16}(X) - 2.572175663851646\cdot10^{-44}B_{11,16}(X) + 4.185932103151714\cdot10^{-44}B_{12,16}(X) + 1.094403987015507\cdot10^{-43}B_{13,16}(X) + 1.770214763715843\cdot10^{-43}B_{14,16}(X) + 2.446025540416179\cdot10^{-43}B_{15,16}(X) + 3.121836317116515\cdot10^{-43}B_{16,16}(X)$$



Degree reduction and raising:

$$q_2 = 3.696477973747613\cdot10^{-79}X^{2} + 1.081297242720538\cdot10^{-42}X - 7.691136110088861\cdot10^{-43}$$
$$= -7.691136110088861\cdot10^{-43}B_{0,2} - 2.284649896486173\cdot10^{-43}B_{1,2} + 3.121836317116515\cdot10^{-43}B_{2,2}$$

$$\widetilde{q}_2 = -3.382791032187499\cdot10^{-339}X^{16} + 2.195568979483387\cdot10^{-338}X^{15} - 4.960588406067856\cdot10^{-338}X^{14} + 1.548747367109897\cdot10^{-338}X^{13} + 1.473652152363228\cdot10^{-337}X^{12} - 3.454811461973597\cdot10^{-337}X^{11} + 3.907683672823046\cdot10^{-337}X^{10} - 2.575671844035123\cdot10^{-337}X^{9} + 9.729121737498621\cdot10^{-338}X^{8} - 1.617643528105738\cdot10^{-338}X^{7} - 1.791534296229099\cdot10^{-339}X^{6} + 1.387056472791705\cdot10^{-339}X^{5} - 2.649535616707909\cdot10^{-340}X^{4} + 1.484406069547834\cdot10^{-341}X^{3} + 3.696477973747613\cdot10^{-79}X^{2} + 1.081297242720538\cdot10^{-42}X - 7.691136110088861\cdot10^{-43}$$
$$= -7.691136110088861\cdot10^{-43}B_{0,16} - 7.015325333388525\cdot10^{-43}B_{1,16} - 6.33951455668189\cdot10^{-43}B_{2,16} - 5.663703779987853\cdot10^{-43}B_{3,16} - 4.987893003287517\cdot10^{-43}B_{4,16} - 4.312082226587181\cdot10^{-43}B_{5,16} - 3.636271449886845\cdot10^{-43}B_{6,16} - 2.960460673186509\cdot10^{-43}B_{7,16} - 2.284649896486173\cdot10^{-43}B_{8,16} - 1.608839119785837\cdot10^{-43}B_{9,16} - 9.330283430855006\cdot10^{-44}B_{10,16} - 2.572175663851646\cdot10^{-44}B_{11,16} + 4.185932103151714\cdot10^{-44}B_{12,16} + 1.094403987015507\cdot10^{-43}B_{13,16} + 1.770214763715843\cdot10^{-43}B_{14,16} + 2.446025540416179\cdot10^{-43}B_{15,16} + 3.121836317116515\cdot10^{-43}B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 2.589974857019897 \cdot 10^{-126}$.
Bounding polynomials M and m:

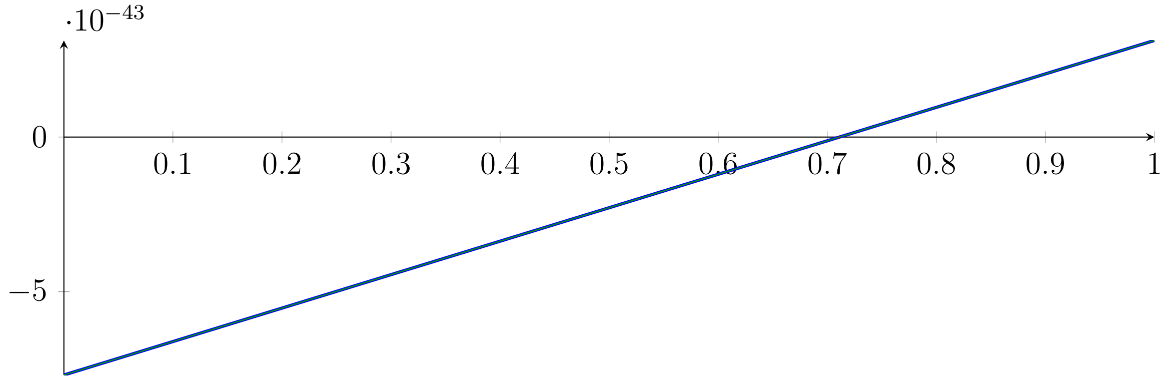
$$M = 3.696477973747613 \cdot 10^{-79} X^2 + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$

$$m = 3.696477973747613 \cdot 10^{-79} X^2 + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$

Root of M and m:

$N(M) = \{-2.925209484271003 \cdot 10^{36}, 0.7112878685178191\}$ $N(m) = \{-2.925209484271003 \cdot 10^{36}, 0.71128786$

Intersection intervals:



$[0.7112878685178191, 0.7112878685178191]$

Longest intersection interval: $4.790495628202167 \cdot 10^{-84}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

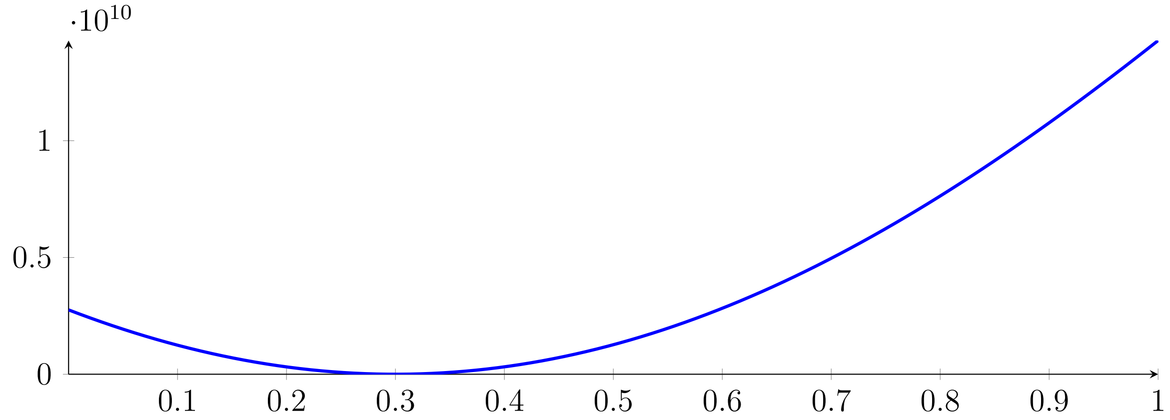
80.11 Recursion Branch 1 1 1 1 1 2 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 8!

80.12 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$
\begin{aligned}
p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\
& + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\
& - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\
& - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\
& + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822
\end{aligned}
$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

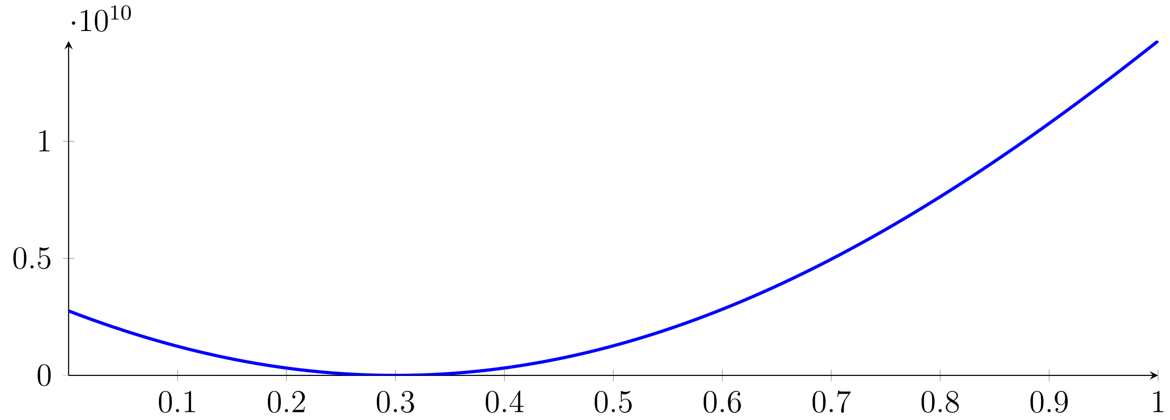
with precision $\varepsilon = 1{\cdot}10^{-64}$.

81 Running CubeClip on h_{16} with epsilon 64

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called CubeClip with input polynomial on interval $[0, 1]$:

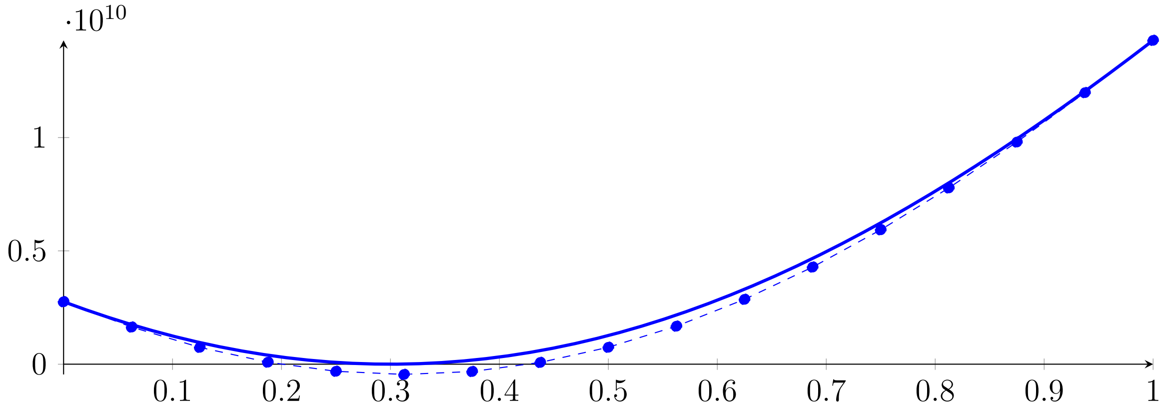
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



81.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

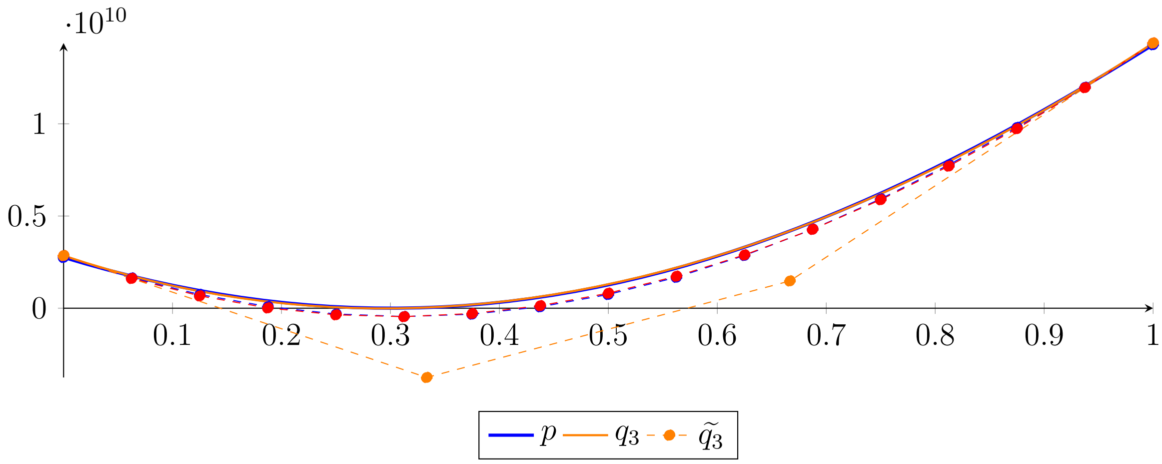
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3}$$
$$+ 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3}$$

$$\widetilde{q}_3 = 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756$$
$$\cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12}$$
$$- 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948$$
$$\cdot 10^{-285}X^{9} + 6.984121997801746\cdot 10^{-286}X^{8} - 2.400829205648148\cdot 10^{-286}X^{7}$$
$$+ 9.243609934541251\cdot 10^{-287}X^{6} - 2.7161427533585\cdot 10^{-287}X^{5} + 4.31464378661806\cdot 10^{-288}X^{4}$$
$$- 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16}$$
$$+ 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16}$$
$$- 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16}$$
$$+ 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16}$$
$$+ 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16}$$
$$+ 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

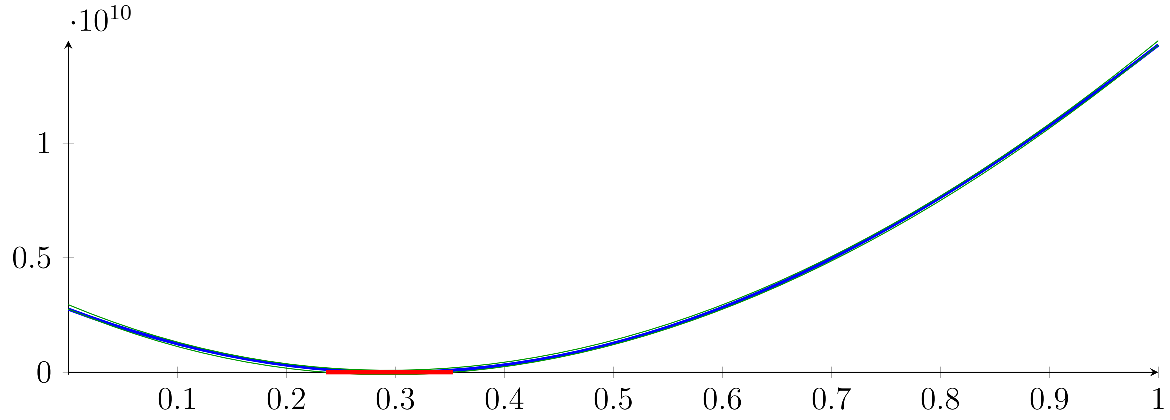
Bounding polynomials M and m:

$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$
$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



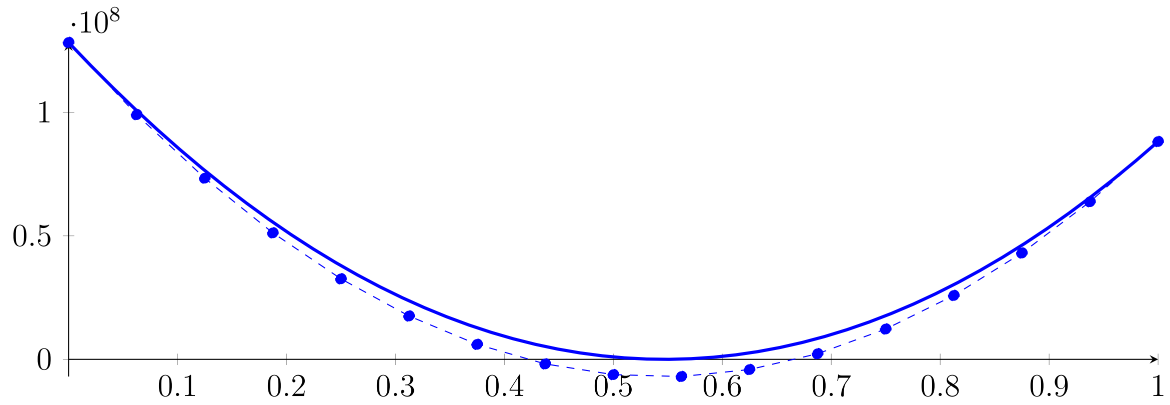
$[0.2362027155340351, 0.3527550629571673]$

Longest intersection interval: 0.1165523474231321
$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

81.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

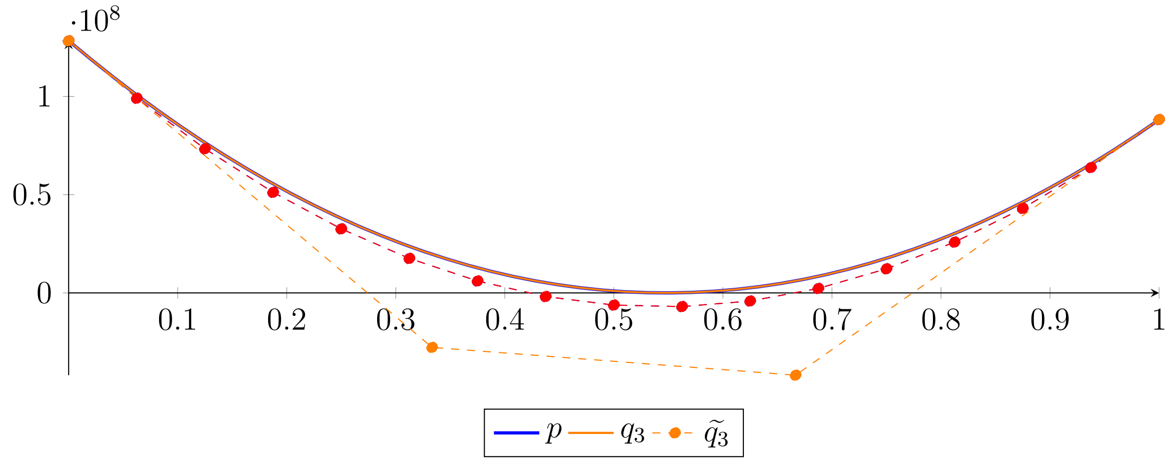
$$\begin{aligned}
p = & -1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14} \\
& - 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321 \\
& \cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8} \\
& + 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4} \\
& + 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316 \\
= & \ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X) \\
& + 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X) \\
& + 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X) \\
& - 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X) \\
& + 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X) \\
& + 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = & \ 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166 \\
= & \ 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3} \\
& - 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= 1.029539679280458\cdot 10^{-288}X^{16} - 8.072523277872609\cdot 10^{-288}X^{15} + 2.888405037194055\\ &\quad\cdot 10^{-287}X^{14} - 6.260623433882179\cdot 10^{-287}X^{13} + 9.183807049175627\cdot 10^{-287}X^{12}\\ &\quad - 9.610199535206024\cdot 10^{-287}X^{11} + 7.321003139036714\cdot 10^{-287}X^{10} - 4.031610440228076\\ &\quad\cdot 10^{-287}X^{9} + 1.537754605851679\cdot 10^{-287}X^{8} - 3.606689105481158\cdot 10^{-288}X^{7}\\ &\quad + 3.223581314526531\cdot 10^{-289}X^{6} + 5.410058694892297\cdot 10^{-290}X^{5} - 1.24173721337985\cdot 10^{-290}X^{4}\\ &\quad + 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128303546.2426166\\ &= 128303546.2426166B_{0,16} + 99049684.91666918B_{1,16} + 73342858.56410606B_{2,16}\\ &\quad + 51187170.59978822B_{3,16} + 32586724.4385766B_{4,16} + 17545623.49533216B_{5,16}\\ &\quad + 6067971.184915845B_{6,16} - 1842129.077811386B_{7,16} - 6180573.877988583B_{8,16}\\ &\quad - 6943259.800754793B_{9,16} - 4126083.431249065B_{10,16} + 2275058.645389556B_{11,16}\\ &\quad + 12264269.84402202B_{12,16} + 25845653.57950928B_{13,16} + 43023313.26671229B_{14,16}\\ &\quad + 63801352.320492B_{15,16} + 88183874.15570936B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

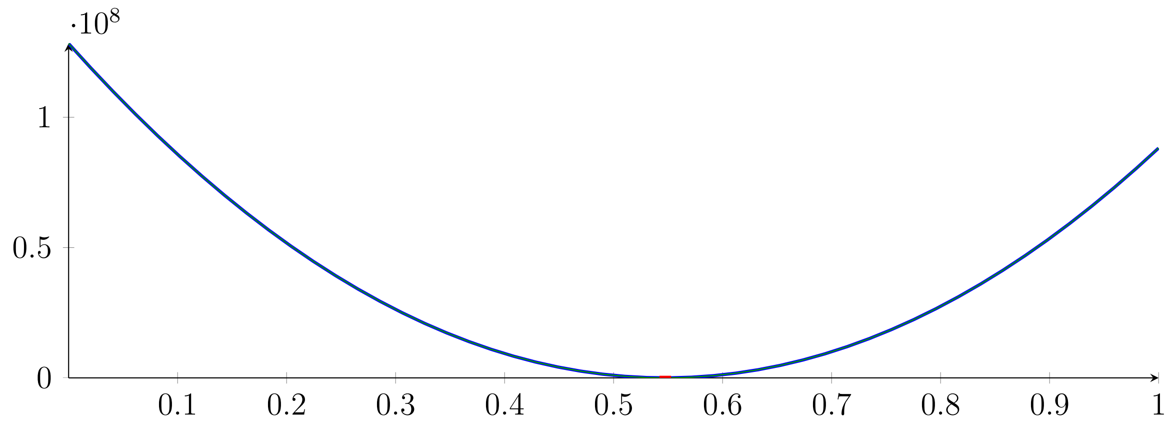
Bounding polynomials M and m:

$$M = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128322140.7478178$$
$$m = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$$[0.5420611331202266, 0.552740643902378]$$

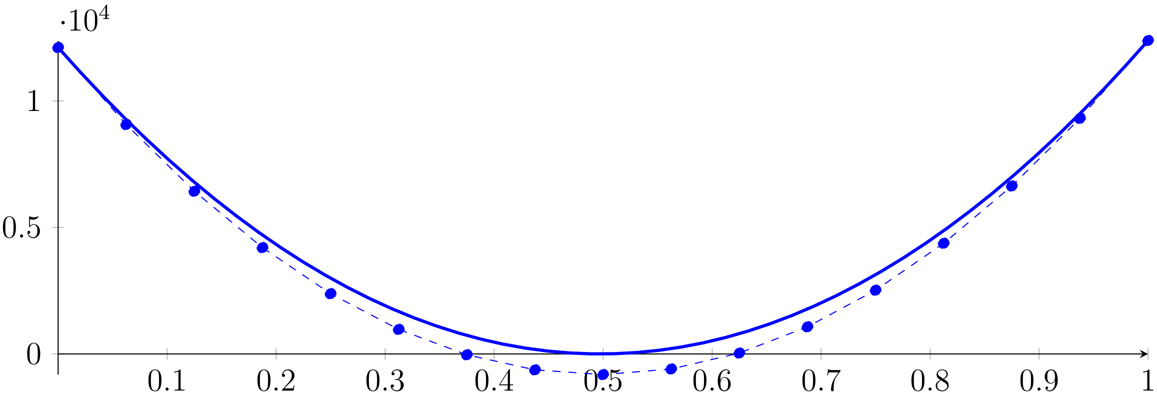
Longest intersection interval: 0.01067951078215144

$\implies$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

81.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.30062593509703$

Normalized monomial und Bézier representations and the Bézier polygon:

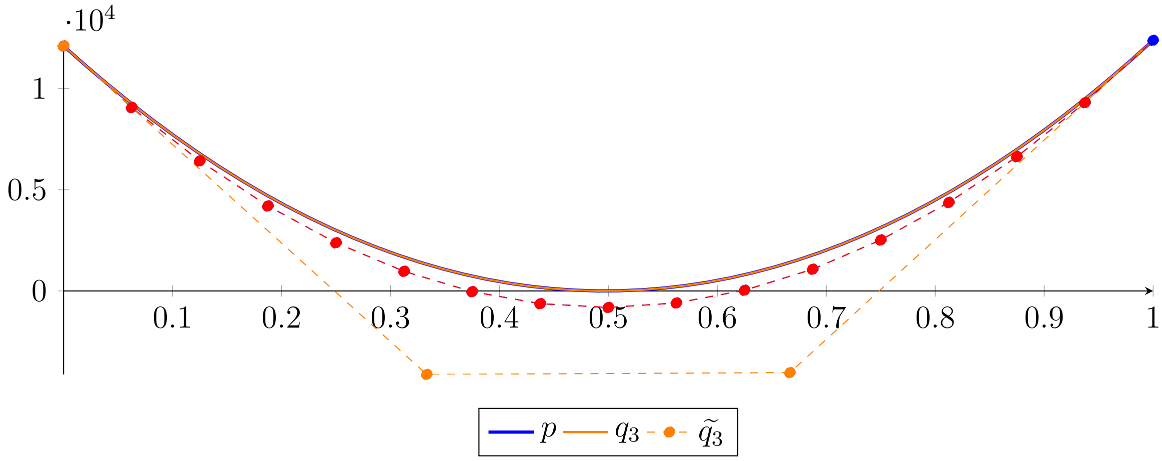
$$\begin{aligned}p = & -3.320153282917824\cdot 10^{-47}X^{16} + 2.160317373028451\cdot 10^{-44}X^{15} + 4.806705749215272\\ & \cdot 10^{-39}X^{14} - 3.045075915978909\cdot 10^{-36}X^{13} - 2.96255923979525\cdot 10^{-31}X^{12}\\ & + 1.726566936989629\cdot 10^{-28}X^{11} + 1.009356568315285\cdot 10^{-23}X^{10} - 5.095799973170639\\ & \cdot 10^{-21}X^{9} - 2.055755460332604\cdot 10^{-16}X^{8} + 8.312655296209706\cdot 10^{-14}X^{7}\\ & + 2.505544083540367\cdot 10^{-09}X^{6} - 7.139662163470987\cdot 10^{-07}X^{5} - 0.01693489925927299X^{4}\\ & + 2.534105689487521X^{3} + 49001.97220419171X^{2} - 48729.12904702137X + 12114.14082112651\\ = & \, 12114.14082112651B_{0,16}(X) + 9068.570255687672B_{1,16}(X) + 6431.349458617101B_{2,16}(X)\\ & + 4202.482955103525B_{3,16}(X) + 2381.975261030786B_{4,16}(X) + 969.830882977672B_{5,16}(X)\\ & - 33.94568178224417B_{6,16}(X) - 629.34994528077B_{7,16}(X) - 816.3774288552541B_{8,16}(X)\\ & - 595.0236631487491B_{9,16}(X) + 34.71581188982676B_{10,16}(X) + 1072.845447005528B_{11,16}(X)\\ & + 2519.36968363722B_{12,16}(X) + 4374.29295391742B_{13,16}(X) + 6637.619680672133B_{14,16}(X)\\ & + 9309.354277420696B_{15,16}(X) + 12389.50114837561B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 = & \, 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619\\ = & \, 12114.14106307619B_{0,3} - 4128.903565586097B_{1,3}\\ & - 4037.950201139886B_{2,3} + 12389.5013903309B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 = & \, 1.099600357934752\cdot 10^{-292}X^{16} - 8.377614657428995\cdot 10^{-292}X^{15} + 2.905985967667976\\ & \cdot 10^{-291}X^{14} - 6.091421693711012\cdot 10^{-291}X^{13} + 8.615207440059339\cdot 10^{-291}X^{12}\\ & - 8.654253929665817\cdot 10^{-291}X^{11} + 6.288120919613368\cdot 10^{-291}X^{10} - 3.270943230058189\\ & \cdot 10^{-291}X^{9} + 1.159364872956392\cdot 10^{-291}X^{8} - 2.426897197847659\cdot 10^{-292}X^{7}\\ & + 1.434105777365326\cdot 10^{-293}X^{6} + 4.843078869312239\cdot 10^{-294}X^{5} - 7.632080003106212\cdot 10^{-295}X^{4}\\ & + 2.500233916081606X^{3} + 49001.99397932548X^{2} - 48729.13388598685X + 12114.14106307619\\ = & \, 12114.14106307619B_{0,16} + 9068.570195202008B_{1,16} + 6431.349277155543B_{2,16}\\ & + 4202.482773640211B_{3,16} + 2381.975149359434B_{4,16} + 969.8308690166351B_{5,16}\\ & - 33.94560268476556B_{6,16} - 629.349801041346B_{7,16} - 816.3772613496847B_{8,16}\\ & - 595.0235189063601B_{9,16} + 34.71589099204953B_{10,16}\\ & + 1072.845433048966B_{11,16} + 2519.36957196781B_{12,16} + 4374.292772452004B_{13,16}\\ & + 6637.619499204969B_{14,16} + 9309.354216930127B_{15,16} + 12389.5013903309B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00024195528529 18186$.

Bounding polynomials M and m:

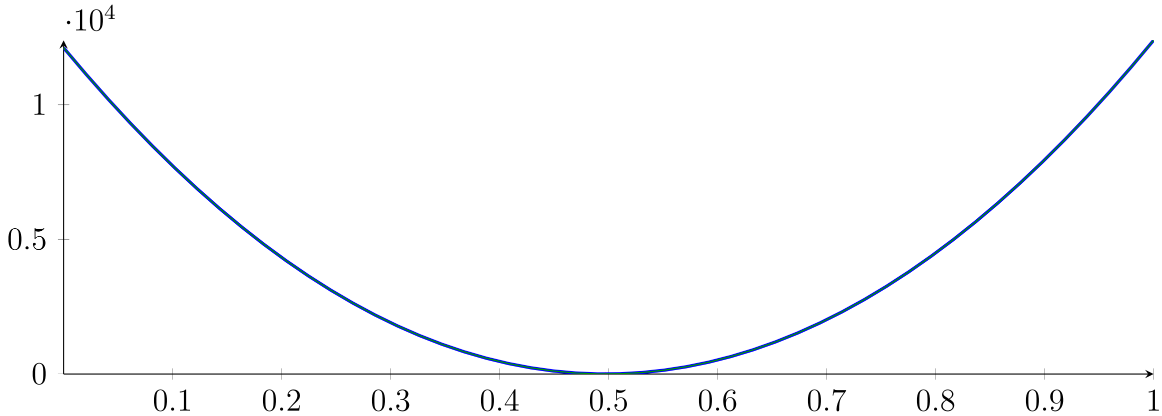
$$M = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.14130503147$$

$$m = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.1408211209$$

Root of M and m:

$$N(M) = \{-19599.95818041899\} \quad N(m) = \{-19599.95818041899, 0.4971412064138645, 0.4972526073815481\}$$

Intersection intervals:



$$[0.4971412064138645, 0.4972526073815481]$$

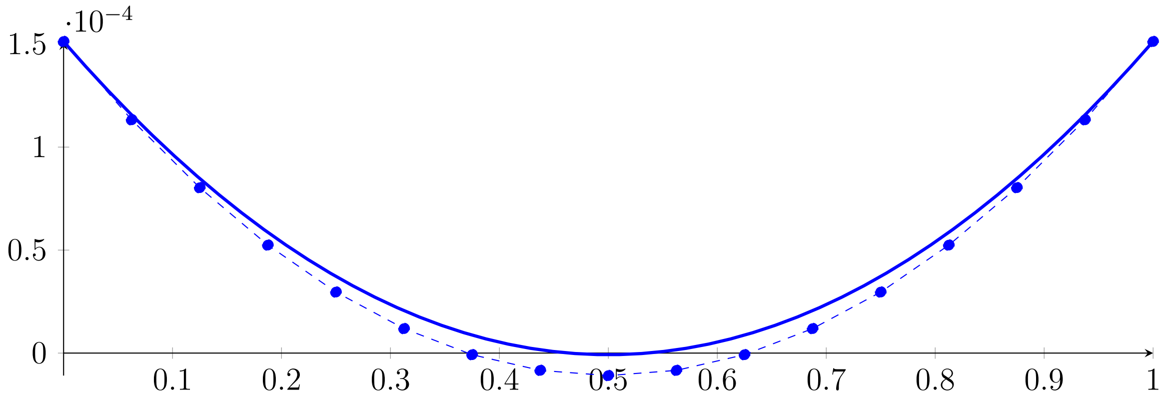
Longest intersection interval: 0.0001114009676835966

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000156681198, 0.3000001543313607]$,

81.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.300000156681198, 0.30000015433136$

Normalized monomial und Bézier representations and the Bézier polygon:

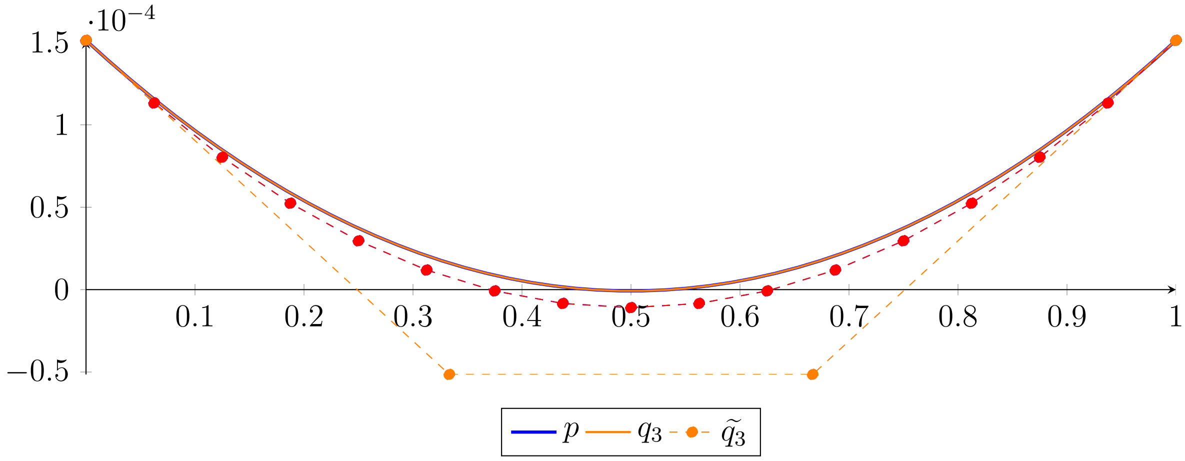
$$
\begin{aligned}
p = {} & -1.868020113181866\cdot 10^{-110}X^{16} + 1.077730427531422\cdot 10^{-103}X^{15} + 2.179252184416492 \\
& \cdot 10^{-94}X^{14} - 1.225622742394903\cdot 10^{-87}X^{13} - 1.082338964896179\cdot 10^{-78}X^{12} \\
& + 5.603938665480187\cdot 10^{-72}X^{11} + 2.971492663890366\cdot 10^{-63}X^{10} - 1.333260671370819 \\
& \cdot 10^{-56}X^{9} - 4.876756665473299\cdot 10^{-48}X^{8} + 1.752546617551927\cdot 10^{-41}X^{7} + 4.789450848811937 \\
& \cdot 10^{-33}X^{6} - 1.212138221580256\cdot 10^{-26}X^{5} - 2.608457365264229\cdot 10^{-18}X^{4} + 3.456855344293596 \\
& \cdot 10^{-12}X^{3} + 0.0006081696715066215X^{2} - 0.0006081719526708613X + 0.0001512528027912101 \\
= {} & 0.0001512528027912101B_{0,16}(X) + 0.0001132420557492813B_{1,16}(X) + 8.029938930324099 \\
& \cdot 10^{-05}B_{2,16}(X) + 5.242480345926214\cdot 10^{-05}B_{3,16}(X) + 2.961829822351771\cdot 10^{-05}B_{4,16}(X) \\
& + 1.187987360218066\cdot 10^{-05}B_{5,16}(X) - 7.904703985760706\cdot 10^{-07}B_{6,16}(X) - 8.392733772579519 \\
& \cdot 10^{-06}B_{7,16}(X) - 1.092691651365674\cdot 10^{-05}B_{8,16}(X) - 8.393018615634788\cdot 10^{-06}B_{9,16}(X) \\
& - 7.910400723407154\cdot 10^{-07}B_{10,16}(X) + 1.187901912239842\cdot 10^{-05}B_{11,16}(X) \\
& + 2.961715897475557\cdot 10^{-05}B_{12,16}(X) + 5.242337949090366\cdot 10^{-05}B_{13,16}(X) + 8.029768067701564 \\
& \cdot 10^{-05}B_{14,16}(X) + 0.0001132400625392645B_{15,16}(X) + 0.0001512505250838823B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = {} & 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} \\
& - 0.0006081719526708621X + 0.0001512528027912102 \\
= {} & 0.0001512528027912102B_{0,3} - 5.147118143241051\cdot 10^{-05}B_{1,3} \\
& - 5.147194182048959\cdot 10^{-05}B_{2,3} + 0.0001512505250838231B_{3,3}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q_3} = {} & 1.301341971574797\cdot 10^{-300}X^{16} - 9.903701542140468\cdot 10^{-300}X^{15} + 3.435393711081806 \\
& \cdot 10^{-299}X^{14} - 7.213305599572634\cdot 10^{-299}X^{13} + 1.023930542727086\cdot 10^{-298}X^{12} \\
& - 1.03405078726607\cdot 10^{-298}X^{11} + 7.556175284428161\cdot 10^{-299}X^{10} - 3.94396828718345\cdot 10^{-299}X^{9} \\
& + 1.391742955019986\cdot 10^{-299}X^{8} - 2.822114462221502\cdot 10^{-300}X^{7} + 1.110580839880333 \\
& \cdot 10^{-301}X^{6} + 7.65504767714651\cdot 10^{-302}X^{5} - 1.17007493272161\cdot 10^{-302}X^{4} + 3.456850127378832 \\
& \cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102 \\
= {} & 0.0001512528027912102B_{0,16} + 0.0001132420557492813B_{1,16} + 8.029938930324096\cdot 10^{-05}B_{2,16} \\
& + 5.242480345926211\cdot 10^{-05}B_{3,16} + 2.961829822351769\cdot 10^{-05}B_{4,16} + 1.187987360218065\cdot 10^{-05}B_{5,16} \\
& - 7.904703985760584\cdot 10^{-07}B_{6,16} - 8.392733772579497\cdot 10^{-06}B_{7,16} - 1.092691651365671\cdot 10^{-05}B_{8,16} \\
& - 8.393018615634766\cdot 10^{-06}B_{9,16} - 7.910400723407032\cdot 10^{-07}B_{10,16} + 1.187901912239842 \\
& \cdot 10^{-05}B_{11,16} + 2.961715897475555\cdot 10^{-05}B_{12,16} + 5.242337949090363\cdot 10^{-05}B_{13,16} \\
& + 8.029768067701561\cdot 10^{-05}B_{14,16} + 0.0001132400625392644B_{15,16} + 0.0001512505250838231B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.726367712763873 \cdot 10^{-20}$.

Bounding polynomials M and m:

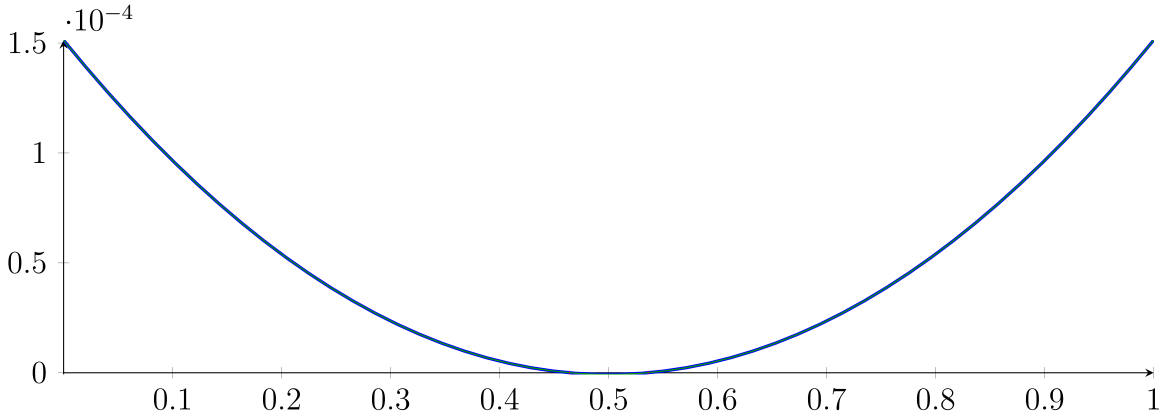
$$\begin{aligned} M = {} & 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ & - 0.0006081719526708621 X + 0.0001512528027912102 \end{aligned}$$

$$\begin{aligned} m = {} & 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ & - 0.0006081719526708621 X + 0.0001512528027912101 \end{aligned}$$

Root of M and m:

$N(M) = \{-175931744.9566824, 0.4639432736155652, 0.5360604563964476\}$ $N(m) = \{-175931744.9566824,$

Intersection intervals:



$$[0.4639432736155635, 0.4639432736155652], [0.5360604563964476, 0.5360604563964493]$$

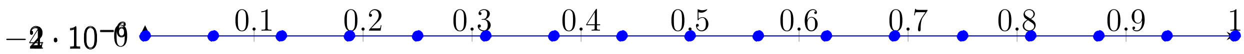
Longest intersection interval: $1.699230475659398 \cdot 10^{-15}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999977, 0.3000000799999977]$, interval 2: $[0.30000009, 0.3000$

81.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000799999977, 0.300000079999$

Normalized monomial und Bézier representations and the Bézier polygon:

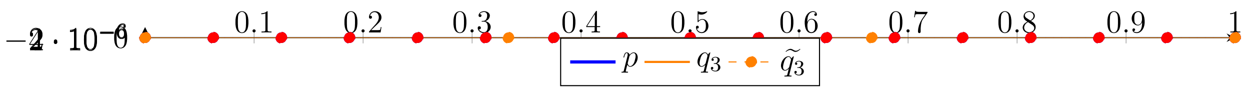
$$\begin{aligned}p &= 5.528728962819218\cdot 10^{-317}X^{16} + 6.592381943367454\cdot 10^{-316}X^{15} + 3.646191975443216\\&\quad\cdot 10^{-301}X^{14} - 1.206802108791304\cdot 10^{-279}X^{13} - 6.271779178309597\cdot 10^{-256}X^{12}\\&\quad + 1.911032531933869\cdot 10^{-234}X^{11} + 5.963449582578663\cdot 10^{-211}X^{10} - 1.574654273552139\\&\quad\cdot 10^{-189}X^{9} - 3.389607402559108\cdot 10^{-166}X^{8} + 7.168613242382433\cdot 10^{-145}X^{7} + 1.152920753962593\\&\quad\cdot 10^{-121}X^{6} - 1.717169281481298\cdot 10^{-100}X^{5} - 2.174667687267227\cdot 10^{-77}X^{4} + 1.696045363009562\\&\quad\cdot 10^{-56}X^{3} + 1.7560195200423\cdot 10^{-33}X^{2} - 7.452738853523238\cdot 10^{-20}X + 7.27439202738246\cdot 10^{-13}\\&= 7.27439202738246\cdot 10^{-13}B_{0,16}(X) + 7.274391980802842\cdot 10^{-13}B_{1,16}(X) + 7.274391934223224\\&\quad\cdot 10^{-13}B_{2,16}(X) + 7.274391887643606\cdot 10^{-13}B_{3,16}(X) + 7.274391841063988\cdot 10^{-13}B_{4,16}(X)\\&\quad + 7.27439179448437\cdot 10^{-13}B_{5,16}(X) + 7.274391747904753\cdot 10^{-13}B_{6,16}(X) + 7.274391701325135\\&\quad\cdot 10^{-13}B_{7,16}(X) + 7.274391654745517\cdot 10^{-13}B_{8,16}(X) + 7.274391608165899\cdot 10^{-13}B_{9,16}(X)\\&\quad + 7.274391561586281\cdot 10^{-13}B_{10,16}(X) + 7.274391515006663\cdot 10^{-13}B_{11,16}(X) + 7.274391468427046\\&\quad\cdot 10^{-13}B_{12,16}(X) + 7.274391421847428\cdot 10^{-13}B_{13,16}(X) + 7.27439137526781\cdot 10^{-13}B_{14,16}(X)\\&\quad + 7.274391328688192\cdot 10^{-13}B_{15,16}(X) + 7.274391282108574\cdot 10^{-13}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 1.696045363009562\cdot 10^{-56}X^{3} + 1.7560195200423\cdot 10^{-33}X^{2}\\&\quad - 7.452738853523238\cdot 10^{-20}X + 7.27439202738246\cdot 10^{-13}\\&= 7.27439202738246\cdot 10^{-13}B_{0,3} + 7.274391778957831\cdot 10^{-13}B_{1,3}\\&\quad + 7.274391530533203\cdot 10^{-13}B_{2,3} + 7.274391282108574\cdot 10^{-13}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q_3} &= 5.719971833076747\cdot 10^{-308}X^{16} - 4.598890442375195\cdot 10^{-307}X^{15} + 1.652591351302372\\&\quad\cdot 10^{-306}X^{14} - 3.497346441216264\cdot 10^{-306}X^{13} + 4.849354007960553\cdot 10^{-306}X^{12}\\&\quad - 4.665113144336927\cdot 10^{-306}X^{11} + 3.252638633884502\cdot 10^{-306}X^{10} - 1.725306268115569\\&\quad\cdot 10^{-306}X^{9} + 7.334647684203053\cdot 10^{-307}X^{8} - 2.545384960198585\cdot 10^{-307}X^{7} + 6.841253439376372\\&\quad\cdot 10^{-308}X^{6} - 1.288279056767672\cdot 10^{-308}X^{5} + 1.476493526426595\cdot 10^{-309}X^{4} + 1.696045363009562\\&\quad\cdot 10^{-56}X^{3} + 1.7560195200423\cdot 10^{-33}X^{2} - 7.452738853523238\cdot 10^{-20}X + 7.27439202738246\cdot 10^{-13}\\&= 7.27439202738246\cdot 10^{-13}B_{0,16} + 7.274391980802842\cdot 10^{-13}B_{1,16} + 7.274391934223224\\&\quad\cdot 10^{-13}B_{2,16} + 7.274391887643606\cdot 10^{-13}B_{3,16} + 7.274391841063988\cdot 10^{-13}B_{4,16}\\&\quad + 7.27439179448437\cdot 10^{-13}B_{5,16} + 7.274391747904753\cdot 10^{-13}B_{6,16} + 7.274391701325135\\&\quad\cdot 10^{-13}B_{7,16} + 7.274391654745517\cdot 10^{-13}B_{8,16} + 7.274391608165899\cdot 10^{-13}B_{9,16}\\&\quad + 7.274391561586281\cdot 10^{-13}B_{10,16} + 7.274391515006663\cdot 10^{-13}B_{11,16} + 7.274391468427046\\&\quad\cdot 10^{-13}B_{12,16} + 7.274391421847428\cdot 10^{-13}B_{13,16} + 7.27439137526781\cdot 10^{-13}B_{14,16}\\&\quad + 7.274391328688192\cdot 10^{-13}B_{15,16} + 7.274391282108574\cdot 10^{-13}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.106668124667468\cdot 10^{-79}$.
Bounding polynomials M and m:

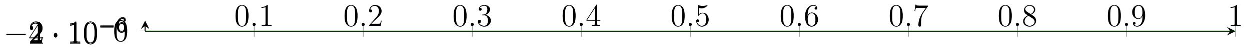
$$\begin{aligned}M &= 1.696045363009562\cdot 10^{-56}X^{3} + 1.7560195200423\cdot 10^{-33}X^{2}\\&\quad - 7.452738853523238\cdot 10^{-20}X + 7.27439202738246\cdot 10^{-13}\end{aligned}$$

$$m = 1.696045363009562{\cdot}10^{-56}X^3 + 1.7560195200423{\cdot}10^{-33}X^2 - 7.452738853523238{\cdot}10^{-20}X + 7.27439202738246{\cdot}10^{-13}$$

Root of M and m:

$N(M) = \{-1.035361175508972{\cdot}10^{23}, 1.384450244011441, 42441083663415.11\}$ $N(m) = \{-1.0353611755089$

Intersection intervals:

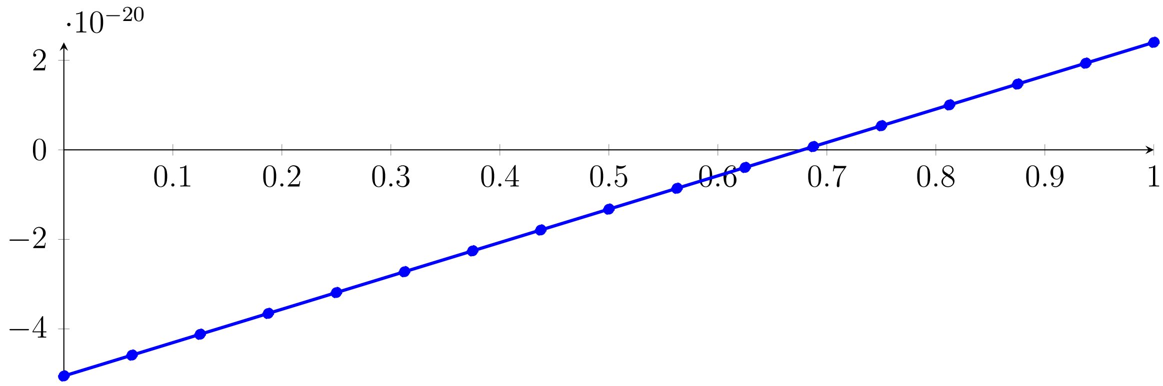


No intersection intervals with the x axis.

81.6 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

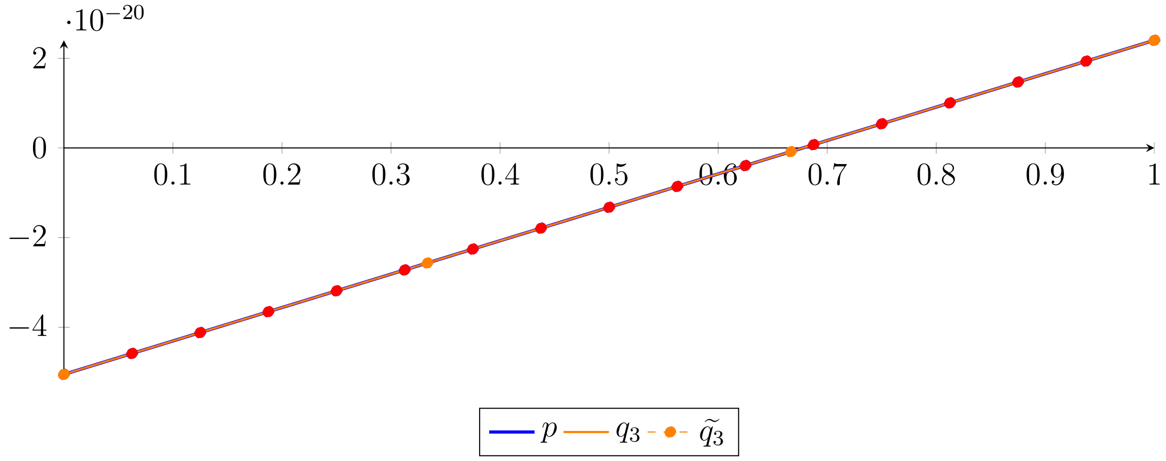
$$\begin{aligned}
p = {} & 7.285538332229319{\cdot}10^{-325}X^{16} - 3.937085615297433{\cdot}10^{-324}X^{15} + 3.646191956469014 \\
& {\cdot}10^{-301}X^{14} - 1.206801885712699{\cdot}10^{-279}X^{13} - 6.271779154117127{\cdot}10^{-256}X^{12} \\
& + 1.911032203899561{\cdot}10^{-234}X^{11} + 5.963449567055239{\cdot}10^{-211}X^{10} - 1.574654014647597{\cdot}10^{-189}X^9 \\
& - 3.389607397458184{\cdot}10^{-166}X^8 + 7.168612070943855{\cdot}10^{-145}X^7 + 1.152920753256697{\cdot}10^{-121}X^6 \\
& - 1.717168984374588{\cdot}10^{-100}X^5 - 2.174667687345432{\cdot}10^{-77}X^4 + 1.696044991742842{\cdot}10^{-56}X^3 \\
& + 1.75601952076212{\cdot}10^{-33}X^2 + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20} \\
= {} & -5.051512577078811{\cdot}10^{-20}B_{0,16}(X) - 4.585716612983338{\cdot}10^{-20}B_{1,16}(X) - 4.119920648887863 \\
& {\cdot}10^{-20}B_{2,16}(X) - 3.654124684792386{\cdot}10^{-20}B_{3,16}(X) - 3.188328720696908{\cdot}10^{-20}B_{4,16}(X) \\
& - 2.722532756601429{\cdot}10^{-20}B_{5,16}(X) - 2.256736792505948{\cdot}10^{-20}B_{6,16}(X) - 1.790940828410466 \\
& {\cdot}10^{-20}B_{7,16}(X) - 1.325144864314982{\cdot}10^{-20}B_{8,16}(X) - 8.593489002194965{\cdot}10^{-21}B_{9,16}(X) \\
& - 3.935529361240098{\cdot}10^{-21}B_{10,16}(X) + 7.224302797147847{\cdot}10^{-22}B_{11,16}(X) \\
& + 5.380389920669682{\cdot}10^{-21}B_{12,16}(X) + 1.003834956162459{\cdot}10^{-20}B_{13,16}(X) + 1.469630920257952 \\
& {\cdot}10^{-20}B_{14,16}(X) + 1.935426884353446{\cdot}10^{-20}B_{15,16}(X) + 2.401222848448942{\cdot}10^{-20}B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = {} & 1.696044991742842{\cdot}10^{-56}X^3 + 1.75601952076212{\cdot}10^{-33}X^2 \\
& + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20} \\
= {} & -5.051512577078811{\cdot}10^{-20}B_{0,3} - 2.567267435236286{\cdot}10^{-20}B_{1,3} \\
& - 8.302229339370134{\cdot}10^{-22}B_{2,3} + 2.401222848448942{\cdot}10^{-20}B_{3,3}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -1.026066912900859\cdot 10^{-315}X^{16} + 8.593538471070521\cdot 10^{-315}X^{15} - 3.223576101816875\\&\quad\cdot 10^{-314}X^{14} + 7.163252241057264\cdot 10^{-314}X^{13} - 1.054054043306296\cdot 10^{-313}X^{12}\\&\quad + 1.091820578452099\cdot 10^{-313}X^{11} - 8.293103170105507\cdot 10^{-314}X^{10} + 4.748156916296184\cdot 10^{-314}X^{9}\\&\quad - 2.062106326583159\cdot 10^{-314}X^{8} + 6.57938063450856\cdot 10^{-315}X^{7} - 1.420328445432424\cdot 10^{-315}X^{6}\\&\quad + 1.834102728564758\cdot 10^{-316}X^{5} - 1.348970337656149\cdot 10^{-317}X^{4} + 1.696044991742842\cdot 10^{-56}X^{3}\\&\quad + 1.75601952076212\cdot 10^{-33}X^{2} + 7.452735425527577\cdot 10^{-20}X - 5.051512577078811\cdot 10^{-20}\\&= -5.051512577078811\cdot 10^{-20}B_{0,16} - 4.585716612983338\cdot 10^{-20}B_{1,16} - 4.119920648887863\\&\quad\cdot 10^{-20}B_{2,16} - 3.654124684792386\cdot 10^{-20}B_{3,16} - 3.188328720696908\cdot 10^{-20}B_{4,16}\\&\quad - 2.722532756601429\cdot 10^{-20}B_{5,16} - 2.256736792505948\cdot 10^{-20}B_{6,16} - 1.790940828410466\\&\quad\cdot 10^{-20}B_{7,16} - 1.325144864314982\cdot 10^{-20}B_{8,16} - 8.593489002194965\cdot 10^{-21}B_{9,16}\\&\quad - 3.935529361240098\cdot 10^{-21}B_{10,16} + 7.224302797147847\cdot 10^{-22}B_{11,16}\\&\quad + 5.380389920669682\cdot 10^{-21}B_{12,16} + 1.003834956162459\cdot 10^{-20}B_{13,16} + 1.469630920257952\\&\quad\cdot 10^{-20}B_{14,16} + 1.935426884353446\cdot 10^{-20}B_{15,16} + 2.401222848448942\cdot 10^{-20}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.106668124779188\cdot 10^{-79}$.

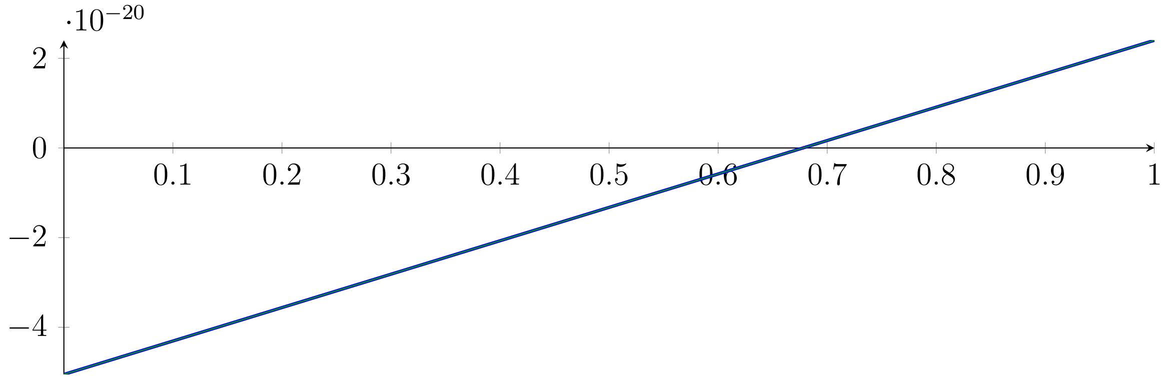
Bounding polynomials M and m:

$$\begin{aligned}M &= 1.696044991742842\cdot 10^{-56}X^3 + 1.75601952076212\cdot 10^{-33}X^2\\&\quad + 7.452735425527577\cdot 10^{-20}X - 5.051512577078811\cdot 10^{-20}\\m &= 1.696044991742842\cdot 10^{-56}X^3 + 1.75601952076212\cdot 10^{-33}X^2\\&\quad + 7.452735425527577\cdot 10^{-20}X - 5.051512577078811\cdot 10^{-20}\end{aligned}$$

Root of M and m:

$N(M) = \{-1.03536140172663\cdot 10^{23}, -42441083680812.44, 0.6778065084366168\}$ $N(m) = \{-1.035361401726$

Intersection intervals:



$$[0.6778065084366168, 0.6778065084366168]$$

Longest intersection interval: $8.336987555302001\cdot 10^{-60}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

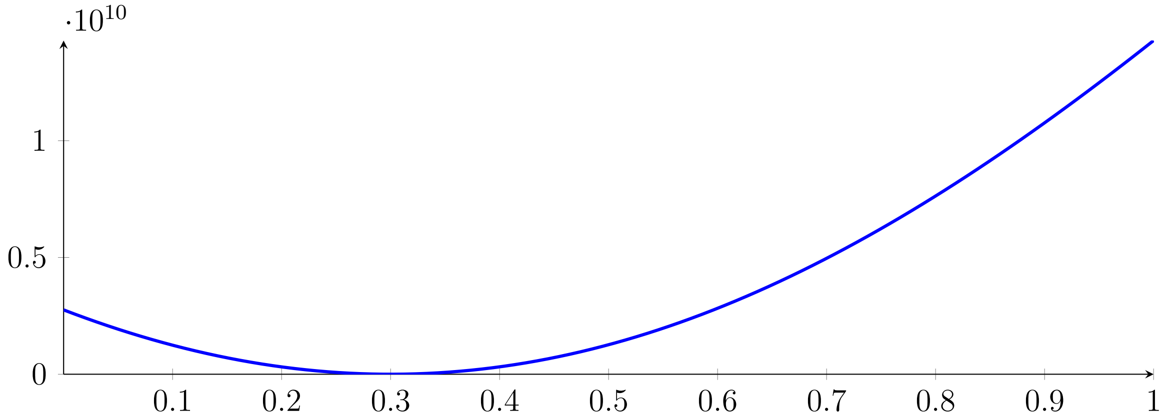
81.7 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 6!

81.8 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



Result: Root Intervals

$$[0.30000009, 0.30000009]$$

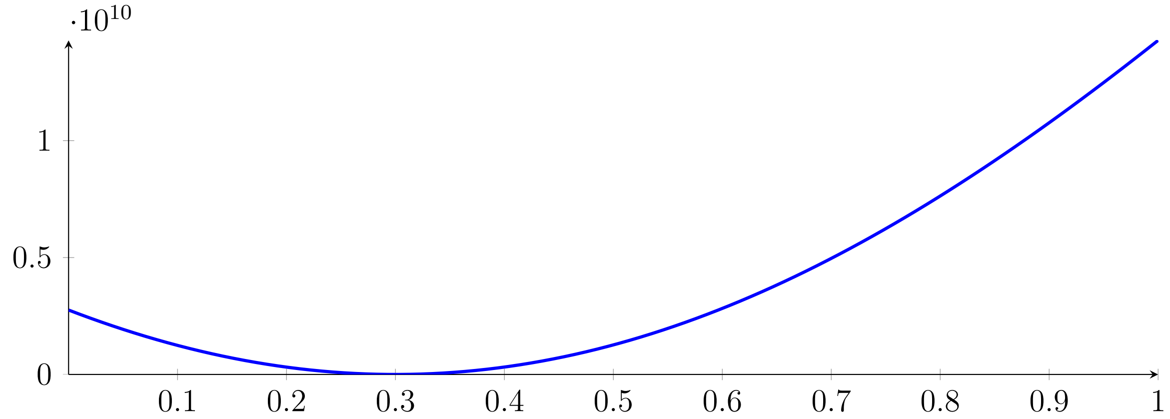
with precision $\varepsilon = 1 \cdot 10^{-64}$.

82 Running `BezClip` on h_{16} with epsilon 128

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `BezClip` with input polynomial on interval $[0, 1]$:

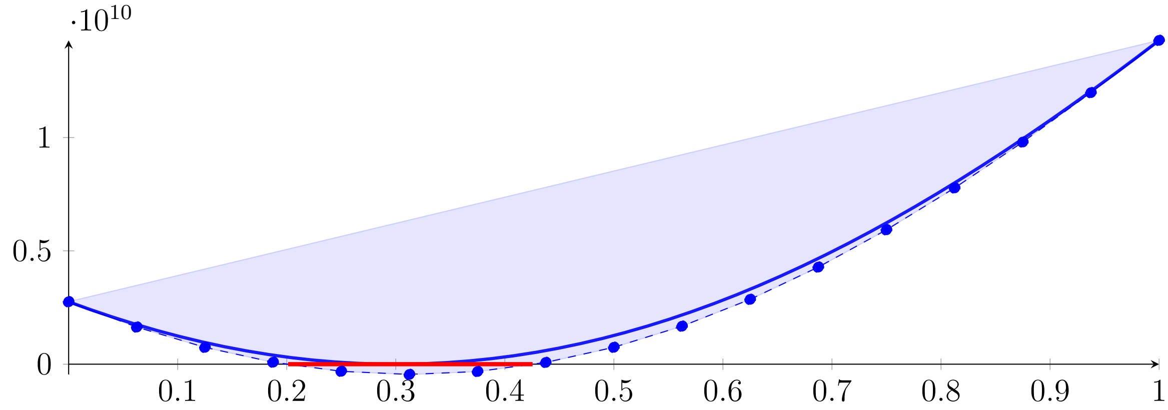
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



82.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.201262666529315, 0.425474032728702\}$$

Intersection intervals with the x axis:

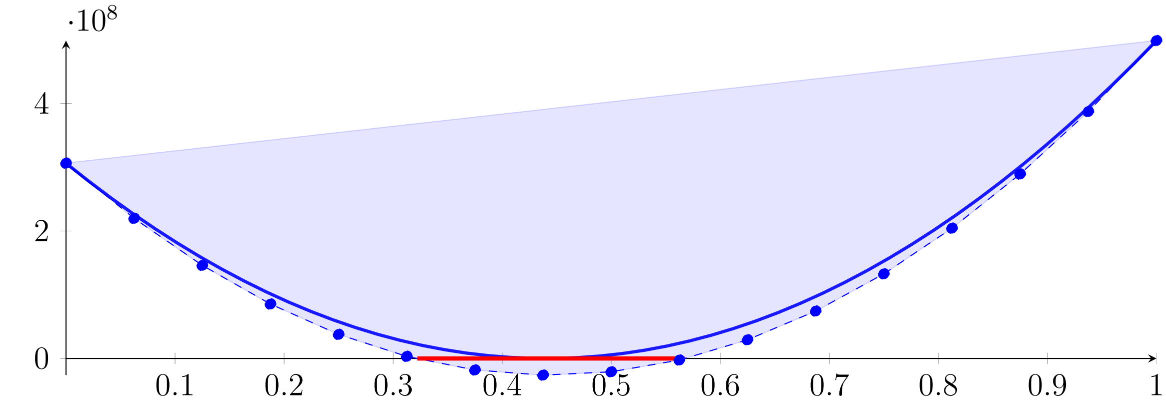
$$[0.201262666529315, 0.425474032728702]$$

Longest intersection interval: 0.2242113661993871
$\Longrightarrow$ Selective recursion: interval 1: $[0.201262666529315, 0.425474032728702]$,

82.2 Recursion Branch 1 1 in Interval 1: $[0.201262666529315, 0.425474032728702]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-4.078702226246453\cdot 10^{-11}X^{16} + 4.329167405007062\cdot 10^{-10}X^{15} + 1.800827856148464\cdot 10^{-07}X^{14}\\ &- 1.7501300195896\cdot 10^{-06}X^{13} - 0.0003388881074603682X^{12} + 0.002918727490860103X^{11}\\ &+ 0.353260574938002X^{10} - 2.587751703352393X^{9} - 220.7388291837263X^{8}\\ &+ 1298.537478880933X^{7} + 82809.42227744751X^{6} - 357003.6974521088X^{5} - 17288824.7550166X^{4}\\ &+ 45617774.75376932X^{3} + 1550181799.359372X^{2} - 1385902556.54429X + 306449533.9978484\\ = &\ 306449533.9978484B_{0,16}(X) + 219830624.2138303B_{1,16}(X) + 146129896.0911402B_{2,16}(X)\\ &+ 85428809.9418386B_{3,16}(X) + 37799326.72372467B_{4,16}(X) + 3303826.308721026B_{5,16}(X)\\ &- 18004963.90790522B_{6,16}(X) - 26084029.94397575B_{7,16}(X) - 20900121.61613177B_{8,16}(X)\\ &- 2429792.29588269B_{9,16}(X) + 29340572.02544693B_{10,16}(X) + 74414734.40659503B_{11,16}(X)\\ &+ 132786599.3869978B_{12,16}(X) + 204440216.186282B_{13,16}(X) + 289349792.6628749B_{14,16}(X)\\ &+ 387479720.0042024B_{15,16}(X) + 498784608.1032447B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3221903269587755, 0.5672799898344481\}$$

Intersection intervals with the x axis:

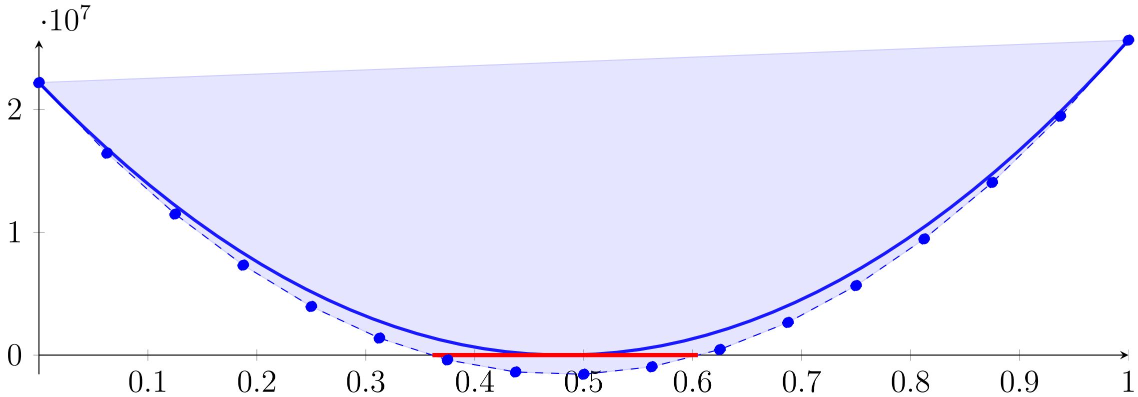
$$[0.3221903269587755, 0.5672799898344481]$$

Longest intersection interval: 0.2450896628756726
$\Longrightarrow$ Selective recursion: interval 1: $[0.2735013999129692, 0.328453288067671]$,

82.3 Recursion Branch 1 1 1 in Interval 1: $[0.2735013999129692, 0.328453288067671]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-6.91388116995714\cdot10^{-21}X^{16} + 1.539971991909181\cdot10^{-19}X^{15} + 5.126557214158853\\&\cdot10^{-16}X^{14} - 1.075268795442723\cdot10^{-14}X^{13} - 1.618466564965893\cdot10^{-11}X^{12}\\&+ 3.060191240732859\cdot10^{-10}X^{11} + 2.825398471644138\cdot10^{-07}X^{10} - 4.580412037193902\\&\cdot10^{-06}X^{9} - 0.002949971804071592X^{8} + 0.0383187988748973X^{7} + 18.44286791446417X^{6}\\&- 172.012783850635X^{5} - 63987.92092437637X^{4} + 338934.074545568X^{3}\\&+ 95113195.48898588X^{2} - 91937923.73553449X + 22182509.73328641\\= &\ 22182509.73328641B_{0,16}(X) + 16436389.49981551B_{1,16}(X) + 11482879.22875282B_{2,16}(X)\\&+ 7322584.159517174B_{3,16}(X) + 3956074.373329099B_{4,16}(X) + 1383884.753830588B_{5,16}(X)\\&- 393485.0499920519B_{6,16}(X) - 1375570.658578954B_{7,16}(X) - 1561942.994257926B_{8,16}(X)\\&- 952208.311393262B_{9,16}(X) + 453991.7757805191B_{10,16}(X) + 2656980.267642443B_{11,16}(X)\\&+ 5657044.755988558B_{12,16}(X) + 9454437.403157084B_{13,16}(X) + 14049374.92347699B_{14,16}(X)\\&+ 19442038.56704145B_{15,16}(X) + 25632574.10580758B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3611633659064024, 0.604821871549368\}$$

Intersection intervals with the x axis:

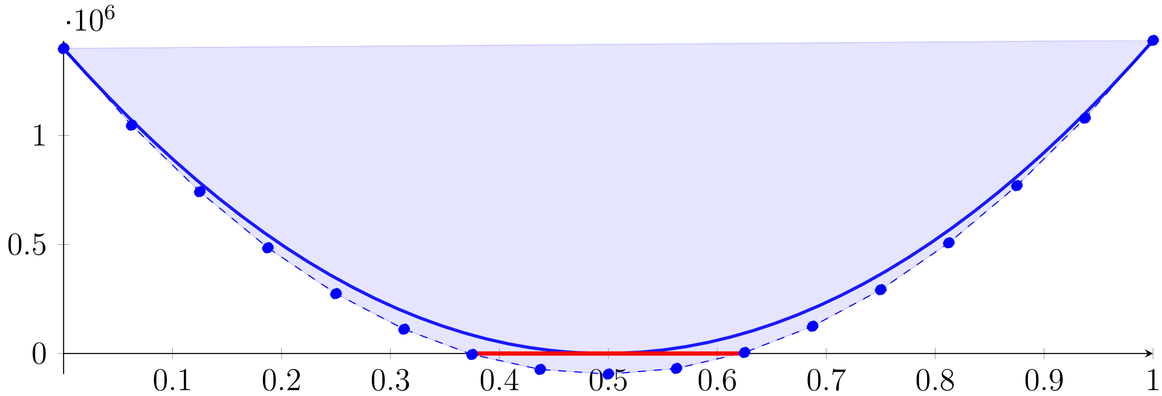
$$[0.3611633659064024, 0.604821871549368]$$

Longest intersection interval: 0.2436585056429656
$\Longrightarrow$ Selective recursion: interval 1: $[0.2933480088018335, 0.3067375037518675]$,

82.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.2933480088018335, 0.30673750375186$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -1.067154088646128 \cdot 10^{-30} X^{16} + 7.224340048814341 \cdot 10^{-29} X^{15} + 1.334696477727887 \\
& \cdot 10^{-24} X^{14} - 8.705175946298654 \cdot 10^{-23} X^{13} - 7.106771480233297 \cdot 10^{-19} X^{12} \\
& + 4.237354105832321 \cdot 10^{-17} X^{11} + 2.091927791167947 \cdot 10^{-13} X^{10} - 1.077046671119229 \\
& \cdot 10^{-11} X^{9} - 3.681404336048099 \cdot 10^{-08} X^{8} + 1.51820465067154 \cdot 10^{-06} X^{7} \\
& + 0.003877397410843758 X^{6} - 0.1133230374256462 X^{5} - 226.5078434192974 X^{4} \\
& + 3562.753220436385 X^{3} + 5665644.405911702 X^{2} - 5632059.446508477 X + 1399245.031427946 \\
= & \ 1399245.031427946 B_{0,16}(X) + 1047241.316021167 B_{1,16}(X) + 742451.3039969843 B_{2,16}(X) \\
& + 484881.3574147218 B_{3,16}(X) + 274537.7138788421 B_{4,16}(X) + 111426.4865130056 B_{5,16}(X) \\
& - 4446.336065390124 B_{6,16}(X) - 73074.88977018565 B_{7,16}(X) - 94453.43507095045 B_{8,16}(X) \\
& - 68576.35701698946 B_{9,16}(X) + 4561.834739135334 B_{10,16}(X) \\
& + 124966.505918568 B_{11,16}(X) + 292642.897593607 B_{12,16}(X) + 507596.1261656377 B_{13,16}(X) \\
& + 769831.1833435517 B_{14,16}(X) + 1079352.93612265 B_{15,16}(X) + 1436166.12676403 B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3726017154160643, 0.6211016992032472\}$$

Intersection intervals with the x axis:

$$[0.3726017154160643, 0.6211016992032472]$$

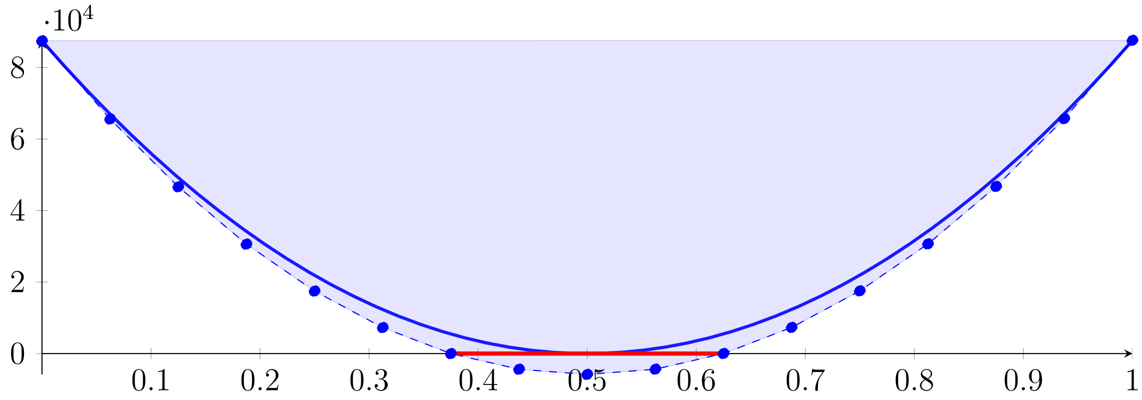
Longest intersection interval: 0.2484999837871829

$\Longrightarrow$ Selective recursion: interval 1: $[0.2983369575887709, 0.3016642468667729]$,

82.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.2983369575887709, 0.301664246866$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -2.256570822524655 \cdot 10^{-40} X^{16} + 5.606069245850654 \cdot 10^{-38} X^{15} + 4.571694482208246 \\
& \cdot 10^{-33} X^{14} - 1.103601619609778 \cdot 10^{-30} X^{13} - 3.943084008767946 \cdot 10^{-26} X^{12} \\
& + 8.746232232432483 \cdot 10^{-24} X^{11} + 1.879997320990413 \cdot 10^{-19} X^{10} - 3.610214442217249 \\
& \cdot 10^{-17} X^{9} - 5.35839719625663 \cdot 10^{-13} X^{8} + 8.241677187819251 \cdot 10^{-11} X^{7} + 9.139570434264648 \\
& \cdot 10^{-07} X^{6} - 9.91682351975725 \cdot 10^{-05} X^{5} - 0.864525557836322 X^{4} + 49.4891850901473 X^{3} \\
& + 350100.5152669434 X^{2} - 350028.3308017828 X + 87482.91293301892 \\
= & \ 87482.91293301892 B_{0,16}(X) + 65606.1422579075 B_{1,16}(X) + 46646.87587668727 B_{2,16}(X) \\
& + 30605.20216290304 B_{3,16}(X) + 17481.20901508557 B_{4,16}(X) + 7274.983856728878 B_{5,16}(X) \\
& - 13.38636373236149 B_{6,16}(X) - 4383.815172945278 B_{7,16}(X) - 5836.216572661172 B_{8,16}(X) \\
& - 4370.505039757766 B_{9,16}(X) + 13.40447373867094 B_{10,16}(X) + 7315.596540631298 B_{11,16}(X) \\
& + 17536.1552585308 B_{12,16}(X) + 30675.1642498336 B_{13,16}(X) + 46732.70666170018 B_{14,16}(X) \\
& + 65708.86516603353 B_{15,16}(X) + 87603.72195945767 B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3748852078437339, 0.6248088966923045\}$$

Intersection intervals with the x axis:

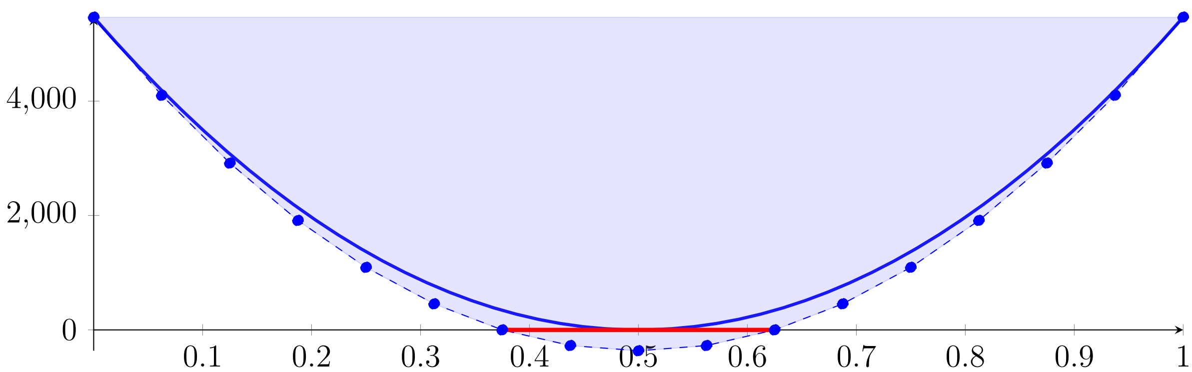
$$[0.3748852078437339, 0.6248088966923045]$$

Longest intersection interval: 0.2499236888485706
$\Longrightarrow$ Selective recursion: interval 1: $[0.2995843091213109, 0.3004158775315354]$,

82.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.2995843091213109, 0.30041587753$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-5.228387005637929 \cdot 10^{-50} X^{16} + 5.071723175262542 \cdot 10^{-47} X^{15} + 1.695940836337518 \\
&\cdot 10^{-41} X^{14} - 1.602367332110541 \cdot 10^{-38} X^{13} - 2.341982733873566 \cdot 10^{-33} X^{12} \\
&+ 2.036120445711374 \cdot 10^{-30} X^{11} + 1.787779362171969 \cdot 10^{-25} X^{10} - 1.346594590666257 \\
&\cdot 10^{-22} X^{9} - 8.158156485257833 \cdot 10^{-18} X^{8} + 4.921695226295225 \cdot 10^{-15} X^{7} \\
&+ 2.227778844339431 \cdot 10^{-10} X^{6} - 9.46914992513699 \cdot 10^{-08} X^{5} - 0.003373649242827386 X^{4} \\
&+ 0.7523208927331248 X^{3} + 21871.35693828986 X^{2} - 21871.48147756637 X + 5467.807676357308 \\
= &\ 5467.807676357308 B_{0,16}(X) + 4100.84008400941 B_{1,16}(X) + 2916.133799480594 B_{2,16}(X) \\
&+ 1913.690166201026 B_{3,16}(X) + 1093.510525747217 B_{4,16}(X) + 455.5962178420057 B_{5,16}(X) \\
&- 0.05141964546683865 B_{6,16}(X) - 273.4310506997833 B_{7,16}(X) - 364.541341159257 B_{8,16}(X) \\
&- 273.3809587159691 B_{9,16}(X) + 0.05142708421757425 B_{10,16}(X) \\
&+ 455.7571448416359 B_{11,16}(X) + 1093.737521302792 B_{12,16}(X) + 1913.993881360346 B_{13,16}(X) \\
&+ 2916.527548053087 B_{14,16}(X) + 4101.339842565916 B_{15,16}(X) + 5468.43208422982 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749929469011197, 0.6249882450180355\}$$

Intersection intervals with the x axis:

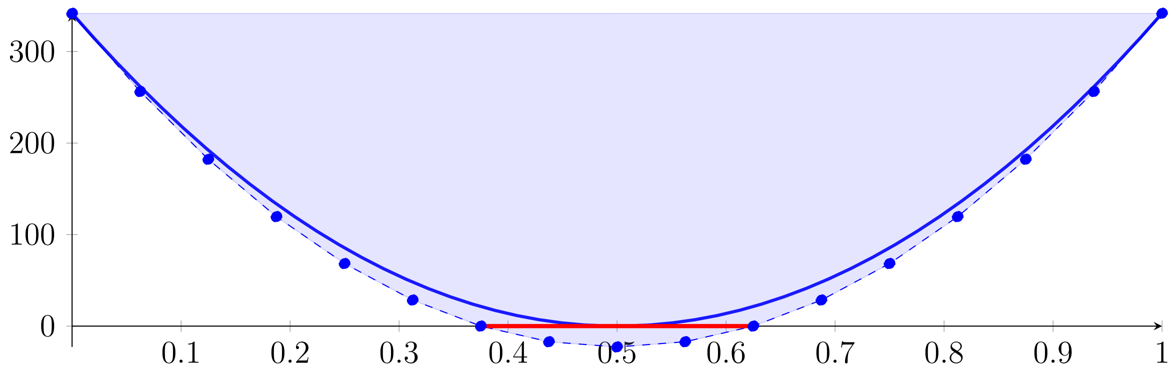
$$[0.3749929469011197, 0.6249882450180355]$$

Longest intersection interval: 0.2499952981169158
$\Longrightarrow$ Selective recursion: interval 1: $[0.2998961414100109, 0.3001040296026296]$,

82.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.2998961414100109, 0.300104029$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.216962444275584\cdot 10^{-59}X^{16} + 4.692870826723641\cdot 10^{-56}X^{15} + 6.316314535763934\\ &\cdot 10^{-50}X^{14} - 2.373865516252114\cdot 10^{-46}X^{13} - 1.395661893422859\cdot 10^{-40}X^{12}\\ &+ 4.828363722587868\cdot 10^{-37}X^{11} + 1.70471850273246\cdot 10^{-31}X^{10} - 5.110411085982617\cdot 10^{-28}X^{9}\\ &- 1.244717766622564\cdot 10^{-22}X^{8} + 2.988632715177156\cdot 10^{-19}X^{7} + 5.438614058423278\cdot 10^{-14}X^{6}\\ &- 9.197401057829169\cdot 10^{-11}X^{5} - 1.317801761681953\cdot 10^{-05}X^{4} + 0.01167528468045732X^{3}\\ &+ 1366.961107526186X^{2} - 1366.963198735845X + 341.7398631167735\\ = &\,341.7398631167735B_{0,16}(X) + 256.3046631957831B_{1,16}(X) + 182.260805837511B_{2,16}(X)\\ &+ 119.6083118906798B_{3,16}(X) + 68.34720219677136B_{4,16}(X) + 28.47749759002708B_{5,16}(X)\\ &- 0.0007811025525137025B_{6,16}(X) - 17.08761306120757B_{7,16}(X) - 22.782977473419B_{8,16}(X)\\ &- 17.08685353390849B_{9,16}(X) + 0.0007795553614777027B_{10,16}(X)\\ &+ 28.4799425851876B_{11,16}(X) + 68.35065633912575B_{12,16}(X) + 119.6129415934909B_{13,16}(X)\\ &+ 182.2668191173573B_{14,16}(X) + 256.312309672558B_{15,16}(X) + 341.7494340136854B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749982857492878, 0.6249971486858456\}$$

Intersection intervals with the x axis:

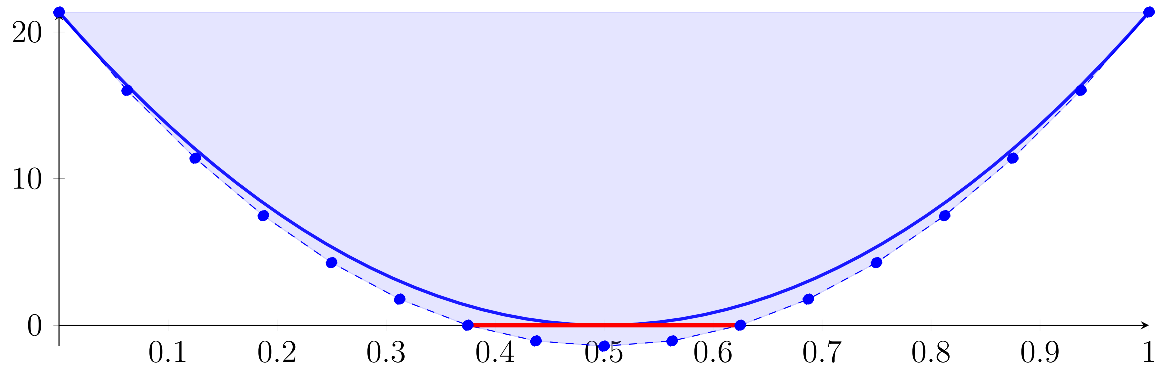
$$[0.3749982857492878, 0.6249971486858456]$$

Longest intersection interval: 0.2499988629365579
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999740991258704, 0.300026070937643]$,

82.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999740991258704, 0.3000260$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-2.833255302235776 \cdot 10^{-69} X^{16} + 4.363478560238502 \cdot 10^{-65} X^{15} + 2.35287052687137\\
&\cdot 10^{-58} X^{14} - 3.532185105967992 \cdot 10^{-54} X^{13} - 8.318408043755505 \cdot 10^{-48} X^{12}\\
&+ 1.149616718196078 \cdot 10^{-43} X^{11} + 1.625691292234855 \cdot 10^{-37} X^{10} - 1.946948798353601 \cdot 10^{-33} X^{9}\\
&- 1.899245778120626 \cdot 10^{-27} X^{8} + 1.821779493317236 \cdot 10^{-23} X^{7} + 1.327769541755276 \cdot 10^{-17} X^{6}\\
&- 8.969682874540076 \cdot 10^{-14} X^{5} - 5.147636798128115 \cdot 10^{-08} X^{4} + 0.000182114977798148 X^{3}\\
&+ 85.43511227371199 X^{2} - 85.43514442584667 X + 21.35877059261705\\
= &\, 21.35877059261705 B_{0,16}(X) + 16.01907406600164 B_{1,16}(X) + 11.39133680833382 B_{2,16}(X)\\
&+ 7.475559144818918 B_{3,16}(X) + 4.271741400633969 B_{4,16}(X) + 1.779883900927722 B_{5,16}(X)\\
&- 1.302917935804413 \cdot 10^{-05} B_{6,16}(X) - 1.067949064595088 B_{7,16}(X) - 1.423923880255569 B_{8,16}(X)\\
&- 1.067937151125187 B_{9,16}(X) + 1.144780338983322 \cdot 10^{-05} B_{10,16}(X)\\
&+ 1.779922241509208 B_{11,16}(X) + 4.271795554943032 B_{12,16}(X) + 7.47563171302734 B_{13,16}(X)\\
&+ 11.39143104065633 B_{14,16}(X) + 16.01919386269591 B_{15,16}(X) + 21.35892050398371 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749995424882778, 0.6249993300354412\}$$

Intersection intervals with the x axis:

$$[0.3749995424882778, 0.6249993300354412]$$

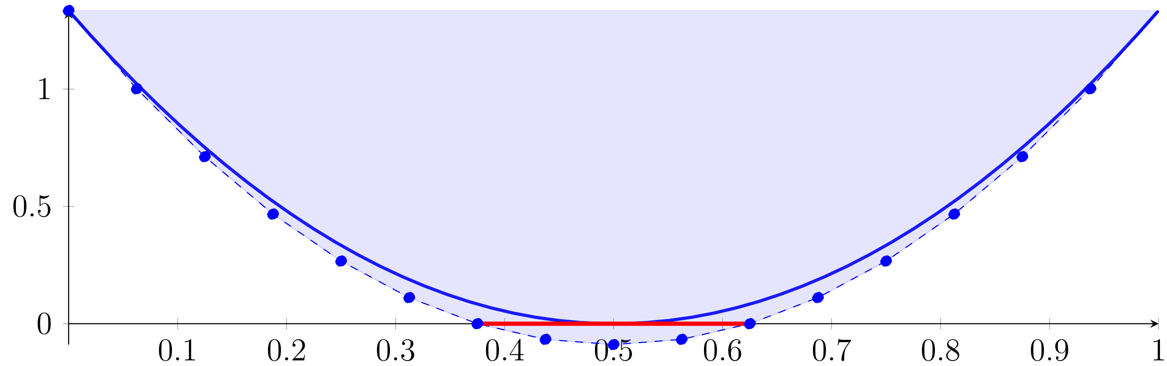
Longest intersection interval: 0.2499997875471635

$\Longrightarrow$ Selective recursion: interval 1: $[0.2999935885315074, 0.300006581473409]$,

82.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999935885315074, 0.30000$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-6.596596862099299 \cdot 10^{-79} X^{16} + 4.062171102620318 \cdot 10^{-74} X^{15} + 8.765030607683244\\
&\cdot 10^{-67} X^{14} - 5.261467161278336 \cdot 10^{-62} X^{13} - 4.958117265353456 \cdot 10^{-55} X^{12}\\
&+ 2.739981763166457 \cdot 10^{-50} X^{11} + 1.550371378259149 \cdot 10^{-43} X^{10} - 7.424637900674377\\
&\cdot 10^{-39} X^{9} - 2.898009393348535 \cdot 10^{-32} X^{8} + 1.111571585405421 \cdot 10^{-27} X^{7} + 3.241620002541647\\
&\cdot 10^{-21} X^{6} - 8.756501261136649 \cdot 10^{-17} X^{5} - 2.010795357557322 \cdot 10^{-10} X^{4} + 2.844332798771609\\
&\cdot 10^{-06} X^{3} + 5.339698243852309 X^{2} - 5.339698356505613 X + 1.33492347100565\\
= &\, 1.33492347100565 B_{0,16}(X) + 1.00119232372405 B_{1,16}(X) + 0.7119586618078846 B_{2,16}(X)\\
&+ 0.4672224903363214 B_{3,16}(X) + 0.266983814388415 B_{4,16}(X) + 0.1112426390431101 B_{5,16}(X)\\
&- 1.030620758846691 \cdot 10^{-06} B_{6,16}(X) - 0.06674718952446819 B_{7,16}(X)\\
&- 0.0889958325894046 B_{8,16}(X) - 0.06674695473706526 B_{9,16}(X) - 5.508890578613216\\
&\cdot 10^{-07} B_{10,16}(X) + 0.1112433840328995 B_{11,16}(X) + 0.2669848551069781 B_{12,16}(X)\\
&+ 0.4672238674112388 B_{13,16}(X) + 0.7119604260236321 B_{14,16}(X)\\
&+ 1.001194536021998 B_{15,16}(X) + 1.334926202484066 B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749994209666256, 0.6250003095051082\}$$

Intersection intervals with the x axis:

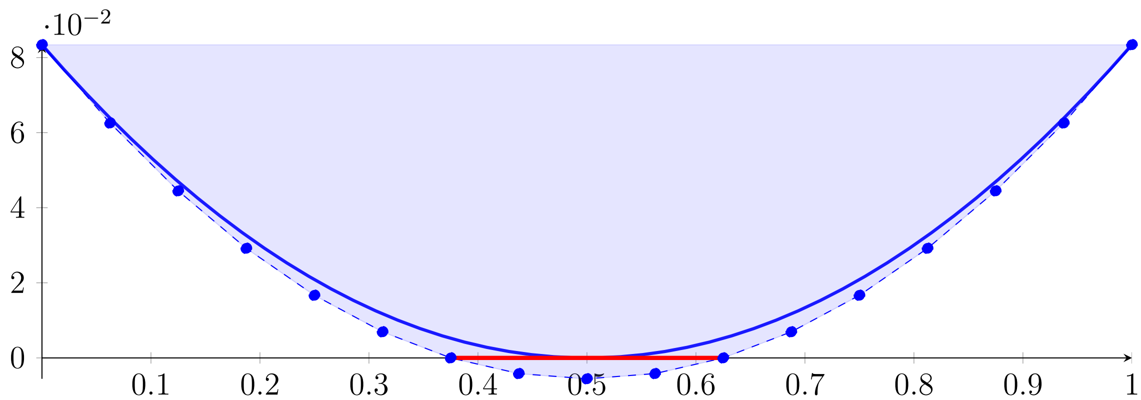
$$[0.3749994209666256, 0.6250003095051082]$$

Longest intersection interval: 0.2500008885384826
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999984608771972, 0.3000017091242173]$,

82.10 Recursion Branch 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999984608771972, 0.300$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.5359772362815\cdot 10^{-88}X^{16} + 3.783024712863664\cdot 10^{-83}X^{15} + 3.265391675561962\cdot 10^{-75}X^{14} \\
& - 7.83987355616511\cdot 10^{-70}X^{13} - 2.955395756595885\cdot 10^{-62}X^{12} + 6.532349021061568 \\
& \cdot 10^{-57}X^{11} + 1.478602992973913\cdot 10^{-49}X^{10} - 2.832143398905245\cdot 10^{-44}X^{9} \\
& - 4.422140960586434\cdot 10^{-37}X^{8} + 6.784132679612849\cdot 10^{-32}X^{7} + 7.914287227230788\cdot 10^{-25}X^{6} \\
& - 8.550710444673422\cdot 10^{-20}X^{5} - 7.854787446136848\cdot 10^{-13}X^{4} + 4.443846100456459 \\
& \cdot 10^{-08}X^{3} + 0.3337337124913315X^{2} - 0.3337336004955191X + 0.08343257581494635 \\
= & \; 0.08343257581494635B_{0,16}(X) + 0.06257422578397641B_{1,16}(X) + 0.04449699002376757B_{2,16}(X) \\
& + 0.02920086861367421B_{3,16}(X) + 0.01668586163305031B_{4,16}(X) \\
& + 0.006951969161249391B_{5,16}(X) - 8.08722375440678\cdot 10^{-07}B_{6,16}(X) \\
& - 0.004172471938471519B_{7,16}(X) - 0.005563020407686608B_{8,16}(X) \\
& - 0.004172454050668901B_{9,16}(X) - 7.727880670249089\cdot 10^{-07}B_{10,16}(X) \\
& + 0.006952023459469962B_{11,16}(X) + 0.01668593477129257B_{12,16}(X) \\
& + 0.02920096122675088B_{13,16}(X) + 0.04449710290519453B_{14,16}(X) \\
& + 0.06257435988597275B_{15,16}(X) + 0.08343273224843432B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3749927302224649, 0.6250069467380417\}$$

Intersection intervals with the x axis:

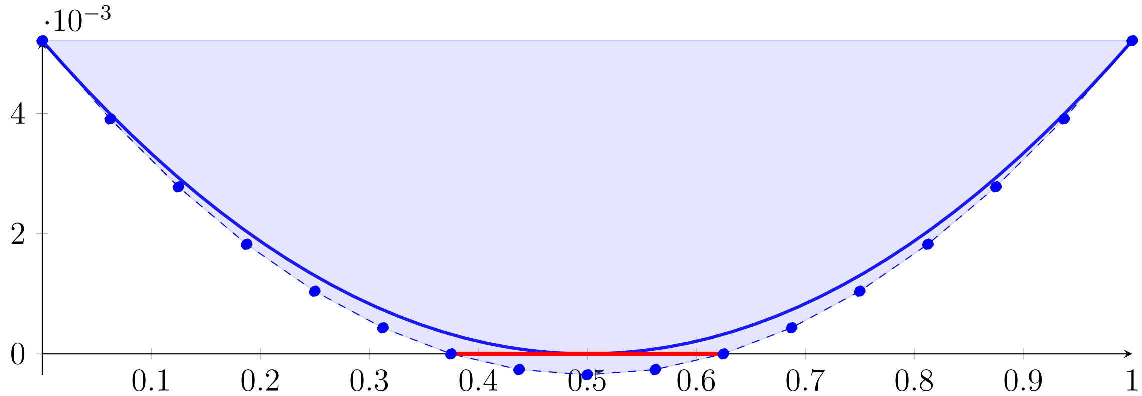
$$[0.3749927302224649, 0.6250069467380417]$$

Longest intersection interval: 0.2500142165155768
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999996789462157, 0.3000004910541495]$,

82.11 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999996789462157, 0.3$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.579480927696053\cdot 10^{-98}X^{16} + 3.526136882584451\cdot 10^{-92}X^{15} + 1.217422078989368\\&\cdot 10^{-83}X^{14} - 1.169070552755016\cdot 10^{-77}X^{13} - 1.762755817503918\cdot 10^{-69}X^{12}\\&+ 1.55837613779483\cdot 10^{-63}X^{11} + 1.410908032488726\cdot 10^{-55}X^{10} - 1.080908862805813\cdot 10^{-49}X^{9}\\&- 6.750723381404434\cdot 10^{-42}X^{8} + 4.142273519440928\cdot 10^{-36}X^{7} + 1.932858818028869\cdot 10^{-28}X^{6}\\&- 8.352503738960762\cdot 10^{-23}X^{5} - 3.06897495525372\cdot 10^{-15}X^{4} + 6.944510025023856\cdot 10^{-10}X^{3}\\&+ 0.02086073248825264X^{2} - 0.0208607236297874X + 0.005214387850787818\\= &\,0.005214387850787818B_{0,16}(X) + 0.003910592623926106B_{1,16}(X)\\&+ 0.002780636834466498B_{2,16}(X) + 0.001824520483649087B_{3,16}(X)\\&+ 0.001042243572713962B_{4,16}(X) + 0.000433806102901211B_{5,16}(X) - 7.919245490808786\\&\cdot 10^{-07}B_{6,16}(X) - 0.0002615505083968289B_{7,16}(X) - 0.0003484696474019504B_{8,16}(X)\\&- 0.0002615493403243644B_{9,16}(X) - 7.895859239916112\cdot 10^{-07}B_{10,16}(X)\\&+ 0.0004338096170392455B_{11,16}(X) + 0.001042248269805423B_{12,16}(X)\\&+ 0.001824526373614615B_{13,16}(X) + 0.002780643929706894B_{14,16}(X)\\&+ 0.00391060093932233B_{15,16}(X) + 0.005214397403700994B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.3748861124966261, 0.6251135508761015\}$$

Intersection intervals with the x axis:

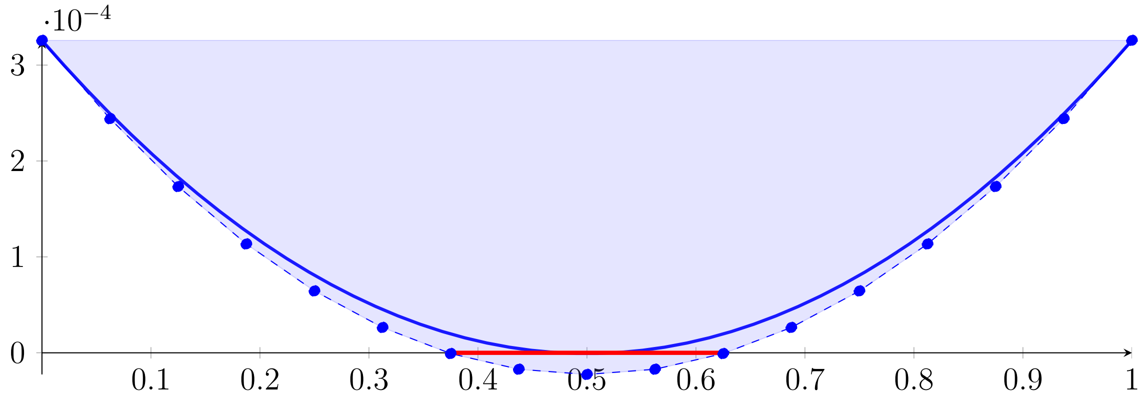
$$[0.3748861124966261, 0.6251135508761015]$$

Longest intersection interval: 0.2502274383794754
$\Longrightarrow$ Selective recursion: interval 1: $[0.2999999833942019, 0.3000001866058899]$,

82.12 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.2999999833942019, 0$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -8.456271781717561 \cdot 10^{-108} X^{16} + 3.329051186798107 \cdot 10^{-101} X^{15} + 4.593357002546748 \\
& \cdot 10^{-92} X^{14} - 1.76275695811523 \cdot 10^{-85} X^{13} - 1.062212310975265 \cdot 10^{-76} X^{12} \\
& + 3.752790404631579 \cdot 10^{-70} X^{11} + 1.357838209904933 \cdot 10^{-61} X^{10} - 4.157203866707784 \\
& \cdot 10^{-55} X^{9} - 1.037599553128416 \cdot 10^{-46} X^{8} + 2.544375234544489 \cdot 10^{-40} X^{7} + 4.744710700950638 \\
& \cdot 10^{-32} X^{6} - 8.193869976131749 \cdot 10^{-26} X^{5} - 1.203186876925119 \cdot 10^{-17} X^{4} + 1.088036641260801 \\
& \cdot 10^{-11} X^{3} + 0.001306169174102627 X^{2} - 0.001306168592474289 X + 0.0003257512461139167 \\
= & \; 0.0003257512461139167 B_{0,16}(X) + 0.0002441157090842736 B_{1,16}(X) \\
& + 0.0001733649151721524 B_{2,16}(X) + 0.0001134988643969824 B_{3,16}(X) \\
& + 6.451755677819264 \cdot 10^{-05} B_{4,16}(X) + 2.642099233521247 \cdot 10^{-05} B_{5,16}(X) \\
& - 7.908289125289409 \cdot 10^{-07} B_{6,16}(X) - 1.71179069456024 \cdot 10^{-05} B_{7,16}(X) - 2.25602417445787 \\
& \cdot 10^{-05} B_{8,16}(X) - 1.711783329002867 \cdot 10^{-05} B_{9,16}(X) - 7.906815625231293 \\
& \cdot 10^{-07} B_{10,16}(X) + 2.64212134573671 \cdot 10^{-05} B_{11,16}(X) + 6.451785178907119 \\
& \cdot 10^{-05} B_{12,16}(X) + 0.0001134992334520183 B_{13,16}(X) + 0.0001733653584656376 B_{14,16}(X) \\
& + 0.0002441162268493581 B_{15,16}(X) + 0.0003257518386226092 B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.3731836273807965, 0.626816029262996\}$$

Intersection intervals with the x axis:

$$[0.3731836273807965, 0.626816029262996]$$

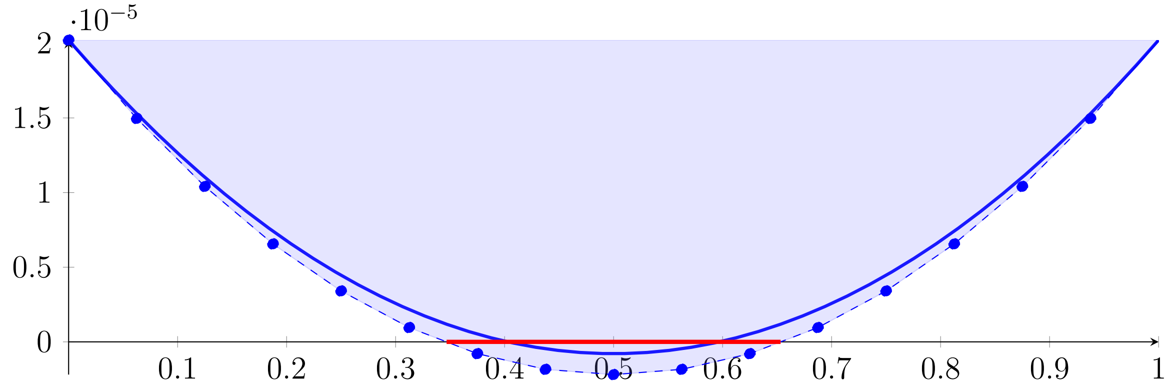
Longest intersection interval: 0.2536324018821995

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000592294767, 0.3000001107705452]$,

82.13 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: [0.3000000592294767

Normalized monomial und Bézier representations and the Bézier polygon:

$$p = -2.480017149294215\cdot 10^{-117}X^{16} + 3.84938040332832\cdot 10^{-110}X^{15} + 2.094095506848417 \cdot 10^{-100}X^{14} - 3.168497686663236\cdot 10^{-93}X^{13} - 7.527801307502411\cdot 10^{-84}X^{12} + 1.048590305184717\cdot 10^{-76}X^{11} + 1.495875245120848\cdot 10^{-67}X^{10} - 1.80569087496117\cdot 10^{-60}X^{9} - 1.776919108083797\cdot 10^{-51}X^{8} + 1.717962865912528\cdot 10^{-44}X^{7} + 1.263101207166261\cdot 10^{-35}X^{6} - 8.600271811104809\cdot 10^{-29}X^{5} - 4.979113541149949\cdot 10^{-20}X^{4} + 1.775239724826758\cdot 10^{-13}X^{3} + 8.402507389292435\cdot 10^{-05}X^{2} - 8.402503818595556\cdot 10^{-05}X + 2.02154953599674\cdot 10^{-05}$$
$$= 2.02154953599674\cdot 10^{-05}B_{0,16}(X) + 1.496393047334518\cdot 10^{-05}B_{1,16}(X) + 1.041257453583066 \cdot 10^{-05}B_{2,16}(X) + 6.561427547740848\cdot 10^{-06}B_{3,16}(X) + 3.410489509392755\cdot 10^{-06}B_{4,16}(X) + 9.59760421103387\cdot 10^{-07}B_{5,16}(X) - 7.907597168102504\cdot 10^{-07}B_{6,16}(X) - 1.84107090403115 \cdot 10^{-06}B_{7,16}(X) - 2.191173140242304\cdot 10^{-06}B_{8,16}(X) - 1.841066425126706\cdot 10^{-06}B_{9,16}(X) - 7.907507583673494\cdot 10^{-07}B_{10,16}(X) + 9.59773860352773\cdot 10^{-07}B_{11,16}(X) + 3.410507431350668 \cdot 10^{-06}B_{12,16}(X) + 6.561449954943343\cdot 10^{-06}B_{13,16}(X) + 1.04126014314478\cdot 10^{-05}B_{14,16}(X) + 1.496396186118106\cdot 10^{-05}B_{15,16}(X) + 2.021553124446011\cdot 10^{-05}B_{16,16}(X)$$



Intersection of the convex hull with the x axis:

$$\{0.3467669730097793, 0.6532326348738207\}$$

Intersection intervals with the x axis:

$$[0.3467669730097793, 0.6532326348738207]$$

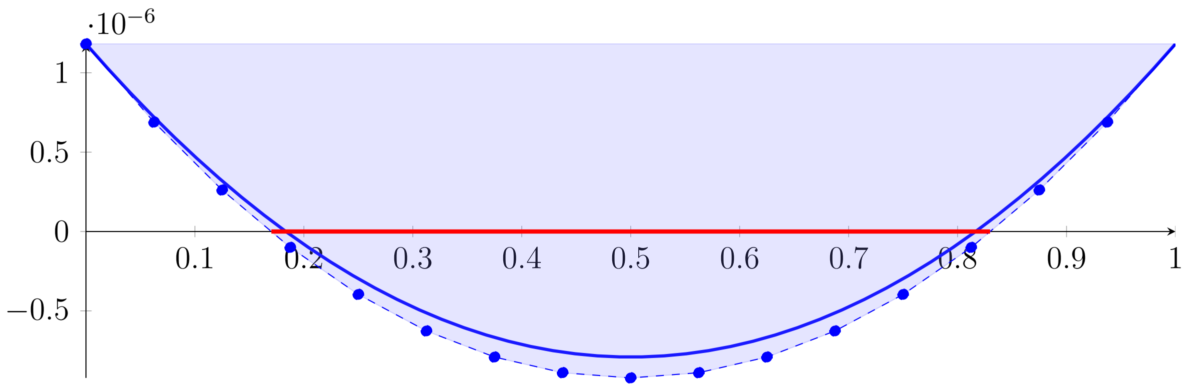
Longest intersection interval: 0.3064656618640414

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000771022171, 0.3000000928977847]$,

82.14 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000771022171, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-1.50163304400299\cdot 10^{-125}X^{16} + 7.605328677353438\cdot 10^{-118}X^{15} + 1.35002551430803\\
&\cdot 10^{-107}X^{14} - 6.665257392422306\cdot 10^{-100}X^{13} - 5.167137254241996\cdot 10^{-90}X^{12}\\
&+ 2.348582048403403\cdot 10^{-82}X^{11} + 1.093235143477049\cdot 10^{-72}X^{10} - 4.306055922184639\cdot 10^{-65}X^{9}\\
&- 1.382681732768409\cdot 10^{-55}X^{8} + 4.362007350883115\cdot 10^{-48}X^{7} + 1.04647552702743\cdot 10^{-38}X^{6}\\
&- 2.324990317134255\cdot 10^{-31}X^{5} - 4.392171749769965\cdot 10^{-22}X^{4} + 5.109781163448487\cdot 10^{-15}X^{3}\\
&+ 7.891735947253277\cdot 10^{-06}X^{2} - 7.891735064633028\cdot 10^{-06}X + 1.182178307271875\cdot 10^{-06}\\
= &\ 1.182178307271875\cdot 10^{-06}B_{0,16}(X) + 6.889448657323109\cdot 10^{-07}B_{1,16}(X) + 2.614758904198573\\
&\cdot 10^{-07}B_{2,16}(X) - 1.00228618656361\cdot 10^{-07}B_{3,16}(X) - 3.961686614872195\cdot 10^{-07}B_{4,16}(X)\\
&- 6.263442380635935\cdot 10^{-07}B_{5,16}(X) - 7.907553483763585\cdot 10^{-07}B_{6,16}(X) - 8.894019924163897\\
&\cdot 10^{-07}B_{7,16}(X) - 9.222841701745627\cdot 10^{-07}B_{8,16}(X) - 8.894018816417527\cdot 10^{-07}B_{9,16}(X)\\
&- 7.907551268088353\cdot 10^{-07}B_{10,16}(X) - 6.263439056666857\cdot 10^{-07}B_{11,16}(X)\\
&- 3.961682182061795\cdot 10^{-07}B_{12,16}(X) - 1.002280644181919\cdot 10^{-07}B_{13,16}(X) + 2.614765557064017\\
&\cdot 10^{-07}B_{14,16}(X) + 6.889456421767258\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.1701811983018366, 0.8298187006126137\}$$

Intersection intervals with the x axis:

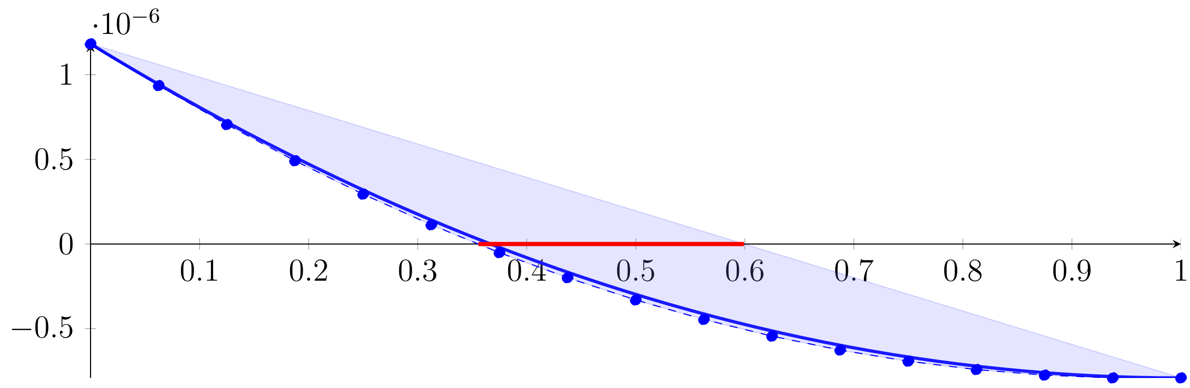
$$[0.1701811983018366, 0.8298187006126137]$$

Longest intersection interval: 0.6596375023107771
$\Longrightarrow$ Bisection: first half $[0.3000000771022171, 0.3000000850000009]$ und second half $[0.3000000850000009, 0.30$

82.15 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 on the First Half $[0.3000000771022171, 0.3000000850000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = & -2.291310186772141\cdot 10^{-130}X^{16} + 2.320962120774364\cdot 10^{-122}X^{15} + 8.239901820727723 \\
& \cdot 10^{-112}X^{14} - 8.13630052785926\cdot 10^{-104}X^{13} - 1.261508118711425\cdot 10^{-93}X^{12} \\
& + 1.146768578321974\cdot 10^{-85}X^{11} + 1.067612444801806\cdot 10^{-75}X^{10} - 8.410265473016874\cdot 10^{-68}X^{9} \\
& - 5.401100518626597\cdot 10^{-58}X^{8} + 3.407818242877434\cdot 10^{-50}X^{7} + 1.635118010980359\cdot 10^{-40}X^{6} \\
& - 7.265594741044545\cdot 10^{-33}X^{5} - 2.745107343606228\cdot 10^{-23}X^{4} + 6.387226454310609\cdot 10^{-16}X^{3} \\
& + 1.972933986813319\cdot 10^{-06}X^{2} - 3.945867532316514\cdot 10^{-06}X + 1.182178307271875\cdot 10^{-06} \\
= & \ 1.182178307271875\cdot 10^{-06}B_{0,16}(X) + 9.35561586502093\cdot 10^{-07}B_{1,16}(X) + 7.053859822890886 \\
& \cdot 10^{-07}B_{2,16}(X) + 4.916514946340023\cdot 10^{-07}B_{3,16}(X) + 2.943581235379749\cdot 10^{-07}B_{4,16}(X) \\
& + 1.135058690021469\cdot 10^{-07}B_{5,16}(X) - 5.090526897234114\cdot 10^{-08}B_{6,16}(X) - 1.988752903843486 \\
& \cdot 10^{-07}B_{7,16}(X) - 3.30404195232735\cdot 10^{-07}B_{8,16}(X) - 4.454919835163597\cdot 10^{-07}B_{9,16}(X) \\
& - 5.441386552340822\cdot 10^{-07}B_{10,16}(X) - 6.263442103847618\cdot 10^{-07}B_{11,16}(X) - 6.921086489672579 \\
& \cdot 10^{-07}B_{12,16}(X) - 7.414319709804301\cdot 10^{-07}B_{13,16}(X) - 7.743141764231377\cdot 10^{-07}B_{14,16}(X) \\
& - 7.907552652942402\cdot 10^{-07}B_{15,16}(X) - 7.907552375925969\cdot 10^{-07}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.355648638833307, 0.599198239772989\}$$

Intersection intervals with the x axis:

$$[0.355648638833307, 0.599198239772989]$$

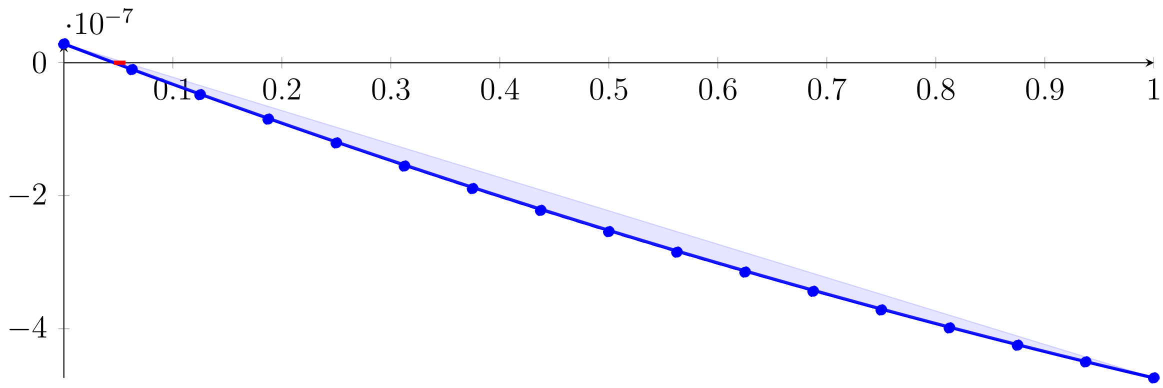
Longest intersection interval: 0.2435496009396821

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799110531, 0.3000000818345552]$,

82.16 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799110531, 0.3000000818345552]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-3.511418903935676\cdot 10^{-140}X^{16} + 1.460425350169266\cdot 10^{-131}X^{15} + 2.12885435080787\\&\cdot 10^{-120}X^{14} - 8.631046367525725\cdot 10^{-112}X^{13} - 5.494638409481287\cdot 10^{-101}X^{12}\\&+ 2.050866587664798\cdot 10^{-92}X^{11} + 7.839490728884222\cdot 10^{-82}X^{10} - 2.535691914683604\cdot 10^{-73}X^{9}\\&- 6.686235030771963\cdot 10^{-63}X^{8} + 1.732161414466012\cdot 10^{-54}X^{7} + 3.412507672068927\cdot 10^{-44}X^{6}\\&- 6.225987736384381\cdot 10^{-36}X^{5} - 9.658485252841958\cdot 10^{-26}X^{4} + 9.227298165814739\cdot 10^{-18}X^{3}\\&+ 1.170273575918748\cdot 10^{-07}X^{2} - 6.192309389580206\cdot 10^{-07}X + 2.838432851654629\cdot 10^{-08}\\= &\,2.838432851654629\cdot 10^{-08}B_{0,16}(X) - 1.031760516832999\cdot 10^{-08}B_{1,16}(X) - 4.804431087327399\\&\cdot 10^{-08}B_{2,16}(X) - 8.479578859826921\cdot 10^{-08}B_{3,16}(X) - 1.205720383432992\cdot 10^{-07}B_{4,16}(X)\\&- 1.553730601083475\cdot 10^{-07}B_{5,16}(X) - 1.891988538933975\cdot 10^{-07}B_{6,16}(X) - 2.220494196984329\\&\cdot 10^{-07}B_{7,16}(X) - 2.539247575234371\cdot 10^{-07}B_{8,16}(X) - 2.848248673683937\cdot 10^{-07}B_{9,16}(X)\\&- 3.147497492332862\cdot 10^{-07}B_{10,16}(X) - 3.436994031180981\cdot 10^{-07}B_{11,16}(X) - 3.71673829022813\\&\cdot 10^{-07}B_{12,16}(X) - 3.986730269474143\cdot 10^{-07}B_{13,16}(X) - 4.246969968918855\cdot 10^{-07}B_{14,16}(X)\\&- 4.497457388562103\cdot 10^{-07}B_{15,16}(X) - 4.738192528403721\cdot 10^{-07}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.04583803348764934, 0.05651956610873593\}$$

Intersection intervals with the x axis:

$$[0.04583803348764934, 0.05651956610873593]$$

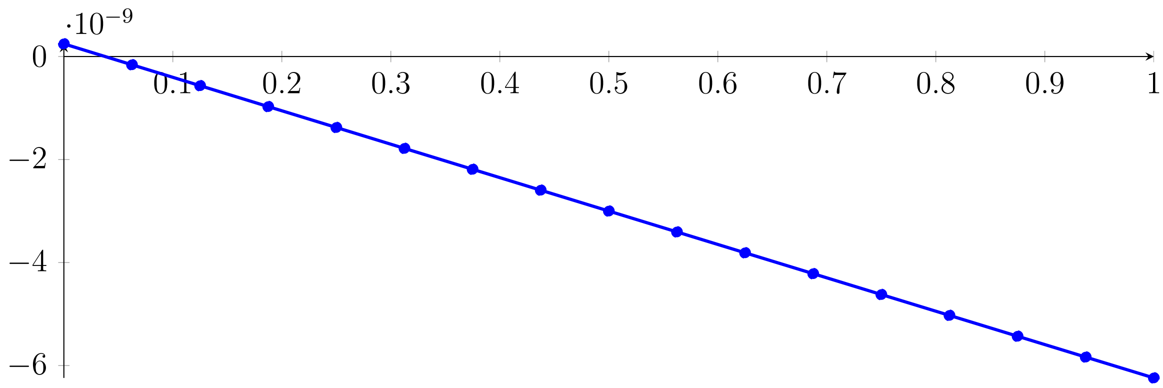
Longest intersection interval: 0.01068153262108659

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799992227, 0.3000000800197686]$,

82.17 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799992227, 0.3000000800197686]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.0083696859672 \cdot 10^{-171} X^{16} + 3.926295021503001 \cdot 10^{-161} X^{15} + 5.358163301918743 \\ &\cdot 10^{-148} X^{14} - 2.033760808321352 \cdot 10^{-137} X^{13} - 1.212109824997298 \cdot 10^{-124} X^{12} \\ &+ 4.235519560164542 \cdot 10^{-114} X^{11} + 1.515735988691705 \cdot 10^{-101} X^{10} - 4.589851404660334 \cdot 10^{-91} X^{9} \\ &- 1.133052955101212 \cdot 10^{-78} X^{8} + 2.748041945976198 \cdot 10^{-68} X^{7} + 5.068448795464027 \cdot 10^{-56} X^{6} \\ &- 8.657173392624336 \cdot 10^{-46} X^{5} - 1.257312709564581 \cdot 10^{-33} X^{4} + 1.124540929737694 \cdot 10^{-23} X^{3} \\ &+ 1.335225264723055 \cdot 10^{-11} X^{2} - 6.499737499496223 \cdot 10^{-09} X + 2.458891434694464 \cdot 10^{-10} \\ = &\ 2.458891434694464 \cdot 10^{-10} B_{0,16}(X) - 1.603444502490676 \cdot 10^{-10} B_{1,16}(X) - 5.664667751955213 \\ &\cdot 10^{-10} B_{2,16}(X) - 9.724778313699147 \cdot 10^{-10} B_{3,16}(X) - 1.378377618772248 \cdot 10^{-09} B_{4,16}(X) \\ &- 1.784166137402521 \cdot 10^{-09} B_{5,16}(X) - 2.189843387260733 \cdot 10^{-09} B_{6,16}(X) - 2.595409368346885 \\ &\cdot 10^{-09} B_{7,16}(X) - 3.000864080660977 \cdot 10^{-09} B_{8,16}(X) - 3.406207524203008 \cdot 10^{-09} B_{9,16}(X) \\ &- 3.811439698972979 \cdot 10^{-09} B_{10,16}(X) - 4.21656060497089 \cdot 10^{-09} B_{11,16}(X) - 4.62157024219674 \\ &\cdot 10^{-09} B_{12,16}(X) - 5.026468610650529 \cdot 10^{-09} B_{13,16}(X) - 5.431255710332258 \cdot 10^{-09} B_{14,16}(X) \\ &- 5.835931541241927 \cdot 10^{-09} B_{15,16}(X) - 6.240496103379535 \cdot 10^{-09} B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03783062677354348, 0.03790850128565779\}$$

Intersection intervals with the x axis:

$$[0.03783062677354348, 0.03790850128565779]$$

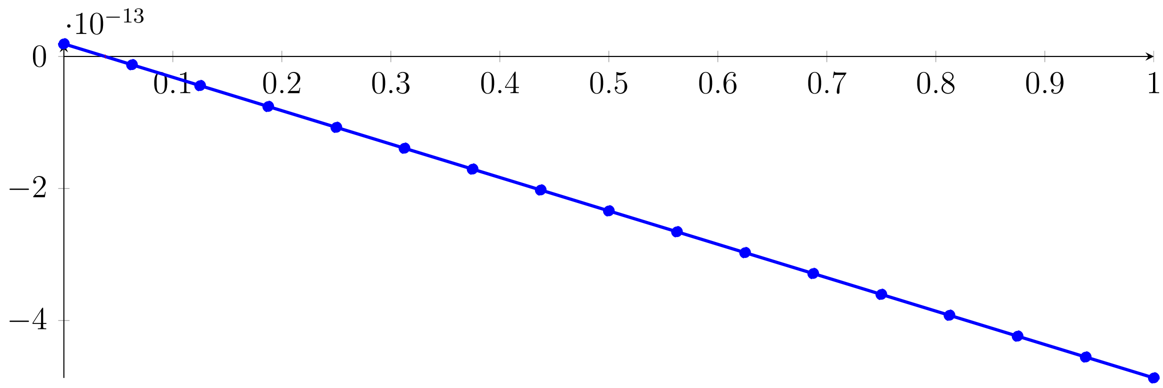
Longest intersection interval: $7.787451211431613 \cdot 10^{-05}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999999, 0.3000000800000015]$,

82.18 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.3000000799999999, 0.3000000800000015]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.844782636201088\cdot 10^{-237}X^{16} + 9.223866560433282\cdot 10^{-223}X^{15} + 1.616406874872468\\ &\cdot 10^{-205}X^{14} - 7.878422704947929\cdot 10^{-191}X^{13} - 6.029565776386014\cdot 10^{-174}X^{12}\\ &+ 2.705549092213795\cdot 10^{-159}X^{11} + 1.243302910922026\cdot 10^{-142}X^{10} - 4.83455688747521\cdot 10^{-128}X^{9}\\ &- 1.5325438805125\cdot 10^{-111}X^{8} + 4.772992249567202\cdot 10^{-97}X^{7} + 1.130438906312722\cdot 10^{-80}X^{6}\\ &- 2.479435471431183\cdot 10^{-66}X^{5} - 4.624072778948578\cdot 10^{-50}X^{4} + 5.310816347750779\cdot 10^{-36}X^{3}\\ &+ 8.097393019768196\cdot 10^{-20}X^{2} - 5.060852140608093\cdot 10^{-13}X + 1.910916079008261\cdot 10^{-14}\\ = &\ 1.910916079008261\cdot 10^{-14}B_{0,16}(X) - 1.252116508871797\cdot 10^{-14}B_{1,16}(X) - 4.41514902927358\\ &\cdot 10^{-14}B_{2,16}(X) - 7.578181482197087\cdot 10^{-14}B_{3,16}(X) - 1.074121386764232\cdot 10^{-13}B_{4,16}(X)\\ &- 1.390424618560928\cdot 10^{-13}B_{5,16}(X) - 1.706727843609796\cdot 10^{-13}B_{6,16}(X) - 2.023031061910837\\ &\cdot 10^{-13}B_{7,16}(X) - 2.33933427346405\cdot 10^{-13}B_{8,16}(X) - 2.655637478269435\cdot 10^{-13}B_{9,16}(X)\\ &- 2.971940676326994\cdot 10^{-13}B_{10,16}(X) - 3.288243867636724\cdot 10^{-13}B_{11,16}(X) - 3.604547052198627\\ &\cdot 10^{-13}B_{12,16}(X) - 3.920850230012703\cdot 10^{-13}B_{13,16}(X) - 4.237153401078951\cdot 10^{-13}B_{14,16}(X)\\ &- 4.553456565397371\cdot 10^{-13}B_{15,16}(X) - 4.869759722967965\cdot 10^{-13}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775878104944304, 0.03775878709087106\}$$

Intersection intervals with the x axis:

$$[0.03775878104944304, 0.03775878709087106]$$

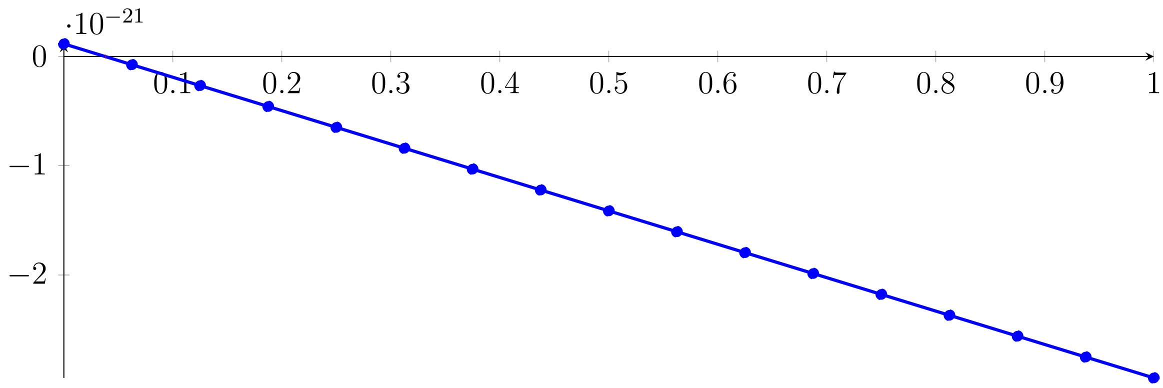
Longest intersection interval: $6.041428015081178\cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

82.19 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-9.719453947461351\cdot10^{-326}X^{16} + 4.850491890476349\cdot10^{-325}X^{15} + 1.39445463768969\\&\cdot10^{-320}X^{14} - 1.125263275537969\cdot10^{-297}X^{13} - 1.425480559690934\cdot10^{-272}X^{12}\\&+ 1.058744275447586\cdot10^{-249}X^{11} + 8.053283759644375\cdot10^{-225}X^{10} - 5.183380864729409\cdot10^{-202}X^{9}\\&- 2.7197548469932\cdot10^{-177}X^{8} + 1.402064409388781\cdot10^{-154}X^{7} + 5.496480949202085\cdot10^{-130}X^{6}\\&- 1.995495881284759\cdot10^{-107}X^{5} - 6.160033605799806\cdot10^{-83}X^{4} + 1.17106256664041\cdot10^{-60}X^{3}\\&+ 2.955455531505519\cdot10^{-36}X^{2} - 3.057477353302275\cdot10^{-21}X + 1.154466008703692\cdot10^{-22}\\= &\ 1.154466008703692\cdot10^{-22}B_{0,16}(X) - 7.564573371102292\cdot10^{-23}B_{1,16}(X) - 2.667380682924151\\&\cdot10^{-22}B_{2,16}(X) - 4.578304028738072\cdot10^{-22}B_{3,16}(X) - 6.489227374551993\cdot10^{-22}B_{4,16}(X)\\&- 8.400150720365913\cdot10^{-22}B_{5,16}(X) - 1.031107406617983\cdot10^{-21}B_{6,16}(X) - 1.222199741199375\\&\cdot10^{-21}B_{7,16}(X) - 1.413292075780767\cdot10^{-21}B_{8,16}(X) - 1.604384410362159\cdot10^{-21}B_{9,16}(X)\\&- 1.795476744943551\cdot10^{-21}B_{10,16}(X) - 1.986569079524943\cdot10^{-21}B_{11,16}(X) - 2.177661414106335\\&\cdot10^{-21}B_{12,16}(X) - 2.368753748687727\cdot10^{-21}B_{13,16}(X) - 2.559846083269119\cdot10^{-21}B_{14,16}(X)\\&- 2.750938417850511\cdot10^{-21}B_{15,16}(X) - 2.942030752431902\cdot10^{-21}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775877546424977, 0.0377587754642498\}$$

Intersection intervals with the x axis:

$$[0.03775877546424977, 0.0377587754642498]$$

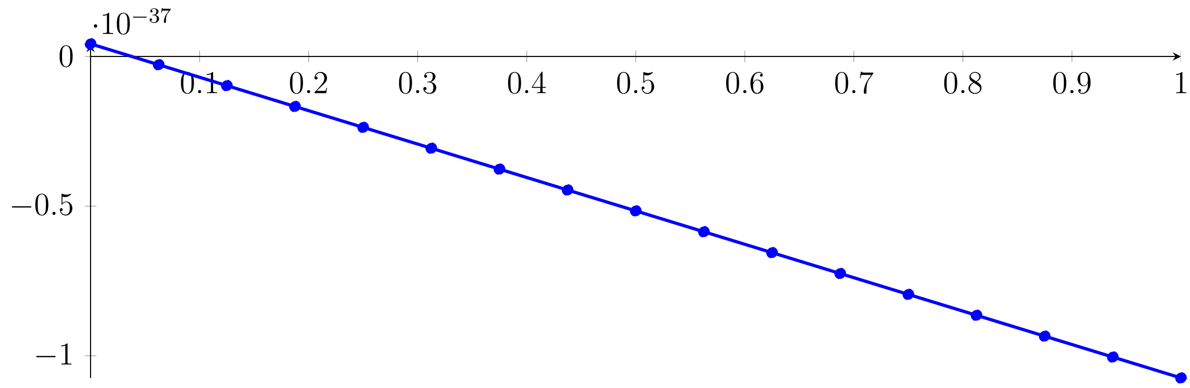
Longest intersection interval: $3.649884166375354\cdot10^{-17}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

82.20 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & 3.068577046082631\cdot 10^{-342}X^{16} + 1.486894544116139\cdot 10^{-341}X^{15} - 7.903181386330787 \\
& \cdot 10^{-341}X^{14} - 4.482026616235252\cdot 10^{-340}X^{13} + 6.545443281471007\cdot 10^{-340}X^{12} \\
& + 6.169377812935582\cdot 10^{-341}X^{11} - 3.267485286110327\cdot 10^{-340}X^{10} + 1.705555506486159\cdot 10^{-340}X^{9} \\
& - 8.565691202159373\cdot 10^{-309}X^{8} + 1.20982216727754\cdot 10^{-269}X^{7} + 1.299448935949168\cdot 10^{-228}X^{6} \\
& - 1.292546704525185\cdot 10^{-189}X^{5} - 1.093199460362076\cdot 10^{-148}X^{4} + 5.693998445459821\cdot 10^{-110}X^{3} \\
& + 3.937155726791203\cdot 10^{-69}X^{2} - 1.11594381808692\cdot 10^{-37}X + 4.213667205786155\cdot 10^{-39} \\
= {} & 4.213667205786155\cdot 10^{-39}B_{0,16}(X) - 2.760981657257093\cdot 10^{-39}B_{1,16}(X) - 9.73563052030034 \\
& \cdot 10^{-39}B_{2,16}(X) - 1.671027938334359\cdot 10^{-38}B_{3,16}(X) - 2.368492824638684\cdot 10^{-38}B_{4,16}(X) \\
& - 3.065957710943008\cdot 10^{-38}B_{5,16}(X) - 3.763422597247333\cdot 10^{-38}B_{6,16}(X) - 4.460887483551658 \\
& \cdot 10^{-38}B_{7,16}(X) - 5.158352369855983\cdot 10^{-38}B_{8,16}(X) - 5.855817256160307\cdot 10^{-38}B_{9,16}(X) \\
& - 6.553282142464632\cdot 10^{-38}B_{10,16}(X) - 7.250747028768957\cdot 10^{-38}B_{11,16}(X) - 7.948211915073282 \\
& \cdot 10^{-38}B_{12,16}(X) - 8.645676801377607\cdot 10^{-38}B_{13,16}(X) - 9.343141687681931\cdot 10^{-38}B_{14,16}(X) \\
& - 1.004060657398626\cdot 10^{-37}B_{15,16}(X) - 1.073807146029058\cdot 10^{-37}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.03775877546424973, 0.03775877546424973\}$$

Intersection intervals with the x axis:

$$[0.03775877546424973, 0.03775877546424973]$$

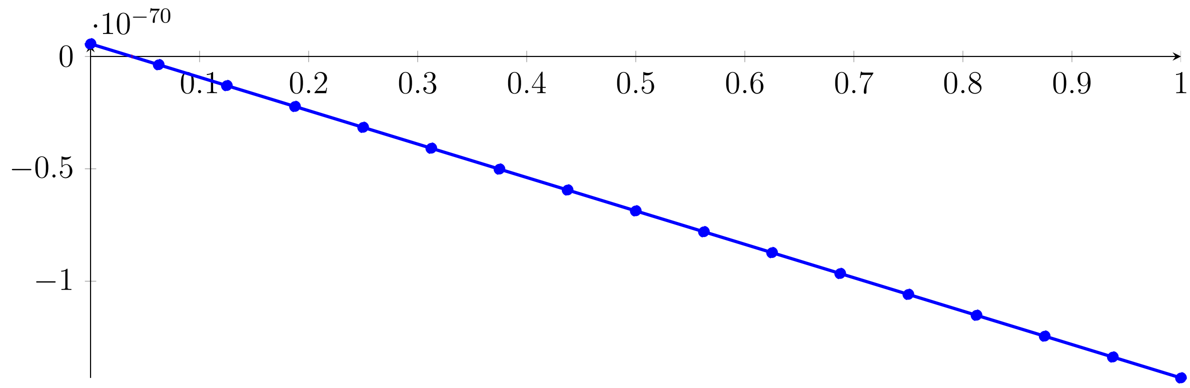
Longest intersection interval: $1.332165442795749\cdot 10^{-33}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

82.21 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.025934731653419\cdot 10^{-374}X^{16} + 4.868132437101619\cdot 10^{-375}X^{15} - 1.47978330372824\\&\cdot 10^{-373}X^{14} + 1.527819842345998\cdot 10^{-373}X^{13} - 8.375928131266133\cdot 10^{-373}X^{12}\\&+ 4.048609744710075\cdot 10^{-373}X^{11} + 4.937543623439895\cdot 10^{-373}X^{10} + 1.106452352591573\\&\cdot 10^{-373}X^{9} + 2.02273320708147\cdot 10^{-373}X^{8} - 4.610218135798222\cdot 10^{-374}X^{7} + 3.227152695058755\\&\cdot 10^{-375}X^{6} - 5.422968261766101\cdot 10^{-354}X^{5} - 3.442960680879394\cdot 10^{-280}X^{4} + 1.346144977055374\\&\cdot 10^{-208}X^{3} + 6.98713155044671\cdot 10^{-135}X^{2} - 1.48662179055694\cdot 10^{-70}X + 5.61330183899004\cdot 10^{-72}\\= &\,5.61330183899004\cdot 10^{-72}B_{0,16}(X) - 3.678084351990836\cdot 10^{-72}B_{1,16}(X) - 1.296947054297171\\&\cdot 10^{-71}B_{2,16}(X) - 2.226085673395259\cdot 10^{-71}B_{3,16}(X) - 3.155224292493346\cdot 10^{-71}B_{4,16}(X)\\&- 4.084362911591434\cdot 10^{-71}B_{5,16}(X) - 5.013501530689521\cdot 10^{-71}B_{6,16}(X) - 5.942640149787609\\&\cdot 10^{-71}B_{7,16}(X) - 6.871778768885697\cdot 10^{-71}B_{8,16}(X) - 7.800917387983784\cdot 10^{-71}B_{9,16}(X)\\&- 8.730056007081872\cdot 10^{-71}B_{10,16}(X) - 9.659194626179959\cdot 10^{-71}B_{11,16}(X)\\&- 1.058833324527805\cdot 10^{-70}B_{12,16}(X) - 1.151747186437613\cdot 10^{-70}B_{13,16}(X) - 1.244661048347422\\&\cdot 10^{-70}B_{14,16}(X) - 1.337574910257231\cdot 10^{-70}B_{15,16}(X) - 1.43048877216704\cdot 10^{-70}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.03775877546424973, 0.03775877546424973\}$$

Intersection intervals with the x axis:

$$[0.03775877546424973, 0.03775877546424973]$$

Longest intersection interval: $1.774664766979194\cdot 10^{-66}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

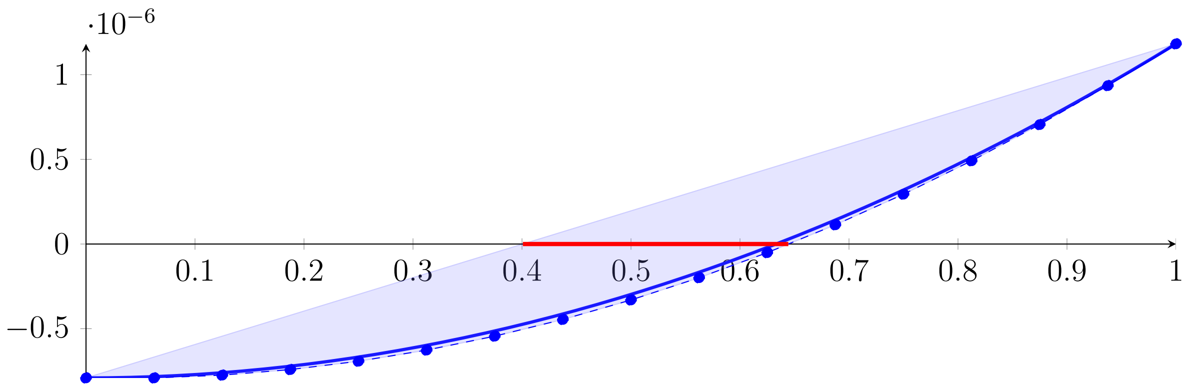
82.22 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 22!

82.23 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 on the Second Half $[0.3000000850000009, 0.3000000928977847]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p &= -2.291310186772141\cdot 10^{-130}X^{16} + 2.320961754164735\cdot 10^{-122}X^{15} + 8.239901824209166\\&\quad\cdot 10^{-112}X^{14} - 8.136299374273005\cdot 10^{-104}X^{13} - 1.261508119769144\cdot 10^{-93}X^{12}\\&\quad + 1.146768426941\cdot 10^{-85}X^{11} + 1.067612446063251\cdot 10^{-75}X^{10} - 8.410264405404428\cdot 10^{-68}X^{9}\\&\quad - 5.401100526195836\cdot 10^{-58}X^{8} + 3.407817810789392\cdot 10^{-50}X^{7} + 1.635118013365832\cdot 10^{-40}X^{6}\\&\quad - 7.265593759973738\cdot 10^{-33}X^{5} - 2.745107347239025\cdot 10^{-23}X^{4} + 6.387225356267671\cdot 10^{-16}X^{3}\\&\quad + 1.972933988729487\cdot 10^{-06}X^{2} + 4.432262923936659\cdot 10^{-13}X - 7.907552375925969\cdot 10^{-07}\\&= -7.907552375925969\cdot 10^{-07}B_{0,16}(X) - 7.907552098909536\cdot 10^{-07}B_{1,16}(X) - 7.743140656165646\\&\quad\cdot 10^{-07}B_{2,16}(X) - 7.414318047682893\cdot 10^{-07}B_{3,16}(X) - 6.921084273449871\cdot 10^{-07}B_{4,16}(X)\\&\quad - 6.263439333455175\cdot 10^{-07}B_{5,16}(X) - 5.441383227687398\cdot 10^{-07}B_{6,16}(X) - 4.454915956135136\\&\quad\cdot 10^{-07}B_{7,16}(X) - 3.304037518786981\cdot 10^{-07}B_{8,16}(X) - 1.988747915631529\cdot 10^{-07}B_{9,16}(X)\\&\quad - 5.090471466573741\cdot 10^{-08}B_{10,16}(X) + 1.13506478814689\cdot 10^{-07}B_{11,16}(X) + 2.943587888792669\\&\quad\cdot 10^{-07}B_{12,16}(X) + 4.916522155291369\cdot 10^{-07}B_{13,16}(X) + 7.053867587654395\cdot 10^{-07}B_{14,16}(X)\\&\quad + 9.355624185893154\cdot 10^{-07}B_{15,16}(X) + 1.182179195001905\cdot 10^{-06}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.4008015798845968, 0.644351143917019\}$$

Intersection intervals with the x axis:

$$[0.4008015798845968, 0.644351143917019]$$

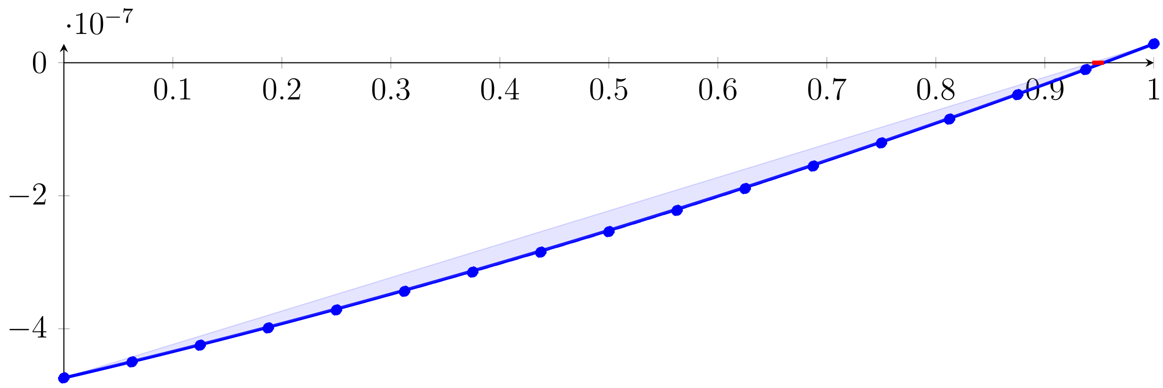
Longest intersection interval: 0.2435495640324222

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000881654451, 0.3000000900889469]$,

82.24 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 in Interval 1: $[0.3000000881654451, 0.3000000900889469]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p &= -3.511410390075519\cdot 10^{-140}X^{16} + 1.460421789403856\cdot 10^{-131}X^{15} + 2.12884983529034\\
&\quad\cdot 10^{-120}X^{14} - 8.631028085335548\cdot 10^{-112}X^{13} - 5.494628422481359\cdot 10^{-101}X^{12}\\
&\quad + 2.050862886067143\cdot 10^{-92}X^{11} + 7.839478858688502\cdot 10^{-82}X^{10} - 2.535688119961682\cdot 10^{-73}X^{9}\\
&\quad - 6.686226934767321\cdot 10^{-63}X^{8} + 1.732159347494186\cdot 10^{-54}X^{7} + 3.412504574505657\cdot 10^{-44}X^{6}\\
&\quad - 6.225982140334265\cdot 10^{-36}X^{5} - 9.658479411653884\cdot 10^{-26}X^{4} + 9.227292313018015\cdot 10^{-18}X^{3}\\
&\quad + 1.170273222422553\cdot 10^{-07}X^{2} + 3.851762081107905\cdot 10^{-07}X - 4.738191826798638\cdot 10^{-07}\\
&= -4.738191826798638\cdot 10^{-07}B_{0,16}(X) - 4.497456696729394\cdot 10^{-07}B_{1,16}(X) - 4.246969289806629\\
&\quad\cdot 10^{-07}B_{2,16}(X) - 3.986729606030177\cdot 10^{-07}B_{3,16}(X) - 3.716737645399875\cdot 10^{-07}B_{4,16}(X)\\
&\quad - 3.436993407915557\cdot 10^{-07}B_{5,16}(X) - 3.147496893577059\cdot 10^{-07}B_{6,16}(X) - 2.848248102384216\\
&\quad\cdot 10^{-07}B_{7,16}(X) - 2.539247034336863\cdot 10^{-07}B_{8,16}(X) - 2.220493689434835\cdot 10^{-07}B_{9,16}(X)\\
&\quad - 1.891988067677968\cdot 10^{-07}B_{10,16}(X) - 1.553730169066096\cdot 10^{-07}B_{11,16}(X) - 1.205719993599056\\
&\quad\cdot 10^{-07}B_{12,16}(X) - 8.479575412766813\cdot 10^{-08}B_{13,16}(X) - 4.804428120988084\cdot 10^{-08}B_{14,16}(X)\\
&\quad - 1.031758060652722\cdot 10^{-08}B_{15,16}(X) + 2.838434768240921\cdot 10^{-08}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9434803899886293, 0.9541619291703946\}$$

Intersection intervals with the x axis:

$$[0.9434803899886293, 0.9541619291703946]$$

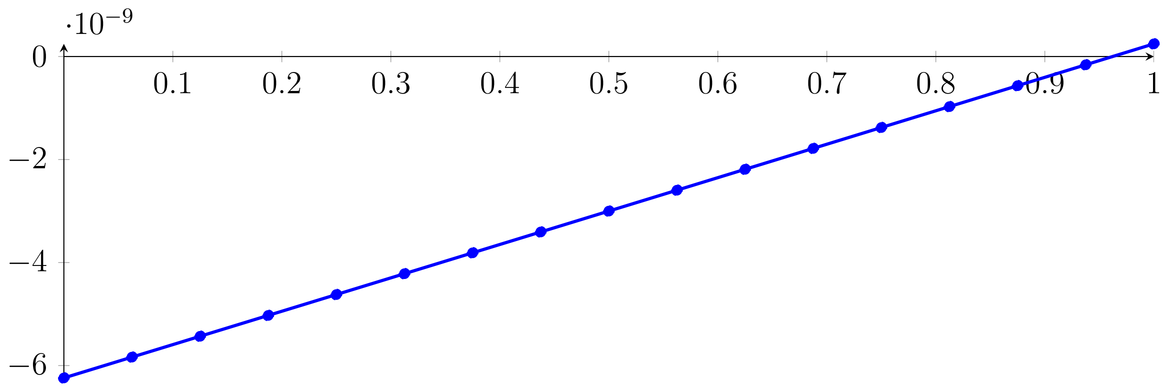
Longest intersection interval: 0.01068153918176529

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899802314, 0.3000000900007773]$,

82.25 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 in Interval 1: $[0.3000000899802314, 0.3000000900007773]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = & -1.008377150647939\cdot 10^{-171} X^{16} + 3.926321486414625\cdot 10^{-161} X^{15} + 5.358198011634222 \\
& \cdot 10^{-148} X^{14} - 2.033772676384319\cdot 10^{-137} X^{13} - 1.212116555959755\cdot 10^{-124} X^{12} \\
& + 4.235540409672438\cdot 10^{-114} X^{11} + 1.515743003803166\cdot 10^{-101} X^{10} - 4.58986978058239\cdot 10^{-91} X^{9} \\
& - 1.133057150938366\cdot 10^{-78} X^{8} + 2.748050405674693\cdot 10^{-68} X^{7} + 5.068462874895849\cdot 10^{-56} X^{6} \\
& - 8.65719194232841\cdot 10^{-46} X^{5} - 1.257315038545109\cdot 10^{-33} X^{4} + 1.12454224629143\cdot 10^{-23} X^{3} \\
& + 1.335226501895653\cdot 10^{-11} X^{2} + 6.473035980688146\cdot 10^{-09} X - 6.240498775828017\cdot 10^{-09} \\
= & -6.240498775828017\cdot 10^{-09} B_{0,16}(X) - 5.835934027035008\cdot 10^{-09} B_{1,16}(X) - 5.43125800936684 \\
& \cdot 10^{-09} B_{2,16}(X) - 5.026470722823515\cdot 10^{-09} B_{3,16}(X) - 4.621572167405032\cdot 10^{-09} B_{4,16}(X) \\
& - 4.216562343111391\cdot 10^{-09} B_{5,16}(X) - 3.811441249942592\cdot 10^{-09} B_{6,16}(X) - 3.406208887898635 \\
& \cdot 10^{-09} B_{7,16}(X) - 3.000865256979519\cdot 10^{-09} B_{8,16}(X) - 2.595410357185246\cdot 10^{-09} B_{9,16}(X) \\
& - 2.189844188515814\cdot 10^{-09} B_{10,16}(X) - 1.784166750971225\cdot 10^{-09} B_{11,16}(X) \\
& - 1.378378044551477\cdot 10^{-09} B_{12,16}(X) - 9.724780692565705\cdot 10^{-10} B_{13,16}(X) - 5.664668250865062 \\
& \cdot 10^{-10} B_{14,16}(X) - 1.603443120412836\cdot 10^{-10} B_{15,16}(X) + 2.458894698790972\cdot 10^{-10} B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9620914659183662, 0.9621693405339302\}$$

Intersection intervals with the x axis:

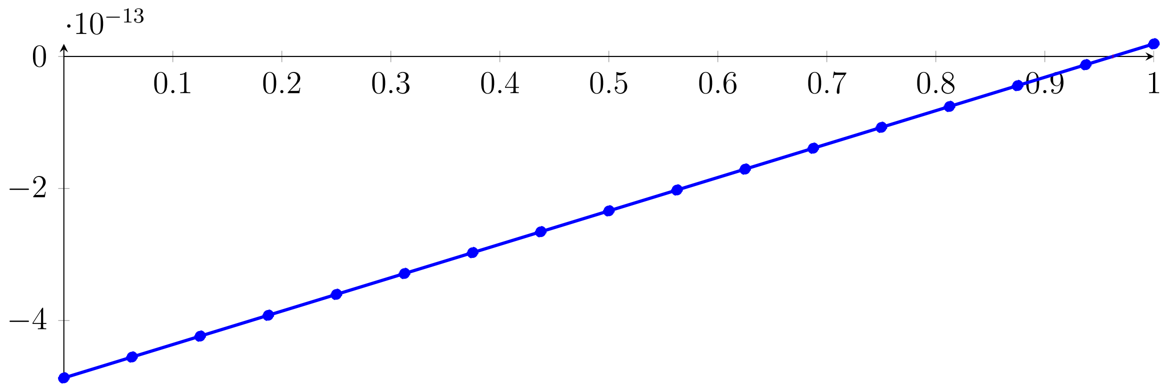
$$[0.9620914659183662, 0.9621693405339302]$$

Longest intersection interval: $7.787461556406489\cdot 10^{-05}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000899999985, 0.3000000900000001]$,

82.26 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.3000000899999985, 0.3000000900000001]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}p = &-1.844835503516189\cdot 10^{-237}X^{16} + 9.224112529673191\cdot 10^{-223}X^{15} + 1.616447407922287\\ &\cdot 10^{-205}X^{14} - 7.878604734573916\cdot 10^{-191}X^{13} - 6.029695377653573\cdot 10^{-174}X^{12}\\ &+ 2.705601945023742\cdot 10^{-159}X^{11} + 1.243325181575566\cdot 10^{-142}X^{10} - 4.834634042887648\cdot 10^{-128}X^{9}\\ &- 1.532565842702471\cdot 10^{-111}X^{8} + 4.773051325459324\cdot 10^{-97}X^{7} + 1.130451056729432\cdot 10^{-80}X^{6}\\ &- 2.479457251994378\cdot 10^{-66}X^{5} - 4.624105915218427\cdot 10^{-50}X^{4} + 5.31084372815498\cdot 10^{-36}X^{3}\\ &+ 8.097422035992938\cdot 10^{-20}X^{2} + 5.060859587615554\cdot 10^{-13}X - 4.869768282121624\cdot 10^{-13}\\ = &-4.869768282121624\cdot 10^{-13}B_{0,16}(X) - 4.553464557895652\cdot 10^{-13}B_{1,16}(X) - 4.237160826921828\\ &\cdot 10^{-13}B_{2,16}(X) - 3.920857089200152\cdot 10^{-13}B_{3,16}(X) - 3.604553344730625\cdot 10^{-13}B_{4,16}(X)\\ &- 3.288249593513246\cdot 10^{-13}B_{5,16}(X) - 2.971945835548016\cdot 10^{-13}B_{6,16}(X) - 2.655642070834933\\ &\cdot 10^{-13}B_{7,16}(X) - 2.339338299374\cdot 10^{-13}B_{8,16}(X) - 2.023034521165214\cdot 10^{-13}B_{9,16}(X)\\ &- 1.706730736208576\cdot 10^{-13}B_{10,16}(X) - 1.390426944504087\cdot 10^{-13}B_{11,16}(X)\\ &- 1.074123146051747\cdot 10^{-13}B_{12,16}(X) - 7.578193408515542\cdot 10^{-14}B_{13,16}(X) - 4.4151552890351\\ &\cdot 10^{-14}B_{14,16}(X) - 1.252117102076142\cdot 10^{-14}B_{15,16}(X) + 1.910921152361334\cdot 10^{-14}B_{16,16}(X)\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411803068306, 0.9622411863482746\}$$

Intersection intervals with the x axis:

$$[0.9622411803068306, 0.9622411863482746]$$

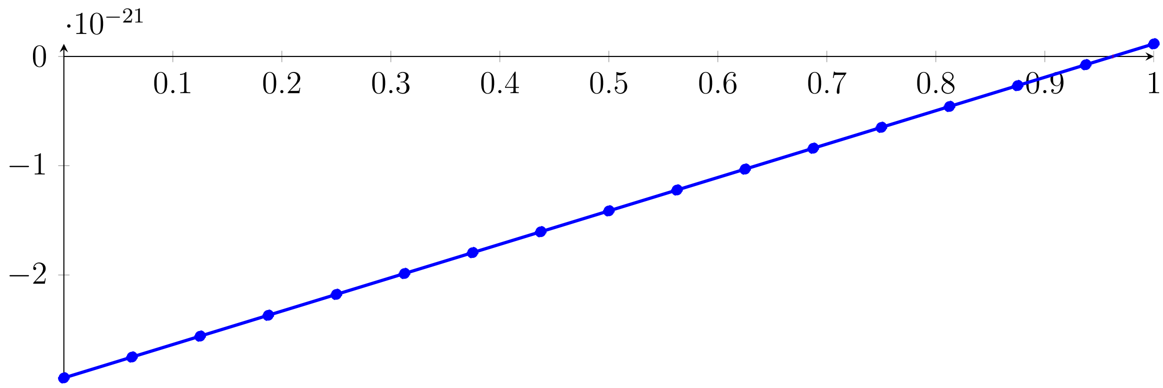
Longest intersection interval: $6.041444057142176\cdot 10^{-09}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

82.27 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p &= 6.121540900010562\cdot 10^{-326}X^{16} - 8.347007493216372\cdot 10^{-325}X^{15} + 1.39439413847082\\
&\quad \cdot 10^{-320}X^{14} - 1.125328119545448\cdot 10^{-297}X^{13} - 1.425556622701337\cdot 10^{-272}X^{12}\\
&\quad + 1.058795883612058\cdot 10^{-249}X^{11} + 8.05364186268366\cdot 10^{-225}X^{10} - 5.183587463234939\cdot 10^{-202}X^{9}\\
&\quad - 2.719851598975594\cdot 10^{-177}X^{8} + 1.402107824165868\cdot 10^{-154}X^{7} + 5.496627599374482\cdot 10^{-130}X^{6}\\
&\quad - 1.995539904687186\cdot 10^{-107}X^{5} - 6.160143177546872\cdot 10^{-83}X^{4} + 1.171077932951323\cdot 10^{-60}X^{3}\\
&\quad + 2.955481817665439\cdot 10^{-36}X^{2} + 3.057490949422222\cdot 10^{-21}X - 2.942043735497876\cdot 10^{-21}\\
&= -2.942043735497876\cdot 10^{-21}B_{0,16}(X) - 2.750950551158987\cdot 10^{-21}B_{1,16}(X) - 2.559857366820099\\
&\quad \cdot 10^{-21}B_{2,16}(X) - 2.36876418248121\cdot 10^{-21}B_{3,16}(X) - 2.177670998142321\cdot 10^{-21}B_{4,16}(X)\\
&\quad - 1.986577813803432\cdot 10^{-21}B_{5,16}(X) - 1.795484629464543\cdot 10^{-21}B_{6,16}(X) - 1.604391445125654\\
&\quad \cdot 10^{-21}B_{7,16}(X) - 1.413298260786765\cdot 10^{-21}B_{8,16}(X) - 1.222205076447875\cdot 10^{-21}B_{9,16}(X)\\
&\quad - 1.031111892108986\cdot 10^{-21}B_{10,16}(X) - 8.400187077700972\cdot 10^{-22}B_{11,16}(X) - 6.489255234312081\\
&\quad \cdot 10^{-22}B_{12,16}(X) - 4.578323390923189\cdot 10^{-22}B_{13,16}(X) - 2.667391547534297\cdot 10^{-22}B_{14,16}(X)\\
&\quad - 7.564597041454045\cdot 10^{-23}B_{15,16}(X) + 1.154472139243488\cdot 10^{-22}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.9622411919334823, 0.9622411919334823\}$$

Intersection intervals with the x axis:

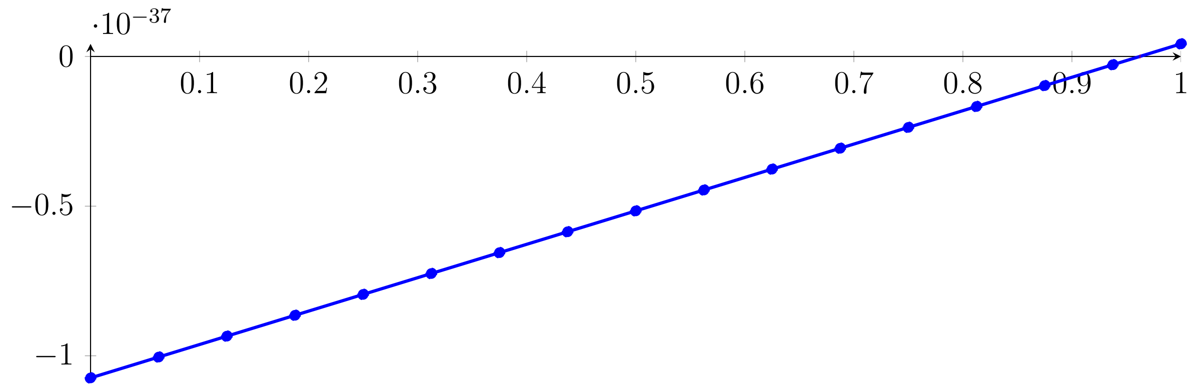
$$[0.9622411919334823, 0.9622411919334823]$$

Longest intersection interval: $3.649903549784843\cdot 10^{-17}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

82.28 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$
\begin{aligned}
p = {} & 6.227129416394075\cdot 10^{-342}X^{16} - 5.276320408729603\cdot 10^{-341}X^{15} - 2.711814423268387 \\
& \cdot 10^{-341}X^{14} + 1.499867730400293\cdot 10^{-340}X^{13} + 3.046606327375596\cdot 10^{-341}X^{12} \\
& + 1.0236597259982\cdot 10^{-339}X^{11} - 6.03228052820368\cdot 10^{-340}X^{10} + 5.745029074479695\cdot 10^{-340}X^{9} \\
& - 8.566359853913263\cdot 10^{-309}X^{8} + 1.209904606350681\cdot 10^{-269}X^{7} + 1.299525013697219\cdot 10^{-228}X^{6} \\
& - 1.292609542602263\cdot 10^{-189}X^{5} - 1.093242128853173\cdot 10^{-148}X^{4} + 5.69416387918096\cdot 10^{-110}X^{3} \\
& + 3.937232562828401\cdot 10^{-69}X^{2} + 1.115954706973122\cdot 10^{-37}X - 1.073817587381597\cdot 10^{-37} \\
= {} & -1.073817587381597\cdot 10^{-37}B_{0,16}(X) - 1.004070418195777\cdot 10^{-37}B_{1,16}(X) - 9.343232490099568 \\
& \cdot 10^{-38}B_{2,16}(X) - 8.645760798241366\cdot 10^{-38}B_{3,16}(X) - 7.948289106383165\cdot 10^{-38}B_{4,16}(X) \\
& - 7.250817414524964\cdot 10^{-38}B_{5,16}(X) - 6.553345722666762\cdot 10^{-38}B_{6,16}(X) - 5.855874030808561 \\
& \cdot 10^{-38}B_{7,16}(X) - 5.15840233895036\cdot 10^{-38}B_{8,16}(X) - 4.460930647092159\cdot 10^{-38}B_{9,16}(X) \\
& - 3.763458955233957\cdot 10^{-38}B_{10,16}(X) - 3.065987263375756\cdot 10^{-38}B_{11,16}(X) - 2.368515571517555 \\
& \cdot 10^{-38}B_{12,16}(X) - 1.671043879659353\cdot 10^{-38}B_{13,16}(X) - 9.735721878011521\cdot 10^{-39}B_{14,16}(X) \\
& - 2.761004959429508\cdot 10^{-39}B_{15,16}(X) + 4.213711959152505\cdot 10^{-39}B_{16,16}(X)
\end{aligned}
$$



Intersection of the convex hull with the x axis:

$$\{0.9622411919334824, 0.9622411919334824\}$$

Intersection intervals with the x axis:

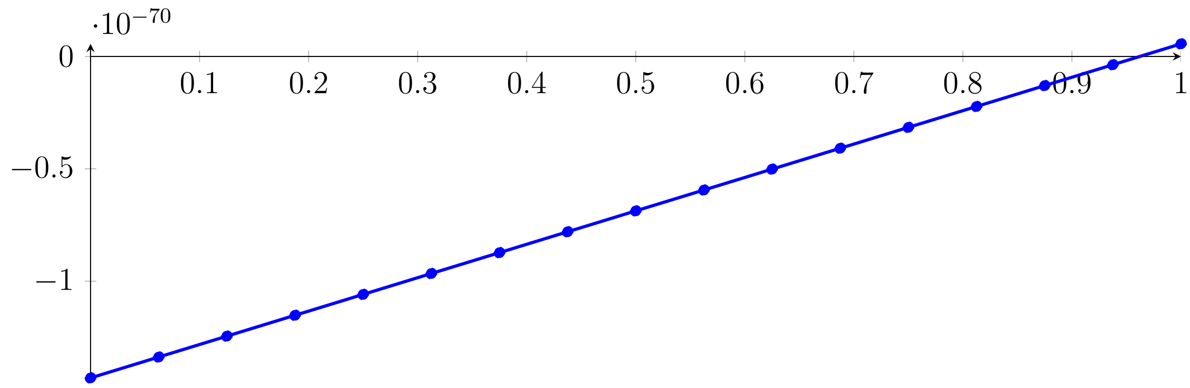
$$[0.9622411919334824, 0.9622411919334824]$$

Longest intersection interval: $1.332179592273198\cdot 10^{-33}$
$\implies$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

82.29 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

$$\begin{aligned}
p = &-6.873416129735531\cdot 10^{-375}X^{16} + 4.374226550105613\cdot 10^{-374}X^{15} - 2.522401866747223\\
&\cdot 10^{-373}X^{14} + 7.943760480144628\cdot 10^{-374}X^{13} - 4.694040283721826\cdot 10^{-374}X^{12}\\
&+ 7.322702842606048\cdot 10^{-373}X^{11} - 1.652302179870083\cdot 10^{-372}X^{10} + 4.425809410366293\cdot 10^{-373}X^{9}\\
&+ 2.48951779333104\cdot 10^{-373}X^{8} + 2.212904705183146\cdot 10^{-373}X^{7} - 5.423519921232731\\
&\cdot 10^{-354}X^{5} - 3.443241346969703\cdot 10^{-280}X^{4} + 1.346226983601526\cdot 10^{-208}X^{3}\\
&+ 6.987416338739455\cdot 10^{-135}X^{2} + 1.48665208653081\cdot 10^{-70}X - 1.430517875733805\cdot 10^{-70}\\
= &-1.430517875733805\cdot 10^{-70}B_{0,16}(X) - 1.337602120325629\cdot 10^{-70}B_{1,16}(X) - 1.244686364917454\\
&\cdot 10^{-70}B_{2,16}(X) - 1.151770609509278\cdot 10^{-70}B_{3,16}(X) - 1.058854854101102\cdot 10^{-70}B_{4,16}(X)\\
&- 9.659390986929268\cdot 10^{-71}B_{5,16}(X) - 8.730233432847512\cdot 10^{-71}B_{6,16}(X) - 7.801075878765756\\
&\cdot 10^{-71}B_{7,16}(X) - 6.871918324684\cdot 10^{-71}B_{8,16}(X) - 5.942760770602244\cdot 10^{-71}B_{9,16}(X)\\
&- 5.013603216520488\cdot 10^{-71}B_{10,16}(X) - 4.084445662438732\cdot 10^{-71}B_{11,16}(X)\\
&- 3.155288108356976\cdot 10^{-71}B_{12,16}(X) - 2.22613055427522\cdot 10^{-71}B_{13,16}(X) - 1.296973000193464\\
&\cdot 10^{-71}B_{14,16}(X) - 3.678154461117079\cdot 10^{-72}B_{15,16}(X) + 5.613421079700481\cdot 10^{-72}B_{16,16}(X)
\end{aligned}$$



Intersection of the convex hull with the x axis:

$$\{0.9622411919334824, 0.9622411919334824\}$$

Intersection intervals with the x axis:

$$[0.9622411919334824, 0.9622411919334824]$$

Longest intersection interval: $1.774702466069183\cdot 10^{-66}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

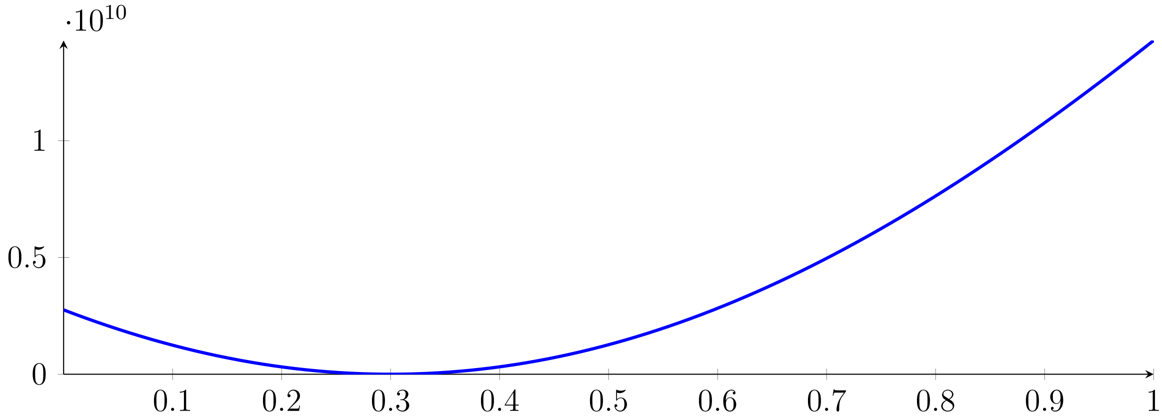
82.30 Recursion Branch 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 22!

82.31 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$\begin{aligned}p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12}\\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9}\\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6}\\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3}\\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822\end{aligned}$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

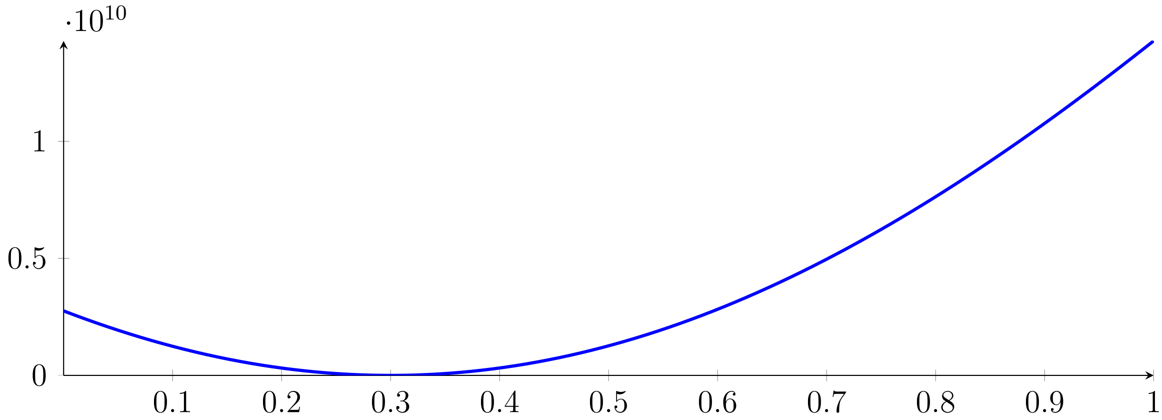
with precision $\varepsilon = 1 \cdot 10^{-128}$.

83 Running `QuadClip` on h_{16} with epsilon 128

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called `QuadClip` with input polynomial on interval $[0, 1]$:

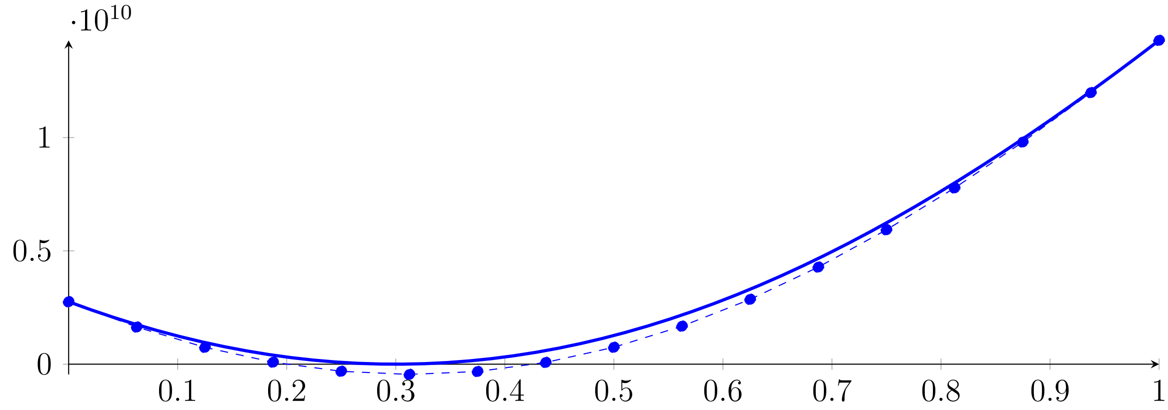
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



83.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

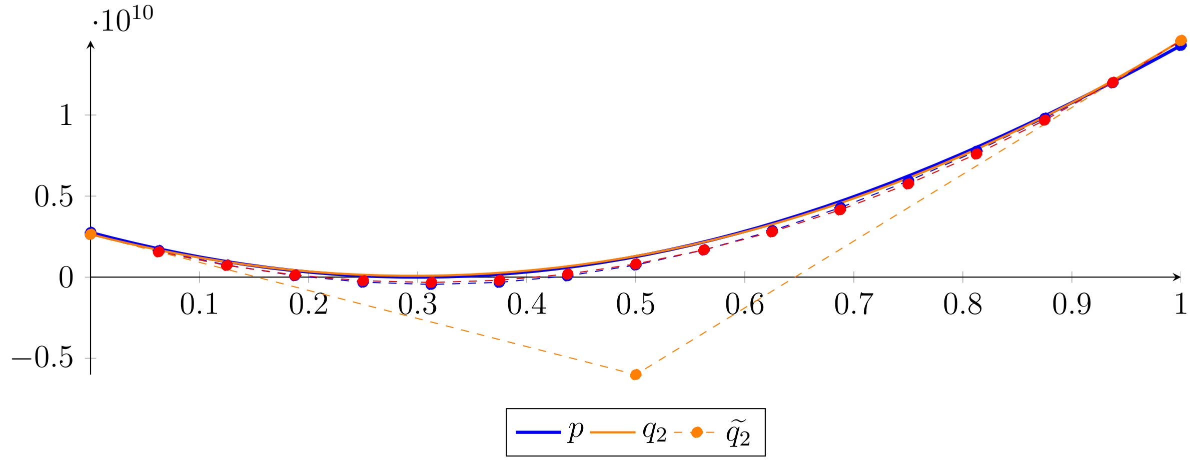
$$\begin{aligned} p = &-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ &+ 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ &- 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ &- 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ &+ 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = &\, 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ &+ 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ &- 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ &+ 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ &+ 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ &+ 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$q_2 = 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328$$
$$= 2643425520.699328B_{0,2} - 6016169000.146027B_{1,2} + 14589651156.70083B_{2,2}$$

$$\widetilde{q}_2 = -4.590615853685512\cdot 10^{-286}X^{16} + 3.626297739922361\cdot 10^{-285}X^{15} - 1.337118562447102 \cdot 10^{-284}X^{14} + 3.057409950495903\cdot 10^{-284}X^{13} - 4.816080313213711\cdot 10^{-284}X^{12} + 5.445450819003583\cdot 10^{-284}X^{11} - 4.453195910004646\cdot 10^{-284}X^{10} + 2.601913123147853 \cdot 10^{-284}X^9 - 1.060485581673177\cdot 10^{-284}X^8 + 2.936863540613122\cdot 10^{-285}X^7 - 5.487274374859613 \cdot 10^{-286}X^6 + 7.23018920152705\cdot 10^{-287}X^5 - 7.254244946204871\cdot 10^{-288}X^4 + 6.659990111567495 \cdot 10^{-289}X^3 + 29265414677.69221X^2 - 17319189041.69071X + 2643425520.699328$$
$$= 2643425520.699328B_{0,16} + 1560976205.593658B_{1,16} + 722405346.1354239B_{2,16} + 127712942.3246248B_{3,16} - 223101005.8387393B_{4,16} - 330036498.3546682B_{5,16} - 193093535.223162B_{6,16} + 187727883.5557793B_{7,16} + 812427757.9821557B_{8,16} + 1681006088.055967B_{9,16} + 2793462873.777214B_{10,16} + 4149798115.145896B_{11,16} + 5750011812.162013B_{12,16} + 7594103964.825565B_{13,16} + 9682074573.136552B_{14,16} + 12013923637.09497B_{15,16} + 14589651156.70083B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 301254626.7406212$.

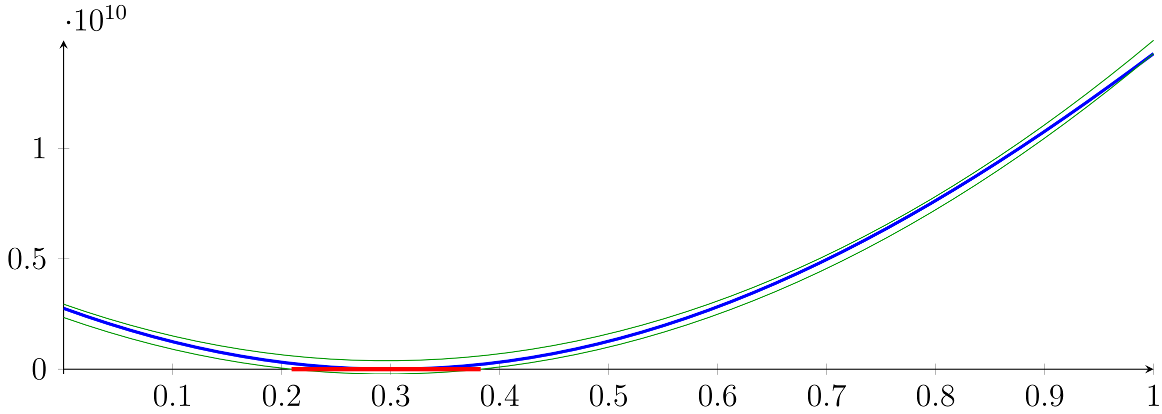
Bounding polynomials M and m:

$$M = 29265414677.69221X^2 - 17319189041.69071X + 2944680147.439949$$
$$m = 29265414677.69221X^2 - 17319189041.69071X + 2342170893.958706$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.2091580131065301, 0.3826391383239738\}$$

Intersection intervals:



$[0.2091580131065301, 0.3826391383239738]$

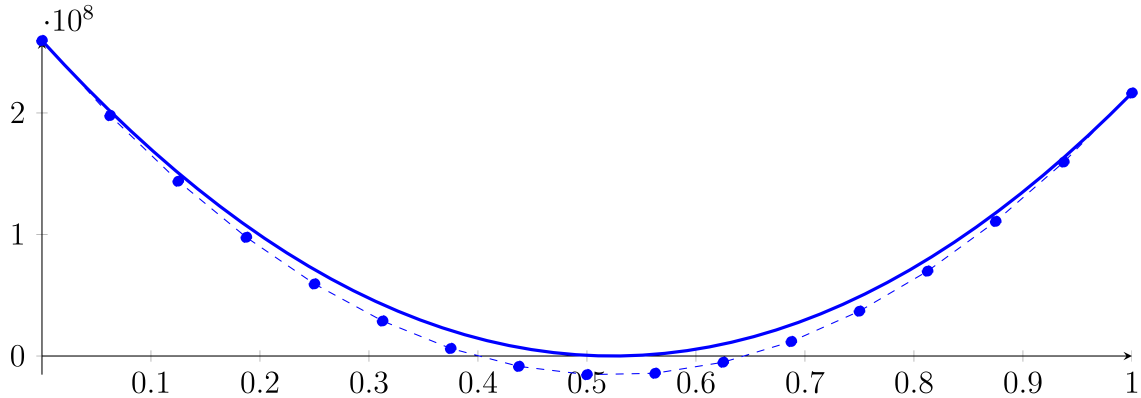
Longest intersection interval: 0.1734811252174437

$\Longrightarrow$ Selective recursion: interval 1: $[0.2091580131065301, 0.3826391383239738]$,

83.2 Recursion Branch 1 1 in Interval 1: $[0.2091580131065301, 0.3826391383239738]$

Normalized monomial und Bézier representations and the Bézier polygon:

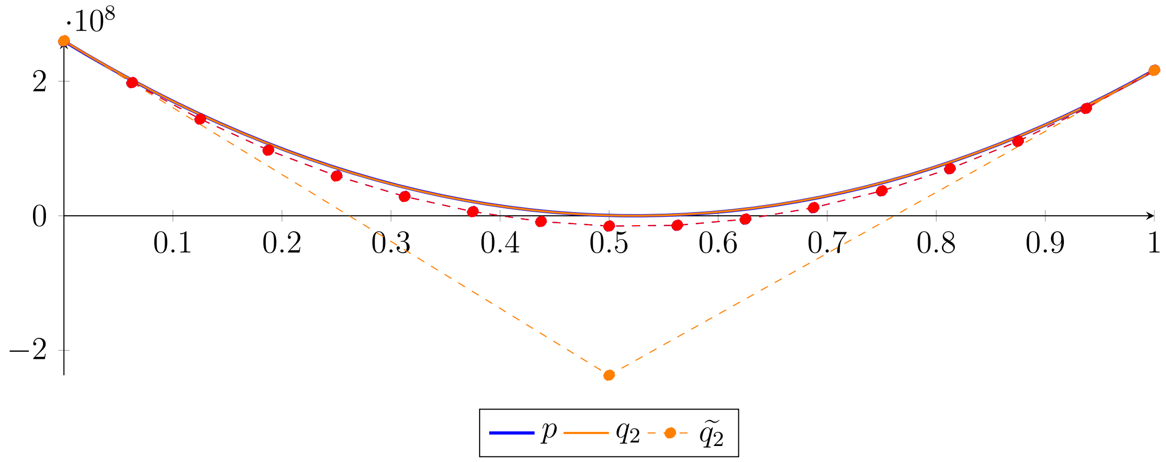
$$\begin{aligned}
p = & -6.730319568869656\cdot 10^{-13}X^{16} + 8.742499458235574\cdot 10^{-12}X^{15} + 4.969734366538428\cdot 10^{-09}X^{14} \\
& - 5.918022439742958\cdot 10^{-08}X^{13} - 1.563835677279211\cdot 10^{-05}X^{12} + 0.000165142833990623X^{11} \\
& + 0.02725178204957372X^{10} - 0.2448183330655992X^{9} - 28.45820232077076X^{8} \\
& + 205.241311077626X^{7} + 17835.39577074806X^{6} - 94141.21233496903X^{5} - 6218439.344794644X^{4} \\
& + 20000843.19616221X^{3} + 930858984.7988209X^{2} - 987724621.0538478X + 259570777.0276934 \\
= & \ 259570777.0276934B_{0,16}(X) + 197837988.2118279B_{1,16}(X) + 143862357.6026193B_{2,16}(X) \\
& + 97679600.99148916B_{3,16}(X) + 59322017.44494463B_{4,16}(X) + 28818467.75210257B_{5,16}(X) \\
& + 6194355.099411763B_{6,16}(X) - 8528392.009487306B_{7,16}(X) - 15331334.57303514B_{8,16}(X) \\
& - 14199535.63666165B_{9,16}(X) - 5121570.552084079B_{10,16}(X) + 11910465.05798682B_{11,16}(X) \\
& + 36900949.63643701B_{12,16}(X) + 69850732.33405701B_{13,16}(X) \\
& + 110757131.9597003B_{14,16}(X) + 159613938.238137B_{15,16}(X) + 216411415.3731615B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 = & \ 950064117.9944898X^2 - 993959420.5216784X + 260029868.8092426 \\
= & \ 260029868.8092426B_{0,2} - 236949841.4515966B_{1,2} + 216134566.282054B_{2,2}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &-1.41520176718356{\cdot}10^{-287}X^{16} + 1.139753408092347{\cdot}10^{-286}X^{15} - 4.276104932493763\\ &{\cdot}10^{-286}X^{14} + 9.896028733517596{\cdot}10^{-286}X^{13} - 1.566233430583374{\cdot}10^{-285}X^{12}\\ &+ 1.767044733566612{\cdot}10^{-285}X^{11} - 1.435814571471649{\cdot}10^{-285}X^{10} + 8.336078570237132\\ &{\cdot}10^{-286}X^{9} - 3.396584250429756{\cdot}10^{-286}X^{8} + 9.548198482934522{\cdot}10^{-287}X^{7} - 1.861944039074976\\ &{\cdot}10^{-287}X^{6} + 2.622613520689986{\cdot}10^{-288}X^{5} - 2.667693132942884{\cdot}10^{-289}X^{4} + 2.048626792988895\\ &{\cdot}10^{-290}X^{3} + 950064117.9944898X^{2} - 993959420.5216784X + 260029868.8092426\\ = &\,260029868.8092426B_{0,16} + 197907405.0266377B_{1,16} + 143702142.2273202B_{2,16}\\ &+ 97414080.41129016B_{3,16} + 59043219.5785475B_{4,16} + 28589559.72909226B_{5,16}\\ &+ 6053100.862924435B_{6,16} - 8566157.019955976B_{7,16} - 15268213.91954897B_{8,16}\\ &- 14053069.83585455B_{9,16} - 4920724.768872719B_{10,16} + 12128821.28139653B_{11,16}\\ &+ 37095568.31495319B_{12,16} + 69979516.33179727B_{13,16}\\ &+ 110780665.3319288B_{14,16} + 159499015.3153477B_{15,16} + 216134566.282054B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 459091.7815492238$.

Bounding polynomials M and m:

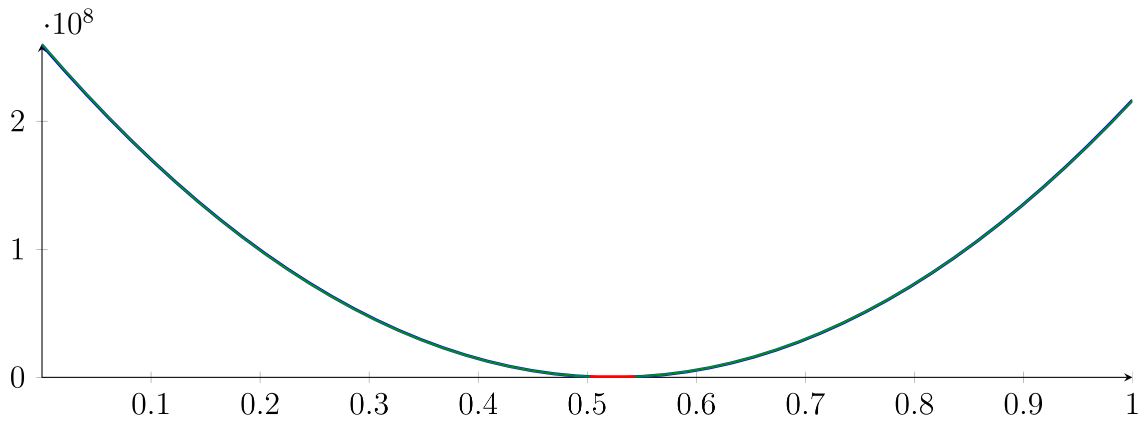
$$M = 950064117.9944898X^2 - 993959420.5216784X + 260488960.5907918$$

$$m = 950064117.9944898X^2 - 993959420.5216784X + 259570777.0276934$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.5025843702721211, 0.5436180930098815\}$$

Intersection intervals:



$$[0.5025843702721211, 0.5436180930098815]$$

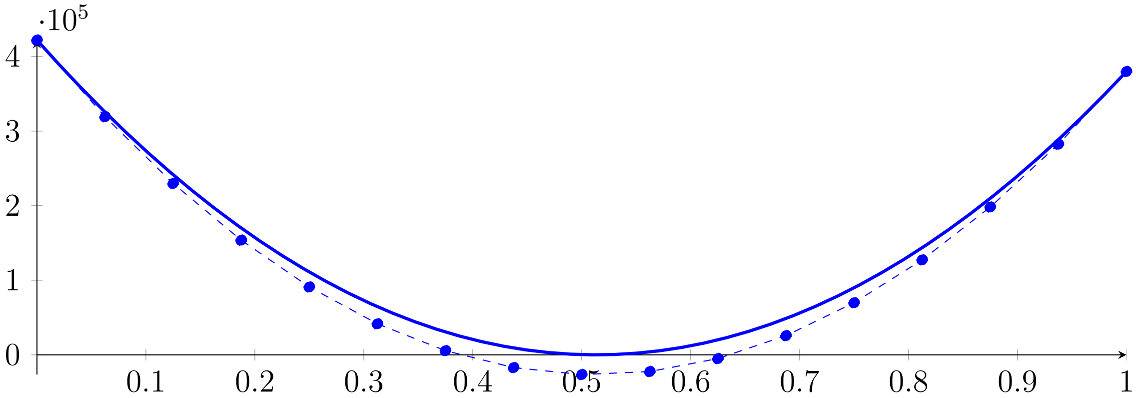
Longest intersection interval: 0.04103372273776039

$\Longrightarrow$ Selective recursion: interval 1: $[0.296346915178038, 0.3034654915704453]$,

83.3 Recursion Branch 1 1 1 in Interval 1: $[0.296346915178038, 0.3034654915704453]$

Normalized monomial und Bézier representations and the Bézier polygon:

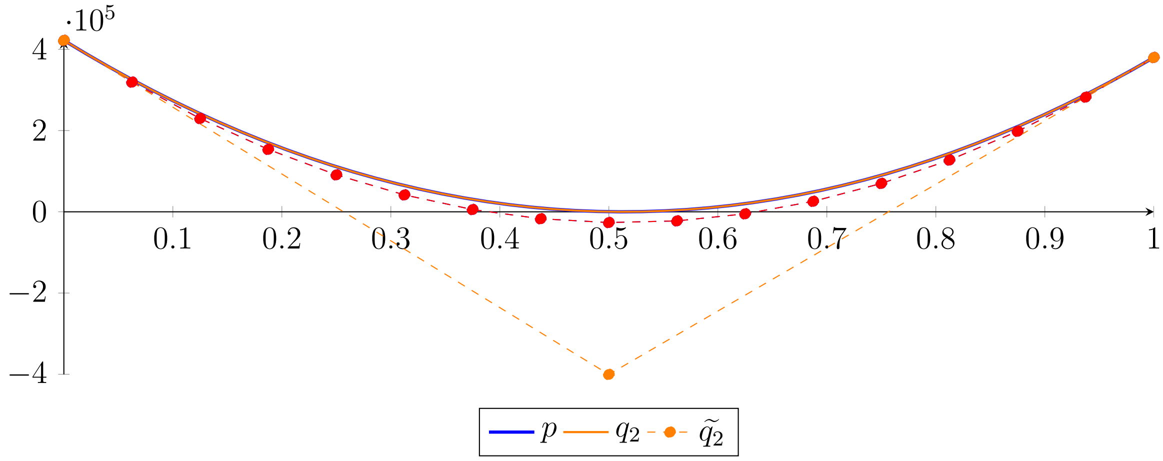
$$\begin{aligned}
p = & -4.348008654913047\cdot 10^{-35}X^{16} + 5.243388239594399\cdot 10^{-33}X^{15} + 1.924264426249969\\
& \cdot 10^{-28}X^{14} - 2.246746658841888\cdot 10^{-26}X^{13} - 3.625525335135332\cdot 10^{-22}X^{12}\\
& + 3.881306066232412\cdot 10^{-20}X^{11} + 3.776139474106064\cdot 10^{-16}X^{10} - 3.496022725744\\
& \cdot 10^{-14}X^{9} - 2.35123279599996\cdot 10^{-10}X^{8} + 1.7435554055155\cdot 10^{-08}X^{7} + 8.761425585865998\\
& \cdot 10^{-05}X^{6} - 0.004592127899352905X^{5} - 18.10659249096443X^{4} + 504.889471438631X^{3}\\
& + 1602084.653024796X^{2} - 1644731.182248784X + 422061.2284789393\\
= & \, 422061.2284789393B_{0,16}(X) + 319265.5295883902B_{1,16}(X) + 229820.5361397145B_{2,16}(X)\\
& + 153727.1497212539B_{3,16}(X) + 90986.26197267314B_{4,16}(X) + 41598.75458390835B_{5,16}(X)\\
& + 5565.499294126806B_{6,16}(X) - 17112.6421093025B_{7,16}(X) - 26434.81779182738B_{8,16}(X)\\
& - 22400.18587271997B_{9,16}(X) - 5007.914426073306B_{10,16}(X) + 25742.81851821299B_{11,16}(X)\\
& + 69852.82497345804B_{12,16}(X) + 127322.906995236B_{13,16}(X) + 198153.8566804232B_{14,16}(X)\\
& + 282346.4561662563B_{15,16}(X) + 379901.4776294016B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 = & \, 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\
= & \, 422084.9204772878B_{0,2} - 400423.8577791184B_{1,2} + 379878.3032798431B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 = & -2.394754881528157\cdot 10^{-290}X^{16} + 1.927007748221533\cdot 10^{-289}X^{15} - 7.222560694913459\\
& \cdot 10^{-289}X^{14} + 1.669823679170844\cdot 10^{-288}X^{13} - 2.640537385502685\cdot 10^{-288}X^{12}\\
& + 2.977245352423787\cdot 10^{-288}X^{11} - 2.4183464566446\cdot 10^{-288}X^{10} + 1.403945843258626\cdot 10^{-288}X^{9}\\
& - 5.721369736991339\cdot 10^{-289}X^{8} + 1.608948017596564\cdot 10^{-289}X^{7} - 3.139550473135121\cdot 10^{-290}X^{6}\\
& + 4.427570920272867\cdot 10^{-291}X^{5} - 4.516145205270189\cdot 10^{-292}X^{4} + 3.478035897244226\\
& \cdot 10^{-293}X^{3} + 1602810.939315368X^{2} - 1645017.556512812X + 422084.9204772878\\
= & \, 422084.9204772878B_{0,16} + 319271.323195237B_{1,16} + 229814.4837408143B_{2,16}\\
& + 153714.4021140197B_{3,16} + 90971.07831485311B_{4,16} + 41584.51234331459B_{5,16}\\
& + 5554.704199404135B_{6,16} - 17118.34611687825B_{7,16} - 26434.63860553258B_{8,16}\\
& - 22394.17326655884B_{9,16} - 4996.95009995704B_{10,16} + 25757.03089427283B_{11,16}\\
& + 69867.76971613076B_{12,16} + 127335.2663656168B_{13,16} + 198159.5208427308B_{14,16}\\
& + 282340.5331474729B_{15,16} + 379878.3032798431B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 23.69199834856345$.
Bounding polynomials M and m:

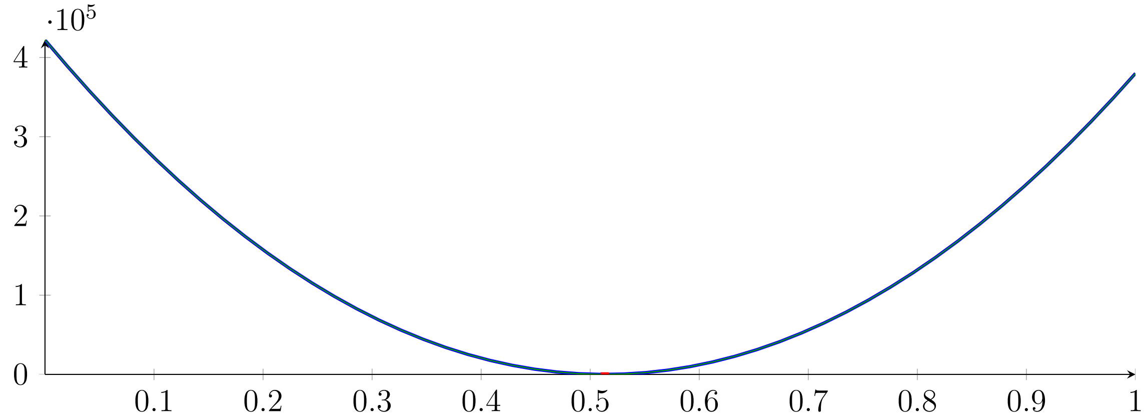
$$M = 1602810.939315368X^2 - 1645017.556512812X + 422108.6124756364$$

$$m = 1602810.939315368X^2 - 1645017.556512812X + 422061.2284789393$$

Root of M and m:

$$N(M) = \{\} \qquad N(m) = \{0.509405571919099, 0.5169273012614853\}$$

Intersection intervals:



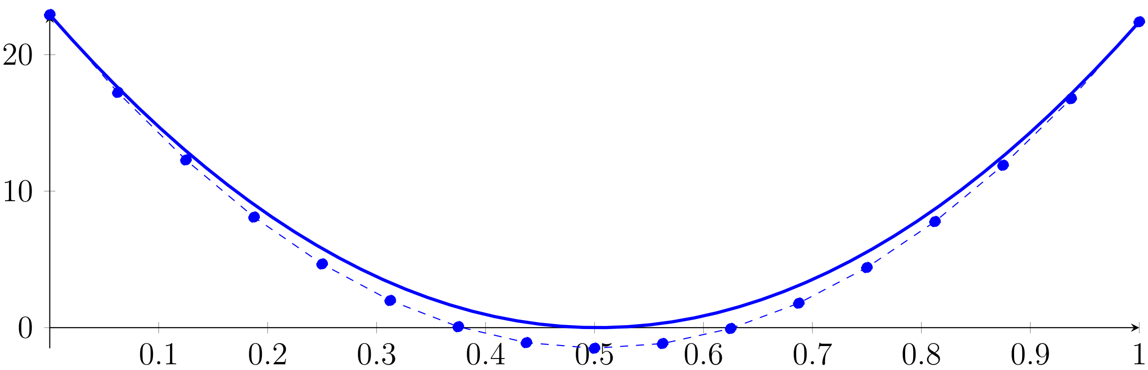
$$[0.509405571919099, 0.5169273012614853]$$

Longest intersection interval: 0.007521729342386311
$\Longrightarrow$ Selective recursion: interval 1: $[0.299973157656462, 0.3000267016613888]$,

83.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.299973157656462, 0.300026701661388$

Normalized monomial und Bézier representations and the Bézier polygon:

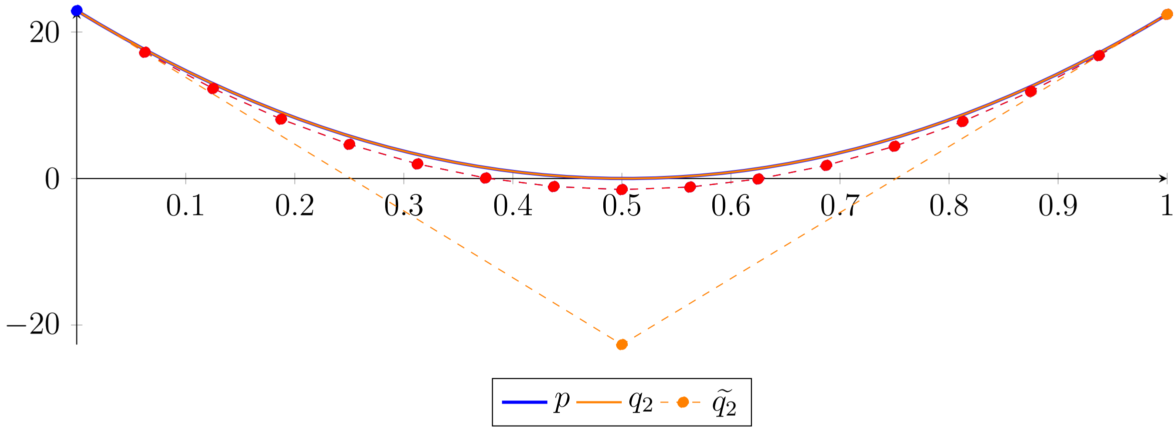
$$\begin{aligned}
p = & -4.564294214760651\cdot 10^{-69}X^{16} + 6.823166131086062\cdot 10^{-65}X^{15} + 3.571082953880241\\
& \cdot 10^{-58}X^{14} - 5.203669764436854\cdot 10^{-54}X^{13} - 1.189477187926378\cdot 10^{-47}X^{12}\\
& + 1.595632405197401\cdot 10^{-43}X^{11} + 2.190120291778079\cdot 10^{-37}X^{10} - 2.545939358967532\cdot 10^{-33}X^{9}\\
& - 2.410599475027015\cdot 10^{-27}X^{8} + 2.244415896590929\cdot 10^{-23}X^{7} + 1.587744426227316\cdot 10^{-17}X^{6}\\
& - 1.041115833763881\cdot 10^{-13}X^{5} - 5.799356524856294\cdot 10^{-08}X^{4} + 0.0001991514832499812X^{3}\\
& + 90.68225982541923X^{2} - 91.20858295780928X + 22.93446410530317\\
= & \, 22.93446410530317B_{0,16}(X) + 17.23392767044009B_{1,16}(X) + 12.28907673412217B_{2,16}(X)\\
& + 8.099911651977055B_{3,16}(X) + 4.666432779600538B_{4,16}(X) + 1.988640472556532B_{5,16}(X)\\
& + 0.06653508637709412B_{6,16}(X) - 1.099883023437587B_{7,16}(X)\\
& - 1.510613501419185B_{8,16}(X) - 1.165655992131239B_{9,16}(X) - 0.0650101401691539B_{10,16}(X)\\
& + 1.791324409839802B_{11,16}(X) + 4.403348013236495B_{12,16}(X) + 7.771061025329927B_{13,16}(X)\\
& + 11.89446380139723B_{14,16}(X) + 16.77355669668369B_{15,16}(X) + 22.4083400664027B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 = & \, 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\
= & \, 22.93447405790644B_{0,2} - 22.66987713993177B_{1,2} + 22.40833011545639B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_2 = & -1.359001790090183\cdot 10^{-294}X^{16} + 1.092598333426672\cdot 10^{-293}X^{15} - 4.09106565145402\\
& \cdot 10^{-293}X^{14} + 9.449139850665356\cdot 10^{-293}X^{13} - 1.493001927703419\cdot 10^{-292}X^{12}\\
& + 1.682443127957849\cdot 10^{-292}X^{11} - 1.366228291670638\cdot 10^{-292}X^{10} + 7.931222707714909\\
& \cdot 10^{-293}X^{9} - 3.23261546930867\cdot 10^{-293}X^{8} + 9.093117628361849\cdot 10^{-294}X^{7} - 1.775022991079505\\
& \cdot 10^{-294}X^{6} + 2.505263643899121\cdot 10^{-295}X^{5} - 2.56169827233206\cdot 10^{-296}X^{4} + 1.979687487137548\\
& \cdot 10^{-297}X^{3} + 90.68255845322638X^{2} - 91.20870239567643X + 22.93447405790644\\
= & \, 22.93447405790644B_{0,16} + 17.23393015817666B_{1,16} + 12.28907424555711B_{2,16}\\
& + 8.099906320047769B_{3,16} + 4.666426381648652B_{4,16} + 1.988634430359754B_{5,16}\\
& + 0.06653046618107656B_{6,16} - 1.099885510887381B_{7,16} - 1.510613500845619B_{8,16}\\
& - 1.165653503693637B_{9,16} - 0.06500551943143578B_{10,16} + 1.791330451940986B_{11,16}\\
& + 4.403354410423627B_{12,16} + 7.771066356016488B_{13,16} + 11.89446628871957B_{14,16}\\
& + 16.77355420853287B_{15,16} + 22.40833011545639B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 9.952603274324834 \cdot 10^{-06}$.
Bounding polynomials M and m:

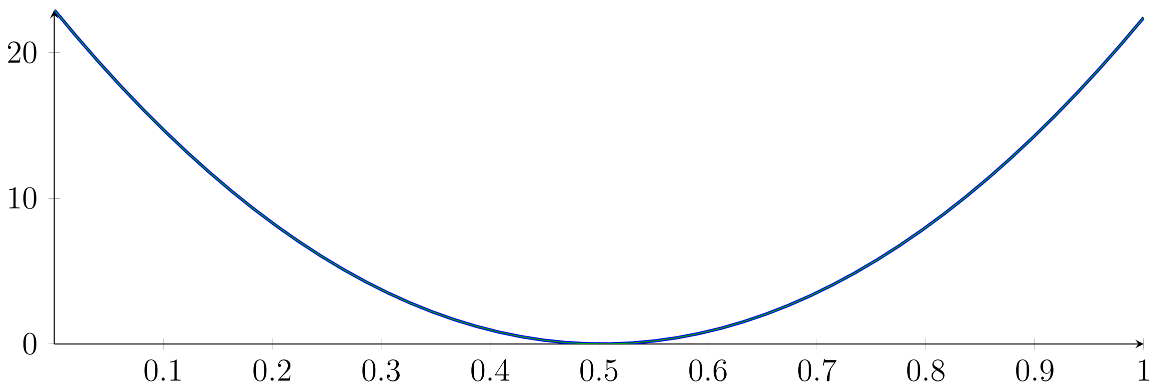
$$M = 90.68255845322638X^2 - 91.20870239567643X + 22.93448401050971$$

$$m = 90.68255845322638X^2 - 91.20870239567643X + 22.93446410530317$$

Root of M and m:

$$N(M) = \{\} \qquad\qquad N(m) = \{0.5025582179560734, 0.5032438232679998\}$$

Intersection intervals:



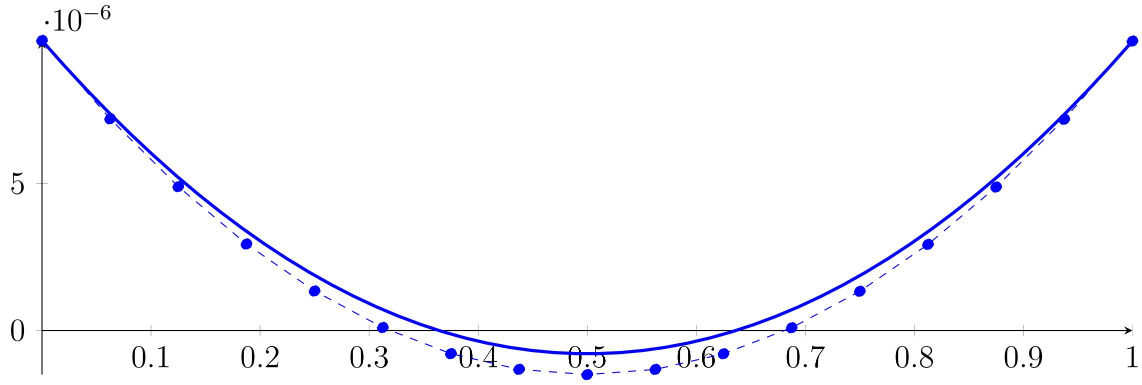
$$[0.5025582179560734, 0.5032438232679998]$$

Longest intersection interval: 0.0006856053119264748
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000666361603, 0.3000001033462145]$,

83.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000666361603, 0.300000103346$

Normalized monomial und Bézier representations and the Bézier polygon:

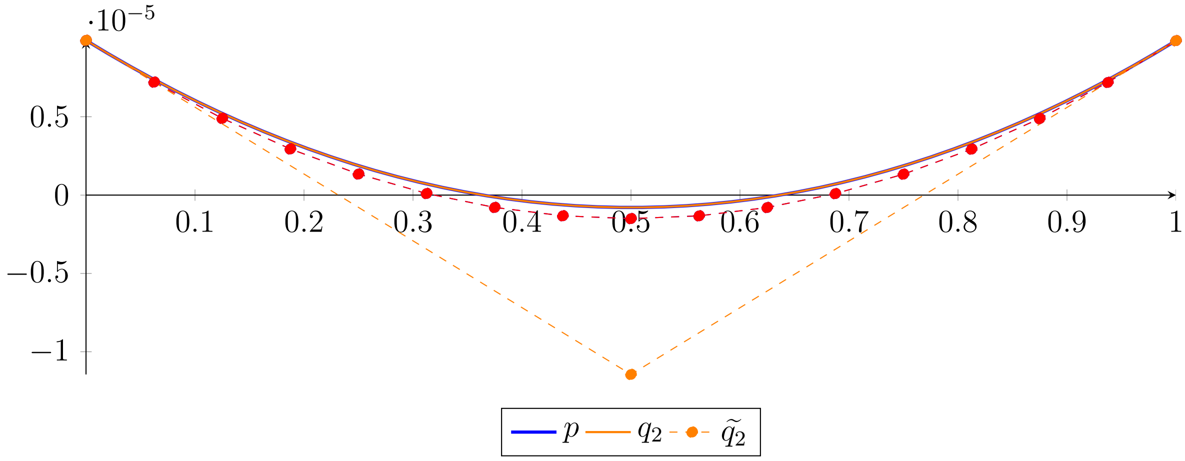
$$
\begin{aligned}
p = {} & -1.087828446441416\cdot 10^{-119}X^{16} + 2.370636059353804\cdot 10^{-112}X^{15} + 1.810668709595505 \\
& \cdot 10^{-102}X^{14} - 3.846486744130667\cdot 10^{-95}X^{13} - 1.283061410949319\cdot 10^{-85}X^{12} \\
& + 2.509305114999168\cdot 10^{-78}X^{11} + 5.0258723654725\cdot 10^{-69}X^{10} - 8.517807546222726\cdot 10^{-62}X^{9} \\
& - 1.17684840157971\cdot 10^{-52}X^{8} + 1.597478332578791\cdot 10^{-45}X^{7} + 1.649027121563264\cdot 10^{-36}X^{6} \\
& - 1.576413560673164\cdot 10^{-29}X^{5} - 1.281381537279639\cdot 10^{-20}X^{4} + 6.414336823836069\cdot 10^{-14}X^{3} \\
& + 4.262575843118847\cdot 10^{-05}X^{2} - 4.264622395354259\cdot 10^{-05}X + 9.875919579833914\cdot 10^{-06} \\
= {} & 9.875919579833914\cdot 10^{-06}B_{0,16}(X) + 7.210530582737503\cdot 10^{-06}B_{1,16}(X) + 4.900356239234328 \\
& \cdot 10^{-06}B_{2,16}(X) + 2.945396549438933\cdot 10^{-06}B_{3,16}(X) + 1.345651513465858\cdot 10^{-06}B_{4,16}(X) \\
& + 1.01121131429646\cdot 10^{-07}B_{5,16}(X) - 7.881945965551621\cdot 10^{-07}B_{6,16}(X) - 1.322295670374024 \\
& \cdot 10^{-06}B_{7,16}(X) - 1.501182089912399\cdot 10^{-06}B_{8,16}(X) - 1.324853855055745\cdot 10^{-06}B_{9,16}(X) \\
& - 7.933109656895192\cdot 10^{-07}B_{10,16}(X) + 9.344657830081879\cdot 10^{-08}B_{11,16}(X) \\
& + 1.335418777029811\cdot 10^{-06}B_{12,16}(X) + 2.932605630611999\cdot 10^{-06}B_{13,16}(X) + 4.885007139161925 \\
& \cdot 10^{-06}B_{14,16}(X) + 7.192623302794129\cdot 10^{-06}B_{15,16}(X) + 9.855454121623155\cdot 10^{-06}B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 = {} & 4.26257585274035\cdot 10^{-05}X^{2} - 4.264622399202859\cdot 10^{-05}X + 9.875919583041081\cdot 10^{-06} \\
= {} & 9.875919583041081\cdot 10^{-06}B_{0,2} - 1.144719241297322\cdot 10^{-05}B_{1,2} + 9.855454118415988\cdot 10^{-06}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q}_2 = {} & -6.428697903766316\cdot 10^{-301}X^{16} + 5.168486014171287\cdot 10^{-300}X^{15} - 1.931525871486767 \\
& \cdot 10^{-299}X^{14} + 4.443748026156872\cdot 10^{-299}X^{13} - 6.983462617461767\cdot 10^{-299}X^{12} \\
& + 7.823576270761152\cdot 10^{-299}X^{11} - 6.321034979462039\cdot 10^{-299}X^{10} + 3.658988671435299\cdot 10^{-299}X^{9} \\
& - 1.493155040297548\cdot 10^{-299}X^{8} + 4.235526792785023\cdot 10^{-300}X^{7} - 8.432417338595836\cdot 10^{-301}X^{6} \\
& + 1.226905231702478\cdot 10^{-301}X^{5} - 1.286361467975875\cdot 10^{-302}X^{4} + 9.604810259325032\cdot 10^{-304}X^{3} \\
& + 4.26257585274035\cdot 10^{-05}X^{2} - 4.264622399202859\cdot 10^{-05}X + 9.875919583041081\cdot 10^{-06} \\
= {} & 9.875919583041081\cdot 10^{-06}B_{0,16} + 7.210530583539294\cdot 10^{-06}B_{1,16} + 4.900356238432536\cdot 10^{-06}B_{2,16} \\
& + 2.945396547720807\cdot 10^{-06}B_{3,16} + 1.345651511404108\cdot 10^{-06}B_{4,16} + 1.011211294824374\cdot 10^{-07}B_{5,16} \\
& - 7.881945980442039\cdot 10^{-07}B_{6,16} - 1.322295671175816\cdot 10^{-06}B_{7,16} - 1.501182089912399\cdot 10^{-06}B_{8,16} \\
& - 1.324853854253953\cdot 10^{-06}B_{9,16} - 7.933109642004772\cdot 10^{-07}B_{10,16} + 9.34465802480274\cdot 10^{-08}B_{11,16} \\
& + 1.335418779091561\cdot 10^{-06}B_{12,16} + 2.932605632330124\cdot 10^{-06}B_{13,16} + 4.885007139963716 \\
& \cdot 10^{-06}B_{14,16} + 7.192623301992338\cdot 10^{-06}B_{15,16} + 9.855454118415988\cdot 10^{-06}B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 3.207167313591001 \cdot 10^{-15}$.
Bounding polynomials M and m:

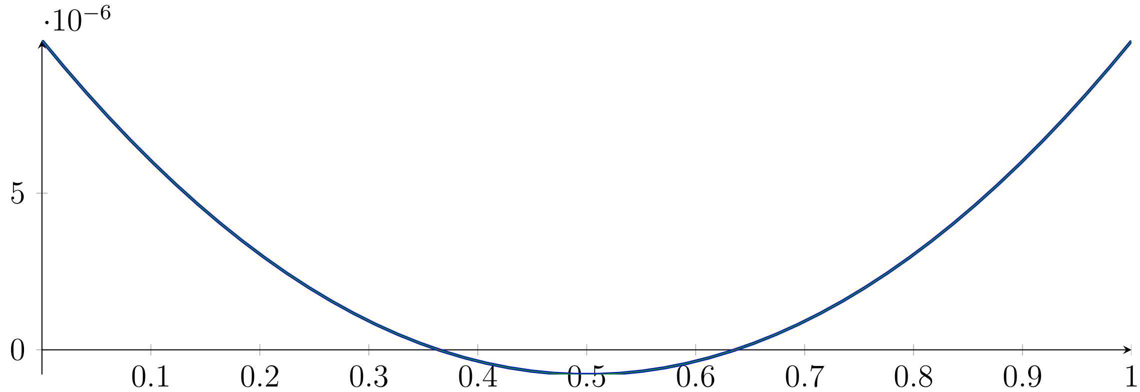
$$M = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919586248249 \cdot 10^{-06}$$

$$m = 4.26257585274035 \cdot 10^{-05} X^2 - 4.264622399202859 \cdot 10^{-05} X + 9.875919579833914 \cdot 10^{-06}$$

Root of M and m:

$N(M) = \{0.3640375916978796, 0.6364425279605644\}$ $N(m) = \{0.3640375911454658, 0.6364425285129782\}$

Intersection intervals:



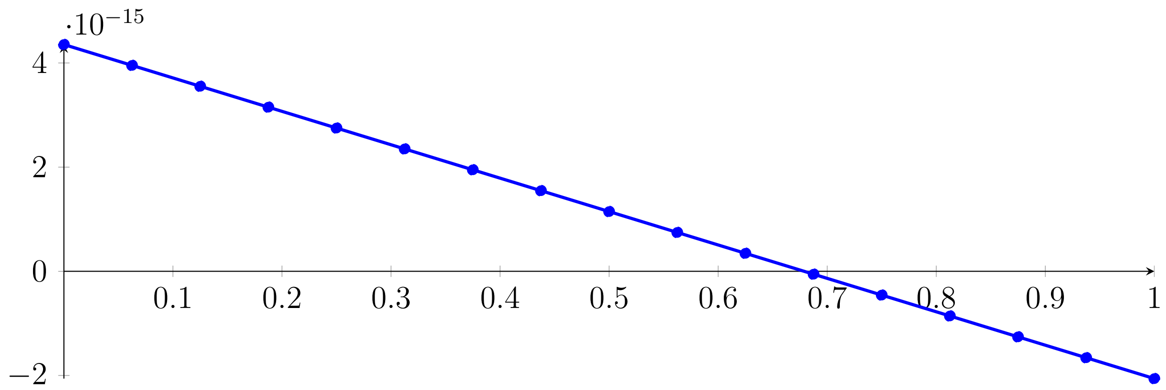
$$[0.3640375911454658, 0.3640375916978796],\ [0.6364425279605644, 0.6364425285129782]$$

Longest intersection interval: $5.524137987858897 \cdot 10^{-10}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$, interval 2: $[0.30000009, 0.30000009]$,

83.6 Recursion Branch 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

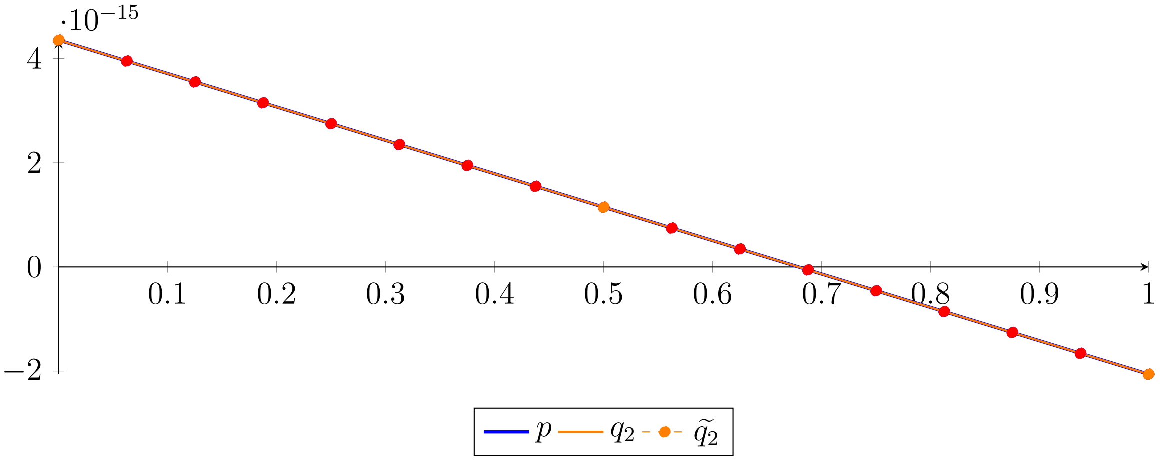
$$\begin{aligned}
p = & -8.180742151883693\cdot 10^{-268}X^{16} + 3.227249519181262\cdot 10^{-251}X^{15} + 4.462130353465124 \\
& \cdot 10^{-232}X^{14} - 1.715943065651804\cdot 10^{-215}X^{13} - 1.03614769176239\cdot 10^{-196}X^{12} + 3.668285054157079 \\
& \cdot 10^{-180}X^{11} + 1.330015307195714\cdot 10^{-161}X^{10} - 4.080453140608508\cdot 10^{-145}X^{9} \\
& - 1.02055559825224\cdot 10^{-126}X^{8} + 2.507763120804761\cdot 10^{-110}X^{7} + 4.686136227440808\cdot 10^{-92}X^{6} \\
& - 8.109473416022201\cdot 10^{-76}X^{5} - 1.19326399448035\cdot 10^{-57}X^{4} + 1.081297478088303\cdot 10^{-41}X^{3} \\
& + 1.300771930692191\cdot 10^{-23}X^{2} - 6.414334643539715\cdot 10^{-15}X + 4.354113798188606\cdot 10^{-15} \\
= & \; 4.354113798188606\cdot 10^{-15}B_{0,16}(X) + 3.953217882967374\cdot 10^{-15}B_{1,16}(X) + 3.552321967854539 \\
& \cdot 10^{-15}B_{2,16}(X) + 3.151426052850102\cdot 10^{-15}B_{3,16}(X) + 2.750530137954063\cdot 10^{-15}B_{4,16}(X) \\
& + 2.349634223166422\cdot 10^{-15}B_{5,16}(X) + 1.948738308487178\cdot 10^{-15}B_{6,16}(X) + 1.547842393916332 \\
& \cdot 10^{-15}B_{7,16}(X) + 1.146946479453883\cdot 10^{-15}B_{8,16}(X) + 7.460505650998323\cdot 10^{-16}B_{9,16}(X) \\
& + 3.451546508541791\cdot 10^{-16}B_{10,16}(X) - 5.574126328307647\cdot 10^{-17}B_{11,16}(X) - 4.566371773119344 \\
& \cdot 10^{-16}B_{12,16}(X) - 8.575330912323946\cdot 10^{-16}B_{13,16}(X) - 1.258429005044457\cdot 10^{-15}B_{14,16}(X) \\
& - 1.659324918748122\cdot 10^{-15}B_{15,16}(X) - 2.060220832343389\cdot 10^{-15}B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_2 = & \; 1.300771930692191\cdot 10^{-23}X^{2} - 6.414334643539715\cdot 10^{-15}X + 4.354113798188606\cdot 10^{-15} \\
= & \; 4.354113798188606\cdot 10^{-15}B_{0,2} + 1.146946476418749\cdot 10^{-15}B_{1,2} - 2.060220832343389\cdot 10^{-15}B_{2,2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q_2} = & \; 1.912651228969383\cdot 10^{-311}X^{16} - 1.223741742808027\cdot 10^{-310}X^{15} + 2.733522262624021 \\
& \cdot 10^{-310}X^{14} - 8.679644486233328\cdot 10^{-311}X^{13} - 7.862443329477107\cdot 10^{-310}X^{12} \\
& + 1.83195702234036\cdot 10^{-309}X^{11} - 2.057972005087252\cdot 10^{-309}X^{10} + 1.349302407721643\cdot 10^{-309}X^{9} \\
& - 5.089946779834537\cdot 10^{-310}X^{8} + 8.605298284676211\cdot 10^{-311}X^{7} + 8.242572388969257\cdot 10^{-312}X^{6} \\
& - 6.927894069632637\cdot 10^{-312}X^{5} + 1.356821367314693\cdot 10^{-312}X^{4} - 8.054206617575827\cdot 10^{-314}X^{3} \\
& + 1.300771930692191\cdot 10^{-23}X^{2} - 6.414334643539715\cdot 10^{-15}X + 4.354113798188606\cdot 10^{-15} \\
= & \; 4.354113798188606\cdot 10^{-15}B_{0,16} + 3.953217882967374\cdot 10^{-15}B_{1,16} + 3.552321967854539 \\
& \cdot 10^{-15}B_{2,16} + 3.151426052850102\cdot 10^{-15}B_{3,16} + 2.750530137954063\cdot 10^{-15}B_{4,16} \\
& + 2.349634223166422\cdot 10^{-15}B_{5,16} + 1.948738308487178\cdot 10^{-15}B_{6,16} + 1.547842393916332 \\
& \cdot 10^{-15}B_{7,16} + 1.146946479453883\cdot 10^{-15}B_{8,16} + 7.460505650998323\cdot 10^{-16}B_{9,16} \\
& + 3.451546508541791\cdot 10^{-16}B_{10,16} - 5.574126328307647\cdot 10^{-17}B_{11,16} - 4.566371773119344 \\
& \cdot 10^{-16}B_{12,16} - 8.575330912323946\cdot 10^{-16}B_{13,16} - 1.258429005044457\cdot 10^{-15}B_{14,16} \\
& - 1.659324918748122\cdot 10^{-15}B_{15,16} - 2.060220832343389\cdot 10^{-15}B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.406487390441514 \cdot 10^{-43}$.
Bounding polynomials M and m:

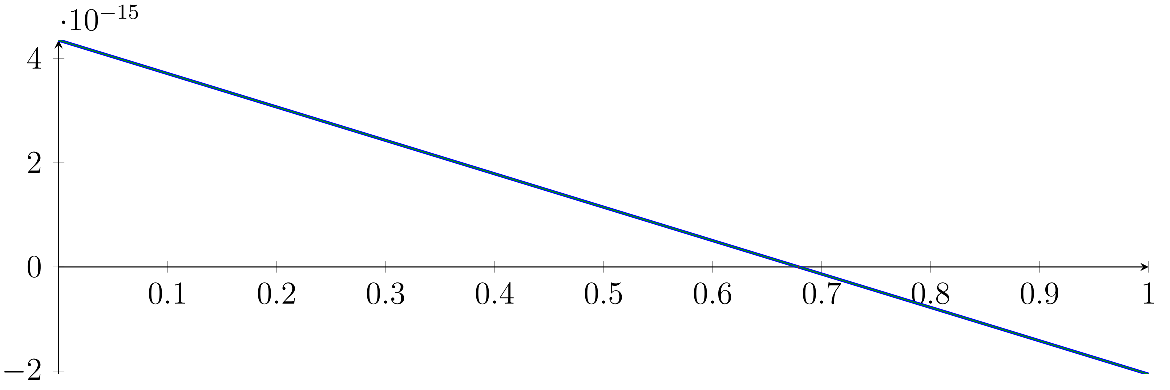
$$M = 1.300771930692191 \cdot 10^{-23} X^2 - 6.414334643539715 \cdot 10^{-15} X + 4.354113798188606 \cdot 10^{-15}$$

$$m = 1.300771930692191 \cdot 10^{-23} X^2 - 6.414334643539715 \cdot 10^{-15} X + 4.354113798188606 \cdot 10^{-15}$$

Root of M and m:

$N(M) = \{0.678809891617932, 493117546.8474808\}$ $N(m) = \{0.678809891617932, 493117546.8474808\}$

Intersection intervals:



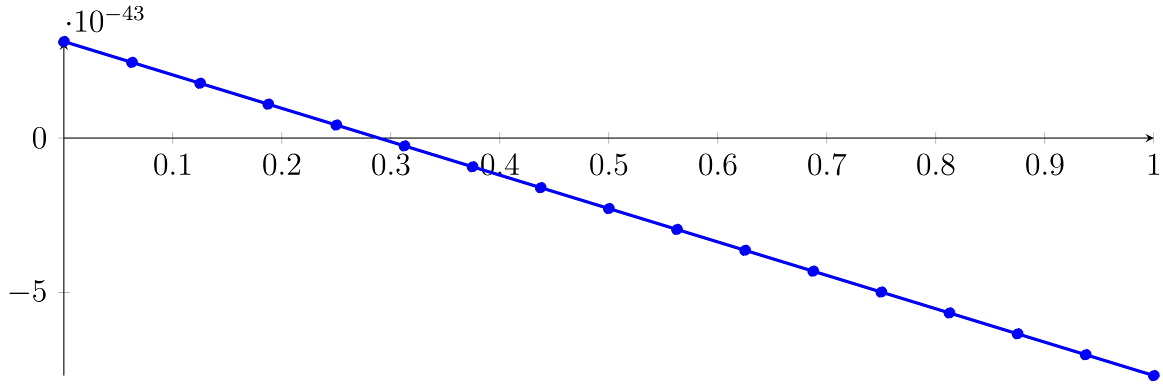
$$[0.678809891617932, 0.678809891617932]$$

Longest intersection interval: $1.685751587897438 \cdot 10^{-28}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

83.7 Recursion Branch 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

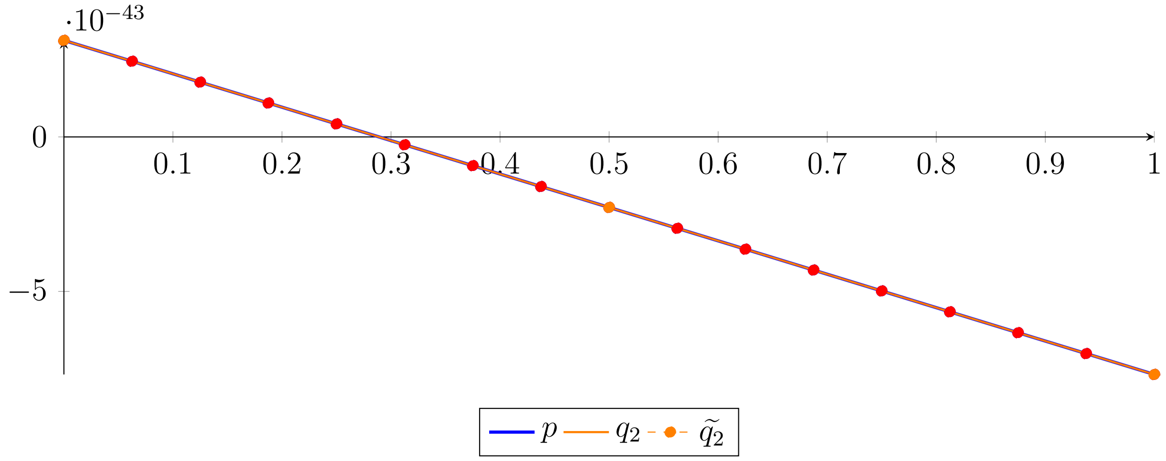
$$\begin{aligned}p = &-1.309058005179934\cdot10^{-347}X^{16} + 1.062571961277761\cdot10^{-346}X^{15} + 3.448250355108118\\&\cdot10^{-347}X^{14} - 6.257935829640659\cdot10^{-346}X^{13} - 1.627063315706571\cdot10^{-345}X^{12}\\&- 2.789251398354122\cdot10^{-346}X^{11} - 1.022725512729845\cdot10^{-345}X^{10} + 1.278406890912306\cdot10^{-345}X^{9}\\&- 4.109165006503841\cdot10^{-346}X^{8} + 9.701559540649497\cdot10^{-305}X^{7} + 1.075415704126648\cdot10^{-258}X^{6}\\&- 1.103978312017599\cdot10^{-214}X^{5} - 9.636311849715274\cdot10^{-169}X^{4} + 5.1799542305243\cdot10^{-125}X^{3}\\&+ 3.696479581469533\cdot10^{-79}X^{2} - 1.081297478088303\cdot10^{-42}X + 3.12447403488504\cdot10^{-43}\\= &\,3.12447403488504\cdot10^{-43}B_{0,16}(X) + 2.448663111079851\cdot10^{-43}B_{1,16}(X) + 1.772852187274661\\&\cdot10^{-43}B_{2,16}(X) + 1.097041263469472\cdot10^{-43}B_{3,16}(X) + 4.212303396642828\cdot10^{-44}B_{4,16}(X)\\&- 2.545805841409065\cdot10^{-44}B_{5,16}(X) - 9.303915079460958\cdot10^{-44}B_{6,16}(X) - 1.606202431751285\\&\cdot10^{-43}B_{7,16}(X) - 2.282013355556474\cdot10^{-43}B_{8,16}(X) - 2.957824279361664\cdot10^{-43}B_{9,16}(X)\\&- 3.633635203166853\cdot10^{-43}B_{10,16}(X) - 4.309446126972042\cdot10^{-43}B_{11,16}(X)\\&- 4.985257050777232\cdot10^{-43}B_{12,16}(X) - 5.661067974582421\cdot10^{-43}B_{13,16}(X) - 6.33687889838761\\&\cdot10^{-43}B_{14,16}(X) - 7.0126898221928\cdot10^{-43}B_{15,16}(X) - 7.688500745997989\cdot10^{-43}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 3.696479581469533\cdot10^{-79}X^{2} - 1.081297478088303\cdot10^{-42}X + 3.12447403488504\cdot10^{-43}\\&= 3.12447403488504\cdot10^{-43}B_{0,2} - 2.282013355556474\cdot10^{-43}B_{1,2} - 7.688500745997989\cdot10^{-43}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 = &\,1.321235643657083\cdot10^{-339}X^{16} - 4.707039676382068\cdot10^{-339}X^{15} + 3.761577682517833\\&\cdot10^{-339}X^{14} - 4.359613352194916\cdot10^{-339}X^{13} + 4.54005469441269\cdot10^{-338}X^{12}\\&- 1.312163583808635\cdot10^{-337}X^{11} + 1.79382514864333\cdot10^{-337}X^{10} - 1.339511978482456\cdot10^{-337}X^{9}\\&+ 5.208651878296088\cdot10^{-338}X^{8} - 5.540207059869677\cdot10^{-339}X^{7} - 3.436463990345086\cdot10^{-339}X^{6}\\&+ 1.463153300127776\cdot10^{-339}X^{5} - 2.06132658730541\cdot10^{-340}X^{4} + 1.627315868116839\cdot10^{-342}X^{3}\\&+ 3.696479581469533\cdot10^{-79}X^{2} - 1.081297478088303\cdot10^{-42}X + 3.12447403488504\cdot10^{-43}\\= &\,3.12447403488504\cdot10^{-43}B_{0,16} + 2.448663111079851\cdot10^{-43}B_{1,16} + 1.772852187274661\cdot10^{-43}B_{2,16}\\&+ 1.097041263469472\cdot10^{-43}B_{3,16} + 4.212303396642828\cdot10^{-44}B_{4,16} - 2.545805841409065\cdot10^{-44}B_{5,16}\\&- 9.303915079460958\cdot10^{-44}B_{6,16} - 1.606202431751285\cdot10^{-43}B_{7,16} - 2.282013355556474\cdot10^{-43}B_{8,16}\\&- 2.957824279361664\cdot10^{-43}B_{9,16} - 3.633635203166853\cdot10^{-43}B_{10,16} - 4.309446126972042\\&\cdot10^{-43}B_{11,16} - 4.985257050777232\cdot10^{-43}B_{12,16} - 5.661067974582421\cdot10^{-43}B_{13,16}\\&- 6.33687889838761\cdot10^{-43}B_{14,16} - 7.0126898221928\cdot10^{-43}B_{15,16} - 7.688500745997989\cdot10^{-43}B_{16,16}\end{aligned}$$



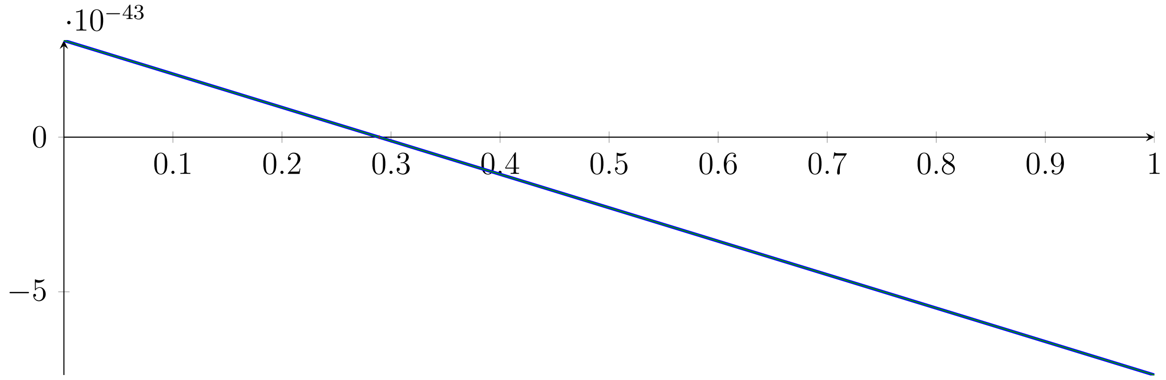
The maximum difference of the Bézier coefficients is $\delta = 2.58997711526215 \cdot 10^{-126}$.
Bounding polynomials M and m:

$$M = 3.696479581469533 \cdot 10^{-79} X^2 - 1.081297478088303 \cdot 10^{-42} X + 3.12447403488504 \cdot 10^{-43}$$
$$m = 3.696479581469533 \cdot 10^{-79} X^2 - 1.081297478088303 \cdot 10^{-42} X + 3.12447403488504 \cdot 10^{-43}$$

Root of M and m:

$N(M) = \{0.2889560087025269, 2.925208848735027 \cdot 10^{36}\}$ $N(m) = \{0.2889560087025269, 2.92520884873502$

Intersection intervals:



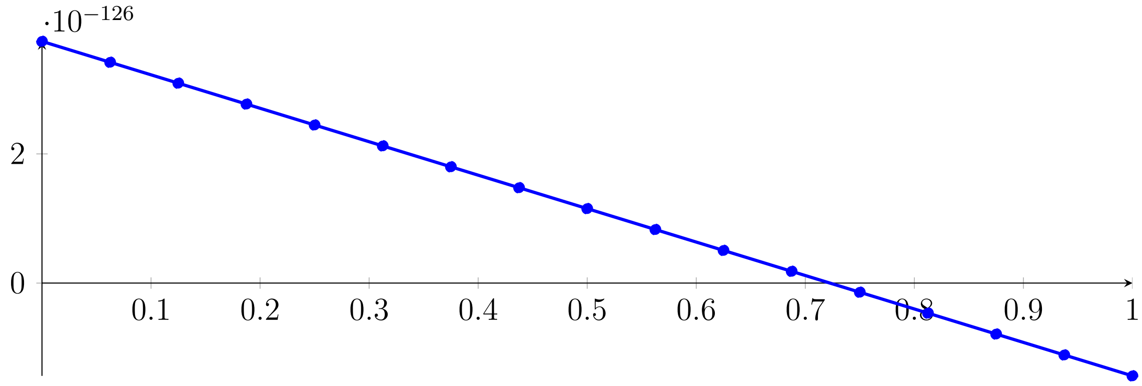
$$[0.2889560087025269, 0.2889560087025269]$$

Longest intersection interval: $4.790498762359349 \cdot 10^{-84}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

83.8 Recursion Branch 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Normalized monomial und Bézier representations and the Bézier polygon:

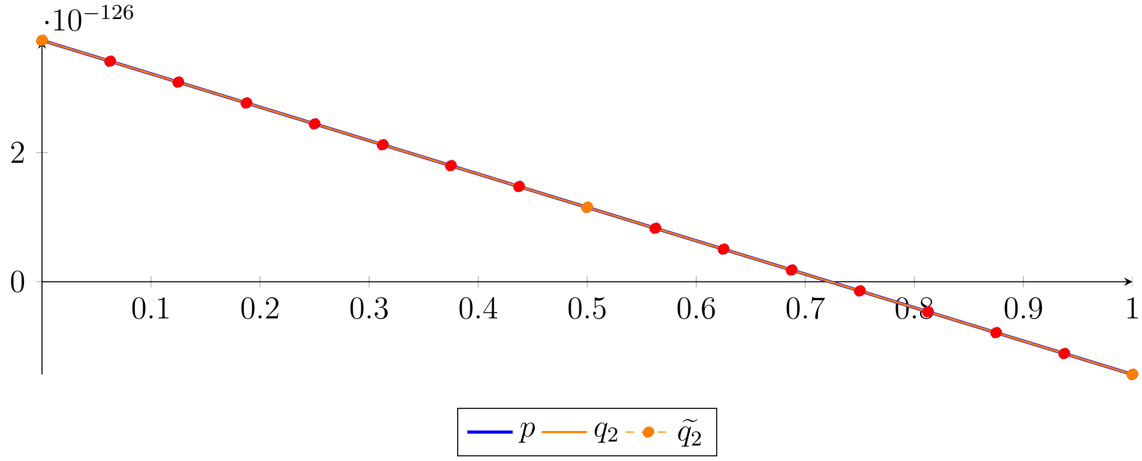
$$
\begin{aligned}
p = {} & 6.521506202791125\cdot 10^{-431}X^{16} - 8.49899518041166\cdot 10^{-430}X^{15} - 2.398231313284478 \\
& \cdot 10^{-429}X^{14} + 2.650676714682844\cdot 10^{-429}X^{13} + 3.445879729087698\cdot 10^{-428}X^{12} \\
& - 1.148626576362566\cdot 10^{-428}X^{11} + 2.526978467997645\cdot 10^{-428}X^{10} + 6.016615399994393\cdot 10^{-429}X^{9} \\
& - 6.768692324993692\cdot 10^{-429}X^{8} + 6.016615399994393\cdot 10^{-429}X^{7} + 5.694664191730495\cdot 10^{-375}X^{3} \\
& + 8.483006039427066\cdot 10^{-246}X^{2} - 5.1799542305243\cdot 10^{-126}X + 3.742868694986836\cdot 10^{-126} \\
= {} & 3.742868694986836\cdot 10^{-126}B_{0,16}(X) + 3.419121555579067\cdot 10^{-126}B_{1,16}(X) + 3.095374416171298 \\
& \cdot 10^{-126}B_{2,16}(X) + 2.77162727676353\cdot 10^{-126}B_{3,16}(X) + 2.447880137355761\cdot 10^{-126}B_{4,16}(X) \\
& + 2.124132997947992\cdot 10^{-126}B_{5,16}(X) + 1.800385858540223\cdot 10^{-126}B_{6,16}(X) + 1.476638719132455 \\
& \cdot 10^{-126}B_{7,16}(X) + 1.152891579724686\cdot 10^{-126}B_{8,16}(X) + 8.291444403169172\cdot 10^{-127}B_{9,16}(X) \\
& + 5.053973009091485\cdot 10^{-127}B_{10,16}(X) + 1.816501615013797\cdot 10^{-127}B_{11,16}(X) - 1.42096977906389 \\
& \cdot 10^{-127}B_{12,16}(X) - 4.658441173141578\cdot 10^{-127}B_{13,16}(X) - 7.895912567219265\cdot 10^{-127}B_{14,16}(X) \\
& - 1.113338396129695\cdot 10^{-126}B_{15,16}(X) - 1.437085535537464\cdot 10^{-126}B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_2 = {} & 8.483006039427066\cdot 10^{-246}X^{2} - 5.1799542305243\cdot 10^{-126}X + 3.742868694986836\cdot 10^{-126} \\
= {} & 3.742868694986836\cdot 10^{-126}B_{0,2} + 1.152891579724686\cdot 10^{-126}B_{1,2} - 1.437085535537464\cdot 10^{-126}B_{2,2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{q_2} = {} & 1.646903409476928\cdot 10^{-422}X^{16} - 1.073856782324668\cdot 10^{-421}X^{15} + 2.435027189672383 \\
& \cdot 10^{-421}X^{14} - 7.561653122498893\cdot 10^{-422}X^{13} - 7.30617276371247\cdot 10^{-421}X^{12} \\
& + 1.716020484636705\cdot 10^{-420}X^{11} - 1.94492618238779\cdot 10^{-420}X^{10} + 1.283970110196866\cdot 10^{-420}X^{9} \\
& - 4.851863553393144\cdot 10^{-421}X^{8} + 8.027185211148974\cdot 10^{-422}X^{7} + 9.251262906800903\cdot 10^{-423}X^{6} \\
& - 7.009373650775329\cdot 10^{-423}X^{5} + 1.329535142491068\cdot 10^{-423}X^{4} - 7.321794940178316\cdot 10^{-425}X^{3} \\
& + 8.483006039427066\cdot 10^{-246}X^{2} - 5.1799542305243\cdot 10^{-126}X + 3.742868694986836\cdot 10^{-126} \\
= {} & 3.742868694986836\cdot 10^{-126}B_{0,16} + 3.419121555579067\cdot 10^{-126}B_{1,16} + 3.095374416171298 \\
& \cdot 10^{-126}B_{2,16} + 2.77162727676353\cdot 10^{-126}B_{3,16} + 2.447880137355761\cdot 10^{-126}B_{4,16} \\
& + 2.124132997947992\cdot 10^{-126}B_{5,16} + 1.800385858540223\cdot 10^{-126}B_{6,16} + 1.476638719132455 \\
& \cdot 10^{-126}B_{7,16} + 1.152891579724686\cdot 10^{-126}B_{8,16} + 8.291444403169172\cdot 10^{-127}B_{9,16} \\
& + 5.053973009091485\cdot 10^{-127}B_{10,16} + 1.816501615013797\cdot 10^{-127}B_{11,16} - 1.42096977906389 \\
& \cdot 10^{-127}B_{12,16} - 4.658441173141578\cdot 10^{-127}B_{13,16} - 7.895912567219265\cdot 10^{-127}B_{14,16} \\
& - 1.113338396129695\cdot 10^{-126}B_{15,16} - 1.437085535537464\cdot 10^{-126}B_{16,16}
\end{aligned}
$$



The maximum difference of the Bézier coefficients is $\delta = 2.847332095865248 \cdot 10^{-376}$.
Bounding polynomials M and m:

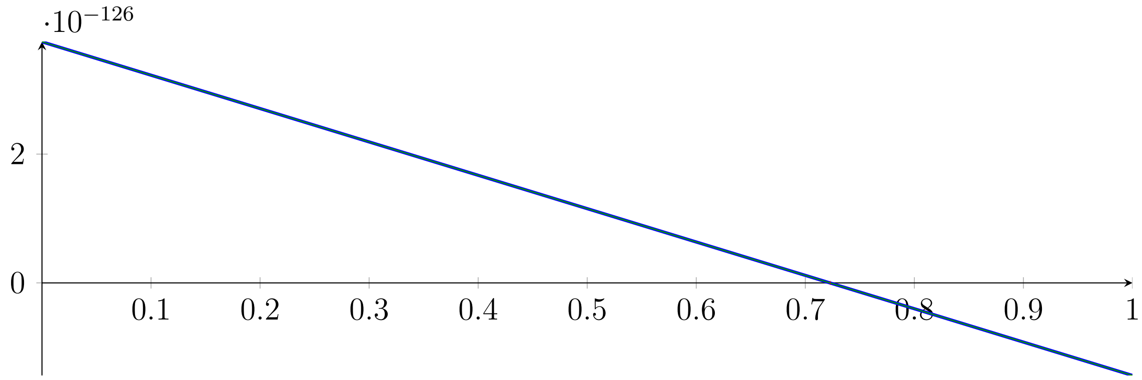
$$M = 8.483006039427066 \cdot 10^{-246} X^2 - 5.1799542305243 \cdot 10^{-126} X + 3.742868694986836 \cdot 10^{-126}$$

$$m = 8.483006039427066 \cdot 10^{-246} X^2 - 5.1799542305243 \cdot 10^{-126} X + 3.742868694986836 \cdot 10^{-126}$$

Root of M and m:

$N(M) = \{0.7225679085986815, 6.106272005995353 \cdot 10^{119}\}$ $\quad N(m) = \{0.7225679085986815, 6.10627200599535$

Intersection intervals:



$$[0.7225679085986815, 0.7225679085986815]$$

Longest intersection interval: 0
$\implies$ Selective recursion: interval 1: $[0.30000008, 0.30000008]$,

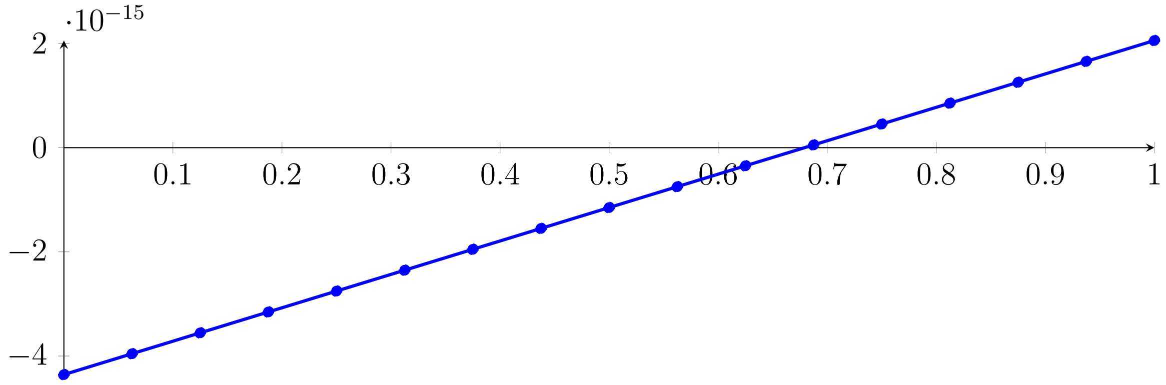
83.9 Recursion Branch 1 1 1 1 1 1 1 1 1 in Interval 1: $[0.30000008, 0.30000008]$

Found root in interval $[0.30000008, 0.30000008]$ at recursion depth 9!

83.10 Recursion Branch 1 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

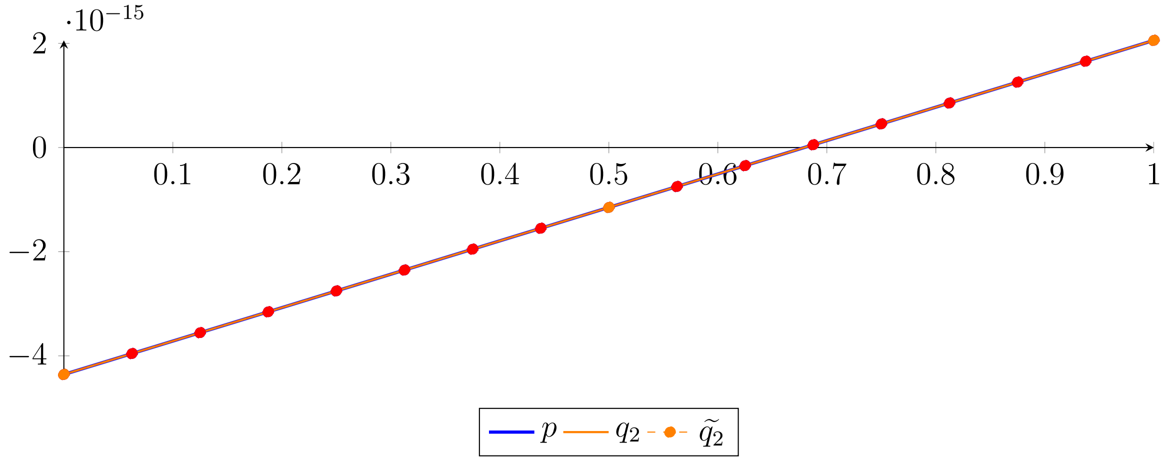
$$\begin{aligned}p &= -8.180742151883693\cdot 10^{-268}X^{16} + 3.227248873730463\cdot 10^{-251}X^{15} + 4.462130355852244\\&\quad\cdot 10^{-232}X^{14} - 1.715942757602136\cdot 10^{-215}X^{13} - 1.0361476928624\cdot 10^{-196}X^{12} + 3.66828444102595\\&\quad\cdot 10^{-180}X^{11} + 1.330015309185499\cdot 10^{-161}X^{10} - 4.080452484754623\cdot 10^{-145}X^{9}\\&\quad - 1.020555600063168\cdot 10^{-126}X^{8} + 2.507762718201662\cdot 10^{-110}X^{7} + 4.686136236097161\cdot 10^{-92}X^{6}\\&\quad - 8.109472029532602\cdot 10^{-76}X^{5} - 1.193263996479812\cdot 10^{-57}X^{4} + 1.081297242720538\cdot 10^{-41}X^{3}\\&\quad + 1.300771932291811\cdot 10^{-23}X^{2} + 6.414334610838192\cdot 10^{-15}X - 4.357019382395145\cdot 10^{-15}\\&= -4.357019382395145\cdot 10^{-15}B_{0,16}(X) - 3.956123469217758\cdot 10^{-15}B_{1,16}(X) - 3.555227555931974\\&\quad\cdot 10^{-15}B_{2,16}(X) - 3.154331642537791\cdot 10^{-15}B_{3,16}(X) - 2.753435729035212\cdot 10^{-15}B_{4,16}(X)\\&\quad - 2.352539815424234\cdot 10^{-15}B_{5,16}(X) - 1.951643901704859\cdot 10^{-15}B_{6,16}(X) - 1.550747987877086\\&\quad\cdot 10^{-15}B_{7,16}(X) - 1.149852073940915\cdot 10^{-15}B_{8,16}(X) - 7.489561598963468\cdot 10^{-16}B_{9,16}(X)\\&\quad - 3.480602457433808\cdot 10^{-16}B_{10,16}(X) + 5.283566851798276\cdot 10^{-17}B_{11,16}(X)\\&\quad + 4.53731582887744\cdot 10^{-16}B_{12,16}(X) + 8.546274973659029\cdot 10^{-16}B_{13,16}(X) + 1.255523411952459\\&\quad\cdot 10^{-15}B_{14,16}(X) + 1.656419326647414\cdot 10^{-15}B_{15,16}(X) + 2.057315241450766\cdot 10^{-15}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 1.300771932291811\cdot 10^{-23}X^{2} + 6.414334610838192\cdot 10^{-15}X - 4.357019382395145\cdot 10^{-15}\\&= -4.357019382395145\cdot 10^{-15}B_{0,2} - 1.14985207697605\cdot 10^{-15}B_{1,2} + 2.057315241450766\cdot 10^{-15}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -1.913962948891774\cdot 10^{-311}X^{16} + 1.224839229220605\cdot 10^{-310}X^{15} - 2.736439272631334\\&\quad\cdot 10^{-310}X^{14} + 8.686727754411293\cdot 10^{-311}X^{13} + 7.87470789022501\cdot 10^{-310}X^{12} - 1.83498999880619\\&\quad\cdot 10^{-309}X^{11} + 2.061599642193156\cdot 10^{-309}X^{10} - 1.351793491536379\cdot 10^{-309}X^{9}\\&\quad + 5.099450965546735\cdot 10^{-310}X^{8} - 8.61911462129453\cdot 10^{-311}X^{7} - 8.275840669030232\cdot 10^{-312}X^{6}\\&\quad + 6.946029354848196\cdot 10^{-312}X^{5} - 1.359818722878116\cdot 10^{-312}X^{4} + 8.064687133307928\cdot 10^{-314}X^{3}\\&\quad + 1.300771932291811\cdot 10^{-23}X^{2} + 6.414334610838192\cdot 10^{-15}X - 4.357019382395145\cdot 10^{-15}\\&= -4.357019382395145\cdot 10^{-15}B_{0,16} - 3.956123469217758\cdot 10^{-15}B_{1,16} - 3.555227555931974\\&\quad\cdot 10^{-15}B_{2,16} - 3.154331642537791\cdot 10^{-15}B_{3,16} - 2.753435729035212\cdot 10^{-15}B_{4,16}\\&\quad - 2.352539815424234\cdot 10^{-15}B_{5,16} - 1.951643901704859\cdot 10^{-15}B_{6,16} - 1.550747987877086\\&\quad\cdot 10^{-15}B_{7,16} - 1.149852073940915\cdot 10^{-15}B_{8,16} - 7.489561598963468\cdot 10^{-16}B_{9,16}\\&\quad - 3.480602457433808\cdot 10^{-16}B_{10,16} + 5.283566851798276\cdot 10^{-17}B_{11,16} + 4.53731582887744\\&\quad\cdot 10^{-16}B_{12,16} + 8.546274973659029\cdot 10^{-16}B_{13,16} + 1.255523411952459\cdot 10^{-15}B_{14,16}\\&\quad + 1.656419326647414\cdot 10^{-15}B_{15,16} + 2.057315241450766\cdot 10^{-15}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 5.406486213602688 \cdot 10^{-43}$.

Bounding polynomials M and m:

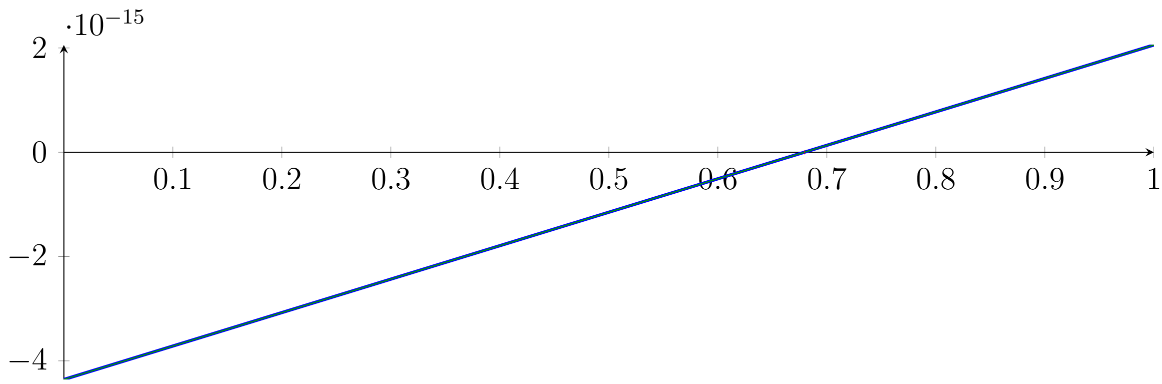
$$M = 1.300771932291811 \cdot 10^{-23} X^2 + 6.414334610838192 \cdot 10^{-15} X - 4.357019382395145 \cdot 10^{-15}$$

$$m = 1.300771932291811 \cdot 10^{-23} X^2 + 6.414334610838192 \cdot 10^{-15} X - 4.357019382395145 \cdot 10^{-15}$$

Root of M and m:

$N(M) = \{-493117545.0851349, 0.6792628761573223\}$ $N(m) = \{-493117545.0851349, 0.6792628761573223\}$

Intersection intervals:



$$[0.6792628761573223, 0.6792628761573223]$$

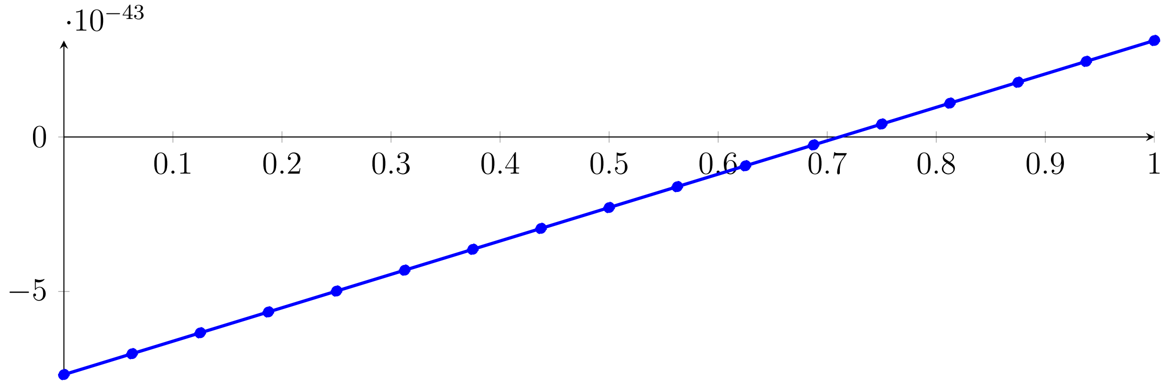
Longest intersection interval: $1.685751220266157 \cdot 10^{-28}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

83.11 Recursion Branch 1 1 1 1 1 2 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

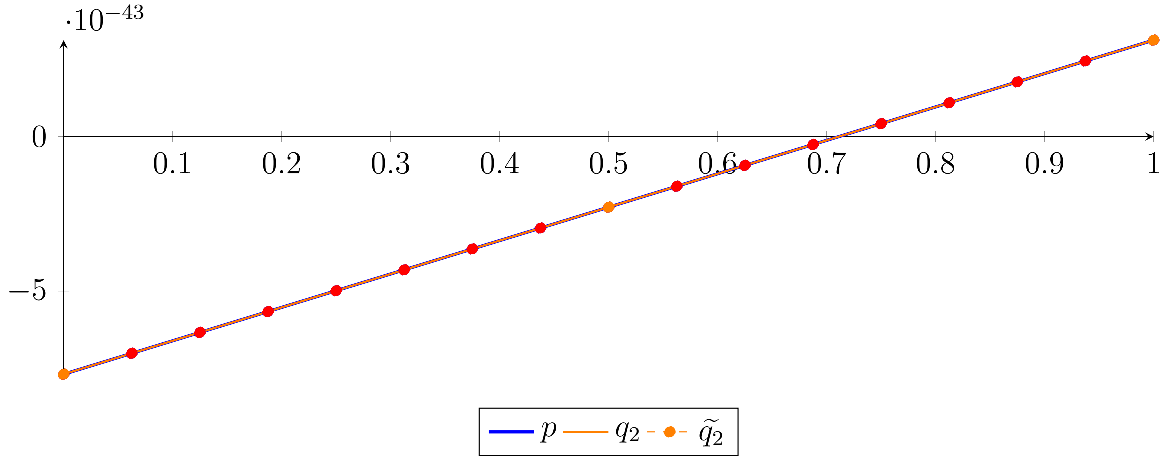
$$p = -6.385648805755774 \cdot 10^{-349} X^{16} + 2.55425952230231 \cdot 10^{-347} X^{15} + 5.44057278250392 \cdot 10^{-346} X^{14} - 1.394625699177061 \cdot 10^{-345} X^{12} - 1.394625699177061 \cdot 10^{-345} X^{11} - 1.022725512729845 \cdot 10^{-345} X^{10} - 2.922072893513842 \cdot 10^{-345} X^{9} - 2.465499003902304 \cdot 10^{-345} X^{8} + 9.701543173030387 \cdot 10^{-305} X^{7} + 1.075414298944151 \cdot 10^{-258} X^{6} - 1.103976919483014 \cdot 10^{-214} X^{5} - 9.636303459858408 \cdot 10^{-169} X^{4} + 5.179949714039793 \cdot 10^{-125} X^{3} + 3.696477973747613 \cdot 10^{-79} X^{2} + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$
$$= -7.691136110088861 \cdot 10^{-43} B_{0,16}(X) - 7.015325333388525 \cdot 10^{-43} B_{1,16}(X) - 6.339514556688189 \cdot 10^{-43} B_{2,16}(X) - 5.663703779987853 \cdot 10^{-43} B_{3,16}(X) - 4.987893003287517 \cdot 10^{-43} B_{4,16}(X) - 4.312082226587181 \cdot 10^{-43} B_{5,16}(X) - 3.636271449886845 \cdot 10^{-43} B_{6,16}(X) - 2.960460673186509 \cdot 10^{-43} B_{7,16}(X) - 2.284649896486173 \cdot 10^{-43} B_{8,16}(X) - 1.608839119785837 \cdot 10^{-43} B_{9,16}(X) - 9.330283430855006 \cdot 10^{-44} B_{10,16}(X) - 2.572175663851646 \cdot 10^{-44} B_{11,16}(X) + 4.185932103151714 \cdot 10^{-44} B_{12,16}(X) + 1.094403987015507 \cdot 10^{-43} B_{13,16}(X) + 1.770214763715843 \cdot 10^{-43} B_{14,16}(X) + 2.446025540416179 \cdot 10^{-43} B_{15,16}(X) + 3.121836317116515 \cdot 10^{-43} B_{16,16}(X)$$



Degree reduction and raising:

$$q_2 = 3.696477973747613 \cdot 10^{-79} X^{2} + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$
$$= -7.691136110088861 \cdot 10^{-43} B_{0,2} - 2.284649896486173 \cdot 10^{-43} B_{1,2} + 3.121836317116515 \cdot 10^{-43} B_{2,2}$$

$$\widetilde{q}_2 = -3.382791032187499 \cdot 10^{-339} X^{16} + 2.195568979483387 \cdot 10^{-338} X^{15} - 4.960588406067856 \cdot 10^{-338} X^{14} + 1.548747367109897 \cdot 10^{-338} X^{13} + 1.473652152363228 \cdot 10^{-337} X^{12} - 3.454811461973597 \cdot 10^{-337} X^{11} + 3.907683672823046 \cdot 10^{-337} X^{10} - 2.575671844035123 \cdot 10^{-337} X^{9} + 9.729121737498621 \cdot 10^{-338} X^{8} - 1.617643528105738 \cdot 10^{-338} X^{7} - 1.791534296229099 \cdot 10^{-339} X^{6} + 1.387056472791705 \cdot 10^{-339} X^{5} - 2.649535616707909 \cdot 10^{-340} X^{4} + 1.484406069547834 \cdot 10^{-341} X^{3} + 3.696477973747613 \cdot 10^{-79} X^{2} + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$
$$= -7.691136110088861 \cdot 10^{-43} B_{0,16} - 7.015325333388525 \cdot 10^{-43} B_{1,16} - 6.339514556688189 \cdot 10^{-43} B_{2,16} - 5.663703779987853 \cdot 10^{-43} B_{3,16} - 4.987893003287517 \cdot 10^{-43} B_{4,16} - 4.312082226587181 \cdot 10^{-43} B_{5,16} - 3.636271449886845 \cdot 10^{-43} B_{6,16} - 2.960460673186509 \cdot 10^{-43} B_{7,16} - 2.284649896486173 \cdot 10^{-43} B_{8,16} - 1.608839119785837 \cdot 10^{-43} B_{9,16} - 9.330283430855006 \cdot 10^{-44} B_{10,16} - 2.572175663851646 \cdot 10^{-44} B_{11,16} + 4.185932103151714 \cdot 10^{-44} B_{12,16} + 1.094403987015507 \cdot 10^{-43} B_{13,16} + 1.770214763715843 \cdot 10^{-43} B_{14,16} + 2.446025540416179 \cdot 10^{-43} B_{15,16} + 3.121836317116515 \cdot 10^{-43} B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 2.589974857019897 \cdot 10^{-126}$.

Bounding polynomials M and m:

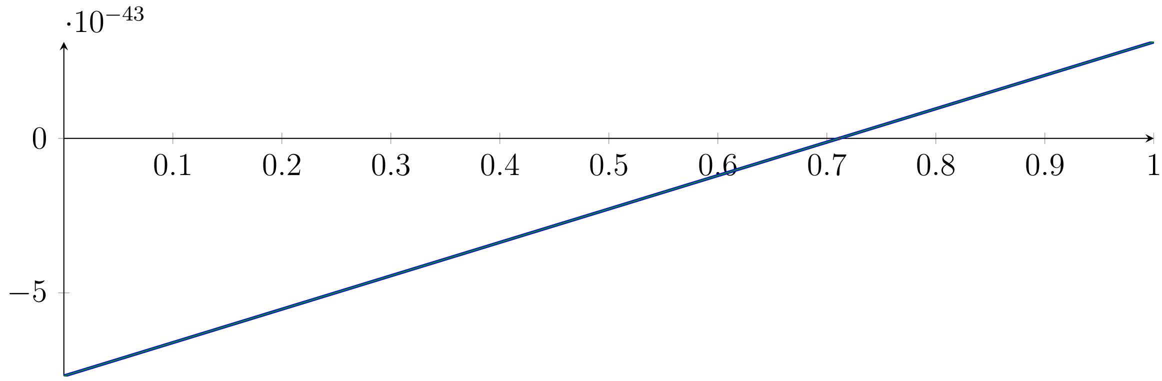
$$M = 3.696477973747613 \cdot 10^{-79} X^2 + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$

$$m = 3.696477973747613 \cdot 10^{-79} X^2 + 1.081297242720538 \cdot 10^{-42} X - 7.691136110088861 \cdot 10^{-43}$$

Root of M and m:

$N(M) = \{-2.925209484271003 \cdot 10^{36}, 0.7112878685178191\}$ $\quad N(m) = \{-2.925209484271003 \cdot 10^{36}, 0.71128786$

Intersection intervals:



$[0.7112878685178191, 0.7112878685178191]$

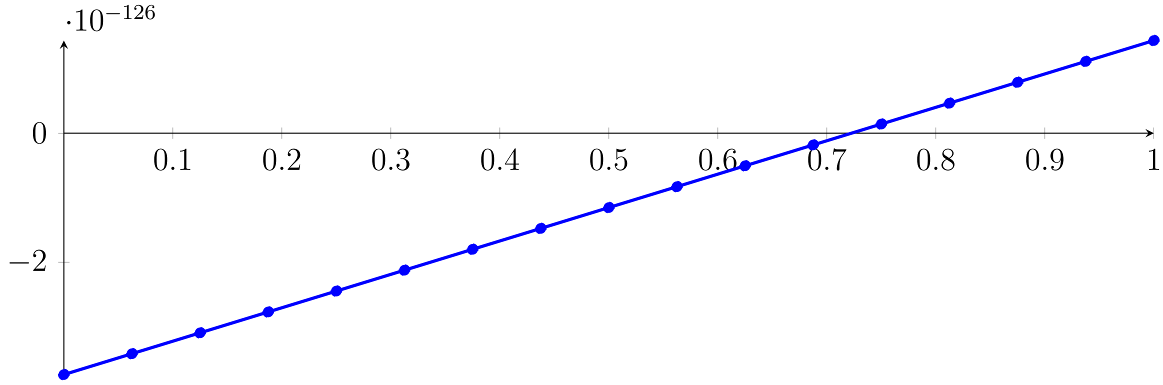
Longest intersection interval: $4.790495628202167 \cdot 10^{-84}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

83.12 Recursion Branch 1 1 1 1 1 2 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

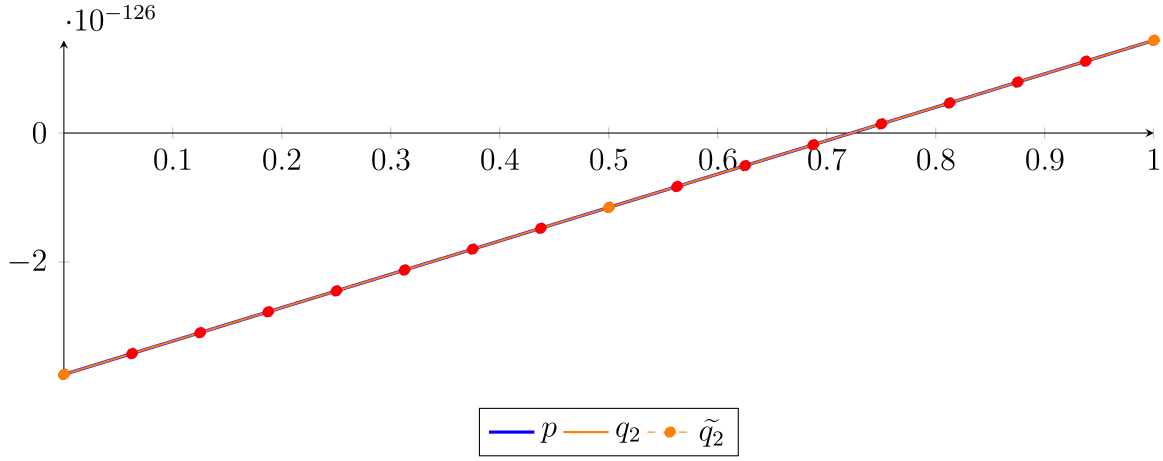
$$\begin{aligned}p &= 3.260753101395563\cdot 10^{-431}X^{16} + 5.385501896498478\cdot 10^{-430}X^{15} - 3.786681020975492\\ &\quad\cdot 10^{-430}X^{14} - 8.099289961530913\cdot 10^{-429}X^{13} + 2.823707000224641\cdot 10^{-428}X^{12}\\ &\quad - 1.263489233998823\cdot 10^{-428}X^{11} - 2.105815389998038\cdot 10^{-428}X^{10} - 6.016615399994393\cdot 10^{-429}X^{9}\\ &\quad + 6.016615399994393\cdot 10^{-429}X^{7} - 3.158723084997056\cdot 10^{-429}X^{6} + 5.74313288181283\\ &\quad\cdot 10^{-430}X^{5} - 1.196486017044339\cdot 10^{-430}X^{4} + 5.694648049372083\cdot 10^{-375}X^{3}\\ &\quad + 8.482991249974965\cdot 10^{-246}X^{2} + 5.179949714039793\cdot 10^{-126}X - 3.743070420543972\cdot 10^{-126}\\ &= -3.743070420543972\cdot 10^{-126}B_{0,16}(X) - 3.419323563416485\cdot 10^{-126}B_{1,16}(X) - 3.095576706288998\\ &\quad\cdot 10^{-126}B_{2,16}(X) - 2.771829849161511\cdot 10^{-126}B_{3,16}(X) - 2.448082992034024\cdot 10^{-126}B_{4,16}(X)\\ &\quad - 2.124336134906536\cdot 10^{-126}B_{5,16}(X) - 1.800589277779049\cdot 10^{-126}B_{6,16}(X) - 1.476842420651562\\ &\quad\cdot 10^{-126}B_{7,16}(X) - 1.153095563524075\cdot 10^{-126}B_{8,16}(X) - 8.293487063965881\cdot 10^{-127}B_{9,16}(X)\\ &\quad - 5.05601849269101\cdot 10^{-127}B_{10,16}(X) - 1.818549921416139\cdot 10^{-127}B_{11,16}(X) + 1.418918649858732\\ &\quad\cdot 10^{-127}B_{12,16}(X) + 4.656387221133603\cdot 10^{-127}B_{13,16}(X) + 7.893855792408473\cdot 10^{-127}B_{14,16}(X)\\ &\quad + 1.113132436368334\cdot 10^{-126}B_{15,16}(X) + 1.436879293495822\cdot 10^{-126}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_2 &= 8.482991249974965\cdot 10^{-246}X^{2} + 5.179949714039793\cdot 10^{-126}X - 3.743070420543972\cdot 10^{-126}\\ &= -3.743070420543972\cdot 10^{-126}B_{0,2} - 1.153095563524075\cdot 10^{-126}B_{1,2} + 1.436879293495822\cdot 10^{-126}B_{2,2}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_2 &= -1.646994498148055\cdot 10^{-422}X^{16} + 1.073933267401417\cdot 10^{-421}X^{15} - 2.435230867118915\\ &\quad\cdot 10^{-421}X^{14} + 7.562144944518816\cdot 10^{-422}X^{13} + 7.307031667182236\cdot 10^{-421}X^{12}\\ &\quad - 1.716232982563429\cdot 10^{-420}X^{11} + 1.945180389577771\cdot 10^{-420}X^{10} - 1.284144687557235\cdot 10^{-420}X^{9}\\ &\quad + 4.852529787317828\cdot 10^{-421}X^{8} - 8.028153209359141\cdot 10^{-422}X^{7} - 9.253596676706869\cdot 10^{-423}X^{6}\\ &\quad + 7.010646616178582\cdot 10^{-423}X^{5} - 1.329745484732865\cdot 10^{-423}X^{4} + 7.322527557770291\cdot 10^{-425}X^{3}\\ &\quad + 8.482991249974965\cdot 10^{-246}X^{2} + 5.179949714039793\cdot 10^{-126}X - 3.743070420543972\cdot 10^{-126}\\ &= -3.743070420543972\cdot 10^{-126}B_{0,16} - 3.419323563416485\cdot 10^{-126}B_{1,16} - 3.095576706288998\\ &\quad\cdot 10^{-126}B_{2,16} - 2.771829849161511\cdot 10^{-126}B_{3,16} - 2.448082992034024\cdot 10^{-126}B_{4,16}\\ &\quad - 2.124336134906536\cdot 10^{-126}B_{5,16} - 1.800589277779049\cdot 10^{-126}B_{6,16} - 1.476842420651562\\ &\quad\cdot 10^{-126}B_{7,16} - 1.153095563524075\cdot 10^{-126}B_{8,16} - 8.293487063965881\cdot 10^{-127}B_{9,16}\\ &\quad - 5.05601849269101\cdot 10^{-127}B_{10,16} - 1.818549921416139\cdot 10^{-127}B_{11,16} + 1.418918649858732\\ &\quad\cdot 10^{-127}B_{12,16} + 4.656387221133603\cdot 10^{-127}B_{13,16} + 7.893855792408473\cdot 10^{-127}B_{14,16}\\ &\quad + 1.113132436368334\cdot 10^{-126}B_{15,16} + 1.436879293495822\cdot 10^{-126}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 2.847324024686041 \cdot 10^{-376}$.
Bounding polynomials M and m:

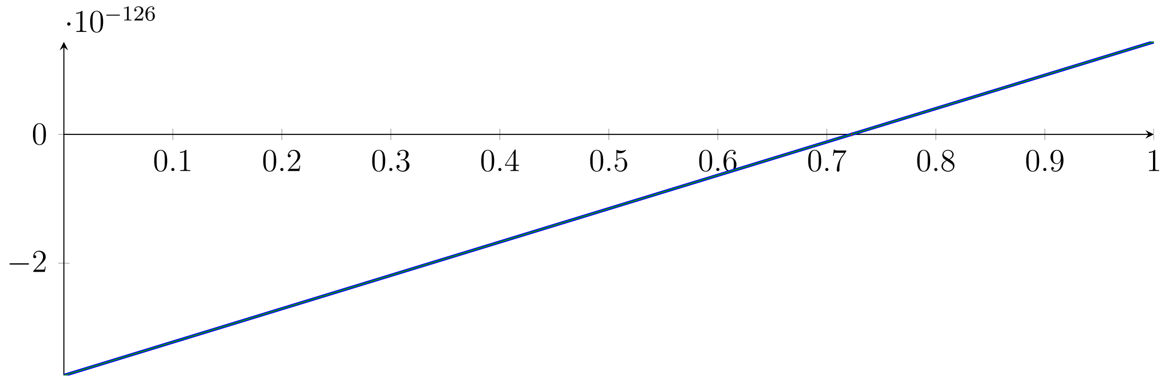
$$M = 8.482991249974965 \cdot 10^{-246} X^2 + 5.179949714039793 \cdot 10^{-126} X - 3.743070420543972 \cdot 10^{-126}$$

$$m = 8.482991249974965 \cdot 10^{-246} X^2 + 5.179949714039793 \cdot 10^{-126} X - 3.743070420543972 \cdot 10^{-126}$$

Root of M and m:

$N(M) = \{-6.106277327652649 \cdot 10^{119}, 0.7226074821534873\}$ $N(m) = \{-6.106277327652649 \cdot 10^{119}, 0.722607$

Intersection intervals:



$[0.7226074821534873, 0.7226074821534873]$

Longest intersection interval: $8.625752293763078 \cdot 10^{-190}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

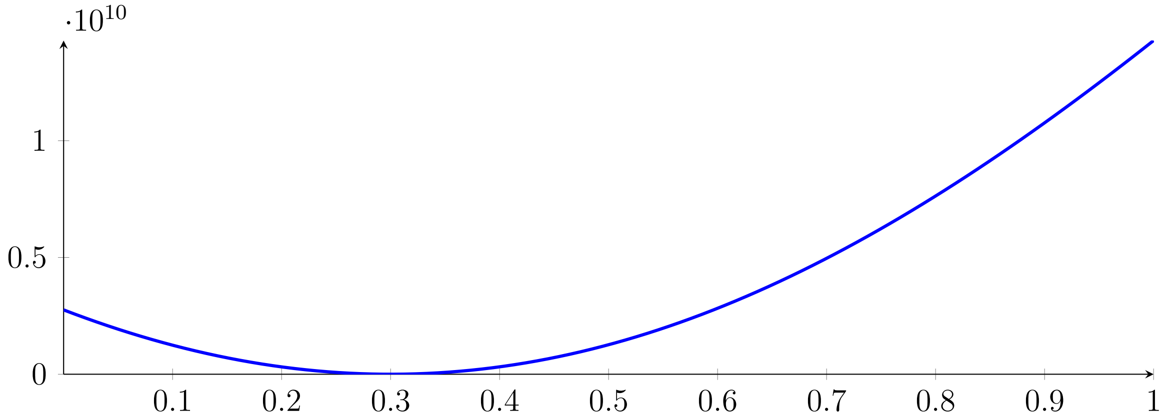
83.13 Recursion Branch 1 1 1 1 1 2 1 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 9!

83.14 Result: 2 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000008, 0.30000008],\ [0.30000009, 0.30000009]$$

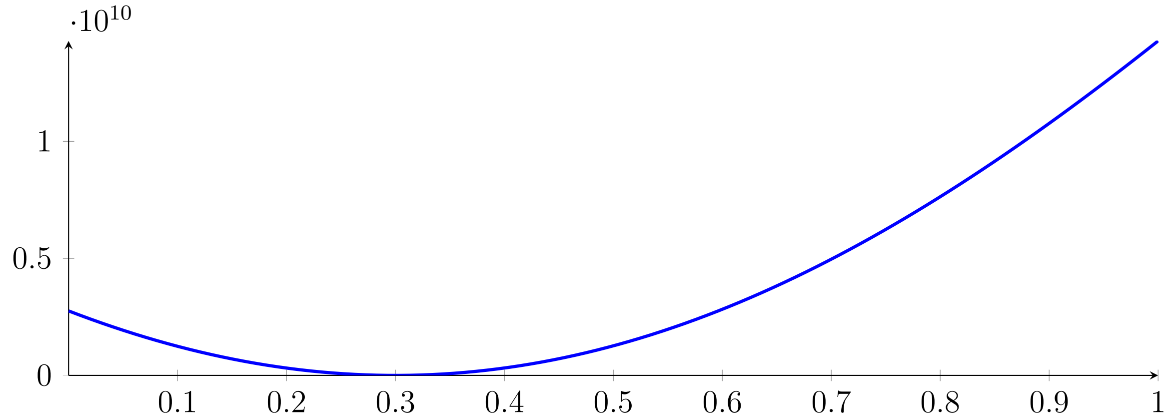
with precision $\varepsilon = 1 \cdot 10^{-128}$.

84 Running CubeClip on h_{16} with epsilon 128

$$-1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$

Called CubeClip with input polynomial on interval $[0, 1]$:

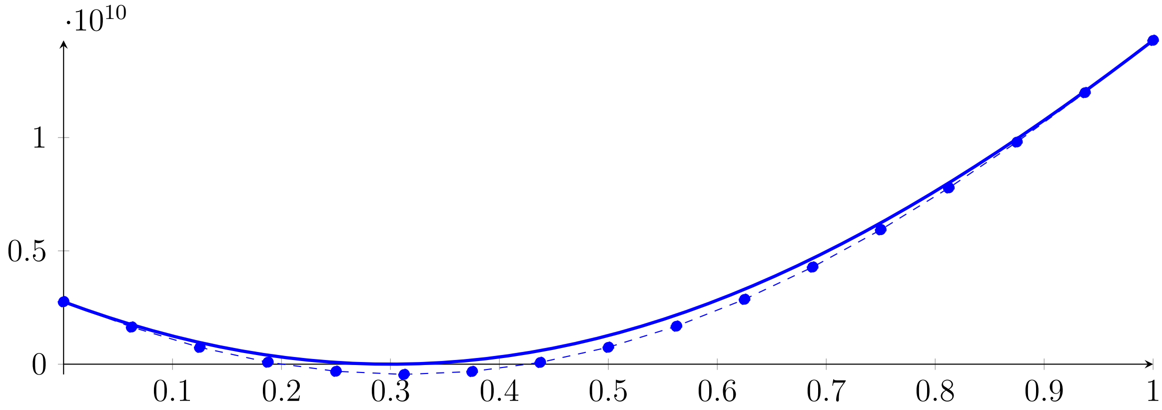
$$\begin{aligned} p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \end{aligned}$$



84.1 Recursion Branch 1 for Input Interval $[0, 1]$

Normalized monomial und Bézier representations and the Bézier polygon:

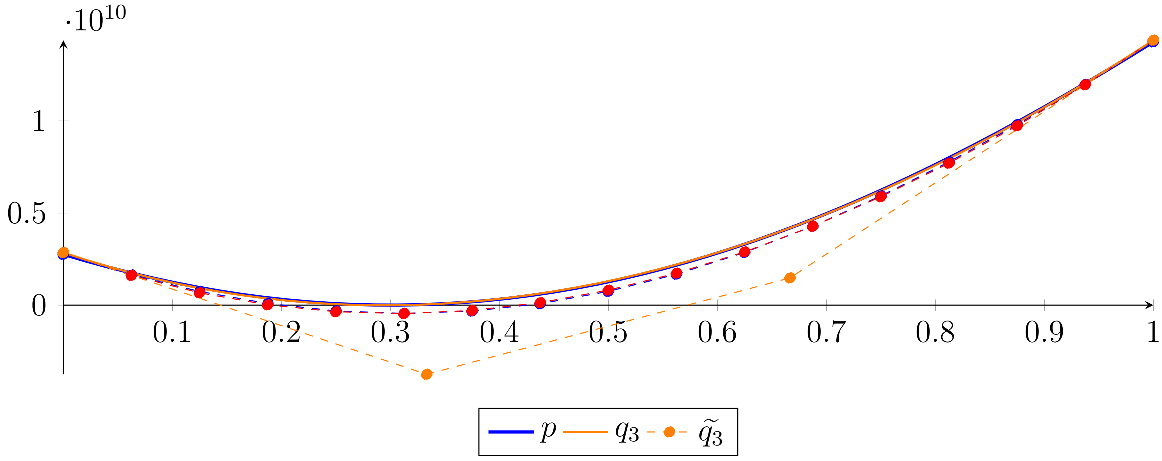
$$\begin{aligned} p = & -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} \\ & + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} \\ & - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} \\ & - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} \\ & + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822 \\ = & \ 2755621561.51822B_{0,16}(X) + 1637790878.39726B_{1,16}(X) + 743271109.8765074B_{2,16}(X) \\ & + 88484675.15500339B_{3,16}(X) - 313348224.4075923B_{4,16}(X) - 452521670.9166005B_{5,16}(X) \\ & - 323087412.8681766B_{6,16}(X) + 76978962.10919518B_{7,16}(X) + 745695311.2415886B_{8,16}(X) \\ & + 1677077302.504944B_{9,16}(X) + 2861230645.190485B_{10,16}(X) + 4284529044.191787B_{11,16}(X) \\ & + 5929872881.820451B_{12,16}(X) + 7777020991.577765B_{13,16}(X) \\ & + 9802985597.278462B_{14,16}(X) + 11982478655.1439B_{15,16}(X) + 14288396529.96021B_{16,16}(X) \end{aligned}$$



Degree reduction and raising:

$$q_3 = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,3} - 3756391006.130182B_{1,3} + 1479191231.735532B_{2,3} + 14380733319.05663B_{3,3}$$

$$\widetilde{q}_3 = 1.707920089479313\cdot 10^{-286}X^{16} - 1.285149559201605\cdot 10^{-285}X^{15} + 4.317602100312756\cdot 10^{-285}X^{14} - 8.501431718860268\cdot 10^{-285}X^{13} + 1.081511666906834\cdot 10^{-284}X^{12} - 9.244250811775947\cdot 10^{-285}X^{11} + 5.383611309108254\cdot 10^{-285}X^{10} - 2.184025259692948\cdot 10^{-285}X^{9} + 6.984121997801746\cdot 10^{-286}X^{8} - 2.400829205648148\cdot 10^{-286}X^{7} + 9.243609934541251\cdot 10^{-287}X^{6} - 2.7161427533585\cdot 10^{-287}X^{5} + 4.31464378661806\cdot 10^{-288}X^{4} - 4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2852343358.34353$$
$$= 2852343358.34353B_{0,16} + 1613205665.004709B_{1,16} + 670175886.7243734B_{2,16} + 15792672.15808792B_{3,16} - 357405330.0385835B_{4,16} - 456879471.2100766B_{5,16} - 290091102.7008273B_{6,16} + 135498424.1447288B_{7,16} + 812427757.9821557B_{8,16} + 1733235547.467018B_{9,16} + 2890460441.254879B_{10,16} + 4276641088.001304B_{11,16} + 5884316136.361857B_{12,16} + 7706024234.992101B_{13,16} + 9734304032.547602B_{14,16} + 11961694177.68392B_{15,16} + 14380733319.05663B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 96721796.82530918$.

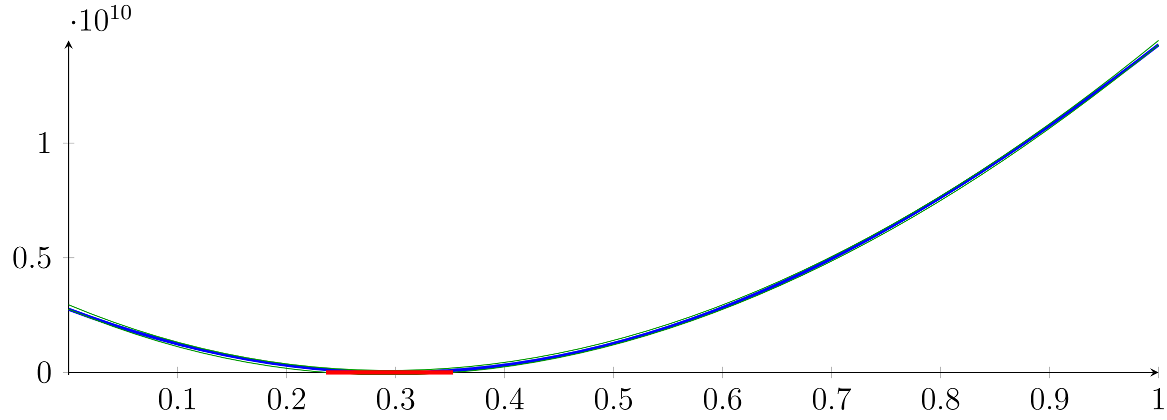
Bounding polynomials M and m:

$$M = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2949065155.168839$$
$$m = -4178356752.884042X^3 + 35532949807.01828X^2 - 19826203093.42114X + 2755621561.51822$$

Root of M and m:

$$N(M) = \{7.915888123074339\} \quad N(m) = \{0.2362027155340351, 0.3527550629571673, 7.91509103986119\}$$

Intersection intervals:



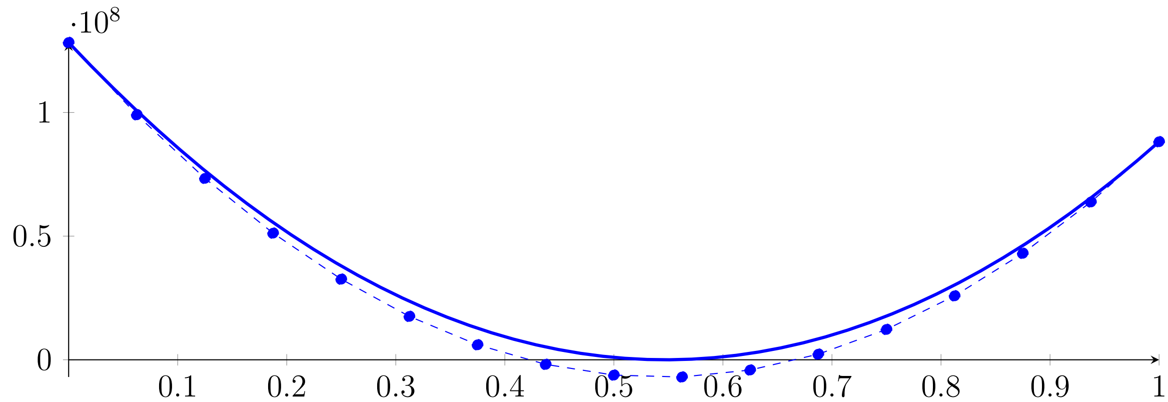
$[0.2362027155340351, 0.3527550629571673]$

Longest intersection interval: 0.1165523474231321
$\Longrightarrow$ Selective recursion: interval 1: $[0.2362027155340351, 0.3527550629571673]$,

84.2 Recursion Branch 1 1 in Interval 1: $[0.2362027155340351, 0.3527550629571673]$

Normalized monomial und Bézier representations and the Bézier polygon:

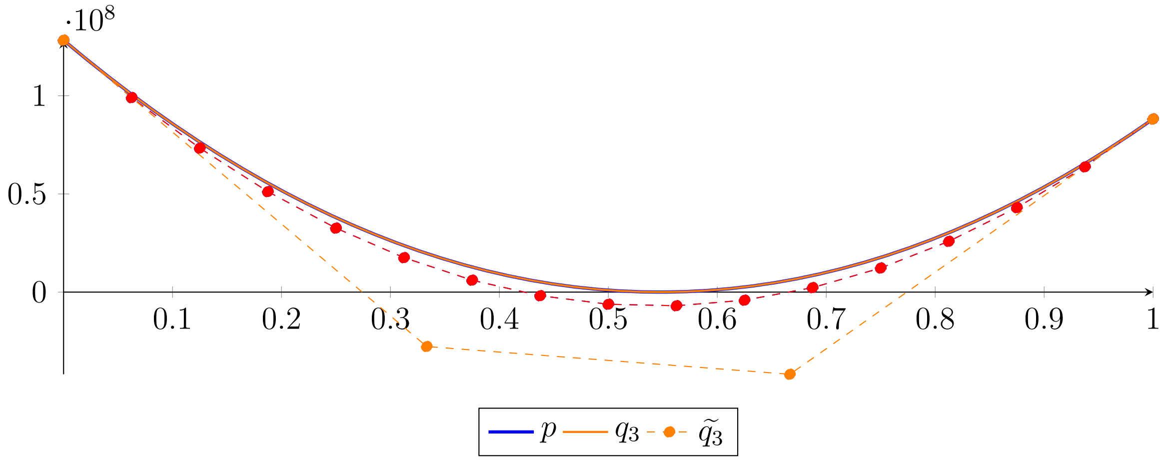
$$
\begin{aligned}
p = & -1.159675274588483\cdot 10^{-15}X^{16} + 1.811620784648044\cdot 10^{-14}X^{15} + 1.904181313086375\cdot 10^{-11}X^{14} \\
& - 2.745089899311131\cdot 10^{-10}X^{13} - 1.331776333869238\cdot 10^{-07}X^{12} + 1.709228363827321 \\
& \cdot 10^{-06}X^{11} + 0.0005154373075671375X^{10} - 0.005636932569987317X^{9} - 1.194310547615576X^{8} \\
& + 10.4754741130729X^{7} + 1659.004456279384X^{6} - 10589.12155320371X^{5} - 1280562.047878962X^{4} \\
& + 4882886.493340317X^{3} + 423977915.1149561X^{2} - 467691034.9555877X + 128284995.8867316 \\
= & \ 128284995.8867316B_{0,16}(X) + 99054306.20200739B_{1,16}(X) + 73356765.8099078B_{2,16}(X) \\
& + 51201094.15059951B_{3,16}(X) + 32595307.05872841B_{4,16}(X) + 17546714.33917011B_{5,16}(X) \\
& + 6061917.549948907B_{6,16}(X) - 1853192.006759158B_{7,16}(X) - 6193435.084716337B_{8,16}(X) \\
& - 6954346.084076609B_{9,16}(X) - 4132174.43156667B_{10,16}(X) + 2276114.250147318B_{11,16}(X) \\
& + 12272837.18435464B_{12,16}(X) + 25859593.69380939B_{13,16}(X) + 43037264.66611236B_{14,16}(X) \\
& + 63806012.23112849B_{15,16}(X) + 88165279.65050818B_{16,16}(X)
\end{aligned}
$$



Degree reduction and raising:

$$
\begin{aligned}
q_3 = & \ 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128303546.2426166 \\
= & \ 128303546.2426166B_{0,3} - 27717047.49576971B_{1,3} \\
& - 41856242.29878324B_{2,3} + 88183874.15570936B_{3,3}
\end{aligned}
$$

$$\begin{aligned}\widetilde{q}_3 &= 1.029539679280458\cdot 10^{-288}X^{16} - 8.072523277872609\cdot 10^{-288}X^{15} + 2.888405037194055\\&\quad\cdot 10^{-287}X^{14} - 6.260623433882179\cdot 10^{-287}X^{13} + 9.183807049175627\cdot 10^{-287}X^{12}\\&\quad - 9.610199535206024\cdot 10^{-287}X^{11} + 7.321003139036714\cdot 10^{-287}X^{10} - 4.031610440228076\\&\quad\cdot 10^{-287}X^{9} + 1.537754605851679\cdot 10^{-287}X^{8} - 3.606689105481158\cdot 10^{-288}X^{7}\\&\quad + 3.223581314526531\cdot 10^{-289}X^{6} + 5.410058694892297\cdot 10^{-290}X^{5} - 1.24173721337985\cdot 10^{-290}X^{4}\\&\quad + 2297912.322133328X^{3} + 425644196.8061184X^{2} - 468061781.215159X + 128303546.2426166\\&= 128303546.2426166B_{0,16} + 99049684.91666918B_{1,16} + 73342858.56410606B_{2,16}\\&\quad + 51187170.59978822B_{3,16} + 32586724.4385766B_{4,16} + 17545623.49533216B_{5,16}\\&\quad + 6067971.184915845B_{6,16} - 1842129.077811386B_{7,16} - 6180573.877988583B_{8,16}\\&\quad - 6943259.800754793B_{9,16} - 4126083.431249065B_{10,16} + 2275058.645389556B_{11,16}\\&\quad + 12264269.84402202B_{12,16} + 25845653.57950928B_{13,16} + 43023313.26671229B_{14,16}\\&\quad + 63801352.320492B_{15,16} + 88183874.15570936B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 18594.50520117899$.

Bounding polynomials M and m:

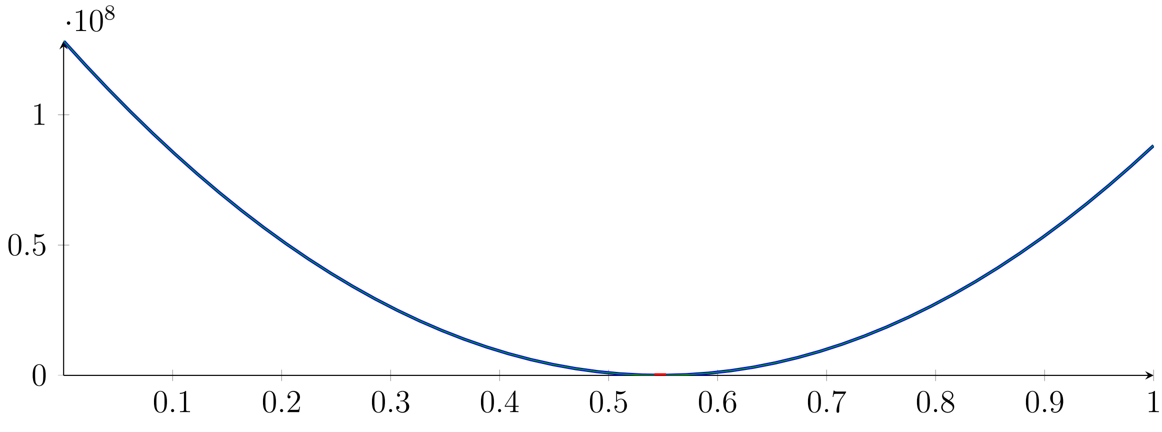
$$M = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128322140.7478178$$

$$m = 2297912.322133328X^3 + 425644196.8061184X^2 - 468061781.215159X + 128284951.7374154$$

Root of M and m:

$$N(M) = \{-186.3256279366086\} \quad N(m) = \{-186.3256274731746, 0.5420611331202266, 0.552740643902378\}$$

Intersection intervals:



$$[0.5420611331202266, 0.552740643902378]$$

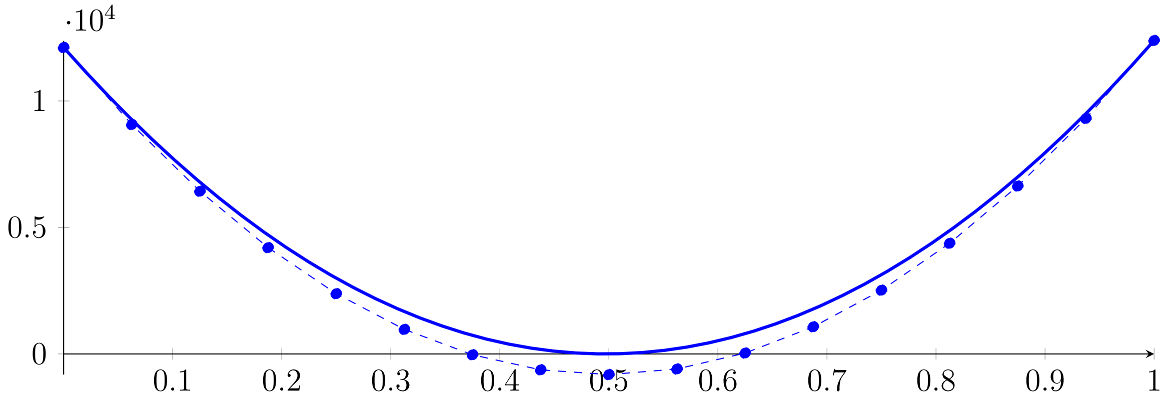
Longest intersection interval: 0.01067951078215144

$\Longrightarrow$ Selective recursion: interval 1: $[0.2993812130460404, 0.3006259350970308]$,

84.3 Recursion Branch 1 1 1 in Interval 1: $[0.2993812130460404, 0.300625935097030$

Normalized monomial und Bézier representations and the Bézier polygon:

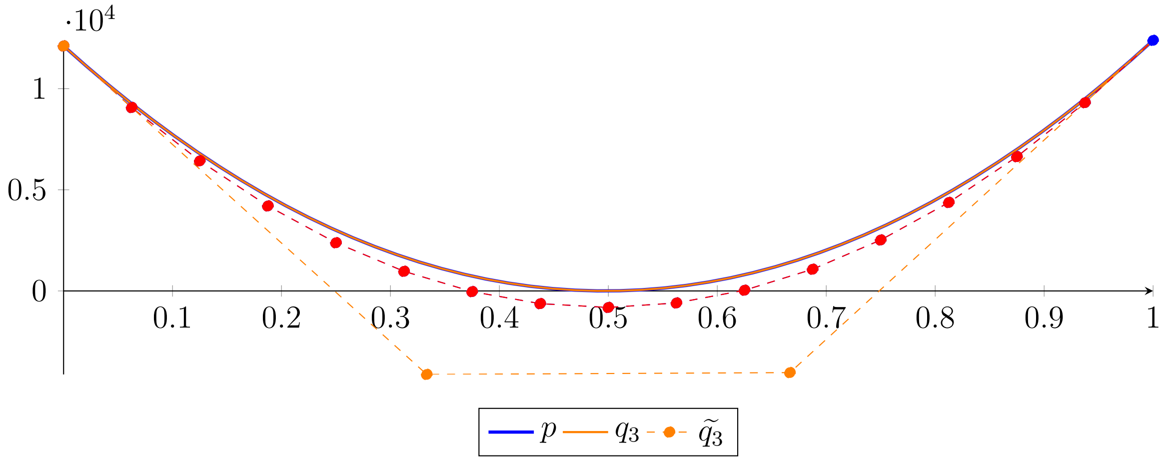
$$\begin{aligned}
p = & -3.320153282917824 \cdot 10^{-47} X^{16} + 2.160317373028451 \cdot 10^{-44} X^{15} + 4.806705749215272 \\
& \cdot 10^{-39} X^{14} - 3.045075915978909 \cdot 10^{-36} X^{13} - 2.96255923979525 \cdot 10^{-31} X^{12} \\
& + 1.726566936989629 \cdot 10^{-28} X^{11} + 1.009356568315285 \cdot 10^{-23} X^{10} - 5.095799973170639 \\
& \cdot 10^{-21} X^{9} - 2.055755460332604 \cdot 10^{-16} X^{8} + 8.312655296209706 \cdot 10^{-14} X^{7} \\
& + 2.505544083540367 \cdot 10^{-09} X^{6} - 7.139662163470987 \cdot 10^{-07} X^{5} - 0.01693489925927299 X^{4} \\
& + 2.534105689487521 X^{3} + 49001.97220419171 X^{2} - 48729.12904702137 X + 12114.14082112651 \\
= & \; 12114.14082112651 B_{0,16}(X) + 9068.570255687672 B_{1,16}(X) + 6431.349458617101 B_{2,16}(X) \\
& + 4202.482955103525 B_{3,16}(X) + 2381.975261030786 B_{4,16}(X) + 969.830882977672 B_{5,16}(X) \\
& - 33.94568178224417 B_{6,16}(X) - 629.34994528077 B_{7,16}(X) - 816.3774288552541 B_{8,16}(X) \\
& - 595.0236631487491 B_{9,16}(X) + 34.71581188982676 B_{10,16}(X) + 1072.845447005528 B_{11,16}(X) \\
& + 2519.36968363722 B_{12,16}(X) + 4374.29295391742 B_{13,16}(X) + 6637.619680672133 B_{14,16}(X) \\
& + 9309.354277420696 B_{15,16}(X) + 12389.50114837561 B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 = & \; 2.500233916081606 X^{3} + 49001.99397932548 X^{2} - 48729.13388598685 X + 12114.14106307619 \\
= & \; 12114.14106307619 B_{0,3} - 4128.903565586097 B_{1,3} \\
& - 4037.950201139886 B_{2,3} + 12389.5013903309 B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 = & \; 1.099600357934752 \cdot 10^{-292} X^{16} - 8.377614657428995 \cdot 10^{-292} X^{15} + 2.905985967667976 \\
& \cdot 10^{-291} X^{14} - 6.091421693711012 \cdot 10^{-291} X^{13} + 8.615207440059339 \cdot 10^{-291} X^{12} \\
& - 8.654253929665817 \cdot 10^{-291} X^{11} + 6.288120919613368 \cdot 10^{-291} X^{10} - 3.270943230058189 \\
& \cdot 10^{-291} X^{9} + 1.159364872956392 \cdot 10^{-291} X^{8} - 2.426897197847659 \cdot 10^{-292} X^{7} \\
& + 1.434105777365326 \cdot 10^{-293} X^{6} + 4.843078869312239 \cdot 10^{-294} X^{5} - 7.632080003106212 \cdot 10^{-295} X^{4} \\
& + 2.500233916081606 X^{3} + 49001.99397932548 X^{2} - 48729.13388598685 X + 12114.14106307619 \\
= & \; 12114.14106307619 B_{0,16} + 9068.570195202008 B_{1,16} + 6431.349277155543 B_{2,16} \\
& + 4202.482773640211 B_{3,16} + 2381.975149359434 B_{4,16} + 969.8308690166351 B_{5,16} \\
& - 33.94560268476556 B_{6,16} - 629.349801041346 B_{7,16} - 816.3772613496847 B_{8,16} \\
& - 595.0235189063601 B_{9,16} + 34.71589099204953 B_{10,16} \\
& + 1072.845433048966 B_{11,16} + 2519.36957196781 B_{12,16} + 4374.292772452004 B_{13,16} \\
& + 6637.619499204969 B_{14,16} + 9309.354216930127 B_{15,16} + 12389.5013903309 B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 0.00024195528529181\,86$.

Bounding polynomials M and m:

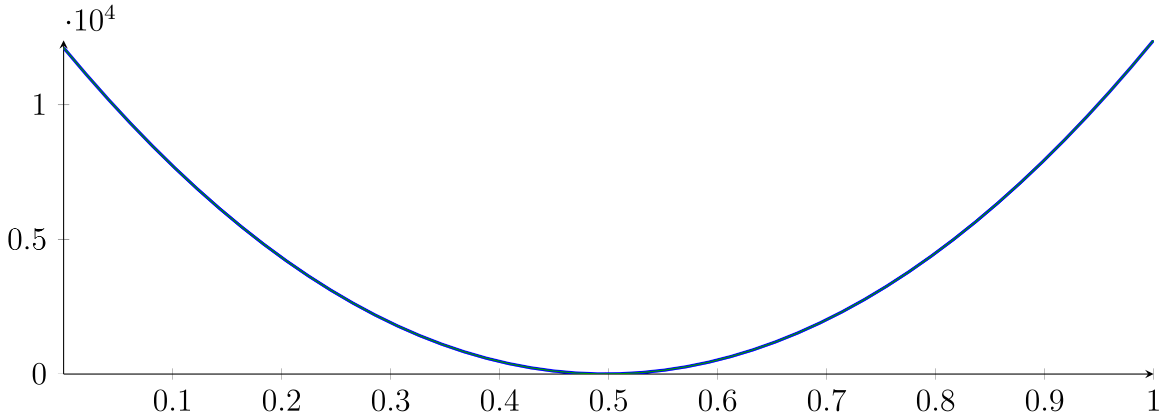
$$M = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.14130503147$$

$$m = 2.500233916081606X^3 + 49001.99397932548X^2 - 48729.13388598685X + 12114.1408211209$$

Root of M and m:

$$N(M) = \{-19599.95818041899\} \quad N(m) = \{-19599.95818041899, 0.4971412064138645, 0.4972526073815481\}$$

Intersection intervals:



$$[0.4971412064138645, 0.4972526073815481]$$

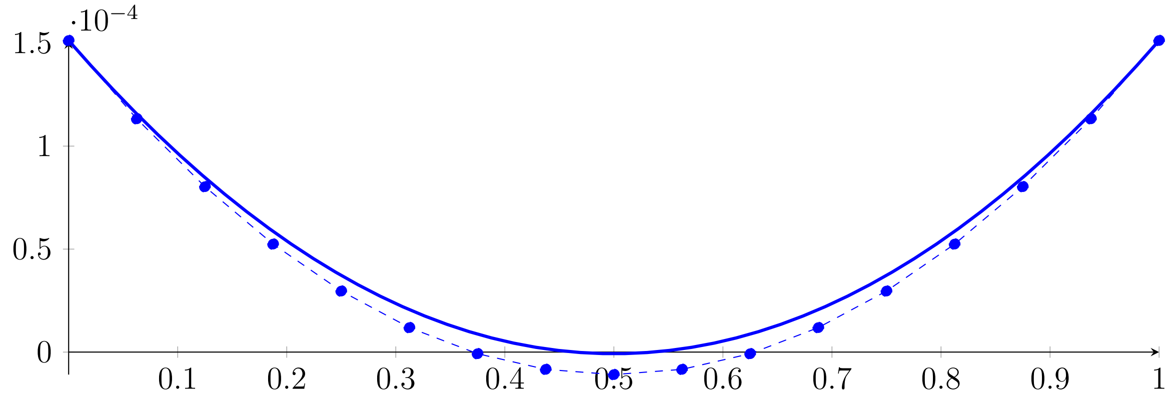
Longest intersection interval: 0.0001114009676835966

$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000156681198, 0.3000001543313607]$,

84.4 Recursion Branch 1 1 1 1 in Interval 1: $[0.3000000156681198, 0.30000015433136$

Normalized monomial und Bézier representations and the Bézier polygon:

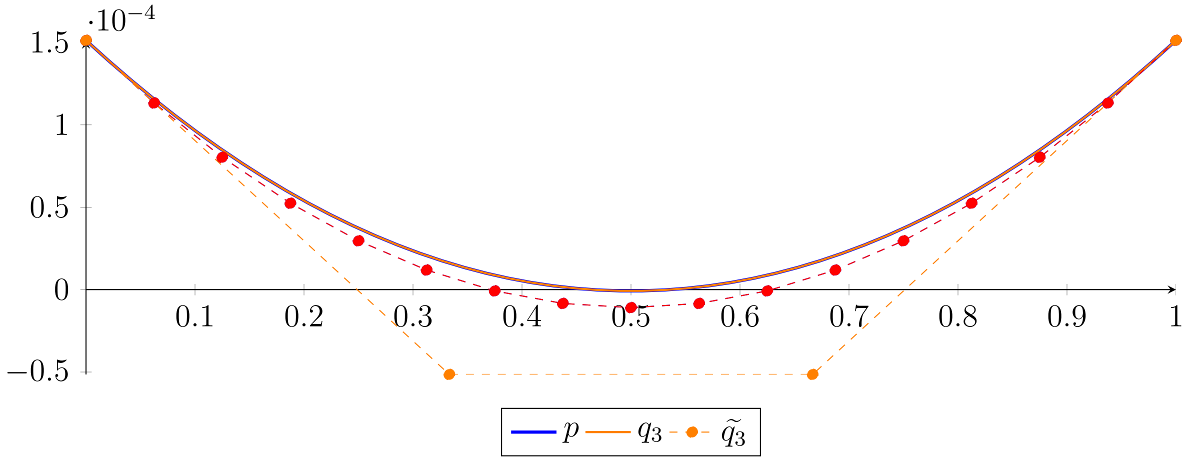
$$p = -1.868020113181866\cdot 10^{-110}X^{16} + 1.077730427531422\cdot 10^{-103}X^{15} + 2.179252184416492\cdot 10^{-94}X^{14} - 1.225622742394903\cdot 10^{-87}X^{13} - 1.082338964896179\cdot 10^{-78}X^{12} + 5.603938665480187\cdot 10^{-72}X^{11} + 2.971492663890366\cdot 10^{-63}X^{10} - 1.333260671370819\cdot 10^{-56}X^{9} - 4.876756665473299\cdot 10^{-48}X^{8} + 1.752546617551927\cdot 10^{-41}X^{7} + 4.789450848811937\cdot 10^{-33}X^{6} - 1.212138221580256\cdot 10^{-26}X^{5} - 2.608457365264229\cdot 10^{-18}X^{4} + 3.456855344293596\cdot 10^{-12}X^{3} + 0.0006081696715066215X^{2} - 0.0006081719526708613X + 0.0001512528027912101$$
$$= 0.0001512528027912101B_{0,16}(X) + 0.0001132420557492813B_{1,16}(X) + 8.029938930324099\cdot 10^{-05}B_{2,16}(X) + 5.242480345926214\cdot 10^{-05}B_{3,16}(X) + 2.961829822351771\cdot 10^{-05}B_{4,16}(X) + 1.187987360218066\cdot 10^{-05}B_{5,16}(X) - 7.904703985760706\cdot 10^{-07}B_{6,16}(X) - 8.392733772579519\cdot 10^{-06}B_{7,16}(X) - 1.092691651365674\cdot 10^{-05}B_{8,16}(X) - 8.393018615634788\cdot 10^{-06}B_{9,16}(X) - 7.910400723407154\cdot 10^{-07}B_{10,16}(X) + 1.187901912239842\cdot 10^{-05}B_{11,16}(X) + 2.961715897475557\cdot 10^{-05}B_{12,16}(X) + 5.242337949090366\cdot 10^{-05}B_{13,16}(X) + 8.029768067701564\cdot 10^{-05}B_{14,16}(X) + 0.0001132400625392645B_{15,16}(X) + 0.0001512505250838230B_{16,16}(X)$$



Degree reduction and raising:

$$q_3 = 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,3} - 5.147118143241051\cdot 10^{-05}B_{1,3} - 5.147194182048959\cdot 10^{-05}B_{2,3} + 0.0001512505250838231B_{3,3}$$

$$\widetilde{q_3} = 1.301341971574797\cdot 10^{-300}X^{16} - 9.903701542140468\cdot 10^{-300}X^{15} + 3.435393711081806\cdot 10^{-299}X^{14} - 7.213305599572634\cdot 10^{-299}X^{13} + 1.023930542727086\cdot 10^{-298}X^{12} - 1.03405078726607\cdot 10^{-298}X^{11} + 7.556175284428161\cdot 10^{-299}X^{10} - 3.94396828718345\cdot 10^{-299}X^{9} + 1.391742955019986\cdot 10^{-299}X^{8} - 2.822114462221502\cdot 10^{-300}X^{7} + 1.110580839880333\cdot 10^{-301}X^{6} + 7.65504767714651\cdot 10^{-302}X^{5} - 1.17007493272161\cdot 10^{-302}X^{4} + 3.456850127378832\cdot 10^{-12}X^{3} + 0.0006081696715066248X^{2} - 0.0006081719526708621X + 0.0001512528027912102$$
$$= 0.0001512528027912102B_{0,16} + 0.0001132420557492813B_{1,16} + 8.029938930324096\cdot 10^{-05}B_{2,16} + 5.242480345926211\cdot 10^{-05}B_{3,16} + 2.961829822351769\cdot 10^{-05}B_{4,16} + 1.187987360218065\cdot 10^{-05}B_{5,16} - 7.904703985760584\cdot 10^{-07}B_{6,16} - 8.392733772579497\cdot 10^{-06}B_{7,16} - 1.092691651365671\cdot 10^{-05}B_{8,16} - 8.393018615634766\cdot 10^{-06}B_{9,16} - 7.910400723407032\cdot 10^{-07}B_{10,16} + 1.187901912239842\cdot 10^{-05}B_{11,16} + 2.961715897475555\cdot 10^{-05}B_{12,16} + 5.242337949090363\cdot 10^{-05}B_{13,16} + 8.029768067701561\cdot 10^{-05}B_{14,16} + 0.0001132400625392644B_{15,16} + 0.0001512505250838231B_{16,16}$$



The maximum difference of the Bézier coefficients is $\delta = 3.726367712763873 \cdot 10^{-20}$.
Bounding polynomials M and m:

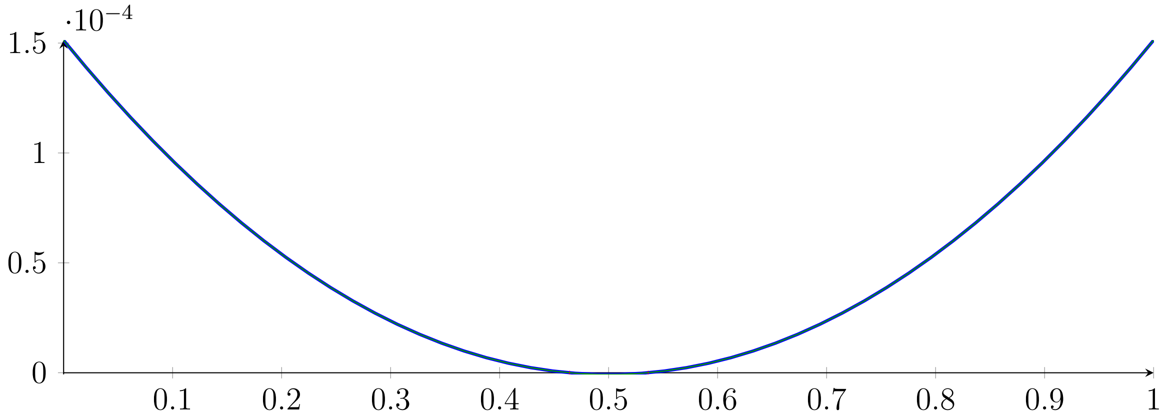
$$M = 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ - 0.0006081719526708621 X + 0.0001512528027912102$$

$$m = 3.456850127378832 \cdot 10^{-12} X^3 + 0.0006081696715066248 X^2 \\ - 0.0006081719526708621 X + 0.0001512528027912101$$

Root of M and m:

$N(M) = \{-175931744.9566824, 0.4639432736155652, 0.5360604563964476\}$ $N(m) = \{-175931744.9566824,$

Intersection intervals:



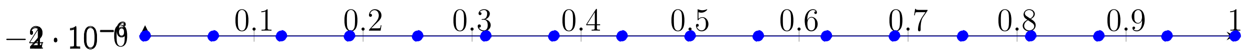
$$[0.4639432736155635, 0.4639432736155652],\ [0.5360604563964476, 0.5360604563964493]$$

Longest intersection interval: $1.699230475659398 \cdot 10^{-15}$
$\Longrightarrow$ Selective recursion: interval 1: $[0.3000000799999977, 0.3000000799999977]$, interval 2: $[0.30000009, 0.3000$

84.5 Recursion Branch 1 1 1 1 1 in Interval 1: $[0.3000000799999977, 0.300000079999$

Normalized monomial und Bézier representations and the Bézier polygon:

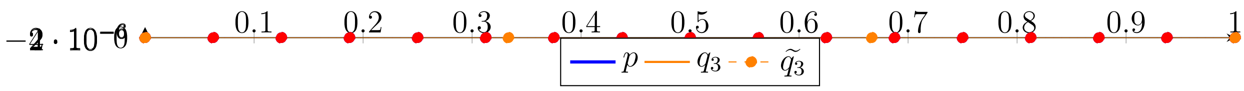
$$\begin{aligned}p &= 5.528728962819218\cdot10^{-317}X^{16} + 6.592381943367454\cdot10^{-316}X^{15} + 3.646191975443216\\ &\quad\cdot10^{-301}X^{14} - 1.206802108791304\cdot10^{-279}X^{13} - 6.271779178309597\cdot10^{-256}X^{12}\\ &\quad + 1.911032531933869\cdot10^{-234}X^{11} + 5.963449582578663\cdot10^{-211}X^{10} - 1.574654273552139\\ &\quad\cdot10^{-189}X^{9} - 3.389607402559108\cdot10^{-166}X^{8} + 7.168613242382433\cdot10^{-145}X^{7} + 1.152920753962593\\ &\quad\cdot10^{-121}X^{6} - 1.717169281481298\cdot10^{-100}X^{5} - 2.174667687267227\cdot10^{-77}X^{4} + 1.696045363009562\\ &\quad\cdot10^{-56}X^{3} + 1.7560195200423\cdot10^{-33}X^{2} - 7.452738853523238\cdot10^{-20}X + 7.27439202738246\cdot10^{-13}\\ &= 7.27439202738246\cdot10^{-13}B_{0,16}(X) + 7.274391980802842\cdot10^{-13}B_{1,16}(X) + 7.274391934223224\\ &\quad\cdot10^{-13}B_{2,16}(X) + 7.274391887643606\cdot10^{-13}B_{3,16}(X) + 7.274391841063988\cdot10^{-13}B_{4,16}(X)\\ &\quad + 7.27439179448437\cdot10^{-13}B_{5,16}(X) + 7.274391747904753\cdot10^{-13}B_{6,16}(X) + 7.274391701325135\\ &\quad\cdot10^{-13}B_{7,16}(X) + 7.274391654745517\cdot10^{-13}B_{8,16}(X) + 7.274391608165899\cdot10^{-13}B_{9,16}(X)\\ &\quad + 7.274391561586281\cdot10^{-13}B_{10,16}(X) + 7.274391515006663\cdot10^{-13}B_{11,16}(X) + 7.274391468427046\\ &\quad\cdot10^{-13}B_{12,16}(X) + 7.274391421847428\cdot10^{-13}B_{13,16}(X) + 7.27439137526781\cdot10^{-13}B_{14,16}(X)\\ &\quad + 7.274391328688192\cdot10^{-13}B_{15,16}(X) + 7.274391282108574\cdot10^{-13}B_{16,16}(X)\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}q_3 &= 1.696045363009562\cdot10^{-56}X^{3} + 1.7560195200423\cdot10^{-33}X^{2}\\ &\quad - 7.452738853523238\cdot10^{-20}X + 7.27439202738246\cdot10^{-13}\\ &= 7.27439202738246\cdot10^{-13}B_{0,3} + 7.274391778957831\cdot10^{-13}B_{1,3}\\ &\quad + 7.274391530533203\cdot10^{-13}B_{2,3} + 7.274391282108574\cdot10^{-13}B_{3,3}\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= 5.719971833076747\cdot10^{-308}X^{16} - 4.598890442375195\cdot10^{-307}X^{15} + 1.652591351302372\\ &\quad\cdot10^{-306}X^{14} - 3.497346441216264\cdot10^{-306}X^{13} + 4.849354007960553\cdot10^{-306}X^{12}\\ &\quad - 4.665113144336927\cdot10^{-306}X^{11} + 3.252638633884502\cdot10^{-306}X^{10} - 1.725306268115569\\ &\quad\cdot10^{-306}X^{9} + 7.334647684203053\cdot10^{-307}X^{8} - 2.545384960198585\cdot10^{-307}X^{7} + 6.841253439376372\\ &\quad\cdot10^{-308}X^{6} - 1.288279056767672\cdot10^{-308}X^{5} + 1.476493526426595\cdot10^{-309}X^{4} + 1.696045363009562\\ &\quad\cdot10^{-56}X^{3} + 1.7560195200423\cdot10^{-33}X^{2} - 7.452738853523238\cdot10^{-20}X + 7.27439202738246\cdot10^{-13}\\ &= 7.27439202738246\cdot10^{-13}B_{0,16} + 7.274391980802842\cdot10^{-13}B_{1,16} + 7.274391934223224\\ &\quad\cdot10^{-13}B_{2,16} + 7.274391887643606\cdot10^{-13}B_{3,16} + 7.274391841063988\cdot10^{-13}B_{4,16}\\ &\quad + 7.27439179448437\cdot10^{-13}B_{5,16} + 7.274391747904753\cdot10^{-13}B_{6,16} + 7.274391701325135\\ &\quad\cdot10^{-13}B_{7,16} + 7.274391654745517\cdot10^{-13}B_{8,16} + 7.274391608165899\cdot10^{-13}B_{9,16}\\ &\quad + 7.274391561586281\cdot10^{-13}B_{10,16} + 7.274391515006663\cdot10^{-13}B_{11,16} + 7.274391468427046\\ &\quad\cdot10^{-13}B_{12,16} + 7.274391421847428\cdot10^{-13}B_{13,16} + 7.27439137526781\cdot10^{-13}B_{14,16}\\ &\quad + 7.274391328688192\cdot10^{-13}B_{15,16} + 7.274391282108574\cdot10^{-13}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.106668124667468\cdot10^{-79}$.
Bounding polynomials M and m:

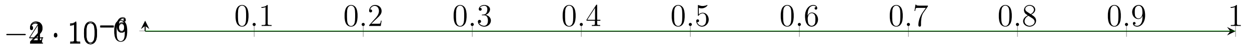
$$\begin{aligned}M &= 1.696045363009562\cdot10^{-56}X^{3} + 1.7560195200423\cdot10^{-33}X^{2}\\ &\quad - 7.452738853523238\cdot10^{-20}X + 7.27439202738246\cdot10^{-13}\end{aligned}$$

$$m = 1.696045363009562{\cdot}10^{-56}X^3 + 1.7560195200423{\cdot}10^{-33}X^2 - 7.452738853523238{\cdot}10^{-20}X + 7.27439202738246{\cdot}10^{-13}$$

Root of M and m:

$N(M) = \{-1.035361175508972{\cdot}10^{23}, 1.384450244011441, 42441083663415.11\}$ $\quad N(m) = \{-1.0353611755089$

Intersection intervals:

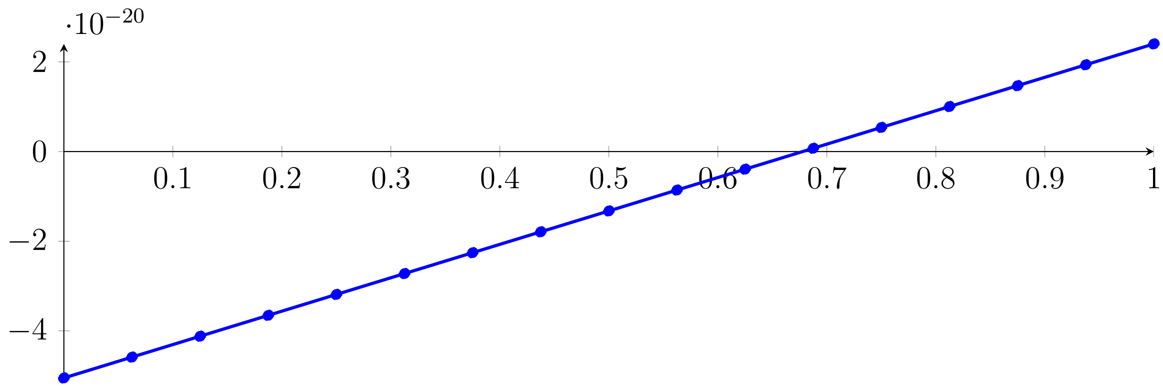


No intersection intervals with the x axis.

84.6 Recursion Branch 1 1 1 1 2 in Interval 2: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

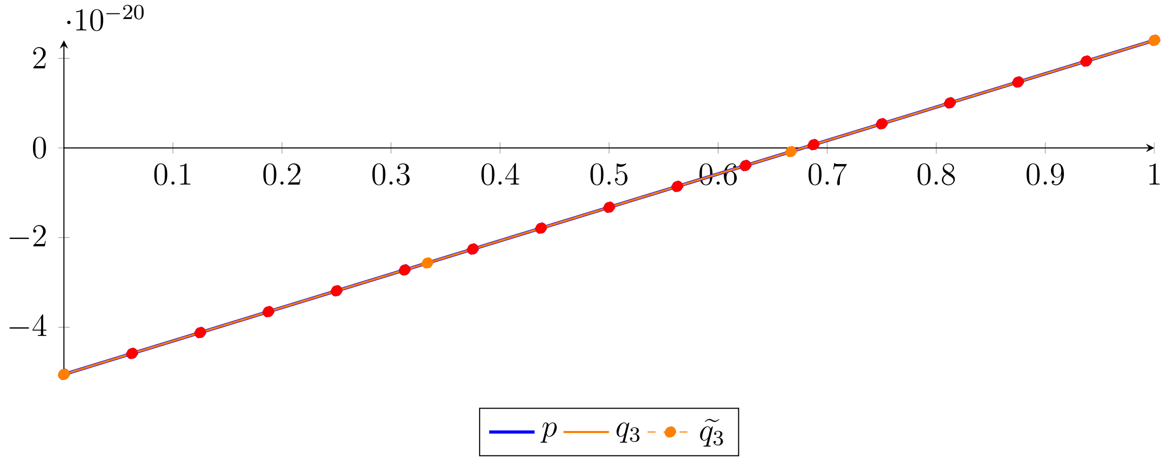
$$\begin{aligned}
p &= 7.285538332229319{\cdot}10^{-325}X^{16} - 3.937085615297433{\cdot}10^{-324}X^{15} + 3.646191956469014 \\
&\quad {\cdot}10^{-301}X^{14} - 1.206801885712699{\cdot}10^{-279}X^{13} - 6.271779154117127{\cdot}10^{-256}X^{12} \\
&\quad + 1.911032203899561{\cdot}10^{-234}X^{11} + 5.963449567055239{\cdot}10^{-211}X^{10} - 1.574654014647597{\cdot}10^{-189}X^9 \\
&\quad - 3.389607397458184{\cdot}10^{-166}X^8 + 7.168612070943855{\cdot}10^{-145}X^7 + 1.152920753256697{\cdot}10^{-121}X^6 \\
&\quad - 1.717168984374588{\cdot}10^{-100}X^5 - 2.174667687345432{\cdot}10^{-77}X^4 + 1.696044991742842{\cdot}10^{-56}X^3 \\
&\quad + 1.75601952076212{\cdot}10^{-33}X^2 + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20} \\
&= -5.051512577078811{\cdot}10^{-20}B_{0,16}(X) - 4.585716612983338{\cdot}10^{-20}B_{1,16}(X) - 4.119920648887863 \\
&\quad {\cdot}10^{-20}B_{2,16}(X) - 3.654124684792386{\cdot}10^{-20}B_{3,16}(X) - 3.188328720696908{\cdot}10^{-20}B_{4,16}(X) \\
&\quad - 2.722532756601429{\cdot}10^{-20}B_{5,16}(X) - 2.256736792505948{\cdot}10^{-20}B_{6,16}(X) - 1.790940828410466 \\
&\quad {\cdot}10^{-20}B_{7,16}(X) - 1.325144864314982{\cdot}10^{-20}B_{8,16}(X) - 8.593489002194965{\cdot}10^{-21}B_{9,16}(X) \\
&\quad - 3.935529361240098{\cdot}10^{-21}B_{10,16}(X) + 7.224302797147847{\cdot}10^{-22}B_{11,16}(X) \\
&\quad + 5.380389920669682{\cdot}10^{-21}B_{12,16}(X) + 1.003834956162459{\cdot}10^{-20}B_{13,16}(X) + 1.469630920257952 \\
&\quad {\cdot}10^{-20}B_{14,16}(X) + 1.935426884353446{\cdot}10^{-20}B_{15,16}(X) + 2.401222848448942{\cdot}10^{-20}B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= 1.696044991742842{\cdot}10^{-56}X^3 + 1.75601952076212{\cdot}10^{-33}X^2 \\
&\quad + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20} \\
&= -5.051512577078811{\cdot}10^{-20}B_{0,3} - 2.567267435236286{\cdot}10^{-20}B_{1,3} \\
&\quad - 8.302229339370134{\cdot}10^{-22}B_{2,3} + 2.401222848448942{\cdot}10^{-20}B_{3,3}
\end{aligned}$$

$$\begin{aligned}\widetilde{q}_3 &= -1.026066912900859{\cdot}10^{-315}X^{16} + 8.593538471070521{\cdot}10^{-315}X^{15} - 3.223576101816875\\ &\quad {\cdot}10^{-314}X^{14} + 7.163252241057264{\cdot}10^{-314}X^{13} - 1.054054043306296{\cdot}10^{-313}X^{12}\\ &\quad + 1.091820578452099{\cdot}10^{-313}X^{11} - 8.293103170105507{\cdot}10^{-314}X^{10} + 4.748156916296184{\cdot}10^{-314}X^{9}\\ &\quad - 2.062106326583159{\cdot}10^{-314}X^{8} + 6.57938063450856{\cdot}10^{-315}X^{7} - 1.420328445432424{\cdot}10^{-315}X^{6}\\ &\quad + 1.834102728564758{\cdot}10^{-316}X^{5} - 1.348970337656149{\cdot}10^{-317}X^{4} + 1.696044991742842{\cdot}10^{-56}X^{3}\\ &\quad + 1.75601952076212{\cdot}10^{-33}X^{2} + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20}\\ &= -5.051512577078811{\cdot}10^{-20}B_{0,16} - 4.585716612983338{\cdot}10^{-20}B_{1,16} - 4.119920648887863\\ &\quad {\cdot}10^{-20}B_{2,16} - 3.654124684792386{\cdot}10^{-20}B_{3,16} - 3.188328720696908{\cdot}10^{-20}B_{4,16}\\ &\quad - 2.722532756601429{\cdot}10^{-20}B_{5,16} - 2.256736792505948{\cdot}10^{-20}B_{6,16} - 1.790940828410466\\ &\quad {\cdot}10^{-20}B_{7,16} - 1.325144864314982{\cdot}10^{-20}B_{8,16} - 8.593489002194965{\cdot}10^{-21}B_{9,16}\\ &\quad - 3.935529361240098{\cdot}10^{-21}B_{10,16} + 7.224302797147847{\cdot}10^{-22}B_{11,16}\\ &\quad + 5.380389920669682{\cdot}10^{-21}B_{12,16} + 1.003834956162459{\cdot}10^{-20}B_{13,16} + 1.469630920257952\\ &\quad {\cdot}10^{-20}B_{14,16} + 1.935426884353446{\cdot}10^{-20}B_{15,16} + 2.401222848448942{\cdot}10^{-20}B_{16,16}\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 3.106668124779188{\cdot}10^{-79}$.

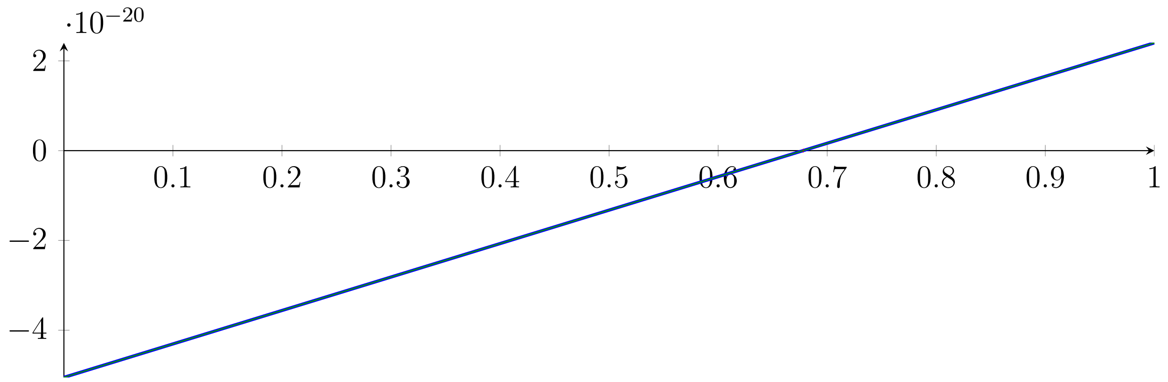
Bounding polynomials M and m:

$$\begin{aligned}M &= 1.696044991742842{\cdot}10^{-56}X^3 + 1.75601952076212{\cdot}10^{-33}X^2\\ &\quad + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20}\\ m &= 1.696044991742842{\cdot}10^{-56}X^3 + 1.75601952076212{\cdot}10^{-33}X^2\\ &\quad + 7.452735425527577{\cdot}10^{-20}X - 5.051512577078811{\cdot}10^{-20}\end{aligned}$$

Root of M and m:

$N(M) = \{-1.03536140172663{\cdot}10^{23}, -42441083680812.44, 0.6778065084366168\}$ $N(m) = \{-1.035361401726$

Intersection intervals:



$$[0.6778065084366168, 0.6778065084366168]$$

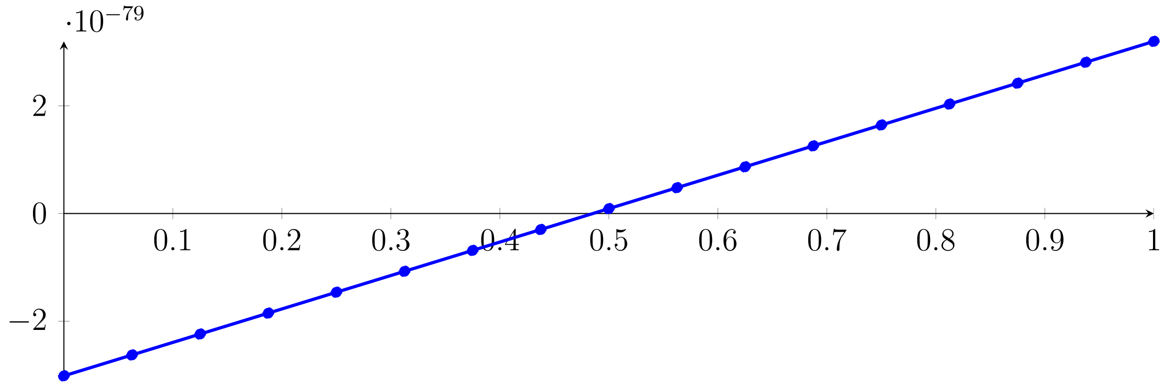
Longest intersection interval: $8.336987555302001{\cdot}10^{-60}$

$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

84.7 Recursion Branch 1 1 1 1 2 1 in Interval 1: $[0.30000009, 0.30000009]$

Normalized monomial und Bézier representations and the Bézier polygon:

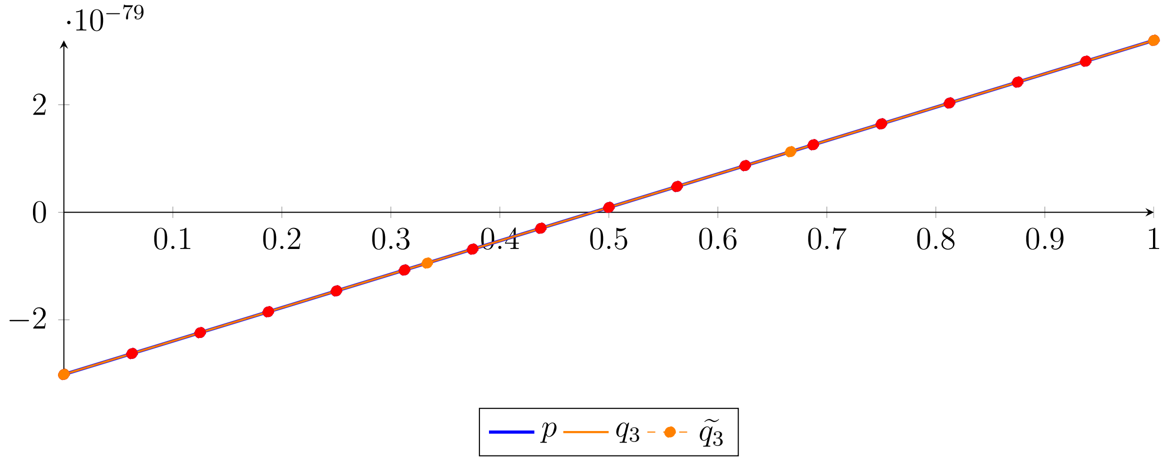
$$\begin{aligned}
p &= -6.005035278000799\cdot 10^{-385}X^{16} + 8.070767413633074\cdot 10^{-384}X^{15} - 2.824768594771576\cdot 10^{-382}X^{14} \\
&\quad - 2.690255804544358\cdot 10^{-383}X^{13} + 2.185832841192291\cdot 10^{-382}X^{12} + 1.15411974014953\cdot 10^{-381}X^{11} \\
&\quad - 1.099161657285266\cdot 10^{-381}X^{9} - 1.391126472501665\cdot 10^{-381}X^{8} + 9.61766450124608\cdot 10^{-383}X^{6} \\
&\quad - 1.0491997637723\cdot 10^{-382}X^{5} - 1.050580932631639\cdot 10^{-313}X^{4} + 9.827992773299941\cdot 10^{-234}X^{3} \\
&\quad + 1.220527715868172\cdot 10^{-151}X^{2} + 6.213336249558376\cdot 10^{-79}X - 3.015763730746062\cdot 10^{-79} \\
&= -3.015763730746062\cdot 10^{-79}B_{0,16}(X) - 2.627430215148664\cdot 10^{-79}B_{1,16}(X) - 2.239096699551265 \\
&\quad \cdot 10^{-79}B_{2,16}(X) - 1.850763183953867\cdot 10^{-79}B_{3,16}(X) - 1.462429668356468\cdot 10^{-79}B_{4,16}(X) \\
&\quad - 1.07409615275907\cdot 10^{-79}B_{5,16}(X) - 6.857626371616713\cdot 10^{-80}B_{6,16}(X) - 2.974291215642728 \\
&\quad \cdot 10^{-80}B_{7,16}(X) + 9.09043940331257\cdot 10^{-81}B_{8,16}(X) + 4.792379096305242\cdot 10^{-80}B_{9,16}(X) \\
&\quad + 8.675714252279227\cdot 10^{-80}B_{10,16}(X) + 1.255904940825321\cdot 10^{-79}B_{11,16}(X) + 1.64423845642272 \\
&\quad \cdot 10^{-79}B_{12,16}(X) + 2.032571972020118\cdot 10^{-79}B_{13,16}(X) + 2.420905487617517\cdot 10^{-79}B_{14,16}(X) \\
&\quad + 2.809239003214915\cdot 10^{-79}B_{15,16}(X) + 3.197572518812314\cdot 10^{-79}B_{16,16}(X)
\end{aligned}$$



Degree reduction and raising:

$$\begin{aligned}
q_3 &= 9.827992773299941\cdot 10^{-234}X^{3} + 1.220527715868172\cdot 10^{-151}X^{2} \\
&\quad + 6.213336249558376\cdot 10^{-79}X - 3.015763730746062\cdot 10^{-79} \\
&= -3.015763730746062\cdot 10^{-79}B_{0,3} - 9.44651647559937\cdot 10^{-80}B_{1,3} \\
&\quad + 1.126460435626188\cdot 10^{-79}B_{2,3} + 3.197572518812314\cdot 10^{-79}B_{3,3}
\end{aligned}$$

$$\begin{aligned}
\widetilde{q}_3 &= 8.47486821414738\cdot 10^{-376}X^{16} - 3.94674357732252\cdot 10^{-375}X^{15} + 2.883725615942849 \\
&\quad \cdot 10^{-375}X^{14} + 2.234818330885562\cdot 10^{-374}X^{13} - 8.168572400159058\cdot 10^{-374}X^{12} \\
&\quad + 1.434549107423008\cdot 10^{-373}X^{11} - 1.567658109397744\cdot 10^{-373}X^{10} + 1.122683929370254\cdot 10^{-373}X^{9} \\
&\quad - 5.135952868254142\cdot 10^{-374}X^{8} + 1.301427266137831\cdot 10^{-374}X^{7} - 5.964405941203546\cdot 10^{-376}X^{6} \\
&\quad - 5.884272281904001\cdot 10^{-376}X^{5} + 1.302246036028008\cdot 10^{-376}X^{4} + 9.827992773299941\cdot 10^{-234}X^{3} \\
&\quad + 1.220527715868172\cdot 10^{-151}X^{2} + 6.213336249558376\cdot 10^{-79}X - 3.015763730746062\cdot 10^{-79} \\
&= -3.015763730746062\cdot 10^{-79}B_{0,16} - 2.627430215148664\cdot 10^{-79}B_{1,16} - 2.239096699551265 \\
&\quad \cdot 10^{-79}B_{2,16} - 1.850763183953867\cdot 10^{-79}B_{3,16} - 1.462429668356468\cdot 10^{-79}B_{4,16} \\
&\quad - 1.07409615275907\cdot 10^{-79}B_{5,16} - 6.857626371616713\cdot 10^{-80}B_{6,16} - 2.974291215642728 \\
&\quad \cdot 10^{-80}B_{7,16} + 9.09043940331257\cdot 10^{-81}B_{8,16} + 4.792379096305242\cdot 10^{-80}B_{9,16} \\
&\quad + 8.675714252279227\cdot 10^{-80}B_{10,16} + 1.255904940825321\cdot 10^{-79}B_{11,16} + 1.64423845642272 \\
&\quad \cdot 10^{-79}B_{12,16} + 2.032571972020118\cdot 10^{-79}B_{13,16} + 2.420905487617517\cdot 10^{-79}B_{14,16} \\
&\quad + 2.809239003214915\cdot 10^{-79}B_{15,16} + 3.197572518812314\cdot 10^{-79}B_{16,16}
\end{aligned}$$



The maximum difference of the Bézier coefficients is $\delta = 1.500829903759485 \cdot 10^{-315}$.
Bounding polynomials M and m:

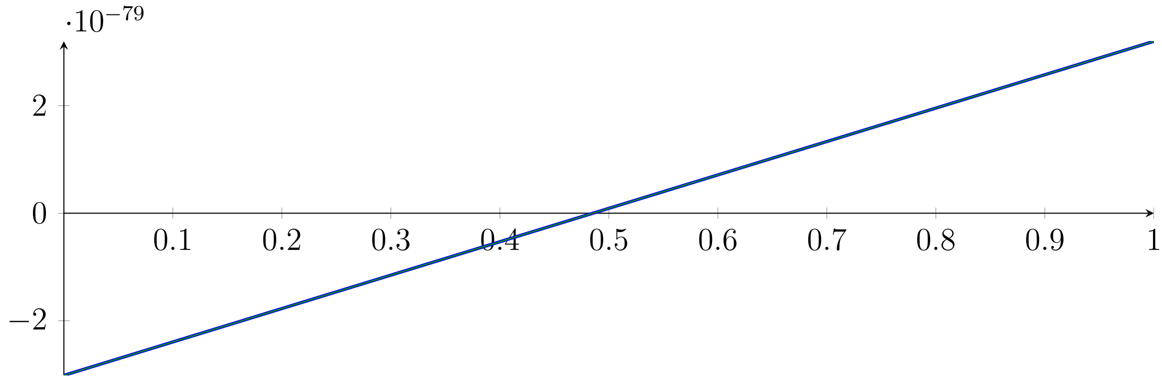
$$M = 9.827992773299941 \cdot 10^{-234} X^3 + 1.220527715868172 \cdot 10^{-151} X^2 + 6.213336249558376 \cdot 10^{-79} X - 3.015763730746062 \cdot 10^{-79}$$

$$m = 9.827992773299941 \cdot 10^{-234} X^3 + 1.220527715868172 \cdot 10^{-151} X^2 + 6.213336249558376 \cdot 10^{-79} X - 3.015763730746062 \cdot 10^{-79}$$

Root of M and m:

$N(M) = \{-1.241889105457739 \cdot 10^{82}, -5.090697736956826 \cdot 10^{72}, 0.4853694713464788\}$ $\quad N(m) = \{-1.241889$

Intersection intervals:



$$[0.4853694713464788, 0.4853694713464788]$$

Longest intersection interval: 0
$\Longrightarrow$ Selective recursion: interval 1: $[0.30000009, 0.30000009]$,

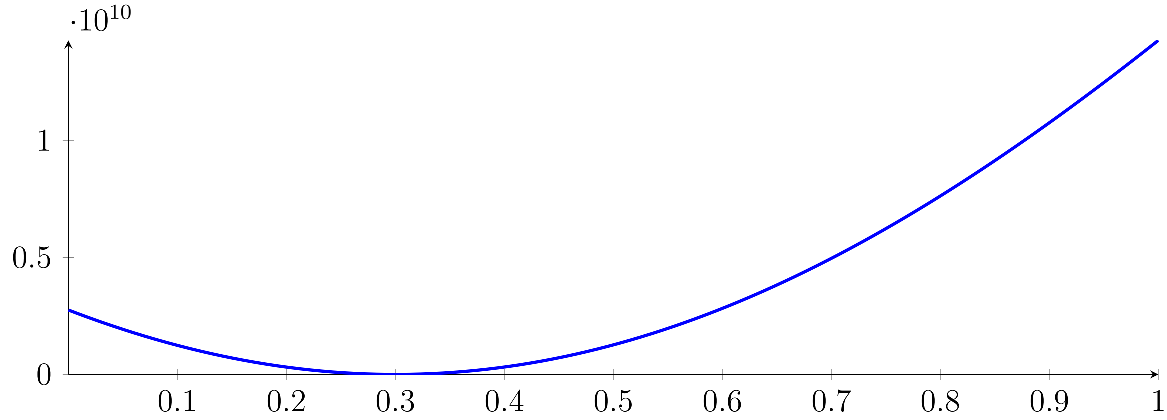
84.8 Recursion Branch 1 1 1 1 2 1 1 in Interval 1: $[0.30000009, 0.30000009]$

Found root in interval $[0.30000009, 0.30000009]$ at recursion depth 7!

84.9 Result: 1 Root Intervals

Input Polynomial on Interval $[0, 1]$

$$p = -1X^{16} + 5.60000017X^{15} + 209.909999099X^{14} - 1094.350035955X^{13} - 18925.629824747X^{12} + 89078.97326993299X^{11} + 955907.1358375549X^{10} - 3894443.576752948X^{9} - 29355705.4379705X^{8} + 97554424.00407885X^{7} + 550011059.5479722X^{6} - 1363102652.61149X^{5} - 5828036467.676009X^{4} + 9196554751.463423X^{3} + 26797309752.02485X^{2} - 17885290929.93536X + 2755621561.51822$$



Result: Root Intervals

$$[0.30000009, 0.30000009]$$

with precision $\varepsilon = 1 \cdot 10^{-128}$.